$\pi^{-\frac{s}{2}} f(\frac{1}{2}s-1)\zeta(s) = \pi^{-\frac{1}{2}-1/2} f(-\frac{1}{2}s-\frac{1}{2})\zeta(1-s)$

implies

$f(-\frac{3}{4}+ai)\zeta(\frac{1}{2}+2ai) = \pi^{2ai} f(-\frac{3}{4}-ai)\zeta(\frac{1}{2}-2ai)$
(we assume $a \in \mathbb{R}$)

$$\frac{\zeta(\frac{1}{2}+2ai)}{\zeta(\frac{1}{2}-2ai)} = \pi^{2ai}\frac{f(-\frac{3}{4}-ai)}{f(-\frac{3}{4}+ai)}$$

if one of $\zeta(\frac{1}{2}+2ai)$ or $\zeta(\frac{1}{2}-2ai)$ is real,

$$1 = \pi^{2ai}\frac{f(-\frac{3}{4}-ai)}{f(-\frac{3}{4}+ai)}$$

$\Big($note that $\overline{\zeta(\frac{1}{2}+2ai)} = \zeta(\frac{1}{2}-2ai)\Big)$

$$\frac{f(-\frac{3}{4}+ai)}{f(-\frac{3}{4}-ai)} = \pi^{2ai}$$

for $a \in \mathbb{R}$, this implies $\zeta(\frac{1}{2}+2ai) = \zeta(\frac{1}{2}-2ai) \in \mathbb{R}$.
However, $\zeta(\frac{1}{2}+2ai) = \zeta(\frac{1}{2}-2ai) \neq 0$ for these a.