2013_1_midterm 2015104027 박경권

1. (a) oversampling 의 이점

1. higher sampling rate eliminates aliasing

2. increase resolution

3. leads to the use of simple anti-aliasing filters

4. improved SNR(signal noise ratio) (reduces noise)
   ↳ aliasing component

5. To achieve easily the requirements of variable cutoff frequency

※ down sampling 의 이점

1. reducing datasize

2. compression or image reduction)

but have a possibility to break up Nyquist rule

→ should pass pre lowpass filter first

(b) Aliasing 이란?

If frequency break up the nyquist rule ($f_s \geq 2f_m$)

(f_s : sampling frequency, f_m : maximum frequency)

the we ~~defined~~ define ~~there~~ Aliasing is occurred

↳ aliasing is occurred

(c) fast convolution 방법?

$$x_1[n] * x_2[n] = F^{-1}(X_1(k) \cdot X_2(k))$$

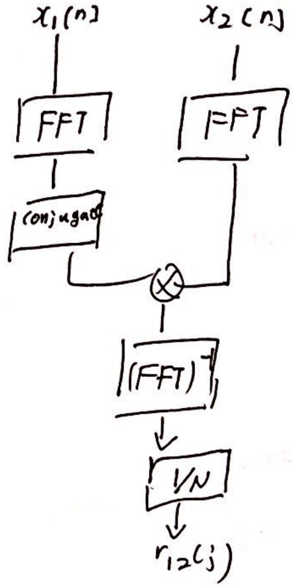
using Fast Fourier Transform

The convolution of two signal is obtained by taking a fast fourier transform, ~~and~~ multiplying each signal, and performing inverse transformation

※ Fast correlation.

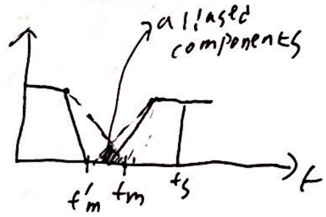
$$r_{12}(j) = \frac{1}{N} F^{-1}[X_1^*(k) X_2(k)]$$

The correlation of two signals is obtained by taking a fast fourier transform, conjugate $X_1(k)$ multiplying $X_2(k)$ and performing inverse transformation, Finally ~~fast~~ take normalized by N



(d) Aliasing 막는 방법?

1. increase sampling frequency
2. use anti aliasing filter ex pre filter.
eliminates aliased portion of spectrum



2. (a) DWHT [1, 1, 0, 0, -1, 0, -2, 1]

Definition of DWHT

$x^{WH}(k) = \frac{1}{N} x_0 \, {}^{N}H$ (∵ N is length of data sequence)

(N is even $N = 2m$)

$${}^{2}H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad {}^{4}H = \begin{bmatrix} {}^{2}H & {}^{2}H \\ {}^{2}H & -{}^{2}H \end{bmatrix} \quad {}^{8}H = \begin{bmatrix} {}^{4}H & {}^{4}H \\ {}^{4}H & -{}^{4}H \end{bmatrix}$$

$${}^{8}H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

	1	1	0	0	-1	0	-2	1	sum
$x^{WH}(0)$	1	1	0	0	-1	0	-2	1	0
$x^{WH}(1)$	1	-1	0	0	-1	0	-2	-1	-4
$x^{WH}(2)$	1	1	0	0	-1	0	2	-1	2
$x^{WH}(3)$	1	-1	0	0	-1	0	2	1	2
$x^{WH}(4)$	1	1	0	0	-1	0	-2	1	0
$x^{WH}(5)$	1	-1	0	0	-1	0	-2	-1	-4
$x^{WH}(6)$	1	1	0	0	-1	0	2	-1	2
$x^{WH}(7)$	1	-1	0	0	-1	0	2	1	2

$x^{WH}(k) = \{0, -\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, -\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$

(b) z-transform 을 구하시오

$x[n] = (\frac{1}{2})^{|n|}$ 1) using definition

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{-1} (\tfrac{1}{2})^{-n} z^{-n} + \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n} z^{-n} = \sum_{k=1}^{\infty} (\tfrac{1}{2})^{k} z^{k} + \sum_{n=0}^{\infty} (\tfrac{1}{2} z^{-1})^{n}$$

$$= \sum_{k=1}^{\infty} (\tfrac{1}{2} z)^{k} + \sum_{n=0}^{\infty} (\tfrac{1}{2} z^{-1})^{n} = \frac{\frac{1}{2} z}{1 - \frac{1}{2} z} + \frac{1}{1 - \frac{1}{2} z^{-1}} = \frac{z}{z - \frac{1}{2}} - \frac{z}{z - 2}$$

ROC $|\frac{1}{2} z| < 1 \Rightarrow |z| < 2$
$|\frac{1}{2} z^{-1}| < 1 \Rightarrow |z| > \frac{1}{2}$
$\Rightarrow \frac{1}{2} < |z| < 2$

$\therefore X(z) = \frac{z}{z - \frac{1}{2}} - \frac{z}{z-2}$ ROC $\frac{1}{2} < |z| < 2$

ii) using pairs

$$x[n] = \left(\tfrac{1}{2}\right)^{|n|} = \left(\tfrac{1}{2}\right)^{n} u[n] + \left(\tfrac{1}{2}\right)^{-n} u[-n-1] = \underbrace{\left(\tfrac{1}{2}\right)^{n} u[n]}_{ROC\ |z|>\frac{1}{2}} + \underbrace{(2)^{n} u[-n-1]}_{ROC\ |z|<2}$$

$$= \left(\tfrac{1}{2}\right)^{n} u[n] - \left(-(2)^{n} u[-n-1]\right)$$

$$X(z) = \frac{z}{z-\frac{1}{2}} - \frac{z}{z-2} \quad ROC\ \tfrac{1}{2} < |z| < 2$$

3. $B = 12$ (a) SQNR?

$f_s = 100KHz$ (b) ADR?

$BW = 20KHz$

step size $q = \frac{V_{pp}}{2^B - 1} \approx \frac{V_{pp}}{2^B}$

Averaged quantization Noise power $N_q = \frac{q^2}{12}$

RMS quantization Noise power $N_{q\_rms} = \frac{0.5V_{pp}}{\sqrt{3\times 2^{2B}}} = \sqrt{\frac{q^2}{12}}\Big|_{q=\frac{V_{pp}}{2^B}}$

$SQNR = 10\log \frac{\text{signal power}}{\text{quantization Noise power}}$

(signal to quantization Noise Ratio)

$$= 10\log \tfrac{3}{2}\cdot 2^{2B} = \underbrace{6.02B}_{10\log 2^{2B}} + \underbrace{1.76}_{10\log\frac{3}{2}} [dB]$$

(dynamic range) The ratio of the maximum and minimum signal levels that the ADC can handle

Quantization Noise power per unit bandwidth $N_{oq} = \frac{q^2/12}{f_s/2}$

ADR (Actual Dynamic range)

$$= 10\log \frac{\text{signal power}}{N_{oq}\cdot BW} = 10\log \frac{P_s}{N_{oq}} - 10\log BW$$

$$= 10\log\left(\tfrac{3}{4} f_s 2^{2B}\right) - 10\log BW = \underbrace{6.02B}_{10\log 2^{2B}} + 10\log f_s \underbrace{- 1.249}_{10\log\frac{3}{4}} - 10\log BW\ [dB]$$

(a) $SQNR = 6.02B + 1.76 [dB]$

$= 10\log \frac{3}{2}\cdot 2^{2B} = 74.008 [dB]$

(b) $ADR = 6.02B + 10\log f_s - 1.249 - 10\log BW\ [dB]$

$= 10\log\left(\frac{3}{4} f_s 2^{2B}\right) - 10\log BW = 77.9875 [dB]$

Scanned with CamScanner

4. $X(z) = \dfrac{1-\frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})}$ $\quad IZT?$

※ order of Numerator M $\quad N > M \quad c_0 = 0$

order of Denominator N $\quad N = M \quad c_0 = X(z)|_{z=0}$

$$C_k = X(z)\,\frac{z-p_k}{z}\Big|_{z=p_k}$$

multiple order

$$\sum_{i=1}^{m} \frac{D_i z}{(z-p_k)^i} = \frac{D_1 z}{z-p_k} + \frac{D_2 z}{(z-p_k)^2} + \cdots + \frac{D_m z}{(z-p_k)^m}$$

$$D_i = \frac{1}{(m-i)!}\,\frac{d^{m-i}}{dz^{m-i}}\left[\frac{(z-p_k)^m}{z}X(z)\right]\Big|_{z=p_k}$$

pair $\quad a^n u[n] \longleftrightarrow \dfrac{z}{z-a} \quad |z|>|a| \qquad n a^n u[n] \longleftrightarrow \dfrac{az}{(z-a)^2} \quad |z|>|a|$

$\quad -a^n u[-n-1] \longleftrightarrow \dfrac{z}{z-a} \quad |z|<|a| \qquad -n a^n u[-n-1] \longleftrightarrow \dfrac{az}{(z-a)^2} \quad |z|<|a|$

$$X(z) = \frac{z(z-\frac{1}{3})}{(z-1)(z+2)}$$

$c_0 = X(z)|_{z=0} = 0$

$$c_1 = \frac{z(z-\frac{1}{3})}{(z-1)(z+2)}\cdot\frac{z-1}{z}\Big|_{z=1} = \frac{z-\frac{1}{3}}{z+2}\Big|_{z=1} = \frac{\frac{2}{3}}{3} = \frac{2}{9}$$

$$c_2 = \frac{z(z-\frac{1}{3})}{(z-1)(z+2)}\cdot\frac{z+2}{z}\Big|_{z=-2} = \frac{z-\frac{1}{3}}{z-1}\Big|_{z=-2} = \frac{-\frac{7}{3}}{-3} = \frac{7}{9}$$

$$X(z) = \frac{2}{9}\cdot\frac{z}{z-1} + \frac{7}{9}\cdot\frac{z}{(z+2)}$$

i) ROC $\quad |z|>2 \rightarrow$ unstable

$$x[n] = \frac{2}{9}(1)^n u[n] + \frac{7}{9}(-2)^n u[n] = \left(\frac{2}{9} + \frac{7}{9}(-2)^n\right)u[n]$$

ii) ROC $\quad 1<|z|<2 \rightarrow$ unstable

$1<|z|$: causal, $|z|<2$: non causal

$$x[n] = \frac{2}{9}(1)^n u[n] - \frac{7}{9}(-2)^n u[-n-1]$$

iii) ROC $\quad |z|<1 \rightarrow$ unstable.

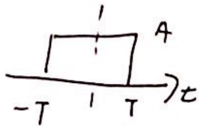
$$x[n] = -\frac{2}{9}(1)^n u[-n-1] - \frac{7}{9}(-2)^n u[-n-1]$$

$$= \left(-\frac{2}{9} - \frac{7}{9}(-2)^n\right)u[-n-1]$$

5. CTFT 구하시오, 필요한 경우 X(w) 도식화 하시오

(a) $4\,\mathrm{sinc}\left(\frac{t}{20}\right)$

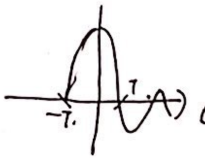
※ $e^{j2\pi f_0 t} = e^{j\omega_0 t} \longleftrightarrow \delta(f-f_0)$, $2\pi\delta(\omega-\omega_0)$

※ 

$\triangleq \quad A\,\mathrm{rect}\left(\frac{t}{2T}\right) = A\Pi\left(\frac{t}{2T}\right)$

$A\,\mathrm{rect}\left(\frac{t}{\tau}\right)$

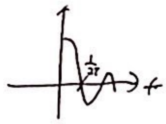
duration $2T = \tau$



$\triangleq \quad A\,\mathrm{sinc}\left(\frac{t}{T}\right)$

$A\,\mathrm{rect}\left(\frac{t}{2T}\right) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right) \longrightarrow 2T\cdot A\,\mathrm{sinc}(f\cdot 2T) = 2T\cdot A\,\mathrm{sinc}\left(\frac{\omega\cdot 2T}{2\pi}\right)$

scale $\frac{1}{2T}$ 의 역수 → 진폭, scale에 모두 곱함



$\rightarrow \quad \tau A\,\mathrm{sinc}(f\cdot\tau) = \tau A\,\mathrm{sinc}\left(\frac{\omega\cdot\tau}{2\pi}\right)$

$= 2Tf$

$A\,\mathrm{sinc}\left(\frac{f}{\frac{1}{2T}}\right) \longrightarrow A\cdot\frac{1}{2T}\,\mathrm{rect}\left(t\cdot\frac{1}{2T}\right) = \frac{A}{2T}\,\mathrm{rect}\left(\frac{t}{2T}\right)$

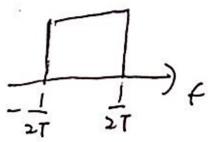
scale $\frac{1}{\frac{1}{2T}} = 2T$ 의 역수

→ 진폭, scale에 모두 곱함

$A\,\mathrm{sinc}\left(\frac{t}{T}\right) \longrightarrow A\cdot T\,\mathrm{rect}(f\cdot T) = AT\,\mathrm{rect}(f\cdot T)$

↳ scale $\frac{1}{T}$ 의 역수

진폭, scale에 모두 곱함



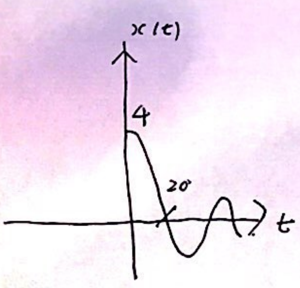
(a) $4\,\mathrm{sinc}\left(\frac{t}{20}\right) \longrightarrow 4\cdot 20\,\mathrm{rect}(f\cdot 20)$

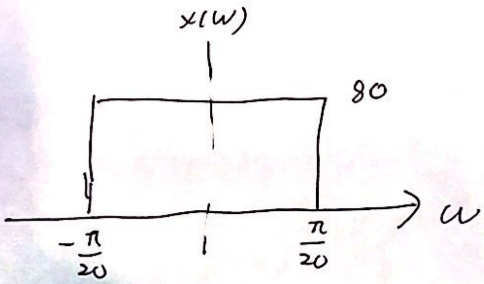
scale $\frac{1}{20}$ 의 역수

진폭, scale 모두 곱

$= 80\,\mathrm{rect}(20\cdot f) = 80\,\mathrm{rect}\left(\frac{10}{\pi}\cdot\omega\right)$

$= 80\,\mathrm{rect}\left(\frac{\omega}{\frac{\pi}{10}}\right)$





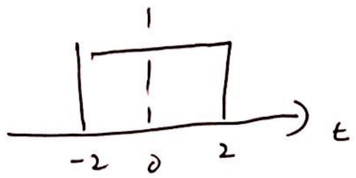
(b) $\Pi(\frac{t}{4}) = rect(\frac{t}{4})$

$rect(\frac{t}{4}) \longrightarrow 4\ sinc(f \cdot 4)$

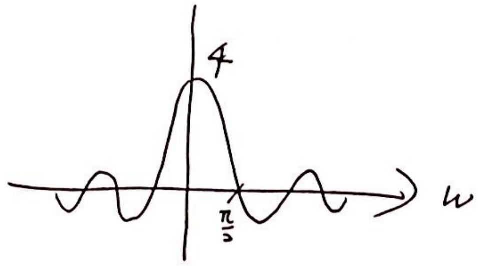
scale $\frac{1}{4}$ 되어서

진폭, scale 모두 4

$= 4 sinc(\frac{4}{2\pi} \cdot w) = 4 sinc(\frac{w}{\frac{\pi}{2}})$



$\rightarrow$



(c) $\cos 100\pi(t) \cdot sinc(\frac{t}{20})$

$\cos 100\pi t = \frac{e^{j100\pi t} + e^{-j100\pi t}}{2} \longrightarrow \frac{2\pi(\delta(w-100\pi) + \delta(w+100\pi))}{2} = \pi\{\delta(w-100\pi) + \delta(w+100\pi)\}$

$sinc(\frac{t}{20}) \longrightarrow 20 \cdot rect(f \cdot 20)$

$= 20\ rect(\frac{w}{\frac{\pi}{10}})$

$\mathcal{F}(\cos 100\pi t \cdot sinc(\frac{t}{20})) = \frac{1}{2\pi}\mathcal{F}(\cos 100\pi t) * \mathcal{F}(sinc(\frac{t}{20}))$

※ shifting properties

$x(t) * \delta(t-t_0) = x(t-t_0)$

※ modulation property

$f(t)\cos w_0 t \longleftrightarrow \frac{1}{2}(F(w-w_0) + F(w+w_0))$

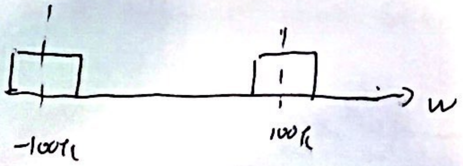
* multiplying, convolution

t	w
$x_1(t) \cdot x_2(t)$	$\frac{1}{2\pi} X_1(w) * X_2(w)$
$x_1(t) * x_2(t)$	$X(w) \cdot X_2(w)$

$\cos 100\pi t \cdot sinc(\frac{t}{20}) \longrightarrow \frac{1}{2\pi}(\pi(\delta(w-100\pi) + \delta(w+100\pi)) * 20\ rect(\frac{w}{\frac{\pi}{10}}))$

$= \frac{1}{2\pi}(20\pi\ rect(\frac{w-100\pi}{\frac{\pi}{10}}) + 20\pi\ rect(\frac{w+100\pi}{\frac{\pi}{10}}))$

$= 10\ rect(\frac{w-100\pi}{\frac{\pi}{10}}) + 10 rect(\frac{w+100\pi}{\frac{\pi}{10}})$



* multiplication

$\mathcal{F}(x(t) \cdot y(t)) = \frac{1}{2\pi} X(w) * Y(w)$

$\mathcal{F}(x(t) \cdot y(t)) = X(f) * Y(f)$

$\mathcal{F}(x(t) * y(t)) = X(w) \cdot Y(w)$

$X(f) \cdot Y(f)$