

2. a. $63 = 2^6 - 1 \Rightarrow$ integer 6 bit

Sign \Rightarrow 1 bit (MSB)

0. --- \Rightarrow decimal 3 bit

1 bit for preventing overflow
+ (carry, bit extension)

11 bit \Rightarrow // of full adders
are required

ex)

$$\begin{array}{r} 011111 \quad 6\text{bit} + 1\text{bit} \\ + 011111 \quad 6\text{bit} + 1\text{bit} \\ \hline 0100000 \quad 16\text{bit} + 1\text{bit} \\ \text{sign} \end{array}$$

b.

To calculate to n th decimal point $\Rightarrow x2^n \div 2^n$

$$(23.76)_{10} = 23 + 0.76 = 23 + 6.08 \div 2^3 = 23 + 6 \div 2^3 \\ = (00010111.110)_2 \quad (\because 11\text{bit})$$

$$(34.872)_{10} = 34 + 0.872 = 34 + 6.972 \div 2^3 = 34 + 7 \div 2^3 \\ = (00100010.111)_2 \quad \downarrow 2\text{'s complement}$$

$$(-34.872)_{10} = (11011101.001)_2$$

$$(31.25)_{10} = 31 + 0.25 = 31 + 2 \div 2^3 \\ = (10001111.010)_2 \quad \downarrow 2\text{'s complement}$$

$$(-31.25)_{10} = (11100000.110)_2$$

$$(56.125)_{10} = 56 + 0.125 = 56 + 1 \div 2^3 \\ = (00111000.001)_2$$

$$\therefore 23.76 - 34.872 = -11.112$$

$$\therefore -31.25 - (-56.125) \\ = -31.25 + 56.125 = 24.875$$

11 of full adders (11 bit)

$$\begin{array}{r} 00010111.110 \quad (23.76)_{10} \\ + 11011101.001 \quad (-34.872)_{10} \\ \hline (11110100.111)_2 \quad \downarrow 2\text{'s complement} \end{array}$$

$$(00001011.001)_2$$

$$= 11.125$$

$$\therefore (11110100.111)_2 = -11.125$$

It has discrepancy with calculation

in decimal and binary

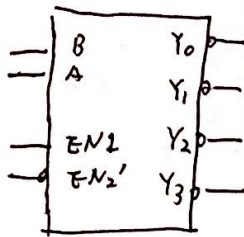
This is because During the conversion from decimal to binary decimal number were

rounded to integers, and 3 bit binary decimal number only can be represented by 0.125 size

$$\begin{array}{r} 11100000.110 \quad (-31.25)_{10} \\ + 00111000.001 \quad (+56.125)_{10} \\ \hline (1)00011000.111 \\ \downarrow \text{discard end carry} \end{array}$$

$$(00011000.111)_2 = (24.875)_{10}$$

2.



2x4 decoder (complement) with EN1, EN2 switches

Decoder works only when $EN1 = 1$, $EN2' = 0$

Truth table (B is MSB)

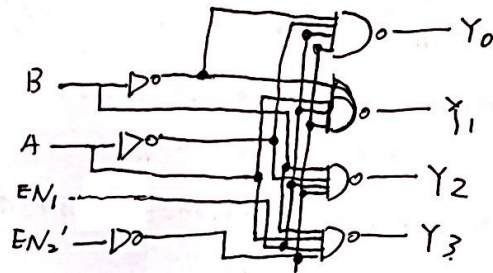
| | EN1 | EN2' | B | A | Y ₀ | Y ₁ | Y ₂ | Y ₃ |
|----------------------|-----|------|---|---|----------------|----------------|----------------|----------------|
| | 0 | x | x | x | 1 | 1 | 1 | 1 |
| | 1 | x | x | x | 1 | 1 | 1 | 1 |
| don't care condition | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |

$$Y_0 = (EN_2 (EN_2')' B' A')'$$

$$Y_1 = (EN_1 (EN_2')' B' A)'$$

$$Y_2 = (EN_1 (EN_2')' B A')'$$

$$Y_3 = (EN_1 (EN_2')' B A)'$$



Verilog

1. Gate-level modeling.

```

module dec_2 (
    output [0:3] Y,
    input A, B, EN1, EN2_prime,
);
    wire A_not, B_not, EN2_prime_not;

```

not

G0 (A_not, A),

G1 (B_not, B);

not (EN2_prime_not, EN2_prime); // optional gate instance name (primitives)

nand

G2 (Y[0], B_not, A_not, EN1, EN2_prime_not),

G3 (Y[1], B_not, A, EN1, EN2_prime_not),

G4 (Y[2], B, A_not, EN1, EN2_prime_not),

G5 (Y[3], B, A, EN1, EN2_prime_not);

end module

2. Data-Flow modelling

```

module dec_2 (
    output [0:3] Y,
    input B, A, EN1, EN2-Prime
);
    assign Y[0] = ~(~B & ~A & EN1 & ~EN2-Prime),
           Y[1] = ~(~B & A & EN1 & ~EN2-Prime),
           Y[2] = ~(B & ~A & EN1 & ~EN2-Prime),
           Y[3] = ~(B & A & EN1 & ~EN2-Prime);
end module
    
```

3. Behavioral modelling

```

module dec_2 (
    output reg [0:3] Y,
    input B, A, EN1, EN2-Prime
);
    always @ (B, A, EN1, EN2-Prime)
        case (EN1, EN2-Prime, B, A)
            4'b1000: Y = 4'b0111;
            4'b1001: Y = 4'b1011;
            4'b1010: Y = 4'b1101;
            4'b1011: Y = 4'b1110;
            default: Y = 4'b1111;
        endcase
end module
    
```

3. $f(x, y, z) = x'y'z + y'z + x'z + xy'z'$

a. input: x, y, z
output: f

b. literal - a variable or its complement
1074

c. product term - one or more literals connected by AND(.)
 $x'y'z, y'z, x'z, xy'z'$
474

d. canonical sop - sum of minterms

* minterm - a product term that includes all the variable either complement or not

Solution 1

$$\begin{aligned}
 & x'y'z + y'z + x'z + xy'z' \\
 &= x'y'z + (x+x')y'z + x'(y+y')z + xy'z' \quad (\because x+x'=1) \\
 &= x'y'z + xy'z + x'y'z + x'y'z + x'y'z + xy'z' \\
 &= x'y'z + xy'z + x'y'z + xy'z' \quad (\because x+x=x)
 \end{aligned}$$

Solution 2

$$\begin{aligned}
 x'y'z &\Rightarrow 001 \Rightarrow 1 \\
 xy'z' &\Rightarrow 100 \Rightarrow 4 \\
 y'z &\Rightarrow \begin{cases} 001 \\ 101 \end{cases} \Rightarrow 1, 5 \\
 x'z &\Rightarrow \begin{cases} 001 \\ 011 \end{cases} \Rightarrow 1, 3
 \end{aligned}$$

$$f = \Sigma(1, 3, 4, 5)$$

$$= x'y'z + x'y'z + xy'z' + xy'z'$$

| x \ yz | 00 | 01 | 11 | 10 |
|--------|----|----|----|----|
| 0 | 0 | 1 | 1* | 0 |
| 1 | 1* | 1 | 0 | 0 |

e. minterm (\because (d) - definition)

$$x'y'z, xy'z, x'y'z, xy'z'$$

g. canonical pos - product of maxterm

$$\begin{aligned}
 f &= \Sigma(1, 3, 4, 5) = \Pi(0, 2, 6, 7) \\
 &= (x+y+z)(x+y'+z)(x'+y'+z)(x'+y'+z')
 \end{aligned}$$

h. Using K-map in (d)

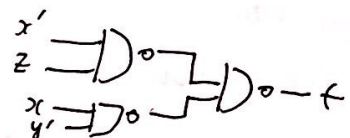
$$\begin{aligned}
 \text{marked } * &\Rightarrow \text{EPI: } x'z, xy' \\
 \text{PI: } &x'z, xy', y'z
 \end{aligned}$$

$$\begin{aligned}
 f(x, y, z) &= \text{EPI} + \text{PI} \\
 &= x'z + xy'
 \end{aligned}$$

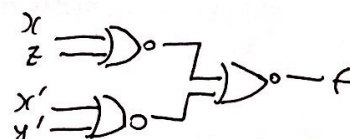
$$\begin{aligned}
 \therefore f &= x'y'z + y'z + x'z + xy'z' \\
 &= x'z(1+y') + xy'(1+z') \\
 &= x'z + xy'
 \end{aligned}$$

i. Following h
EPI: $x'z, xy'$
PI: $x'z, xy', y'z$

$$f = x'z + xy' = ((x'z + xy')')' = ((x'z)'(xy')')'$$



$$\begin{aligned}
 k. f &= (x+z)(x'+y') \quad (\because \text{K-map } o) \\
 &= ((x+z)' + (x'+y')')'
 \end{aligned}$$



4. Latch \rightarrow output changes as input changes (non-clocked memory)
 Flip-Flop \rightarrow output changes as clockedge (clocked memory)
 (shortly after)
 (clock transition)

* 4 kinds of Flip-Flop (RS, D, JK, T), characteristic table, K-map
 excitation table, Verilog (Behavioral modeling) (asynchronous reset)
 (assume Flip-Flop is positive triggered)

RS Flip-Flop

| CLK | RS | Q(t+1) (characteristic table) |
|-----|-----|--|
| 0 | - | Q |
| 1 | - | Q |
| ↑ | 0 0 | Q (no change) |
| ↑ | 0 1 | 1 (set) |
| ↑ | 1 0 | 0 (reset) |
| ↑ | 1 1 | - or? (illegal input) (unpredictable) |

K-map

| | | | | | |
|---|----|----|----|----|----|
| | RS | 00 | 01 | 11 | 10 |
| Q | 0 | 0 | 1 | - | 0 |
| | 1 | 1 | 1 | - | 0 |

$Q(t+1) = S + R'Q$

excitation table

| Q(t) | Q(t+1) | S R | |
|------|--------|-----|--------------|
| 0 | 0 | 0 X | no change 00 |
| 0 | 1 | 1 0 | Reset 01 |
| 1 | 0 | 0 1 | - see 10 |
| 1 | 1 | X 0 | - Reset 01 |
| | | | no change 00 |
| | | | see 10 |

```

Module RSFF (
    output reg Q,
    input S, R, CLK, rst
);

```

```

always @ (posedge CLK, negedge rst)
    if (~rst) Q <= 1'b0; // if (rst == 0)
    else case ({S, R})
        2'b00: Q <= Q;
        2'b01: Q <= 1'b0;
        2'b10: Q <= 1'b1;
        2'b11: Q <= 1'bZ; // High Impedance
    endcase
endmodule

```

or

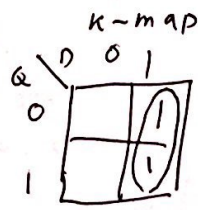
```

always @ (posedge CLK, negedge rst)
    if (~rst) Q <= 1'b0;
    else begin
        if (S != R) begin
            // S=1, R=0 or S=0, R=1
            Q <= S;
        end
        else if (S == 1 && R == 1) begin
            Q <= 1'bZ;
        end
    end
end

```

D-Flip Flop

| CLK | D | Q(t+1) (characteristic table) |
|-----|---|-------------------------------|
| 0 | - | Q |
| 1 | - | Q |
| ↑ | 0 | 0 (Reset) |
| ↑ | 1 | 1 (Set) |



$$Q(t+1) = D$$

Excitation table

| Q(t) | Q(t+1) | D |
|------|--------|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

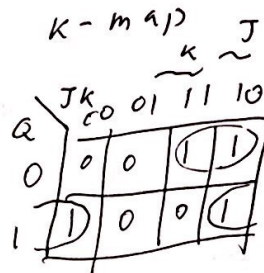
```

module DFF (
    output reg Q,
    input D, CLK, rst
);
always @(posedge CLK, negedge rst)
    if (~rst) Q <= 1'b0;
    else Q <= D;
endmodule
    
```

JK-Flip Flop

characteristic table

| CLK | J | K | Q(t+1) |
|-----|---|---|-----------------|
| 0 | - | - | Q |
| 1 | - | - | Q |
| ↑ | 0 | 0 | Q (no change) |
| ↑ | 0 | 1 | 0 (Reset) |
| ↑ | 1 | 0 | 1 (Set) |
| ↑ | 1 | 1 | Q' (complement) |



$$Q = JQ' + KQ$$

Excitation table

| Q(t) | Q(t+1) | J | K | |
|------|--------|---|---|---------------|
| 0 | 0 | 0 | X | no change 00 |
| 0 | 1 | 1 | X | reset 01 |
| 1 | 0 | X | 1 | set 10 |
| 1 | 1 | X | 0 | complement 11 |

```

module JKFF (
    output reg Q,
    input J, K, CLK, rst
);
    
```

```

always @(posedge CLK, negedge rst)
    if (~rst) Q <= 1'b0;
    else case (J, K)
        2'b00: Q <= Q;
        2'b01: Q <= 1'b0;
        2'b10: Q <= 1'b1;
        2'b11: Q <= ~Q;
    endcase
    
```

endcase

endmodule

Wire JK;
 assign JK = (J & ~Q) | (~K & Q);
 DFF D1(Q, JK, CLK, rst);

T-Flip Flop Characteristic table

| CLK | T | Q(t+1) |
|-----|---|-----------------|
| 0 | - | Q |
| 1 | 0 | Q (no change) |
| 1 | 1 | Q' (complement) |

K-map

| T \ Q | 0 | 1 |
|-------|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

$$Q(t+1) = TQ' + T'Q = T \oplus Q$$

Excitation Table

| Q(t) | Q(t+1) | T |
|------|--------|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

```

module TFF (
    output reg Q;
    input T, CLK, rst;
);
    wire D;
    assign D = T ^ Q;
    DFF D1(Q, D, CLK, rst);
end module.
    
```

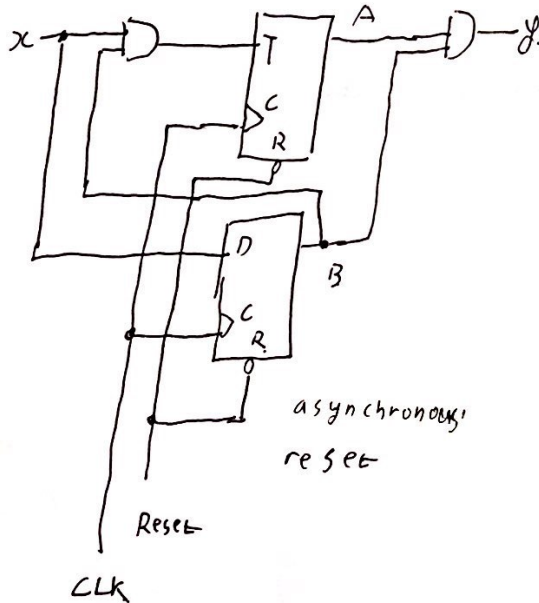
JKFF(Q, T, T, CLK, rst);

```

always @(posedge CLK, negedge rst)
    if (~rst) Q <= 1'b0;
    else begin
        if (~T) Q <= ~Q;
    end
    
```

5. circuit Analysis

circuit diagram → state equations → state table → state diagram
 state equations
 $y = AB$ (combinational machine)
 output is a function of the present state only)



$$\begin{aligned}
 T &= Bx \\
 D &= x \\
 B(t+1) &= x \\
 A(t+1) &= A(t) \oplus T = A(t)(Bx)' + A'(t)Bx \\
 &= A(t)B'(t) + A(t)x' + A'(t)B(t)x.
 \end{aligned}$$

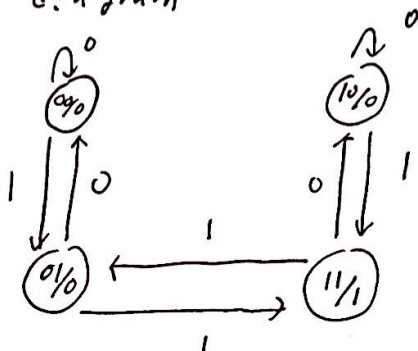
*: $A \oplus B \Rightarrow$ Xor is 1 only when the number of 1 input is odd

$$A \oplus Bx = \begin{cases} A=0, Bx=1 \Rightarrow 011 \\ A=1, Bx=0 \Rightarrow 100 \end{cases}$$

State table

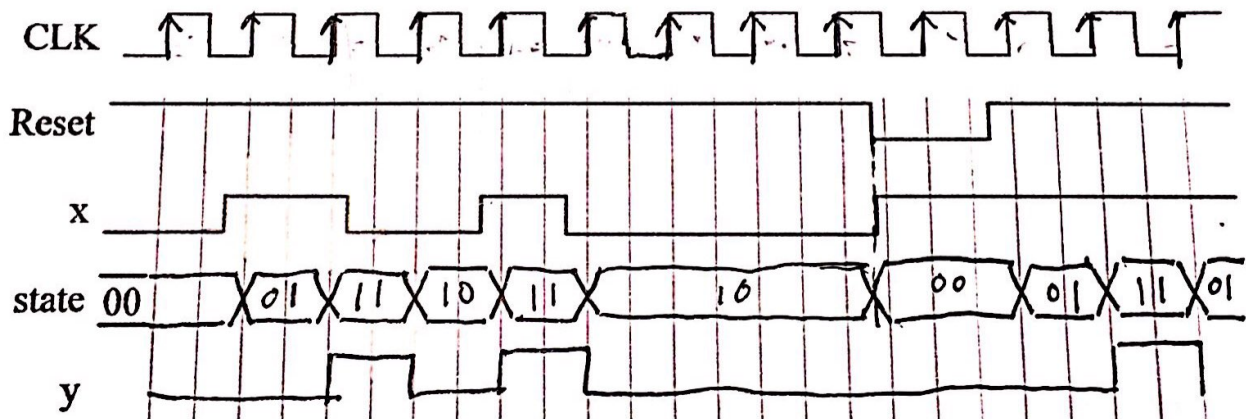
| Present state | | input | next state | | output |
|---------------|------|-------|------------|--------|--------|
| A(t) | B(t) | x | A(t+1) | B(t+1) | y |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 |

State diagram



Positive edge trigger and asynchronous reset

moore model



1. X' verilog moore machine.

```
module moore_1 (  
    output reg y,  
    input      x, clk, rst  
);  
    reg [1:0] state;  
    parameter s0 = 2'b00, s1 = 2'b01, s2 = 2'b10, s3 = 2'b11;  
  
    always @ (posedge clk, negedge rst)  
    if (~rst) state <= s0;  
    else case (state)  
        s0: if (~x) state <= s0; else state <= s1;  
        s1: if (~x) state <= s0; else state <= s3;  
        s2: if (~x) state <= s2; else state <= s3;  
        s3: if (~x) state <= s2; else state <= s1;  
    endcase  
    assign y = (state == s3);  
endmodule
```