

$$\zeta_K(s) = \prod_{p:\text{ramified}} \left(1 - \frac{1}{p^s}\right)^{-1} \prod_{p:\text{split}} \left(1 - \frac{1}{p^s}\right)^{-l} \prod_{p:\text{inert}} \left(1 - \frac{1}{p^{ls}}\right)^{-1} = \zeta(s) \prod_{k=1}^{l-1} L(s, \chi^k)$$

for some Dirichlet character χ with conductor N . Or equivalently,

$$\frac{\zeta_K(s)}{\zeta(s)} = \prod_{p:\text{split}} \left(1 - \frac{1}{p^s}\right)^{-(l-1)} \prod_{p:\text{inert}} \frac{\left(1 - \frac{1}{p^{ls}}\right)^{-1}}{\left(1 - \frac{1}{p^s}\right)^{-1}} = \prod_{k=1}^{l-1} L(s, \chi^k)$$

if $l = 2$,

$$\prod_{p:\text{split}} \left(1 - \frac{1}{p^s}\right) \prod_{p:\text{inert}} \left(1 + \frac{1}{p^s}\right) = L(s)$$

Let

$$f_s(s) = \prod_{p:\text{split}} \left(1 - \frac{1}{p^s}\right), \quad f_i(s) = \prod_{p:\text{inert}} \left(1 + \frac{1}{p^s}\right)$$

which means

$$f_s(s)f_i(s) = L(s)$$