Solution 1

(a) First, we can reexpress our problem as:

$$\hat{x} = arg\min_x (x-y)^2 + \lambda|x|$$
$$\hat{x} = arg\min_x x^2 - 2xy + y^2 + \lambda|x|$$

since y^2 is not related with this optimization problem,

$$\hat{x} = arg\min_x x^2 - 2xy + \lambda|x|$$

To solve this problem, we need to obtain its first-order derivate and second-order derivate. Let $f(x) = x^2 - 2xy + \lambda|x|$

$$\frac{\partial f}{\partial x} = 2x - 2y + \lambda sgn(x)$$
$$\frac{\partial^2 f}{\partial^2 x} = 2$$

because $\frac{\partial^2 f}{\partial^2 x} > 0$ for all x, $f(x)$ is a convex function. Let $\frac{\partial f}{\partial x} = 0$, we can get the global minimum.

$$2x - 2y + \lambda sgn(x) = 0$$

We then can divide the problem into two parts based on whether x is greater than zero.

Case 1: $x > 0$,

$$\hat{x} = y - \frac{1}{2}\lambda$$

this is only feasible only under condition that right side of equation is non-negative. So,

$$\hat{x} = (y - \frac{1}{2}\lambda)^+ = sgn(y)(|y| - \frac{1}{2}\lambda)^+$$

Case 2: $x < 0$,

It's similar with last case,

$$\hat{x} = (y + \frac{1}{2}\lambda)^+ = sgn(y)(|y| - \frac{1}{2}\lambda)^+$$

In conclusion, $\hat{x} = sgn(y)(|y| - \frac{1}{2}\lambda)^+$.

(b) Result with $\lambda = 0.58$ is shown as figure [1]

Figure 1: Problem 1 (b)

Solution 2

(a) Result of *Gray World* is shown as figure [2], figure [3] and figure [4].

Figure 2: Problem 2 (a1)

Figure 3: Problem 2 (a2)

(b) Result of *Gray World with 10% brightest intensities* is shown as figure [5], figure [6] and figure [7].

Figure 4: Problem 2 (a3)

Figure 5: Problem 2 (b1)

Figure 6: Problem 2 (b2)

Figure 7: Problem 2 (b3)

Solution 3

(a) Result of normal vector image is shown as figure [8].

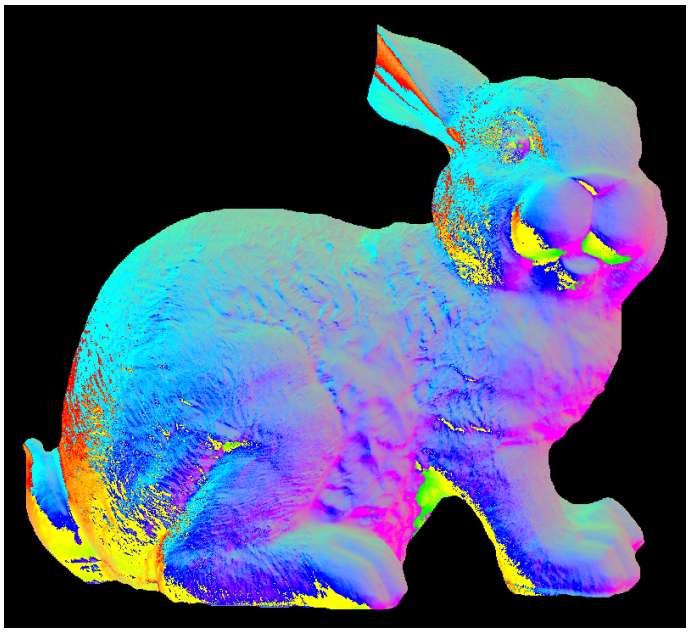

Figure 8: Problem 3 (a)

(b) Result of surface color albedos image is shown as figure [9].

Figure 9: Problem 3 (b)

Solution 4

Result of 3D surface plot image is shown as figure [10].

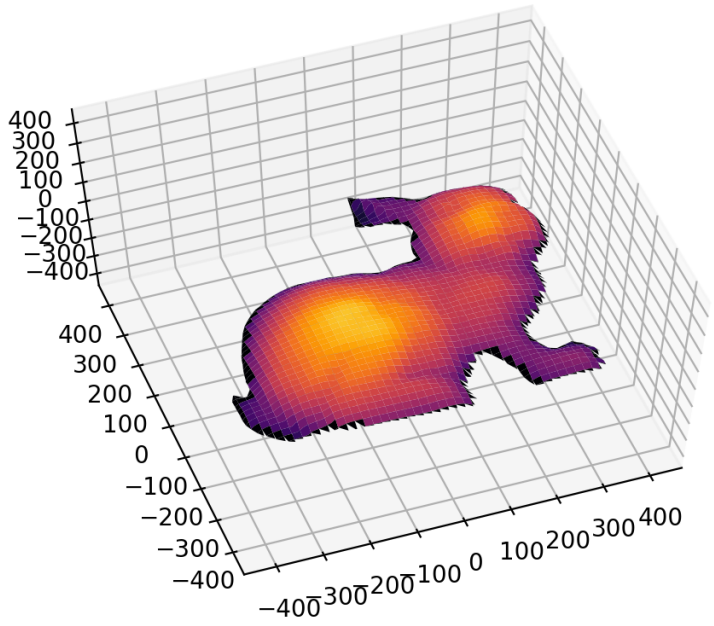

Figure 10: Problem 4

Solution 5

Result of 3D surface plot image is shown as figure [11].

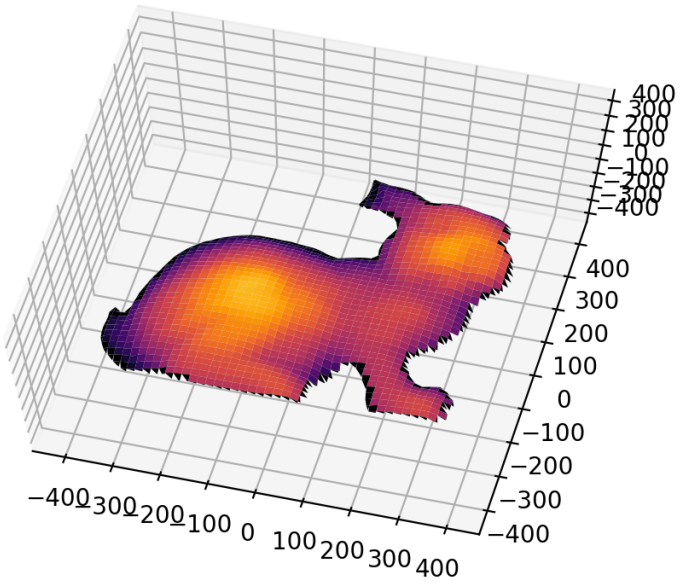

Figure 11: Problem 5

Information

This problem set took approximately 14 hours of effort.

I also got hints from the following sources:

- Derivation of Closed Form Lasso Solution: https://stats.stackexchange.com/questions/17781/derivation-of-closed-form-lasso-solution