2011-1 : midterm 2015104027 박경린

1. Compute the Discrete Walsh-Hadamard transform (DWHT) of the sequence

$\{1, -1, 0, 0, 2, 0, -2, 1\}$

Definition of DWHT

$X^{WH}(k) = \frac{1}{N} x_i \, {}^{N}H$ (∵ N is length of data sequence)

(N is even $N = 2m$)

$${}^{2m}H = \begin{bmatrix} {}^{m}H & {}^{m}H \\ {}^{m}H & -{}^{m}H \end{bmatrix}$$

ex) $${}^{2}H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$${}^{4}H = \begin{bmatrix} {}^{2}H & {}^{2}H \\ {}^{2}H & -{}^{2}H \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$${}^{8}H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$\therefore X^{WH}(k) = \{1/8, 1/8, 3/8, 7/8, 1/8, -1/8, -1/8, 7/8\}$

	1	-1	0	0	2	0	-2	1	sum	$X^{WH}(k)$
$X^{WH}(0)$	1	-1	0	0	2	0	-2	1	1	$\frac{1}{8}$
$X^{WH}(1)$	1	1	0	0	2	0	-2	-1	1	1/8
$X^{WH}(2)$	1	-1	0	0	2	0	2	-1	3	3/8
$X^{WH}(3)$	1	1	0	0	2	0	2	1	7	7/8
$X^{WH}(4)$	1	-1	0	0	2	0	-2	1	1	1/8
$X^{WH}(5)$	1	1	0	0	2	0	-2	-1	-1	-1/8
$X^{WH}(6)$	1	-1	0	0	2	0	2	-1	-1	-1/8
$X^{WH}(7)$	1	1	0	0	2	0	2	1	7	7/8

2. 다음에 주어진 함수의 z-transform을 계산하시오

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$ $(z = re^{j\theta})$

$a^n u[n] \longleftrightarrow \frac{z}{z-a}$ ROC $|z| > |a|$

$-a^n u(-n-1) \longleftrightarrow \frac{z}{z-a}$ ROC $|z| < |a|$

ROC : causal $|z| > |a|$

noncausal $|z| < |a|$

$x(n) = (-3)^n u(-n-1)$

(a) i) using definition

$x[n] = (-3)^n u(-n-1)$

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{-1} (-3)^n z^{-n}$

Let $n = -k$

$\sum_{k=1}^{\infty} (-3)^{-k} z^k = \sum_{k=1}^{\infty} (-\frac{1}{3} z)^k = \frac{-\frac{1}{3}z}{1-(-\frac{1}{3}z)} = -\frac{z}{z+3}$ ROC $|-\frac{1}{3}z| < 1$, $|z| < 3$ → stable

※ $\sum_{k=1}^{\infty} (-\frac{1}{3} z)^k = \sum_{k=1}^{\infty} (-\frac{1}{3} z)^{2k-1} + \sum_{k=1}^{\infty} (-\frac{1}{3} z)^{2k} = \frac{-\frac{1}{3}z}{1-\frac{1}{9}z^2} + \frac{\frac{1}{9}z^2}{1-\frac{1}{9}z^2} = \frac{z^2-3z}{9-z^2} = -\frac{z(z-3)}{(z-3)(z+3)}$

$= -\frac{z}{z+3}$ ROC $|\frac{1}{9}z^2| < 1$

$|z| < 3$ → stable

ii) using pair

$x[n] = (-3)^n u(-n-1)$

$= -\underbrace{(-(-3)^n u(-n-1))}$

$a = -3$

$-a^n u[-n-1] \longleftrightarrow \frac{z}{z-a}$ ROC $|z| < |a|$

$X(z) = -\frac{z}{z-(-3)} = -\frac{z}{z+3}$

(b) $(\frac{1}{4})^n u(n) + 2^n u(-n-1) = x[n]$

i) using definition

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} (\frac{1}{4})^n z^{-n} + \sum_{n=-\infty}^{-1} 2^n z^{-n}$

$= \sum_{n=0}^{\infty} (\frac{1}{4} z^{-1})^n + \sum_{k=1}^{\infty} 2^{-k} z^k = \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{k=1}^{\infty} (\frac{1}{2}z)^k = \frac{z}{z-\frac{1}{4}} + \frac{\frac{1}{2}z}{1-\frac{1}{2}z}$

$= \frac{z}{z-\frac{1}{4}} + (-\frac{z}{z-2}) = \frac{z}{z-\frac{1}{4}} - \frac{z}{z-2}$ ROC $|\frac{1}{4}z^{-1}| < 1 \Rightarrow |z| > \frac{1}{4}$

$|\frac{1}{2}z| < 1 \Rightarrow |z| < 2$

$\therefore \frac{1}{4} < |z| < 2$

ii) using pair $a^n u[n] \longleftrightarrow \frac{z}{z-a}$ ROC $|z|>|a|$

$-a^n u[-n-1] \longleftrightarrow \frac{z}{z-a}$ ROC $|z|<|a|$

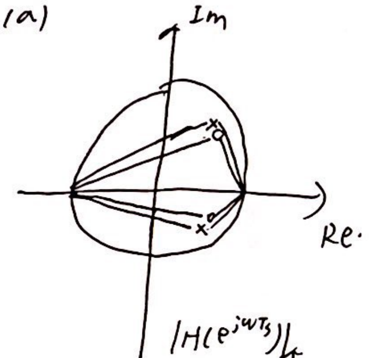
$$x[n] = \left(\tfrac{1}{4}\right)^n u[n] + 2^n u[-n-1]$$

$$= \underbrace{\left(\tfrac{1}{4}\right)^n u[n]}_{\substack{a=\frac{1}{4} \\ \text{ROC } |z|>\frac{1}{4}}} + \left(-\underbrace{(-2^n u[-n-1])}_{\substack{a=2 \\ \text{ROC } |z|<2}}\right)$$

$$X(z) = \frac{z}{z-\frac{1}{4}} - \frac{z}{z-2} \qquad \text{ROC } \tfrac{1}{4}<|z|<2$$

3. By considering the geometric determination of the frequency response sketch, for each of the pole-zero plots in figure 1, the magnitude of the associated Fourier transform

(a)



In phase $\theta = 0, \pi, 2\pi$

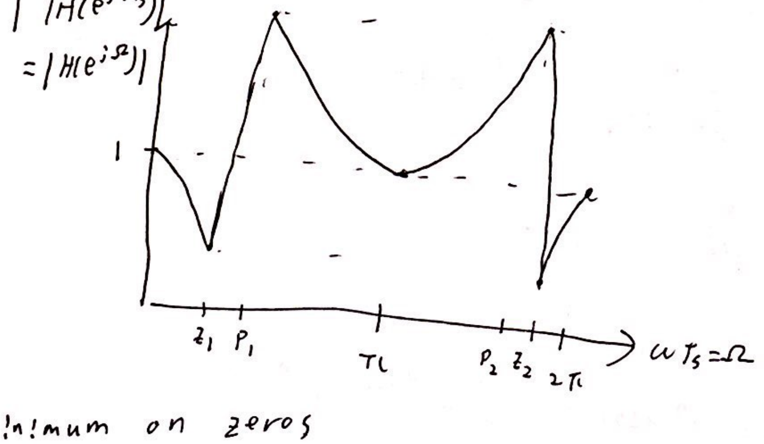
$U_1 \approx V_1$, $U_2 \approx V_2$ $\Rightarrow$ $H(e^{j\cdot 0}) = \frac{U_1 U_2}{V_1 V_2} = 1$

$H(e^{j\pi}) = 1$

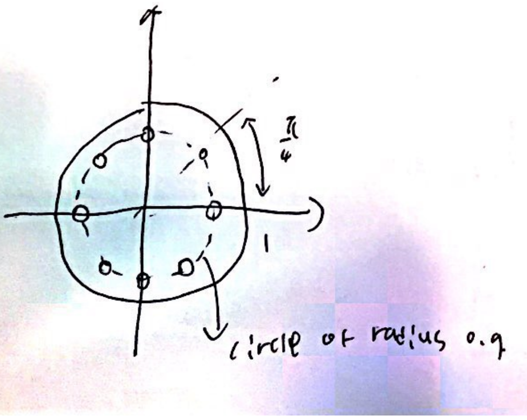
$H(e^{j2\pi}) = 1$

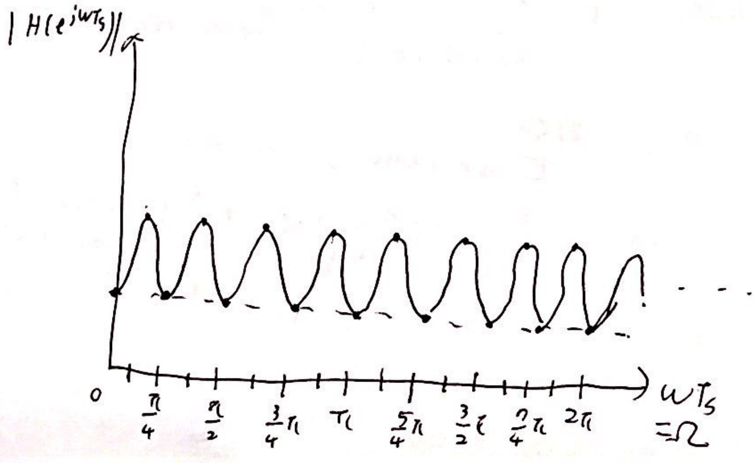
maximum on poles

minimum on zeros



(b) minimum on zeros





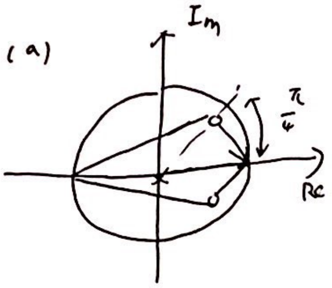
(c) ROC $\underbrace{\tfrac{1}{3}<}_{\text{causal}}|z|\underbrace{<\tfrac{1}{2}}_{\text{noncausal}}$

$$h[n] = \underbrace{-\frac{3}{5}\left\{-\left(\frac{1}{2}\right)^n u[-n-1]\right\}}_{ROC\ |z|<\frac{1}{2}} + \underbrace{\frac{8}{5}\left(-\frac{1}{3}\right)^n u[n]}_{ROC\ |z|>\frac{1}{3}} \Rightarrow ROC\ \frac{1}{3}<|z|<\frac{1}{2}$$

$$= \frac{3}{5}\left(\frac{1}{2}\right)^n u[-n-1] + \frac{8}{5}\left(-\frac{1}{3}\right)^n u[n]$$

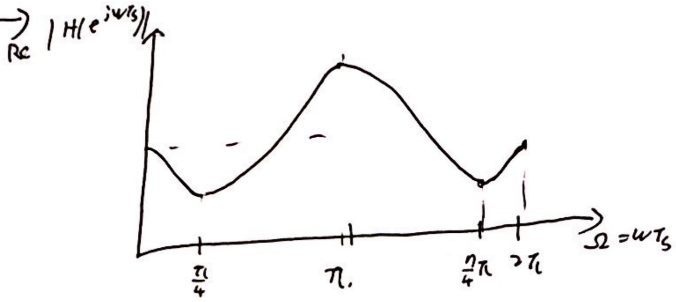
※ Addition of problem 3   2013_1_Quiz#1_problem 1

(a)

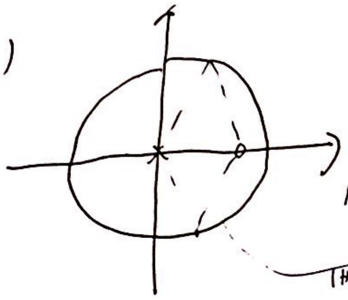


$|H(e^{j\pi})| > |H(e^{j0})|$ → maximum

$\Omega = \frac{\pi}{4},\ 2\pi - \frac{\pi}{4}$ → minimum



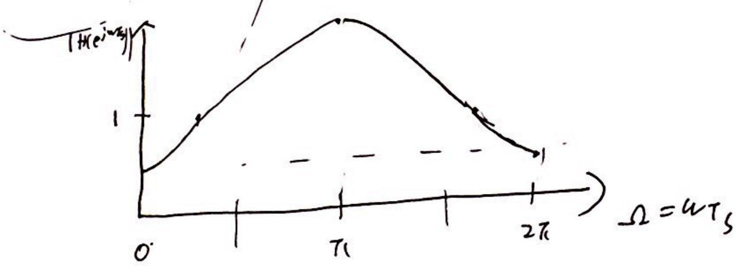
(b)



$|H(e^{j0})| \neq 0$

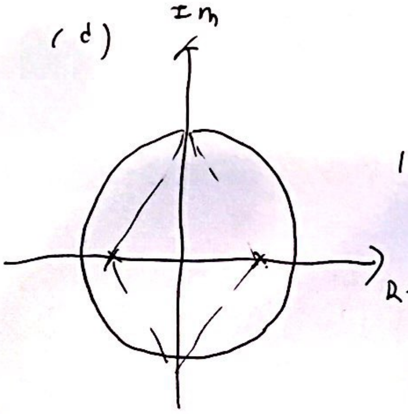
$|H(e^{j\pi})|$ → maximum

$|H(e^{j\Omega})|$ → 1



(c), (e) is same one with (a), (b) of problem 3

(d)



$|H(e^{j0})| = |H(e^{j\pi})|$ → ~~minimum~~ maximum

$|H(e^{j\frac{\pi}{2}})|$ → ~~[illegible]~~ minimum

$= |H(e^{-j\frac{\pi}{2}})|$