Solution 1

(a) Since there are J outliers and total N correspondences, amount of inliers is $N - J$. And because K points are selected randomly from all N correspondences, the possibility is: Especially, $C(n, k)$ means all choice when we select disordered k samples within n data.

$$\begin{aligned} P_1 &= \frac{C(N-J,K)}{C(N,K)} \\ &= \frac{\frac{(N-J)!}{K!(N-J-K)!}}{\frac{N!}{K!(N-K)!}} \\ &= \frac{(N-J)!(N-K)!)}{N!(N-J-K)!} \end{aligned}$$

(b) It's equivalent between $at\ least\ once$ with contrary of $None$. The result is:

$$\begin{aligned} P &= 1 - P(All\ K\ Points\ are\ outliers) \\ P &= 1 - (1-P_1)^T \\ T \times In(1-P_1) &= In(1-P) \\ T &= \frac{In(1-P)}{In(1-P_1)} \\ T &= \frac{In(1-P)}{In(1-\frac{(N-J)!(N-K)!}{N!(N-J-K)!})} \end{aligned}$$

(c) Total amount of correspondences and inliers points are same with above problem. And all possible combination are still $C(K, N)$. The correct combination is sum of K samples in I_1 area, which means $C(I_1, K)$ and K samples in I_2 area, which means $C(I_2, K)$. So, the result is:

$$\begin{aligned} P_3 &= \frac{C(I_1,K)+C(I_2,K)}{C(N,K)} \\ P_3 &= \frac{\frac{I_1!}{K!(I_1-k)!}+\frac{I_2!}{K!(I_2-k)!}}{\frac{N!}{K!(N-K)!}} \end{aligned}$$

Solution 2

(a) Code Output: (Top Left) No outliers, simple fit Error = 0.00. (Top Right) 10pc outliers, simple fit Error = 0.41. (Bottom Left) 10pc outliers, 10 iters fit Error = 0.01. (Bottom Right) 50pc outliers, 10 iters fit Error = 1.26.

The output figure is shown as figure [1]. The choice when $outliers$ = 10pc and $iters$ = 1, have the least fit error, since it has no outliers. When $outliers$ = 10pc and $iters$ = 1, fit error is large since some outliers point affect its first estimation. If we drop all outliers point, fit error become acceptable as shown in choice when $outliers$ = 10pc and $iters$ = 10. The finial choice when $outliers$ = 100pc and $iters$ = 10, isn' t good enough, since too many outliers have a really negative effective on its first estimation. The first estimation leaves the true fit line too far. Drop some outliers based on the first estimation can not lead it return to the right fit line.

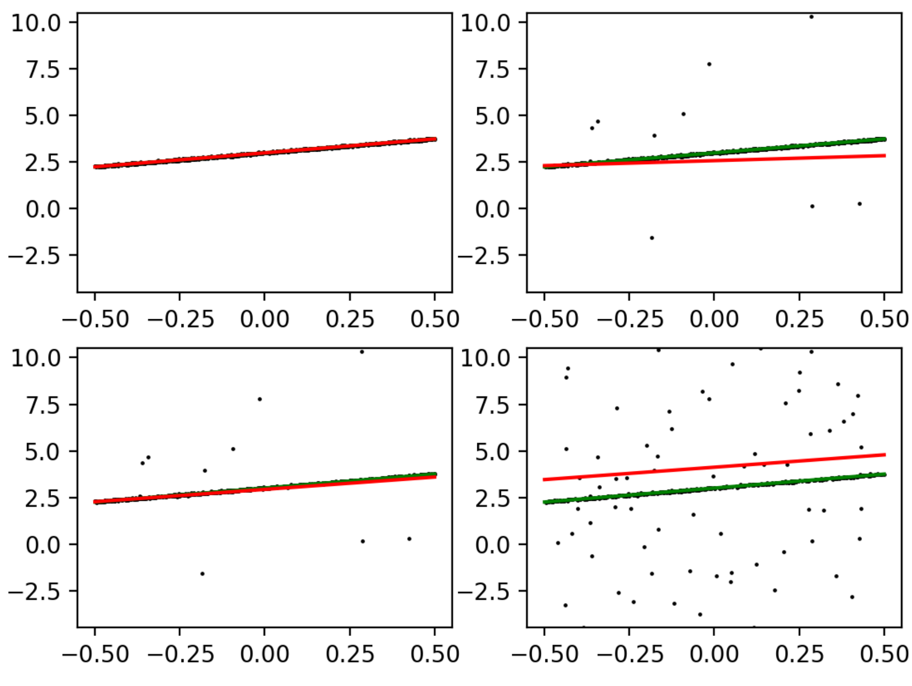

Figure 1: Output Figure in Prob2a

(b) I call the script 10 times. The result is shown as table [1]. Since mean and variation are all the least, the choice when K = 5 and N = 1000, returns the best fit most consistently.

The output figure is shown as figure [2].

Count	K = 50, N = 4, Error =	K = 5, N = 40, Error =	K = 50, N = 100, Error =	K = 5, N = 1000, Error =
1	3.92	32.64	13.95	0.19
2	4	2.32	30.36	0.19
3	2.61	0.3	0.84	1.13
4	29.81	0.98	2.08	0.64
5	66.61	0.85	8.91	0.16
6	79.83	3.34	1.25	1.15
7	52.13	1.56	0.71	0.89
8	84.46	0.33	7.88	4.55
9	5.26	1.05	14.98	0.27
10	125.08	6.37	3.03	0.2
Mean	45.371	4.974	8.399	0.937
Variation	1852.773	97.832	88.118	1.770

Table 1: Output Table in Prob2b

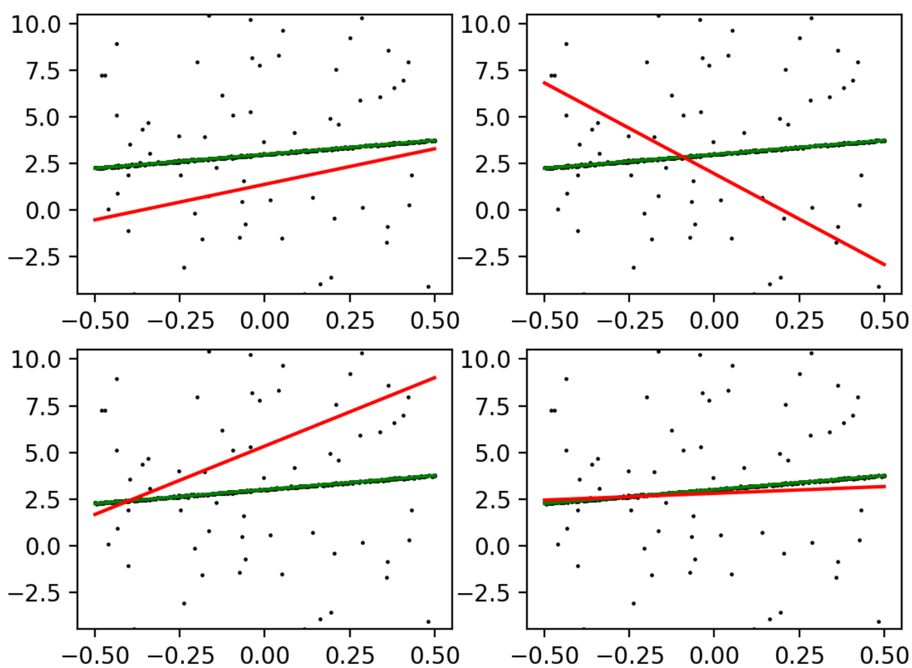

Figure 2: Output Figure in Prob2b

Solution 3

(a) Since there is no rotation and translation, matrix $[R|t]$ is same for 2 camera and we can ignore it in both camera 1 and camera 2. So,

$$p^{(1)} = [K_1 0]p' = \begin{bmatrix} f_1 & 0 & W/2 & 0 \\ 0 & f_1 & H/2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} p'$$

and

$$p^{(2)} = [K_2 0]p' = \begin{bmatrix} f_2 & 0 & W/2 & 0 \\ 0 & f_2 & H/2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} p'$$

Suppose $p' = [p'_1, p'_2, p'_3, 1]^T$, then

$$p^{(1)} = [K_1 0]p' = \begin{bmatrix} f_1 & 0 & W/2 & 0 \\ 0 & f_1 & H/2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p'_1 \\ p'_2 \\ p'_3 \\ 1 \end{bmatrix} = \begin{bmatrix} f_1 p'_1 + \frac{W}{2} p'_3 \\ f_1 p'_2 + \frac{H}{2} p'_3 \\ p'_3 \end{bmatrix} = p'_3 \begin{bmatrix} f_1 \frac{p'_1}{p'_3} + \frac{W}{2} \\ f_1 \frac{p'_2}{p'_3} + \frac{H}{2} \\ 1 \end{bmatrix}$$

and

$$p^{(2)} = [K_2 0]p' = \begin{bmatrix} f_2 & 0 & W/2 & 0 \\ 0 & f_2 & H/2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p'_1 \\ p'_2 \\ p'_3 \\ 1 \end{bmatrix} = \begin{bmatrix} f_2 p'_1 + \frac{W}{2} p'_3 \\ f_2 p'_2 + \frac{H}{2} p'_3 \\ p'_3 \end{bmatrix} = p'_3 \begin{bmatrix} f_2 \frac{p'_1}{p'_3} + \frac{W}{2} \\ f_2 \frac{p'_2}{p'_3} + \frac{H}{2} \\ 1 \end{bmatrix}$$

Because $p_1^{(1)} = f_1 \frac{p'_1}{p'_3} + \frac{W}{2}$ and $p_1^{(2)} = f_2 \frac{p'_1}{p'_3} + \frac{W}{2}$, let $p_1^{(1)} - p_1^{(2)}$:

$$p_1^{(1)} - p_1^{(2)} = (f_1 - f_2)\frac{p'_1}{p'_3}$$
$$\frac{p'_1}{p'_3} = \frac{p_1^{(1)} - p^{(2)_1}}{f_1 - f_2}$$

So, $p_1^{(1)} = f_1 \frac{p'_1}{p'_3} + \frac{W}{2} = f_1 \frac{p_1^{(1)} - p_1^{(2)}}{f_1 - f_2} + \frac{W}{2}$ and

$$p_1^{(1)} = \frac{f_1}{f_2} p_1^{(2)} - \frac{1}{2}(\frac{f_1}{f_2} - 1)W$$

Similarly, $p_2^{(1)} = \frac{f_1}{f_2} p_2^{(2)} - \frac{1}{2}(\frac{f_1}{f_2} - 1)H$.

(b) Suppose two cameras relate with same rotation matrix R and translation matrix t, and extrinsic matrices are K_1 and K_2. Consider a plane in world 3D co-ordinate $p' = (p'_1, p'_2, 0, 1)$, projected plane $p^{(1)}$ in camera 1 is:

$$p^{(1)} = K_1[R|t]p' = K_1 \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_1 \\ R_{21} & R_{22} & R_{23} & t_2 \\ R_{31} & R_{32} & R_{33} & t_3 \end{bmatrix} [p'_1, p'_2, 0, 1]^T = K_1 \begin{bmatrix} R_{11}p'_1 + R_{12}p'_2 + t_1 \\ R_{21}p'_1 + R_{22}p'_2 + t_2 \\ R_{31}p'_1 + R_{32}p'_2 + t_3 \end{bmatrix}$$

Similarly, projected plane $p^{(2)}$ in camera 1 is $p^{(2)} = K_2 \begin{bmatrix} R_{11}p'_1 + R_{12}p'_2 + t_1 \\ R_{21}p'_1 + R_{22}p'_2 + t_2 \\ R_{31}p'_1 + R_{32}p'_2 + t_3 \end{bmatrix}$.

Since there are no R_{31}, R_{31} and R_{31} in $p^{(1)}$ and $p^{(2)}$. R_{31}, R_{31} and R_{31} are redundant. **Matrix with rotation and translation can be represented with 9 elements. We can see $[R|t]$ matrix as a homography matrix that exist a extra zero column. I will represent this special $[R|t]$ matrix as H_s in my following statement.**

Then, $p^{(1)} = K_1 H_s p'$ and $p^{(2)} = K_2 H_s p'$. So, $p' = H_s^{-1} K_2^{-1} p^{(2)}$ and $p' = H_s^{-1} K_1^{-1} p^{(1)}$.

$$\begin{aligned} H_s^{-1} K_1^{-1} p^{(1)} &= H_s^{-1} K_2^{-1} p^{(2)} \\ p^{(1)} &= K_1 H_s H_s^{-1} K_2^{-1} p^{(2)} \end{aligned}$$

In conclusion, projected plane $p^{(1)}$ in camera 1 and projected plane $p^{(2)}$ in camera 2 can be related with 9 valid elements matrix H_s. These 9 elements also can be seen as a homography matrix.

Solution 4

The result figure is shown as figure [3].

Figure 3: Problem 4

Solution 5

The result figure is shown as figure [4].

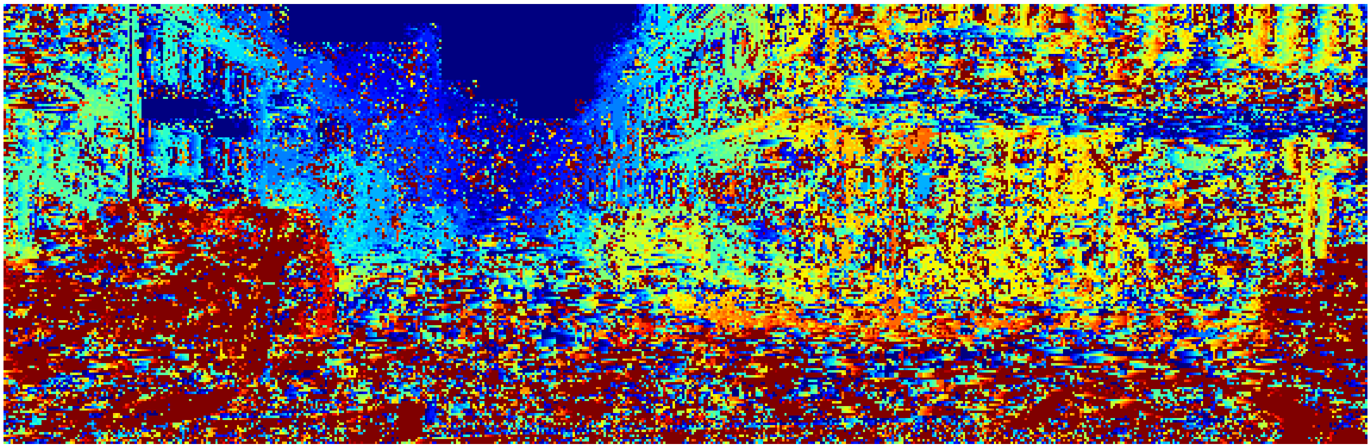

Figure 4: Problem 5

Information

This problem set took approximately 16.5 hours of effort.

I also got hints from the following sources:

- Wikipedia article on ordinary least squares at https://en.wikipedia.org/wiki/Ordinary_least_squares