The Macroeconomics of Clean Energy Subsidies

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Abstract

We study clean energy subsidies in a quantitative climate-economy model. Clean energy subsidies decrease carbon emissions if and only if they lower the marginal product of dirty energy. The constrained-efficient subsidy equals the marginal external cost of dirty energy multiplied by the marginal impact of clean energy production on dirty energy production. With standard functional forms, two factors determine the impact of clean subsidies on dirty energy production: the elasticity of substitution between clean and dirty energy and the price elasticity of demand for energy services. With some commonly used parameter values, clean production subsidies increase emissions and decrease welfare relative to laissez faire. With greater substitutability between clean and dirty energy, the subsidies in the Inflation Reduction Act can generate modest emissions reductions. Even in this more optimistic scenario, a clean subsidy generates significantly higher emissions and lower welfare than a tax on dirty energy.

Keywords Climate Change Mitigation, Second-Best Policies, Economic Growth

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1 Introduction

Most macroeconomic analyses of climate change mitigation policies focus on carbon taxes, because they are the first-best approach to addressing the negative externality from carbon emissions. In real-world policymaking, however, a much wider set of policy options are considered and implemented. For example, subsidies for the production of clean energy are a central component of the Inflation Reduction Act (IRA), which was recently passed in the United States. We study the effectiveness of these clean energy subsidies in a macroeconomic climate-economy model.

In the first step of our analysis, we study a static model for which we derive analytical results on the welfare impacts of clean energy subsidies. Output is a function of labor, clean energy, and dirty energy. Both types of energy are extracted from the environment using the final good. There is only one market failure: a negative externality from dirty energy use. The social planner only has access to one instrument: a subsidy for clean energy production. Clean energy subsidies increase the quantity of clean energy extracted from the environment. We show that their impact on dirty energy production is ambiguous. If the expansion in clean energy increases the marginal product of dirty energy, subsidies lead to greater extraction of dirty energy, increasing emissions and decreasing welfare. Formally, the condition that marginal productivity of one input increases with the level of the other input is known as supermodularity (a positive cross-partial derivative). Importantly, the crucial criterion for whether emissions increase with a clean subsidy is *not* simply whether clean and dirty energy are complements or substitutes in the production of energy services, *but* whether clean and dirty energy are supermodular or not in the production of final good. This intuition is independent of other market failures, such as learning-by-doing (LBD) spillovers or distortionary taxation. We show that the constrained-efficient subsidy equals the marginal external cost of dirty energy times the marginal impact of clean energy production on dirty energy production. We refer to this quantity as the *indirect externality* associated with clean energy use. If clean and dirty energy inputs are supermodular, then the indirect externality reduces welfare, and the social planner would prefer to tax clean energy.

In the second step of our analysis, we decompose the condition of supermodularity into a substitution effect and a production scale effect. Drawing an analogy to consumer theory, the substitution effect represents a shift from dirty to clean energy while maintaining a constant level of production. The production scale effect increases overall energy consumption in response to the clean subsidies. In a parsimoniously abstract environment, we analytically demonstrate that the substitution effect decreases with the degree of supermodularity, while the production scale effect increases in the level of supermodularity, both favoring an increase in dirty energy use.[1] As a result, supermodularity holds the key to signing the welfare impact of clean production subsidies on emissions by reducing substitution and increasing the production scale in response to the clean subsidies. While our analytical results are robust to functional form assumptions, the literature commonly models the trade-off between clean and dirty energy inputs using a constant elasticity of substitution (CES) function, where the CES aggregate captures overall energy input. In many integrated assessment models (IAMs), this energy aggregate is incorporated as an additional input to a final good production function that follows a Cobb-Douglas structure. Importantly, a CES function in isolation is always (weakly) supermodular, even if the two inputs are substitutes.[2]

In this nested Cobb-Douglas-CES production framework, the CES function determines the strength of the substitution effect, while the energy intensity of the final good production sector dictates the scale effect. For a clean energy subsidy to be effective, the substitution effect must dominate the production scale effect, which is easier the lower the overall energy share in the economy. We show that in this setting, clean subsidies increase dirty energy production if and only if the magnitude of the elasticity of substitution between clean and dirty energy is less than the magnitude of the price elasticity of demand for energy services.

In the third step of our analysis, we map the substitution and scale effects in our simple theory model to the key parameters used in the existing IAM literature. In influential work, Golosov et al. (2014) use an elasticity of substitution between dirty and clean energy (ϵ) just below one and a price elasticity of demand just above one (see also Hassler et al., 2016; Hassler and Krusell, 2018; Hassler et al., 2021b). In other words, commonly used parameter values imply that clean energy subsidies *increase* dirty energy use and *decrease* welfare. However, there is considerable uncertainty about the elasticity of substitution, ϵ. Several studies use values closer to 2, often based on the evidence in Papageorgiou et al. (2017). With this higher value, the impacts of the subsidy flip signs.

In the fourth step of our analysis, we study the full dynamic model. We calibrate the model to data from the United States and recent estimates of the social cost of carbon emissions from Rennert et al. (2022). We show that the intuition from the simple model holds in a dynamic setting. In particular, we simulate a 20 percent subsidy to clean energy production, which is in line with estimates of the subsidies in the IRA (Bistline et al., 2023).

[1] The other crucial determinant of the production scale and substitution effect is decreasing marginal returns. Decreasing marginal returns suppresses the magnitudes of both the (positive) substitution and the (generally negative) production scale effect, impacting dirty energy use in opposite directions.

[2] Substitutes have an elasticity of substitution larger than unity, whereas we define complements as goods with an elasticity of substitution below unity.

The calibration with $\epsilon \approx 1$ implies that the subsidy increases US emissions by approximately 1.6 percent and reduces welfare (measured as consumption equivalent variation) by approximately 0.16 percent, relative to a no-policy scenario. The calibration with $\epsilon \approx 2$ implies that the subsidy decreases emissions by approximately 6.6 percent and has virtually no effect on welfare. With the higher elasticity, the best constant clean subsidy is about half as large as the IRA subsidy. It reduces emissions by 3.4 percent and increases welfare by 0.05 percent. The best constant dirty energy tax, however, reduces emissions by 40 percent and increases welfare by 0.7 percent.

These results have several important policy implications. First, a standard macro climate-economy model suggests limited environmental and economic benefits from the clean energy subsidies in the IRA. At some commonly used parameter values, the model predicts that the subsidies will increase emissions and decrease welfare. At alternate plausible values, emissions decrease slightly. Second, policy or technological process that raises the elasticity of substitution between energy sources or decrease the elasticity of energy demand increase the effectiveness of clean energy subsidies. Third, even when subsidies decrease emissions, the best possible subsidy yields economic and environmental outcomes that are significantly worse than those that could be attained with a tax on dirty energy. Fourth, there is considerable need for better estimates of the elasticity of substitution between different sources of energy. The impacts of a clean energy subsidy change signs within the range of parameters used in the existing literature.

Related literature Following Nordhaus (1993), a rapidly expanding literature uses growth models to study climate policy (e.g., Golosov et al., 2014; Rezai and van der Ploeg, 2016; Gerlagh and Liski, 2018; Lemoine and Rudik, 2017; Dietz and Venmans, 2019; Hassler et al., 2021b; Krusell and Smith Jr, 2022; Traeger, 2023; Barrage and Nordhaus, 2024). This literature primarily focuses on carbon taxes. Several studies, however, examine the role of clean energy subsidies in the presence of other market failures, such as learning-by-doing or R&D spillovers (Popp, 2006; Bosetti et al., 2009; Rezai and van der Ploeg, 2017; Baldwin et al., 2020; Hassler et al., 2020). In contrast, we build upon the existing *analytical* IAM literature by extending Hoel (2012) within a macro climate-economy model to *theoretically* characterize the constrained-efficient renewable energy production subsidy in the absence of other market failures and *quantitatively* assess the welfare impacts of production subsidies for clean energy, a focal point of recent legislation in the US. We focus on production subsidies for clean energy as a policy tool to replace carbon taxes in a second-best setting without considering any additional positive externalities that might justify them. We also shed light on the energy production structure with a particular focus on the interplay between energy demand elasticity and the elasticity of substitution between green and brown energy sources by showing that subsidies increase emissions in a standard macro climate-economy model.

Bistline et al. (2023), on the other hand, *quantitatively* assess the impacts of the IRA using the *numerical* energy system model — the US Regional Economy, Greenhouse Gas, and Energy (US-REGEN). They find that subsidies decrease dirty energy use. As discussed in Section 5.2, our results complement theirs in that we focus on macroeconomic dynamics, while they utilize a very detailed model of the energy sector. They also use the first-order conditions of a simplified macro climate-economy to show that the intuition from the static second-best environmental economics literature holds in a dynamic setting, but they do not solve the model quantitatively. Our analysis builds on their work in several ways. First, and the most importantly, we characterize the conditions under which subsidies decrease emissions and increase welfare. In the US-REGEN model, energy service demand is exogenous, which dampens a channel through which subsidies could lead to an increase in energy service consumption and, thus, higher use of dirty energy. Second, we solve a quantitative macro model to study the effects of the IRA. Third, we calculate the best possible subsidy and compare the implications of the subsidy to those of a dirty energy tax.

Our analysis relates to the long microeconomics literature on second-best environmental policies (e.g., Bennear and Stavins, 2007; Goulder and Parry, 2008; Holland et al., 2009). Palmer and Burtraw (2005) and Fischer and Newell (2008) study static models where clean energy subsidies reduce emissions, but are less efficient than other options, like emissions pricing or renewable portfolio standards. Gerlagh and van der Zwaan (2006) and Kalkuhl et al. (2013) arrive at similar findings in dynamic settings (see also, Gugler et al., 2021; Cruz and Rossi-Hansberg, 2024; Airaudo et al., 2023). Closely related, Baumol and Oates (1988) theoretically show that, in a model of firm entry, subsidies designed to promote clean alternatives to polluting inputs can unintendedly increase pollution by expanding the scale of production. Indeed, Fullerton and Wolverton (2000, 2005) show that optimal policy requires both subsidizing the clean alternative and taxing output, a 'two-part instrument.' We build on this literature by providing a simple and novel characterization of constrained-efficient subsidies and the conditions under which subsidies increase emissions. We also quantify these effects in a macro climate-economy model and show that subsidies increase emissions at standard parameter values.

2 Static model

This section presents a simple static model that builds the intuition underlying our general findings.

2.1 Model structure

Gross output (q) is given by a constant-return-to-scale (CRS) function[3]

$$q = f(l, e_d, e_c), \tag{1}$$

where l is the quantity of labor, e_d is the quantity of dirty energy, and e_c is the quantity of clean energy. We assume that the production function is twice continuously differentiable, has increasing and diminishing marginal products, and satisfies the Inada conditions. We use f_j, $j \in \{l, d, c\}$, to denote partial derivatives.

It takes p_c (p_d) units of the final good to extract one unit of e_c (e_d) from the environment. As noted by Hassler et al. (2021b), this is equivalent to assuming that the production functions for both primary energy sectors are symmetric to the production function for final goods and differ only in total factor productivity.

Labor supply is inelastic: $l = 1$. Final output (y) is gross output minus extraction costs:

$$y = f(l, e_d, e_c) - p_c e_c - p_d e_d. \tag{2}$$

A representative consumer has the utility function

$$U = u(y) - m e_d, \tag{3}$$

where m is the marginal external cost of dirty energy use, $u' > 0$, and $u'' < 0$.

[3]The general approach of this section follows from a working paper by Hoel (2012). We extend his work by considering a concave utility function and a constant-return-to-scale production function with three inputs. We also do not make direct assumptions about the sign of f_{cd}. Hoel (2012) considers the case of $f_{cd} < 0$ in a two-factor production function, but this assumption is inconsistent with standard neoclassical production functions. By including a third factor, we can consider a wider range of possible outcomes and map the results to production functions used in macroeconomic climate-economy models. Hassler et al. (2021b) study a similar static model to investigate the role of carbon taxes that are set above or below their optimal level.

2.2 Competitive equilibrium

There is perfect competition, and the price of the final good is normalized to one. The representative firm solves

$$\max_{l,e_c,e_d} f(l,e_d,e_c) - (p_c+\tau_c)e_c - p_d e_d - wl, \tag{4}$$

where τ_c is a tax/subsidy on e_c ($\tau_c > 0$ is a tax), and w is the wage. After imposing $l = 1$, the firm's first-order conditions for energy use are

$$f_c(1,e_d,e_c) = p_c + \tau_c \tag{5}$$

$$f_d(1,e_d,e_c) = p_d. \tag{6}$$

The Inada conditions imply that there exists a unique e_d that satisfies (6) for a given e_c. Thus, there exists some single-valued function $e_d = D(e_c)$ with $D'(e_c) = \frac{f_{cd}}{-f_{dd}}$. Since $f_{dd} < 0$, the sign of $D'(e_c)$ matches the sign of f_{cd}. This follows intuitively from equation (6). If clean and dirty energy are supermodular ($f_{cd} > 0$), then an increase in clean energy increases the return to extracting dirty energy from the environment. So, the producer extracts more dirty energy until the marginal product is equal to the price.

Equations (5) and (6) also imply that $\frac{de_c}{d\tau_c} < 0$. In other words, clean energy taxes (subsidies) decrease (increase) clean energy use (see Appendix section A.1). We can also rearrange (5) to get $\tau_c = f_c(1,D(e_c),e_c) - p_c \equiv \tau(e_c)$, where $\tau'(e_c) < 0$.

Since the production function is CRS, total wages are equal to gross output not paid to energy producers:

$$wl = q - (p_c+\tau_c)e_c - p_d e_d. \tag{7}$$

The government pays subsidies (or collects taxes) of $T = -\tau_c e_c$ to energy producers. To balance the budget, it uses lump sum taxation that does not affect incentives. Thus, the consumer's after-tax income is

$$q - p_c e_c - p_d e_d = y, \tag{8}$$

i.e. the representative consumer simply consumes all of the final output.

2.3 Constrained-efficient subsidies

The constrained social planner chooses τ_c to maximize (3), subject to the competitive equilibrium equations (5)–(6). This differs from a first-best equilibrium in which the social planner would freely choose both e_c and e_d. We will use τ_c^* to denote the constrained-efficient tax/subsidy on the quantity of clean energy. To more easily derive τ_c^*, we re-frame the optimization problem as one where the social planner chooses e_c subject to $e_d = D(e_c)$. This is equivalent to choosing a tax/subsidy τ_c, because of the one-to-one mapping $\tau_c = \tau(e_c)$. We will use e_c^* for the constrained-efficient quantity of clean energy.

The social planner's maximization problem is

$$\max_{e_c} u\Big(f(1, D(e_c), e_c) - p_d D(e_c) - p_c e_c\Big) - mD(e_c). \tag{9}$$

The first order condition is

$$\Big(f_c(1, D(e_c^*), e_c^*) - p_c\Big) + \Big(f_d(1, D(e_c^*), e_c^*) - p_d\Big) D'(e_c^*) = \frac{m}{u'(y)} D'(e_c^*). \tag{10}$$

The right-hand side of (10) is the marginal external cost of dirty energy measured in units of output, $\frac{m}{u'(y)}$, scaled by the impact of clean energy production on dirty energy production, $D'(e_c^*)$. We refer to this as the marginal *indirect externality* from clean energy use. At the constrained optimum, this quantity should equal the marginal change in y from increasing e_c, which is on the left-hand side of the equation.

To determine the tax/subsidy that implements e_c^*, we compare the social planner's solution to the competitive equilibrium. From the firm's FOCs, $f_c(1, D(e_c^*), e_c^*) - p_c = \tau_c^*$ and $f_d(1, D(e_c^*), e_c^*) - p_d = 0$. Plugging in these results, the social planner's optimality condition becomes

$$\tau_c^* = \frac{m}{u'(y)} D'(e_c) = \left(\frac{m}{u'(y)}\right)\left(\frac{f_{cd}}{-f_{dd}}\right). \tag{11}$$

The optimal tax/subsidy for clean energy is equal to the indirect externality. When $f_{cd} > 0$, an increase in clean energy leads to greater extraction of dirty energy, implying that the indirect externality from clean energy reduces welfare, and the constrained social planner would prefer to tax clean energy. The reverse occurs when $f_{cd} < 0$.

Proposition 1. *Consider the static model presented in this section. If $f_{cd} > 0$ (i.e., increases in clean energy raise the marginal product of dirty energy, indicating supermodularity), then the constrained social planner chooses to tax clean energy. If $f_{cd} < 0$ (i.e., submodularity), then the constrained social planner chooses to subsidize clean energy.*

If the subsidy-induced expansion in clean energy increases the marginal product of dirty energy (supermodularity), then firms respond by extracting more dirty energy. This happens until the marginal product falls back to the level of the extraction cost. In this case, subsidies increase pollution and decrease welfare, and the constrained social planner could increase welfare by taxing clean energy. The opposite occurs if the increase in clean energy decreases the marginal product of dirty energy (submodularity).

2.4 Additional results

The analysis in the preceding section also has implications for the welfare effects of non-constrained-efficient taxes/subsidies for clean energy. Appendix section A.2 derives a more general expression for the welfare impacts of clean energy taxes/subsidies. The marginal welfare impact starting from laissez faire is given by

$$\frac{dU}{d\tau_c}\frac{1}{u'(y)}\Big|_{\tau_c=0} = -\frac{m}{u'(y)}\frac{de_d}{d\tau_c} = -\frac{m}{u'(y)}\frac{de_d}{de_c}\frac{de_c}{d\tau_c} = \frac{m}{u'(y)}\frac{f_{cd}}{-f_{dd}}\left(-\frac{de_c}{d\tau_c}\right). \tag{12}$$

The consumers' utility function is linearly-separable in final output and the external cost of pollution. The firm already chooses energy inputs to maximize output, implying that the only first-order welfare effect of subsidies comes from pollution. A small subsidy will decrease welfare if and only if it increases dirty energy use, which occurs when $f_{cd} > 0$.

2.5 Substitution and scale effects

This section further examines why f_{cd} plays such an important role for the effectiveness of clean energy production subsidies. It also pinpoints the crucial difference between dirty energy taxes and clean energy subsidies, connecting back to the existing literature's intuition regarding the inefficiency of clean subsidies (e.g., Newell et al., 2019). To accomplish these goals, we decompose the response of dirty energy to clean subsidies into a *substitution effect* and a production *scale effect*. We examine how f_{cd} operates through both channels.

Our approach closely relates to the well-known decomposition of demand into income and substitution effects using the Slutsky equation. We are interested in the representative firm's factor demand for dirty energy, which is a function of the energy prices: $e_d(p_d, p_c + \tau_c)$.[4] We define the *conditional* factor demand for dirty energy as the function $e_d^\dagger(p_d, p_c + \tau_c; q)$, which characterizes the firm's demand for dirty energy given energy input prices and conditional

[4]We let the labor market clear. As a result, the price of labor is endogenous, and the representative firm's demand for dirty energy determines equilibrium emissions.

on a fixed output level q. Conditional factor demand takes the role of Hicksian demand in consumer theory and allows us to single out the pure substitution effect (Puu, 1966). We write the representative firm's profit-maximizing production level as $q(p_d, p_c + \tau_c)$. Then, we can decompose the impact of a clean energy subsidy on dirty energy use as

$$\underbrace{\frac{de_d(p_d, p_c + \tau_c)}{d\tau_c}}_{\text{Overall effect}} = \underbrace{\frac{\partial e_d^\dagger(p_d, p_c + \tau_c; q)}{\partial \tau_c}}_{\text{Substitution effect}} + \underbrace{\frac{\partial e_d^\dagger(p_d, p_c + \tau_c; q)}{\partial q} \frac{\partial q(p_d, p_c + \tau_c)}{\partial \tau_c}}_{\text{Scale effect}}. \tag{13}$$

The *substitution effect* captures the reallocation between clean and dirty energy, keeping overall production constant. The *scale effect* has two components that jointly characterize the consequence of changing production levels. The first component of the scale effect characterizes the increase in dirty energy resulting from a production increase. The second component captures the overall change in output resulting from a change in policy.

It is straightforward that the substitution effect is directly linked to the complementarity/substitutability of clean and dirty energy. We will show that the complementarity/substitutability embodied in f_{cd} also influences the production scale effect. The scale effect highlights the difference between a dirty energy tax and a clean energy subsidy. The tax and the subsidy both decrease the relative price of clean energy, triggering similar substitution effects. But, the two policies have opposite implications for the scale effect. A clean subsidy increases energy use while a dirty tax decreases energy use.

In our setting, it is easier to interpret the different components of equation (13) if we multiply the decomposition by the price of dirty energy, stating the changes in absolute units. Appendix A.3 derives a general expression for substitution and scale effects. We find

$$\text{Substitution effect: } \Bigg(\underbrace{-\frac{f_{cc}}{f_c} + \left(-\frac{f_{dd}}{f_d}\right)\frac{f_c}{f_d}}_{\text{decreasing marginal returns}} + \underbrace{2\,\frac{f_{cd}}{f_d}}_{\text{supermodularity}} \Bigg)^{-1} \geq 0.$$

The substitution effect is positive for an increase in the relative price of dirty energy and, hence, always reduces dirty energy use in response to a clean subsidy. The first two terms of the substitution effect measure the (normalized) concavity of the production function in clean and dirty energy. Substituting away from either input is harder when marginal productivity falls more quickly in the individual inputs. We are particularly interested in the last term. Supermodularity accelerates the decrease of marginal utility when substituting away from one of the inputs, here dirty. Thus, substituting clean for dirty energy is harder if the two energy sources are supermodular.

The scale effect is composed of two terms:

$$\text{Scale effect: } \Bigg(1+\frac{\overbrace{-f_{dd}\frac{f_c}{f_d}+f_{cd}}^{\substack{\text{decreasing}\\\text{returns dirty}}}}{\underbrace{-f_{cc}\frac{f_d}{f_c}+f_{cd}}_{\substack{\text{decreasing}\\\text{returns clean}}}}\Bigg)^{-1}\times\Bigg(-\frac{\overbrace{f_c+f_d\frac{f_{cd}}{-f_{dd}}}^{\substack{\text{marginal productivity}\\\text{clean energy}}}}{\underbrace{(-f_{cc})-(-f_{dd})\left(\frac{f_{cd}}{-f_{dd}}\right)^2}_{\substack{\text{decreasing returns}\\\text{clean energy}}}}\Bigg) \tag{14}$$

Overall, the scale effect is negative when f exhibits constant returns to scale. In response to a clean subsidy, it increases dirty emissions, counteracting the substitution effect. For a dirty energy tax, by contrast, it reinforces the substitution effect. The final term in expression (14) explains why the scale effect is so sensitive to f_{cd}. First, in the numerator, supermodularity between clean and dirty (f_{cd}) increases the effective marginal productivity of the subsidized clean energy because an increase in clean energy also increases the productivity of dirty energy. Second, in the denominator, the supermodularity moderates the decrease in the returns to scale because the dirty energy input increases together with the subsidized clean energy input. Both of the channels amplify the increase in energy use.[5]

The trade-off between the two effects characterizes the

$$\text{Overall effect: } -\frac{f_d f_{cd}}{f_{dd}f_{cc}-f_{cd}^2} \gtreqless 0,$$

whose sign depends only on the supermodularity embodied in f_{cd}. As we observed previously, emissions decrease under a clean subsidy if and only if $f_{cd}<0$.

2.6 Calibration with nested CES-in-CD production

As is standard in macro climate-economy models (e.g., Golosov et al., 2014; Hassler et al., 2021b), consider a nested production structure where gross output,

$$q = g(l,e) = l^{1-\nu}e^{\nu}, \tag{15}$$

[5]Interestingly, the dependence of the first term in expression (14) on f_{cd} is ambiguous. This term pins down the increase in dirty energy use in response to a production increase. If clean energy's productivity falls faster than dirty energy's productivity, then dirty energy use expands more strongly with a production increase. Taken together, under the assumption of constant returns, the magnitude of the joint expression (14) always increases in the magnitude of f_{cd}, qualitatively following the described impact of f_{cd} on the second term (see Appendix A.3).

relies on labor and *energy services*,

$$e = h(e_d, e_c) = \left(\omega e_d^{\frac{\epsilon-1}{\epsilon}} + (1-\omega)e_c^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{16}$$

and satisfies the Inada condition.[6] Here, $\omega \in (0,1)$ is a distribution parameter, $\frac{\partial \ln q}{\partial \ln e} = \nu \in (0,1)$ is the elasticity of output with respect to energy services, $\frac{\partial \ln \frac{e_d}{e_c}}{\partial \ln \frac{p_c}{p_d}} = \epsilon \in (0,\infty)$ is the elasticity of substitution between clean and dirty energy, and $f(l, e_d, e_c) = g(l, h(e_d, e_c))$. We ignore capital because this is a static model. If p_e is the price of energy services, then $\frac{\partial \ln e}{\partial \ln p_e} = (1-\nu)^{-1}$ is the price elasticity of demand for energy services.

This production structure directly separates the two channels discussed in the previous section, and it shows the close relationship between these channels and the existence of supermodularity in the aggregate production function. The substitution effect is proportional to ϵ, and the production scale effect is proportional to $(1-\nu)^{-1}$. In addition, [7]

$$f_{cd} > 0 \iff (1-\nu)^{-1} > \epsilon. \tag{18}$$

When ϵ is low, the two goods are more complementary in energy service production (h_{cd} is larger), and an increase in clean energy has a greater impact on the marginal product of dirty energy. When ν is low, an increase in clean energy decreases the marginal product of energy services more significantly (g_{ee} is more negative). The relative strength of these two forces determines the overall emissions impact of an increase in clean energy subsidies. To further highlight these competing forces, we note that equation (13) becomes

$$\frac{d \ln e_d}{d \ln(p_c + \tau_c)} = \left(\epsilon - (1-\nu)^{-1}\right) \frac{(p_c+\tau_c)e_c}{(p_c+\tau_c)e_c + p_d e_d} \tag{19}$$

when utilizing these standard functional forms.

[6] In general, the Inada conditions do not hold for CES production functions. However, nested CES-in-CD production functions do satisfy the Inada conditions. See Appendix section A.4.2 for proof.

[7] It is tedious but straightforward to show that:

$$f_{cd} = \left((1-\nu)^{-1} - \epsilon\right) \left(\frac{\nu(1-\nu)}{\epsilon}\right) \left(\frac{1}{e_d e_c}\right) \left(\frac{\omega e_d^{\frac{\epsilon-1}{\epsilon}}}{\omega e_d^{\frac{\epsilon-1}{\epsilon}} + (1-\omega)e_c^{\frac{\epsilon-1}{\epsilon}}}\right) \left(\frac{(1-\omega)e_c^{\frac{\epsilon-1}{\epsilon}}}{\omega e_d^{\frac{\epsilon-1}{\epsilon}} + (1-\omega)e_c^{\frac{\epsilon-1}{\epsilon}}}\right) q. \tag{17}$$

See Appendix A.4 for derivation.

Given the small number of parameters, it is straightforward to compare ϵ and $(1-\nu)^{-1}$ to estimates from the existing literature. With perfect competition, ν is the energy share of gross output. Based on Casey (2024), we consider a value of $\nu = 0.08$. In this case, $f_{cd} > 0$ if and only if $\epsilon < 1.09$. Many macroeconomic climate-economy models, including the influential work of Golosov et al. (2014) and handbook chapters by Hassler et al. (2016) and Hassler and Krusell (2018), use a value at or slightly below $\epsilon = 1$, based on evidence from a meta-study by Stern (2012). At these values, $f_{cd} > 0$ and it is optimal to tax, rather than subsidize, clean energy production.

Papageorgiou et al. (2017) estimate the elasticity of substitution between clean and dirty sources of energy for several sectors. Unfortunately, they do not provide an economy-wide estimate that is directly applicable to a macroeconomic model. They do, however, find an elasticity of around 2 for the electricity sector and values of in the range $(1.5, 3)$ for non-electricity sectors.[8] Acemoglu et al. (2019) use $\epsilon = 1.85$ as a summary of the findings from Papageorgiou et al. (2017) that can be used in a macro model. At this value, $f_{cd} < 0$ and the social planner prefers to subsidize clean energy.[9]

2.7 Scale effects in the presence of other market failures

We have, thus far, focused on the case where there is only one externality. Clean subsidies are sometimes justified by learning-by-doing (LBD) externalities in clean energy production (e.g., Gillingham and Stock, 2018; Newell et al., 2019; Bistline et al., 2023). Appendix section A.5 analyzes an extension of the static model that includes LBD. LBD does not affect the relationship between the quantity of clean energy and the marginal product of dirty energy. With LBD and $f_{cd} > 0$, subsidies can increase welfare relative to laissez-faire, but only at the expense of worsening environmental outcomes. In other words, the subsidy addresses the LBD externality, but not the climate externality. The key mechanism we highlight in the simple model is independent of LBD. We also stress that the detrimental outcomes associated with clean energy subsidies do not rely on the presence of distortionary taxation, a market failure that would decrease the effectiveness of subsidies relative to carbon taxes.

[8] The average across sectors is different than the economy-wide elasticity, because the economy-wide elasticity must take into account reallocation across sectors (Oberfield and Raval, 2021).

[9] With perfect substitution, when clean energy becomes relatively cheaper than dirty energy due to subsidies, the economy will only use renewable energy. However, the overall energy service might still be oversupplied because of production scale effects, leading to inefficiencies. In practice, both clean and dirty energy are used even though their prices differ. Moreover, due to the intermittency of clean energy, dirty energy remains valuable when clean energy is unavailable. Therefore, perfect substitution is not considered in this paper.

In addition, the common assumptions of complete pass-through and perfect competition in the incidence of energy prices is not empirically supported (Ganapati et al., 2020). In Appendix section A.6, we extend the static model to incorporate firms enjoying monopolistic power when combining dirty and clean energy to produce energy composites. In this environment, monopolists set energy prices where marginal cost equals marginal revenue and charge markups. With monopolistic pricing, a decline in the overall energy price is not completely passed through to output producers, leading to smaller scale effects. Accordingly, the secondary environmental degradation from the subsidy is smaller due to market power.

3 Quantitative model

We now specify a dynamic model that is more amenable to quantitative analysis.

3.1 Structure

Final output (Y_t) is a Cobb-Douglas combination of capital ($K_{y,t}$), labor ($L_{y,t}$), and energy services ($E_{y,t}$):

$$Y_t = K_{y,t}^{\alpha} E_{y,t}^{\nu} \left(A_{y,t} L_{y,t}\right)^{1-\alpha-\nu}, \tag{20}$$

where $\alpha, \nu \in (0,1)$, and the productivity term ($A_{y,t}$) grows at a constant rate g_y. The production function for energy services is

$$E_t = \left(\omega^{\frac{1}{\epsilon}} Z_{d,t}^{\frac{\epsilon-1}{\epsilon}} + (1-\omega)^{\frac{1}{\epsilon}} Z_{c,t}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{21}$$

where $\omega \in (0,1)$, $\epsilon \in [0,\infty)$, $Z_{d,t}$ is dirty energy, and $Z_{c,t}$ is clean energy. The energy extraction technology is

$$Z_{j,t} = K_{j,t}^{\alpha} E_{j,t}^{\nu} (A_{j,t} L_{j,t})^{1-\alpha-\nu}, \quad j = c, d, \tag{22}$$

where $L_{j,t}$ is labor in energy sector j at time t, and $K_{j,t}$ is capital used in energy sector j at time t, and $E_{j,t}$ is energy services used in energy sector j at time t. Productivity terms $A_{d,t}$ and $A_{c,t}$ grow at exogenous rates g_d and g_c, respectively. Including energy services in the primary energy production is somewhat unusual, but increases realism. We follow the structural change literature and assume factor shares are the same in all sectors (Herrendorf et al., 2014). This specification is isomorphic to one in which there is an exogenous extraction cost paid in final goods, implying that this is the dynamic analogue of our simple model.

The market clearing conditions for aggregate capital (K_t) and aggregate labor (L_t) are

$$K_t = K_{y,t} + K_{d,t} + K_{c,t} \tag{23}$$

$$L_t = L_{y,t} + L_{d,t} + L_{c,t}. \tag{24}$$

Aggregate labor grows at exogenous rate n. Capital evolves according to

$$K_{t+1} = Y_t - C_t + (1-\delta)K_t, \tag{25}$$

where $\delta \in (0,1)$ is the depreciation rate. The market clearing condition for output is

$$Y_t = w_t L_t + \rho_t K_t, \tag{26}$$

where w_t is the wage, and ρ_t is the rental rate, and $r_t = \rho_t - \delta$ is the real interest rate.

The representative household has lifetime utility

$$U = \sum_{t=0}^{\infty} \beta^t \Big(L_t \ln(C_t) + m \sum_{\tilde{t}=0}^{t} \eta_{\tilde{t}} Z_{d,\tilde{t}} \Big), \tag{27}$$

where $\beta \in (0,1)$ is the discount rate, $m > 0$ is the marginal damage from carbon emissions per period, and

$$\eta_t = (1 + g_\eta)\eta_{t-1} \tag{28}$$

captures exogenous changes in the carbon-intensity of fossil fuel use (Krusell and Smith Jr, 2022). In other words, g_η captures substitution between different fossil fuels, which happens outside of our model and is unaffected by a clean energy subsidy. The representative household ignores the utility cost of dirty energy when making consumption and investment decisions. The household budget constraint is

$$C_t + K_{t+1} = w_t L_t + (1 + r_t)K_t. \tag{29}$$

Golosov et al. (2014) specify a simple climate model where production declines exponentially in carbon concentrations. Combined with log utility and full depreciation, they find that damages from CO_2 are linear in welfare. Traeger (2023) derives the same result for a fully calibrated climate model and partial capital persistence. The intuition for this potentially surprising result is that warming is strongly concave in CO_2, while damages are convex in

Table 1: Model Parameters

Parameter	Value	Description	Source
ϵ	0.95	Clean-Dirty EoS	Golosov et al. (2014)
	1.85		Papageorgiou et al. (2017)
ν	0.08	Energy share of income	Casey (2024)
α	0.27	Capital share of income	Jones (2016)
ω	0.60	Distribution parameter	Golosov et al. (2014)
δ	0.27	Depreciation rate	Standard
n	0.05	Population growth	EIA
g	0.10	Income per capita growth	Jones (2016)
β	0.88	Discount factor	Golosov et al. (2014)
g_η	-0.14	Fall in carbon intensity of Z_d	EIA
m	0.24	Flow damages	Calibrated

the temperature increase. Our specification directly assumes this linearity of damages from carbon emissions. For our purposes, there are several benefits to this simple specification. First, we study outcomes in the United States. With linear damages, the welfare cost of US emissions is independent of emissions from elsewhere in the world and emissions that occur before $t = 0$. Second, recent evidence suggests that non-market damages account for a large portion of the social cost of carbon (e.g., Climate Impact Lab, 2022; Rennert et al., 2022; EPA, 2022), which are easily incorporated into m.

There are two possible policy interventions. We mainly study constant value-added subsidies for clean energy, $\tau_c \in (-1, 0)$. So, $(1 + \tau_c)p_{c,t}$ is the policy-inclusive price of energy paid by the energy service producer, and $p_{c,t}$ is the price received by the primary energy producer. We also compare the impacts of these clean energy subsidies to a constant value-added tax on dirty energy, where $(1 + \tau_d)p_{d,t}$ is the policy-inclusive price of dirty energy and $\tau_d > 0$. All agents in the economy take policy as given.

3.2 Calibration

We calibrate the model to the United States. The time step is five years. We simulate the model over 1000 years and study outcomes over the first 50 years. All parameter values are shown in Table 1.

We consider both $\epsilon = 0.95$ from Golosov et al. (2014) and $\epsilon = 1.85$ from Papageorgiou et al. (2017). Based on Golosov et al. (2014), we set $\omega = 0.60$. In our model, the effective discount rate for consumption is $\beta(1 + n)$, which we calibrate to to $0.985^5 = 0.93$. We take $n = 0.05$ (1%/year), which gives $\beta = 0.88$. We also take $\nu = 0.08$ from Casey (2024), and

$\alpha = 0.27$, which gives a standard labor share of $(1 - \alpha - \nu) = 0.63$ (Gollin, 2002; Jones, 2016). We set $\delta = 0.27$ (6%/year). We assume that all technologies ($A_{j,t}$ for $j = y, d, c$) grow at $g_j = 0.10$ (2%/year), which matches estimates of long-run income per capital growth from Jones (2016). To set units, we normalize $A_{y,0} = A_{d,0} = A_{c,0} = L_0 = 1$.

We set $g_\eta = -0.14$ (2.3%/year) to match data on the declining carbon intensity of fossil fuels (Energy Information Administration, 2019). Combining the social cost of carbon estimate of \$185/tCO_2 from Rennert et al. (2022) with data on emissions from Energy Information Administration (2019) implies that the monetary cost of US emissions in 2020 was equal to 4.0% of GDP. We set $m = 0.24$ to match this value.

3.3 Solution method

We solve the model using techniques from the structural change literature (Herrendorf et al., 2014). Since the steps are standard, the full characterization of the equilibrium is included in Appendix section B. Since climate damages do not affect the dynamics of the macroeconomic variables, we can solve the underlying growth model with standard computational tools. We use *Dynare* (Adjemian et al., 2011). Appendix section B shows the equations used in the computational solution.

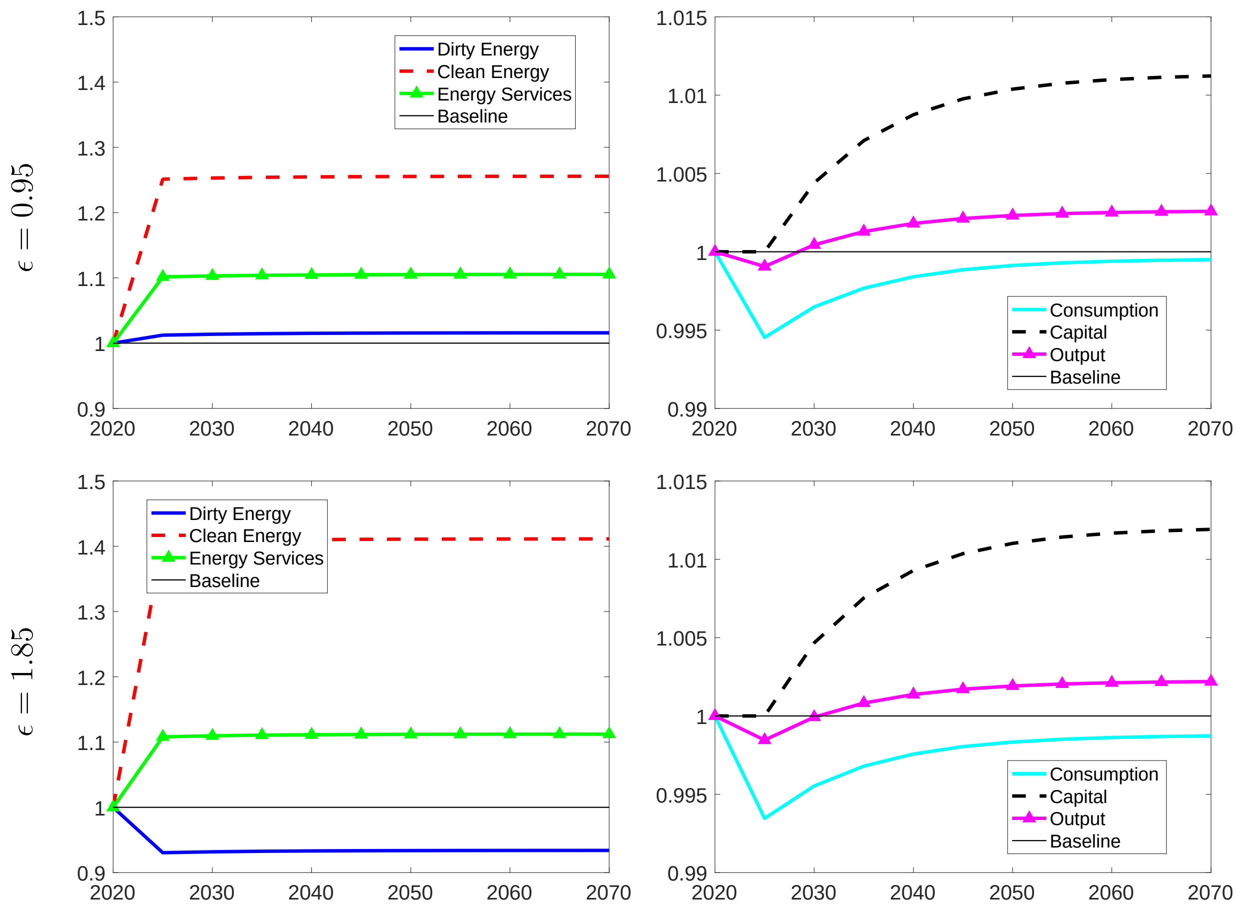
4 Quantitative results

4.1 Impacts of the Inflation Reduction Act

Clean energy subsidies are a central component of the Inflation Reduction Act, which was recently passed in the United States (White House, 2023). The subsidies take two forms: tax credits per kWh of energy production (*production tax credits*, PTC) or tax credits for investment by clean energy firms (*investment tax credits*, ITC). Producers can choose which subsidy to take. The size of the subsidies increases substantially if firms meet certain labor requirements. The subsidies also increase in size if clean energy production takes place in vulnerable communities or meets domestic content requirements. The policies are currently set to expire in 2035.

We model the IRA as a PTC. We assume that it is announced and implemented with full commitment in 2025 and that it lasts forever. Bistline et al. (2023) calculate the effects of the IRA on clean energy prices, allowing the form of the credit to differ by sector and assuming that all firms receive the labor bonus, but not the other two bonuses. They find that the IRA will lower the prices of utility-scale solar and offshore wind by approximately 20 percent in 2023 and have a slightly smaller effect on the price of onshore wind. We simulate a subsidy of 20 percent. Given that not all firms will receive the bonus, we think this is likely to be an overestimate of the effect of the IRA on clean energy prices.

4.1.1 Results with baseline parameters



(a) Energy (b) Macro Aggregates

Figure 1: Impacts of the IRA clean energy subsidies

Note: All outcomes are shown relative to laissez-faire levels.

The top row of Figure 1 shows the simulated impacts of the IRA clean energy subsidies using the value of $\epsilon = 0.95$ from Golosov et al. (2014) and Stern (2012). In all figures, outcomes are shown relative to laissez-faire levels. The panel on the top left shows that the subsidies increase clean energy use by approximately twenty-five percent and dirty energy use by 1.6 percent. The change in total energy use is just over ten percent. These results are consistent with our earlier analysis of the static model. Since clean and dirty energy are supermodular, clean energy subsidies increase dirty energy use. The strength of this force is weak and the overall change in dirty energy is small, because the parameter values are close to $\epsilon \approx (1-\nu)^{-1}$ where $f_{cd} \approx 0$.

The panel on the top right shows the path of macroeconomic aggregates. The energy sector is a small fraction of the economy, and even large changes in energy prices have little spillover to long-run economic dynamics. The small change in macro aggregates explains why energy use quickly adjusts to the new BGP level. Since the policy is unexpected, the capital stock is unaffected in the year the policy is implemented. In the absence of policy intervention, the competitive equilibrium would maximize output in the initial year given the quantity of available inputs. So, the policy intervention, which alters relative prices, leads to a small decrease in output in the initial year of the policy. The clean energy subsidy increases the tax-inclusive return to investment, which increases the saving rate immediately and capital and output in subsequent years, though the effect is again quantitatively small. Consumption initially dips due to the increased saving rate and decreased output, and it then converges back almost to its initial level.

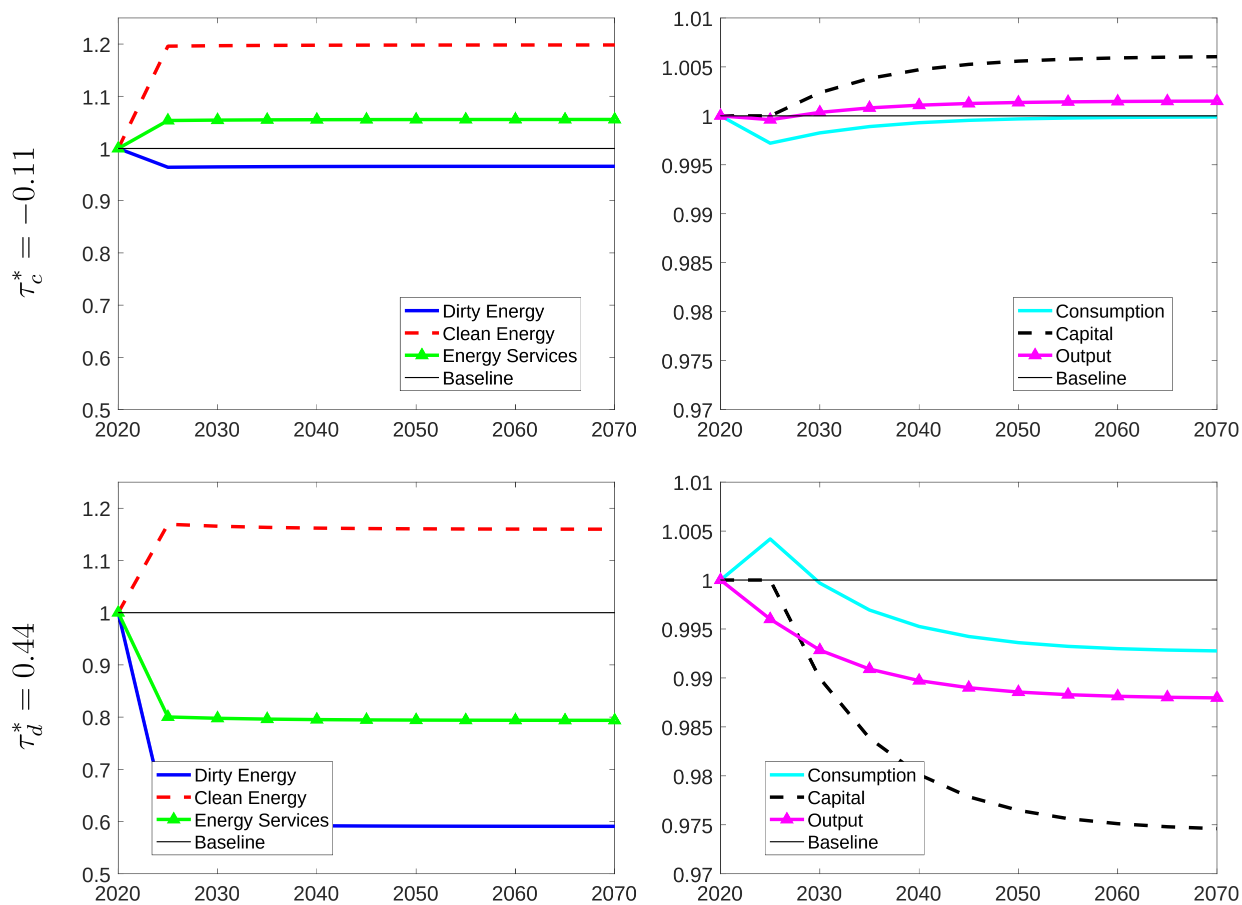
These results imply that the subsidy decreases welfare. It simultaneously increases dirty energy production and decreases consumption. Given that neither quantity deviates much from its laissez-faire value, the welfare impacts of the policy are small. With our calibrated value of m, moving from the laissez-faire equilibrium to the equilibrium with clean energy subsidies decreases welfare as much as decreasing consumption by 0.16 percent in every period in the laissez-faire equilibrium.

4.1.2 Results with alternate parameters

The bottom row of Figure 1 simulates the impacts of the twenty percent subsidy using the value of $\epsilon = 1.85$ from Papageorgiou et al. (2017). The panel on the left shows the path of energy use. With this higher elasticity, clean and dirty energy are submodular. The policy increases clean energy use and decreases dirty energy use. This is again consistent with the static model. Clean energy use is about forty percent higher than on the laissez-faire BGP, but dirty energy use is only 6.6 percent lower. Total energy services production is about ten percent higher.

The panel on the right shows the macroeconomic aggregates. Capital increases by around one percent, while output initially decreases and subsequently increases relative to laissez-faire. Consumption is slightly below its laissez-faire level. Moving from the laissez-faire equilibrium to the equilibrium with clean energy subsidies has essentially no impact on welfare. The results from the simple model indicate that a subsidy can increase welfare when the elasticity of substitution between clean and dirty energy is sufficiently high, but they do not imply that *any* subsidy will increase welfare. As we discuss in the next section, the twenty percent subsidy is too high and the benefits of lower emissions are almost exactly offset by the inefficient reallocation of inputs away from final good production and towards energy production.

4.2 Best constant subsidy with $\epsilon = 1.85$



(a) Energy (b) Macro Aggregates

Figure 2: Best constant subsidies and taxes with $\epsilon = 1.85$

Note: All outcomes are shown relative to laissez-faire levels.

In this section, we investigate the impacts of the constant clean energy subsidy that maximizes welfare in the absence of a carbon price. With $\epsilon = 0.95$, subsidies necessarily decrease welfare. So, we focus on the higher value of $\epsilon = 1.85$. Welfare is maximized at $\tau_c^* = -0.11$, which is smaller than the IRA subsidies of $\tau_c = -0.20$.[10] As a result, the change in dirty energy use, 3.4 percent, is smaller in this case. The top row of Figure 2 shows the resulting dynamics, which are similar to the bottom row in Figure 1, but more muted. The welfare gain is 0.05 percent CEV. The increase in welfare and decrease in emissions are both consistent with the results from the simple model.

4.3 Dirty energy tax with $\epsilon = 1.85$

In this section, we study the impacts of a dirty energy tax with $\epsilon = 1.85$. To ensure a reasonable comparison to the constant subsidy for clean energy used in the previous section, we examine a constant tax on dirty energy, despite the changing carbon-intensity of dirty energy. The constant tax that maximizes welfare is $\tau_d^* = 0.44$. Although it is not the first-best policy response, the tax is a significant improvement over a clean energy subsidy. Moving from laissez-faire to a dirty energy tax increases welfare as much as increasing consumption by 0.7 percent CEV. This is an order of magnitude larger than the gain from the best clean energy subsidy.

The bottom row of Figure 2 shows the impact of the dirty energy tax. Dirty energy use falls by 40 percent, which is an order of magnitude larger than the impacts of the best clean subsidy. Unlike a subsidy, a tax on dirty energy increases the price of the energy services, and the consumption of energy services decreases by 20 percent relative to laissez faire. This difference between clean subsidies and dirty taxes comes from the differing signs on the scale effect. Clean energy use increases with the both the tax and the subsidy, because of the substitution effect.

As shown in the panel on the right, the tax also leads to qualitatively different macroeconomic dynamics. The tax leads to a fall of approximately 2.5 percent in capital and a 1 percent fall in output. Consumption falls by about 0.75 percent with the tax and is virtually unchanged with the subsidy. As noted above, however, welfare is significantly higher under the tax. For a given reduction in consumption, the tax can achieve a much greater reduction in dirty energy use, leading to higher welfare.

[10] Appendix Figure B.1 plots the objective function for the best constant clean subsidy and dirty tax. It shows that there is a unique optimum in each case.

5 Discussion and conclusion

5.1 Policy implications

Our results suggest that the clean energy subsidies in the IRA will have limited impacts on emissions and welfare. The sign of the impacts differs across plausible parameter values, but the magnitude is consistently small in our simulations. These outcomes reflect the limited effectiveness of clean energy subsidies. At standard parameter values, any subsidy decreases welfare. At alternative plausible values, the best possible constant subsidy yields only modest emission reductions and welfare increases relative to a no-policy scenario. The best possible subsidy yields large increases in emissions and decreases in welfare relative to the best constant tax on dirty energy. Together, these results suggest that moving US climate policy to a carbon pricing approach could generate large emissions reductions and welfare gains.

5.2 Comparison with earlier work

There have been several engineering analyses of the Inflation Reduction Act, with one of the most prominent likely being the REGEN model constructed by the Electric Power Research Institute (EPRI, 2020), which is used in the economic analysis by Bistline et al. (2023). The REGEN model predicts that the IRA will decrease emissions by 6-11 percent relative to laissez-faire. This is similar to our predictions with $\epsilon = 1.85$, but our predictions with $\epsilon = 0.95$ have the opposite sign. It is tempting to conclude that the REGEN must implicitly have a high elasticity of substitution, but this conclusion overlooks another important difference, namely the treatment of energy demand. In our model, subsidies decrease the price of energy services, which in turn increases the equilibrium quantity of energy services produced. This effect is determined by ν, which is calibrated to match long-run patterns in aggregate energy use and expenditure (Atkeson and Kehoe, 1999; Hassler et al., 2021a; Casey, 2024). In REGEN, the demand for energy services is exogenous, implying that it dampens an important channel by which subsidies increase the quantity of energy services and, consequently, dirty energy use.[11]

[11] In REGEN, the demand for energy services is exogenous, but this demand can be satisfied with different levels of efficiency (e.g., miles-per-gallon). This incorporates some, but not all, of the ways that total energy production may increase as a result of clean energy subsidies. For example, it does not incorporate the increase in miles driven that may occur when electricity is cheaper.

5.3 The elasticity of substitution between clean and dirty energy

The welfare impacts of a clean energy subsidy depend importantly on the elasticity of substitution between clean and dirty energy. Unfortunately, there is considerable uncertainty surrounding this parameter. This is true for at least two reasons. First, estimating any substitution elasticity is notoriously difficult and generally requires exogenous variation in one or more prices (León-Ledesma et al., 2010). Indeed, climate-economy models require macro elasticities that are complicated aggregates of firm and sector-level elasticities, which can be easier to estimate (Papageorgiou et al., 2017; Oberfield and Raval, 2021).[12] Second, knowing this elasticity is often not required to determine the first-best carbon tax (e.g., Golosov et al., 2014; Traeger, 2023), which may explain why this important parameter has not received more attention.

Our results imply that subsidies will be a more effective way to combat climate change if there are complementary innovations that increase the elasticity of substitution between energy sources. We have treated the elasticity of substitution between energy types as a structural parameter. The elasticity of substitution between energy sources is determined in part by the facts that renewables are intermittent and their efficiency varies across space. Therefore, significant breakthroughs in storage and transmission technology are expected to raise the elasticity. The IRA includes incentives meant to improve storage technology, and future analyses quantifying the impact of these policies would make an important contribution to our overall understanding of the US climate policy. Indeed, our results suggest that improvements in storage and transmission are essential to making clean energy subsidies an effective tool for reducing emissions and increasing welfare.

[12] In concurrent work in progress, Casey and Gao (2023) examine the elasticity of substitution between clean and dirty energy using aggregated macro-level time series data, rather than sectoral data. They find that, at the macro level, the share of dirty energy remains relatively constant in the US despite rising fossil fuel prices over time, suggesting a relatively low elasticity of substitution, approximately equal to unity.

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Online Appendix

The Macroeconomics of Clean Energy Subsidies

A Derivations for static model

A.1 Impact of τ_c on energy use

Differentiating (5) and (6) with respect to τ_c gives

$$f_{cc}\frac{de_c}{d\tau_c} + f_{cd}\frac{de_d}{d\tau_c} = 1 \tag{A.1}$$

$$f_{cd}\frac{de_c}{d\tau_c} + f_{dd}\frac{de_d}{d\tau_c} = 0, \tag{A.2}$$

which implies

$$\begin{pmatrix} \frac{de_c}{d\tau_c} \\ \frac{de_d}{d\tau_c} \end{pmatrix} = \begin{pmatrix} f_{cc} & f_{cd} \\ f_{cd} & f_{dd} \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{f_{dd}}{f_{cc}f_{dd}-(f_{cd})^2} \\ \frac{-f_{cd}}{f_{cc}f_{dd}-(f_{cd})^2} \end{pmatrix}. \tag{A.3}$$

Note that the denominator is always non-negative. This is because when f is concave with respect to (l, e_d, e_c), the k^{th} order leading principal minors of its Hessian

$$\begin{pmatrix} f_{dd} & f_{dc} & f_{dl} \\ f_{cd} & f_{cc} & f_{cl} \\ f_{ld} & f_{lc} & f_{ll} \end{pmatrix} \tag{A.4}$$

have the same sign as $(-1)^k$. Therefore,

$$\begin{vmatrix} f_{dd} & f_{dc} \\ f_{cd} & f_{cc} \end{vmatrix} = f_{cc}f_{dd} - (f_{cd})^2 \geq 0. \tag{A.5}$$

Thus, $\frac{de_c}{d\tau_c} < 0$ and $\operatorname{sgn}\left(\frac{de_d}{d\tau_c}\right) = -\operatorname{sgn}(f_{cd})$. In addition, since $\frac{de_c}{d\tau_c} < 0$, the inverse relationship $\tau_c = \tau(e_c)$ is well-defined and monotonic, ruling out the possibility of multiple equilibria.

A.2 Derivation of the welfare impact of τ_c

Combining utility function (3) with market clearing condition (8) gives

$$U = u(f(1, e_d, e_c) - p_d e_d - p_c e_c) - m e_d. \tag{A.6}$$

The welfare effect of clean energy subsidies, measured in units of output, is given by

$$\frac{dU}{d\tau_c}\frac{1}{u'(y)} = (f_c - p_c)\frac{de_c}{d\tau_c} + (f_d - p_d)\frac{de_d}{d\tau_c} - \frac{m}{u'(y)}\frac{de_d}{d\tau_c}. \tag{A.7}$$

Applying the firm's first-order conditions gives

$$\frac{dU}{d\tau_c}\frac{1}{u'(y)} = \tau_c\frac{de_c}{d\tau_c} - \frac{m}{u'(y)}\frac{de_d}{d\tau_c}. \tag{A.8}$$

Totally differentiating (6) delivers

$$f_{cd}de_c + f_{dd}de_d = 0, \tag{A.9}$$

$$\frac{de_d}{de_c} = \frac{-f_{cd}}{f_{dd}}. \tag{A.10}$$

Evaluated at the laissez-faire equilibrium ($\tau_c = 0$), (A.8) and (A.10) yield

$$\frac{dU}{d\tau_c}\frac{1}{u'(y)}\Big|_{\tau_c=0} = -\frac{m}{u'(y)}\frac{de_d}{d\tau_c} = -\frac{m}{u'(y)}\frac{de_d}{de_c}\frac{de_c}{d\tau_c} = \frac{m}{u'(y)}\frac{f_{cd}}{-f_{dd}}\left(-\frac{de_c}{d\tau_c}\right). \tag{A.11}$$

Evaluated at the constrained-efficient subsidy ($\frac{dU}{d\tau_c} = 0$), they yield

$$\tau_c^* = \frac{m}{u'(y)}\frac{de_d}{de_c} = \frac{m}{u'(y)}\frac{f_{cd}}{-f_{dd}}. \tag{A.12}$$

A.3 Decomposition

A.3.1 Cost-minimizing input choices

We analyze the conditional factor demand of the firms given an arbitrary output level $q > 0$:

$$(l^\dagger, e_d^\dagger, e_c^\dagger) = \arg\min_{l,e_d,e_c} \; \{ \; wl + p_d e_d + (p_c + \tau_c) e_c \; \} \quad \text{subject to} \quad f(l, e_d, e_c) = q. \tag{A.13}$$

$$\Leftrightarrow \; (l^\dagger, e_d^\dagger, e_c^\dagger, \lambda^\dagger) = \arg\max_{l,e_d,e_c,\lambda} \; \left\{ \; -(wl + p_d e_d + (p_c + \tau_c) e_c) + \lambda\big(f(l, e_d, e_c) - q\big) \; \right\}. \tag{A.14}$$

After imposing $l^\dagger = 1$, the first-order conditions are

$$p_d = \lambda^\dagger f_d \tag{A.15}$$

$$p_c + \tau_c = \lambda^\dagger f_c \tag{A.16}$$

$$q = f. \tag{A.17}$$

Note that an equilibrium wage is endogenously determined such that $w = f_l(1, e_d^\dagger, e_c^\dagger)$ for market clearing. Therefore, $(e_d^\dagger, e_c^\dagger, \lambda^\dagger)$ is a function of exogenous prices $(p_d, p_c + \tau_c)$ and an exogenous output level q. Let $c(p_d, p_c + \tau_c, q)$ represent the associated indirect cost function. Now, consider a comparative statics analysis with respect to a change in τ_c

$$0 = f_d \frac{\partial \lambda^\dagger}{\partial \tau_c} + \lambda^\dagger \left(f_{dc} \frac{\partial e_c^\dagger}{\partial \tau_c} + f_{dd} \frac{\partial e_d^\dagger}{\partial \tau_c} \right) \tag{A.18}$$

$$1 = f_c \frac{\partial \lambda^\dagger}{\partial \tau_c} + \lambda^\dagger \left(f_{cc} \frac{\partial e_c^\dagger}{\partial \tau_c} + f_{cd} \frac{\partial e_d^\dagger}{\partial \tau_c} \right) \tag{A.19}$$

$$0 = f_c \frac{\partial e_c^\dagger}{\partial \tau_c} + f_d \frac{\partial e_d^\dagger}{\partial \tau_c}, \tag{A.20}$$

which is equivalent to

$$\begin{pmatrix} \frac{\partial e_c^\dagger}{\partial \tau_c} \\ \frac{\partial e_d^\dagger}{\partial \tau_c} \\ \frac{\partial \lambda^\dagger}{\partial \tau_c} \end{pmatrix} = \begin{pmatrix} \lambda^\dagger f_{cd} & \lambda^\dagger f_{dd} & f_d \\ \lambda^\dagger f_{cc} & \lambda^\dagger f_{cd} & f_c \\ f_c & f_d & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \tag{A.21}$$

The Lagrange multiplier $\lambda^\dagger (> 0)$ represents the marginal change in costs arising from a marginal change in output from the envelop theorem. In equilibrium, firms make zero profits

$$\lambda^{\dagger} = \frac{\partial(wl^{\dagger} + p_d e_d^{\dagger} + (p_c + \tau_c)e_c^{\dagger})}{\partial q} = 1. \tag{A.22}$$

The impact of clean energy subsidies on conditional clean energy demand is determined by

$$\frac{\partial e_d^{\dagger}}{\partial \tau_c} = \left(\frac{1}{\lambda^{\dagger}}\right)\left(\frac{-f_c f_d}{f_{cc}f_d^2 - 2f_{cd}f_c f_d + f_{dd}f_c^2}\right) = \frac{f_c f_d}{-\left(f_{cc}f_d^2 - 2f_{cd}f_c f_d + f_{dd}f_c^2\right)}. \tag{A.23}$$

Note that the denominator is always non-negative. This is because when f is concave with respect to (l, e_d, e_c), it is quasi-concave with respect to (l, e_d, e_c). Then, the largest two leading principal minors of its bordered Hessian matrix

$$\begin{pmatrix} 0 & f_d & f_c & f_l \\ f_d & f_{dd} & f_{dc} & f_{dl} \\ f_c & f_{cd} & f_{cc} & f_{cl} \\ f_l & f_{ld} & f_{lc} & f_{ll} \end{pmatrix} \tag{A.24}$$

alternate in sign, with the smallest being non-negative. Thus,

$$\begin{vmatrix} 0 & f_d & f_c \\ f_d & f_{dd} & f_{dc} \\ f_c & f_{cd} & f_{cc} \end{vmatrix} = -\left(f_{cc}f_d^2 - 2f_{cd}f_c f_d + f_{dd}f_c^2\right) \geq 0. \tag{A.25}$$

Therefore, $\frac{de_d^{\dagger}}{d\tau_c}$ is always non-negative. When the price of clean energy decreases, the use of dirty energy also declines. Similarly, the impact of a change in output level on conditional dirty energy demand is determined by

$$\frac{\partial e_d^{\dagger}}{\partial q} = \frac{-f_{cc}f_d + f_{cd}f_c}{-\left(f_{cc}f_d^2 - 2f_{cd}f_c f_d + f_{dd}f_c^2\right)}. \tag{A.26}$$

When f is homothetic, the cost function can be written as $c(p_d, p_c+\tau_c, q) = C(p_d, p_c+\tau_c)h(q)$ with convex h and concave C. According to the Shephard's lemma, $e_d^{\dagger} = \frac{\partial c}{\partial p_d}$. Therefore,

$$\frac{\partial e_d^{\dagger}}{\partial q} = \frac{\partial C}{\partial p_d}\frac{\partial h}{\partial q} > 0, \tag{A.27}$$

and $-f_{cc}f_d + f_{cd}f_c > 0$.

A.3.2 Profit-maximizing output and input choices

Here, we turn to the profit-maximizing output and input decisions:

$$\max_{q,l,e_d,e_c} \; q - (wl + p_d e_d + (p_c + \tau_c)e_c) \quad \text{subject to} \quad f(l, e_d, e_c) = q \tag{A.28}$$

$$\Leftrightarrow \max_{q,l,e_d,e_c,\mu} \; \left\{ q - (wl + p_d e_d + (p_c + \tau_c)e_c) + \mu\big(f(l, e_d, e_c) - q\big) \right\}. \tag{A.29}$$

After imposing $l = 1$, the first-order conditions are

$$\begin{aligned} p_d &= \mu f_d \\ p_c + \tau_c &= \mu f_c \\ 1 &= \mu \\ 0 &= f - q. \end{aligned}$$

Note that an equilibrium wage is endogenously determined such that $w = f_l(1, e_d, e_c)$ for market clearing. Therefore, (e_d, e_c, q, μ) is a function of exogenous prices $(p_d, p_c + \tau_c)$.

Now, consider a comparative statics analysis with respect to a change in τ_c

$$0 = f_d \frac{\partial \mu}{\partial \tau_c} + \mu \left(f_{dc} \frac{\partial e_c}{\partial \tau_c} + f_{dd} \frac{\partial e_d}{\partial \tau_c} \right) \tag{A.30}$$

$$1 = f_c \frac{\partial \mu}{\partial \tau_c} + \mu \left(f_{cc} \frac{\partial e_c}{\partial \tau_c} + f_{cd} \frac{\partial e_d}{\partial \tau_c} \right) \tag{A.31}$$

$$0 = \frac{\partial \mu}{\partial \tau_c} \tag{A.32}$$

$$0 = f_c \frac{\partial e_c}{\partial \tau_c} + f_d \frac{\partial e_d}{\partial \tau_c} - \frac{\partial q}{\partial \tau_c}, \tag{A.33}$$

which is equivalent to

$$\begin{pmatrix} \frac{\partial e_c}{\partial \tau_c} \\ \frac{\partial e_d}{\partial \tau_c} \\ \frac{\partial \mu}{\partial \tau_c} \\ \frac{\partial q}{\partial \tau_c} \end{pmatrix} = \begin{pmatrix} \mu f_{cd} & \mu f_{dd} & f_d & 0 \\ \mu f_{cc} & \mu f_{cd} & f_c & 0 \\ 0 & 0 & 1 & 0 \\ f_c & f_d & 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}. \tag{A.34}$$

The impact of clean subsidies on a profit-maximizing output decision is determined by

$$\begin{aligned}\frac{\partial q}{\partial \tau_c} &= \left(\frac{1}{\mu}\right)\left(\frac{-f_{dd}f_c + f_{cd}f_d}{f_{cd}^2 - f_{cc}f_{dd}}\right) \\ &= -\frac{-f_{dd}f_c + f_{cd}f_d}{f_{cc}f_{dd} - (f_{cd})^2}.\end{aligned} \tag{A.35}$$

Combined with the homotheticity of f, profit maximization implies

$$1 = C(p_d, p_c + \tau_c)h'(q). \tag{A.36}$$

Totally differentiating this equation with respect to τ_c yields

$$\frac{\partial C(p_d, p_c + \tau_c)}{\partial \tau_c}h'(q) + C(p_d, p_c + \tau_c)h''(q)\frac{\partial q}{\partial \tau_c} = 0. \tag{A.37}$$

Therefore,

$$\frac{\partial q}{\partial \tau_c} = -\frac{h'(q)}{h''(q)}\frac{\frac{\partial C(p_d, p_c+\tau_c)}{\partial \tau_c}}{C(p_d, p_c + \tau_c)} < 0, \tag{A.38}$$

and $-f_{dd}f_c + f_{cd}f_d > 0$.

A.3.3 Substitution and production scale effects

The substitution effect is given by

$$\frac{\partial e_d^\dagger}{\partial \tau_c} = \frac{f_c f_d}{-\left(f_{cc}f_d^2 - 2f_{cd}f_c f_d + f_{dd}f_c^2\right)}, \tag{A.39}$$

which is always positive from the concavity of f. Therefore, when clean energy prices decline due to clean energy production subsidies, conditional dirty energy demand always decreases. Multiplying the substitution effect by the price of dirty energy $p_d = f_d$, we get

$$\begin{aligned}f_d\frac{\partial e_d^\dagger}{\partial \tau_c} &= \frac{f_c(f_d)^2}{-f_{cc}f_d^2 - f_{dd}f_c^2 + 2f_{cd}f_c f_d} \\ &= \left(-\frac{f_{cc}}{f_c} + \left(-\frac{f_{dd}}{f_d}\right)\frac{f_c}{f_d} + 2\frac{f_{cd}}{f_d}\right)^{-1}\end{aligned} \tag{A.40}$$

The production scale effect is given by

$$\frac{\partial e_d^\dagger}{\partial q}\frac{\partial q}{\partial \tau_c} = \left(\frac{-f_{cc}f_d + f_{cd}f_c}{-f_{cc}f_d^2 + 2f_{cd}f_cf_d - f_{dd}f_c^2}\right)\left(-\frac{-f_{dd}f_c + f_{cd}f_d}{f_{cc}f_{dd} - (f_{cd})^2}\right), \tag{A.41}$$

which is always negative from the concavity and homotheticity of f. Therefore, when clean energy prices decline due to clean energy production subsidies, profit-maximizing output choice always increases, leading to a higher dirty energy use through the production scale effect. Multiplying the scale effect by the price of dirty energy $p_d = f_d$, we get

$$\begin{aligned} f_d\frac{\partial e_d^\dagger}{\partial q}\frac{\partial q}{\partial \tau_c} &= \left(\frac{-f_{cc}f_d^2 + f_{cd}f_cf_d}{-f_{cc}f_d^2 + 2f_{cd}f_cf_d - f_{dd}f_c^2}\right)\left(-\frac{-f_{dd}f_c + f_{cd}f_d}{f_{cc}f_{dd} - (f_{cd})^2}\right) \\ &= \left(1 + \frac{-f_{dd}\frac{f_c}{f_d} + f_{cd}}{-f_{cc}\frac{f_d}{f_c} + f_{cd}}\right)^{-1}\left(-\frac{f_c + f_d\frac{f_{cd}}{-f_{dd}}}{(-f_{cc}) - (-f_{dd})\left(\frac{f_{cd}}{-f_{dd}}\right)^2}\right). \end{aligned} \tag{A.42}$$

Note that this equation can also be rewritten as

$$f_d\frac{\partial e_d^\dagger}{\partial q}\frac{\partial q}{\partial \tau_c} = \left(\frac{1}{-f_{cc}f_d^2 + f_{cd}f_cf_d} + \frac{1}{-f_{dd}f_c^2 + f_{cd}f_cf_d}\right)^{-1}\left(-\frac{1}{f_{cc}f_{dd} - (f_{cd})^2}\right)\frac{1}{f_c}. \tag{A.43}$$

It is important to emphasize that the homotheticity and concavity of f ensure the positivity of all denominators, as demonstrated in earlier sections. Consequently, with an increase in supermodularity, the production scale effect also increases in magnitude.

A.4 Nested CES-in-CD production function

A.4.1 Derivatives

To simplify expressions, we define $\rho \equiv \frac{\epsilon-1}{\epsilon}$. The partial derivatives of $f(l, e_d, e_c)$ with respect to energy inputs are given by

$$f_d = \nu\omega e_d^{\rho-1} \left(\omega e_d^\rho + (1-\omega)e_c^\rho\right)^{\frac{\nu}{\rho}-1} > 0, \tag{A.44}$$

$$f_c = \nu(1-\omega) e_c^{\rho-1} \left(\omega e_d^\rho + (1-\omega)e_c^\rho\right)^{\frac{\nu}{\rho}-1} > 0, \tag{A.45}$$

where we use $l = 1$. For the second derivatives, we find

$$\begin{aligned} f_{cd} &= \omega(1-\omega)\nu(\nu-\rho)e_c^{\rho-1}e_d^{\rho-1}\left(\omega e_d^\rho + (1-\omega)e_c^\rho\right)^{\frac{\nu}{\rho}-2}, \\ &= \nu\left(\nu-\rho\right)\left(\frac{1}{e_d e_c}\right)\left(\frac{\omega e_d^\rho}{\omega e_d^\rho + (1-\omega)e_c^\rho}\right)\left(\frac{(1-\omega)e_c^\rho}{\omega e_d^\rho + (1-\omega)e_c^\rho}\right)\left(\omega e_d^\rho + (1-\omega)e_c^\rho\right)^{\frac{\nu}{\rho}} \end{aligned} \tag{A.46}$$

which is equivalent to (17) since $\nu - \frac{\epsilon-1}{\epsilon} = \frac{1}{\epsilon}\big(1-\epsilon(1-\nu)\big) = \frac{1-\nu}{\epsilon}\big((1-\nu)^{-1}-\epsilon\big)$, and

$$f_{dd} = -\nu\omega e_d^{\rho-2}\big((1-\nu)\omega e_d^\rho + (1-\rho)(1-\omega)e_c^\rho\big)\left(\omega e_d^\rho + (1-\omega)e_c^\rho\right)^{\frac{\nu}{\rho}-2} < 0. \tag{A.47}$$

A.4.2 The Inada conditions

The Inada condition holds when $\lim_{x_i \to 0} \frac{\partial f(x_1,x_2,x_3)}{\partial x_i} = \infty$ and $\lim_{x_i \to \infty} \frac{\partial f(x_1,x_2,x_3)}{\partial x_i} = 0$ for any $i \in \{1, 2, 3\}$. The Inada condition holds for l as $f_l = \frac{1-\nu}{l}q$. We consider the following three cases to show that the Inada condition holds for e_d: (i) $1 > \nu > \rho > 0$, (ii) $1 > \nu > 0 > \rho$, and (iii) $1 > \rho > \nu > 0$. If we factor out $e_d^{\nu-\rho}$, then f_d can be rewritten as follows:

$$\frac{\partial f}{\partial e_d} = \nu\omega \underbrace{\frac{1}{e_d^{1-\nu}}}_{=\text{term (A)}} Al^{1-\nu} \underbrace{\left(\left(\omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^\rho\right)^{\frac{1}{\rho}}\right)^{\nu-\rho}}_{=\text{term (B)}}. \tag{A.48}$$

Case i: $1 > \nu > \rho > 0$

1. When $e_d \to 0$, the term (A) approaches ∞ and the term (B) approaches ∞ because

$$
\begin{aligned}
& e_d \to 0 \\
& \frac{e_c}{e_d} \to \infty \\
& \left(\frac{e_c}{e_d}\right)^{\rho} \to \infty && (\because \rho > 0) \\
& \omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho} \to \infty \\
& \left(\omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho}\right)^{\frac{1}{\rho}} \to \infty && (\because \rho > 0) \\
& \left(\left(\omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu-\rho} \to \infty && (\because \nu - \rho > 0)
\end{aligned}
$$

Therefore, f_d approaches ∞.

2. When $e_d \to \infty$, term (A) approaches 0 and term (B) approaches $\omega^{\frac{\nu}{\rho}-1}$ because

$$
\begin{aligned}
& e_d \to \infty \\
& \frac{e_c}{e_d} \to 0 \\
& \left(\frac{e_c}{e_d}\right)^{\rho} \to 0 && (\because \rho > 0) \\
& \omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho} \to \omega \\
& \left(\omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho}\right)^{\frac{1}{\rho}} \to \omega^{\frac{1}{\rho}} \\
& \left(\left(\omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu-\rho} \to \omega^{\frac{\nu}{\rho}-1}
\end{aligned}
$$

Therefore, f_d approaches 0.

Case ii: $1 > \nu > 0 > \rho$

1. When $e_d \to 0$, term (A) approaches ∞ and term (B) approaches $\omega^{\frac{\nu}{\rho}-1}$ because

$$
\begin{aligned}
& e_d \to 0 \\
& \frac{e_c}{e_d} \to \infty \\
& \left(\frac{e_c}{e_d}\right)^{\rho} \to 0 && (\because \rho < 0) \\
& \omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho} \to \omega \\
& \left(\omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho}\right)^{\frac{1}{\rho}} \to \omega^{\frac{1}{\rho}} \\
& \left(\left(\omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu-\rho} \to \omega^{\frac{\nu}{\rho}-1}
\end{aligned}
$$

   Therefore, f_d approaches ∞.

2. When $e_d \to \infty$, term (A) approaches 0 and term (B) approaches 0 because

$$
\begin{aligned}
& e_d \to \infty \\
& \frac{e_c}{e_d} \to 0 \\
& \left(\frac{e_c}{e_d}\right)^{\rho} \to \infty && (\because \rho < 0) \\
& \omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho} \to \infty \\
& \left(\omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho}\right)^{\frac{1}{\rho}} \to 0 && (\because \rho < 0) \\
& \left(\left(\omega + (1-\omega)\left(\frac{e_c}{e_d}\right)^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu-\rho} \to 0 && (\because \nu - \rho > 0)
\end{aligned}
$$

   Therefore, f_d approaches 0.

Alternatively, if we factor out $e_c^{\nu-\rho}$, then f_d can be rewrtten as follows:

$$
\frac{\partial f}{\partial e_d} = \nu\omega \underbrace{\frac{1}{e_d^{1-\rho}}}_{=\text{term (C)}} A l^{1-\nu} e_c^{\nu-\rho} \underbrace{\left(\omega\left(\frac{e_d}{e_c}\right)^{\rho} + (1-\omega)\right)^{\frac{\nu-\rho}{\rho}}}_{=\text{term (D)}}. \tag{A.49}
$$

Case iii: $1 > \rho > \nu > 0$

1. When $e_d \to 0$, term (C) approaches ∞ and term (D) approaches $(1-\omega)^{\frac{\nu}{\rho}-1}$ because

$$
\begin{aligned}
& e_d \to 0 \\
& \frac{e_d}{e_c} \to 0 \\
& \left(\frac{e_d}{e_c}\right)^{\rho} \to 0 && (\because \rho > 0) \\
& \omega\left(\frac{e_d}{e_c}\right)^{\rho} + (1-\omega) \to 1-\omega \\
& \left(\omega\left(\frac{e_d}{e_c}\right)^{\rho} + (1-\omega)\right)^{\frac{1}{\rho}} \to (1-\omega)^{\frac{1}{\rho}} \\
& \left(\left(\omega\left(\frac{e_d}{e_c}\right)^{\rho} + (1-\omega)\right)^{\frac{1}{\rho}}\right)^{\nu-\rho} \to (1-\omega)^{\frac{\nu}{\rho}-1}
\end{aligned}
$$

   Therefore, f_d approaches ∞.

2. When $e_d \to \infty$, term (C) approaches 0 and term (D) approaches 0 because

$$
\begin{aligned}
& e_d \to \infty \\
& \frac{e_d}{e_c} \to \infty \\
& \left(\frac{e_d}{e_c}\right)^{\rho} \to \infty && (\because \rho > 0) \\
& \omega\left(\frac{e_d}{e_c}\right)^{\rho} + (1-\omega) \to \infty \\
& \left(\omega\left(\frac{e_d}{e_c}\right)^{\rho} + (1-\omega)\right)^{\frac{1}{\rho}} \to \infty && (\because \rho > 0) \\
& \left(\left(\omega\left(\frac{e_d}{e_c}\right)^{\rho} + (1-\omega)\right)^{\frac{1}{\rho}}\right)^{\nu-\rho} \to 0 && (\because \nu-\rho < 0)
\end{aligned}
$$

   Therefore, f_d approaches 0.

Similarly, the Inada condition holds for e_c.

A.4.3 Substitution and production scale effects

In a nested CES-in-CD production function,

$$g(l,e) = l^{1-\nu}e^{\nu} \quad \text{where} \quad h(e_d,e_c) = (\omega e_d^{\frac{\epsilon-1}{\epsilon}} + (1-\omega)e_c^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}, \tag{A.50}$$

which yields the substitution and production scale effects as follows:

$$\frac{\partial e_d^{\dagger}}{\partial(p_c+\tau_c)} = \frac{e_c e_d}{g_e e}\epsilon \tag{A.51}$$

$$\frac{\partial e_d^{\dagger}}{\partial q}\frac{\partial q}{\partial(p_c+\tau_c)} = -\frac{e_c e_d}{g_e e}(1-\nu)^{-1}. \tag{A.52}$$

Since h exhibits constant returns to scale, energy producers make zero profits in equilibrium: $g_e e = (p_c+\tau_c)e_c + p_d e_d$. Multiplying each by $\frac{(p_c+\tau_c)}{e_d}$ and summing them up gives

$$\frac{d\ln e_d}{d\ln(p_c+\tau_c)} = \left(\epsilon - (1-\nu)^{-1}\right)\frac{(p_c+\tau_c)e_c}{(p_c+\tau_c)e_c + p_d e_d}. \tag{A.53}$$

A.5 Baseline model with learning-by-doing

We now consider learning-by-doing (LBD) in clean energy extraction. Let the price of clean energy be a function of the produced quantity, $p_c = p(e_c)$, with $p'(e_c) < 0$. To ensure a unique interior equilibrium, we assume $p(0) > 0$, $p''(e_c) > 0$, and $\lim_{e_c\to\infty} p(e_c) = \underline{p} > 0$.

Since each firm is too small to affect the overall energy price, their first-order conditions are unaffected by the externality. We start by discussing the first-best implementation, which also involves a tax on dirty energy ($\tau_d > 0$). In this case, the firms' first-order conditions are

$$f_d = p_d + \tau_d \tag{A.54}$$

$$f_c = p_c + \tau_c. \tag{A.55}$$

The social planner's problem is

$$\max_{e_d,e_c} u(f(1,e_d,e_c) - p_d e_d - p(e_c)e_c) - m e_d, \tag{A.56}$$

resulting in the first-order conditions

$$f_d = p_d + \frac{m}{u'(y)}, \tag{A.57}$$

$$f_c = p(e_c) + p'(e_c)e_c. \tag{A.58}$$

The first-best outcome requires a clean energy subsidy of $\tau_c = p'(e_c)e_c$ and a tax on dirty energy equal to $\tau_d = \frac{m}{u'(y)}$. Note that neither the tax nor the subsidy alone can implement the first-best allocation. A single policy that generates the correct *relative* price between clean and dirty energy will deliver an inefficient price *level*.

Now, we consider the constrained-efficient subsidy. As in the baseline case, equation (6) delivers the relationship $e_d = D(e_c)$ with $D'(e_c) = \frac{f_{cd}}{-f_{dd}}$. The presence of LBD has not changed our finding that, for $f_{cd} > 0$, an increase in clean energy also increases dirty energy use.

As in the baseline case, we have $\tau_c = \tau(e_c)$, and we think of the planner as choosing e_c^* to maximize utility subject to $e_d = D(e_c)$. The social planner's problem is

$$\max_{e_c} u\big(f(1, D(e_c), e_c) - p_d D(e_c) - p(e_c)e_c\big) - mD(e_c). \tag{A.59}$$

The first order condition is

$$f_c(1, D(e_c^*), e_c^*) - p(e_c^*) - p'(e_c^*)e_c^* + f_d(1, D(e_c^*), e_c^*)D'(e_c^*) - p_d D'(e_c^*) = D'(e_c^*)\frac{m}{u'(y)}. \tag{A.60}$$

Applying the competitive equilibrium conditions gives

$$\tau_c^* = D'(e_c^*)\frac{m}{u'(y)} + p'(e_c^*)e_c^*. \tag{A.61}$$

Since $p'(e_c^*) < 0$, the LBD externality makes it more likely that it is optimal to subsidize, rather than tax, clean energy, holding all else equal. If $f_{cd} > 0$, it may be optimal to subsidize clean energy, but doing so increases emissions. Put differently, if $f_{cd} > 0$, the subsidy addresses the LBD externality, but does nothing to address the climate externality, which is the focus of our analysis. In this sense, adding LBD does not alter the key intuition from the simple model.

A.6 Baseline model with energy producers under monopoly

We now consider energy producers under monopoly in nested CES-in-CD production. We start by deriving the downward-sloping demand for energy services. Consider the following profit maximization problem for output producers under perfect competition:

$$\max_{l,e} \; l^{1-\nu}e^{\nu} - wl - pe. \tag{A.62}$$

The first order conditions are

$$(1-\nu)\left(\frac{e}{l}\right)^{\nu} = w \tag{A.63}$$
$$\nu\left(\frac{l}{e}\right)^{1-\nu} = p. \tag{A.64}$$

After imposing $l = 1$, the latter first order condition can be rewritten to derive the demand for energy services with respect to its own price:

$$e(p) = \left(\frac{1}{\nu}\right)^{\frac{1}{\nu-1}} p^{\frac{1}{\nu-1}} \tag{A.65}$$
$$= \nu^{\frac{1}{1-\nu}} p^{\frac{1}{\nu-1}} \tag{A.66}$$

Note that

$$e'(p) = \nu^{\frac{1}{1-\nu}} \frac{1}{\nu-1} p^{\frac{1}{\nu-1}-1} \tag{A.67}$$
$$\frac{e(p)}{e'(p)} = (\nu-1)p. \tag{A.68}$$

Before analyzing the profit maximization problem for energy producers under monopoly, we first consider their cost-minimization problem:

$$c(p_c, p_d; e) = \min_{e_c, e_d} \; p_c e_c + p_d e_d \quad \text{where} \quad e = h(e_c, e_d) = \left(\omega_c e_c^{\frac{\epsilon-1}{\epsilon}} + \omega_d e_d^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{A.69}$$

Since $e = h(e_c, e_d)$ exhibits constant returns to scale,

$$c(p_c, p_d; e) = c(p_c, p_d)e \tag{A.70}$$
$$\frac{\partial c(p_c, p_d; e)}{\partial e} = c(p_c, p_d). \tag{A.71}$$

Then, the profit maximization for energy producers under monopoly can be written as

$$\max_{p} \quad pe(p) - c\Big(p_c, p_d; e(p)\Big) \tag{A.72}$$

The first order condition is

$$e(p^{\text{monopolistic}}) + p^{\text{monopolistic}} e'(p^{\text{monopolistic}}) = \frac{\partial c\Big(p_c, p_d; e(p^{\text{monopolistic}})\Big)}{\partial e} e'(p^{\text{monopolistic}}). \tag{A.73}$$

Combining (A.68) with (A.71) yields

$$p^{\text{monopolistic}} = \underbrace{\frac{1}{\nu}}_{\text{Markup}} \underbrace{c(p_c, p_d)}_{\text{MC}}, \tag{A.74}$$

which is different from the equilibrium energy price under perfect competition due to price markup — $p^{\text{competitive}} = c(p_c, p_d)$.

Plugging both prices back to $e(p)$ yields the equilibrium energy service productions:

$$e\left(p^{\text{competitive}}\right) = \nu^{\frac{1}{1-\nu}} \left(c(p_c, p_d)\right)^{\frac{1}{\nu-1}} \tag{A.75}$$

$$e\left(p^{\text{monopolistic}}\right) = \nu^{\frac{2}{1-\nu}} \left(c(p_c, p_d)\right)^{\frac{1}{\nu-1}}. \tag{A.76}$$

Clean energy production subsidies lower p_c, leading to higher equilibrium energy service production due to lower energy prices. But this scale effect is smaller under monopoly due to the price markup chosen by the monopolist:

$$\left| \frac{\partial e\left(p^{\text{competitive}}\right)}{\partial p_c} \right| > \left| \frac{\partial e\left(p^{\text{monopolistic}}\right)}{\partial p_c} \right|, \tag{A.77}$$

because $\nu^{\frac{1}{1-\nu}} > \nu^{\frac{2}{1-\nu}}$ when $\nu \in (0,1)$.

B Analysis of quantitative model

B.1 Characterization of competitive equilibrium

Let $j = y, c, d$ index a 'sector' and $p_{j,t}$ be the price of output from sector j. The first-order conditions are:

$$\rho_t = \alpha p_{j,t} K_{j,t}^{\alpha-1} E_{j,t}^{\nu} \left(A_{j,t} L_{j,t}\right)^{1-\alpha-\nu} \tag{B.1}$$

$$w_t = (1-\alpha-\nu) p_{j,t} K_{j,t}^{\alpha} E_{j,t}^{\nu} A_{j,t}^{1-\alpha-\nu} L_{j,t}^{-\alpha-\nu} \tag{B.2}$$

$$p_{E,t} = \nu p_{j,t} K_{j,t}^{\alpha} E_{j,t}^{\nu-1} \left(A_{j,t} L_{j,t}\right)^{1-\alpha-\nu}, \tag{B.3}$$

where ρ_t is the rental rate, w_t is the wage and $p_{E,t}$ is the price of energy services. For a given j, divide through to get

$$\frac{\rho_t}{w_t} = \frac{\alpha}{1-\alpha-\nu} (K_{j,t}/L_{j,t})^{-1}. \tag{B.4}$$

The only term that varies across sectors is the capital-labor ratio. So, the capital-labor ratio is the same in all sectors and equals the aggregate ratio, i.e., $(K_{j,t}/L_{j,t}) = (K_t/L_t)\ \forall j, t$. Similarly,

$$\frac{p_{E,t}}{w_t} = \frac{\nu}{1-\alpha-\nu} (E_{j,t}/L_{j,t})^{-1}. \tag{B.5}$$

So, $(E_{j,t}/L_{j,t}) = (E_t/L_t)\ \forall j, t$. Together, these results imply that all production factors are used in constant ratios in each sector. We will use $n_{j,t}$ to denote these ratios, which implies the following market clearing condition:

$$1 = n_{y,t} + n_{c,t} + n_{d,t}. \tag{B.6}$$

Here, (B.6) is market clearing condition for labor, and $n_{j,t}$ as the share of labor used in each sector j, which will also be the share of capital and share of energy used in sector j.

The price index for sector j is

$$p_{j,t} = \tilde{\alpha} A_{j,t}^{\alpha+\nu-1} w_t^{1-\alpha-\nu} \rho_t^{\alpha} p_{E,t}^{\nu}, \tag{B.7}$$

where $\tilde{\alpha} \equiv \alpha^{-\alpha} \nu^{-\nu} (1-\alpha-\nu)^{\nu+\alpha-1}$ is a collection of constants. We normalize the price of the final good to one in every period:

$$1 = \tilde{\alpha} A_{y,t}^{\alpha+\nu-1} w_t^{1-\alpha-\nu} \rho_t^{\alpha} p_{E,t}^{\nu}. \tag{B.8}$$

Combined with (B.7), the normalization implies that

$$p_{c,t} = \left(\frac{A_{y,t}}{A_{c,t}}\right)^{1-\alpha-\nu} \tag{B.9}$$

$$p_{d,t} = \left(\frac{A_{y,t}}{A_{d,t}}\right)^{1-\alpha-\nu}. \tag{B.10}$$

Since all firms have constant-returns-to-scale (CRS) production, differences in prices only reflect differences in unit costs. Since the producer in each sector j has a symmetric production function, differences in unit costs are driven entirely by productivity levels. Importantly, (B.8), (B.9), and (B.10) imply that the sector-level prices can be found independently of the rest of the model.

The first order conditions for the energy aggregator are:

$$(1+\tau_d) p_{d,t} = p_{E,t} \omega^{\frac{1}{\epsilon}} \left(\frac{Z_{d,t}}{E_t}\right)^{\frac{-1}{\epsilon}} \tag{B.11}$$

$$(1+\tau_c) p_{c,t} = p_{E,t} (1-\omega)^{\frac{1}{\epsilon}} \left(\frac{Z_{c,t}}{E_t}\right)^{\frac{-1}{\epsilon}}. \tag{B.12}$$

In addition, the price index for energy services is:

$$p_{E,t} = \left(\omega((1+\tau_d)p_{d,t})^{1-\epsilon} + (1-\omega)((1+\tau_c)p_{c,t})^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{B.13}$$

Combining (B.13) with (B.9) and (B.10) yields

$$p_{E,t} = \left(\omega\left((1+\tau_d)\left(\frac{A_{y,t}}{A_{d,t}}\right)^{1-\alpha-\nu}\right)^{1-\epsilon} + (1-\omega)\left((1+\tau_c)\left(\frac{A_{y,t}}{A_{c,t}}\right)^{1-\alpha-\nu}\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{B.14}$$

All of the variables on the right-hand side (RHS) of this equation evolve exogenously, and $p_{E,t}$ can be found independently of the rest of the model.

Household optimization yields the Euler equation

$$C_{t+1} = \beta(1+n)(1+r_{t+1})C_t \tag{B.15}$$

where $r_{t+1} = \rho_{t+1} - \delta$ is the real interest rate. The transversality condition is

$$\lim_{T\to\infty} \beta^T K_{T+1} C_T^{-1} = 0. \tag{B.16}$$

B.2 Intensive form

Let $d_t = \frac{D_t}{A_{y,t}L_t}$ for any variable D_t. It is straightforward to re-write production functions (20) and (22) as:

$$y_t = k_t^{\alpha} e_t^{\nu} n_{y,t} \tag{B.17}$$

$$z_{d,t} = k_t^{\alpha} e_t^{\nu} n_{d,t} p_{d,t}^{-1} \tag{B.18}$$

$$z_{c,t} = k_t^{\alpha} e_t^{\nu} n_{c,t} p_{c,t}^{-1}. \tag{B.19}$$

In addition, using first order conditions (B.1) and (B.3) along with price normalization (B.8) gives factor demands:

$$\rho_t = \alpha k_t^{\alpha-1} e_t^{\nu} \tag{B.20}$$

$$p_{E,t} = \nu k_t^{\alpha} e_t^{\nu-1}. \tag{B.21}$$

The demand equations for primary energy, (B.11) and (B.12), can be re-written as:

$$(1+\tau_d) p_{d,t} = p_{E,t} \omega^{\frac{1}{\epsilon}} \left(\frac{z_{d,t}}{e_t}\right)^{\frac{-1}{\epsilon}} \tag{B.22}$$

$$(1+\tau_c) p_{c,t} = p_{E,t} (1-\omega)^{\frac{1}{\epsilon}} \left(\frac{z_{c,t}}{e_t}\right)^{\frac{-1}{\epsilon}}. \tag{B.23}$$

To complete the static equations, we have the labor-market clearing condition from above:

$$1 = n_{c,t} + n_{d,t} + n_{y,t}. \tag{B.24}$$

Then, to close the model, we have the two standard dynamic equations. The law of motion for capital (29) is

$$k_{t+1} = \frac{y_t - c_t + (1-\delta) k_t}{(1+g_y)(1+n)} \tag{B.25}$$

and the Euler equation is

$$c_{t+1} = \frac{\beta(1 + r_{t+1})}{(1 + g_y)} c_t. \tag{B.26}$$

Recall that the price variables, $\{p_{E,t}, p_{c,t}, p_{d,t}\}_{t=0}^{\infty}$, can be determined prior to solving the model. The period-to-period dynamics of $\{y_t, k_t, r_t, c_t, e_t, z_{d,t}, z_{c,t}, n_{y,t}, n_{d,t}, n_{c,t}\}$ are pinned down by the ten equations (B.17) – (B.26). Note that only the consumption Euler equation (B.15) and the capital stock's equation of motion have intertemporal components. For boundary conditions, we have k_0 given and a tranversality condition for c_t. In practice, the latter is satisfied by the fact that the economy converges to a BGP.

B.3 Intermediate results

In this section, we derive some helpful intermediate results that are useful for characterizing the balanced growth path (BGP) and to simplifying the computational solution.

To start, we take the ratio of the intensive-form inverse demand functions for primary energy, (B.22) and (B.23), to get:

$$\frac{(1 + \tau_d) p_{d,t}}{(1 + \tau_c) p_{c,t}} = \tilde{\omega} \left(\frac{z_{c,t}}{z_{d,t}} \right)^{\frac{1}{\epsilon}}, \tag{B.27}$$

where $\tilde{\omega} \equiv \left(\frac{\omega}{1-\omega}\right)^{\frac{1}{\epsilon}}$ is a collection of constants. Then, we plug in the intensive form primary energy production functions, (B.18) and (B.19), to get:

$$\frac{(1 + \tau_d) p_{d,t}}{(1 + \tau_c) p_{c,t}} = \tilde{\omega} \left(\frac{n_{c,t} p_{c,t}^{-1}}{n_{d,t} p_{d,t}^{-1}} \right)^{\frac{1}{\epsilon}} \tag{B.28}$$

and solve for

$$\frac{n_{c,t}}{n_{d,t}} = \tilde{\omega}^{-\epsilon} \left(\frac{p_{d,t}}{p_{c,t}} \right)^{\epsilon - 1} \left(\frac{1 + \tau_d}{1 + \tau_c} \right)^{\epsilon} \equiv \tilde{n}_{cd,t}, \tag{B.29}$$

which, by equations (B.9) and (B.10), can be found independently of the rest of the model.

Now, we combine this with the labor market clearing condition to write:

$$1 - n_{y,t} = \tilde{n}_{cd,t} n_{d,t} + n_{d,t} \;\; \Rightarrow \tag{B.30}$$
$$n_{d,t} = \frac{1 - n_{y,t}}{1 + \tilde{n}_{cd,t}}, \tag{B.31}$$
$$n_{c,t} = \tilde{n}_{cd,t} \left(\frac{1 - n_{y,t}}{1 + \tilde{n}_{cd,t}} \right). \tag{B.32}$$

Thus, we have written all of the labor market allocation in terms of one variable, $n_{y,t}$.

We now move to expressing $n_{y,t}$ as a function of prices only. Using the intensive form production functions for primary energy, (B.18) and (B.19), we can re-write the production function for energy services, (21), as

$$E_t = A_{y,t} L_t k_t^{\alpha} e_t^{\nu} \left(\omega^{\frac{1}{\epsilon}} (n_{d,t} p_{d,t}^{-1})^{\frac{\epsilon-1}{\epsilon}} + (1-\omega)^{\frac{1}{\epsilon}} (n_{c,t} p_{c,t}^{-1})^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \;\; \Rightarrow \tag{B.33}$$
$$e_t = \frac{y_t}{n_{y,t}} \left(\omega^{\frac{1}{\epsilon}} (n_{d,t} p_{d,t}^{-1})^{\frac{\epsilon-1}{\epsilon}} + (1-\omega)^{\frac{1}{\epsilon}} (n_{c,t} p_{c,t}^{-1})^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{B.34}$$

Factoring out $n_{d,t}$ and utilizing (B.29) gives

$$e_t = \frac{y_t}{n_{y,t}} \left(\frac{1 - n_{y,t}}{1 + \tilde{n}_{cd,t}} \right) \tilde{p}_{E,t}^{-1}, \tag{B.35}$$

where

$$\tilde{p}_{E,t}^{-1} \equiv \left(\omega^{\frac{1}{\epsilon}} (p_{d,t}^{-1})^{\frac{\epsilon-1}{\epsilon}} + (1-\omega)^{\frac{1}{\epsilon}} (\tilde{n}_{cd,t} p_{c,t}^{-1})^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{B.36}$$

Rearranging the intensive form of the first-order condition for energy, (B.21), gives

$$e_t = \left(\frac{\nu}{p_{E,t}} \right) \left(\frac{y_t}{n_{y,t}} \right). \tag{B.37}$$

Combining (B.35) and (B.37) gives

$$\frac{y_t}{n_{y,t}} \left(\frac{1 - n_{y,t}}{1 + \tilde{n}_{cd,t}} \right) \tilde{p}_{E,t}^{-1} = \left(\frac{\nu}{p_{E,t}} \right) \left(\frac{y_t}{n_{y,t}} \right). \tag{B.38}$$

which we can solve for

$$n_{y,t} = 1 - \nu \left(\frac{\tilde{p}_{E,t}}{p_{E,t}} \right) (1 + \tilde{n}_{cd,t}). \tag{B.39}$$

We can then plug back into (B.31) and (B.32) to get clean and dirty labor allocations as a function of prices.

$$n_{d,t} = \frac{1 - n_{y,t}}{1 + \tilde{n}_{cd,t}}, \tag{B.40}$$

$$n_{c,t} = \tilde{n}_{cd,t} \left(\frac{1 - n_{y,t}}{1 + \tilde{n}_{cd,t}} \right). \tag{B.41}$$

We have now written all of the labor allocations as functions only of prices, which we can take as given in the computational solution.

B.4 Balanced growth path (BGP)

On the BGP, all intensive form variables are constant. We use $\bar{d}$ to denote the BGP value of some variable d_t. In addition, we assume that, after some future date, all technologies grow at the same constant rate and policies reach a constant level. Together, these assumptions imply that energy prices are constant.

From the Euler equation (B.26),

$$\bar{r} = \frac{(1+g)}{\beta} - 1. \tag{B.42}$$

We use factor demands (B.20) and (B.21), to get

$$\bar{k} = \alpha^{\frac{1-\nu}{1-\alpha-\nu}} (\bar{\rho})^{\frac{\nu-1}{1-\alpha-\nu}} \left(\frac{\nu}{\bar{p}_E} \right)^{\frac{\nu}{1-\alpha-\nu}} \tag{B.43}$$

$$\bar{e} = \left(\frac{\nu}{\bar{p}_E} \right)^{\frac{1}{1-\nu}} \bar{k}^{\frac{\alpha}{1-\nu}}. \tag{B.44}$$

From (B.39), (B.40), and (B.41), we know the labor allocations as a function of prices:

$$\bar{n}_y = \left[1 + \nu \left(\frac{\bar{\tilde{p}}_E}{\bar{p}_E} \right) (1 + \bar{\tilde{n}}_{cd}) \right]^{-1} \tag{B.45}$$

$$\bar{n}_d = \frac{1 - \bar{n}_y}{1 + \bar{\tilde{n}}_{cd}}, \tag{B.46}$$

$$\bar{n}_c = \bar{\tilde{n}}_{cd} \left(\frac{1 - \bar{n}_y}{1 + \bar{\tilde{n}}_{cd}} \right). \tag{B.47}$$

Production functions (B.17), (B.18), and (B.19) yield

$$\bar{y} = \bar{k}^{\alpha}\bar{e}^{\nu}\bar{n}_y \tag{B.48}$$

$$\bar{z}_d = \bar{k}^{\alpha}\bar{e}^{\nu}\bar{n}_d\bar{p}_d^{-1} \tag{B.49}$$

$$\bar{z}_c = \bar{k}^{\alpha}\bar{e}^{\nu}\bar{n}_c\bar{p}_c^{-1}. \tag{B.50}$$

Finally, we rearrange the law of motion for capital, (B.25) to arrive at

$$\bar{c} = \bar{y} + (1-\delta)\bar{k} - (1+g)(1+n)\bar{k}. \tag{B.51}$$

The ten equations (B.42) – (B.51) give the BGP values for the ten intensive form variables that define the evolution of the economy. The tranversality condition (B.16) can be rewritten as

$$\lim_{T\to\infty} \beta^T (1+g)(1+n)\bar{k}\bar{c}^{-1} = 0, \tag{B.52}$$

which is clearly satisfied, since $\beta < 1$ and all of the other terms are constant.

B.5 Solution method

A key result from this appendix is that all prices and labor allocations can be solved separately from the rest of the model. Indeed, we have closed form solutions for these relations. After solving for prices and labor allocations, we can separate out the following intensive form equations:

$$\begin{aligned}
k_{t+1} &= \frac{y_t - c_t + (1-\delta)k_t}{(1+g)(1+n)} \\
c_{t+1} &= \frac{\beta(1+r_{t+1})}{(1+g)}c_t. \\
y_t &= k_t^{\alpha}e_t^{\nu}n_{y,t} \\
\rho_t &= \alpha k_t^{\alpha-1}e_t^{\nu} \\
p_{E,t} &= \nu k_t^{\alpha}e_t^{\nu-1},
\end{aligned}$$

which give the dynamics for $\{k_t, c_t, y_t, r_t, e_t\}$ independently of the other variables (recall, $r_t = \rho_t - \delta$). These equations represent a fairly standard growth model with two exogenously evolving parameters, $n_{y,t}$ and $p_{E,t}$. We solve this set of equations using the perfect foresight solver in *Dynare* (Adjemian et al., 2011). To find the primary energy allocations, we then plug back into (B.18) and (B.19).

To check the accuracy of the model, we can test the intensive form equations that were excluded from this solution method, because they were used to derive the closed form labor allocations. These are:

$$(1+\tau_d)p_{d,t} = p_{E,t}\omega^{\frac{1}{\epsilon}}\left(\frac{z_{d,t}}{e_t}\right)^{\frac{-1}{\epsilon}}$$
$$(1+\tau_c)p_{c,t} = p_{E,t}(1-\omega)^{\frac{1}{\epsilon}}\left(\frac{z_{c,t}}{e_t}\right)^{\frac{-1}{\epsilon}}$$
$$1 = n_{c,t} + n_{d,t} + n_{y,t}.$$

For the sake of convenience, we also use Dynare to (i) perform these checks, (ii) to solve for the intermediate variables, $p_{E,t}$, $\tilde{p}_{E,t}$ and $\tilde{n}_{cd,t}$, and (iii) solve for the labor allocations, $\{n_{y,t}, n_{d,t}, n_{c,t}\}$.

B.6 Calibration

B.6.1 Damages

Dirty energy use in period $t = 0$ $(Z_{d,0})$ causes lifetime utility damages of

$$\sum_{t=0}^{\infty}\beta^t m\eta_0 Z_{d,0} = m\eta_0\frac{Z_{d,0}}{1-\beta}. \tag{B.53}$$

To convert this value to dollar, we divide by the marginal utility of consumption, $1/C_0$. Then, we multiply by $1/Y_0$ to express this cost as a fraction of GDP, which we match to a given value Ξ. Thus, we solve:

$$\Xi = m\frac{\eta_0 Z_{d,0}}{(1-\beta)}\frac{C_0}{Y_0} \tag{B.54}$$

to calibrate m.

To calculate Ξ, we take the social cost of carbon from Rennert et al. (2022) (185\$/tCO_2, measured in 2020 US dollars) and multiply it by tonnes of CO_2 emitted in the US in 2020. Then, we divide it by US GDP in 2020. We find that the external cost of dirty energy use is 4 percent of US GDP in 2020.

B.6.2 Lifetime welfare calculation

To calculate welfare, we start with the definition

$$U = \sum_{t=0}^{\infty} \beta^t \big(L_t \ln(C_t) - m \sum_{v=0}^{t} \eta_v Z_{d,v} \big). \tag{B.55}$$

Then, we note that the total marginal cost of carbon is $\frac{m}{1-\beta}$, which lets us re-write the utility function as

$$U = \sum_{t=0}^{\infty} \beta^t \left(L_t \ln(C_t) - \left(\frac{m}{1-\beta} \right) \eta_t Z_{d,t} \right). \tag{B.56}$$

We simulate the model for $T+1$ periods. It is then straightforward to calculate the flow utility in the first T periods.

To calculate the continuation values, we assume that consumption and dirty energy grow at constant rates g_c and g_d in $T+1$ and beyond (in practice the economy is on the BGP well before we stop simulating). The continuation value is

$$\begin{aligned}
&\beta^T \sum_{v=0}^{\infty} \left(L_T \big(\beta(1+n)\big)^v \ln\left(C_T(1+g_c)^v\right) - \left(\frac{m}{1-\beta} \right) \eta_T Z_{d,T} \beta^v (1+g_\eta)^v (1+g_d)^v \right) \\
&= \beta^T \left(\frac{L_T \ln(C_T)}{1-\beta(1+n)} + L_T \ln(1+g_c) \sum_{v=0}^{\infty} \big(\beta(1+n)\big)^v v - \frac{m\eta_T Z_{d,T}}{(1-\beta)\big(1-\beta(1+g_\eta)(1+g_d)\big)} \right) \\
&= \beta^T \left(\frac{L_T \ln(C_T)}{1-\beta(1+n)} + \frac{L_T \ln(1+g_c)\beta(1+n)}{\big(1-\beta(1+n)\big)^2} - \frac{m\eta_T Z_{d,T+1}}{(1-\beta)\big(1-\beta(1+g_\eta)(1+g_d)\big)} \right).
\end{aligned} \tag{B.57}$$

B.6.3 CEV calculation

Let variables with a tilde ($\tilde{}$) denote the outcomes with policy and variables without the tilde denote outcomes along the laissez-faire BGP. Using the procedure outlined in the previous subsection, we calculate lifetime utility $\tilde{U}$ and U. Then, for the CEV, we find ψ such that

$$\tilde{U} = \sum_{t=0}^{\infty} \beta^t \big(L_t \ln(\psi C_t) - m \sum_{v=0}^{t} \eta_v Z_{k,v} \big). \tag{B.58}$$

To start, we pull out ψ to get

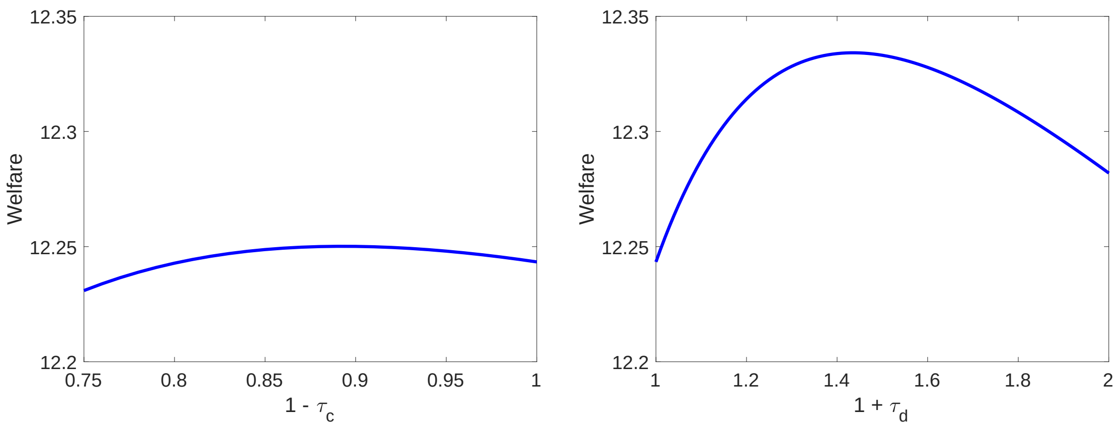
$$\tilde{U} = \sum_{t=0}^{\infty} \beta^t L_t \ln(\psi) + \sum_{t=0}^{\infty} L_t \beta^t \big(L_t \ln(C_t) - m \sum_{v=0}^{t} \eta_v Z_{d,v}\big) \tag{B.59}$$

$$= L_0 \sum_{t=0}^{\infty} \big(\beta(1+n)\big)^t \ln(\psi) + U. \tag{B.60}$$

So, to calculate the CEV, we take

$$\psi = \exp\left[\big(1 - \beta(1+n)\big)\left(\frac{\tilde{U} - U}{L_0}\right)\right]. \tag{B.61}$$

B.7 Additional simulation results



(a) Clean Energy Subsidy

(b) Dirty Energy Tax

Figure B.1: Constrained Planner's Objective Function

During the preparation of this work, the authors used ChatGPT for minor language editing.