Group-Based Sparse Representation for Image Denoising

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1 Introduction

Vision is one of the most important and effective ways for a machine to sense our world and capture data for human to analysis. Due to interface of environmental factors or instability of imaging device, the obtained image usually contain unexpected noises. Compared with reducing environmental interface or improve the imaging device, it's more feasible to reduce these noise by digital technology after obtaining the image. Therefore, image denoising is always a popular problem in image processing area.

Basically, transform-based method, like Fourier transform and Wavelet transform[1], and filter-based method, like BM3D[2], are two common and traditional way in image denoising. And recent state-of-art method can be divided into sparse coding method(K-SVD[3]), effective prior method(EPLL[4]) and deep learning method(TNRD[5]).

Among these method, sparse coding method is more prevalent due to its simplicity and stability. Thus, many image denoising works are based on sparse coding. In sparse based method, image is firstly sliced into many patches and then sparse algorithm is applied into each patch. For example, in K-SVD method, the learned dictionary is obtained from and sparse representation is computed for each patch iteratively.

Honestly, different patch may have similar structure and information and these patches may be far away from each other in the image. In the left part of figure [1], patches with blue boarder are far away in the image, but have similar content. Naturally, we can use their similarity as a prior knowledge in sparse image denoising problem. Since these patches are not in a neighbor area, these prior is called non-local self-similarity (NSS).

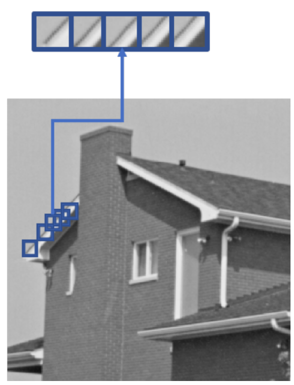

Figure 1: Group Explanation

To make full use of the similarity information, the patches are gathered together as groups. Similar with patch-based method in K-SVD, those groups are seen as basic units in sparse algorithm, which is called as group-based sparse representation method.

Traditional patch-wise sparse method is optimized for all patches in one iterations. However, group-based method is need to be optimized for each group. Due to huge amount of groups, a efficient optimization method is required.

In our project, we study group-based sparse method and implement a simple algorithm for image denoising. This algorithm is concluded from two literatures[6, 7]. In our team, Jane and Zhengzhao studied the first literature, while me and Xiangyu learned the second one. We mainly learned how algorithm works and test its source code. Based on our work, we had a discussion about commonality from these two papers and concluded our simple algorithm. To implement our idea, I am responsible for the part about how to produce groups from image by Matlab. The other teammates are in charge of optimization method, experiment comparison with other image denoising method.

The related work will be introduced in Section 2 and flowchart with our algorithm will be explained in Section 3. In Section 4, experiment results are shown and compared with other popular method. Finial, we will conclude our method in Section 5.

Our contribution include:

- A simple group-based method is introduced.
- The effective optimization method, ADMM, is used into our optimization problem.

2 Prior Work

Shutao Li firstly introduced group-based method in sparse dictionary learning in 2012[8]. S/He used a similar structure with K-SVD, but did not consider self-similarity among group members. This innovative dictionary learning method can own a better result than K-SVD. But this method showed no different in sparse representation part.

Then, in 2014 Jian Zhang introduced an advanced method[9]. Her contributions not only included taking non-local self-similarity into her algorithm, but indicated a untrained dictionary learning method. She thought it was not necessary to learn an over-complete dictionary, and it was even possible to learn a dictionary by a more efficient and effective manner, by SVD decomposition.

Based on Jian' s work, Xu improved dictionary method by combining Gaussian component[6]. S/He trained servals Gaussian component dictionary at the beginning. When computing non-trained dictionary for group, the algorithm estimates gaussian component for the group, and combine the non-trained dictionary with certain pre-trained Gaussian dictionary.

To make a better result, Zha[7] in 2016 added a residual term in group-based method optimization problem as prior. The prior mainly computed L2 Norm of subtraction between finial output and denoising result obtained by BM3D. On the other work, the finial result is based on estimation of BM3D and will be improved.

With the aim of implementing a simple algorithm, we only used an easy group-based algorithm structure, which means non-local similarity group and simple non-trained dictionary learning. Additionally, we apply ADMM method in optimization phase.

3 Proposed Method

3.1 Flowchart

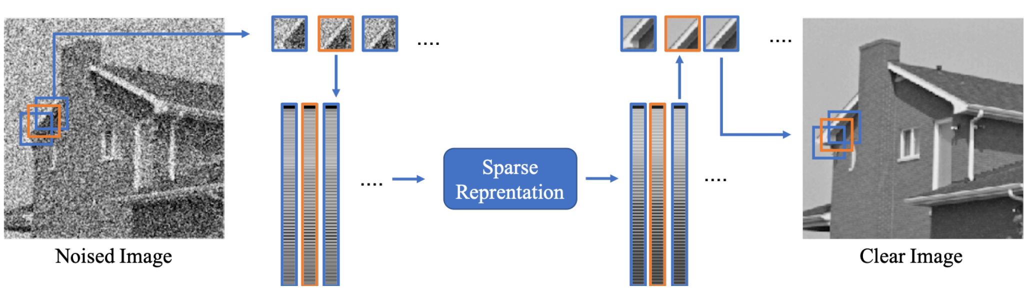

Figure 2: Flowchart of our Algorithm

In our method, we firstly compute non-overlapped patches with $N \times N$ size for input image, which is similar with patch-wise sparse method. For each patch, we will search its similar patch using a searching window with $S \times S$ size. We compute L2 Norm distance between the patch and each patch in searching window. Then we will pick up a fixed M amount of patches with the lowest distance and take them into a group. Thus, in the group, we have M matrixes with $N \times N$ dimensions.

Then, we vectorize each patch in the group as $N^2 \times 1$ vectors. If we stack these vectors into a whole matrix, then we obtain a $N^2 \times M$ matrix G represented each group. This matrix is the basic unit in group-based representation problem.

What we need to do in the next step is to compute sparse representation for this matrix. The problem can be expressed as a LASSO formulation:

$$\hat{\alpha_i} = \arg\min_{\alpha} \frac{1}{2}||G_i - D_i\alpha||_2^2 + \lambda_i||\alpha||_1 \tag{1}$$

where G_i is the i th group of given image, D_i is related non-trained dictionary, $\hat{\alpha}_i$ is sparse coefficient corresponded wih G_i. Sparse representation for G_i can be computed by $D_i\hat{\alpha}_i$.

In the next step, we recover the group that is sparse represented, into patches by doing reverse operation. Then, put each patch into their original position in the image. Though one patch will be seen as non-local similar patch for other different patches. So, at each time patch is processed and put back, the value is accumulated.

Finally, based on the amount of being picked up, we compute average value of each patch in the image. After that, the first iteration is finished.

3.2 Algorithm

The above subsection remains us three problem: (1)How to compute non-trained dictionary, (2)How to choice λ for each group and (3)How to solve this problem.

For the first problem, we compute dictionary by SVD method. Concretely, we firstly obtain covariance matrix of given group, shown as Ω. Then, apply SVD decomposition into Ω:

$$\Omega = PDP^T \tag{2}$$

We choice the first matrix P as the aimed dictionary matrix for this group.

For the second problem, it should be clear that λ control sparsity of finial result. Basically, with image have more noised, sparsity should also be larger. That means value of λ should have relation with both noise level in the group and also whole input image. In our project, noise level is computed by standard deviation, expressed as std. Mathematically,

$$\lambda_i = \mathrm{std}(\text{Whole Image}) * \mathrm{std}(G_i) * 0.04 \tag{3}$$

where 0.04 is chosen by experiment result.

For the third problem, the LASSO problem can be solved efficiently by ADMM method. In the first step, we re-write this problem as follow:

$$\begin{aligned} &\text{minimize } \frac{1}{2}||G_i - D_i\alpha||_2^2 + \lambda_i||z||_1 \\ &\text{subject to } x - \alpha = 0 \end{aligned} \tag{4}$$

The augmented Lagrangian with penalty parameter $(1/\tau) > 0$ for it is:

$$L_{\frac{1}{\tau}}(\alpha, y, z) = \frac{1}{2}||G_i - D_i\alpha||_2^2 + \lambda_i||z||_1 + \frac{1}{\tau} < y, \alpha - z > + \frac{1}{2\tau}||\alpha - z||_2^2 \tag{5}$$

To apply ADMM[10] into this problem, shown as Algorithm [1], we can get the solution.

Algorithm 1: ADMM for LASSO

Initialization: $k = 1$, y, z and $\tau > 0$;
while *convergence criterion is not satisfied* **do**
  $x_k \leftarrow (D^TD + \frac{1}{\tau}I)^{-1}(D^TG_i + \frac{1}{\tau}(z_{k-1} - y_{k-1}))$;
  $z_k \leftarrow S_{\lambda\tau}(x_k + y_{k-1})$;
  $y_k \leftarrow y_{k-1} + \frac{1}{\tau}(x_k - z_k)$;
  $k \leftarrow k + 1$;
end

In conclusion, our proposed method is shown as Algorithm [2].

Algorithm 2: Group-Based Sparse Representation Method

Initialization: $M_0 \leftarrow$ Noised Image and $k = 1$;
while *stop criterion is not satisfied* **do**
    Compute Pathes for matrix M_{k-1};
    Group non-local similarity patches using L2 Norm distance;
    For **Each Group**: compute its sparse representation as section [3.2];
    Update M_{k-1} to M_k by group-based result;
    $k \leftarrow k + 1$;
end

4 Experimental Results

In this part, experiment results are shown as figure [3], figure [4], figure [5] and figure [6]. To obtain noised image, we add gaussian noise into ground-truth image with standard deviation of noise data, $\sigma = 0.01$ and $\sigma = 0.1$. And in order to compare our method with other popular method, total variation denoising, BM3D and K-SVD are also implemented as comparison.

Based on our experiment result, we found that:

- In **low noise** situation, which means $\sigma = 0.01$, all algorithm have good performances, except total variation algorithm, which can not totally clean out noise data. For BM3D method, it has the best performance. For K-SVD method, it exits some block-like texture. For our proposed method, it clean out noises perfectly and also smooth out some details at the same time. When it comes to PSNR result,

- In **high noise** situation, which means $\sigma = 0.1$, only BM3D have acceptable performance. For total variation method, it fails to clean out noises. For K-SVD method, we can still see some noises in the result. For our proposed method, it also clean out noises perfectly and most details are smoothed out, which means we can only see a proximate contour of image. In PSNR estimation,

Combine with above description, we conclude the pros and cons of our method:

- **pros**:(1)Our method can make full use of non-local similarity information to gain a better result; (2)a trained dictionary is not necessary in our method, which means time-saving; (3)though some details are smoothed out, it can clean out noise data perfectly; and (4))the results look well in high noise situation.

- **cons**:(1)Based on our experiment, even though with the help of ADMM, we still found that our algorithm runs slow, due to optimization in each group and huge amount of groups in image. and (2) the results shown in the section are still not good enough, especially compared with BM3D.

	House $\sigma = 0.01$	House $\sigma = 0.1$	Lena $\sigma = 0.01$	Lena $\sigma = 0.1$
Proposed Method	29.962	21.733	27.524	21.167
Total Variation	28.46	21.03	27.06	20.56
K-SVD	30.43	23.85	27.92	23.12
BM3D	30.31	25.845	32.80	23.982
Wiener	27.05	18.65	26.73	18.51

Table 1: Experiment PSNR Result

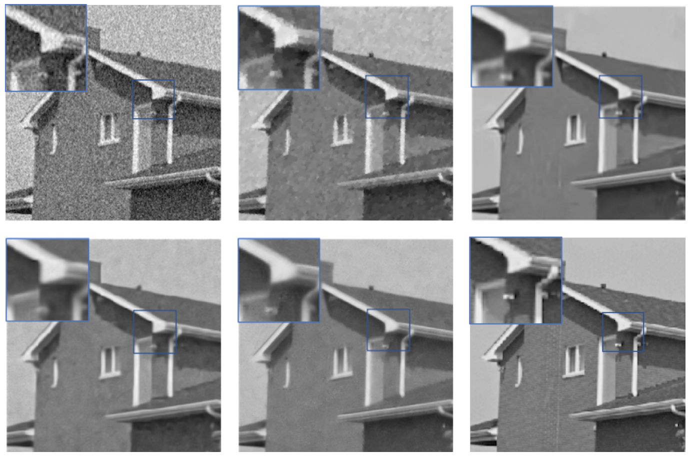

Figure 3: Experiment Result of House with $\sigma = 0.01$

From left to right and up to bottom, are noised image, total variation, BM3D, K-SVD and proposed method

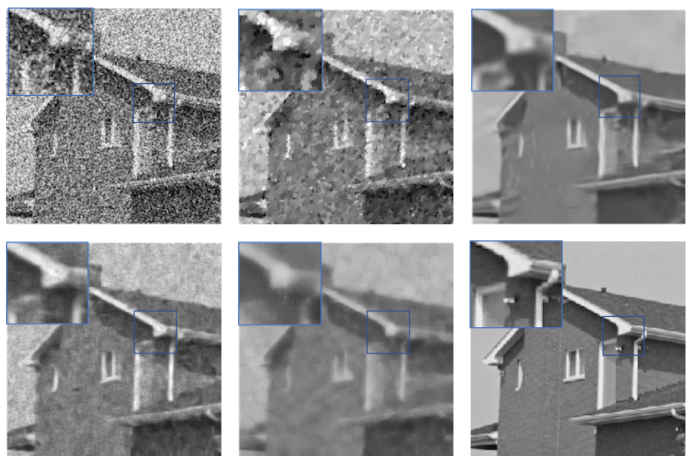

Figure 4: Experiment Result of House with $\sigma = 0.1$

From left to right and up to bottom, are noised image, total variation, BM3D, K-SVD and proposed method

Figure 5: Experiment Result of Lena with $\sigma = 0.01$

From left to right and up to bottom, are noised image, total variation, BM3D, K-SVD and proposed method

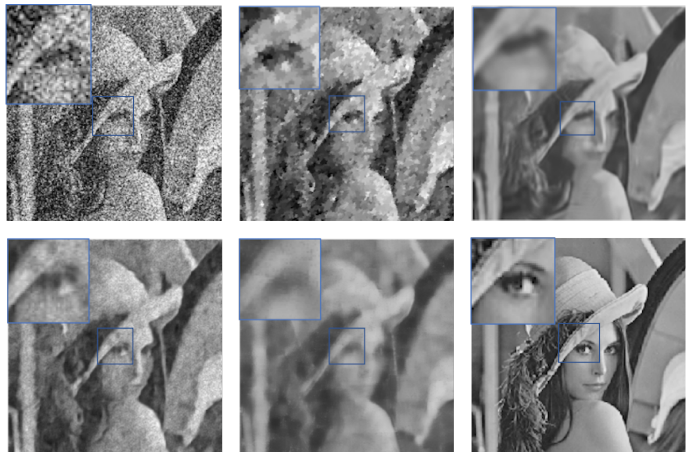

Figure 6: Experiment Result of Lena with $\sigma = 0.1$

From left to right and up to bottom, are noised image, total variation, BM3D, K-SVD and proposed method

5 Conclusion

All in all, we studied literatures about group-based method and introduced a simple group-based method in this project. We implemented our idea and compared it with other popular method. The experiment showed that our algorithm is still not good enough, and the running time is slow.

About the future works, our team have two thinkings:

- For slow running-time, maybe we can optimize every group parallelly, due to independence for each group.
- For the poor results, we can add a prior knowledge into our simple LASSO problem.

That's my report for ese585 finial project. Thinks for reading.

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