APPENDIX

Proof of Theorem 5.6. The primary time overhead of LSBM lies in (i) estimating the influence of seeds of misinformation; (ii) generating the CP sequences; (iii) executing the Max-Coverage algorithm and computing $\sigma^L(B_L)$ and $\sigma^U(B_L^o)$ in all iterations.

As shown in [47], the time complexity of influence estimation is $O(\frac{m\cdot\ln 1/\delta}{\beta^2})$. Then. we analyze the number of CP sequences generated by LSBM.

Let $\epsilon_1 = \epsilon$, $\tilde{\epsilon}_1 = \epsilon/e$, $\hat{\epsilon}_1 = \sqrt{\frac{2a_3\mathbb{E}[I_G(s)]}{D_s^L(B_L^o)\theta_1}}$, $\epsilon_2 = \sqrt{\frac{2a_3\mathbb{E}[I_G(s)]}{D_s^L(B_L)\theta_2}}$, $\tilde{\epsilon}_2 = (\sqrt{\frac{2a_3D_s^L(B_L)\theta_2}{\mathbb{E}[I_G(s)]}+\frac{a_3^2}{9}}+\frac{a_3}{3})\cdot\frac{\mathbb{E}[I_G(s)]}{D_s^L(B_L)\theta_2}$, $a_3 = c\ln(\frac{3i_{\max}}{\delta})$ for any $c \geq 1$. In addition, let $\theta_a = \frac{2\mathbb{E}[I_G(s)]\ln\frac{6}{\delta}}{(1-1/e-\epsilon)\epsilon_1^2D_s^L(B_L^o)}$, $\theta_b = \frac{(2+2\tilde{\epsilon}_1/3)\mathbb{E}[I_G(s)]\ln\frac{6\binom{n-|S|}{k}}{\delta}}{\tilde{\epsilon}_1^2D_s^L(B_L^o)}$, $\theta_c = \frac{27\mathbb{E}[I_G(s)]\ln\frac{3i_{\max}}{\delta}(1+\beta)^2}{(1-1/e-\epsilon)(\epsilon_1+\epsilon_1\beta-2\beta)^2D_s^L(B_L^o)}$, $\theta_d = \frac{5\ln\frac{3i_{\max}}{\delta}\mathbb{E}[I_G(s)]}{18(1-\epsilon_1)(1-1/e-\epsilon)D_s^L(B_L^o)}$ and $\theta' = \max\{\theta_a,\theta_b,\theta_c,\theta_d\}$. It is easy to verify that

$$\theta' = O\left(\frac{(k\ln(n-|S|)+\ln 1/\delta)\mathbb{E}[I_G(s)]}{(\epsilon+\epsilon\beta-2\beta)^2D_s^L(B_L^o)}\right). \tag{18}$$

When $\theta_1 = \theta_2 = c\theta'$, based on Eq. (8) and Eq. (9), we have:

$$\Pr[\frac{Cov_{\mathbb{C}_1^s}(B_L^o)}{\theta_1} < (1-\epsilon_1)\cdot D_s^L(B_L^o)] \leq \left(\frac{\delta}{6}\right)^c, \tag{19}$$

$$\Pr[\frac{Cov_{\mathbb{C}_2^s}(B_L)}{\theta_2} < (1-\epsilon_1)\cdot D_s^L(B_L)] \leq \left(\frac{\delta}{6}\right)^c, \tag{20}$$

$$\Pr[\frac{Cov_{\mathbb{C}_1^s}(B_L)}{\theta_1} > D_s^L(B_L)+\tilde{\epsilon}_1\cdot D_s^L(B_L^o)] \leq \left(\frac{\delta}{6\binom{n-|S|}{k}}\right)^c, \tag{21}$$

$$\Pr[\frac{Cov_{\mathbb{C}_1^s}(B_L^o)}{\theta_1} < (1-\hat{\epsilon}_1)\cdot D_s^L(B_L^o)] \leq \left(\frac{\delta}{3i_{\max}}\right)^c, \tag{22}$$

$$\Pr[\frac{Cov_{\mathbb{C}_2^s}(B_L)}{\theta_2} < (1-\epsilon_2)\cdot D_s^L(B_L)] \leq \left(\frac{\delta}{3i_{\max}}\right)^c, \tag{23}$$

$$\Pr[\frac{Cov_{\mathbb{C}_2^s}(B_L)}{\theta_2} > (1+\tilde{\epsilon}_2)\cdot D_s^L(B_L)] \leq \left(\frac{\delta}{3i_{\max}}\right)^c. \tag{24}$$

Specially, when $\theta_1 \geq c\theta_a$, Eq. (19) holds; when $\theta_1 \geq c\theta_b$, Eq. (21) holds. Eq. (22)-Eq. (24) are obtained based on the definition of $\hat{\epsilon}_1, \epsilon_2$ and $\tilde{\epsilon}_2$. When the event in Eq. (19) and Eq. (21) not happen, we get:

$$D_s^L(B_L) \geq (1-1/e-\epsilon)D_s^L(B_L^o). \tag{25}$$

Based on Eq. (25), when when $\theta_1 \geq c\theta_a$, Eq. (20) holds. Since B_L is not independent of $\mathbb{C}_1^s$ and there are at most $\binom{n-|S|}{k}$ blocker sets, based on the union bound, the probability that none of the events in Eq. (19)-Eq. (24) happens is at least:

$$1-\left((\frac{\delta}{6})^c\cdot 2+(\frac{\delta}{6\binom{n-|S|}{k}})^c\cdot\binom{n-|S|}{k}+3\cdot(\frac{\delta}{3i_{\max}})^c\right) \geq 1-\delta^c.$$

And we have:

$$\hat{\epsilon}_1 \leq \sqrt{\frac{2(1-1/e-\epsilon)(\epsilon_1+\epsilon_1\beta-2\beta)^2}{27(1+\beta)^2}} \leq \frac{\epsilon_1+\epsilon_1\beta-2\beta}{3(1+\beta)},$$

$$\epsilon_2 \leq \sqrt{\frac{2(1-1/e-\epsilon)D_s^L(B_L^o)(\epsilon_1+\epsilon_1\beta-2\beta)^2}{27(1+\beta)^2D_s^L(B_L)}} \leq \frac{\epsilon_1+\epsilon_1\beta-2\beta}{3(1+\beta)},$$

$$\tilde{\epsilon}_2 \leq \sqrt{\frac{(\epsilon_1+\epsilon_1\beta-2\beta)^2(2+2\tilde{\epsilon}_2/3)}{27(1+\beta)^2}} \leq \frac{\epsilon_1+\epsilon_1\beta-2\beta}{3(1+\beta)}.$$

In addition, when the event in Eq. (22) not happen, we have:

$$\left(\sqrt{\frac{Cov_{\mathbb{C}_1^s}(B_L^o)\cdot(1+\beta)}{\hat{I}_G(s)}+\frac{a_3}{2}}+\sqrt{\frac{a_3}{2}}\right)^2\cdot\frac{1}{\theta_1} \geq \frac{D_s^L(B_L^o)}{\mathbb{E}[I_G(s)]}.$$

Thus, it holds that:

$$\begin{aligned}1-\hat{\epsilon}_1 &= 1-\sqrt{\frac{2a_3\mathbb{E}[I_G(s)]}{D_s^L(B_L^o)\theta_1}}\\ &\leq 1-\frac{\sqrt{2a_3}}{\sqrt{Cov_{\mathbb{C}_1^s}(B_L^o)\cdot(1+\beta)/\hat{I}_G(s)+\frac{a_3}{2}}+\sqrt{\frac{a_3}{2}}}\\ &\leq \frac{Cov^u_{\mathbb{C}_1^s}(B_L^o)\cdot(1+\beta)/\hat{I}_G(s)}{(\sqrt{Cov^u_{\mathbb{C}_1^s}(B_L^o)\cdot(1+\beta)/\hat{I}_G(s)+\frac{a_3}{2}}+\sqrt{\frac{a_3}{2}})^2}.\end{aligned}$$

Since $a_2 = \ln(\frac{3i_{\max}}{\delta}) \leq a_3$, based on Line 19 of Algorithm 4, thus

$$\begin{aligned}\sigma^U(B_L^o) &\leq \left(\sqrt{\frac{Cov_{\mathbb{C}_1^s}(B_L^o)\cdot(1+\beta)}{\hat{I}_G(s)}+\frac{a_3}{2}}+\sqrt{\frac{a_3}{2}}\right)^2\cdot\frac{1}{\theta_1}\\ &\leq \frac{Cov^u_{\mathbb{C}_1^s}(B_L^o)\cdot(1+\beta)/\hat{I}_G(s)}{1-\hat{\epsilon}_1}\cdot\frac{1}{\theta_1}.\end{aligned} \tag{26}$$

When $\theta_2 \geq \theta_d$ and according to Eq. (20), we have:

$$\frac{Cov_{\mathbb{C}_2^s}(B_L)}{\mathbb{E}[I_G(s)]} \geq \frac{\theta_2\cdot(1-\epsilon_1)D_s^L(B_L)}{\mathbb{E}[I_G(s)]} \geq \frac{5a_1}{18}.$$

Thus, $f(x) = (\sqrt{x+\frac{2a_1}{9}}-\sqrt{\frac{a_1}{2}})^2-\frac{a_1}{18})$ monotonically increasing. In addition, when the event in Eq. (24) does not happen, we have:

$$\left((\sqrt{\frac{Cov_{\mathbb{C}_2^s}(B_L)\cdot(1-\beta)}{\hat{I}_G(s)}+\frac{2a_3}{9}}-\sqrt{\frac{a_3}{2}})^2-\frac{a_3}{18}\right)\cdot\frac{1}{\theta_2} \leq \frac{D_s^L(B_L)}{\mathbb{E}[I_G(s)]}.$$

Thus, it holds that:

$$\begin{aligned}&\frac{Cov_{\mathbb{C}_2^s}(B_L)\cdot(1-\beta)}{\hat{I}_G(s)}-\frac{\tilde{\epsilon}_2D_s^L(B_L)\cdot\theta_2}{\mathbb{E}[I_G(s)]}\\ =&\frac{Cov_{\mathbb{C}_2^s}(B_L)\cdot(1-\beta)}{\hat{I}_G(s)}-(\sqrt{2a_3\frac{D_s^L(B_L)\cdot\theta_2}{\mathbb{E}[I_G(s)]}+\frac{a_3^2}{9}}+\frac{a_3}{3})\\ \leq&\frac{Cov_{\mathbb{C}_2^s}(B_L)\cdot(1-\beta)}{\hat{I}_G(s)}-(\sqrt{\frac{2Cov_{\mathbb{C}_2^s}(B_L)(1-\beta)a_3}{\hat{I}_G(s)}+\frac{4a_3^2}{9}}-\frac{2a_3}{3})\\ =&\left(\sqrt{\frac{Cov_{\mathbb{C}_2^s}(B_L)\cdot(1-\beta)}{\hat{I}_G(s)}+\frac{2a_3}{9}}-\sqrt{\frac{a_3}{2}}\right)^2-\frac{a_3}{18}.\end{aligned}$$

Since $a_1 \leq a_3$, based on the Line 14 of Algorithm 4, thus

$$\sigma^L(B_L) \geq \frac{Cov_{\mathbb{C}_2^s}(B_L)\cdot(1-\beta)}{\hat{I}_G(s)\cdot\theta_2} - \frac{\tilde{\epsilon}_2 D_s^L(B_L)}{\mathbb{E}[I_G(s)]}. \quad (27)$$

Putting Eq. (26) and Eq. (27) together, when none of the events in Eq. (19)-Eq. (24) happens, we have:

$$\begin{aligned}
\frac{\sigma^L(B_L)}{\sigma^U(B_L^o)} &\geq \frac{\frac{Cov_{\mathbb{C}_2^s}(B_L)\cdot(1-\beta)}{\hat{I}_G(s)\cdot\theta_2} - \frac{\tilde{\epsilon}_2 D_s^L(B_L)}{\mathbb{E}[I_G(s)]}}{\frac{Cov^u_{\mathbb{C}_1^s}(B_L^o)\cdot(1+\beta)/\hat{I}_G(s)}{1-\hat{\epsilon}_1}\cdot\frac{1}{\theta_1}} \\
&\geq \frac{\theta_1\left(\frac{Cov_{\mathbb{C}_2^s}(B_L)\cdot(1-\beta)}{(1+\beta)\cdot\theta_2} - \tilde{\epsilon}_2\cdot D_s^L(B_L)\right)(1-\hat{\epsilon}_1)}{Cov^u_{\mathbb{C}_1^s}(B_L^o)} \\
&\geq \frac{\theta_1\left(\frac{1-\beta}{1+\beta}\cdot(1-\epsilon_2)-\tilde{\epsilon}_2\right)\cdot D_s^L(B_L)(1-\hat{\epsilon}_1)}{Cov^u_{\mathbb{C}_1^s}(B_L^o)} \\
&\geq \frac{\theta_1\left(\frac{1-\beta}{1+\beta}\cdot(1-\epsilon_2)-\tilde{\epsilon}_2\right)\cdot D_s^L(B_L)(1-\hat{\epsilon}_1)}{Cov_{\mathbb{C}_1^s}(B_L)}(1-1/e) \\
&\geq \frac{\theta_1\left(1-\epsilon_2-\frac{2\beta}{1+\beta}-\tilde{\epsilon}_2-\hat{\epsilon}_1\right)\cdot D_s^L(B_L)}{Cov_{\mathbb{C}_1^s}(B_L)}(1-1/e) \\
&\geq \frac{\theta_1\,(1-\epsilon_1)\cdot D_s^L(B_L)}{Cov_{\mathbb{C}_1^s}(B_L)}(1-1/e) \\
&\geq \frac{\theta_1\,(1-\epsilon_1)\cdot\left(Cov_{\mathbb{C}_1^s}(B_L)/\theta_1-\tilde{\epsilon}_1\cdot D_s^L(B_L^o)\right)}{Cov_{\mathbb{C}_1^s}(B_L)}(1-1/e) \\
&\geq \frac{(1-\epsilon_1)\cdot\left(Cov_{\mathbb{C}_1^s}(B_L)-\tilde{\epsilon}_1\cdot\frac{Cov_{\mathbb{C}_1^s}(B_L^o)}{(1-\epsilon_1)}\right)}{Cov_{\mathbb{C}_1^s}(B_L)}(1-1/e) \\
&\geq (1-\epsilon_1)\left(1-\frac{\tilde{\epsilon}_1}{(1-\epsilon_1)(1-1/e)}\right)(1-1/e) \\
&= 1-1/e-\epsilon.
\end{aligned}$$

Therefore, when $\theta_1 = \theta_2 = c\theta'$ CP sequences are generated, LSBM does not stop only if at least one of the events in Eq. (19)-Eq. (24) happens. The probability is at most δ^c.

Let j be the first iteration in which the number of CP sequences generated by LSBM reaches θ'. From this iteration onward, the expected number of CP sequences further generated is at most

$$\begin{aligned}
2\cdot\sum_{z\geq j}\theta_0\cdot 2^z\cdot\delta^{2^{z-j}} &= 2\cdot 2^j\cdot\theta_0\sum_{z=0}2^z\cdot\delta^{2^z} \\
&\leq 4\theta'\sum_{z=0}2^{-2^z+z} \\
&\leq 4\theta'\sum_{z=0}2^{-z}\leq 8\theta'.
\end{aligned}$$

If the algorithm stops before this iteration, there are at most $2\theta'$ CP sequences generated. Therefore, the expected number of CP sequences generated is less than $10\theta'$, which is

$$O\left(\frac{(k\ln(n-|S|)+\ln 1/\delta)\mathbb{E}[I_G(s)]}{(\epsilon+\epsilon\beta-2\beta)^2 D_s^L(B_L^o)}\right). \quad (28)$$

We have shown that the expected time required to generate a CP sequence is $O(m\cdot\alpha(m,n))$. Based on Wald's equation [40], LSBM requires $O(\frac{(k\ln(n-|S|)+\ln 1/\delta)\mathbb{E}[I_G(s)]m\cdot\alpha(m,n)}{(\epsilon+\epsilon\beta-2\beta)^2 D_s^L(B_L^o)})$ in CP sequences generation. In addition, the total expected time used for executing the Max-Coverage and computing $\sigma^L(B_L)$ and $\sigma^U(B_L^o)$ in all the iterations is

$$\begin{aligned}
&O(k(n-|S|)\cdot i_{\max}+2\mathbb{E}[|\mathbb{C}_1^s\cup\mathbb{C}_2^s|]\cdot\mathbb{E}[|C^s|]) \\
=&O\left(\frac{(k\ln(n-|S|)+\ln 1/\delta)\mathbb{E}[I_G(s)]}{(\epsilon+\epsilon\beta-2\beta)^2}\right).
\end{aligned}$$

In summary, LSBM runs in $O(\frac{(k\ln(n-|S|)+\ln 1/\delta)\mathbb{E}[I_G(s)]m\cdot\alpha(m,n)}{(\epsilon+\epsilon\beta-2\beta)^2 D_s^L(B_L^o)}+\frac{m\cdot\ln 1/\delta}{\beta^2})$ expected time. □

Lemma 8.1. *Given a blocker set B, a seed node s and a fixed number of θ random LRR sets $\mathbb{L}$. For any $\lambda > 0$,*

$$\Pr[Cov_{\mathbb{L}}(B)-\frac{D_s^U(B)\cdot\theta}{|V_s'|}\geq\lambda]\leq\exp\left(-\frac{\lambda^2}{\frac{2D_s^U(B)}{|V_s'|}\cdot\theta+\frac{2}{3}\lambda}\right), \quad (29)$$

$$\Pr[Cov_{\mathbb{L}}(B)-\frac{D_s^U(B)\cdot\theta}{|V_s'|}\leq-\lambda]\leq\exp\left(-\frac{\lambda^2}{\frac{2D_s^U(B)}{|V_s'|}\cdot\theta}\right). \quad (30)$$

Proof. Let $L_1, L_2, \ldots, L_\theta$ denote θ random LRR sets in $\mathbb{L}$, $Cov_{L_i}(B) = x_i$ and $\frac{D_s^U(B)}{|V_s'|} = p$. Since each L_i is generated from a random realization, for any $i \in [1,\theta]$, we have

$$\mathbb{E}[x_i \mid x_1, x_2, \ldots, x_{i-1}] = \mathbb{E}[x_i] = p.$$

Let $M_i = \sum_{j=1}^{i}(x_j - p)$, thus $\mathbb{E}[M_i] = \sum_{j=1}^{i}\mathbb{E}[(x_j-p)] = 0$, and

$$\begin{aligned}
\mathbb{E}[M_i \mid M_1, M_2, \ldots, M_{i-1}] &= \mathbb{E}[M_{i-1}+(x_i-p)\mid M_1, M_2, \ldots, M_{i-1}] \\
&= M_{i-1}+\mathbb{E}[x_i]-p = M_{i-1}.
\end{aligned}$$

Therefore, $M_1, M_2, \ldots, M_\theta$ is a martingale. Since $x_i, p \in [0,1]$, we have $M_1 \leq 1$ and $M_k - M_{k-1} \leq 1$ for any $k \in [2,\theta]$. In addition,

$$\mathrm{Var}[M_1]+\sum_{k=2}^{\theta}\mathrm{Var}[M_k \mid M_1, M_2, \ldots, M_{k-1}] = \sum_{k=1}^{\theta}\mathrm{Var}[x_k].$$

We know that $\mathrm{Var}[x_i] = \mathbb{E}[x_i^2] - (\mathbb{E}[x_i])^2$ and $x_i \in [0,1]$, thus we have $\mathrm{Var}[x_i] \leq \mathbb{E}[x_i] - (\mathbb{E}[x_i])^2 = p(1-p)$. Therefore,

$$\mathrm{Var}[M_1]+\sum_{k=2}^{\theta}\mathrm{Var}[M_k \mid M_1, M_2, \ldots, M_{k-1}] \leq \theta p(1-p).$$

Then by Lemma 5.2 and $M_\theta = \sum_{i=1}^{\theta}(x_i - p)$, we can get that Eq. (29) holds. In addition, by applying Lemma 5.2 on the martingale $-M_1, -M_2, \ldots, -M_\theta$, we can deduce that Eq. (30) holds. □

Proof of Theorem 5.7. The proof of Theorem 5.7 is similar to that of Theorem 5.5 and the differences between them are that we use Lemma 8.1 as the concentration bounds and we set:

$$\theta_1 = \frac{2|V_s'|\ln(6/\delta)}{\epsilon_1^2 \cdot D_s^U(B_U^o)},$$

$$\theta_2 = \frac{(2-2/e)|V_s'|\ln(\binom{n-|S|}{k}\cdot 6/\delta)}{D_s^U(B_U^o)(\epsilon-(1-1/e)\epsilon_1)^2}, \epsilon_2 = \epsilon - (1-1/e)\cdot\epsilon_1,$$

$$\theta_{\max} = \frac{2|V_s'|\left((1-1/e)\sqrt{\ln\frac{6}{\delta}} + \sqrt{(1-1/e)(\ln\binom{|V_s'|-|S|}{k} + \ln\frac{6}{\delta})}\right)^2}{\epsilon^2\mathrm{OPT}^L},$$

$$\theta_0 = \theta_{\max}\cdot\epsilon^2\mathrm{OPT}^L/|V_s'|,$$

$$\sigma^L(B_U) = \left((\sqrt{Cov_{\mathbb{L}_2}(B_U) + \frac{2a_1}{9}} - \sqrt{\frac{a_1}{2}})^2 - \frac{a_1}{18}\right)\cdot\frac{|V_s'|}{|\mathbb{L}_2|},$$

$$\sigma^U(B_U^o) \leftarrow \left(\sqrt{Cov^u_{\mathbb{L}_1}(B_U^o) + \frac{a_2}{2}} + \sqrt{\frac{a_2}{2}}\right)^2\cdot\frac{|V_s'|}{|\mathbb{L}_1|}.$$

Note that, $\sigma^L(B_U)$ is the lower bound of $D_s^U(B_U)$ with at least $1-\frac{\delta}{3i_{\max}}$ probability, and $\sigma^U(B_U^o)$ is the upper bound of $D_s^U(B_U^o)$ with at least $1-\frac{\delta}{3i_{\max}}$ probability. □

Proof of Theorem 5.8. The proof of Theorem 5.8 is similar to that of Theorem 5.6 and the differences between them are that we use Lemma 8.1 as the concentration bounds and we set: $\epsilon_1 = \epsilon$, $\tilde{\epsilon}_1 = \epsilon/e$, $\hat{\epsilon}_1 = \sqrt{\frac{2a_3|V_s'|}{D_s^U(B_U^o)\theta_1}}$, $\epsilon_2 = \sqrt{\frac{2a_3|V_s'|}{D_s^U(B_U)\theta_2}}$, $\tilde{\epsilon}_2 = (\sqrt{\frac{2a_3 D_s^U(B_U)\theta_2}{|V_s'|} + \frac{a_3^2}{9}} + \frac{a_3}{3})\cdot\frac{|V_s'|}{D_s^U(B_U)\theta_2}$, $a_3 = c\ln(\frac{3i_{\max}}{\delta})$ for any $c \geq 1$. In addition, let $\theta_a = \frac{2|V_s'|\ln\frac{6}{\delta}}{\epsilon_1^2 D_s^U(B_U^o)}$, $\theta_b = \frac{(2+2\tilde{\epsilon}_1/3)|V_s'|\ln\frac{6\binom{n-|S|}{k}}{\delta}}{\tilde{\epsilon}_1^2 D_s^U(B_U^o)}$, $\theta_c = \frac{27|V_s'|\ln\frac{3i_{\max}}{\delta}}{(1-1/e-\epsilon)\epsilon_1^2 D_s^U(B_U^o)}$, and $\theta' = \max\{\theta_a,\theta_b,\theta_c\}$. □