

# Pricing Externalities in the Presence of Adaptation

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## Abstract

I study the optimal taxation on externality-generating activities in a general equilibrium model in which households compete against final-goods producers for pollutant-intensive intermediate goods to adapt to environmental degradation—such as the use of energy for cooling. I show theoretically that an increase in market demand for such goods due to adaptation shifts up the marginal profit of polluting firms. In response, forward-looking polluting industries increase their production leading to higher pollution levels. Using a macroeconomic climate-economy model as an example, I find that about 7% of the Pigou tax for correcting heat-related discomfort is due to the unintended warming that is caused by residential energy use for cooling in 2020.

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# 1 Introduction

When people anticipate an adverse change in environmental conditions, they may engage in *private adaptation*—the process of adjusting one’s own behavior to reduce related negative externalities. Some examples are reroofing with asphalt shingles against wildfires, building a house on concrete stilts to prevent flooding, installing air purifiers to reduce the amount of pollutants inhaled, etc. A failure to consider such behaviors may lead to the overstatement of the social cost of externality-generating activities ([Graff Zivin and Neidell, 2013](#); [Kahn, 2016](#)). As such, the conventional wisdom in partial equilibrium analyses is that considering private adaptation in a cost-benefit analysis will lead to a lower Pigouvian tax because they tend to focus narrowly on the role of adaptation in reducing external marginal costs. But when people increase their demand for pollutant-intensive inputs—such as asphalt, cement, or energy—for adaptation, its market demand increases leading to higher pollution levels. A key question is, “how should policymakers account for such adaptation-driven general equilibrium effects when pricing externalities?”

This paper examines the role of pollutant-intensive adaptation in setting optimal pollution taxes. To this end, I compare allocations in a general equilibrium model in which households adapt relative to the benchmark without adaptation. The key ingredient of this comparative statics is an increase in market demand for pollutant-intensive inputs for adaptive purposes. I show theoretically that endogenizing pollutant-intensive adaptation in resource constraint and utility not only reduces the external marginal costs, but also shifts up the marginal profit of polluting firms—such as cement industries—which leads to a higher level of pollution. The second channel has been previously overlooked as the existing studies implicitly capture adaptation by netting out its net benefit from pollution damages. Neglecting such general equilibrium effects in a cost-benefit analysis may lead to a lower-than-optimal pollution tax. I use a dynamic climate-economy model with heat-related discomfort and cooling energy as a stylized example to quantify how much of the Pigou tax is due to an adaptation-driven upward shift in the marginal profit of intermediate firms.

This paper identifies a general equilibrium channel through which pollutant-intensive adaptation shifts up the “marginal benefit of pollution” (or the marginal cost of pollution abatement). While some studies balance the cost and benefit of investing in abatement projects such as scrubbers on smokestacks, I compare the cost and benefit of emitting pollutants as a byproduct of the production of intermediate goods throughout this paper. Provided that the economy’s total production of a pollutant-intensive input is fixed in the short run, transferring some from manufacturers to individuals for adaptation will crowd out this factor that can be used to produce other consumer goods. The crowding-out of a pollutant-intensive input has unintended consequences on factor prices that are determined by their marginal products in a competitive factor market. First, this scarcity raises the price of a pollutant-intensive input because of the diminishing marginal product. Second, this shortage makes wages go down because the value of marginal product of labor declines due to the complementarity between labor and pollutant-intensive inputs. As a consequence, the marginal profit of polluting firms shifts up because they can produce at a lower cost but can sell at a higher price. In response, polluting firms will increase their production in the long run.

The magnitude of such effects relies on how much pollutant-intensive goods households demand for self-protection in response to endogenously changing pollution levels. I use a macroeconomic climate-economy model à la [Goloso, Hassler, Krusell, and Tsyvinski \(2014\)](#), which is augmented with the use of energy for cooling against heat-related discomfort. The key ingredient is the nonseparability between non-market goods and market goods in utility, which is captured by the constant elasticity of substitution (CES) between carbon abatement and cooling adaptation. Since emissions abatement is a substitute to cooling, an increase in temperature may increase the residential energy demand for cooling, which thereby shifts up the marginal profit of power generators and increases carbon emissions. To pin down the magnitude of substitutability, I calibrate a quantitative climate-economy model to match some recent reduced-form causal evidence on the global mortality cost of climate change and electricity consumption by [Carleton et al. \(2022\)](#) and [Rode et al. \(2021\)](#).

I first compare the competitive equilibria with endogenous cooling to the ones without adaptation to examine how fossil fuel-based energy producers react to the changes in their marginal profits. For example, when there are no carbon taxes, about 3.4% of energy is used for cooling by the end of this century. Because of an increase in the aggregate demand for energy, the share of capital and labor in the fossil fuel-based energy sector rises by about 0.17 percentage point and about 0.06 percentage point in 2100 to clear the market, which leads to an increase in emissions per period (5 years) by about 33.3 giga ton of CO<sub>2</sub>.

To quantify the role of such general equilibrium effects on environmental policies, I next compare the Pigou tax with endogenous adaptation to the case in which a damage function implicitly includes the benefits and costs of adaptation. Since adaptation is embedded in a damage function in the second case, the resource constraint for energy do not change as temperature rises and, thus, the general equilibrium effects do not arise. I find that about 7% of the Pigou tax for correcting heat-related mortality is due to the unintended warming that is additionally caused by the use of energy for cooling for the base period (2020).

Finally, I examine whether energy efficiency improvements can be accepted as a carbon abatement strategy in the absence of government intervention. In my model, the elasticity of substitution between environmental quality and cooling measures how efficient adaptation technology is in reducing marginal climate impacts. An increase in the substitutability will reduce expenditures from energy savings, which may, however, increase energy use due to income effects. I find that a 0.1% increase in the substitutability boosts the use of energy for cooling by about 1% in 2100 in a competitive equilibrium without carbon taxes, which means income effects dominate the direct energy savings. But the adaptation benefit of cooling from the enhanced efficacy dominates the unintended warming, which decreases the mortality costs of carbon from \$202 per ton of CO<sub>2</sub> to \$199 per ton of CO<sub>2</sub> in 2010 USD in 2100. The associated welfare gain is about 1% in consumption equivalent variation of households alive in 2020, which implies that the problem of an increase in energy consumption resulting from an efficiency improvement is not as consequential in this case as the literature contends.

This paper offers a structural framework for reconciling the two seemingly contradictory strands of reduced-form studies on cooling energy consumption. One focuses on its role as self-protective measures in reducing mortality sensitivities to weather variations ([Deschênes and Greenstone, 2011](#); [Barreca, Clay, Deschênes, Greenstone, and Shapiro, 2016](#); [Heutel, Miller, and Molitor, 2021](#); [Carleton et al., 2022](#)). The other underscores the adverse effects of climate-driven cooling energy demand by showing that electricity consumption responses to heat waves are much larger in areas with higher levels of long-run average temperature ([Davis and Gertler, 2015](#); [Biardeau, Davis, Gertler, and Wolfram, 2020](#); [Rode et al., 2021](#); [Auffhammer, 2022](#); [Deschênes, 2022](#)). Depending on which perspective an economic modeler takes, the welfare implication of cooling energy usage will be substantially different. This paper brings the two seemingly incongruous aspects of cooling energy usage into a single framework by specifying household preferences for adaptation in a climate-economy model.

This paper also conducts a consistent welfare analysis of climate change and adaptation by taking a dynamic general equilibrium approach in line with other macroeconomic studies on endogenous climate such as [Acemoglu, Aghion, Bursztyn, and Hémous \(2012\)](#), [Golosov et al. \(2014\)](#), and [Barrage \(2020b\)](#). In general, it may not be innocuous to extrapolate a dose-response relation between economic outcomes and weather fluctuations using exogenously given emissions pathways, which is a common practice in reduced-form studies; see [Hsiang \(2016\)](#) for a review. For example, as much as mortality sensitivities to temperature fluctuations decline due to cooling, an ensuing increase in emissions can feed back into the economy by heightening the risks of heat-related discomfort, which can further elevate the use of energy for cooling leading to a different trajectory of carbon emissions. This type of analysis will be valid if exogenously given scenarios are in line with its implied emissions, but it may not be ideal for simulating various policy counterfactuals that can endogenously change emissions pathways. This paper assumes that people adjust their behaviors under rational expectations about climate and equilibrium temperature turns out to be consistent with their expectations.

This paper also contributes to our understanding of the interrelation between private and public adjustments to climate externalities. Most structural cost-benefit studies on climate change lump all the relevant welfare effects of adaptation into one stylized damage function; see [Fankhauser \(2017\)](#) for a review. Specifically, each locus on this damage curve represents the least-cost combination of adaptation costs and ceteris paribus temperature effects net of adaptation benefits. But there is an emerging literature that explicitly addresses adaptation. An earlier strand of the literature decomposes the climate damage in [Nordhaus and Boyer \(2000\)](#) on an ad-hoc basis to model adaptation as a decision variable ([de Bruin, Dellink, and Tol, 2009](#)).<sup>1</sup> More recently, a couple of studies have used micro-data to build an empirically-grounded damage function with adaptation in a quantitative macroeconomic model such as [Fried \(2021\)](#), [Balboni \(2021\)](#), [Conte \(2022\)](#), [Cruz and Rossi-Hansberg \(2022\)](#), [Nath \(2022\)](#), and [Rudik, Lyn, Tan, and Ortiz-Bobea \(2022\)](#). In particular, [Barrage \(2020a\)](#) develops a climate-economy model with distortionary taxes in which climate change affects public investments in adaptation, government consumption requirements, tax revenue, and transfer payment to examine the interplay between optimal carbon taxes and the fiscal burden caused by climate change and public adaptation. But none of the previous studies examine private adaptation using pollutant-intensive intermediate goods and, thus, general equilibrium effects in factor markets do not arise in response to endogenously evolving climate.

The remainder of this paper proceeds as follows. In section 2, I illustrate how pollutant-intensive adaptation shifts up the marginal profit of polluting industries using a static model. Section 3 introduces a macroeconomic climate-economy model enriched with cooling against heat-related discomfort as a stylized example, which is calibrated in section 4. Section 5 presents quantitative results and I conclude with a discussion of the implication of this paper for reduced-form environmental studies in section 6.

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<sup>1</sup>See also [Settle, Shogren, and Kane \(2007\)](#), [Bosello \(2010\)](#), [Bosello, Carraro, and de Cian \(2010\)](#), [Agrawala et al. \(2011\)](#), and [Millner and Dietz \(2015\)](#) that use climate damages from [Nordhaus and Yang \(1996\)](#) or [Nordhaus and Boyer \(2000\)](#). Other integrated assessment models that account for adaptation are the Climate Framework for Uncertainty, Negotiation and Distribution (FUND) by [Tol \(2007\)](#) and the Policy Analysis of the Greenhouse Effect (PAGE) by [Hope \(2011\)](#).

## 2 Static model of externalities with private adaptation

I build a static model to illustrate how pollutant-intensive adaptation shifts the marginal profit of polluters by altering factor prices. For simplicity, I consider a cutback in the production of pollutant-intensive intermediate goods as emissions abatement strategies and abstract from carbon-free technologies. In section 3, I build a dynamic climate-economy model with carbon-free technologies, capital accumulation, and carbon dynamics for quantitative analyses.

### 2.1 Environment

**Household** A representative household has preferences over non-durable consumption  $C$ , pollution  $T$ , and pollutant-intensive goods for private adaptation  $E^H$ . For simplicity, I assume quasi-linearity (to be relaxed in section 3). The literature has not reached a consensus on how the shape of utility function varies with health status (Finkelstein, Luttmer, and Notowidigdo, 2009). I assume a state independence of consumption with respect to pollution externalities and focus on the interdependence of private adaptation and pollution as follows

$$u(C, T, E^H) = C - h(T, E^H),$$

where  $\frac{\partial h(T, E^H)}{\partial T} \geq 0$ ,  $\frac{\partial^2 h(T, E^H)}{\partial (T)^2} \geq 0$ ,  $\frac{\partial h(T, E^H)}{\partial E^H} \leq 0$ , and  $\frac{\partial^2 h(T, E^H)}{\partial (E^H)^2} \geq 0$ . The household takes  $T$  as given and, thus, it is an externality. Utility damages are determined by the pollution level  $T$  and adaptation  $E^H$ . I consider heat-related discomfort from climate change  $T$  and cooling  $E^H$  as an example, but the framework is general enough to capture a wide set of pollutant-intensive goods for adaptation such as cement stilts or asphalt shingles. Specifically, I model adaptation as a flow decision and do not explicitly consider investments in durable goods—such as air conditioners—to focus on the pecuniary effects caused by the use of energy for space cooling. Following the common practice in the environmental macroeconomics literature, I define  $T$  as a change in the global mean surface temperature relative to the pre-industrial level. The size of population is normalized to one and the household supplies one unit of labor inelastically.

Importantly, I assume that the sign of the cross-partial derivative of utility damages with respect to climate  $T$  and household energy consumption  $E^H$  is nonpositive.

**Assumption 1** For any  $(T, E^H) \in \mathbb{R}_+^2$ ,  $\frac{\partial}{\partial E^H} \left( \frac{\partial h(T, E^H)}{\partial T} \right) \leq 0$ .

This assumption implies that the marginal impacts of climate are smaller when a household takes an additional self-protective measure. It is consistent with the empirical observation that the dose-response relationship between extreme heat events and mortality rates becomes less sensitive as either per capita income or long-run average temperature rises, which are key indicators of private adaptation (Barreca et al., 2016; Heutel et al., 2021; Carleton et al., 2022). As an extreme case, if the cross-partial derivative becomes zero, cooling will lessen damage to utility in level, but the slope of the marginal damage curve will remain unchanged. If the cross-partial derivative is negative, the marginal damage curve becomes flatter as a household increases its energy use for self-protection. Many empirical studies find a similar negative association between key determinants of adaptation and the marginal effect of environmental stresses such as extreme heat events or hurricanes.<sup>2</sup>

In other words, this assumption implies that damage reductions from adaptive responses are stronger in magnitude with a more drastic change in the climate.<sup>3</sup> It is aligned with the empirical evidence that dose-response relations between extreme heat events and electricity usage become more sensitive as either per capita income or a long-run average temperature rises (Rode et al., 2021; Auffhammer, 2022). Since abating carbon curbs global warming, this complementarity captures how much emissions abatement can be substituted for cooling.

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<sup>2</sup>See for example Sadowski and Sutter (2008), Keefer, Neumayer, Plümper (2011), Hsiang and Narita (2012), Deryugina and Hsiang (2017), Gourio and Fries (2020), Fried (2021), Cruz and Rossi-Hansberg (2022) and Nath (2022).

<sup>3</sup>It follows from the Young's theorem that the Assumption 1 can be rewritten as  $\frac{\partial}{\partial T} \left( -\frac{\partial h(T, E^H)}{\partial E^H} \right) \geq 0$ .



**Production** A representative firm in the final goods sector combines labor  $L^Y$  and energy  $E^Y$  to produce output  $Y$ . Its technology  $F^Y$  exhibits constant returns to scale with positive and diminishing marginal returns, satisfying the Inada condition

$$Y = F^Y(L^Y, E^Y). \quad (1)$$

A representative firm in the intermediate goods sector hires labor  $L^E$  to generate energy  $E$ . The production of energy is linear in labor  $L^E$

$$E = A^E L^E \quad \text{where} \quad A^E > 0. \quad (2)$$

Both labor and energy are perfectly mobile across the sectors

$$L^Y + L^E = 1 \quad \text{and} \quad E^H + E^Y = E. \quad (3)$$

Energy can be substituted for any pollutant-intensive intermediate goods such as cement.

**Carbon cycle** Producing energy yields carbon as its byproduct. I normalize  $E$  such that it can be expressed in the same unit of its carbon content. That is, one unit of energy makes one unit of carbon. I assume a linear model of warming with respect to carbon emissions (to be discussed further in section 3)

$$T = \zeta E \quad \text{where} \quad \zeta > 0. \quad (4)$$

This can be generalized to a relationship between any other pollutants and pollution such as cement production and particulate matter levels in the atmosphere.

## 2.2 Social planner's problem

The main goal of this paper is to improve our understanding of the general equilibrium effects of pollutant-intensive adaptation on the marginal profit of polluting firms. I compare the two otherwise identical economies that differ in households' ability to adapt to uncover the role of pollutant-intensive adaptation on both the external marginal costs and private marginal benefit of pollution.

To provide comparative statics for the changes in the availability of energy as adaptation measures, I generate an objective function with a dummy parameter  $\theta \in \{0, 1\}$  as follows

$$d(T, E^H; \theta) = \theta \cdot h(T, E^H) + [1 - \theta] \cdot g(T) = \begin{cases} h(T, E^H) & \text{if } \theta = 1 \\ g(T) & \text{otherwise} \end{cases}.$$

Here,  $g$  denotes the utility damage caused by climate change without private adaptation, which is increasing and convex in  $T$ . I assume that marginal climate damages are smaller when a household adapts;  $\frac{dg(T)}{dT} \geq \frac{\partial h(T, E^H)}{\partial T}$  for all  $(T, E^H) \in \mathbb{R}_+^2$ . This parameterization is along the lines of the monotone comparative statics (Milgrom and Shannon, 1994), which allows for a discrete change in parameter space. If  $\theta = 1$ , households can use energy for space cooling. Otherwise, energy is not available as an adaptive measure (benchmark case).

Given  $\theta \in \{0, 1\}$ , a planner solves the following problem:  $\max_{\{C, T, E^H, L^Y, E^Y, L^E\}} C - d(T, E^H; \theta)$  subject to (1), (2), (3), (4), and  $C = Y$ , as well as nonnegativity constraints for choice variables. Substituting the constraints into the objective function, I can transform the planner's problem into a unconstrained optimization with two choice variables and one dummy parameter  $\theta$

$$\max_{\{L^E, E^H\}} W(L^E, E^H; \theta) = F^Y(1 - L^E, A^E L^E - E^H) - d(\zeta A^E L^E, E^H; \theta).$$

The planner decides how much carbon to release into the atmosphere by adjusting labor in the energy sector  $L^E$ , while protecting households from climate damages using energy  $E^H$ .

The first order conditions are given by;

$$\underbrace{-\theta \cdot \frac{\partial h(T, E^H)}{\partial E^H}}_{\text{Marginal benefit of adaptation}} = \underbrace{\frac{\partial F^Y(L^Y, E^Y)}{\partial E^Y}}_{\text{Marginal cost of adaptation}}, \text{ and} \quad (5)$$

$$\underbrace{\left[ \theta \cdot \frac{\partial h(T, E^H)}{\partial T} + [1 - \theta] \cdot \frac{dg(T)}{dT} \right] \zeta}_{\text{Marginal external cost from carbon emissions}} = \underbrace{\left[ \underbrace{\frac{\partial F^Y(L^Y, E^Y)}{\partial E^Y}}_{\text{Gains from energy}} - \underbrace{\frac{\partial F^Y(L^Y, E^Y)}{\partial L^Y} \cdot \frac{1}{A^E}}_{\text{Losses from labor reallocation}} \right]}_{\text{Marginal private profit from carbon emissions}}. \quad (6)$$

Without clean energy, the carbon inventory is one-to-one related to the energy production in the economy, which is determined by the employment in the energy sector. Given any  $L^E \in [0, 1]$ , the climate  $T$  is determined according to (4). The planner then decides how much energy to allocate for households— $E^H(L^E)$ —balancing its damage reductions and the losses from foregone consumption as in (5). While taking as given this contingent plan  $E^H(L^E)$ , the planner balances the external cost and private benefit from emissions as in (6). If  $\theta = 0$ , only the condition (6) becomes relevant; the planner would not allocate any energy for households.

To examine how efficient allocations change as adaptation becomes relevant ( $\theta = 0 \rightarrow 1$ ), I use the monotone comparative statics method by [Milgrom and Shannon \(1994\)](#).

**Proposition 1**  $W(L^E, E^H; \theta)$  has increasing differences in  $(L^E, \theta)$ ,  $(E^H, \theta)$ , and  $(L^E, E^H)$ .

*Proof.* A function has increasing differences if an incremental return from one argument is larger when the other variable is higher. For any  $(L^E, E^H) \in [0, 1] \times \mathbb{R}_+$ ,

$$\frac{\partial W(L^E, E^H; \theta = 1)}{\partial E^H} - \frac{\partial W(L^E, E^H; \theta = 0)}{\partial E^H} = -\frac{\partial h(T, E^H)}{\partial E^H} \geq 0 \quad (7)$$

$$\frac{\partial W(L^E, E^H; \theta = 1)}{\partial L^E} - \frac{\partial W(L^E, E^H; \theta = 0)}{\partial L^E} = \left[ \frac{dg(T)}{dT} - \frac{\partial h(T, E^H)}{\partial T} \right] \zeta A^E \geq 0 \quad (8)$$

$$\frac{\partial^2 W(L^E, E^H; \theta)}{\partial L^E \partial E^H} = \left[ -\frac{\partial^2 F^Y(L^Y, E^Y)}{\partial (E^Y)^2} + \frac{\partial^2 F^Y(L^Y, E^Y)}{\partial L^H \partial E^Y} \frac{1}{A^E} \right] - \theta \cdot \frac{\partial^2 h(T, E^H)}{\partial E^H \partial T} \zeta \geq 0 \quad (9)$$

■

Adaptation benefits are positive when it is available as in (7). The returns to fossil fuel use are higher with adaptation as the marginal damage curve becomes flatter with adaptation as in (8). Provided that the economy's total energy volume is fixed in the short run, transferring some from firms to households for cooling will crowd out energy that can be used to produce other consumption goods. This scarcity raises the marginal profit of carbon-emitting firms via general equilibrium effects on factor prices; energy price increases and wage decreases. When a factor market is competitive, equilibrium factor price equals its marginal product because there do not exist arbitrage opportunities. First, the energy scarcity in the final goods sector increases energy prices because of the diminishing marginal product of energy. Second, the energy shortage in the final goods sector makes wage go down because the value of marginal product of labor declines due to the complementarity between labor and energy. On the other hand, cooling energy modulates the marginal impacts of climate. In sum, adaptation and carbon emissions complement each other as in (9).

Proposition 1 establishes the sufficient condition for monotone comparative statics.

**Proposition 2** *It follows from monotone comparative statics (Milgrom and Shannon, 1994) that*

$$E^H(\theta = 1) \geq E^H(\theta = 0) = 0 \quad \text{and} \quad L^E(\theta = 1) \geq L^E(\theta = 0).$$

When  $\theta = 0$ , the planner would not allocate any energy for households because it decreases output without any benefits. Now, let  $L_0^E := L^E(\theta = 0)$  be an optimal labor allocation when adaptation is not available. Then, by construction, given any  $l \in [0, L_0^E]$ , the planner prefers  $L_0^E$  to  $l$  under  $\theta = 0$ . That is, the incremental returns to choosing  $L_0^E$  over  $l$  is always positive under  $\theta = 0$ ;  $W(L_0^E, E^H; \theta = 0) - W(l, E^H; \theta = 0) \geq 0$ . It follows from Proposition 1 that these positive incremental returns to emitting more carbon are further sustained even under  $\theta = 1$ . Therefore, even though self-protective measures directly contribute to negative externalities, the planner would never choose lower carbon emissions, which leads to a higher temperature.

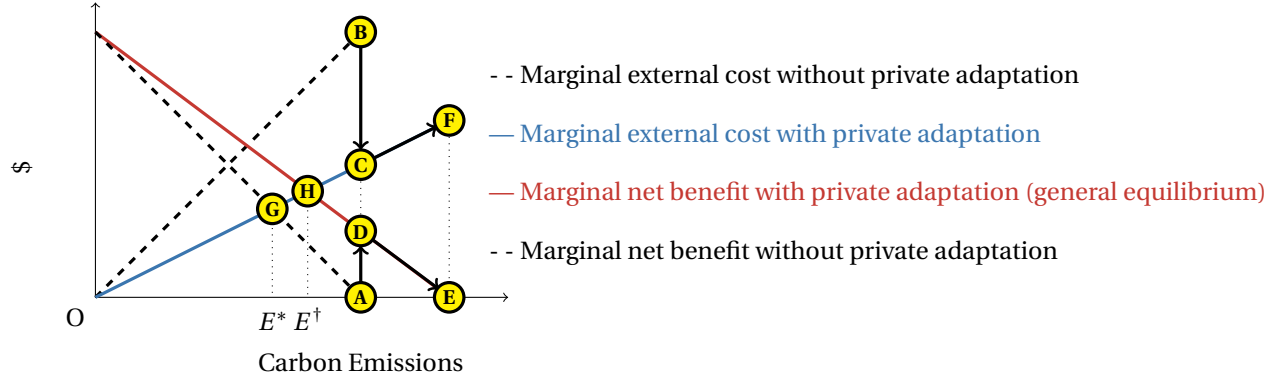


Figure 1: Graphical cost-benefit analysis

The optimal Pigouvian tax is determined at the point where the marginal external cost and marginal private profit from carbon emissions intersect according to the equation (6).

$$\text{Pigou Tax} = \begin{cases} \frac{dg(T)}{dT} & \text{if } \theta = 0 \\ \frac{\partial h(T, E^H)}{\partial T} & \text{if } \theta = 1 \end{cases}$$

On one hand, adaptation— $E^H(\theta = 1) \geq 0$ —reduces marginal damages decreasing optimal carbon taxes. On the other hand, an increase in emissions from general equilibrium effects— $L^E(\theta = 1) \geq L^E(\theta = 0)$ —offsets the direct impact of adaptation, which increases the Pigou tax. In section 3, I provide a proof for the optimal carbon taxes that decentralize the efficient allocations in a dynamic climate-economy model with private adaptation.

## 2.3 A graphical representation of cost-benefit analysis and Pigou taxes

Figure 1 illustrates the intuition of the model's general equilibrium effects using a graphical cost-benefit analysis. As a benchmark, consider an economy without cooling. With no carbon taxes, energy producers will increase their production until their marginal profit becomes zero (point A), which leads to external damages (point B). Now, suppose that energy is available for cooling. Private adaptation reduces the marginal impacts of climate (point C). The crowding-out of industrial energy by residential energy increases energy prices, but decreases wages.

Consequently, the profit of carbon-emitting firms rises in the short run (point D). With no carbon taxes, energy producers will increase their production capacity in the long run (point E), which leads to a higher external cost (point F) along the solid line since forward-looking households adapt to an endogenously evolving climate.

In principle, climate externalities can be internalized by regulating carbon emissions at the point where its private marginal net benefit equals its external marginal cost. However, such cost-benefit analyses may not be straightforward with endogenous adaptation because both curves shift. When it comes to optimal climate policies, failing to account for such effects may lead to inefficient levels of emissions. Much of the existing literature has focused on the shift in marginal damage curve. Specifically, many Integrated Assessment Models (IAMs) implicitly incorporate the costs and benefits of adaptation into a damage function by calibrating it to the least cost combination of residual damage and adaptation costs ([Fankhauser, 2017](#))

$$\text{Damage}(T) := \arg\min_{E^H} \{ \text{Residual Damage}(T, E^H) + \text{Adaptation Cost}(T, E^H) \}, \quad (10)$$

where  $T$  is a global mean surface temperature change relative to pre-industrial level and  $E^H$  is adaptation. But since most of the existing damage functions lump all the relevant welfare effects of adaptive responses into one stylized damage function in a reduced-form way, the previous research has overlooked the general equilibrium effects of pollutant-intensive adaptation on the marginal profit of polluting firms. In this paper, I specify endogenous adaptive decisions in households' preference and budget constraint to shed some light on its general equilibrium effects on factor markets. In section 3, I construct a quantitative dynamic climate-economy model with private adaptation using the global mortality costs of climate change and electricity consumption for cooling as an example. I then recalibrate a damage function as in equation (10) to quantify how much of the Pigou tax with endogenous cooling is due to its general equilibrium effects on factor markets (point G to point H in Figure 1).

### 3 Dynamic climate-economy model with private adaptation

To quantify the role of adaptation in determining optimal carbon taxes, I extend the stylized framework to a much richer dynamic climate-economy model. First, I assume that damages are inversely related to environmental qualities, which is a constant elasticity of substitution aggregate of temperature and cooling energy. Second, I model the technology for producing an energy composite as a constant elasticity of substitution production function of carbon-free and fossil fuel-based energy. Third, I assume that the global mean surface temperature change is linear in the cumulative emissions of carbon dioxide. I then characterize optimal carbon taxes in a setting in which the government has access to lump-sum transfers.

**Household** The economy is populated by an infinitely-lived representative household with the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[ v(C_t) - \left[ \theta \cdot h(T_t, E_t^H) + [1 - \theta] \cdot g(T_t) \right] \right],$$

where  $v(C_t) = \frac{C_t^{1-\eta_c}}{1-\eta_c}$ ,

$$h(T_t, E_t^H) = \frac{1}{\eta_h - 1} \left( \omega \left( \frac{1}{1 + \gamma_h T_t^2} \right)^{1-\rho_h} + [1 - \omega] (\epsilon E_t^H)^{1-\rho_h} \right)^{\frac{1-\eta_h}{1-\rho_h}}, \quad \text{and}$$

$$g(T_t) = \frac{\omega}{\eta_h - 1} \left( \frac{1}{1 + \gamma_h T_t^2} \right)^{1-\eta_h}. \quad (11)$$

Here, I consider a power function with a constant elasticity of marginal utility for non-durable consumption and climate impacts;  $\eta_c \geq 1$  and  $\eta_h \geq 1$ . The parameter  $\gamma_h > 0$  is used to scale gross impacts—ceteris paribus ambient temperature effects that would occur without private adaptation. The parameter  $\epsilon > 0$  determines the efficiency by augmenting energy use, which reduces the severity of climate impacts. The parameter  $\beta \in (0, 1)$  is the discount factor.

Climate impacts net of adaptation are inversely proportional to ingested environmental quality  $Q$ , which I model as a constant elasticity of substitution aggregate of gross impacts and cooling in line with [Gerlagh and van der Zwaan \(2002\)](#) and [Hoel and Sterner \(2007\)](#)

$$Q(T, E^H) = \left( \omega \left( \frac{1}{1 + \gamma_h T^2} \right)^{1-\rho_h} + [1-\omega] (\epsilon E^H)^{1-\rho_h} \right)^{\frac{1}{1-\rho_h}} \quad \text{where } \rho_h \in [0, \eta_h].$$

I restrict the parameter space for  $\rho_h$  to ensure that the aforementioned assumptions on  $h$  in section 2 hold (see the appendix 7.1 for derivations). Carbon mitigation improves ceteris paribus ambient temperature effects by curbing climate change. Therefore,  $\frac{1}{\rho_h}$  measures the ease with which the planner can switch between carbon mitigation and private adaptation along an indifference curve. This functional form is general enough to nest a wide range of the climate damages in the literature (see the appendix 7.2 for comparison to other studies). Note that  $g(T_t)$  is a special case of  $h(T_t, E_t^H)$  when  $\epsilon = 0$  and  $\rho_h = \eta_h$ .

Let  $V_t(K_t, S_t)$  denote the household's value function in period  $t$  with capital  $K_t$  and carbon stock  $S_t$ . Taking climate and prices as given, the dynastic household solves

$$V_t(K_t, S_t) = \max_{\{C_t, K_{t+1}, E_t^H\}} \left\{ v(C_t) - [\theta \cdot h(T_t, E_t^H) + [1-\theta] \cdot g(T_t)] + \beta V_{t+1}(K_{t+1}, S_{t+1}) \right\}$$

subject to  $C_t + p_t^E E_t^H + K_{t+1} = w_t L_t + [1 + r_t] K_t + G_t,$

where  $p_t^E$  is energy price,  $w_t$  is wage,  $r_t$  is the rental rate of capital, and  $G_t$  is a transfer from the government.



**Production** I assume that the final and intermediate goods markets are complete. There are four types of firms in the economy: final goods producers, energy aggregators, carbon-free energy producers, and fossil fuel-based energy producers. I assume that the final goods are produced à la Cobb-Douglas and that output depends on climate change à la [Nordhaus \(2017\)](#)

$$Y_t = (1 - D(T_t)) \cdot F_t(K_t^Y, L_t^Y, E_t^Y) = \frac{1}{1 + \gamma_y T_t^2} A_t^Y (K_t^Y)^\alpha (L_t^Y)^{1-\alpha-\nu} (E_t^Y)^\nu. \quad (12)$$

A representative firm in the final goods sector solves

$$\max_{K_t^Y, L_t^Y, E_t^Y} Y_t - p_t^E E_t^Y - w_t L_t^Y - (r_t + \delta) K_t^Y,$$

subject to non-negativity constraints, where  $\delta$  is the depreciation rate of capital.

There are two types of energy in the economy: dirty (D) and carbon-free (R). Carbon-free energy is not associated with climate externalities whereas dirty energy emits carbon into the atmosphere. Energy from a source  $i \in \{D, R\}$  is produced according to the Cobb-Douglas function

$$E_t^i = G_t^i(K_t^i, L_t^i) = A_t^i (K_t^i)^{\alpha_i} (L_t^i)^{1-\alpha_i}. \quad (13)$$

I assume that dirty energy is in unlimited supply and its producers do not collect the Hotelling rents à la [Goloso et al. \(2014\)](#), in which they show that when a non-fossil alternative becomes economically profitable in the distant future, coal is not depleted even under laissez-faire equilibria. I calibrate the economy to justify this assumption in section 4. I normalize dirty energy production such that one unit of  $E^D$  generates one unit of carbon. Both renewable and dirty energy are expressed in the same unit.

A representative firm in the energy sector  $i \in \{R, D\}$  solves

$$\max_{K_t^i, L_t^i} (p_t^i - \tau_t^i) E_t^i - w_t L_t^i - (r_t + \delta) K_t^i$$

subject to non-negativity constraints for choice variables, where  $p_t^i$  is the price of energy of type  $i$  and  $\tau_t^i$  is its corresponding per-unit tax on output.

Energy composites are made according to the following constant elasticity of substitution production function

$$E_t = \left[ \kappa_R (E_t^R)^{\frac{\sigma_e - 1}{\sigma_e}} + \kappa_D (E_t^D)^{\frac{\sigma_e - 1}{\sigma_e}} \right]^{\frac{\sigma_e}{\sigma_e - 1}} \quad \text{where} \quad \sum_{i \in \{R, D\}} \kappa_i = 1. \quad (14)$$

Here,  $\kappa_i \in (0, 1)$  measures the relative energy-efficiency of the source  $i \in \{R, D\}$ , and  $\sigma_e > 0$  determines the elasticity of substitution between carbon-free and dirty energy along an isoquant. A representative aggregator solves

$$\max_{E_t^R, E_t^D} p_t^E E_t - p_t^R E_t^R - p_t^D E_t^D.$$

**Government** I assume that the government redistributes carbon tax revenues to households using lump-sum transfer

$$G_t = \sum_{i \in \{R, D\}} \tau_t^i E_t^i. \quad (15)$$

**Carbon cycle** Most existing Earth System Models—a framework widely used to calculate the state of global and regional earth system responses under various environmental conditions—generate a near proportional relation between the cumulative emissions of carbon dioxide and global mean surface temperature changes over the pre-industrial level (MacDougall, 2017). But it follows from Dietz, van der Ploeg, Rezai, and Venmans (2021) that most of the existing climate models in economics research exhibit excessive delays in temperature responses to emissions and they fail to account for carbon sinks’ declining abilities to remove carbon from the atmosphere as it becomes more saturated. They argue that unless cumulative emissions are too high in the future, a linear mapping will suffice to bring climate dynamics into line with the Earth System Models predictions. Thus, I specify climate change as a linear function of carbon stock

$$T_t = \zeta S_t \quad \text{and} \quad S_{t+1} = S_t + \vartheta_t E_t^D \quad \text{where} \quad \vartheta_t = \frac{1}{1 + \exp\{-(a + 5bt)\}}. \quad (16)$$

The parameter  $\zeta$  captures the relationship between cumulative emissions and warming, which is defined as the transient climate response to cumulative carbon emissions (TCRE) by Collins et al. (2013). Following Dietz and Venmans (2019), I assume a relatively short delay—5 years—for the temperature response to carbon emissions. Furthermore, I introduce an exogenously declining emissions intensity à la Golosov et al. (2014). The parameter  $\vartheta_t \in (0, 1)$  captures the fraction of carbon emitted in period  $t$ . This is also consistent with Nordhaus (2017) in which the economy becomes less carbon-intensive even without carbon taxes because abatement costs decline over time due to technological progresses. The quantitative implications of this assumption are further discussed in the calibration section.

Let  $W_t(K_t, S_t)$  denote a benevolent planner's value function in period  $t$  with capital  $K_t$  and carbon stock  $S_t$ . Then each period  $t$ , the planner solves

$$W_t(K_t, S_t) = \max_{\{C_t, K_{t+1}, L_t^Y, L_t^R, L_t^D, K_t^Y, K_t^R, K_t^D, E_t^H, E_t^Y\}} \left\{ v(C_t) - \left[ \theta \cdot h(T_t, E_t^H) + [1 - \theta] \cdot g(T_t) \right] + \beta W_{t+1}(K_{t+1}, S_{t+1}) \right\}$$

subject to (11), (12), (13), (14), (16), and

$$\begin{aligned} C_t + K_{t+1} &= Y_t + (1 - \delta)K_t, \\ L_t^Y + L_t^R + L_t^D &= 1, \\ K_t^Y + K_t^R + K_t^D &= K_t, \\ E_t^H + E_t^Y &= E_t, \end{aligned} \tag{17}$$

as well as non-negativity constraints for each choice variable. The envelope and the first order conditions are fully characterized in the appendix 7.3.

I define a recursive competitive equilibrium in this economy as follows.

**Definition 1** *A recursive competitive equilibrium consists of prices  $\{p_t^E, w_t, r_t, p_t^R, p_t^D\}$ , climate change  $\{T_t\}$ , tax/transfer  $\{\tau_t^R, \tau_t^D, G_t\}$ , policy functions  $\{C_t, K_{t+1}, E_t^H, E_t^Y, L_t^Y, L_t^R, L_t^D, K_t^Y, K_t^R, K_t^D\}$ , and a value function  $\{V_t\}$  such that each period  $t = 0, 1, 2, \dots$*

1. *taking prices, climate change, and tax/transfer as given, the policy functions and the value function solve the households' and the producers' maximization problem,*
2. *the government budget is balanced as in (15),*
3. *the climate change  $\{T_t\}$  is consistent with the policy functions through (16), and*
4. *the markets clear as in (17).*

The envelope and the first order conditions are fully characterized in the appendix 7.3.

**Proposition 3** *Suppose that the government plans to maximize economic efficiency using tax and transfer systems. Then the following output taxes on energy sectors decentralize the first best allocation from the planning problem*

$$\tau_t^R = 0 \quad \text{and} \quad \tau_t^D = \begin{cases} \frac{1}{v'(C_t)} \sum_{s=t+1}^{\infty} \beta^{s-t} \left( \frac{dg(T_s)}{dT_s} - v'(C_s) \frac{\partial Y_s}{\partial T_s} \right) \zeta \vartheta_t & \text{if } \theta = 0 \\ \frac{1}{v'(C_t)} \sum_{s=t+1}^{\infty} \beta^{s-t} \left( \frac{\partial h(T_s, E_s^H)}{\partial T_s} - v'(C_s) \frac{\partial Y_s}{\partial T_s} \right) \zeta \vartheta_t & \text{if } \theta = 1 \end{cases} \quad \forall t = 0, 1, \dots$$

where policy functions are the solutions for the planning problem.

See the appendix 7.3 for a proof. Under the linear warming model, an additional unit of carbon emissions in period  $t$  translates into a temperature rise by  $\zeta \vartheta_t$  from period  $t+1$  onwards. The optimal tax on dirty energy equals the sum of the present values of all future climate impacts discounted by the marginal utility of non-durable consumption in period  $t$ . With adaptation, marginal damage curve becomes flatter, which lowers Pigouvian carbon taxes. On the other hand, a rise in temperature from general equilibrium effects increases optimal carbon taxes because the damage function  $h$  is convex in  $T$ .

## 4 Calibration

I calibrate the model's laissez-faire equilibrium to match the projected impacts of climate change on heat-related mortality costs and electricity consumption under a high emissions scenario (Representative Concentration Pathway [RCP] 8.5); for details of this scenario, see [Riahi et al. \(2011\)](#). I choose this scenario because it does not include any carbon mitigation targets, which corresponds to the model's competitive equilibrium with no carbon taxes. I calibrate six parameters  $\{\gamma_h, \epsilon, 1/\rho_h, \omega, a, b\}$  to justify some recent reduced-form evidence on the benefits and costs of the use of energy for cooling ([Rode et al., 2021](#); [Carleton et al., 2022](#)). I adopt other parameters directly from the existing literature. The time step is 5 years. The year 2015 is a base period ( $t = -1$ ), and simulations begin in 2020 ( $t = 0$ ).

Using the external parameters in Table 2, I relate my model to some observables in the base period to initialize allocations. The 2015 world gross saving rate is used to calculate the base-period non-durable consumption.<sup>4</sup> The 2016 world final energy consumption for space cooling is used to calculate the base-period household cooling energy, which is about 3% of world total primary energy use (IEA, 2018a). The 2014 primary global fossil-fuel based energy demand was 11.085 giga ton of oil equivalents of coal, oil, and gas and the 2014 primary global carbon-free energy demand was 2.599 giga ton of oil equivalents of nuclear, hydro, bioenergy, and other renewables (IEA, 2016). Using the guidelines on national greenhouse gas inventories by IPCC (2006), I express the amount of energy demand in carbon units.

## 4.1 Internal parameters

**Preferences** Carleton et al. (2022) find that ceteris paribus effects of climate change on global heat-related mortality costs in 2100 are projected to be 221 deaths per 100,000 people under the RCP8.5, which is about 8.32% of the 2100 world gross domestic product.<sup>5</sup> I calibrate the parameter  $\gamma_h$  such that the model-simulated disutility caused by climate change from 2015 to 2100 without cooling equals the utility loss from 8.32% consumption

$$g(T_{2015}) - g(T_{2100}) = \nu(C_{2100}) - \nu([1 - 0.0832]C_{2100}). \quad (\text{Ceteris paribus climate impacts})$$

In addition, Carleton et al. (2022) project that the mortality costs net of adaptation benefits are expected to be 73 deaths per 100,000 people, which is about 5.57% of the 2100 world gross domestic product. I calibrate the parameter  $\rho_h$  such that the model-simulated adaptation benefits match the empirical moments as follows

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<sup>4</sup>World Bank, “Gross savings (% of GDP).” The World Bank Group, accessed July 15, 2022, <https://data.worldbank.org/indicator/NY.GNS.ICTR.ZS?end=2021&start=1960&view=chart>.

<sup>5</sup>Carleton et al. (2022) report climate damages as percentage of global GDP only for the full mortality costs of climate change that include both the benefits and costs of adaptation. To calculate the ceteris paribus climate impacts as percentage of global GDP, I assume that both benefits and costs of adaptation occur proportionally to all age groups.

$$\frac{h^{\max}(T_{2100}, E_{2100}^H) - h(T_{2100}, E_{2100}^H)}{h^{\max}(T_{2100}, E_{2100}^H) - h^{\min}(T_{2100}, E_{2100}^H)} = \frac{v(C_{2100}) - v([1 - 0.0557]C_{2100})}{v(C_{2100}) - v([1 - 0.0832]C_{2100})}, \quad (\text{Adaptation benefits})$$

where  $h^{\max}$  equals to  $h$  when  $1/\rho_h = 1/\eta_h$  and  $h^{\min}$  equals to  $h$  when  $1/\rho_h \rightarrow \infty$ . If cooling reduces deaths by 221 per 100,000 people, then the parameter  $1/\rho_h$  becomes infinity. If there are no benefits of cooling, then the parameter  $1/\rho_h$  equals to its lower bound  $1/\eta_h$ .

The parameter  $\omega$  governs the efficiency of greenhouse gas abatement efforts relative to space cooling in determining an environmental quality  $Q$ . All else being equal, as  $\omega$  increases, households need to use more energy to attain the same level of well-being. [Rode et al. \(2021\)](#) project future global electricity consumption relative to 2000-2010 under the RCP8.5. They first identify a causal effect of temperature fluctuations on electricity consumption using the standard two-way fixed effects model. They allow their dose-response functions to become steeper as either the long-run average temperature or income per capita rises, which are two key indicators of adaptation. It is worth noting that their projection does not include secular trends in energy consumption, but it just captures an increase in electricity usage attributable to climate change.<sup>6</sup> Therefore, I calibrate the parameter  $\omega$  such that the changes in the model-simulated cooling energy net of secular trends match the changes in electricity consumption projections in [Rode et al. \(2021\)](#), which is 1.21 giga joule per capita per year by the end of this century. I multiply this estimate by the 2015 world population to derive climate-driven cooling loads, which is 3.3214 giga ton of CO<sub>2</sub> per period. To derive the secular trends of household energy consumption, I simulate the competitive equilibrium without climate externalities.

$$\left[ E_{2100}^H - E_{2015}^H \right] - \left[ E_{2100}^{H, \text{Secular}} - E_{2015}^H \right] = 3.3214 \quad (\text{Climate-driven cooling demands})$$

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<sup>6</sup>In projecting the impact of climate change on future energy consumption, [Rode et al. \(2021\)](#) do not include time fixed effects. This is because the standard two-way fixed effects model estimate time fixed effects non-parametrically and it is not possible to extrapolate them.

Moment	Model-simulated	Empirical	Sources
Ceteris paribus climate impacts on mortality (% of world GDP)	8.32	8.32	<a href="#">Carleton et al. (2022)</a>
Mortality reduction due to cooling (% of world GDP)	5.57	5.57	<a href="#">Carleton et al. (2022)</a>
Climate-driven cooling demands (giga ton of CO <sub>2</sub> )	3.3215	3.3214	<a href="#">Rode et al. (2021)</a>
Asymptotic cumulative carbon emissions (giga ton of carbon)	4,889	5,000	<a href="#">Tokarska et al. (2016)</a>

Table 1: Model’s fit for targeted moments

Lastly, I calibrate the parameter  $\epsilon$  such that the marginal rate of substitution between  $C$  and  $E^H$  equals its price ratio in the base year. It is worth noting that [Carleton et al. \(2022\)](#) identify the benefits of adaptation by estimating reduced mortalities to weather fluctuations, which result from all of the actions people take to alleviate their mortality costs. In calibration, I assume that all the benefits result from the use of energy for cooling.

**Climate model** I introduce a declining emissions intensity  $\{\vartheta_t\}$  so that a temperature change can reach a steady state in the distant future à la [Goloso et al. \(2014\)](#). This assumption serves to validate the linear warming model. At high cumulative emissions, the transient climate response to cumulative carbon emissions (TCRE) is no longer constant and it starts to decline ([MacDougall, 2017](#); [Dietz et al., 2021](#)). I set  $a = 8$  and calibrate  $b$  such that the atmospheric carbon concentration in laissez-faire converges to five trillion tons of carbon, which validates the TCRE parameter in [Tokarska et al. \(2016\)](#). Five trillion tons of carbon also corresponds to the lower end of the range of estimates of the total fossil fuel resource ([IEA, 2013](#)). Therefore, my calibration justifies the assumption on the unlimited supply of fossil fuel-based energy since the depletion of fossil fuels does not arise.

I numerically solve for the model’s laissez-faire equilibrium to match all the moments jointly. Model-simulated moments are compared to the empirical moments in Table 1 and the resulting parameter values are summarized in Table 2. In both [Carleton et al. \(2022\)](#) and [Rode et al. \(2021\)](#), the median warming in 2100 relative to 2001-2010 under the RCP8.5 across all the climate models both studies consider is 3.7°C. A simulated temperature change in 2100 relative to 2015 is about 3.6°C in the laissez-faire equilibrium. The computational procedures are provided in the appendix 7.4.



Parameter		Description	Sources and notes
Preferences			
$\eta_c$	2	Elasticity of marginal utility of consumption	Weitzman (2007)
$\eta_h$	2	Elasticity of marginal utility of environmental quality	
$\beta$	(0.985) <sup>5</sup>	Discount factor	Nordhaus (2017)
$\gamma_h$	8.6801e-05	Utility damage	Internally calibrated
$\epsilon$	2.0700	Effectiveness of adaptation	Internally calibrated
$1/\rho_h$	0.5601	Substitutability between abatement and adaptation	Internally calibrated
$\omega$	0.0214	Relative efficiency	Internally calibrated
Technology			
$\gamma_y$	.0021	Production damage	Barrage (2020b)
$\alpha$	0.3	Capital expenditure share in final good sector	Golosov et al. (2014)
$\nu$	0.04	Energy expenditure share in final good sector	Golosov et al. (2014)
$\alpha_R$	0.597	Capital expenditure share in renewable energy sector	Barrage (2020b)
$\alpha_D$	0.597	Capital expenditure share in dirty energy sector	Barrage (2020b)
$\sigma_e$	1.949	Substitutability between dirty and renewable energy	Papageorgiou et al. (2017)
$\kappa_R$	0.442	Relative energy efficiency of renewable energy source	Papageorgiou et al. (2017)
$\kappa_D$	0.558	Relative energy efficiency of dirty energy source	$1 - \kappa_R$
$\delta$	0.4095	Capital depreciation rate	Nordhaus (2017)
$gA_1^Y$	0.076	Initial growth rate in output productivity	Nordhaus (2017)
$\delta_A$	0.005	Decline rate in productivity growth	Nordhaus (2017)
$gA_t^Y$		$gA_1^Y \exp(-5\delta_A(t-1))$	Nordhaus (2017)
$gA_t^R$		Growth rate in clean energy sector productivity	$((1 + gA_t^Y)^{\frac{1}{1-\alpha-\nu}})^{1-\alpha_R} - 1$
$gA_t^D$		Growth rate in dirty energy sector productivity	$((1 + gA_t^Y)^{\frac{1}{1-\alpha-\nu}})^{1-\alpha_D} - 1$
Climate model			
$S_{2015}$	851	2015 atmospheric carbon concentration (giga ton of carbon)	Nordhaus (2017)
$\zeta$	1.63e-03	Transient climate response to cumulative carbon emissions (°C per giga ton of carbon)	Tokarska et al. (2016)
$\vartheta_t$		$[1 + \exp\{-(a + 5b\,t)\}]^{-1}$	Golosov et al. (2014)
$a$	8		
$b$	-0.0798		Internally calibrated

Table 2: Calibration Summary

## 4.2 External parameters

**Climate model** The atmospheric carbon concentration in 2015 is from [Nordhaus \(2017\)](#). The transient climate response to cumulative carbon emissions (TCRE) is set to 0.00163 °C per giga ton of carbon from [Tokarska et al. \(2016\)](#).

**Preferences** I assume that the elasticity of marginal utility equals 2 for both non-durable goods and environmental qualities. The rate of pure time preference is set to be 0.015 per year, or  $\beta = (0.985)^5$  in the quantitative model in line with [Nordhaus \(2017\)](#).

**Technology** Following [Golosov et al. \(2014\)](#), I assume a capital expenditure share of 0.3 and an energy income share of 0.04 for final goods sector. Based on [Barrage \(2020b\)](#), I assume a capital expenditure share of 0.597 in both fossil fuel-based and carbon-free energy sector. Following [Papageorgiou, Saam, and Schulte \(2017\)](#), I set the elasticity of substitution between renewable and dirty energy to be 1.949 and the relative efficiency of renewable energy to be 0.442. The capital depreciation rate is set to 0.1 per year, or  $\delta = 0.4095$  in the quantitative model in line with [Nordhaus \(2017\)](#). The path of total factor productivity is also taken from [Nordhaus \(2017\)](#). The productivities in both dirty and renewable energy sectors are set such that the labor augmenting technological progress is the same across all sectors.

## 5 Quantitative results

I use the calibrated model to quantify the economic impacts of heat-related mortalities and ensuing adjustments in cooling. First, I quantify the general equilibrium effects of cooling on factor markets in competitive equilibria. Second, I quantify how much of the Pigou tax for correcting heat-related discomfort can be attributable to cooling-driven general equilibrium effects. Third, I compare the household saving with and without cooling in the first-best to quantify ex-ante adaptation to climate change. Fourth, I run a counterfactual experiment to examine the welfare impacts of exogenous advances in cooling technology.

### 5.1 General equilibrium effects of cooling on factor markets

To isolate the general equilibrium effects of household energy consumption for cooling on the marginal profit of dirty energy producers, I compare competitive equilibrium allocations from the two otherwise identical economies that differ in the ability to adapt. When I do not include endogenous adaptive decisions in utility and resource constraints, adaptation-driven general equilibrium effects do not arise. Therefore, I attribute an increase in factor shares in the dirty energy sector to the general equilibrium effects of cooling on factor prices.

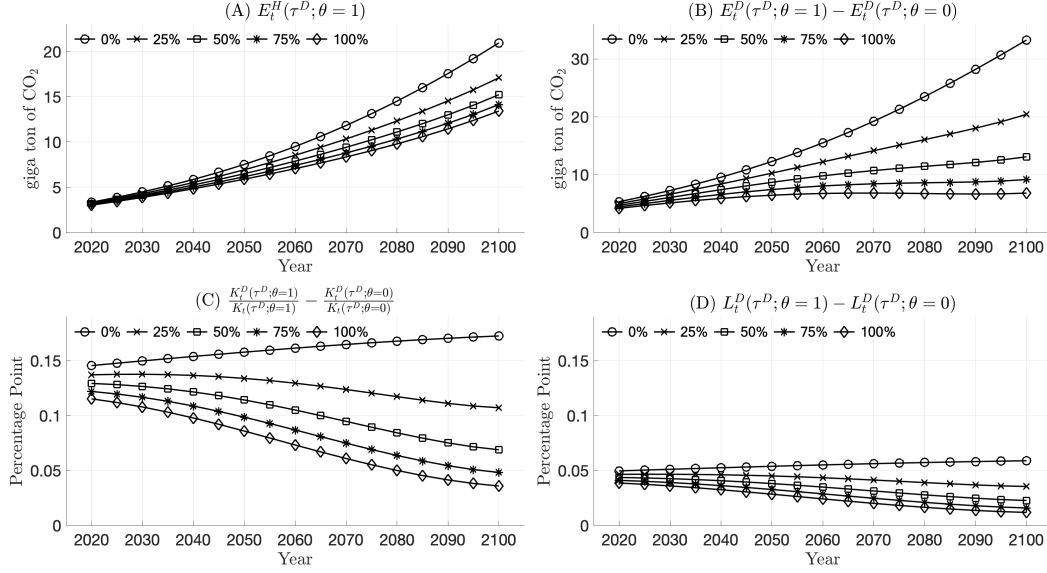


Figure 2: General equilibrium effects of cooling on factor markets in competitive equilibria

Each panel compares the time paths of competitive equilibrium allocations with cooling to the ones without adaptation as output taxes on dirty energy sector ( $\tau^D$ ) vary. I solve for the optimal carbon taxes with endogenous adaptation ( $\tau^*$ ). I then solve for competitive equilibria when  $\tau^D$  equals to  $x\%$  of  $\tau^*$  where  $x \in \{0, 25, 50, 75, 100\}$ . Note that I convert all the energy sources into tonnes of oil equivalent and then convert them into CO<sub>2</sub> for presentation. Panel (A): The use of energy for cooling. Panel (B): The difference in CO<sub>2</sub> emissions per period. Panel (C): The difference in the share of capital in dirty energy sector. Panel (D): The difference in the share of labor in dirty energy sector.

Figure 2 compares the simulated paths of competitive equilibrium allocations with cooling to the ones without adaptation as output taxes on the dirty energy sector— $\{\tau_t^D\}$ —change. I solve for the socially optimal allocation to derive the optimal carbon taxes— $\{\tau_t^*\}$ . I then solve for competitive equilibria when  $\{\tau_t^D\}$  equals to  $x\%$  of  $\{\tau_t^*\}$  where  $x \in \{0, 25, 50, 75, 100\}$ . Panel (A) shows that as output taxes on dirty energy sector vary from optimal carbon taxes to zero taxes, the time path of cooling shifts up. For example, when there are no carbon taxes, about 21 gigaton of CO<sub>2</sub>—3.4% of energy—is used for cooling in 2100. Having less regulation in dirty sector leads to more emissions. Because of nonseparability between environmental quality and cooling, a higher temperature change makes household demand more energy for cooling. Note that equilibrium cooling is not zero when the government imposes optimal carbon taxes. This is because the planner reduces carbon until its private marginal benefit equals its external marginal costs, which leads to non-zero carbon emissions. Since the efficient pollution level is not zero, the efficient cooling does not equal to zero either.

In response to endogenous cooling, the marginal profit of dirty energy sector shifts up because of an increase in its market demand. If there were no shifts, factor shares between two economies should be the same. Panel (C) and (D) shows that the differences in factor shares in dirty energy sector increase as output tax rates for dirty sector decline. For example, when there are no carbon taxes, the share of capital and labor in dirty energy sector increases by about 0.17 percentage point and about 0.06 percentage point by the end of this century. This is because increases in cooling loads due to a higher temperature amplifies general equilibrium effects. In response to increases in factor shares in dirty energy sector, emissions also increase as output tax rates decline for dirty energy sector as in Panel (B). For example, when there are no carbon taxes, emissions per period increases by about 33.3 giga ton of CO<sub>2</sub> in 2100.

## 5.2 General equilibrium effects of cooling on Pigou taxes

To quantify the general equilibrium effects of cooling on optimal climate policies, I compare the Pigou tax with endogenous cooling to the case in which a damage function is recalibrated to the least cost combination of residual damage and adaptation costs (point G to point H in Figure 1). When adaptation is implicit in a damage function, general equilibrium effects do not arise because household energy consumption do not show up in resource constraints.

Figure 3 shows the global mean surface temperature change over the pre-industrial level and mortality social costs of carbon in the first-best. To compare the simulated outcomes with endogenous cooling to the ones with implicit adaptation, I use upward-pointing triangle and circle symbols, respectively. In the social optima, equilibrium climate change is higher with endogenous cooling (Panel A of Figure 3). This is because the marginal profit of fossil fuel-based energy producers shifts up. Since increased profits flow back to the households' budget constraint, the planner increases optimal carbon emissions. Equilibrium temperature keeps going up over time in both cases because it is linear in the cumulative carbon emissions.

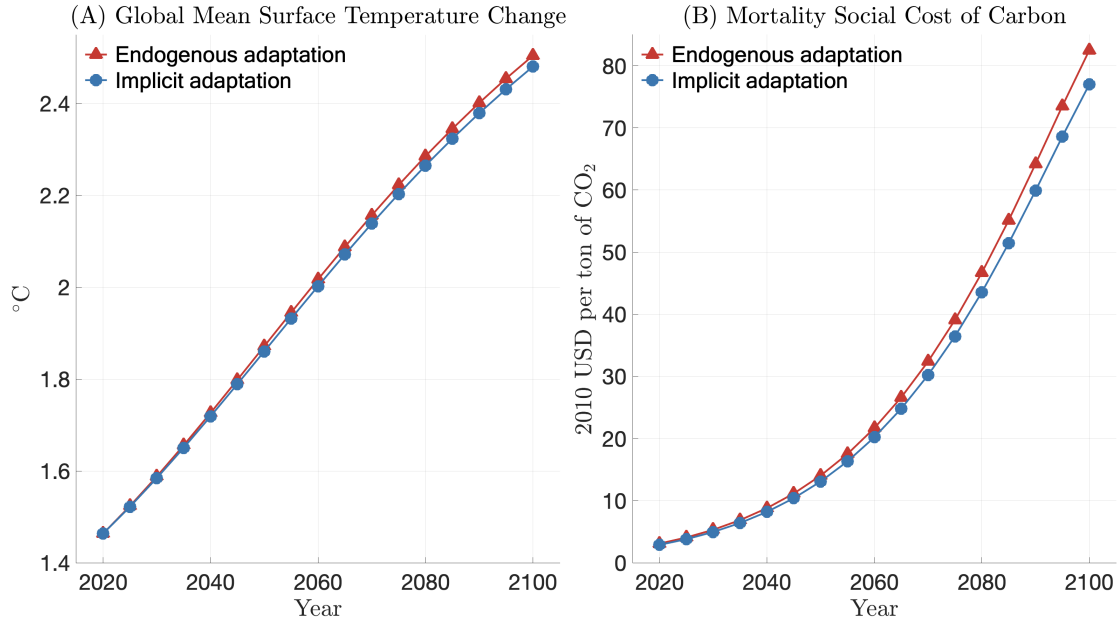


Figure 3: Simulated paths of climate change and Pigou tax correcting heat-related discomfort

Each panel compares the time paths of the first-best allocations with endogenous adaptation to the ones with implicit adaptation.

The social cost of carbon in this environment is defined as the present discounted sum of climate externalities that would result from emitting an additional ton of carbon dioxide. To highlight the general equilibrium effects of endogenous cooling, I isolate the mortality social costs of carbon from the social cost of carbon that additionally includes production damages. In the social optima, the mortality social cost of carbon is higher with endogenous cooling (Panel B of Figure 3) because of cooling-driven upward shifts in the marginal profit of dirty energy producers. It follows from the Proposition 3 that the efficient allocations can be fully decentralized when the government prices carbon at its social cost in the first-best solution of the model. For example, when I account for endogenous cooling, Pigou tax correcting for mortality costs increase by about 7% for the base period (2020).

### 5.3 Saving as ex-ante adaptation

[Fankhauser, Smith, and Tol \(1999\)](#) claim that it is crucial to distinguish between anticipatory (ex-ante) and reactive (ex-post) adaptation. In this paper, I model adaptation as an ex-post flow decision variable using residential energy use for cooling and do not explicitly consider investments in durable goods such as air conditioners. Despite my modeling assumption on adaptive measures, the use of energy for cooling has intertemporal implications via saving. In the social optima, the household saving in 2020 increases by 0.09% with endogenous cooling. Consequently, capital stock rises by 0.1% with endogenous cooling by the end of this century, which implies household energy expenditures due to climate change do not divert resources from productive capital accumulation. Equilibrium temperature keeps rising over time since it is proportional to the cumulative carbon emissions. As a result, as soon as cooling energy is available as a self-protective measure, households derive higher utility from converting the marginal unit of final goods to the very first unit of cooling services in the following period compared to the current period. This is because avoided damages from cooling are higher when the climate is worse, which provides higher incentives to save all else being equal.

### 5.4 Welfare impacts of advances in cooling technology

Energy efficiency improvements have received attention in the global discourse on climate change as an effective greenhouse gas mitigation strategy ([IEA, 2018b](#)). Yet, such advances can create income from energy savings and potentially lead to an increased energy use, which is referred to as “rebound” effects in the energy efficiency literature ([Borenstein, 2015](#); [Lemoine, 2020](#)). I use the calibrated model to see if such rebound effects more than offset the energy savings from technological advances. In particular, I examine the competitive equilibria with no carbon taxes to focus on the role of energy efficiency improvements as greenhouse gas mitigation strategy in the worst case. In this paper, I consider an increase in the efficacy of cooling at reducing marginal damages via the parameter  $1/\rho_h$ .

‡ Simulated outcomes using the parameters in Table 2

Competitive equilibrium without carbon taxes	Baseline <sup>‡</sup>	$(1/\rho_h) \times 1.001$
Cooling energy use in 2100 (giga ton CO <sub>2</sub> )	20.9	21.2
Cumulative carbon dioxide emissions in 2100 (giga ton)	12,076	12,078
Mortality social cost of carbon <sup>†</sup> in 2100 (2010 US\$/ton CO <sub>2</sub> )	202	199

Table 3: Welfare impacts of the advances in cooling technology

Table 3 summarizes the welfare impacts of such technological advances in cooling. A 0.1% increase in the efficacy of cooling at reducing marginal damages  $(1/\rho_h)$  boosts energy use by about 1% in the year 2100, which means that rebound effects reverse energy savings. Thus, related general equilibrium effects also increase leading to higher carbon emissions. But the benefits of cooling from the enhanced efficacy dominate the unintended consequences, which decreases the mortality social cost of carbon from \$202 per ton of CO<sub>2</sub> to \$199 per ton of CO<sub>2</sub> in 2010 USD by the end of this century.

To convert the decrease in the mortality social cost of carbon into more interpretable units, I compute the constant consumption stream  $w$  a household must receive to attain the life-time utility à la Bénabou (2002)

$$\frac{1}{1-\beta} \frac{w^{1-\eta_c}}{1-\eta_c} = \sum_{t=0}^{\infty} \beta^t [\nu(C_t) - h(T_t, E_t^H)].$$

A 0.1% increase in the parameter  $1/\rho_h$  provides welfare gains that are equivalent to about 1% of consumption for households. Therefore, even though rebound effects overturn the direct energy efficiency gains leading to a worse climate, the enhanced cooling services more than offset this vicious cycle effects providing welfare gains.

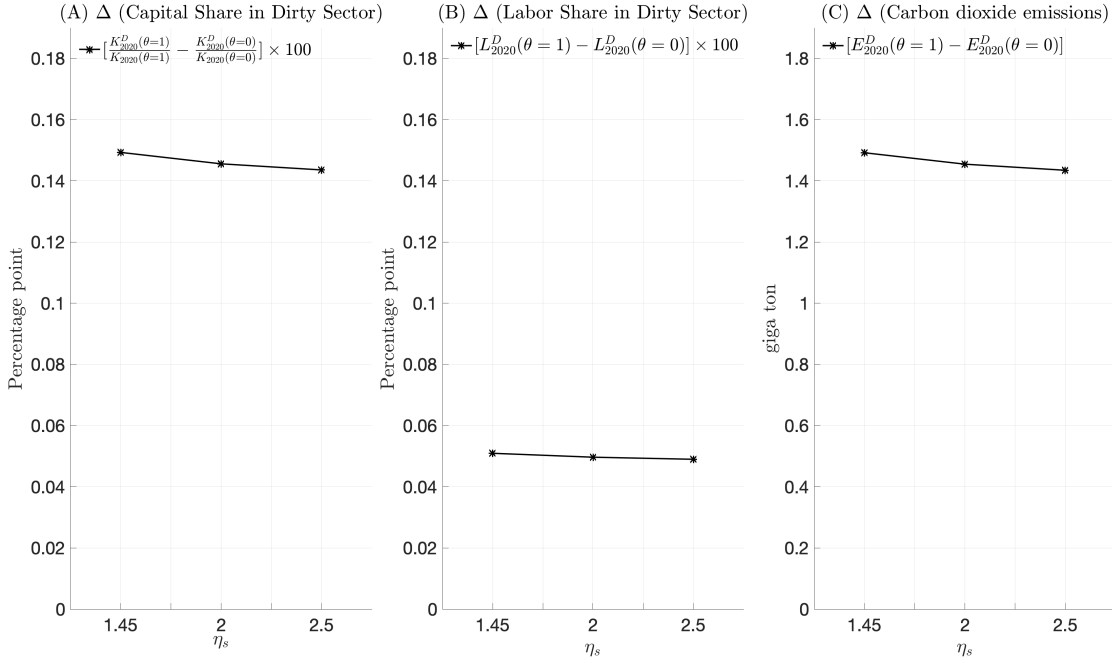


Figure 4: Differences of factor shares in the dirty sector and per period emissions in 2020

Each panel compares the laissez-faire allocations with cooling to the ones without adaptation in 2020 as the elasticity of marginal utility changes from 1.45 to 2.5 relative to the baseline calibration ( $\eta_c = 2$ ).

## 5.5 Sensitivity analysis

The main theoretical insight in this paper is that cooling-driven energy scarcity increases the marginal benefit of fossil fuel use by changing factor prices, which are determined by their marginal products in a competitive factor market. Here, I examine the robustness of adaptation-driven general equilibrium effects to alternative assumptions on the elasticity of marginal utility— $\eta_c$ . Figure 4 shows the difference of factor shares in the dirty sector between the laissez-faire with endogenous cooling and the laissez-faire without cooling across a range of parameter values. I find that the general equilibrium effects are in line with the baseline quantitative analysis.



## 6 Summary and concluding remarks

This paper develops a theory of how pollutant-intensive adaptation affects optimal Pigou taxes. The most apparent effect, which has been highlighted previously in the literature, is that adaptation reduces the external marginal costs of pollution; this effect always works to decrease optimal pollution taxes. However, I show that adaptation using pollutant-intensive intermediate goods comes with unintended consequences on factor prices. An increase in demand for pollutant-intensive inputs raises the marginal profit of polluting industries. This second effect works in the opposite direction, and increases the optimal pollution price. I use a macroeconomic climate-economy model with heat-related discomfort and cooling energy as a stylized example. I find that about 7% of the Pigouvian tax for correcting heat-related mortality is due to the unintended warming that is additionally caused by the use of energy for cooling for the base period.

The theoretical insight in this paper provides a guiding principle in interpreting reduced-form studies on adaptation to derive their policy implications. It is becoming more popular, in econometric analyses, to account for adaptation by allowing the dose-response relation between economic outcomes and weather fluctuations to depend on the long-run average temperature and per capita income, which are key indicators of adaptation (see [Kolstad and Moore \(2020\)](#) for a review). Policymakers should be concerned about general equilibrium effects when potential mechanisms explaining the benefit of adaptation are from carbon-intensive intermediate goods. In such cases, a back-of-the-envelope calculation based on engineering-based or partial equilibrium costs of pollution abatement could prescribe a lower-than-optimal tax rate.

Lastly, I conclude with a discussion of potential extensions of my model for future research. First, while a multi-sector growth model with various sources of energy as intermediate goods elucidates the general equilibrium effects of private adaptation, additional efforts need to be undertaken to generalize the framework to a multi-sector model with input-output linkages.

Adaptive behaviors that can lead to increased greenhouse gas emissions include, but are not limited to cooling energy use. The macroeconomic literature on production networks can provide a useful framework to examine the transmission of such risks over the input-output networks. Second, future research should probe the distributional effects of climate-driven adaptation. In this paper, I focus on the wedge between potential and realized exposures to climate change, which is driven by self-protective behaviors of a representative household. But economically disadvantaged people may not enjoy the same benefits because of their lack of resources to adapt to even worse climate conditions caused by the wealthy households' adjustments to climate change. The heterogeneous agent macro model with idiosyncratic income shocks can be a useful framework to design climate policies that reduce both carbon emissions and inequality among households.

## 7 Appendix

### 7.1 Regularity conditions on $h$

Given any  $(T, E^H) \in \mathbb{R}_+^2$ , if  $\eta_h \geq 1$ ,  $\rho_h \in [0, \eta_h]$ , and  $\gamma_h$  is small enough,

$$\frac{\partial h(T, E^H)}{\partial T} = \omega \cdot \frac{2\gamma_h T}{(1 + \gamma_h T^2)^{2-\rho_h}} \left( \omega \cdot \left( \frac{1}{1 + \gamma_h T^2} \right)^{1-\rho_h} + [1 - \omega] \cdot (\epsilon E^H)^{1-\rho_h} \right)^{\frac{1-\eta_h}{1-\rho_h}-1} > 0$$

$$\begin{aligned} \frac{\partial^2 h(T, E^H)}{\partial (T)^2} &= 2\omega\gamma_h \left( \omega \cdot \left( \frac{1}{1 + \gamma_h T^2} \right)^{1-\rho_h} + [1 - \omega] \cdot (\epsilon E^H)^{1-\rho_h} \right)^{\frac{1-\eta_h}{1-\rho_h}-2} \\ &\quad \times \left[ \frac{1 + \omega\gamma_h T^2(2\eta_h - 3)}{(1 + \gamma_h T^2)^{4-2\rho_h}} + [1 - \omega] \cdot \frac{1 + \gamma_h T^2(-3 + 2\rho_h)}{(1 + \gamma_h T^2)^{1+\rho_h}} \cdot (\epsilon E^H)^{1-\rho_h} \right] > 0 \end{aligned}$$

$$\frac{\partial h(T, E^H)}{\partial E^H} = -[1 - \omega] \cdot \frac{\epsilon}{(\epsilon E^H)^{\rho_h}} \left( \omega \cdot \left( \frac{1}{1 + \gamma_h T^2} \right)^{1-\rho_h} + [1 - \omega] \cdot (\epsilon E^H)^{1-\rho_h} \right)^{\frac{1-\eta_h}{1-\rho_h}-1} < 0$$

$$\begin{aligned} \frac{\partial^2 h(T, E^H)}{\partial (E^H)^2} &= [1 - \omega] \cdot \frac{\epsilon^2}{(\epsilon E^H)^{1+\rho_h}} \left( \omega \cdot \left( \frac{1}{1 + \gamma_h T^2} \right)^{1-\rho_h} + [1 - \omega] \cdot (\epsilon E^H)^{1-\rho_h} \right)^{\frac{1-\eta_h}{1-\rho_h}-2} \\ &\quad \times \left[ \rho_h \cdot \omega \cdot \left( \frac{1}{1 + \gamma_h T^2} \right)^{1-\rho_h} + \eta_h [1 - \omega] \cdot (\epsilon E^H)^{1-\rho_h} \right] > 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 h(T, E^H)}{\partial T \partial E^H} &= -[\eta_h - \rho_h] \cdot \omega \cdot [1 - \omega] \cdot \frac{2\gamma_h T}{(1 + \gamma_h T^2)^{2-\rho_h}} \cdot \frac{\epsilon}{(\epsilon E^H)^{\rho_h}} \\ &\quad \times \left( \omega \cdot \left( \frac{1}{1 + \gamma_h T^2} \right)^{1-\rho_h} + [1 - \omega] \cdot (\epsilon E^H)^{1-\rho_h} \right)^{\frac{1-\eta_h}{1-\rho_h}-2} < 0 \end{aligned}$$

## 7.2 Climate damage functions in the literature

The specification in section 3 nests a wide range of damage functions that have been used to study the role of adaptation in the literature.

1. When  $\rho_h \rightarrow 1$ , the environmental quality  $Q$  becomes multiplicative (de Bruin et al., 2009; Bosello, 2010; Bosello et al., 2010; Bréchet et al., 2013; Millner and Dietz, 2015; Barrage, 2020a; Fried, 2021);

$$h(T, E^H) = \frac{1}{\eta_h - 1} \left( \left( \frac{1}{1 + \gamma_h T^2} \right)^\omega (\epsilon E^H)^{1-\omega} \right)^{1-\eta_h}.$$

2. When  $\rho_h \downarrow 0$ , the environmental quality  $Q$  becomes additive (Bretschger and Valente, 2011; Zemel, 2015):

$$h(T, E^H) = \frac{1}{\eta_h - 1} \left( \omega \left( \frac{1}{1 + \gamma_h T^2} \right) + [1 - \omega] (\epsilon E^H) \right)^{1-\eta_h}.$$

3. When  $\rho_h \uparrow \eta_h$ , the climate impacts  $h$  become separable in  $T$  and  $E^H$ ;

$$h(T, E^H) = \frac{\omega}{\eta_h - 1} \left( \frac{1}{1 + \gamma_h T^2} \right)^{1-\eta_h} + \frac{1-\omega}{\eta_h - 1} (\epsilon E^H)^{1-\eta_h}.$$

### 7.3 Proof for proposition 3

Consider the planner's problem in Section 3. It follows from the Envelope theorem that

$$\frac{\partial W_t}{\partial K_t} = v'(C_t) \left[ 1 - \delta + \frac{\partial Y_t}{\partial K_t^Y} \right], \quad \text{and} \quad (\text{Envelope condition for } K_t)$$

$$\frac{\partial W_t}{\partial S_t} = - \left[ \theta \cdot \frac{\partial h(T_t, E_t^H)}{\partial T_t} + [1 - \theta] \cdot \frac{dg(T_t)}{dT_t} \right] \zeta + v'(C_t) \frac{\partial Y_t}{\partial T_t} \zeta + \beta \frac{\partial W_{t+1}}{\partial S_{t+1}}. \quad (\text{Envelope condition for } S_t)$$

Given  $i \in \{R, D\}$ , let  $\mathcal{S}_i := \kappa_i(E_t^i)^{\frac{\sigma_e - 1}{\sigma_e}} / [\kappa_R(E_t^R)^{\frac{\sigma_e - 1}{\sigma_e}} + \kappa_D(E_t^D)^{\frac{\sigma_e - 1}{\sigma_e}}]$ . Using the envelope condition for  $K_t$ , the first order conditions can be written as follows;

$$\begin{aligned} v'(C_t) &= \beta v'(C_{t+1}) \left[ 1 - \delta + \frac{\partial Y_{t+1}}{\partial K_{t+1}^Y} \right], \\ -\frac{\partial Y_t}{\partial L_t^Y} + \frac{\partial Y_t}{\partial E_t^Y} E_t \mathcal{S}_D \frac{1}{E_t^D} \frac{\partial E_t^D}{\partial L_t^D} &= -\beta \frac{1}{v'(C_t)} \frac{\partial W_{t+1}}{\partial S_{t+1}} \vartheta_t \frac{\partial E_t^D}{\partial L_t^D}, \\ -\frac{\partial Y_t}{\partial L_t^Y} + \frac{\partial Y_t}{\partial E_t^Y} E_t \mathcal{S}_R \frac{1}{E_t^R} \frac{\partial E_t^R}{\partial L_t^R} &= 0, \\ -\frac{\partial Y_t}{\partial K_t^Y} + \frac{\partial Y_t}{\partial E_t^Y} E_t \mathcal{S}_D \frac{1}{E_t^D} \frac{\partial E_t^D}{\partial K_t^D} &= -\beta \frac{1}{v'(C_t)} \frac{\partial W_{t+1}}{\partial S_{t+1}} \vartheta_t \frac{\partial E_t^D}{\partial K_t^D}, \\ -\frac{\partial Y_t}{\partial K_t^Y} + \frac{\partial Y_t}{\partial E_t^Y} E_t \mathcal{S}_R \frac{1}{E_t^R} \frac{\partial E_t^R}{\partial K_t^R} &= 0, \\ -\frac{\partial h(T_t, E_t^H)}{\partial E_t} &= v'(C_t) \frac{\partial Y_t}{\partial E_t^Y}. \end{aligned} \quad \text{and}$$

Now, consider the competitive equilibrium. It follows from the Envelope theorem that

$$\frac{\partial V_t}{\partial K_t} = v'(C_t) [1 + r_t]. \quad (\text{Envelope condition for } K_t)$$

Using the envelope condition for  $K_t$ , the first order conditions can be written as follows;

$$\begin{aligned}
[p_t^i - \tau_t^i] \frac{\partial E_t^i}{\partial K_t^i} &= r_t + \delta \quad \forall i \in \{R, D\}, \\
[p_t^i - \tau_t^i] \frac{\partial E_t^i}{\partial L_t^i} &= w_t \quad \forall i \in \{R, D\}, \\
p_t E_t \mathcal{S}_i \frac{1}{E_t^i} &= p_t^i \quad \forall i \in \{R, D\}, \\
\frac{\partial Y_t}{\partial K_t^Y} &= r_t + \delta, \\
\frac{\partial Y_t}{\partial L_t^Y} &= w_t, \\
\frac{\partial Y_t}{\partial E_t^Y} &= p_t, \\
v'(C_t) &= \beta v'(C_{t+1})[1 + r_{t+1}], \quad \text{and} \\
-\frac{\partial h(T_t, E_t^H)}{\partial E_t} &= v'(C_t)p_t.
\end{aligned}$$

By substituting prices, the first order conditions can be rewritten as follows:

$$\begin{aligned}
v'(C_t) &= \beta v'(C_{t+1}) \left[ 1 - \delta + \frac{\partial Y_{t+1}}{\partial K_{t+1}^Y} \right], \\
-\frac{\partial Y_t}{\partial L_t^Y} + \frac{\partial Y_t}{\partial E_t^Y} E_t \mathcal{S}_D \frac{1}{E_t^D} \frac{\partial E_t^D}{\partial L_t^D} &= \tau_t^D \frac{\partial E_t^D}{\partial L_t^D}, \\
-\frac{\partial Y_t}{\partial L_t^Y} + \frac{\partial Y_t}{\partial E_t^Y} E_t \mathcal{S}_R \frac{1}{E_t^R} \frac{\partial E_t^R}{\partial L_t^R} &= \tau_t^R \frac{\partial E_t^R}{\partial L_t^R}, \\
-\frac{\partial Y_t}{\partial K_t^Y} + \frac{\partial Y_t}{\partial E_t^Y} E_t \mathcal{S}_D \frac{1}{E_t^D} \frac{\partial E_t^D}{\partial K_t^D} &= \tau_t^D \frac{\partial E_t^D}{\partial K_t^D}, \\
-\frac{\partial Y_t}{\partial K_t^Y} + \frac{\partial Y_t}{\partial E_t^Y} E_t \mathcal{S}_R \frac{1}{E_t^R} \frac{\partial E_t^R}{\partial K_t^R} &= \tau_t^R \frac{\partial E_t^R}{\partial K_t^R}, \quad \text{and} \\
-\frac{\partial h(T_t, E_t^H)}{\partial E_t} &= v'(C_t) \frac{\partial Y_t}{\partial E_t^Y}.
\end{aligned}$$

Two sets of the first order conditions from planning problem and competitive equilibriums are equivalent if and only if

$$\tau_t^R = 0 \quad \text{and} \quad \tau_t^D = -\beta \frac{1}{v'(C_t)} \frac{\partial W_{t+1}}{\partial S_{t+1}} \vartheta_t.$$

Using the envelope condition for  $S_t$  and iteration,  $\tau_t^D$  can be rewritten as follows:

$$\tau_t^D = \begin{cases} \frac{1}{v'(C_t)} \sum_{s=t+1}^{\infty} \beta^{s-t} \left( \frac{dg(T_s)}{dT_s} - v'(C_s) \frac{\partial Y_s}{\partial T_s} \right) \zeta \vartheta_t & \text{if } \theta = 0 \\ \frac{1}{v'(C_t)} \sum_{s=t+1}^{\infty} \beta^{s-t} \left( \frac{\partial h(T_s, E_s^H)}{\partial T_s} - v'(C_s) \frac{\partial Y_s}{\partial T_s} \right) \zeta \vartheta_t & \text{if } \theta = 1 \end{cases} \quad \forall t = 0, 1, \dots$$

■

## 7.4 Computation

The main computational challenge in my setting is that atmospheric carbon concentrations do not stabilize over time because carbon stock does not depreciate in a linear warming model. Furthermore, productivities systematically evolve over time. Therefore, associated value and policy functions depend both on state and on time. Moreover, it is not possible to convert this environment into a stationary one using a labor augmenting technological progress because climate change is inversely related to utility in a quadratic fashion. To solve this problem, I combine the Extended Function Path approach by [Maliar, Maliar, Taylor, and Tsener \(2020\)](#) with the Envelope Condition Method by [Maliar and Maliar \(2013\)](#) and [Arellano, Maliar, Maliar, and Tsyrennikov\(2016\)](#). [Maliar et al. \(2020\)](#) show that if we are interested in the evolution of an infinite-horizon nonstationary economy during the first  $t_0$  periods, we can approximate its solution by solving a truncated problem. This method relies on the turnpike theorem that the convergence of a truncated economy to the corresponding infinite-horizon one is insensitive to a large enough terminal date ( $T$ ) and specific terminal conditions.

In this paper, I derive the optimal Pigouvian carbon taxes with adaptation towards the end of this century ( $t_0 = 17$ ). I set a large enough terminal period  $T = 100$  (500 years) to reduce approximation errors. I assume that technological progress becomes stationary in the terminal period  $T$  and construct a stationary solution. Given the terminal conditions, I solve for the Bellman equations by backward inductions and construct a sequence of time-inhomogeneous policy functions. Starting from an observable initial state, I simulate the economy forward and derive the optimal carbon taxes with adaptation.



## References

- Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous**, “The Environment and Directed Technical Change,” *American Economic Review*, February 2012, 102 (1), 131–166.
- Agrawala, Shardul, Francesco Bosello, Carlo Carraro, Kelly de Bruin, Enrica de Cian, Rob Dellink, and Elisa Lanzi**, “Plan or React? Analysis of Adaptation Costs and Benefits Using Integrated Assessment Models,” *Climate Change Economics*, 2011, 02 (03), 175–208.
- Arellano, Cristina, Lilia Maliar, Serguei Maliar, and Viktor Tsyrennikov**, “Envelope Condition Method with an Application to Default Risk Models,” *Journal of Economic Dynamics and Control*, August 2016, 69, 436–459.
- Auffhammer, Maximilian**, “Climate Adaptive Response Estimation: Short and Long Run Impacts of Climate Change on Residential Electricity and Natural Gas Consumption,” *Journal of Environmental Economics and Management*, July 2022, 114, 102669.
- Balboni, Clare**, “In Harm’s Way? Infrastructure Investments and the Persistence of Coastal Cities,” *Working Paper*, 2021.
- Barrage, Lint**, “The Fiscal Costs of Climate Change,” *AEA Papers and Proceedings*, May 2020, 110, 107–112.
- , “Optimal Dynamic Carbon Taxes in a Climate-Economy Model with Distortionary Fiscal Policy,” *The Review of Economic Studies*, January 2020, 87 (1), 1–39.
- Barreca, Alan, Karen Clay, Olivier Deschênes, Michael Greenstone, and Joseph S. Shapiro**, “Adapting to Climate Change: The Remarkable Decline in the US Temperature-Mortality Relationship over the Twentieth Century,” *Journal of Political Economy*, February 2016, 124 (1), 105–159.

- Bénabou, Roland**, “Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?,” *Econometrica*, March 2002, 70 (2), 481–517.
- Biardeau, Léopold T., Lucas W. Davis, Paul Gertler, and Catherine Wolfram**, “Heat Exposure and Global Air Conditioning,” *Nature Sustainability*, January 2020, 3, 25–28.
- Borenstein, Severin**, “A Microeconomic Framework for Evaluating Energy Efficiency Rebound and some Implications,” *The Energy Journal*, 2015, 36 (1).
- Bosello, Francesco**, “Adaptation, Mitigation and Green R&D to Combat Global Climate Change. Insights from an Empirical Integrated Assessment Exercise,” *FEEM Working Paper No.22.2010*, 2010.
- , **Carlo Carraro, and Enrica De Cian**, “Climate Policy and the Optimal Balance between Mitigation, Adaptation and Unavoided Damage,” *Climate Change Economics*, 2010, 1 (2), 71–92.
- Bréchet, Thierry, Natali Hritonenko, and Yuri Yatsenko**, “Adaptation and Mitigation in Long-term Climate Policy,” *Environmental and Resource Economics*, 2013, 55, 217–243.
- Bretschger, Lucas and Simone Valente**, “Climate Change and Uneven Development,” *The Scandinavian Journal of Economics*, December 2011, 113 (4), 825–845.
- Carleton, Tamma, Amir Jina, Michael Delgado, Michael Greenstone, Trevor Houser, Solomon Hsiang, Andrew Hultgren, Robert E Kopp, Kelly E. McCusker, Ishan Nath, James Rising, Ashwin Rode, Hee Kwon Seo, Arvid Viaene, Jiacan Yuan, and Alice Tianbo Zhang**, “Valuing the Global Mortality Consequences of Climate Change Accounting for Adaptation Costs and Benefits,” *The Quarterly Journal of Economics*, 2022.
- Collins, Matthew, Reto Knutti, Julie Arblaster, Jean-Louis Dufresne, Thierry Fichet, Pierre Friedlingstein, Xuejie Gao, William J. Gutowski, Tim Johns, Gerhard Krinner, Mxolisi**

- Shongwe, Claudia Tebaldi, Andrew J. Weaver, and Michael Wehner**, *Long-term Climate Change: Projections, Commitments and Irreversibility. In: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]*, Cambridge, United Kingdom and New York, NY, USA: Cambridge University Press, 2013.
- Conte, Bruno**, “Climate Change and Migration: the Case of Africa,” *Working Paper*, 2022.
- Cruz, José-Luis and Esteban Rossi-Hansberg**, “The Economic Geography of Global Warming,” *Working Paper*, 2022.
- Davis, Lucas W. and Paul J. Gertler**, “Contribution of Air Conditioning Adoption to Future Energy Use under Global Warming,” *Proceedings of the National Academy of Sciences*, May 2015, 112 (19), 5962–5967.
- de Bruin, Kelly C., Rob B. Dellink, and Richard S. J. Tol**, “AD-DICE: an Implementation of Adaptation in the DICE Model,” *Climatic Change*, 2009, 95, 63–81.
- Deryugina, Tatyana and Solomon M. Hsiang**, “The Marginal Product of Climate,” *National Bureau of Economic Research Working Paper No.24072*, 2017.
- Deschênes, Olivier**, “The impact of climate change on mortality in the United States: Benefits and costs of adaptation,” *Canadian Journal of Economics/Revue canadienne d'économie*, 2022, 55 (3), 1227–1249.
- **and Michael Greenstone**, “Climate Change, Mortality, and Adaptation: Evidence from Annual Fluctuations in Weather in the US,” *American Economic Journal: Applied Economics*, October 2011, 3 (4), 152–185.

- Dietz, Simon and Frank Venmans**, “Cumulative Carbon Emissions and Economic Policy: In Search of General Principles,” *Journal of Environmental Economics and Management*, July 2019, 96, 108–129.
- , **Frederick van der Ploeg, Armon Rezai, and Frank Venmans**, “Are Economists Getting Climate Dynamics Right and Does It Matter?,” *Journal of the Association of Environmental and Resource Economists*, February 2021, 8 (5), 895–921.
- Fankhauser, Sam**, “Adaptation to Climate Change,” *Annual Review of Resource Economics*, 2017, 9, 209–230.
- Fankhauser, Samuel, Joel B. Smith, and Richard S.J. Tol**, “Weathering Climate Change: Some Simple Rules to Guide Adaptation Decisions,” *Ecological Economics*, 1999, 30 (1), 67–78.
- Finkelstein, Amy, Erzo F. P. Luttmer, and Matthew J. Notowidigdo**, “Approaches to Estimating the Health State Dependence of the Utility Function,” *American Economic Review*, May 2009, 99 (2), 116–21.
- Fried, Stephie**, “Seawalls and Stilts: A Quantitative Macro Study of Climate Adaptation,” *The Review of Economic Studies*, December 2021, pp. 1–42.
- Gerlagh, Reyer and B. C. C. van der Zwaan**, “Long-Term Substitutability between Environmental and Man-Made Goods,” *Journal of Environmental Economics and Management*, September 2002, 44 (2), 329–345.
- Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski**, “Optimal Taxes on Fossil Fuel in General Equilibrium,” *Econometrica*, January 2014, 82 (1), 41–88.
- Gourio, François and Charles Fries**, “Adaptation and the Costs of Rising Temperature for the U.S. Economy,” *Working Paper*, 2020.

**Heutel, Garth, Nolan H. Miller, and David Molitor**, “Adaptation and the Mortality Effects of Temperature across U.S. Climate Regions,” *The Review of Economics and Statistics*, September 2021, 103 (4), 740–753.

**Hoel, Michael and Thomas Sterner**, “Discounting and Relative Prices,” *Climatic Change*, 2007, 84, 265–280.

**Hope, Chris W.**, “The Social Cost of CO<sub>2</sub> from the PAGE09 Model,” *Economics Discussion Papers*, No 2011-39, Kiel Institute for the World Economy, 2011.

**Hsiang, Solomon**, “Climate Econometrics,” *Annual Review of Resource Economics*, 2016, 8, 43–75.

**Hsiang, Solomon M. and Daiju Narita**, “Adaptation to Cyclone Risk: Evidence from the Global Cross-Section,” *Climate Change Economics*, 2012, 3 (2), 1250011.

**IEA**, *Resources to Reserves 2013: Oil, Gas and Coal Technologies for the Energy Markets of the Future*, Paris, France: OECD Publishing, 2013.

—, *World Energy Outlook 2016*, Paris, France: International Energy Agency, 2016.

—, *The Future of Cooling - Opportunities for Energy-Efficient Air Conditioning*, Paris, France: International Energy Agency, 2018.

—, *Perspectives for the Energy Transition - The Role of Energy Efficiency*, Paris, France: International Energy Agency, 2018.

**IPCC**, *2006 IPCC Guidelines for National Greenhouse Gas Inventories*, Hayama, Japan: Institute for Global Environmental Strategies (IGES), 2006.

—, *Climate Change 2022: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change* [H.-O. Pörtner, D.C. Roberts, M. Tignor, E.S. Poloczanska, K. Mintenbeck, A. Alegría, M. Craig,

*S. Langsdorf, S. Löschke, V. Möller, A. Okem, B. Rama (eds.)*], Cambridge, United Kingdom and New York, NY, USA: Cambridge University Press, 2022.

**Kahn, Matthew E.**, “The Climate Change Adaptation Literature,” *Review of Environmental Economics and Policy*, 2016, 10 (1), 166–178.

**Keefer, Philip, Eric Neumayer, and Thomas Plümper**, “Earthquake Propensity and the Politics of Mortality Prevention,” *World Development*, 2011, 39 (9), 1530–1541.

**Kolstad, Charles D. and Frances C. Moore**, “Estimating the Economic Impacts of Climate Change Using Weather Observations,” *Review of Environmental Economics and Policy*, 2020, 14 (1), 1–24.

**Lemoine, Derek**, “General Equilibrium Rebound from Energy Efficiency Innovation,” *European Economic Review*, June 2020, 125, 103431.

**MacDougall, Andrew H.**, “The Oceanic Origin of Path-Independent Carbon Budgets,” *Scientific Reports*, 2017, 7, 10373.

**Maliar, Lilia and Serguei Maliar**, “Envelope Condition Method versus Endogenous Grid Method for Solving Dynamic Programming Problems,” *Economics Letters*, August 2013, 120 (2), 262–266.

—, —, **John B. Taylor, and Inna Tsener**, “A Tractable Framework for Analyzing a Class of Non-stationary Markov Models,” *Quantitative Economics*, November 2020, 11 (4), 1289–1323.

**Milgrom, Paul and Chris Shannon**, “Monotone Comparative Statics,” *Econometrica*, January 1994, 62 (1), 157–180.

**Millner, Antony and Simon Dietz**, “Adaptation to Climate Change and Economic Growth in Developing Countries,” *Environment and Development Economics*, 2015, 20 (3), 380–406.

**Nath, Ishan**, “Climate Change, The Food Problem, and the Challenge of Adaptation through Sectoral Reallocation,” *Working Paper*, 2022.

- Nordhaus, William D.**, “Revisiting the Social Cost of Carbon,” *Proceedings of the National Academy of Sciences*, February 2017, 114 (7), 1518–1523.
- **and Joseph Boyer**, *Warming the World: Economic Models of Global Warming*, Cambridge, Massachusetts and London, England: The MIT press, 2000.
- **and Zili Yang**, “A Regional Dynamic General-Equilibrium Model of Alternative Climate-Change Strategies,” *The American Economic Review*, September 1996, 86 (4), 741–765.
- Papageorgiou, Chris, Marianne Saam, and Patrick Schulte**, “Substitution between Clean and Dirty Energy Inputs: A Macroeconomic Perspective,” *The Review of Economics and Statistics*, May 2017, 99 (2), 281–290.
- Riahi, Keywan, Shilpa Rao, Volker Krey, Cheolhung Cho, Vadim Chirkov, Guenther Fischer, Georg Kindermann, Nebojsa Nakicenovic, and Peter Rafaj**, “RCP 8.5-A Scenario of Comparatively High Greenhouse Gas Emissions,” *Climatic Change*, 2011, 109, 33–57.
- Rode, Ashwin, Tamma Carleton, Michael Delgado, Michael Greenstone, Trevor Houser, Solomon Hsiang, Andrew Hultgren, Amir Jina, Robert E Kopp, Kelly E. McCusker, Ishan Nath, James Rising, and Jiacan Yuan**, “Estimating a Social Cost of Carbon for Global Energy Consumption,” *Nature*, 2021, 598, 308–314.
- Rudik, Ivan, Gary Lyn, Weiliang Tan, and Ariel Ortiz-Bobea**, “The Economic Effects of Climate Change in Dynamic Spatial Equilibrium,” *Working Paper*, 2022.
- Sadowski, Nicole Cornell and Daniel Sutter**, “Mitigation Motivated by Past Experience: Prior Hurricanes and Damages,” *Ocean & Coastal Management*, 2008, 51 (4), 303–313.
- Settle, Chad, Jason F. Shogren, and Sally Kane**, “Assessing Mitigation-Adaptation Scenarios for Reducing Catastrophic Climate Risk,” *Climatic Change*, 2007, 83 (4), 443–456.

**Tokarska, Katarzyna B., Nathan P. Gillett, Andrew J. Weaver, Vivek K. Arora, and Michael Eby,** “The Climate Response to Five Trillion Tonnes of Carbon,” *Nature Climate Change*, September 2016, 6 (9), 851–855.

**Tol, Richard SJ,** “The Double Trade-Off between Adaptation and Mitigation for Sea Level Rise: an Application of FUND,” *Mitigation and Adaptation Strategies for Global Change*, 2007, 12 (5), 741–753.

**Weitzman, Martin L.,** “A Review of the Stern Review on the Economics of Climate Change,” *Journal of Economic Literature*, September 2007, 45 (3), 703–724.

**Zemel, Amos,** “Adaptation, Mitigation and Risk: An Analytic Approach,” *Journal of Economic Dynamics and Control*, February 2015, 51, 133–147.

**Zivin, Joshua Graff and Matthew Neidell,** “Environment, Health, and Human Capital,” *Journal of Economic Literature*, September 2013, 51 (3), 689–730.