2011_2_ Quiz #1 2015104027 박경진

(1) Find the inverse Z-transform of

$$X(z) = \frac{1-\frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})^2} \qquad |z|>2$$

※ order of Numerator M $\quad N>M \quad C_0=0$

order of Denominator N $\quad N=M \quad C_0 = X(z)|_{z=0}$

multiple order

$$C_k = X(z)\frac{z-p_k}{z}\Big|_{z=p_k} \qquad \sum_{i=1}^{m}\frac{D_i z}{(z-p_k)^i} = \frac{D_1 z}{z-p_k} + \frac{D_2 z}{(z-p_k)^2} + \cdots + \frac{D_m z}{(z-p_k)^m}$$

$$D_i = \frac{1}{(m-i)!}\frac{d^{m-i}}{dz^{m-i}}\left[\frac{(z-p_k)^m}{z}X(z)\right]\Big|_{z=p_k}$$

$$a^n u[n] \longleftrightarrow \frac{z}{z-a} \quad |z|>|a| \qquad n a^n u[n] \longleftrightarrow \frac{az}{(z-a)^2} \quad |z|>|a|$$

$$-a^n u[-n-1] \longleftrightarrow \frac{z}{z-a} \quad |z|<|a| \qquad -n a^n u[-n-1] \longleftrightarrow \frac{az}{(z-a)^2} \quad |z|<|a|$$

$$X(z) = \frac{1-\frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})^2} = \frac{z^3-\frac{1}{3}z^2}{(z-1)(z+2)^2} = \frac{z^2(z-\frac{1}{3})}{(z-1)(z+2)^2}$$

$$C_0 = X(z)|_{z=0} = 0$$

$$C_1 = \frac{z^2(z-\frac{1}{3})}{(z-1)(z+2)^2}\cdot\frac{z-1}{z}\Big|_{z=1} = \frac{z(z-\frac{1}{3})}{(z+2)^2}\Big|_{z=1} = \frac{1\cdot\frac{2}{3}}{9} = \frac{2}{27}$$

$$D_1 = \frac{1}{(2-1)!}\frac{d^{2-1}}{dz^{2-1}}\left[\frac{(z+2)^2}{z}\frac{z^2(z-\frac{1}{3})}{(z-1)(z+2)^2}\right]\Big|_{z=-2} = \frac{d}{dz}\left[\frac{z(z-\frac{1}{3})}{z-1}\right]\Big|_{z=-2}$$

$$= \frac{(2z-\frac{1}{3})(z-1)-z(z-\frac{1}{3})}{(z-1)^2}\Big|_{z=-2} = \frac{25}{27}$$

$$D_2 = \left[\frac{(z+2)^2}{z}\cdot\frac{z^2(z-\frac{1}{3})}{(z-1)(z+2)^2}\right]\Big|_{z=-2} = \frac{z(z-\frac{1}{3})}{z-1}\Big|_{z=-2} = -\frac{14}{9}$$

$$X(z) = C_0 + C_1\cdot\frac{z}{z-1} + D_1\cdot\frac{z}{z+2} + D_2\frac{z}{(z+2)^2}$$

$$= \frac{2}{27}\cdot\frac{z}{z-1} + \frac{25}{27}\cdot\frac{z}{z+2} - \underbrace{\frac{14}{9}\cdot\frac{z}{(z+2)^2}}_{-\frac{1}{2}\times-\frac{14}{9}\cdot\frac{-2z}{(z+2)^2} = \frac{7}{9}\cdot\frac{-2z}{(z+2)^2}}$$

ROC $|z|>2$

↳ causal system both

Scanned with CamScanner

$$x[n] = \frac{2}{27}(1)^n u[n] + \frac{25}{27}(-2)^n u[n] + \frac{7}{9} n(-2)^n u[n]$$

$$= \left\{\frac{2}{27} + \frac{25}{27}(-2)^n + \frac{7}{9} n(-2)^n\right\} u[n] \rightarrow \text{unstable}$$

[2] Determine the z-transform of the following sequence. sketch the pole zero plot and indicate the region of convergence. Indicate whether or not the fourier transform of the sequence exists

· $2^n u[-n] + \left(\frac{1}{4}\right)^n u[n-1]$

Using definition of z transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left\{2^n u[-n] + \left(\tfrac{1}{4}\right)^n u[n-1]\right\} z^{-n} = \sum_{n=-\infty}^{0} 2^n z^{-n} + \sum_{n=1}^{\infty} \left(\tfrac{1}{4}\right)^n z^{-n}$$

Let [redacted] $n = -k$

$$\sum_{k=0}^{\infty} 2^{-k} z^{k} + \sum_{n=1}^{\infty} \left(\tfrac{1}{4} z^{-1}\right)^n = \sum_{k=0}^{\infty} \left(\tfrac{1}{2} z\right)^k + \sum_{n=1}^{\infty} \left(\tfrac{1}{4} z^{-1}\right)^n = \frac{1}{1-\frac{1}{2}z} + \frac{\frac{1}{4}z^{-1}}{1-\frac{1}{4}z^{-1}}$$

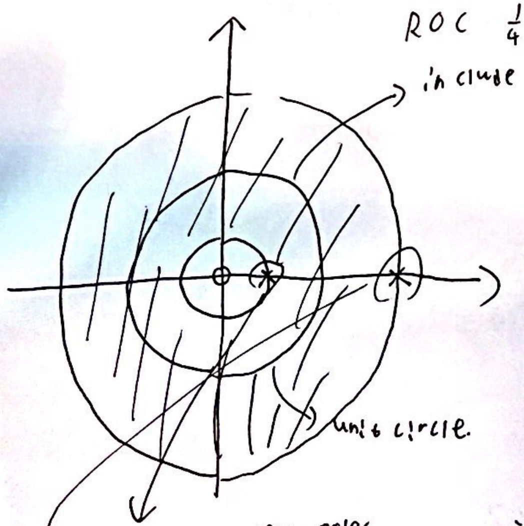
ROC $\left|\frac{1}{2}z\right| < 1 \Rightarrow |z| < 2$, $\left|\frac{1}{4}z^{-1}\right| < 1 \Rightarrow |z| > \frac{1}{4}$ $\Rightarrow \frac{1}{4} < |z| < 2$

$$X(z) = \frac{1}{1-\frac{1}{2}z} + \frac{\frac{1}{4}z^{-1}}{1-\frac{1}{4}z^{-1}} = \frac{-\frac{7}{4}z}{z^2 - \frac{9}{4}z + \frac{1}{2}} = \frac{-\frac{7}{4}z}{(z-2)(z-\frac{1}{4})}$$

ROC $\frac{1}{4} < |z| < 2$

include unit circle → stable

Fourier transform of the sequence is existed



causal system poles inside unit circle → stable

uncausal system poles outside unit circle → stable

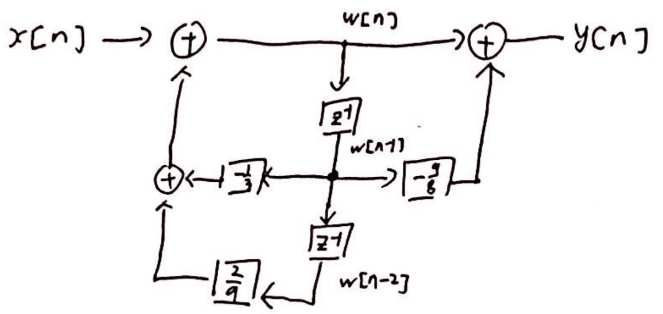
※ If the ROC extends outward from the outermost pole, then the system is causal

※ If the ROC includes the unit circle, then the system is stable

※ when system is stable and causal all of the poles are inside the unit circle

※ when system is stable and uncausal all of the poles are outside the unit circle

[3] The input $x[n]$ and output $y[n]$ of a causal LTI system are related through the block-diagram representation as shown in the following figure



$$w[n] = x[n] - \frac{1}{3}w[n-1] + \frac{2}{9}w[n-2] \Rightarrow X(z) = W(z)\left(1 + \frac{1}{3}z^{-1} - \frac{2}{9}z^{-2}\right)$$

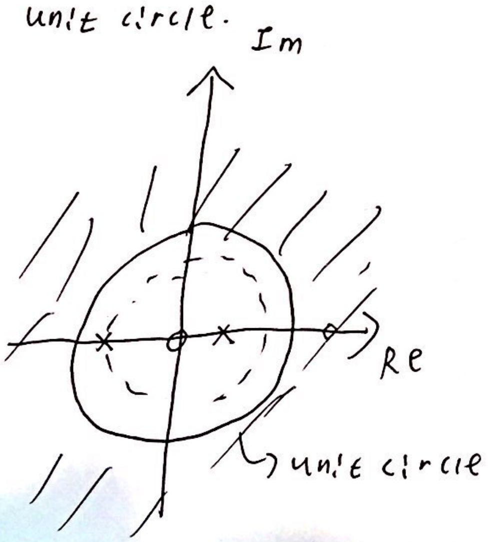
$$y[n] = w[n] - \frac{9}{8}w[n-1] \Rightarrow Y(z) = W(z)\left(1 - \frac{9}{8}z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{9}{8}z^{-1}}{1 + \frac{1}{3}z^{-1} - \frac{2}{9}z^{-2}} = \frac{z\left(z - \frac{9}{8}\right)}{\left(z - \frac{1}{3}\right)\left(z + \frac{2}{3}\right)} \qquad \text{ROC } |z| > \frac{2}{3}$$

$$y[n] + \frac{1}{3}y[n-1] - \frac{2}{9}y[n-2] = x[n] - \frac{9}{8}x[n-1]$$

$$y[n] = x[n] - \frac{9}{8}x[n-1] - \frac{1}{3}y[n-1] + \frac{2}{9}y[n-2]$$

If system is stable and causal all of the poles are inside the unit circle.



$\Rightarrow$ system is stable.