Solution 1

(a) The projection function in the first camera is: $p = K[I|\overline{0}]P$, where $p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ is camera point and $P = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ is world point.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} = \begin{bmatrix} \frac{fX}{Z} \\ \frac{fY}{Z} \\ 1 \end{bmatrix}$$

So, $x = \frac{fX}{Z}$.

The projection function in the second camera is: $p' = K[I|t]P$, where $p' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$ is camera point and $P = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ is the same world point.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X - t_x \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} f(X - t_x) \\ fY \\ Z \end{bmatrix} = \begin{bmatrix} \frac{f(X-t_x)}{Z} \\ \frac{fY}{Z} \\ 1 \end{bmatrix}$$

So, $x' = \frac{f(X-t_x)}{Z}$.

Then, $d = x - x' = \frac{fX}{Z} - \frac{f(X-t_x)}{Z} = \frac{fX}{Z} - \frac{fX}{Z} + \frac{ft_x}{Z} = \frac{ft_x}{Z}$

(b) Based on the above solution:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{fX}{Z} \\ \frac{fY}{Z} \\ 1 \end{bmatrix}$$

which indicates $X = \frac{xZ}{f}$ and $Y = \frac{yZ}{f}$.

We can reexpress $X\alpha + Y\beta + Z\gamma = k$ as:

$$\begin{aligned} X\alpha + Y\beta + Z\gamma &= k \\ \frac{xZ}{f}\alpha + \frac{yZ}{f}\beta + Z\gamma &= k \\ Z(\frac{x}{f}\alpha + \frac{y}{f}\beta + \gamma) &= k \\ \frac{1}{k}(\frac{x}{f}\alpha + \frac{y}{f}\beta + \gamma) &= \frac{1}{Z} \end{aligned}$$

Since $d = \frac{ft_x}{Z}$,

$$
\begin{aligned}
ft_x\left(\frac{1}{k}(\frac{x}{f}\alpha + \frac{y}{f}\beta + \gamma)\right) &= d \\
\frac{\alpha t_x}{k}x + \frac{\beta t_x}{k}y + \frac{fxt_x\gamma}{k} &= d
\end{aligned}
$$

In conclusion,

$$
\begin{cases}
a &= \frac{\alpha t_x}{k} \\
b &= \frac{\beta t_x}{k} \\
c &= \frac{fxt_x\gamma}{k}
\end{cases}
$$

.

(c) From $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{fX}{Z} \\ \frac{fY}{Z} \\ 1 \end{bmatrix}$, we can get $fX = xZ$ and $fY = yZ$.

$$
\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z - t_z \end{bmatrix} = \begin{bmatrix} fX \\ fY \\ Z - t_z \end{bmatrix} = \begin{bmatrix} \frac{fX}{Z-t_z} \\ \frac{fY}{Z-t_z} \\ 1 \end{bmatrix}
$$

Then $x' = \frac{fX}{Z-t_z} = \frac{xZ}{Z-t_z}$ and

$$
\begin{aligned}
x'Z - x't_z &= xZ \\
x' &= \frac{x}{1 - \frac{t_z}{Z}}
\end{aligned}
$$

Also, $y' = \frac{fY}{Z-t_z} = \frac{yZ}{Z-t_z}$ and

$$
\begin{aligned}
y'Z - y't_z &= yZ \\
y' &= \frac{y}{1 - \frac{t_z}{Z}}
\end{aligned}
$$

When $Z \to +\infty$, the co-ordinates (x', y') can match (x, y).

Solution 2

(a) Result is shown as figure [1].

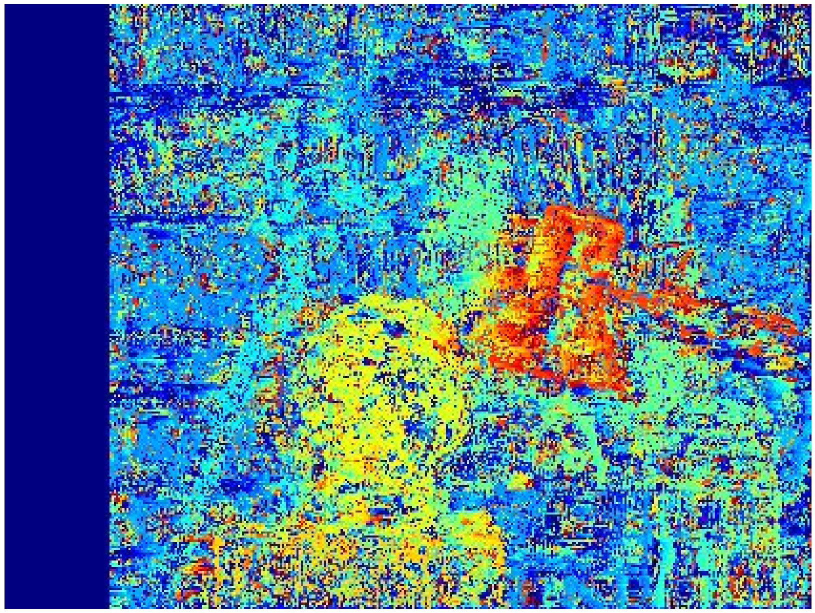

Figure 1: Result of Problem 2. (a)

(b) Result is shown as figure [2].

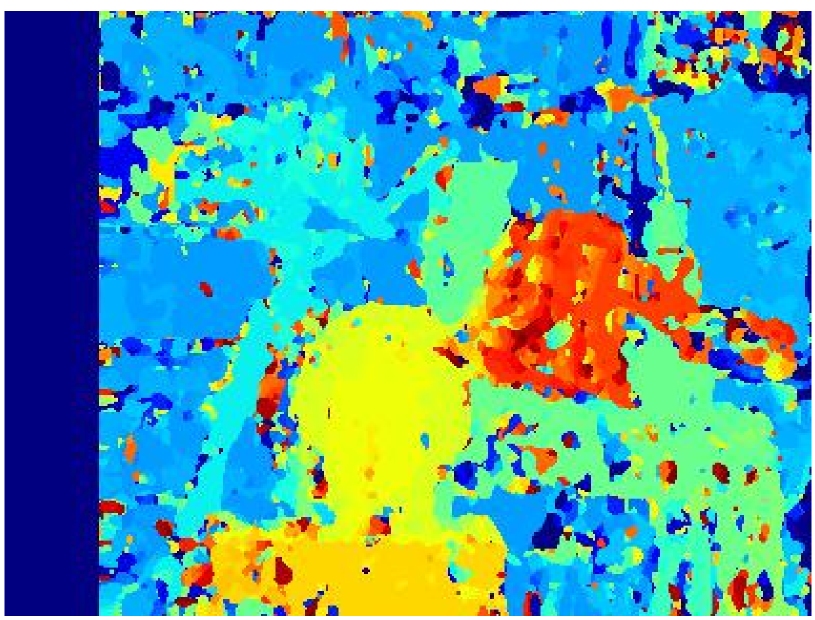

Figure 2: Result of Problem 2. (b)

Solution 3

(a) Result is shown as figure [3].

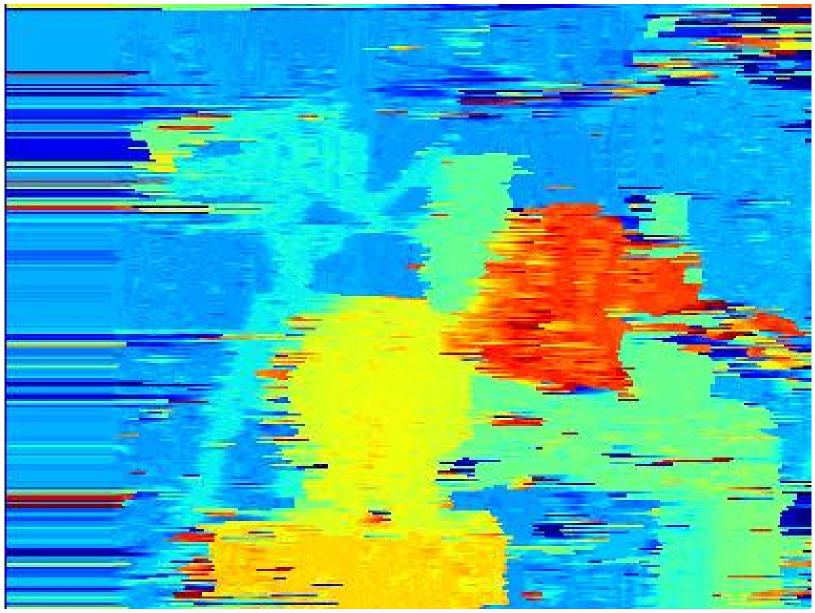

Figure 3: Result of Problem 3. (a)

(b) Result is shown as figure [4].

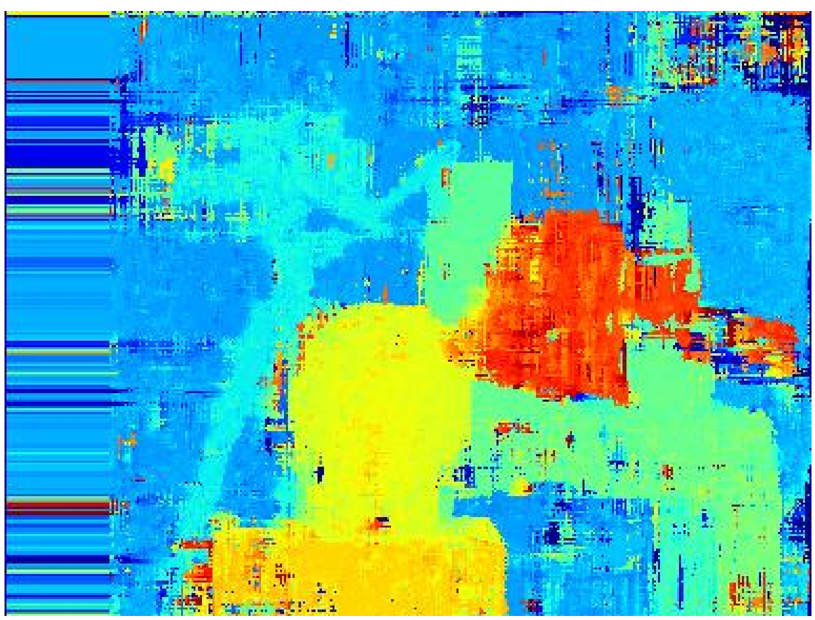

Figure 4: Result of Problem 3. (b)

Solution 4

Result is shown as figure [5].

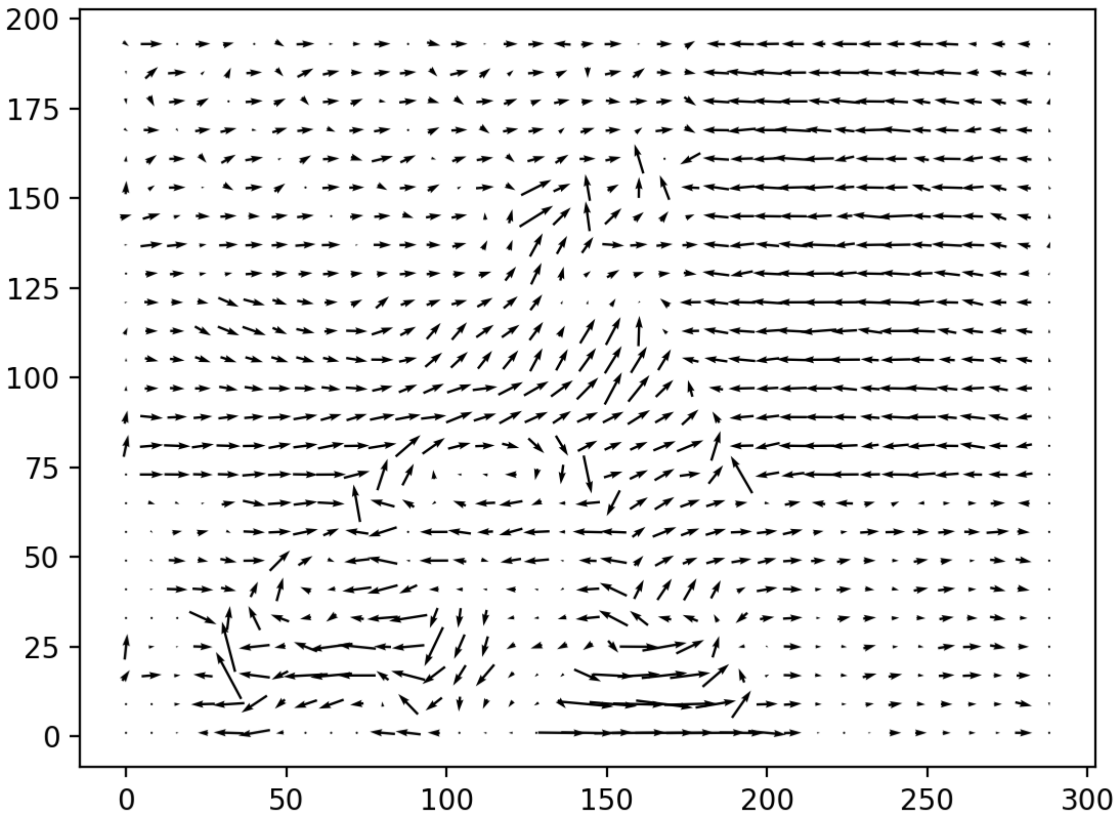

Figure 5: Result of Problem 4

Information

This problem set took approximately 15 hours of effort.

I discussed this problem set with:

- I ask one of the classmate for help about prob3a in the class, but I don't know his name. He just sat behind me. I studied the basic idea of how to solve problem and write the code for him. But I did not refer his code.