1. (1) 3; (2) $\{0, \pm 3\}$. $\square$

2. 取 V 的有序基 $\mathcal{B} = \{\alpha_1, \dots, \alpha_n\}$, 记 $A = [T]_{\mathcal{B}} \in \mathbb{C}^{n\times n}$. 则 $\mathcal{B}_{\mathbb{R}} := \{\alpha_1, \dots, \alpha_n, \sqrt{-1}\alpha_1, \dots, \sqrt{-1}\alpha_n\}$ 为 $V_{\mathbb{R}}$ 的有序基, 并且 $[T_{\mathbb{R}}]_{\mathcal{B}_{\mathbb{R}}} = \begin{pmatrix} \mathrm{Re}(A) & -\mathrm{Im}(A) \\ \mathrm{Im}(A) & \mathrm{Re}(A) \end{pmatrix} \in \mathbb{R}^{2n\times 2n}$. 记 $P = \begin{pmatrix} I_n & \sqrt{-1}I_n \\ \sqrt{-1}I_n & I_n \end{pmatrix}$. 则 $P \begin{pmatrix} \mathrm{Re}(A) & -\mathrm{Im}(A) \\ \mathrm{Im}(A) & \mathrm{Re}(A) \end{pmatrix} P^{-1} = \begin{pmatrix} A & \\ & \overline{A} \end{pmatrix}$. 从而
$$\det(T_{\mathbb{R}}) = \det([T_{\mathbb{R}}]_{\mathcal{B}_{\mathbb{R}}}) = \det(A)\det(\overline{A}) = |\det(A)|^2 = |\det(T)|^2.$$ $\square$

3. (1) 设 $f \in W^0$, 即对任意 $\alpha \in W$ 有 $f(\alpha) = 0$. 为说明 $T^t(f) \in W^0$, 只需注意到: 对任意 $\alpha \in W$ 有 $T\alpha \in W$, 从而 $T^t(f)(\alpha) = f(T\alpha) = 0$.

   (2) 先证明两个辅助结论:

   (i) $(T^t)^{-1}(W^0) = T(W)^0$: 对 $f \in V^*$ 有
$$f \in (T^t)^{-1}(W^0) \iff f\circ T \in W^0 \iff f(T(W)) = \{0\} \iff f \in T(W)^0.$$

   (ii) $\dim(W + T(W)) + \dim(W \cap T^{-1}(W)) = 2\dim W$: 考虑两个满映射
$$W \to T(W), \quad \alpha \mapsto T(\alpha) \qquad 和 \qquad W\cap T^{-1}(W) \to W \cap T(W), \quad \alpha \mapsto T(\alpha).$$
   它们的核均为 $W \cap \mathrm{Ker}(T)$. 所以
$$\begin{aligned}\dim W - \dim T(W) &= \dim(W\cap T^{-1}(W)) - \dim(W \cap T(W)) \\ &= \dim(W\cap T^{-1}(W)) + \dim(W + T(W)) - \dim W - \dim T(W).\end{aligned}$$
   整理即得 (ii).

   原题的证明:
$$\begin{aligned}\dim(W^0 + T^t(W^0)) &= 2\dim W^0 - \dim(W^0 \cap (T^t)^{-1}(W^0)) \\ &= 2\dim W^0 - \dim(W^0 \cap T(W)^0) \\ &= 2\dim W^0 - \dim(W + T(W))^0 \\ &= \dim W^0 + 1.\end{aligned}$$ $\square$

4. 对 $h \in V_n$, 记 $S(h)$ 为 fh 除以 g 的商式, 即 $fh = gS(h) + T_1(h)$. 则 $S(h) \in V_n$, 从而 $S \in L(V_n)$. 由 $gS(h) = fh - T_1(h)$ 可知 $T_2(S(h)) = -T_1(h)$, 从而 $T_2 \circ S = -T_1$. 容易看出, 对 V_n 的有序基 $\mathcal{B} = \{1, x, \dots, x^{n-1}\}$, $[S]_{\mathcal{B}}$ 是对角元为 1 的上三角矩阵. 所以 $\det(S) = 1$. 因此 $\det(T_1) = \det(-T_2) = \det(T_2)$. $\square$