

DSP 실험 프로젝트

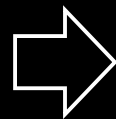
조장 : 이효건

조원 : 안유민 유지나 이종혁 정수현

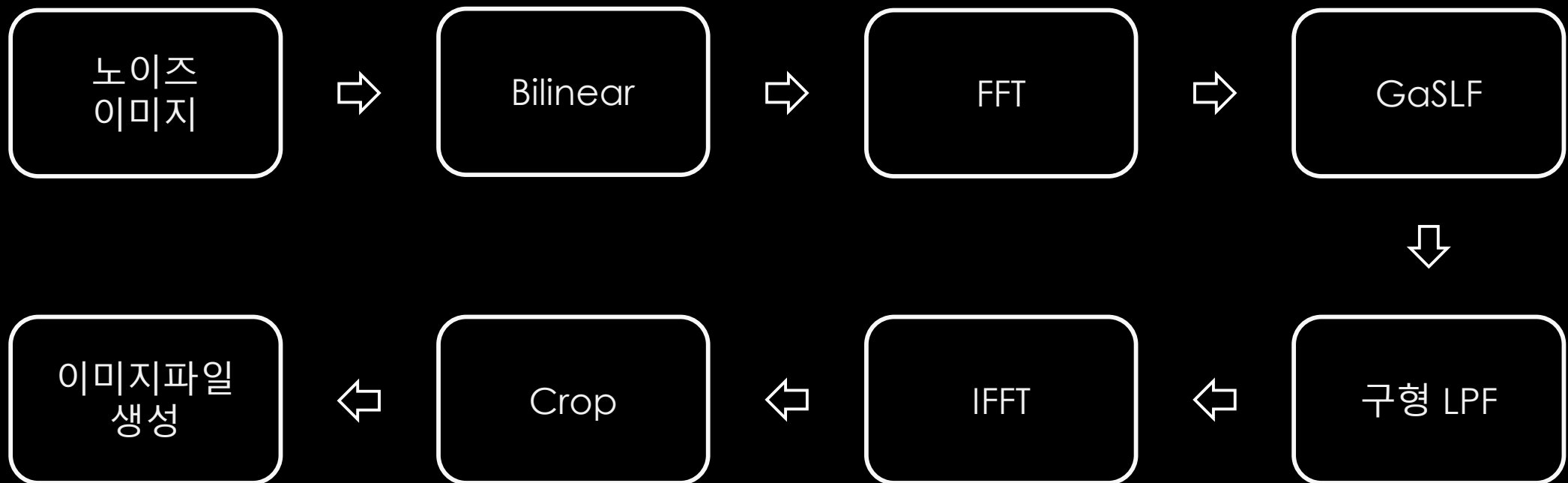
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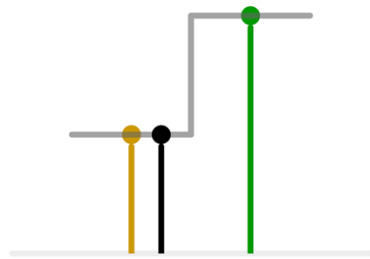
1. 필터링 목표



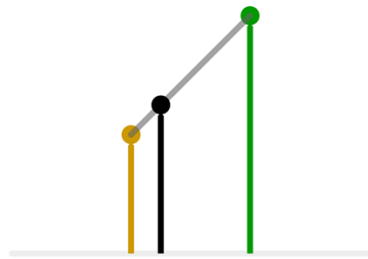
2. 알고리즘



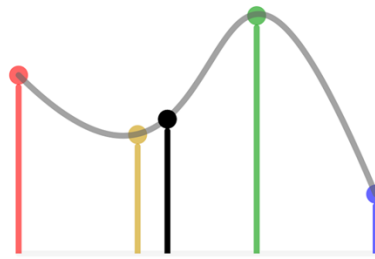
BILINEAR 함수



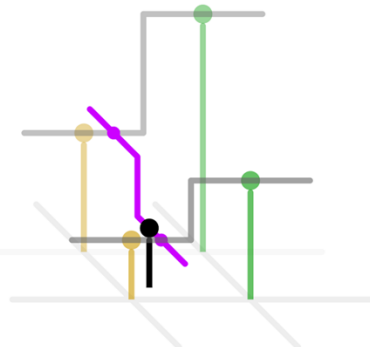
1D nearest-neighbour



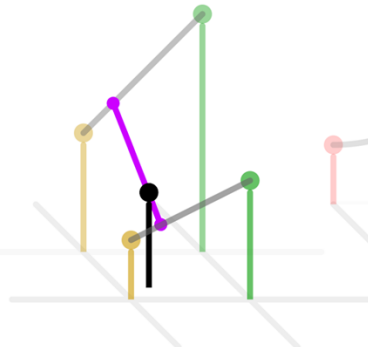
Linear



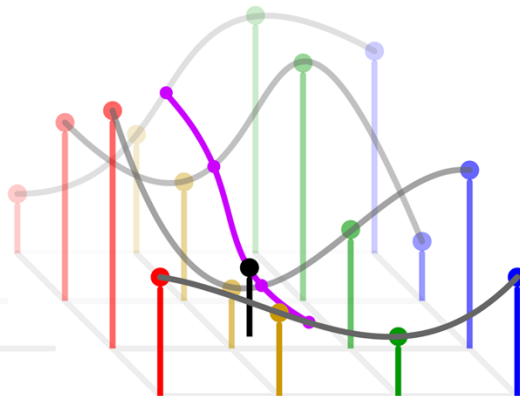
Cubic



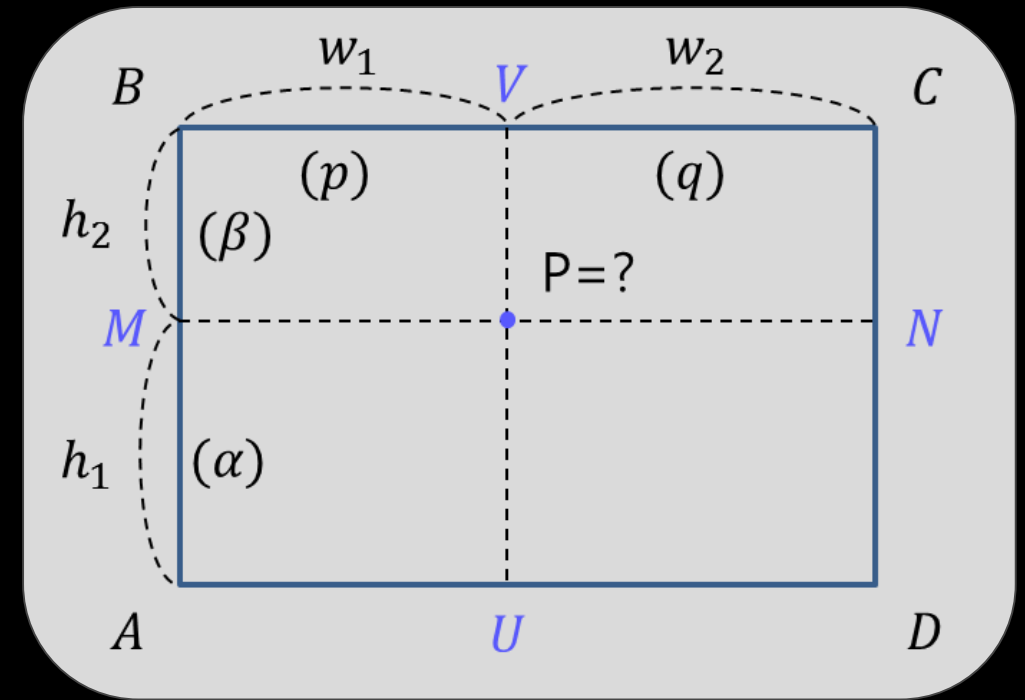
2D nearest-neighbour



Bilinear



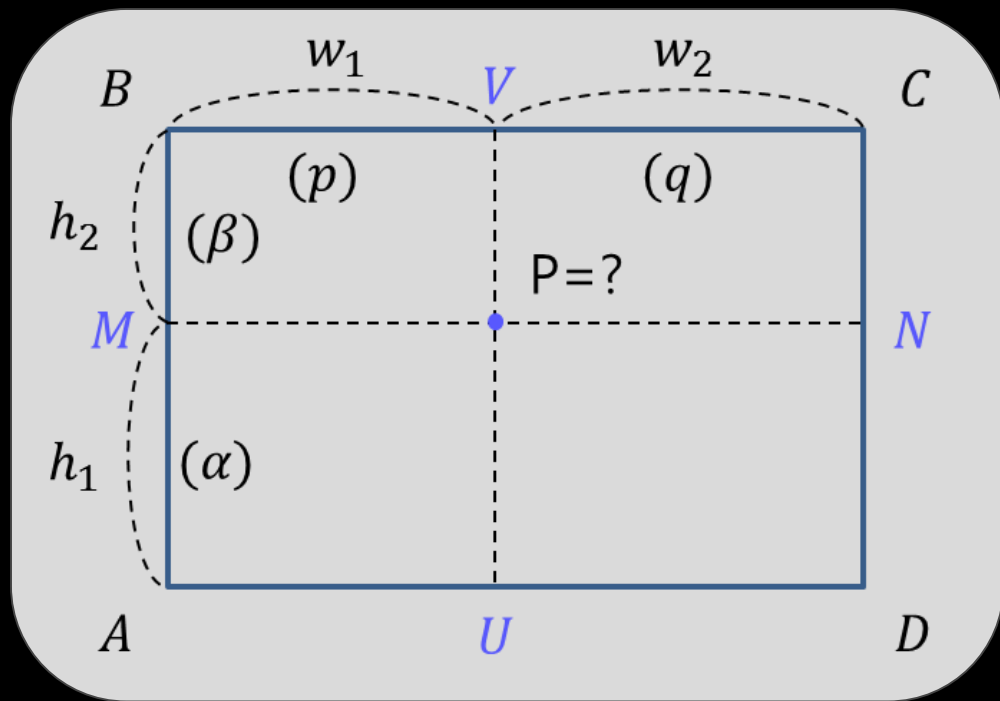
Bicubic



[다크프로그래머 블로그](#)

[Wikipedia : Bilinear Interpolation](#)

BILINEAR 함수



[다크프로그래머 블로그](#)

$$\alpha = \frac{h1}{h1 + h2} \quad \beta = \frac{h2}{h1 + h2}$$
$$p = \frac{w1}{w1 + w2} \quad q = \frac{w2}{w1 + w2}$$

$$P = q(\beta A + \alpha B) + p(\beta D + \alpha C)$$
$$= q\beta A + q\alpha B + p\beta D + p\alpha C$$

BILINEAR 함수

R : Rate

$$\alpha = \frac{h1}{h1 + h2} \quad \beta = \frac{h2}{h1 + h2}$$
$$p = \frac{w1}{w1 + w2} \quad q = \frac{w2}{w1 + w2}$$

$$P = q(\beta A + \alpha B) + p(\beta D + \alpha C)$$
$$= q\beta A + q\alpha B + p\beta D + p\alpha C$$

$$\overline{\begin{matrix} \bullet & & \bullet & & \bullet \\ Rn & & m & & R(n+1) \end{matrix}}$$

BILINEAR 함수

$$\alpha = \frac{h1}{h1 + h2} \quad \beta = \frac{h2}{h1 + h2}$$
$$p = \frac{w1}{w1 + w2} \quad q = \frac{w2}{w1 + w2}$$

$$P = q(\beta A + \alpha B) + p(\beta D + \alpha C)$$
$$= q\beta A + q\alpha B + p\beta D + p\alpha C$$

R : Rate

$$\overbrace{Rn \quad m \quad R(n+1)}$$

$$Rn \leq m \leq R(n+1)$$

$$n \leq \frac{m}{R} \leq n+1$$

BILINEAR 함수

$$\alpha = \frac{h1}{h1 + h2} \quad \beta = \frac{h2}{h1 + h2}$$
$$p = \frac{w1}{w1 + w2} \quad q = \frac{w2}{w1 + w2}$$

$$P = q(\beta A + \alpha B) + p(\beta D + \alpha C)$$
$$= q\beta A + q\alpha B + p\beta D + p\alpha C$$

R : Rate

$$\overline{Rn} \quad \overline{m} \quad \overline{R(n+1)}$$

$$Rn \leq m \leq R(n+1)$$

$$n \leq \frac{m}{R} \leq n+1$$

$$\therefore n = \left\lfloor \frac{m}{R} \right\rfloor$$

BILINEAR 함수

R : Rate

```
void bilinear(uchar **x, uchar **y, int Win, int Wout, int Hin, int Hout) {
    int nx, ny;
    double Rx = (Wout-1) / (double)(Win-1), Ry = (Hout-1) / (double)(Hin-1);
    double alphax, betax, alphay, betay;
    double tmp;
    for (int my = 0; my < Hout; my++) {
        for (int mx = 0; mx < Wout; mx++) {
            nx = (int)(mx / Rx);
            ny = (int)(my / Ry);
            alphax = mx / Rx - nx;
            alphay = my / Ry - ny;
            betax = 1 - alphax;
            betay = 1 - alphay;
            tmp = (
                betax * betay * x[ny][nx]
                + alphax * betay * x[ny][min(nx+1, Win-1)]
                + betax * alphay * x[min(ny+1, Hin-1)][nx]
                + alphax * alphay * x[min(ny+1, Hin-1)][min(nx+1, Win-1)]
            );
            y[my][mx] = ((tmp > 255) ? 255 : (tmp < 0) ? 0 : tmp);
        }
    }
}
```

$$\overline{Rn} \quad m \quad \overline{R(n+1)}$$

$$Rn \leq m \leq R(n+1)$$

$$n \leq \frac{m}{R} \leq n+1$$

$$\therefore n = \left\lfloor \frac{m}{R} \right\rfloor$$

BILINEAR 함수

R : Rate

```
void bilinear(uchar **x, uchar **y, int Win, int Wout, int Hin, int Hout) {
    int nx, ny;
    double Rx = (Wout-1) / (double)(Win-1), Ry = (Hout-1) / (double)(Hin-1);
    double alphax, betax, alphay, betay;
    double tmp;
    for (int my = 0; my < Hout; my++) {
        for (int mx = 0; mx < Wout; mx++) {
            nx = (int)(mx / Rx);
            ny = (int)(my / Ry);
            alphax = mx / Rx - nx;
            alphay = my / Ry - ny;
            betax = 1 - alphax;
            betay = 1 - alphay;
            tmp = (
                betax * betay * x[ny][nx]
                + alphax * betay * x[ny][min(nx+1, Win-1)]
                + betax * alphay * x[min(ny+1, Hin-1)][nx]
                + alphax * alphay * x[min(ny+1, Hin-1)][min(nx+1, Win-1)]
            );
            y[my][mx] = ((tmp > 255) ? 255 : (tmp < 0) ? 0 : tmp);
        }
    }
}
```

$$\overline{Rn} \quad m \quad \overline{R(n+1)}$$

$$Rn \leq m \leq R(n+1)$$

$$n \leq \frac{m}{R} \leq n+1$$

$$\therefore n = \left\lfloor \frac{m}{R} \right\rfloor$$

BILINEAR 함수

```
void bilinear(uchar **x, uchar **y, int Win, int Wout, int Hin, int Hout) {
    int nx, ny;
    double Rx = (Wout-1) / (double)(Win-1), Ry = (Hout-1) / (double)(Hin-1);
    double alphas, betas, alphas, betas;
    double tmp;
    for (int my = 0; my < Hout; my++) {
        for (int mx = 0; mx < Wout; mx++) {
            nx = (int)(mx / Rx);
            ny = (int)(my / Ry);
            alphas = mx / Rx - nx;
            alphas = my / Ry - ny;
            betas = 1 - alphas;
            betas = 1 - alphas;
            tmp = (
                betas * betas * x[ny]
                + alphas * betas * x[ny]
                + betas * alphas * x[min(ny+1, Hin-1)]
                + alphas * alphas * x[min(ny+1, Hin-1)]
            );
            y[my][mx] = ((tmp > 255) ? 255 : (tmp < 0) ? 0 : tmp);
        }
    }
}
```

$$\alpha = \frac{h1}{h1 + h2} \quad \beta = \frac{h2}{h1 + h2}$$
$$p = \frac{w1}{w1 + w2} \quad q = \frac{w2}{w1 + w2}$$

$$P = q(\beta A + \alpha B) + p(\beta D + \alpha C)$$
$$= q\beta A + q\alpha B + p\beta D + p\alpha C$$

BILINEAR 함수



BILINEAR 함수



노이즈 원본 이미지



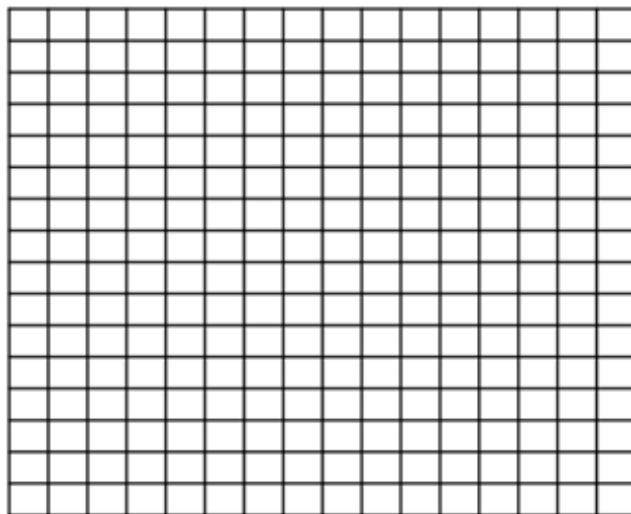
크기가 변경된 이미지

FFT

$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi \left(\frac{ux}{M} + \frac{vy}{N} \right)}$$

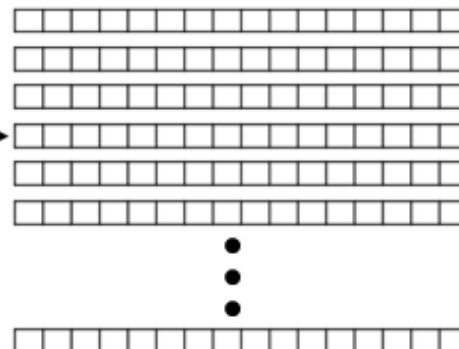


$M \times N$ Image



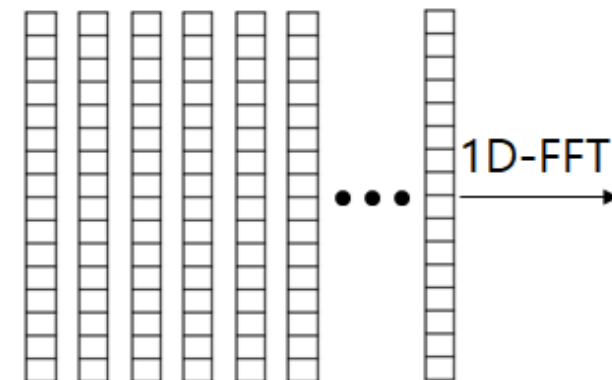
Pixel

가로 성분



1D-FFT

세로 성분



- For FFT, $M = 2^m$, $N = 2^n$ ($m > 1, n > 1$)

Gaussian Star Low Pass Filter

Design of Gaussian Star Filter for Reduction of Periodic Noise And Quasi-Periodic Noise in Gray Level Images

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Abstract—This paper proposes a new filtering method for periodic noise reduction in gray level images. The proposed filter consists of two orthogonal Gaussian filters with elliptic profile for each noise peak provided a star shaped filter. Therefore it is called “Gaussian star filter” (GaSF). The filter parameters for each noise peak are estimated by region growing algorithm in image amplitude spectrum. Obtained results show that the periodic noise and quasi periodic noise are effectively reduced by the GaSF filtering method.

Keywords—periodic noise, quasi-periodic noise, gaussian filter, region growing.

I. INTRODUCTION

The periodic patterns in image are result of undesired effect of scanning of photographs by the scanners. In addition, periodic or quasi-periodic noise caused by electrical interference or some other environmental factors sometimes reduces image quality while recording a video or taking a photograph.

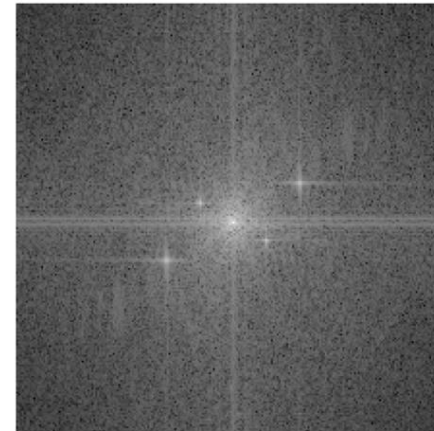


Figure 1. Periodic noise in spectrum.

amplitude spectrum.

As shown Fig. 1, the appearance of noise in the image

주파수 영역 분석 - Region Growing



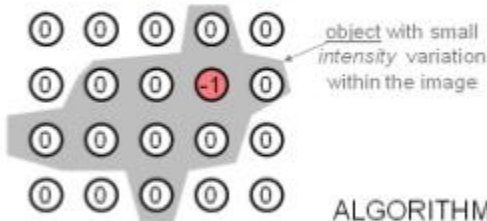
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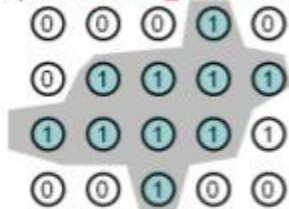
Algorithm Illustration

growRegion: red nodes are the "active_front" (queue or stack)

add seed into active_front

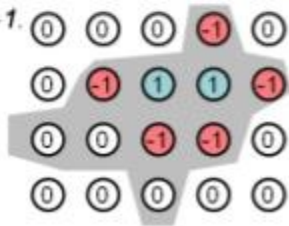
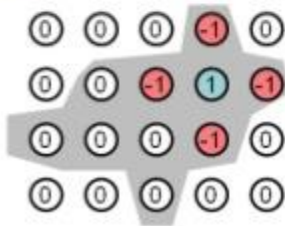


stop when active_front is empty



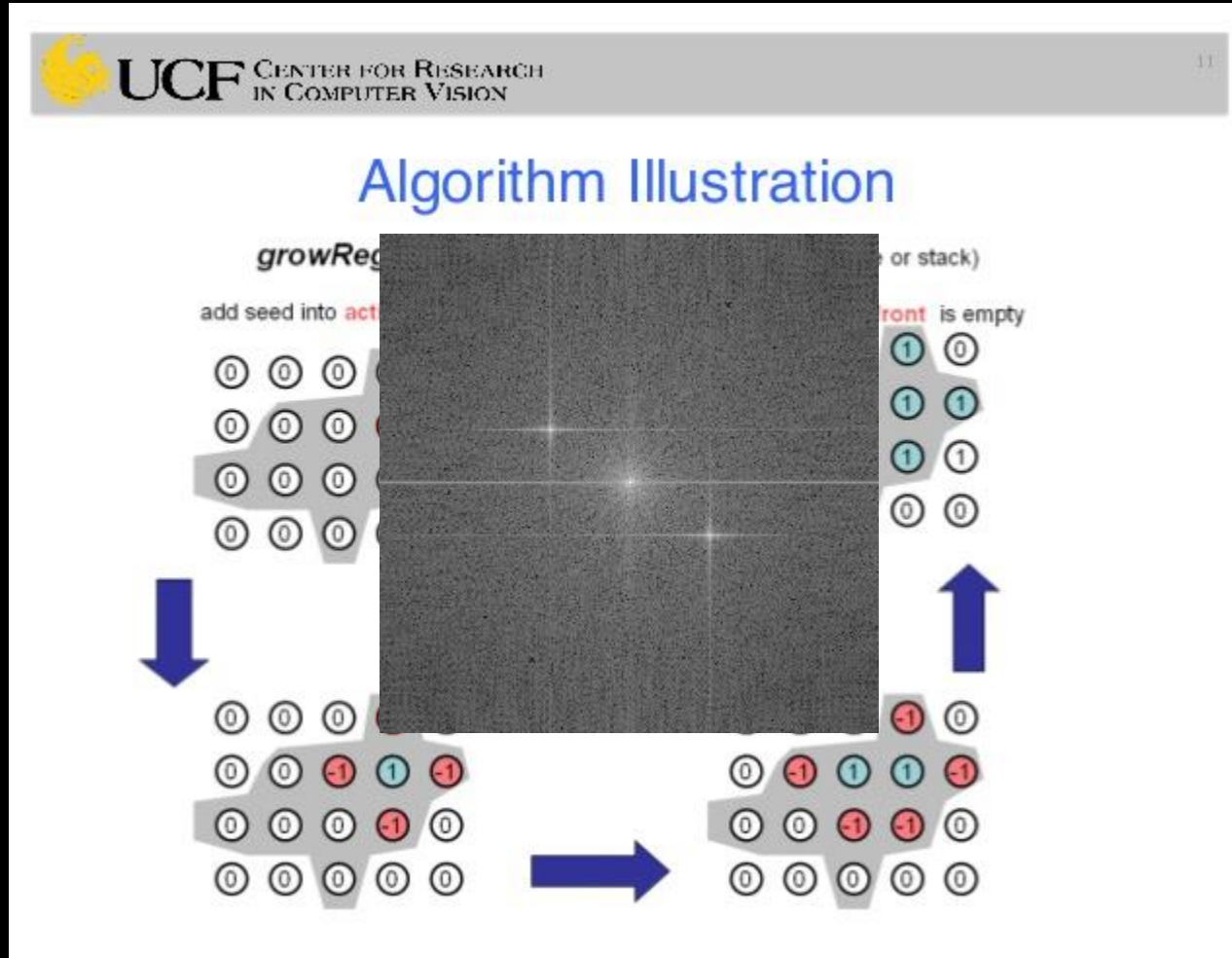
ALGORITHM:

Remove pixel p from active_front
and mark it as $region[p] = 1$.
Add all neighbors q such that:
 $region[q] = 0, |I_p - I_q| < T$
and set $region[q] = -1$.



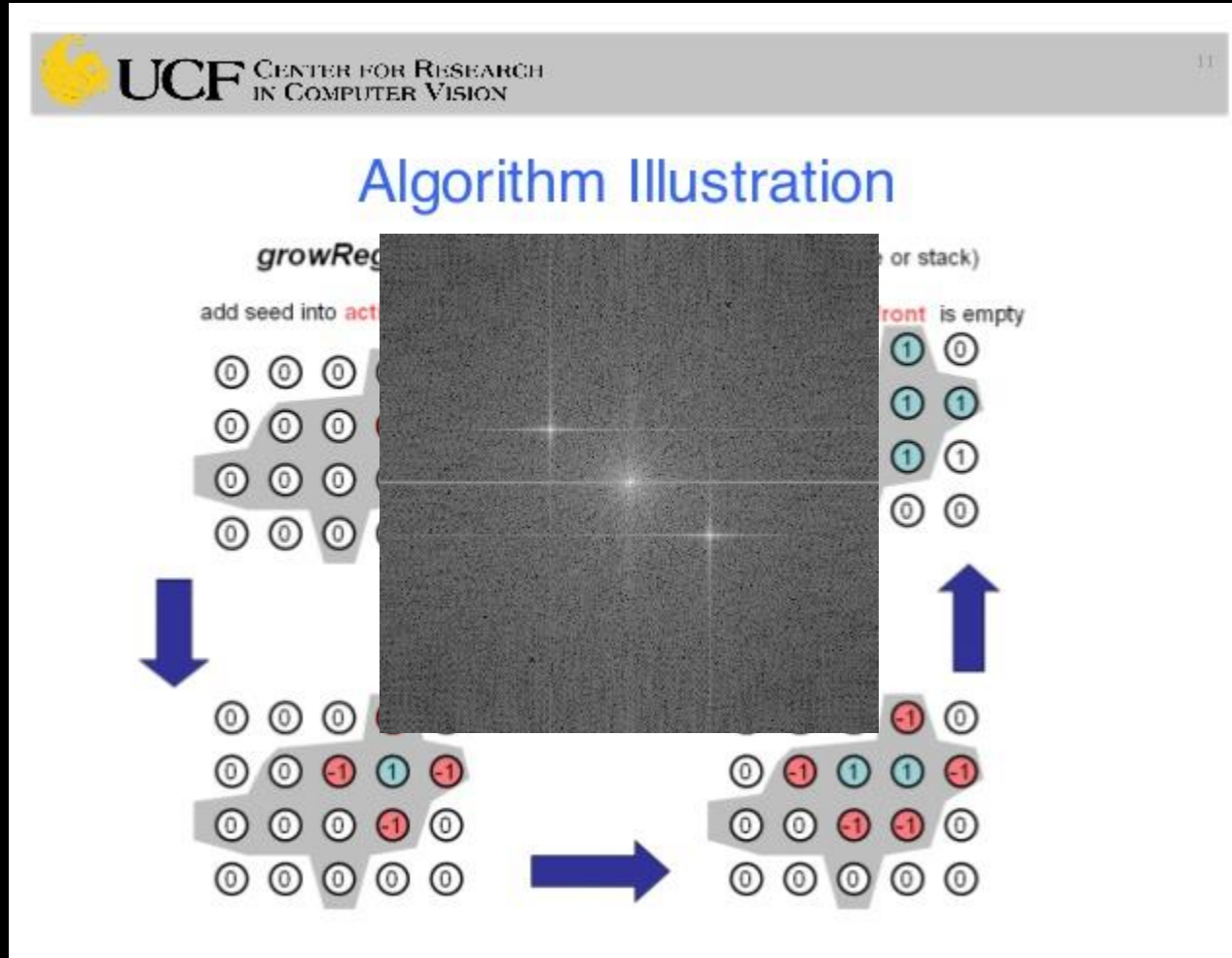
```
for every peak points
while active points exist
  if valid
    mark it as blue zone (1)
    mark all inactive neighbors as active (-1)
  else
    mark it as inactive (0)
```

주파수 영역 분석 - Region Growing



```
for every peak points
while active points exist
  if valid
    mark it as blue zone (1)
    mark all inactive neighbors as active (-1)
  else
    mark it as inactive (0)
```


주파수 영역 분석 - Region Growing



peak points 찾기

low freq peak 제거하기

```
for every peak points
while active points exist
if valid
mark it as blue zone (1)
mark all inactive neighbors as active (-1)
else
mark it as inactive (0)
```

Region Growing

```
// find peaks
peak_points.resize(nNoisyPeaks + 1);
vector<double> max(nNoisyPeaks + 1, -9999);
for (int i = 0; i < H; i++)
    for (int j = 0; j < W; j++)
        for (int k = 0; k < nNoisyPeaks + 1; k++) {
            if (mag[i][j] > max[k]) {
                max.insert(max.begin()+k, mag[i][j]);
                peak_points.insert(peak_points.begin()+k, {j, i});
                max.pop_back();
                peak_points.pop_back();
                break;
            }
        }

// remove low freq area
for (int k = 0; k < nNoisyPeaks + 1; k++)
    if (abs(peak_points[k].x - 128) < 3 && abs(peak_points[k].y - 128) < 3) {
        peak_points.erase(peak_points.begin() + k);
        break;
    }

int xx, yy;
for (const point &pp : peak_points) {

    double peak = mag[pp.y][pp.x];

    queue<point> active_points;
    active_points.push(pp);

    int **states = new int*[H];
    for (int i = 0; i < H; i++) {
        states[i] = new int[W];
        for (int j = 0; j < W; j++) states[i][j] = 0;
    }
    states[pp.y][pp.x] = -1; // active
```

```
    point ap;
    int state;

    while (!active_points.empty()) {
        ap = active_points.front();

        if (mag[ap.y][ap.x] / peak > tau) { // valid?
            states[ap.y][ap.x] = 1; // blue zone
            for (int ii = -1; ii < 2; ii++) for (int jj = -1; jj < 2; jj++) {
                xx = ap.x + jj;
                yy = ap.y + ii;
                if (xx >= W || xx < 0 || yy >= H || yy < 0) continue;
                if (states[yy][xx] == 0) { // non-blue & inactive
                    active_points.push({xx, yy});
                    states[yy][xx] = -1;
                }
            }
        } else states[ap.y][ap.x] = 0; // mark it as inactive (0)

        active_points.pop();
    }

    for (int i = 0; i < H; i++) for (int j = 0; j < W; j++)
        if (states[i][j] == 1)
            noisy_region.push_back({j, i});

    for (int i = 0; i < H; i++) delete[] states[i];
    delete[] states;
}
```

Region Growing

```
// find peaks
peak_points.resize(nNoisyPeaks + 1);
vector<double> max(nNoisyPeaks + 1, -9999);
for (int i = 0; i < H; i++)
    for (int j = 0; j < W; j++)
        for (int k = 0; k < nNoisyPeaks + 1; k++) {
            if (mag[i][j] > max[k]) {
                max.insert(max.begin()+k, mag[i][j]);
                peak_points.insert(peak_points.begin()+k, {j, i});
                max.pop_back();
                peak_points.pop_back();
                break;
            }
        }

// remove low freq area
for (int k = 0; k < nNoisyPeaks + 1; k++)
    if (abs(peak_points[k].x - 128) < 3 && abs(peak_points[k].y - 128) < 3) {
        peak_points.erase(peak_points.begin() + k);
        break;
    }

int xx, yy;
for (const point &pp : peak_points) {

    double peak = mag[pp.y][pp.x];

    queue<point> active_points;
    active_points.push(pp);

    int **states = new int*[H];
    for (int i = 0; i < H; i++) {
        states[i] = new int[W];
        for (int j = 0; j < W; j++) states[i][j] = 0;
    }
    states[pp.y][pp.x] = -1; // active
}
```

```
point ap;
int state;

while (!active_points.empty()) {
    ap = active_points.front();

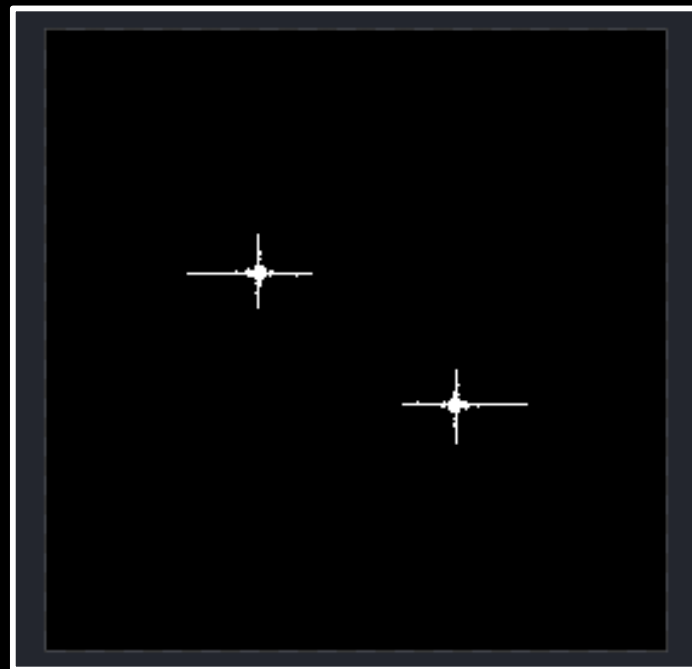
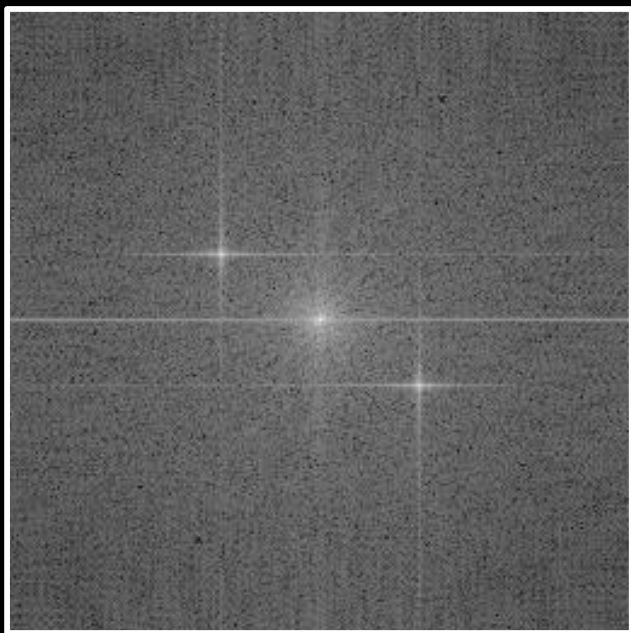
    if (mag[ap.y][ap.x] / peak > tau) { // valid?
        states[ap.y][ap.x] = 1; // blue zone
        for (int ii = -1; ii < 2; ii++) for (int jj = -1; jj < 2; jj++) {
            xx = ap.x + jj;
            yy = ap.y + ii;
            if (xx >= W || xx < 0 || yy >= H || yy < 0) continue;
            if (states[yy][xx] == 0) { // non-blue & inactive
                active_points.push({xx, yy});
                states[yy][xx] = -1;
            }
        }
        else states[ap.y][ap.x] = 0; // mark it as inactive (0)

        active_points.pop();
    }

    for (int i = 0; i < H; i++) for (int j = 0; j < W; j++)
        if (states[i][j] == 1)
            noisy_region.push_back({j, i});

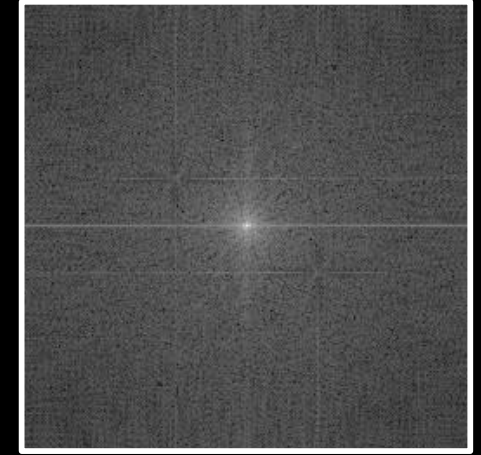
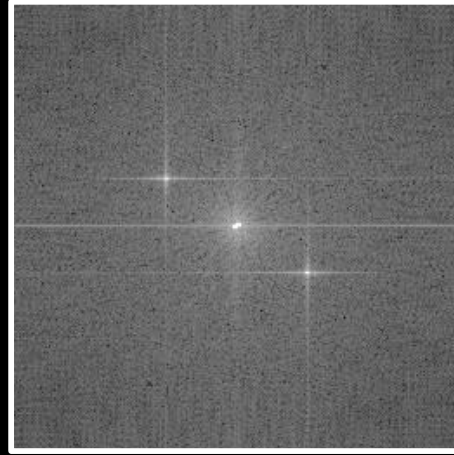
    for (int i = 0; i < H; i++) delete[] states[i];
    delete[] states;
}
```

Region Growing

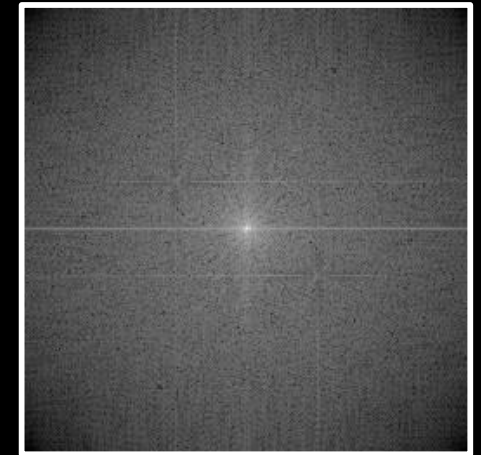
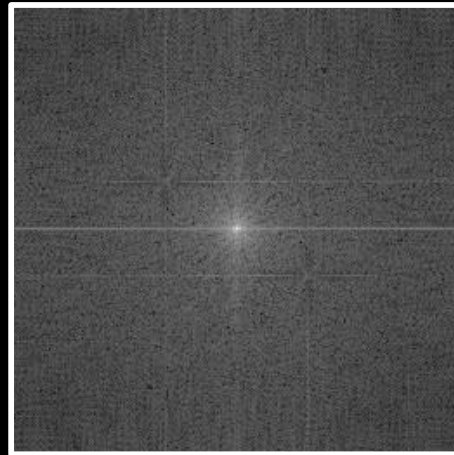


주파수 영역 필터링

1. Kernel Function



2. 구형 Low Pass Filter



Kernel Function

$$GaSLF(u, v) = \begin{cases} \max(H_1, H_2) & \text{if } (H_1 \neq 0 \text{ and } H_2 \neq 0) \\ H_1 + H_2 & \text{else} \end{cases}$$

$$H_1(u, v) = \sum_{n=1}^N e^{-\frac{D_{1n}^2(u, v)}{2}}$$

$$H_2(u, v) = \sum_{n=1}^N e^{-\frac{D_{2n}^2(u, v)}{2}}$$

$$D_{1n}(u, v) = \left(\frac{u - u_n}{a_n} \right)^2 + \left(\frac{v - v_n}{b_n} \right)^2$$

$$D_{2n}(u, v) = \left(\frac{u - u_n}{b_n} \right)^2 + \left(\frac{v - v_n}{a_n} \right)^2$$

$$GaSF(u, v) = 1 - GaSLF(u, v)$$

Kernel Function

$$GaSLF(u, v) = \begin{cases} \max(H_1, H_2) & \text{if } (H_1 \neq 0 \text{ and } H_2 \neq 0) \\ H_1 + H_2 & \text{else} \end{cases}$$

$$GaSF(u, v) = 1 - GaSLF(u, v)$$

$$H_1(u, v) = \sum_{n=1}^N e^{-\frac{D_{1n}^2(u, v)}{2}}$$

$$H_2(u, v) = \sum_{n=1}^N e^{-\frac{D_{2n}^2(u, v)}{2}}$$

$$D_{1n}(u, v) = \left(\frac{u - u_n}{a_n} \right)^2 + \left(\frac{v - v_n}{b_n} \right)^2$$

$$D_{2n}(u, v) = \left(\frac{u - u_n}{b_n} \right)^2 + \left(\frac{v - v_n}{a_n} \right)^2$$

```
int u, v;  
for (v = 0; v < H; v++) for (u = 0; u < W; u++) GaSF[u][v] = 1;  
for (const point &p : noisy_region) {  
    u = p.x; v = p.y;  
    H1 = H2 = 0;  
    for (int n = 0; n < 2; n++) {  
        D1n = sqrt(  
            pow((u - peak_points[n].x) / a[n], 2)  
            + pow((v - peak_points[n].y) / b[n], 2)  
        );  
        D2n = sqrt(  
            pow((u - peak_points[n].x) / b[n], 2)  
            + pow((v - peak_points[n].y) / a[n], 2)  
        );  
        H1 += exp(-pow(D1n, 2)/2.);  
        H2 += exp(-pow(D2n, 2)/2.);  
    }  
    H1 = ((H1 < tol) ? 0 : H1);  
    H2 = ((H2 < tol) ? 0 : H2);  
    GaSLF = ((H1 > tol && H2 > tol) ? max(H1, H2) : H1 + H2);  
    GaSF[v][u] = 1 - GaSLF;  
}
```

Kernel Function

$$GaSLF(u, v) = \begin{cases} \max(H_1, H_2) & \text{if } (H_1 \neq 0 \text{ and } H_2 \neq 0) \\ H_1 + H_2 & \text{else} \end{cases}$$

$$GaSF(u, v) = 1 - GaSLF(u, v)$$

$$H_1(u, v) = \sum_{n=1}^N e^{-\frac{D_{1n}^2(u, v)}{2}}$$

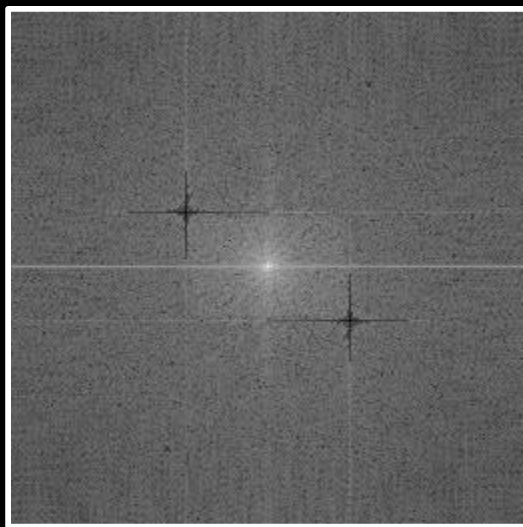
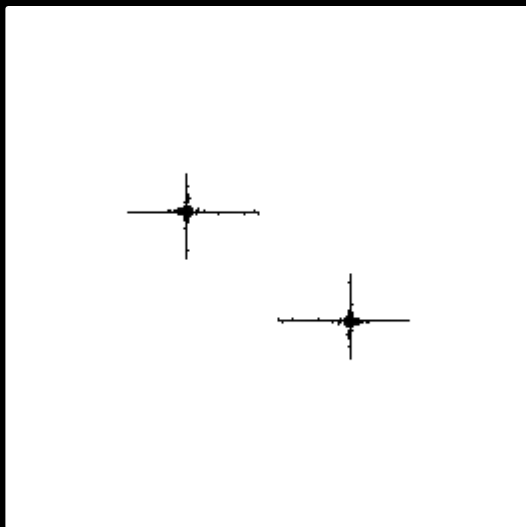
$$H_2(u, v) = \sum_{n=1}^N e^{-\frac{D_{2n}^2(u, v)}{2}}$$

$$D_{1n}(u, v) = \left(\frac{u - u_n}{a_n} \right)^2 + \left(\frac{v - v_n}{b_n} \right)^2$$

$$D_{2n}(u, v) = \left(\frac{u - u_n}{b_n} \right)^2 + \left(\frac{v - v_n}{a_n} \right)^2$$

```
int u, v;  
for (v = 0; v < H; v++) for (u = 0; u < W; u++) GaSF[u][v] = 1;  
for (const point &p : noisy_region) {  
    u = p.x; v = p.y;  
    H1 = H2 = 0;  
    for (int n = 0; n < 2; n++) {  
        D1n = sqrt(  
            pow((u - peak_points[n].x) / a[n], 2)  
            + pow((v - peak_points[n].y) / b[n], 2)  
        );  
        D2n = sqrt(  
            pow((u - peak_points[n].x) / b[n], 2)  
            + pow((v - peak_points[n].y) / a[n], 2)  
        );  
        H1 += exp(-pow(D1n, 2)/2.);  
        H2 += exp(-pow(D2n, 2)/2.);  
    }  
    H1 = ((H1 < tol) ? 0 : H1);  
    H2 = ((H2 < tol) ? 0 : H2);  
    GaSLF = ((H1 > tol && H2 > tol) ? max(H1, H2) : H1 + H2);  
    GaSF[v][u] = 1 - GaSLF;  
}
```

Kernel Function



구형 LPF

$$\tilde{F}(u, v) = \left(1 - \frac{(u - 128)^2 + (v - 128)^2}{2 \times 128^2} \right) F(u, v)$$

이미지의 중심(128, 128)에서 최대값

중심->바깥 : mag감소 (LPF)

성능 향상을 위해 2번 실행

```
for (int v = 0; v < H; v++) for (int u = 0; u < W; u++)  
{  
    Yr[v][u] *= GaSF[v][u];  
    Yi[v][u] *= GaSF[v][u]; //심자노이즈 제거  
  
    Yr[v][u] *= ((32768 - pow(v - 128, 2) - pow(u - 128, 2)) / 32768.);  
    Yi[v][u] *= ((32768 - pow(v - 128, 2) - pow(u - 128, 2)) / 32768.);  
  
    Yr[v][u] *= ((32768 - pow(v - 128, 2) - pow(u - 128, 2)) / 32768.);  
    Yi[v][u] *= ((32768 - pow(v - 128, 2) - pow(u - 128, 2)) / 32768.);  
}
```


IFFT

$$f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi \left(\frac{ux}{M} + \frac{vy}{N} \right)}$$

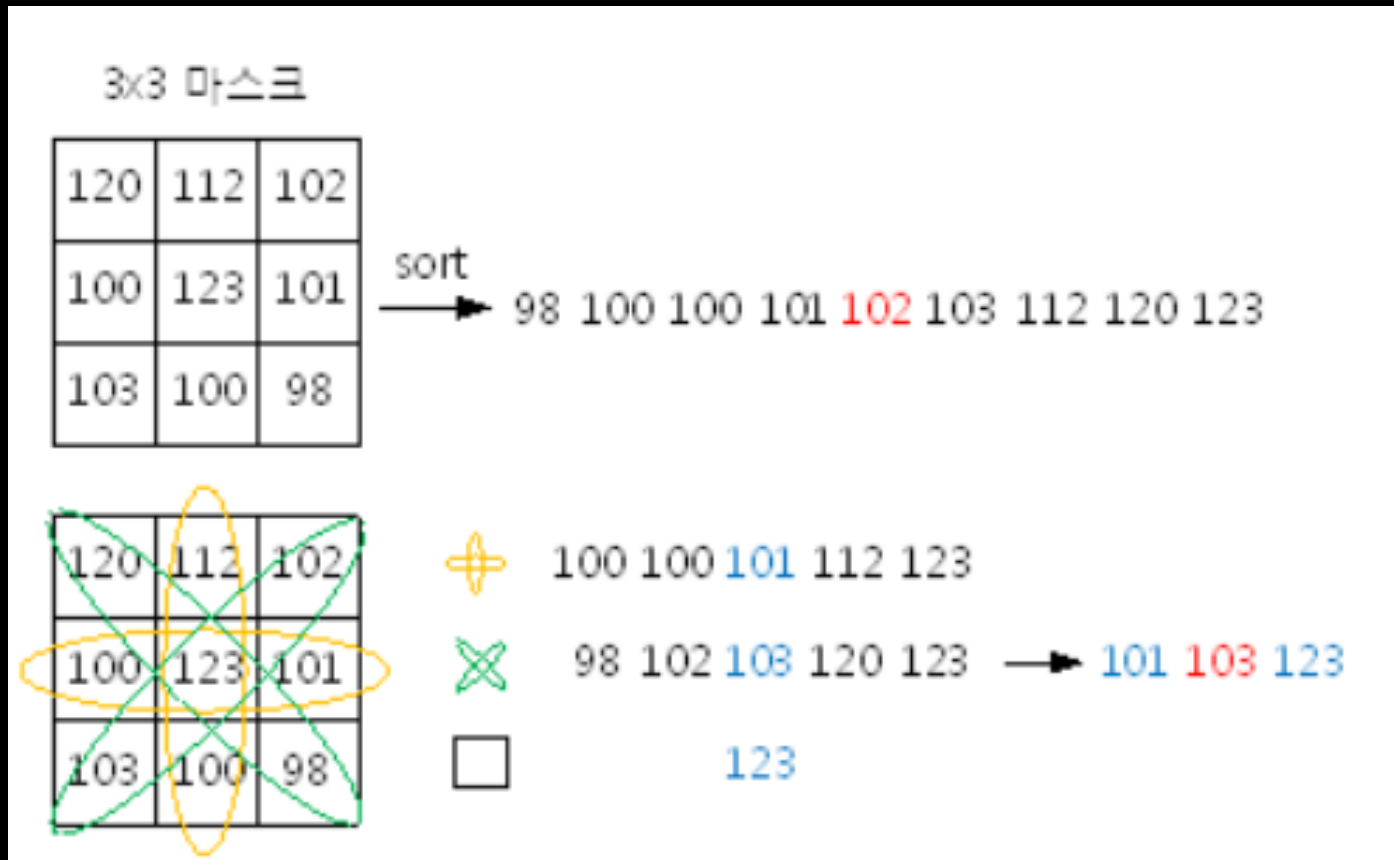
Hybrid Median Filter

1. 마스크 내의
전체 픽셀에 대한
중간값

2. [1,3,4,5,7]에 위치한
픽셀에 대한
중간값

3. [0,2,4,6,8]에 위치한
픽셀에 대한
중간값

4. 구한 3개의 값들의
중간값을 [4]에 입력



Hybrid Median Filter

```
temp3[0] = mid1[4];
mid2[0] = before[row - 1][col]; //1
mid2[1] = before[row][col - 1]; //3
mid2[2] = before[row][col]; //4
mid2[3] = before[row][col + 1]; //5
mid2[4] = before[row + 1][col]; //7
for (i = 1; i < 6; i++) {
    tmp = mid2[i];
    j = i - 1;
    while (j >= 0 && mid2[j] > tmp) {
        mid2[j + 1] = mid2[j];
        j = j - 1;
    }
    mid2[j + 1] = tmp;
}
temp3[1] = mid2[2];
mid3[0] = before[row - 1][col - 1]; //0
mid3[1] = before[row - 1][col + 1]; //2
mid3[2] = before[row][col]; //4
mid3[3] = before[row + 1][col - 1]; //6
mid3[4] = before[row + 1][col + 1]; //8
for (i = 1; i < 5; i++) {
    tmp = mid3[i];
    j = i - 1;
    while (j >= 0 && mid3[j] > tmp) {
        mid3[j + 1] = mid3[j];
        j = j - 1;
    }
    mid3[j + 1] = tmp;
}
temp3[2] = mid3[2];
for (i = 1; i < 3; i++) {
    tmp = temp3[i];
    j = i - 1;
    while (j >= 0 && temp3[j] > tmp) {
        temp3[j + 1] = temp3[j];
        j = j - 1;
    }
    temp3[j + 1] = tmp;
}
after[row][col] = temp3[1];
```

3종류의 알고리즘으로

얻은 3개의 값들 중의

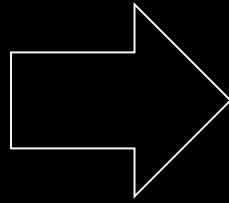
중간값을 고르는 함수 구현

기댓값)

salt & pepper noise와 유사한

노이즈 제거

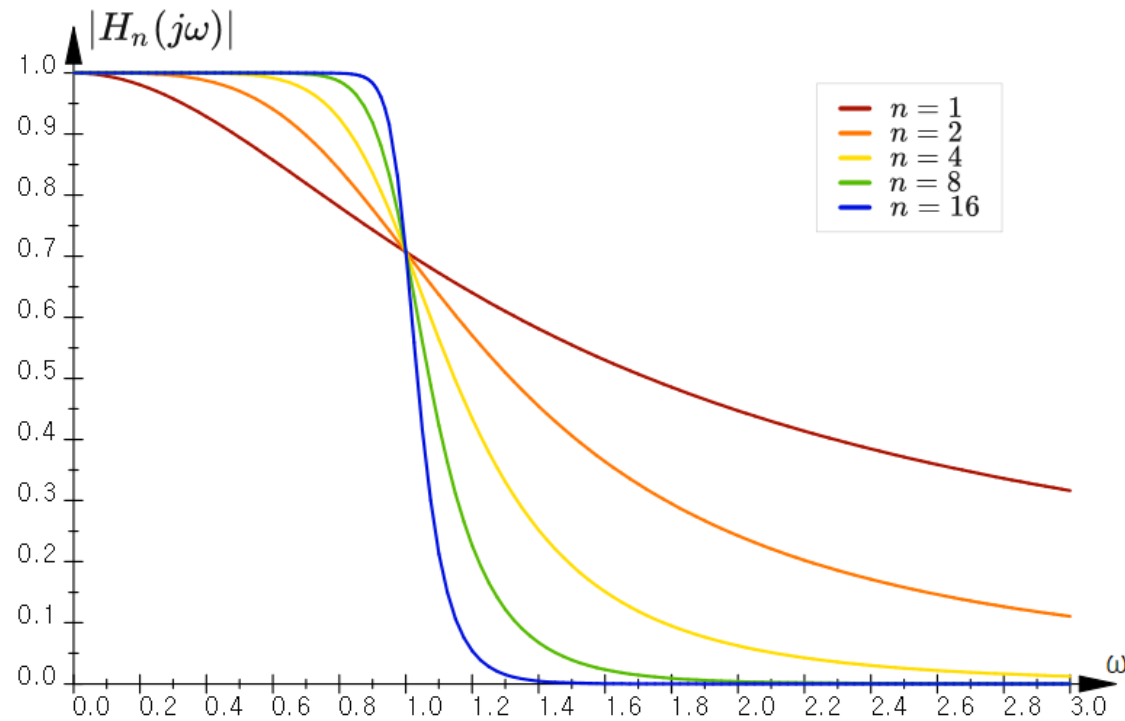
Hybrid Median Filter



Noise 뿐만 아니라 edge 또한 손상

Butter worth filter

$$|H_n(j\omega)| \triangleq \frac{1}{\sqrt{1 + \omega^{2n}}}$$



Butter worth filter

```
double* ComputeLP(int FilterOrder)
{
    double* NumCoeffs;
    int m;
    int i;

    NumCoeffs = (double*)calloc(FilterOrder + 1, sizeof(double));
    if (NumCoeffs == NULL) return(NULL);

    NumCoeffs[0] = 1;
    NumCoeffs[1] = FilterOrder;
    m = FilterOrder / 2;
    for (i = 2; i <= m; ++i)
    {
        NumCoeffs[i] = (double)(FilterOrder - i + 1) * NumCoeffs[i - 1] / i;
        NumCoeffs[FilterOrder - i] = NumCoeffs[i];
    }
    NumCoeffs[FilterOrder - 1] = FilterOrder;
    NumCoeffs[FilterOrder] = 1;

    return NumCoeffs;
}

double* ComputeHP(int FilterOrder)
{
    double* NumCoeffs;
    int i;

    NumCoeffs = ComputeLP(FilterOrder);
    if (NumCoeffs == NULL) return(NULL);

    for (i = 0; i <= FilterOrder; ++i)
        if (i % 2) NumCoeffs[i] = -NumCoeffs[i];

    return NumCoeffs;
}
```

```
double* TrinomialMultiply(int FilterOrder, double* b, double* c)
{
    int i, j;
    double* RetVal;

    RetVal = (double*)calloc(4 * FilterOrder, sizeof(double));
    if (RetVal == NULL) return(NULL);

    RetVal[2] = c[0];
    RetVal[3] = c[1];
    RetVal[0] = b[0];
    RetVal[1] = b[1];

    for (i = 1; i < FilterOrder; ++i)
    {
        RetVal[2 * (2 * i + 1)] += c[2 * i] * RetVal[2 * (2 * i - 1)] - c[2 * i + 1] * RetVal[2 * (2 * i - 1) + 1];
        RetVal[2 * (2 * i + 1) + 1] += c[2 * i] * RetVal[2 * (2 * i - 1) + 1] + c[2 * i + 1] * RetVal[2 * (2 * i - 1)];

        for (j = 2 * i; j > 1; --j)
        {
            RetVal[2 * j] += b[2 * i] * RetVal[2 * (j - 1)] - b[2 * i + 1] * RetVal[2 * (j - 1) + 1] +
                c[2 * i] * RetVal[2 * (j - 2)] - c[2 * i + 1] * RetVal[2 * (j - 2) + 1];
            RetVal[2 * j + 1] += b[2 * i] * RetVal[2 * (j - 1) + 1] + b[2 * i + 1] * RetVal[2 * (j - 1)] +
                c[2 * i] * RetVal[2 * (j - 2) + 1] + c[2 * i + 1] * RetVal[2 * (j - 2)];
        }

        RetVal[2] += b[2 * i] * RetVal[0] - b[2 * i + 1] * RetVal[1] + c[2 * i];
        RetVal[3] += b[2 * i] * RetVal[1] + b[2 * i + 1] * RetVal[0] + c[2 * i + 1];
        RetVal[0] += b[2 * i];
        RetVal[1] += b[2 * i + 1];
    }

    return RetVal;
}
```

Butter worth filter

```
double* ComputeNumCoeffs(int FilterOrder)
{
    double* TCoeffs;
    double* NumCoeffs;
    int i;

    NumCoeffs = (double*)calloc(2 * FilterOrder + 1, sizeof(double));
    if (NumCoeffs == NULL) return(NULL);

    TCoeffs = ComputeHP(FilterOrder);
    if (TCoeffs == NULL) return(NULL);

    for (i = 0; i < FilterOrder; ++i)
    {
        NumCoeffs[2 * i] = TCoeffs[i];
        NumCoeffs[2 * i + 1] = 0.0;
    }
    NumCoeffs[2 * FilterOrder] = TCoeffs[FilterOrder];

    free(TCoeffs);

    return NumCoeffs;
}
```

```
double* ComputeDenCoeffs(int FilterOrder, double Lcutoff, double Ucutoff)
{
    int k; // loop variables
    double theta; // PI * (Ucutoff - Lcutoff) / 2.0
    double cp; // cosine of phi
    double st; // sine of theta
    double ct; // cosine of theta
    double s2t; // sine of 2*theta
    double c2t; // cosine of 2*theta
    double* RCoeffs; // z^-2 coefficients
    double* TCoeffs; // z^-1 coefficients
    double* DenomCoeffs; // dk coefficients
    double PoleAngle; // pole angle
    double SinPoleAngle; // sine of pole angle
    double CosPoleAngle; // cosine of pole angle
    double a; // workspace variables

    cp = cos(PI * (Ucutoff + Lcutoff) / 2.0);
    theta = PI * (Ucutoff - Lcutoff) / 2.0;
    st = sin(theta);
    ct = cos(theta);
    s2t = 2.0 * st * ct; // sine of 2*theta
    c2t = 2.0 * ct * ct - 1.0; // cosine of 2*theta

    RCoeffs = (double*)calloc(2 * FilterOrder, sizeof(double));
    TCoeffs = (double*)calloc(2 * FilterOrder, sizeof(double));

    for (k = 0; k < FilterOrder; ++k) {
        PoleAngle = PI * (double)(2 * k + 1) / (double)(2 * FilterOrder);
        SinPoleAngle = sin(PoleAngle);
        CosPoleAngle = cos(PoleAngle);
        a = 1.0 + s2t * SinPoleAngle;
        RCoeffs[2 * k] = c2t / a;
        RCoeffs[2 * k + 1] = s2t * CosPoleAngle / a;
        TCoeffs[2 * k] = -2.0 * cp * (ct + st * SinPoleAngle) / a;
        TCoeffs[2 * k + 1] = -2.0 * cp * st * CosPoleAngle / a;
    }

    DenomCoeffs = TrinomialMultiply(FilterOrder, TCoeffs, RCoeffs);
    free(TCoeffs);
    free(RCoeffs);

    DenomCoeffs[1] = DenomCoeffs[0];
    DenomCoeffs[0] = 1.0;
    for (k = 3; k <= 2 * FilterOrder; ++k)
        DenomCoeffs[k] = DenomCoeffs[2 * k - 2];

    return DenomCoeffs;
}
```


Butter worth filter

```
void filter(int ord, double* a, double* b, int np, double* x, double* y)
{
    int i, j;
    y[0] = b[0] * x[0];
    for (i = 1; i < ord + 1; i++)
    {
        y[i] = 0.0;
        for (j = 0; j < i + 1; j++)
            y[i] = y[i] + b[j] * x[i - j];
        for (j = 0; j < i; j++)
            y[i] = y[i] - a[j + 1] * y[i - j - 1];
    }
    for (i = ord + 1; i < np + 1; i++)
    {
        y[i] = 0.0;
        for (j = 0; j < ord + 1; j++)
            y[i] = y[i] + b[j] * x[i - j];
        for (j = 0; j < ord; j++)
            y[i] = y[i] - a[j + 1] * y[i - j - 1];
    }
}
```



이미지파일 생성

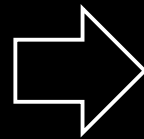
```
void writedata(ofstream &out, double **data, string basepath, string filepath, int W, int H, bool logscale, char *header, int header_size) {  
  
    double **output = new double*[H];  
    for (int i = 0; i < H; i++) {  
        output[i] = new double[W];  
        for (int j = 0; j < W; j++) output[i][j] = data[i][j];  
    }  
  
    if (logscale)  
        for (int i = 0; i < H; i++) for (int j = 0; j < W; j++)  
            output[i][j] = log(1. + output[i][j]);  
  
    if (filepath.substr(filepath.size() - 3) == "csv") {  
        out.open(basepath + filepath);  
        for (int i = 0; i < H; i++) for (int j = 0; j < W; j++)  
            out << output[i][j] << ((j==W-1) ? "\n" : ",");  
    }  
  
    else { // bmp  
        out.open(basepath + filepath, ios::binary);  
        uchar *row = new uchar[3*W];  
        uchar **normalized = new uchar*[H];  
        for (int i = 0; i < H; i++) normalized[i] = new uchar[W];  
        DNormalize2D(output, normalized, W, H);  
        out.write((char*)header, header_size);  
        for (int i = 0; i < H; i++) {  
            for (int j = 0; j < W; j++)  
                row[3*j] = row[3*j+1] = row[3*j+2] = normalized[i][j];  
            out.write((char*)row, 3*W);  
        }  
  
        for (int i = 0; i < H; i++) delete[] normalized[i];  
        delete[] row, normalized;  
    }  
  
    out.close();  
    for (int i = 0; i < H; i++) delete[] output[i];  
    delete[] output;  
}
```

3. 결과 분석

1. 노이즈 이미지 vs 복원 이미지



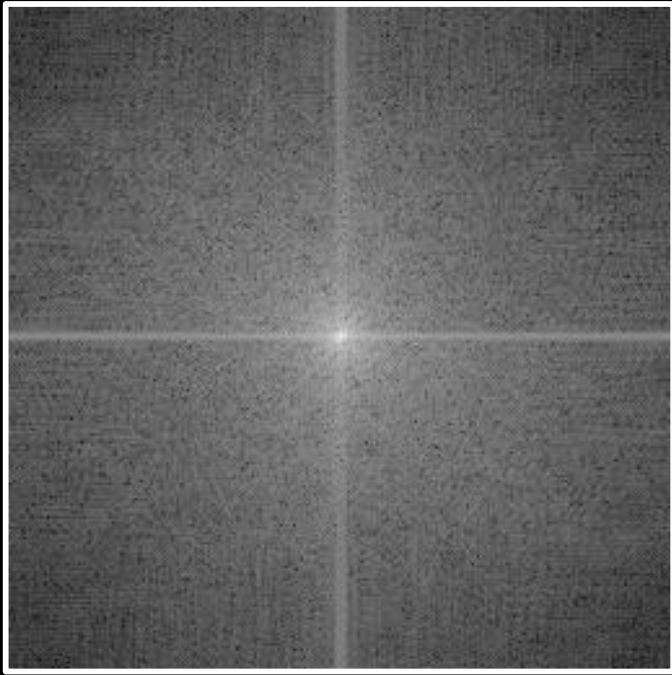
노이즈 이미지



복원 이미지

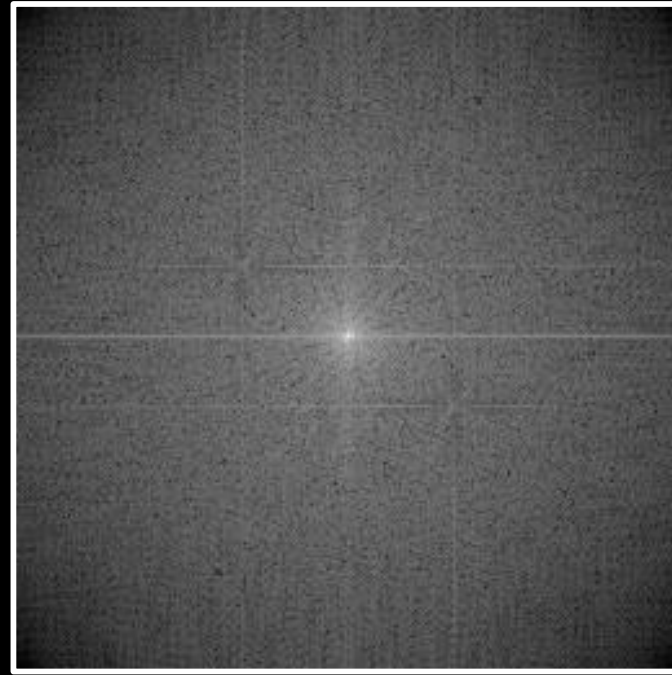
3. 결과 분석

2. 원본 스펙트럼 vs 복원 스펙트럼



원본 스펙트럼

VS



복원 스펙트럼

3. 결과 분석

3. 원본 이미지 vs 복원 이미지



원본 이미지

VS



복원 이미지

4. 결론

5. Q&A

무엇이든
물어보세요~~~~~

