2018_2_midterm 2015104027 박경리

1. $f_s = 10KHz$ $Bit = 8$, $BW = 2KHz$

Averaged quantization Noise power $N_q = \frac{q^2}{12}$

RMS quantization Noise power $N_{q\_rms} = \sqrt{\frac{q^2}{12}} \Big|_{q = \frac{V_{PP}}{2^B}}$ 2^{2B}

SQNR (dynamic range)

SQNR → signal to Quantization Noise ratio

Dynamic range → The ratio of the maximum and minimum signal levels that the ADC can handle

- SQNR is a theoretical maximum
- achievable SQNR is less than this value

$$SQNR = 10 \log \frac{\text{signal power}}{\text{quantization Noise power}} = 10 \log \frac{3}{2} \cdot 2^{2B} = 6.02B + 1.76 \, [dB]$$

N_{0q} (Quantization Noise power per unit Bandwidth)

$$N_{0q} = \frac{q^2/12}{f_s/2}$$

ADR (Actual Dynamic range)

$$= 10 \log \frac{P_s \text{ (signal power)}}{N_{0q} \times BW} = \underline{10 \log \frac{P_s}{N_{0q}}} - 10 \log BW$$

↳ signal to Noise Density Ratio

$$= 10 \log\left(\frac{3}{4} f_s \cdot 2^{2B}\right) - 10 \log BW = 6.02B + 10 \log f_s - 1.249 - 10 \log BW \, [dB]$$

1. Dynamic range

$6.02B + 1.76 = 49.92 \, dB$

2. ADR

$6.02B + 10 \log f_s - 1.249 - 10 \log BW = 53.9 \, dB$

2. $x(t) \leftrightarrow X(\omega)$

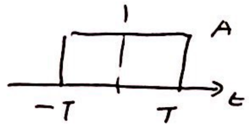
(a) $x(t) = \cos(t/10)$

※ $e^{j2\pi f_0 t} = e^{j\omega_0 t} \longleftrightarrow \begin{matrix} \delta(f-f_0) \\ 2\pi\,\delta(\omega-\omega_0) \end{matrix}$

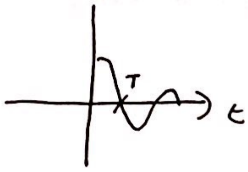
$$x(t) = \frac{e^{j\frac{1}{10}t} + e^{-j\frac{1}{10}t}}{2} \longleftrightarrow X(\omega) = \frac{2\pi\,\delta(\omega-\frac{1}{10}) + 2\pi\,\delta(\omega+\frac{1}{10})}{2}$$

$$= \pi\left(\delta(\omega-\tfrac{1}{10}) + \delta(\omega+\tfrac{1}{10})\right)$$

(b) $x(t) = \mathrm{rect}(8t)$

※ 

$\triangleq A\,\mathrm{rect}\left(\frac{t}{2T}\right)$



$\triangleq A\,\mathrm{sinc}\left(\frac{t}{T}\right)$

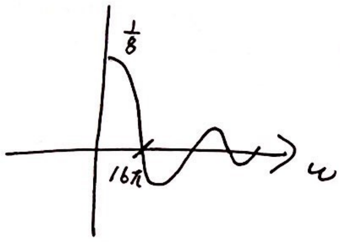
$A\,\mathrm{rect}\left(\frac{t}{2T}\right) \longleftrightarrow A \cdot 2T \cdot \mathrm{sinc}(f \cdot 2T)$

ㄴ> scale $\frac{1}{2T}$ 의 역수 진폭, scale 에 곱함

$= A \cdot 2T\,\mathrm{sinc}(2Tf) = A \cdot 2T\,\mathrm{sinc}\left(\frac{2T \cdot \omega}{2\pi}\right)$

$\mathrm{rect}\left(\frac{t}{\frac{1}{8}}\right) \longrightarrow 1 \cdot \frac{1}{8}\,\mathrm{sinc}\left(\frac{1}{8}f\right)$

$= \frac{1}{8}\,\mathrm{sinc}\left(\frac{\omega}{16\pi}\right)$

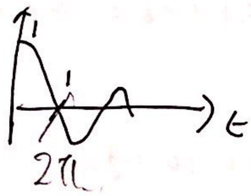


(c) $\mathrm{rect}(\omega) = X(\omega)$ $\quad x(t)$?

$\mathrm{rect}(\omega) = \mathrm{rect}\left(\frac{\omega}{1}\right) \rightarrow 1 \cdot 1\,\mathrm{sinc}(t \cdot 1) = \mathrm{sinc}(t)$

$\frac{2\pi t}{\frac{1}{2\pi}}$

$2\pi\,\mathrm{sinc}\left(\frac{t}{2\pi}\right)$



(d)

※ shifting property

$x(t) * \delta(t-t_0) = x(t-t_0)$

※ multiplying convolution

t	w	f
$x_1(t) \cdot x_2(t)$	$\frac{1}{2\pi} X_1(w) * X_2(w)$	$X_1(f) * X_2(f)$
$x_1(t) * x_2(t)$	$X_1(w)\, X_2(w)$	$X_1(f) X_2(f)$

※ modulation property

$$x(t)\cos w_0 t \longleftrightarrow \frac{1}{2}\left(X(w-w_0) + X(w+w_0)\right)$$

$$\text{rect}(t) \longrightarrow 1 \cdot \text{sinc}(1 \cdot f) = \text{sinc}\left(\frac{w}{2\pi}\right)$$

$$\cos(t) \longrightarrow \pi\left(\delta(w-w_0) + \delta(w+w_0)\right) \quad (w_0 = 1)$$

$$= \pi\left(\delta(w-1) + \delta(w+1)\right)$$

$$\cos(t)\,\text{rect}(t) \longrightarrow \frac{1}{2\pi}\left(\text{sinc}\left(\frac{w}{2\pi}\right) * \pi\left(\delta(w-w_0) + \delta(w+w_0)\right)\right)$$

$$= \frac{1}{2}\left(\text{sinc}\left(\frac{w-1}{2\pi}\right) + \text{sinc}\left(\frac{w+1}{2\pi}\right)\right)$$

or

By modulation property

$$X(w) = \text{sinc}\left(\frac{w}{2\pi}\right) \quad w_0 = 1$$

$$\frac{1}{2}\left(X(w-w_0) + X(w+w_0)\right) = \frac{1}{2}\left(\text{sinc}\left(\frac{w-1}{2\pi}\right) + \text{sinc}\left(\frac{w+1}{2\pi}\right)\right)$$

3. z-transform

Definition 으로 풀기 Σ 이용, ROC 쓰기

(a) $x[n] = (-2)^n u[-n-1]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{-1} (-2)^n z^{-n} = \sum_{k=1}^{\infty} (-2)^{-k} z^{k} = \sum_{k=1}^{\infty} \left(-\tfrac{1}{2}z\right)^k = \frac{-\frac{1}{2}z}{1+\frac{1}{2}z}$$

$$= \frac{-z}{z+2} \quad \text{ROC } \left|-\tfrac{1}{2}z\right| < 1 \Rightarrow |z| < 2 \quad \therefore X(z) = -\frac{z}{z+2} \quad |z| < 2$$

(b) $x[n] = \left(\frac{1}{5}\right)^n u[n] + 2^n u[-n-1]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} \left(\tfrac{1}{5}\right)^n z^{-n} + \sum_{k=1}^{\infty} \left(\tfrac{1}{2}\right)^k z^k = \sum_{n=0}^{\infty} \left(\tfrac{1}{5}z^{-1}\right)^n + \sum_{k=1}^{\infty} \left(\tfrac{1}{2}z\right)^k$$

$$= \frac{1}{1-\frac{1}{5}z^{-1}} + \frac{\frac{1}{2}z}{1-\frac{1}{2}z} = \frac{z}{z-\frac{1}{5}} - \frac{z}{z-2}$$

ROC $\left|\frac{1}{5}z^{-1}\right| < 1 \Rightarrow |z| > \frac{1}{5}$, $\left|\frac{1}{2}z\right| < 1 \Rightarrow |z| < 2$ $\Rightarrow \frac{1}{5} < |z| < 2$

$$\therefore X(z) = \frac{z}{z-\frac{1}{5}} - \frac{z}{z-2} \quad \frac{1}{5} < |z| < 2$$

4. Inverse z-transform

(a) $X(z) = \dfrac{z^2-3z}{z^2+\frac{3}{2}z-1} = \dfrac{z(z-3)}{(z+2)(z-\frac{1}{2})}$ $\quad \underbrace{\tfrac{1}{2} <}_{\text{causal}} |z| \underbrace{< 2}_{\text{uncausal}}$

$c_0 = X(z)|_{z=0} = 0 \quad c_1 = \left.\frac{z-3}{z-\frac{1}{2}}\right|_{z=-2} = \frac{-5}{-\frac{5}{2}} = 2 \quad c_2 = \left.\frac{z-3}{z+2}\right|_{z=\frac{1}{2}} = \frac{-\frac{5}{2}}{\frac{5}{2}} = -1$

$X(z) =$

$$= 2 \cdot \frac{z}{z+2} - \frac{z}{z-\frac{1}{2}}$$

$$x[n] = -2(-2)^n u[-n-1] - \left(\tfrac{1}{2}\right)^n u[n]$$

(b) $X(z) = \dfrac{3z(z-\frac{1}{12})}{(z-4)(z+4)}$ $\quad \underbrace{|z| > 4}_{\text{causal}}$ $\quad c_0 = X(z)|_{z=0} = 0$

$c_1 = \left.\frac{3(z-\frac{1}{12})}{z+4}\right|_{z=4} = \frac{47}{32} \quad c_2 = \left.\frac{3(z-\frac{1}{12})}{z-4}\right|_{z=-4} = \frac{49}{32}$

$$X(z) = \frac{47}{32} \cdot \frac{z}{z-4} + \frac{49}{32} \cdot \frac{z}{z+4}$$

$$= \frac{47}{32} \cdot (4)^n u[n] + \frac{49}{32} (-4)^n u[n]$$

$$= \left(\frac{47}{32} \cdot (4)^n + \frac{49}{32}(-4)^n\right) u[n]$$

5. $y(n-1) - \frac{5}{2}y(n) + y(n+1) = x(n)$

$H(z) = \frac{Y(z)}{X(z)} = ?$

(a) $Z(y(n-1) - \frac{5}{2}y(n) + y(n+1)) = Z(x(n))$

$Y(z)(z^{-1} - \frac{5}{2} + z) = X(z)$

$H(z) = \frac{1}{z^{-1} - \frac{5}{2} + z} = \frac{z}{z^2 - \frac{5}{2}z + 1}$

(b) ROC $|z| > 2$, $h[n]$? causal system → unstable

$H(z) = \frac{z}{(z-2)(z-\frac{1}{2})} =$ $\quad c_0 = 0$ $\quad c_1 = \frac{1}{z-\frac{1}{2}}\Big|_{z=2} = \frac{2}{3}$ $\quad c_2 = \frac{1}{z-2}\Big|_{z=\frac{1}{2}} = -\frac{2}{3}$

$= \frac{2}{3} \cdot \frac{z}{z-2} - \frac{2}{3} \frac{z}{z-\frac{1}{2}}$

$h[n] = \frac{2}{3}(2)^n u[n] - \frac{2}{3}(\frac{1}{2})^n u[n] = \frac{2}{3}((2)^n - (\frac{1}{2})^n) u[n]$

(c) unstable when system is causal, LTI system, If all of poles is inside unit circle system is stable but one of the poles, $P_1 = 2$ P_1 is not inside unit circle

OR

ROC not includes unit circle

(d) system stable for ROC.

ROC should be includes unit circle

⇒ ROC. $\frac{1}{2} < |z| < 2$

causal uncausal