Problem Set III

1. **Point Distribution Models** Read the classic paper by Cootes and Taylor on active shape models (see the PDF file of ASM) and write a program to construct a point distribution model from a set of point sets for faces (see below for a description of the data). A number of matlab utility functions are provided which you shall use for your solution. Document all equations that you use and explain what they are used for. All results should be clearly reported and discussed in the report.

(a) (50%) Write a function that computes a mean point set and aligns all other point sets to it using a similarity transformation model of the form

$$\Phi(x, y) = s \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix},$$

where $s > 0$ denotes scale, θ is a rotation angle and t_x and t_y are the x and y components for the translation respectively. Follow the steps

1) Set the mean to the first point set, $x^\mu = x^1$.
2) Align all x^i to x^μ by minimizing

$$E(s, \theta, t_x, t_y) = \sum_{j=1}^{M} \| \begin{pmatrix} x^\mu \\ y^\mu \end{pmatrix} - s \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_j^i \\ y_j^i \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \end{pmatrix} \|^2$$

to obtain $\{\hat{x}^1, \cdots, \hat{x}^M\}$. Note, getAlignedPts.m (see description below and Figure 1) performs this computation for one point set. See Appendix A in the ASM paper for a description of the solution implemented in getAlignedPts.m. Note that the solution method does not solve directly for θ and s, but instead for $s\sin\theta$ and $s\cos\theta$. This allows for a solution by solving a linear system.

3) Calculate the new mean as

$$x^\mu = \frac{1}{N} \sum_{i=1}^{N} \hat{x}^i$$

4) Align x^μ to x^1 (and use it as the new x^μ)
5) Goto 2 unless x^μ has converged.

The energy function is shown in the problem and how to solve it is provided in the code. Basically, in takes 5 iterations to get the optimal value x^u. The iterate in t epoch does not stop until $||x_t^u - x_{t-1}^u||_2^2$ is less than 1e-6. The iterate data is shown as follow:

Num. of Iter.	$\|\|x_u\|\|_2^2$	$\|\|x_t^u - x_{t-1}^u\|\|_2^2$
1	3820.1176320	50.4778952
2	3820.1148216	0.1356699
3	3820.1148123	0.0004116
4	3820.1148123	0.0000013
5	3820.1148123	0.0000000

The final result of x_u would be:

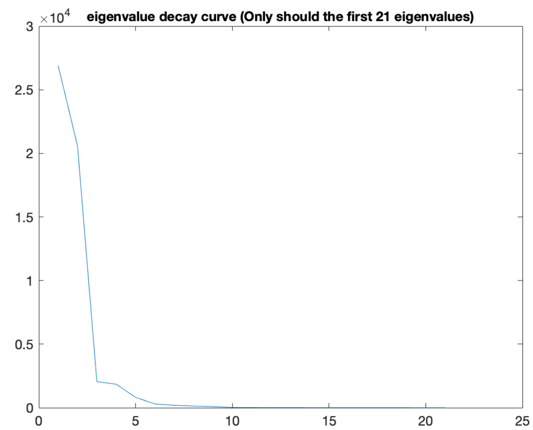
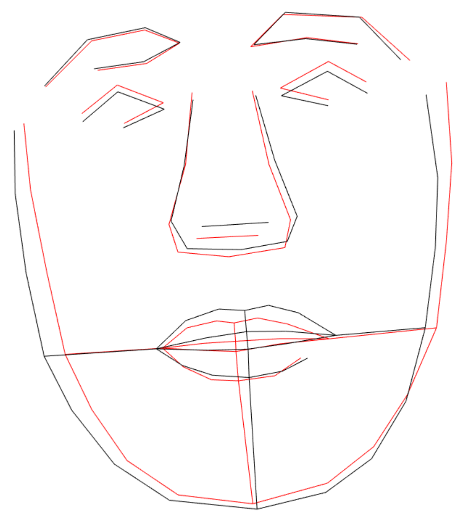


Figure 1: Eigenvalues Curve



where the red face is the first original face and the black face aligned face.

(b) (50%) Write a function to extract the three most important shape variations using principal component analysis. Show the results by plotting shape variations with respect to these three principal components independently by varying between two standard deviations around the mean.

The face totally have 21 faces. Dimension of each face x_i, $i = 1, 2, ..21$, is 68×2. We firstly vectorize each face into 136×1, compute and let each face data minus mean face. Then we concatenate all data into a whole matrix P, with dimension as 136×21.

The next step is to solve covariance matrix C of matrix P and also the corresponding eigenvalues and eigenvectors. All eigenvalues are plotted as figure (1):

Select three eigenvectors that related with the three largest eigenvalues as v_1, v_2 and v_3.

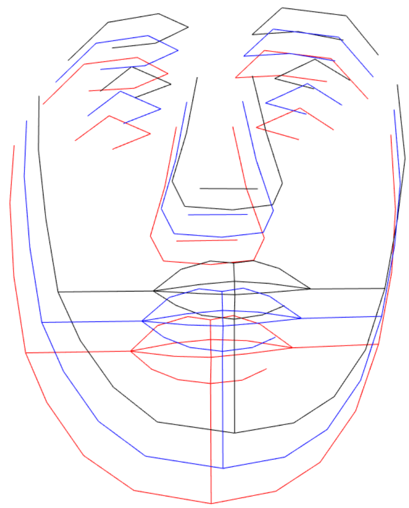


Figure 2: Result of v_1

The reconstructed face of x_i, $i = 1, 2, ..21$, using v_m, $m = 1, 2, 3$, is computed as:

$$\mathrm{rec}(x_i) = (x_i^T v_i) v_i$$

Using this equation, we can reconstruct all 21 face data using different v_m. To visualize it, we solve mean and standard derivation of all reconstructed faces and plot two faces, which are "mean plus standard derivation" and "mean minus standard derivation". The figure is shown as (2), (3) and (4). In all figures, the blue face is the mean face, the red face is the "mean plus standard derivation" and the black face is the "mean minus standard derivation".

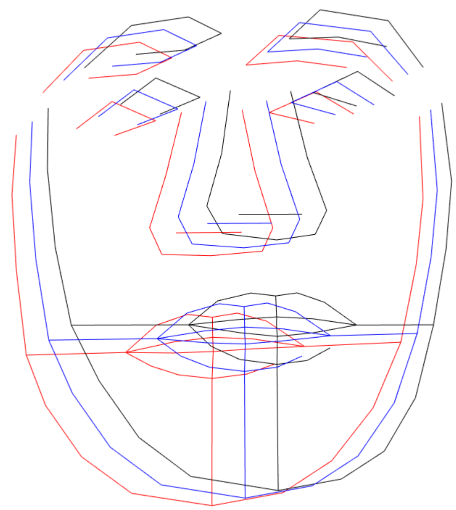


Figure 3: Result of v_2

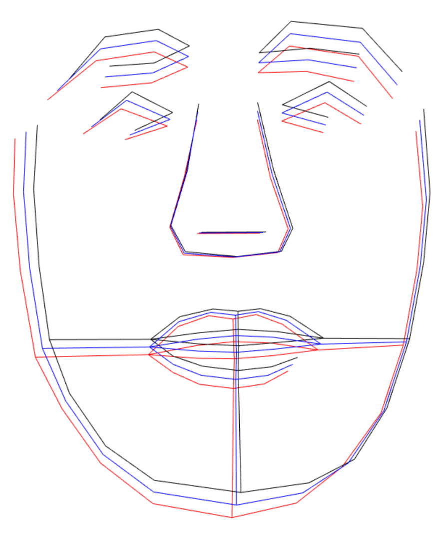


Figure 4: Result of v_3