1. DTFT, Truncated DTFT 2014_1-midterm 2015104027 박정권

CTFT $\quad X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j\omega n T_s} \cdot T_s$$

$$T_s \sum_{n=-\infty}^{\infty} \underset{\downarrow\, x[n]}{x(nT_s)}\, e^{-j\Omega n} = T_s X(\Omega)$$

DTFT $\quad X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} \quad \left(\Omega = \omega T_s = 2\pi \cdot \frac{\omega}{\omega_s} = 2\pi f_d = 2\pi \cdot \frac{f_a}{f_s} \right)$

$\rightarrow 0 \le \Omega < 2\pi$

↳ $-\infty \le n \le \infty$ 현실적 계산 불가

DFT $\quad X(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \quad \left(\Omega = \frac{2\pi}{N} k \;\; (k = 0, 1, 2, \cdots N-1) \right)$

$X(\omega) = T_s X(k)$

$\Rightarrow$ Truncated DTFT $\Rightarrow \quad X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$

$\left(\Omega \rightarrow \frac{2\pi}{N} k \right)$

FFT $\rightarrow$ Reduce the computation redundancy of DFT

	DFT	FFT
complex mul	N^2	$\frac{N}{2}\log_2 N$
// add	$N(N-1)$	$N \log_2 N$

2. Fast convolution

$$x_1[n] * x_2[n] = F^{-1}(X_1(k) \cdot X_2(k))$$

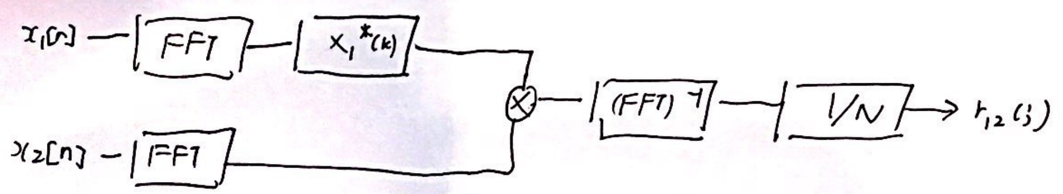
The convolution of two signal is obtained by taking a fast fourier transform, multiplying each signal, and performing inverse transformation

Fast correlation

$$r_{12}(j) = \frac{1}{N} F^{-1}(X_1^*(k) X_2(k))$$

The correlation of two signals is obtained by taking a fast fourier transform, ~~conjugating $X_1(k)$~~ to each signals.

next conjugate $X_1(k)$ and multiply $X_2(k)$, Finally perform inverse transformation, and take normalized by N



3. DWHT $\{1, -1, 0, 0, 2, 0, -2, 1\}$

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$$X^{WH}(k) = \{1/8,\ 1/8,\ 3/8,\ 7/8,\ 1/8,\ -1/8,\ -1/8,\ 7/8\}$$

$$8^H = \begin{bmatrix} 4_H & 4_H \\ 4_H & -4_H \end{bmatrix} \qquad 4^H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

4. Z-transform $(\frac{1}{2})^{n}$

i) using definition

$x[n] = (\frac{1}{2})^{|n|}$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{-1} (\tfrac{1}{2})^{-n} z^{-n} + \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n} z^{-n} = \sum_{k=1}^{\infty} (\tfrac{1}{2})^{k} z^{k} + \sum_{n=0}^{\infty} (\tfrac{1}{2} z^{-1})^{n}$$

$$= \sum_{k=1}^{\infty} (\tfrac{1}{2} z)^{k} + \sum_{n=0}^{\infty} (\tfrac{1}{2} z^{-1})^{n} = \frac{\frac{1}{2} z}{1 - \frac{1}{2} z} + \frac{1}{1 - \frac{1}{2} z^{-1}} = \frac{z}{z - \frac{1}{2}} - \frac{z}{z-2}$$

$$\text{ROC} \quad \left.\begin{array}{l} |\tfrac{1}{2} z| < 1 \Rightarrow |z| < 2 \\ |\tfrac{1}{2} z^{-1}| < 1 \Rightarrow |z| > \tfrac{1}{2} \end{array}\right\} \Rightarrow \tfrac{1}{2} < |z| < 2$$

$$\therefore X(z) = \frac{z}{z - \frac{1}{2}} - \frac{z}{z-2} \quad \text{ROC } \tfrac{1}{2} < |z| < 2$$

ii) using pairs

$$x[n] = (\tfrac{1}{2})^{|n|} = (\tfrac{1}{2})^{n} u[n] + (\tfrac{1}{2})^{-n} u[-n-1] = \underbrace{(\tfrac{1}{2})^{n} u[n]}_{\text{ROC } |z| > \frac{1}{2}} + \underbrace{(2)^{n} u[-n-1]}_{\text{ROC } |z| < 2}$$

$$= (\tfrac{1}{2})^{n} u[n] - (-(2)^{n} u[-n-1])$$

$$X(z) = \frac{z}{z - \frac{1}{2}} - \frac{z}{z - 2} \quad \text{ROC } \tfrac{1}{2} < |z| < 2$$

5. $B=12$, $f_s=10KHz$, $BW=4KHz$

(a) SQNR?

(b) ADR ?

Averaged Quantization Noise power $N_q = \frac{q^2}{12}$

RMS quantization Noise power $N_{q\text{-}rms} = \sqrt{\frac{q^2}{12}}\Big|_{q=\frac{V_{PP}}{2^B}} = \frac{0.5V_{PP}}{\sqrt{3\times 2^{2B}}}$

SQNR (dynamic range) (The ratio of the maximum and minimum signal levels that the ADC can handle)

- SQNR is a theoretical maximum
- achievable SQNR is less than this value

SQNR (signal to quantization Noise Ratio)

$= 10\log\frac{\text{signal power}}{\text{quantization Noise power}} = 10\log\frac{3}{2}\cdot 2^{2B} = 6.02B+1.76$ [dB]

ADR (Actual Dynamic Range)

$= 10\log\frac{P_s(\text{signal power})}{N_{0q}\times BW} = \underline{10\log\frac{P_s}{N_{0q}}} - 10\log BW$

↳ signal to Noise Density Ratio

- N_{0q} (quantization Noise power per unit Bandwidth)

$N_{0q} = \frac{q^2/12}{f_s/2}$

$ADR = 10\log(\frac{3}{4}f_s\cdot 2^{2B}) - 10\log BW = 10\log\frac{3}{4} + 10\log f_s + 10\log 2^{2B} - 10\log BW$

$= 6.02B + 10\log f_s - 1.249 - 10\log BW$ [dB]

$\therefore$ $SQNR = 6.02B + 1.76 = 74$ dB

$ADR = 6.02B + 10\log f_s - 1.249 - 10\log BW = 74.97$ dB