Solution 1

(a) From slide, we know that:

$$I = gI^0 + g\sqrt{I^0}\varepsilon_1 + \sqrt{\left(g^2\sigma_{2a}^2 + \sigma_{2b}^2\right)}\varepsilon_2$$

Where both ε_1 and ε_2 is zero-mean Gaussian noise $\mathcal{N}(0, 1)$. Thus,

$$\begin{aligned} I &= gI^0 + \mathcal{N}(0,\ g^2I^0) + \mathcal{N}(0, (g^2\sigma_{2a}^2 + \sigma_{2b}^2)) \\ &= gI^0 + \mathcal{N}(0,\ g^2I^0 + (g^2\sigma_{2a}^2 + \sigma_{2b}^2)) \end{aligned}$$

Variance of $\varepsilon_{(a)}$ is $g^2I^0 + (g^2\sigma_{2a}^2 + \sigma_{2b}^2)$

(b) Since $g_{(b)} = g \times k$ and $I^0_{(b)} = \frac{I^0}{k}$,

$$\begin{aligned} \varepsilon_{(b)} &= \mathcal{N}\left(0,\ g_{(b)}^2 I_{(b)}^0 + (g_{(b)}^2\sigma_{2a}^2 + \sigma_{2b}^2)\right) \\ &= \mathcal{N}\left(0,\ (g \times k)^2\frac{I^0}{k} + ((g \times k)^2\sigma_{2a}^2 + \sigma_{2b}^2)\right) \\ &= \mathcal{N}\left(0,\ g^2kI_0 + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2\right) \end{aligned}$$

Variance of $\varepsilon_{(b)}$ is $g^2kI_0 + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2$

(c) Similar with above excise (b):

$$I_k = gI^0 + \mathcal{N}\left(0,\ g^2kI_0 + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2\right)$$

Since $I = (I_1 + I_2 + ...I_k)/k$:

$$\begin{aligned} I_1 + I_2 + ...I_k &= k * gI^0 + \sum_k \mathcal{N}\left(0,\ g^2kI_0 + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2\right) \\ I &= \frac{1}{k}\left(k * gI^0 + \sum_k \mathcal{N}\left(0,\ g^2kI_0 + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2\right)\right) \\ I &= \frac{1}{k}\left(k * gI^0 + \mathcal{N}\left(0,\ g^2k^2I_0 + g^2k^3\sigma_{2a}^2 + k\sigma_{2b}^2\right)\right) \\ I &= gI^0 + \mathcal{N}\left(0,\ \frac{1}{k^2}(g^2k^2I_0 + g^2k^3\sigma_{2a}^2 + k\sigma_{2b}^2)\right) \\ I &= gI^0 + \mathcal{N}\left(0,\ g^2I_0 + g^2k\sigma_{2a}^2 + \frac{1}{k}\sigma_{2b}^2\right) \end{aligned}$$

Variance of $\varepsilon_{(c)}$ is $g^2I_0 + g^2k\sigma_{2a}^2 + \frac{1}{k}\sigma_{2b}^2$

(d) Because $\varepsilon_{(c)}$ is smaller than $\varepsilon_{(b)}$, I prefer k shots with exposure time T/k.

Solution 2

The answer is shown as figure [1] and figure [2].

Figure 1: Solution 2 - Original Image

Figure 2: Solution 2 - Histogram Equalized Image

Solution 3

(a) The answer is shown as figure [3] and figure [4].

Figure 3: Solution 3 - Original Image

Figure 4: Solution 3 - Magnitudes H Image

(b) When threshold value is $T_0 = 0.5$, image before NMS is shown as figure [5] and image after NMS is shown as figure [6]. When threshold value is $T_1 = 1.0$, image before NMS is shown as figure [7] and image after NMS is shown as figure [8]. When threshold value is $T_2 = 1.5$, image before NMS is shown as figure [9] and image after NMS is shown as figure [10].

Figure 5: Solution 3 - $T_0 = 0.5$ Before NMS

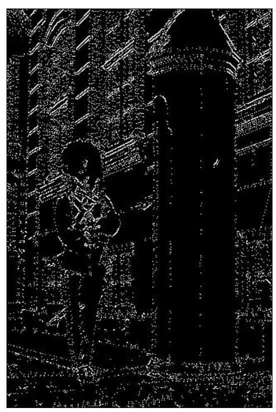

Figure 6: Solution 3 - $T_1 = 0.5$ After NMS

Figure 7: Solution 3 - $T_1 = 1.0$ Before NMS

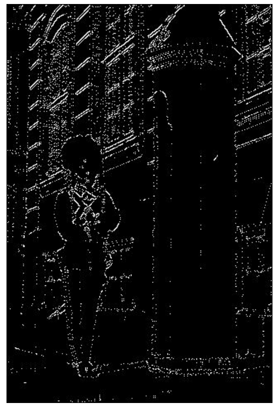

Figure 8: Solution 3 - $T_0 = 1.0$ After NMS

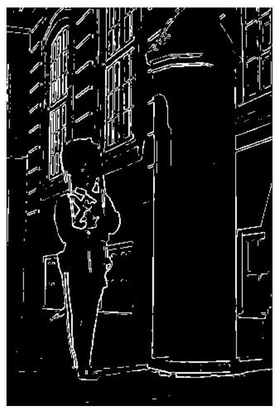

Figure 9: Solution 3 - $T_2 = 1.5$ Before NMS

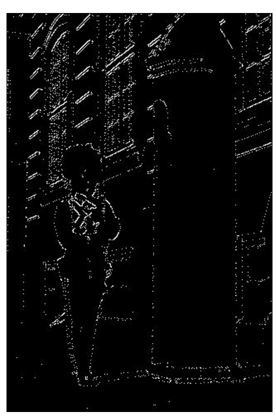

Figure 10: Solution 3 - $T_2 = 1.5$ After NMS

Solution 4

Results are listed as follows.

Figure 11: Solution 4 - prob4_1_a

Figure 12: Solution 4 - prob4_1_b

Figure 13: Solution 4 - prob4_1_c

Figure 14: Solution 4 - prob4_1_rep

Figure 15: Solution 4 - prob4_2_rep

Solution 5

(a) The key idea to solve this problem is: $F[u, v] = \overline{F}[W - u, H - v]$. Then, when only consider imaginary part, $I(F[u, v]) = -I(F[W - u, H - v])$. When only consider real part, $R(F[u, v]) = R(F[W - u, H - v])$. Since both real part and imaginary part in $F[u, v]$ are **central symmetric**, we both can use half of $W \times H$ space to store real part or imaginary part. All in all, we can ues $W \times H$ space to store $F[u, v]$.

The situation that both width and height are odd is shown as figure [16]. In a $W \times H$ space, there is a central point (x_c, y_c), $x_c = int(width/2)$ and $y_c = int(height/2)$, which shown as red area in the figure. The blue area means real part of F and the imaginary part of F is shown as green area. Also, I think the central point can not be stored with one real scalar.

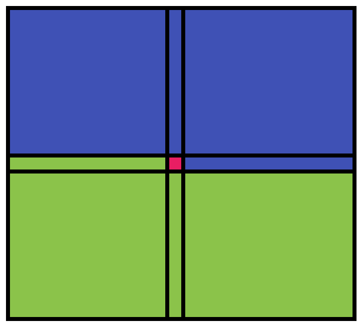

Figure 16: Solution 5 - a- All Odd

The situation that both width and height are even is shown as figure [17]. Because both are even, a $W \times H$ space can be divided into two equal part, both in vertical and horizontal direction. The blue area means real part of F and the imaginary part of F is shown as green area.

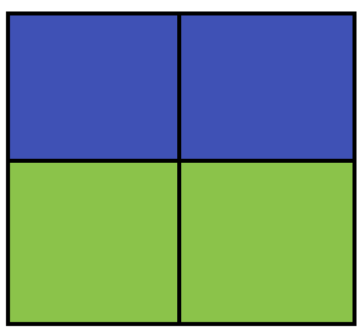

Figure 17: Solution 5 - a - All Even

The situation that width or height is odd and the other is even, is shown as figure [18]. Because width and height is even, a $W \times H$ space can be divided into two equal part, in vertical direction, if height is odd, and horizontal otherwise. The blue area means real part of F and the imaginary part of F is shown as green area.

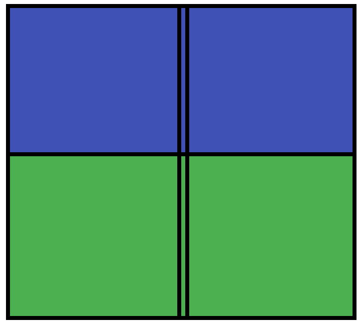

Figure 18: Solution 5 - a - One is Odd and the other is Even

(b) Result is shown as follow.

Figure 19: Solution 5 - b

Solution 6

(a) Results are listed as follows.

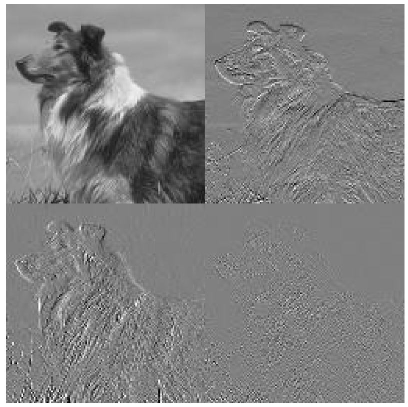

Figure 20: Solution 6 - prob6a_1

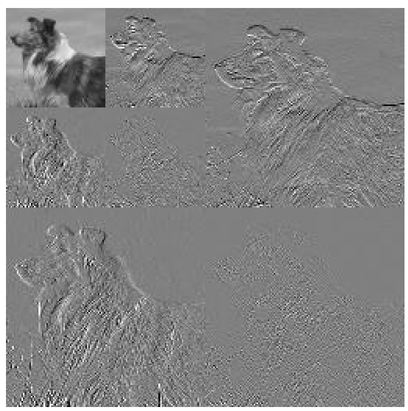

Figure 21: Solution 6 - prob6a_2

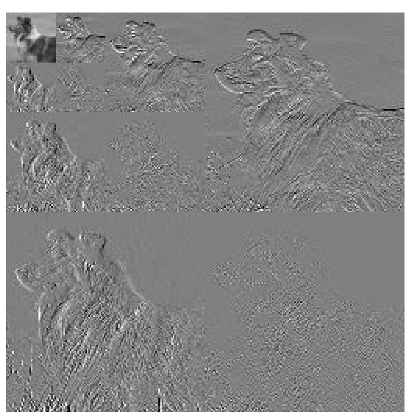

Figure 22: Solution 6 - prob6a_3

(b) Results are listed as follows.

Figure 23: Solution 6 - prob6b_0

Figure 24: Solution 6 - prob6b_1

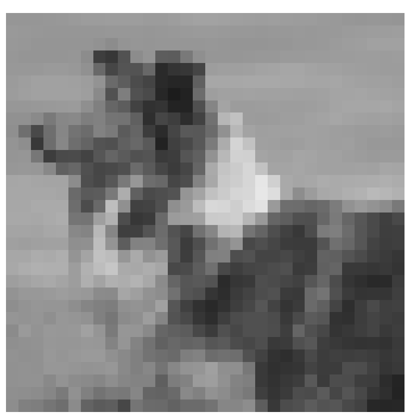

Figure 25: Solution 6 - prob6a_2

Information

This problem set took approximately 20 hours of effort. And I finish it by myself.