$$\zeta_K(s) = \prod_{p:ramified}\left(1-\frac{1}{p^s}\right)^{-1}\prod_{p:split}\left(1-\frac{1}{p^s}\right)^{-l}\prod_{p:inert}\left(1-\frac{1}{p^{ls}}\right)^{-1} = \zeta(s)\prod_{k=1}^{l-1}L(s,\chi^k)$$

for some Dirichlet character χ with conductor N. Or equivalently,

$$\frac{\zeta_K(s)}{\zeta(s)} = \prod_{p:split}\left(1-\frac{1}{p^s}\right)^{-(l-1)}\prod_{p:inert}\frac{\left(1-\frac{1}{p^{ls}}\right)^{-1}}{\left(1-\frac{1}{p^s}\right)^{-1}} = \prod_{k=1}^{l-1}L(s,\chi^k)$$

if $l=2$,

$$\prod_{p:split}\left(1-\frac{1}{p^s}\right)\prod_{p:inert}\left(1+\frac{1}{p^s}\right) = L(s)$$

Let

$$f_s(s) = \prod_{p:split}\left(1-\frac{1}{p^s}\right), \qquad f_i(s) = \prod_{p:inert}\left(1+\frac{1}{p^s}\right)$$

which means

$$f_s(s)f_i(s) = L(s)$$

Now,

$$A(s) = \prod_{p\equiv 1}\left(1+\frac{2}{p^2}\right)$$

has simple pole at $s=1$?

$$\left(1+\frac{2}{p^s}-\frac{2}{p^{3s}}-\frac{1}{p^{4s}}\right) < \left(1+\frac{2}{p^s}\right) < \left(1+\frac{2}{p^s}+\frac{2}{p^{2s}}\right)$$

equivalently,

$$\left(1+\frac{1}{p^s}\right)^3\left(1-\frac{1}{p^s}\right) < \left(1+\frac{2}{p^s}\right) < \left(1+\frac{1}{p^s}\right)^2$$

so for $s>1$

$$\frac{f_1(s)^2}{f_1(2s)^3} < A(s) < \frac{f_1(s)^2}{f_1(2s)^2}$$

$$\frac{f_1(s)^2}{f_1(2s)^3}(s-1) < A(s)(s-1) < \frac{f_1(s)^2}{f_1(2s)^2}(s-1)$$

$s\to 1^+$

$$0 < a \le \lim_{s\to 1^+} A(s)(s-1) \le b$$