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on Kinematic Generalization that
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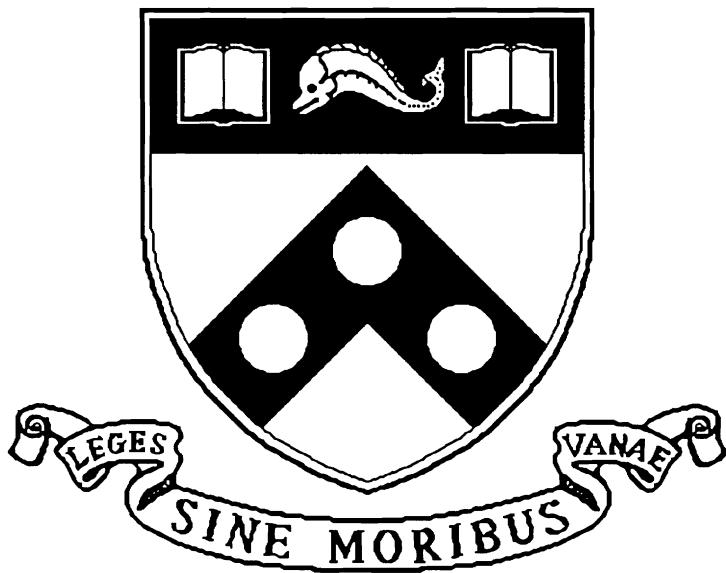
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Generalization that Preserves the Original
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Abstract

The most prominent problems in utilizing the rotoscoping data for human walking animation can be summarized into two: Preservation of the original motion characteristics in the *generalization* process and the Constraint Satisfaction.

Generalization is the process of producing the step of an arbitrary body and step length out of the original measured step which is of one particular subject and step length. If we lose much of the original style in the generalization, it would be meaningless to use the measured data. We present a generalization technique that keeps the original motion characteristics as much as possible.

Two types of generalization are considered. The one is the *body condition generalization*, which handles the differences between the two bodies. The ratio between the corresponding segments of the two bodies may not be uniform, which makes this generalization complicated. The other one is the *step length generalization*, which provides the steps with different step lengths of the same subject. These two generalizations are combined together to generate a step of arbitrary subject and step length.

The constraint satisfaction is enforced inside of our generalization process. Therefore the only thing that concerns us is the *quality* of the generalization. In our work, the preservation of the original characteristics is considered as the criteria determining the quality of the generalization. We prove that our generalization scheme actually preserves the characteristics of the original walk.

1 Introduction

The dynamic model of human walking can be described by [3, 2, 11, 4]

$$M\ddot{q} + D\dot{q} + Kq = f_{internal} + f_{others}$$

where q is the generalized coordinates, and M , D , and K are the matrices of inertia, damp, and stiffness, respectively. Generalized force is decomposed into two parts, the internal force (torque) $f_{internal}$ and the other forces f_{others} . The problem encountered in using the above equation in human walking animation is that there are unknowns in the both sides of the equation, namely \ddot{q} and $f_{internal}$.

Because $f_{internal}$ comes from the muscles, it naturally leads us to the dynamics of human muscles, whose quantitative property is not very well known enough to be used in the animation. The muscles are, in turn, controlled by the neural inputs, therefore the problem of too-many-unknowns remains basically the same [4].

Bruderlin and Calvert built a keyframeless locomotion system [2, 3]. They generated every single frame based on both dynamics (for the movement of the underlying dynamic model of human body) and kinematics (for obtaining the detailed configuration). In the forward dynamics, he approximated $f_{internal}$ according to the general biomechanical knowledge on human walking. Their system could generate a wide gamut of walking by changing the three primary parameters and other attributes. However, the dynamic model was not easy to control. For example, the walking motion is very sensitive to the hip height. 1cm difference in the height produce a big difference in the appearance of the resulting gait. Because their dynamic model was an approximation, it could generate the height error that made the motion unnatural. The dynamic correction phase required several iterations, due to its *undirectness*.

In the kinematic aspects, enormous amount of measurement has been performed on human walking [8, 7], including Winter et. al's work [12, 13, 9]. Therefore we tried to find a solution in a kinematic space that utilizes the measured data, which by its nature promises the realistic human like motion.

There have been several attempts [7, 8, 5, 6] to obtain a general property of human walking, which is basically the average of the subjects considered. This generalization demonstrates the properties of the human walking in general, but it does not provide exact data for one particular subject and step, which makes it difficult to be used in the computer animation.

Boulic et al. tried a generalization of experimental data based on the normalized velocity of walking [1]. Their generalization could produce the parameters which might violate, in its direct application, some of the constraints imposed on walking. They overcame this problem through a correction phase based on the inverse kinematics. Not to lose the original characteristics of the walking data, they introduced *coach concept*, which basically chooses the one among the multiple inverse kinematics solutions that is closest to the original motion. In our approach, the constraints are enforced within the generalization process, obviating the correction phase. Also the original locomotion style is maintained as much as possible, according to the definitions and discussions in the following sections.

Suppose a measured data set $W(S_1, sl_1)$ of the subject S_1 and the step length sl_1 is given. Our goal in this paper is to generate another data set $W(S_2, sl_2)$ of arbitrary subject S_2 and step length sl_2 . In this way, from the data of one particular subject and step, we can produce steps of any subject and step length.

When another step data $W(S_3, sl_3)$ is to be generated, most people may use the original measured one $W(S_1, sl_1)$ rather than the generalized one $W(S_2, sl_2)$ as the input of the generalization process, because some characteristics of the original motion may have been lost in producing $W(S_2, sl_2)$. However, if both of $W(S_1, sl_1)$ and $W(S_2, sl_2)$ produce the same result, then we can consider the original characteristics of $W(S_1, sl_1)$ are maintained in $W(S_2, sl_2)$ during the generalization.

Further more, if the above is true for any S_3 and sl_3 (*transitive*), then $\check{W}(S_n, sl_n)$ after the series of generalizations ($W(S_1, sl_1), W(S_2, sl_2), \dots, \check{W}(S_n, sl_n)$) will be the same with $\tilde{W}(S_n, sl_n)$ after the direct generalization ($W(S_1, sl_1), \tilde{W}(S_n, sl_n)$). Under the transitivity, we can keep any one of the intermediate results instead of the original measured data for the further generalization. The transitivity will be used as the measure of characteristic preservation, in the generalization of experimental data. We will show in the subsequent sections that our generalization scheme is indeed transitive.

One merit of our generalization method is that it can be extended *incrementally*. Because it basically *imitates* the original motion, we can *simulate different locomotion styles* by acquiring multiple sets of measurements. Thus, in one scene, several people can walk in their own walking pattern.

The kinematic generalization in the joint space only has a drawback. For example, if the angle of stance leg is given from the ball of the foot, then ankle, knee, and hip, in that order, the error at the ball affects the position of the upper body more than the one at the hip. If we reverse the order, same problem appears in locating the foot. To avoid this problem, we use Cartesian points for locating the hip and the ankle. Thus even if there is any error in the lower limb extreme angles, it is closed within the foot and the upper body is not affected.

A *timed sequence*, say Q , is a set of 2-tuples

$$Q = \{(t_i, v_i) \mid i = 1, \dots, n\} \quad (1)$$

where each t_i is a real number with $t_i < t_{i+1}$ for $i = 1, \dots, n-1$, and v_i can be any dimensional but should be the same dimensional for every i . For any timed sequence Q , we can define the function *interpolation* as,

$$\text{interpolation}(Q, t) = v_i + \frac{t - t_i}{t_{i+1} - t_i}(v_{i+1} - v_i), \quad t_i \leq t \leq t_{i+1} \quad (2)$$

At a certain moment, if a leg is between its own heelstrike (beginning) and the other leg's heelstrike (ending), it is called the *stance leg*. If a leg is between the other leg's heelstrike (beginning) and its own heelstrike (ending), it is called the *swing leg*. For example, in Figure 1, left leg is the stance leg during the interval 1, and right leg is the stance leg during the interval 2. Thus at each moment we can refer to a specific leg by either stance or swing leg with no ambiguity. The joints and segments in a leg will be referred to using prefixes *swing* or *stance*. For example, swing ankle is the ankle in the swing leg.

Let HSM^- be the Heel Strike Moment just before the current step, HSM^+ be the Heel Strike Moment right after the current step, which is one step after HSM^- , FGM be the moment when the stance foot is put flat on the ground (Flat Ground Moment), MOM be the Meta Off Moment when the toes begin to be off the floor and rotate around the tip of the toe, and TOM be the Toe Off Moment.

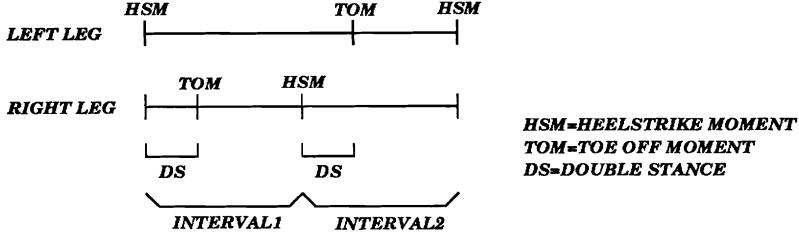


Figure 1: The Phase Diagram of Human Walk

The *body condition* $B(S)$ of a subject S is simply the m -tuple (l_1, \dots, l_m) , where l_i is the length of the i th segment and m is the total number of the segments. We say $S_2 = \alpha S_1$ iff $B(S_2) = \alpha B(S_1)$. In this case S_2 is also denoted as αS_1 .

Among the body condition components, the one that mostly affects the lower body movement in walking would be the length of the leg. The *leg length* $ll(S)$ of the subject S is defined to be the sum of the lengths of the thigh and the shin. We say $S_2 \approx \alpha S_1$ iff $ll(S_2) = ll(\alpha S_1)$. In this case, S_2 is denoted as $\tilde{\alpha} S_1$, and we say

$$\alpha = \frac{S_2}{S_1} \quad (3)$$

Note that $S_2 \approx \alpha S_1$ and $\alpha = \frac{S_2}{S_1}$ hold also when $S_2 = \alpha S_1$. The *walk condition* is simply the tuple (S, sl) of subject and the step length.

The *walk data* $W(S, sl)$ of the subject S and the step length sl is defined as the collection of the six timed sequences

$$W(S, sl) = \{H, F_1, F_2, F_3, M, A\} \quad (4)$$

where each element represents hip trajectory, foot sole angle (simply *foot angle* later on) trajectories during $[HSM^-, FGM]$, $[FGM, TOM]$, and $[TOM, HSM^+]$, the *meta angle* trajectory, and the ankle trajectory, respectively. The meta angle is the angle between the floor and the toes during the meta off phase [2, 3].

Note that H and A are in the Cartesian space, and the other elements are in the joint space. H and F_i , ($i = 1, 2, 3$) govern the stance leg of the current step, from the start to the end of the step. M is for the current stance leg from the *MOM* until the *TOM* of the next step. A is for the swing leg.

All the time values used in defining the timed sequences are normalized according to the step duration. For example, $HSM^- = 0$ and $HSM^+ = 1$. Together with the function *interpolation*, $W(S, sl)$ provides enough information to generate the lower body movement during one step. If it is clear in the context, we denote them simply as W without the arguments. The difference between \bar{W} and W is that \bar{W} is the measured data whereas W is either measured or computed according to the generalization algorithm.

In the following sections, we will assume that we have the measured data of the subject S^* at the step length sl^* , $\bar{W}(S^*, sl^*)$, which will be called *prototype walk data*.

2 Generalization to Other Step Lengths

Suppose that we have the walk data $W(S, sl)$ of the subject S with the step length sl . Our goal in this section is to obtain the walk data $W(S, psl)$ of the same figure with the step length psl .

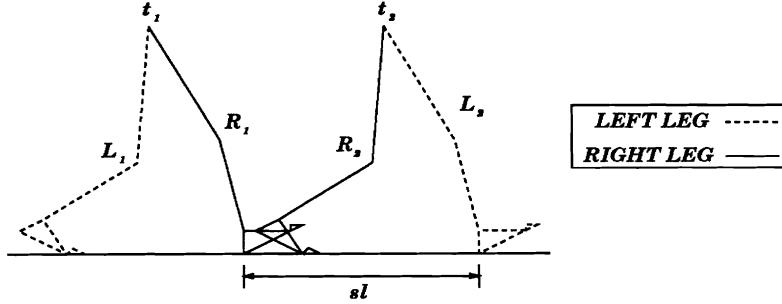


Figure 2: Before and After the Stepping

Through out this paper, because we consider only the sagittal plane movements, every position is looked at from the side.

2.1 The Position of the Hip at the Heel Strike Moment

The problem in this subsection is to find the position of the hip at the HSM^+ (at the time t_2 in the Figure 2) after stepping sl . We will use the *step symmetry concept* which was assumed in Bruderlin's work [3, 2]. A closer look of the leg L_2 of the Figure 2 is shown in the Figure 3.

a_1 is the distance between *HIP* and *ANKLE*₂. It depends on the knee angle at the heel strike moment (KAHSM) and the foot angle at the heel strike moment (FAHSM). Inman showed the KAHSM depends on the subjects and the step length [5]. But within one subject, the KAHSM is bigger for the longer steps. We approximated the KAHSM by the function

$$\mu_1(S, sl) = -\alpha_1 \left(\frac{sl}{ll(S)} - sl^* \right) + \mu_1^* \quad (5)$$

where, μ_1^* is the FAHSM of the prototype walk data. We can increase or decrease α_1 within the range [0,0.3] without affecting the preservation property of our generalization. This specific interval is based on Inman's work [5]. A similar function μ_2 can be defined to approximate FAHSM.

$$\mu_2(S, sl) = \alpha_2 \left(\frac{sl}{ll(S)} - sl^* \right) + \mu_2^* \quad (6)$$

where, μ_2^* is the FAHSM of the prototype walk data, and α_2 is a positive constant we have the control. As demonstrated by the formula, the FAHSM tends to be bigger for the longer steps. This is also justified by Imman's work [5].

Now we can compute a_1 of the figure S by the formula

$$a_1^2 = thigh^2 + shin^2 - 2 \times thigh \times shin \times \cos \mu_1(S, sl) \quad (7)$$

for the step length sl .

Let the position $ANKLE_1 = (x_a, y_a)$. Then $ANKLE_2 = (x_a + sl, y_a)$. The heel position is given by

$$HEEL = (x_a + sl - a_2, 0) \quad (8)$$

The position of A is obtained by rotating $ANKLE_2$ by $\mu_2(S, sl)$ around *HEEL*. Let the resulting position be

$$A = (x'_a, y'_a) \quad (9)$$

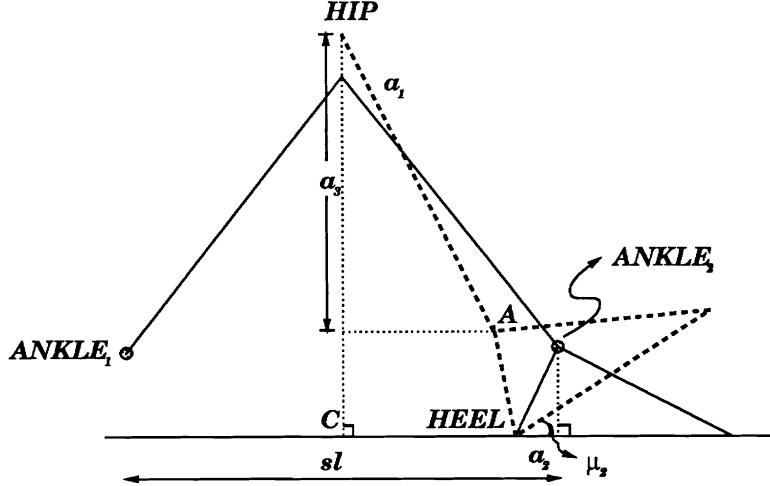


Figure 3: Computation of the Hip Height at Heel Strike

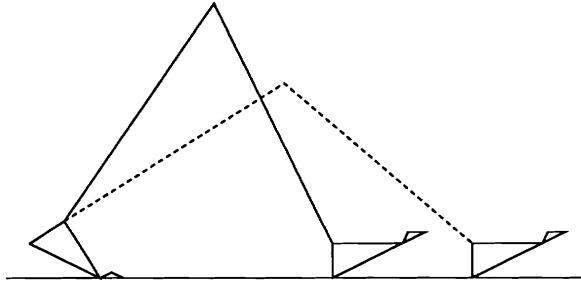


Figure 4: Comparison of Hip Heights at HSM^+ According to the Step Length

The x coordinate of HIP is $x_a + \frac{sl}{2}$. Therefore

$$a_3 = \sqrt{a_1^2 - (x'_a - x_a - \frac{sl}{2})^2} \quad (10)$$

Therefore the height of the hip is $a_3 + y'_a$, and

$$HIP = (x_a + \frac{sl}{2}, a_3 + y'_a) \quad (11)$$

Note that the position of the hip relative to the previous ankle $ANKLE_1$ is completely determined by the given step length sl . That is, the hip position at the HSM^+ is determined independently of the previous steps.

2.2 Hip Trajectory in Cartesian Space

In this subsection, we will show how the hip trajectory H^2 of the step of the Subject S and step length ρsl is generated, which is the first element of $W(S, \rho sl)$. It will be obtained by modifying H^1 of the original walk data $W(S, sl)$. The other elements of $W(S, \rho sl)$ will be discussed in the subsequent subsections.

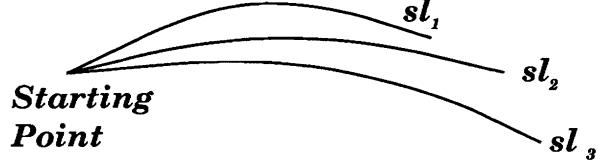


Figure 5: Comparison of Hip Trajectories According to the Step Length

Because the length of the leg is limited, the height of the hip is lower at the HSM^+ of the bigger steps, as shown in the Figure 4. Inman observed that the knee flexion during the stance phase is bigger, and therefore the maximum height of the hip is lower, for the longer steps [5]. The resulting hip trajectory will look like the Figure 5.

Let the height of the hip just before the current step be \tilde{y}_1 which is given by the current posture, and the one right after the step be \tilde{y}_n which is given by the computation in the previous subsection. Let's suppose the hip trajectory H^1 of $W(S, sl)$ be

$$H^1 = \{(t_i, x_i, y_i) \mid i = 1, \dots, n\} \quad (12)$$

x_i is the value relative to the stance ankle at FGM of the current step, and y_i is relative to the height of the floor. For example, the x coordinate of the hip position just before the current step is negative, until the hip pass the current ankle. This convention will be used through out this section.

The hip trajectory H^2 of $W(S, \rho sl)$ is defined as

$$H^2 = \{(t_i, \rho' x_i, y'_i) \mid i = 1, \dots, n\} \quad (13)$$

where

$$y'_i = y_i + (1 - t_i)(\tilde{y}_1 - y_1) + t_i(\tilde{y}_n - y_n) \quad (14)$$

ρ' is to accommodate the step length difference. Note that it may be different from ρ because the step length is defined by the foot movement, not by the hip movement. It is given by

$$\rho' = \frac{sl_{before} + \rho sl}{sl_{before} + sl} \quad (15)$$

where sl_{before} is the step length of the previous step. These particular definitions will be justified in the later sections, by showing its characteristic preservation property.

In the following subsections, we will consider the foot movement, in both angular and positional way. The foot angle is enough to determine the stance foot configuration until MOM , because the toes stay flat on the floor. But from MOM until TOM , both the foot angle and the meta angle are needed to determine the configuration. Again, from the TOM until HSM^+ , because the swing foot is off the ground, we need not only the foot angle, but also at least one point (in our work, ankle) in the foot. That is why the F_1, F_2, F_3, M , and A of $W(S, sl)$ are defined in the specific intervals.

2.3 The Foot Angle Trajectory

Suppose F^1 is the profile of the foot angle of $W(S, sl)$, which is the set union of the timed sequences F_1^1 , F_2^1 , and F_3^1 . F_1^1 represents the foot angle during $[HSM^-, FGM]$, F_2^1 during $[FGM, TOM]$,

and F_3^1 during $[TOM, HSM^+]$. I.e.,

$$F^1 = F_1^1 \cup F_2^1 \cup F_3^1 \quad (16)$$

$$F_1^1 = \{(t_{1i}, f_{1i}) \mid i = 1, \dots, n_1\} \quad (17)$$

$$F_2^1 = \{(t_{2i}, f_{2i}) \mid i = 1, \dots, n_2\} \quad (18)$$

$$F_3^1 = \{(t_{3i}, f_{3i}) \mid i = 1, \dots, n_3\} \quad (19)$$

with $t_{1i_{n_1}} = t_{21}$ and $t_{2i_{n_2}} = t_{31}$

The corresponding foot angle F^2 of $W(S, \rho sl)$ is defined by set union of the timed sequences F_1^2 , F_2^2 , and F_3^2 . F_1^2 is defined as

$$F_1^2 = \{(t_{1i}, f_{1i} \frac{\mu_2(S, sl_{before})}{f_{11}}) \mid i = 1, \dots, n_1\} \quad (20)$$

Inman [5] shows the tendency of bigger foot angles at TOM (FATOM) for the longer steps, which is approximated by the function μ_3 as follows.

$$\mu_3(S, sl) = \alpha_3(\frac{sl}{ll(S)} - sl^*) + \mu_3^* \quad (21)$$

where, μ_3^* is the FATOM of the prototype walk data, and α_3 is a positive constant we have control. μ_3 can be used in defining F_2^2 .

$$F_2^2 = \{(t_{2i}, f_{2i} \frac{\mu_3(S, \rho sl)}{f_{2n_2}}) \mid i = 1, \dots, n_2\} \quad (22)$$

Based on the monotonic property of the foot angle during $[TOM, HSM^+]$ [5], we define the last subset F_3^2 of F^2 as

$$F_3^2 = \{(t_{3i}, f'_{3i}) \mid f'_{3i} = f_{3i} + (1 - t'_{3i})d_1 + t'_{3i}d_2, i = 1, \dots, n_3\} \quad (23)$$

where $t'_{3i} = \frac{t_{3i}}{HSM^+ - TOM}$, $d_1 = \mu_3(S, \rho sl) - f_{31}$, and $d_2 = \mu_2(S, \rho sl) - f_{3n_3}$.

2.4 The Meta Angle Trajectory

After the FGM , meta angle is maintained close to zero until the MOM . From the MOM , it suddenly increases to its maximum linearly until the TOM [5]. The maximum value, i.e, the meta angle at TOM (MATOM), depends on the step length. It is approximated by the function μ_4 as follows.

$$\mu_4(S, sl) = \alpha_4(\frac{sl}{ll(S)} - sl^*) + \mu_4^* \quad (24)$$

where, μ_4^* is the MATOM of the prototype walk data, and α_4 is a positive constant we can control.

Suppose M^1 is the timed sequence that represents the meta angle trajectory of $W(S, sl)$ during $[MOM, TOM]$.

$$M^1 = \{(t_i, m_i) \mid i = 1, \dots, n_m\} \quad (25)$$

The corresponding meta angle trajectory M^2 of $W(S, \rho sl)$ is defined to be

$$M^2 = \{(t_i, m_i \frac{\mu_4(S, \rho sl)}{m_{n_m}}) \mid i = 1, \dots, n_m\} \quad (26)$$

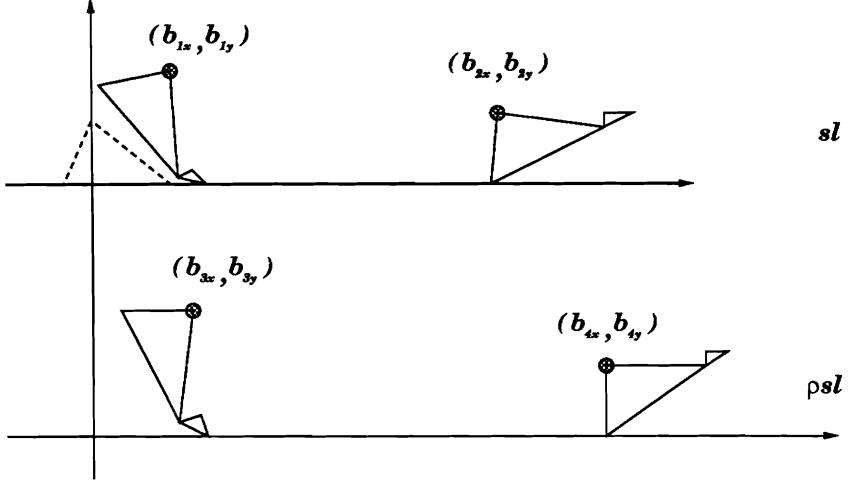


Figure 6: The Ankle Positions Before and After the Step

2.5 Ankle Trajectory During $[TOM, HSM^+]$

The ankle positions at TOM and HSM^+ can be computed using the functions μ_3 , μ_4 , and μ_2 . As shown in the Figure 6, the distance traveled along x axis during $[TOM, HSM^+]$ is slightly shorter than the actual step length. Let the coordinates of the ankle just before the TOM and right after the HSM^+ of the step length sl be (b_{1x}, b_{1y}) and (b_{2x}, b_{2y}) , respectively. Likewise, those of the step length ρsl be (b_{3x}, b_{3y}) , and (b_{4x}, b_{4y}) , respectively.

If the ankle trajectory of $W(S, sl)$ is given by

$$A^1 = \{(t_i, x_i, y_i) \mid i = 1, \dots, n_a\} \quad (27)$$

then that of $W(S, \rho sl)$ is defined as

$$A^2 = \{(t_i, x'_i, y'_i) \mid i = 1, \dots, n_a\} \quad (28)$$

where

$$x'_i = b_{3x} + \frac{b_{4x} - b_{3x}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (29)$$

and

$$y'_i = y_i + (1 - t'_i)d_1 + t'_i d_2 \quad (30)$$

with $d_1 = b_{3y} - y_1$, $d_2 = b_{4y} - y_{n_a}$, and $t'_i = \frac{t_i - TOM}{HSM^+ - TOM}$.

3 Properties of Step Length Generalization

We will denote the whole step length generalization process in the previous section by ϕ_ρ . It can be interpreted as an operator whose input is a walk data, say, $W(S, sl)$, and its output is another walk data of the same subject with step length ρ times that of the original one. It can be compactly written as

$$W(S, \rho sl) = \phi_\rho(W(S, sl)) \quad (31)$$

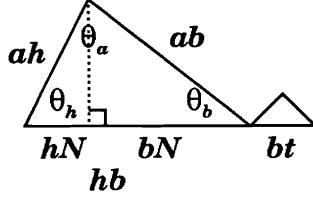


Figure 7: The Constants defined in the Foot

Before introducing the lemmas and theorems about the property of the step length generalization, let's define several lengths in the foot (Figure 7). ah , ab , hb , and bt are defined to be the lengths between the ankle and the heel, the ankle and the ball of the foot, the heel and the ball of the foot, the ball of the foot and the tip of toes, respectively. Let the point on the foot sole which is right below the ankle when the foot is flat on the ground be N . aN , hN , and bN are defined to be the length between the ankle and N , the heel and N , the ball of the foot and N , respectively. θ_a , θ_b , θ_h are defined to be the inner angles of the hindfoot, the ankle, the ball of the foot, and the heel, respectively.

Lemma 1 *The coordinates of the ankle of the subject S just before the TOM and right after the HSM^+ of the step length sl , (b_{1x}, b_{1y}) and (b_{2x}, b_{2y}) , respectively are given by the following formulas.*

$$b_{1x} = bN + bt - bt \cos \mu_4(S, sl) - ab \cos \mu_3(S, sl) \quad (32)$$

$$b_{1y} = bt \sin \mu_4(S, sl) + ab \sin \mu_3(S, sl) \quad (33)$$

$$b_{2x} = sl - (ah \cos \theta_h - ah \cos(\theta_h + \mu_2(S, sl))) \quad (34)$$

$$b_{2y} = ah \sin(\theta_h + \mu_2(S, sl)) \quad (35)$$

Proof 1 Note that the origin of the coordinates is put on the point N (defined above) of the current stance foot. The x axis is toward the walking direction, and y axis is upwards. The derivation of the four formulas is trivial, using the fact that the foot angle at TOM, the meta angle at TOM, and the foot angle at HSM^+ are given by $\mu_3(S, sl)$, $\mu_4(S, sl)$, and $\mu_2(S, sl)$, respectively. \square

Lemma 2 *In the step length generalization algorithm given in the previous section, the step length is generalized correctly. That is, the step length of $\phi_\rho(W(S, sl))$ is ρsl .*

Proof 2 In defining ϕ_ρ the ankle is placed at

$$b_{4x} = \rho sl - (ah \cos \theta_h - ah \cos(\theta_h + \mu_2(S, \rho sl))) \quad (36)$$

in x direction at the end of the step. Because the foot rotates around the heel until the FGM, the heel is fixed during that period. Therefore the x coordinate a_x of the ankle during $[HSM_+, FGM_{next}]$ can be given

$$a_x = \rho sl - (ah \cos \theta_h - ah \cos(\theta_h + \theta_{foot})) \quad (37)$$

where FGM_{next} is the FGM of the next step, and θ_{foot} is the foot angle during that interval. Because θ_{foot} becomes zero at the FGM_{next} , the equation 37 is reduced to ρsl . \square

Theorem 1 *The step length generalization defined in the previous section, can be composed in the following way for any positive number ρ_1 and ρ_2 .*

$$\phi_{\rho_1 \rho_2} = \phi_{\rho_2} \circ \phi_{\rho_1} \quad (38)$$

In the proofs through this paper, whenever a comparison is done between two walk data sets, we will have six paragraphs (i) through (vi), each of them showing the comparison between the corresponding H, F_1, F_2, F_3, M , and A 's of the sets.

Proof 1 (i) *The multiplier on x component of LHS is $\frac{sl_{before} + \rho_1 \rho_2 sl}{sl_{before} + sl}$, by the definition. Meanwhile, in the RHS, the multiplier on x component in ϕ_{ρ_1} is $\frac{sl_{before} + \rho_1 sl}{sl_{before} + sl}$. By lemma 1, the resulting step length of $\phi_{\rho}(W(S, sl))$ is actually $\rho_1 sl$. Therefore the multiplier on x component in ϕ_{ρ_2} out of ϕ_{ρ_1} is $\frac{sl_{before} + \rho_2 \rho_1 sl}{sl_{before} + \rho_1 sl}$. Therefore, the multiplier of the composite transform $\phi_{\rho_2} \circ \phi_{\rho_1}$ is*

$$\frac{sl_{before} + \rho_2 \rho_1 sl}{sl_{before} + \rho_1 sl} \frac{sl_{before} + \rho_1 sl}{sl_{before} + sl} = \frac{sl_{before} + \rho_1 \rho_2 sl}{sl_{before} + sl} \quad (39)$$

Therefore $LHS = RHS$.

Let the height of the hip just before the current step be h_{before} . Let the computed height of the hip after the step $\rho_1 sl$ and $\rho_1 \rho_2 sl$ be $h_{\rho_1 sl}$ and $h_{\rho_1 \rho_2 sl}$. The y component y_i^{LHS} of the LHS is

$$y_i^{LHS} = y_i + (1 - t_i)(h_{before} - y_1) + t_i(h_{\rho_1 \rho_2 sl} - y_n) \quad (40)$$

Meanwhile, by ϕ_{ρ_1} y_i is transformed to

$$y_i^{TEMP} = y_i + (1 - t_i)(h_{before} - y_1) + t_i(h_{\rho_1 sl} - y_n) \quad (41)$$

Note that $y_1^{TEMP} = h_{before}$ and $y_n^{TEMP} = h_{\rho_1 sl}$. Again, by ϕ_{ρ_2} , y_i^{TEMP} is transformed to

$$y_i^{RHS} = y_i^{TEMP} + (1 - t_i)(h_{before} - y_1^{TEMP}) + t_i(h_{\rho_2 \rho_1 sl} - y_n^{TEMP}) \quad (42)$$

which can be rewritten as

$$y_i^{RHS} = y_i + (1 - t_i)(h_{before} - y_1) + t_i(h_{\rho_1 sl} - y_n) \quad (43)$$

$$+ (1 - t_i)(h_{before} - y_1^{TEMP}) + t_i(h_{\rho_2 \rho_1 sl} - y_n^{TEMP}) \quad (44)$$

$$= y_i + (1 - t_i)(h_{before} - y_1) + t_i(h_{\rho_1 sl} - y_n + h_{\rho_2 \rho_1 sl} - y_n^{TEMP}) \quad (45)$$

$$= y_i + (1 - t_i)(h_{before} - y_1) + t_i(h_{\rho_2 \rho_1 sl} - y_n) = y_i^{LHS} \quad (46)$$

(ii) *The foot angle component of F_1 in LHS is*

$$f_{1i}^{LHS} = f_{1i} \frac{\mu_2(S, sl_{before})}{f_{11}} \quad (47)$$

By ϕ_{ρ_1} in RHS,

$$f_{1i}^{TEMP} = f_{1i} \frac{\mu_2(S, sl_{before})}{f_{11}} \quad (48)$$

Then by ϕ_{ρ_2} ,

$$f_{1i}^{RHS} = f_{1i}^{TEMP} \frac{\mu_2(S, sl_{before})}{f_{11}^{TEMP}} \quad (49)$$

$$= f_{1i} \frac{\mu_2(S, sl_{before})}{f_{11}} \frac{\mu_2(S, sl_{before})}{f_{11}^{TEMP}} \quad (50)$$

By noting that

$$f_{11}^{TEMP} = \mu_2(S, sl_{before}) \quad (51)$$

f_{1i}^{RHS} is reduced to

$$f_{1i}^{RHS} = f_{1i} \frac{\mu_2(S, sl_{before})}{f_{11}} \quad (52)$$

(iii) The foot angle component of F_2 in LHS is

$$f_{2i}^{LHS} = f_{2i} \frac{\mu_3(S, \rho_1 \rho_2 sl)}{f_{2n_2}} \quad (53)$$

By ϕ_{ρ_1} in RHS,

$$f_{2i}^{TEMP} = f_{2i} \frac{\mu_3(S, \rho_1 sl)}{f_{2n_2}} \quad (54)$$

Then by ϕ_{ρ_2} ,

$$f_{2i}^{RHS} = f_{2i}^{TEMP} \frac{\mu_3(S, \rho_2 \rho_1 sl)}{f_{2n_2}^{TEMP}} \quad (55)$$

$$= f_{2i} \frac{\mu_3(S, \rho_1 sl)}{f_{2n_2}} \frac{\mu_3(S, \rho_2 \rho_1 sl)}{f_{2n_2}^{TEMP}} \quad (56)$$

Noting that

$$f_{2n_2}^{TEMP} = \mu_3(S, \rho_1 sl) \quad (57)$$

f_{2i}^{RHS} is reduced to

$$f_{2i}^{RHS} = f_{2i} \frac{\mu_3(S, \rho_2 \rho_1 sl)}{f_{2n_2}} \quad (58)$$

(iv)

$$f_{3i}^{LHS} = f_{3i} + (1 - t'_{3i})d_1^{LHS} + t'_{3i}d_2^{LHS} \quad (59)$$

where

$$d_1^{LHS} = \mu_3(S, \rho_1 \rho_2 sl) - f_{31} \quad (60)$$

$$d_2^{LHS} = \mu_2(S, \rho_1 \rho_2 sl) - f_{3n_3} \quad (61)$$

Meanwhile, in the right hand side,

$$f_{3i}^{TEMP} = f_{3i} + (1 - t'_{3i})d_1^{TEMP} + t'_{3i}d_2^{TEMP} \quad (62)$$

where

$$d_1^{TEMP} = \mu_3(S, \rho_1 sl) - f_{31} \quad (63)$$

$$d_2^{TEMP} = \mu_2(S, \rho_1 sl) - f_{3n_3} \quad (64)$$

and finally,

$$f_{3i}^{RHS} = f_{3i}^{TEMP} + (1 - t'_{3i})d_1^{RHS} + t'_{3i}d_2^{RHS} \quad (65)$$

where

$$d_1^{RHS} = \mu_3(S, \rho_2 \rho_1 sl) - f_{31}^{TEMP} \quad (66)$$

$$d_2^{RHS} = \mu_2(S, \rho_2 \rho_1 sl) - f_{3n_3}^{TEMP} \quad (67)$$

Therefore

$$f_{3i}^{RHS} = f_{3i} + (1 - t'_{3i})d_1^{TEMP} + t'_{3i}d_2^{TEMP} + (1 - t'_{3i})d_1^{RHS} + t'_{3i}d_2^{RHS} \quad (68)$$

$$= f_{3i} + (1 - t'_{3i})(d_1^{TEMP} + d_1^{RHS}) + t'_{3i}(d_2^{TEMP} + d_2^{RHS}) \quad (69)$$

Noting the fact that

$$f_{31}^{TEMP} = f_{31} + d_1^{TEMP}, \quad (70)$$

we have

$$d_1^{TEMP} + d_1^{RHS} \quad (71)$$

$$= d_1^{TEMP} + \mu_3(S, \rho_2 \rho_1 sl) - f_{31}^{TEMP} \quad (72)$$

$$= d_1^{TEMP} + \mu_3(S, \rho_2 \rho_1 sl) - (f_{31} + d_1^{TEMP}) \quad (73)$$

$$= \mu_3(S, \rho_2 \rho_1 sl) - f_{31} \quad (74)$$

$$= d_1^{LHS} \quad (75)$$

Similarly, we have

$$d_2^{TEMP} + d_2^{RHS} = d_2^{LHS} \quad (76)$$

Therefore $f_{3i}^{LHS} = f_{3i}^{RHS}$ holds.

(v) Proving this part is very similar to the part (iii).

(vi) By the definition

$$x_i^{LHS} = b_{3x}^{LHS} + \frac{b_{4x}^{LHS} - b_{3x}^{LHS}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (77)$$

where

$$b_{3x}^{LHS} = bN + bt - bt \cos \mu_4(S, \rho_1 \rho_2 sl) - ab \cos \mu_3(S, \rho_1 \rho_2 sl) \quad (78)$$

$$b_{4x}^{LHS} = \rho_1 \rho_2 sl - (ah \cos \theta_h - ah \cos(\theta_h + \mu_2(S, \rho_1 \rho_2 sl))) \quad (79)$$

Meanwhile in the RHS,

$$x_i^{TEMP} = b_{3x}^{TEMP} + \frac{b_{4x}^{TEMP} - b_{3x}^{TEMP}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (80)$$

where

$$b_{3x}^{TEMP} = bN + bt - bt \cos \mu_4(S, \rho_1 sl) - ab \cos \mu_3(S, \rho_1 sl) \quad (81)$$

$$b_{4x}^{TEMP} = \rho_1 sl - (ah \cos \theta_h - ah \cos(\theta_h + \mu_2(S, \rho_1 sl))) \quad (82)$$

and finally,

$$x_i^{RHS} = b_{3x}^{RHS} + \frac{b_{4x}^{RHS} - b_{3x}^{RHS}}{b_{4x}^{TEMP} - b_{3x}^{TEMP}} (x_i^{TEMP} - b_{3x}^{TEMP}) \quad (83)$$

where

$$b_{3x}^{RHS} = bN + bt - bt \cos \mu_4(S, \rho_2 \rho_1 sl) - ab \cos \mu_3(S, \rho_2 \rho_1 sl) \quad (84)$$

$$b_{4x}^{RHS} = \rho_2 \rho_1 sl - (ah \cos \theta_h - ah \cos(\theta_h + \mu_2(S, \rho_2 \rho_1 sl))) \quad (85)$$

By substituting the equation 80 for x_i^{TEMP} in the equation 83

$$x_i^{RHS} = b_{3x}^{RHS} + \frac{b_{4x}^{RHS} - b_{3x}^{RHS}}{b_{4x}^{TEMP} - b_{3x}^{TEMP}} (b_{3x}^{TEMP} + \frac{b_{4x}^{TEMP} - b_{3x}^{TEMP}}{b_{2x} - b_{1x}} (x_i - b_{1x}) - b_{3x}^{TEMP}) \quad (86)$$

$$= b_{3x}^{RHS} + \frac{b_{4x}^{RHS} - b_{3x}^{RHS}}{b_{2x} - b_{1x}} (x_i - b_{1x}) \quad (87)$$

As demonstrated above in the equations 78, 79, 84, and 85,

$$b_{3x}^{RHS} = b_{3x}^{LHS} \quad (88)$$

$$b_{4x}^{RHS} = b_{4x}^{LHS} \quad (89)$$

$$(90)$$

and therefore finally we have

$$x_i^{LHS} = x_i^{RHS} \quad (91)$$

The y component of the LHS is defined as

$$y_i^{LHS} = y_i + (1 - t'_i) d_1^{LHS} + t'_i d_2^{LHS} \quad (92)$$

where

$$d_1^{LHS} = b_{3y}^{LHS} - y_1 \quad (93)$$

$$d_2^{LHS} = b_{4y}^{LHS} - y_{n_a} \quad (94)$$

Meanwhile, in the RHS,

$$y_i^{TEMP} = y_i + (1 - t'_i) d_1^{TEMP} + t'_i d_2^{TEMP} \quad (95)$$

with

$$d_1^{TEMP} = b_{3y}^{TEMP} - y_1 \quad (96)$$

$$d_2^{TEMP} = b_{4y}^{TEMP} - y_{n_a} \quad (97)$$

and finally,

$$y_i^{RHS} = y_i^{TEMP} + (1 - t'_i) d_1^{RHS} + t'_i d_2^{RHS} \quad (98)$$

with

$$d_1^{RHS} = b_{3y}^{RHS} - y_1^{TEMP} \quad (99)$$

$$d_2^{RHS} = b_{4y}^{RHS} - y_{n_a}^{TEMP} \quad (100)$$

By substituting the equation 95 for y_i^{TEMP} in the equation 98,

$$y_i^{RHS} \quad (101)$$

$$= y_i + (1 - t'_i)d_1^{TEMP} + t'_i d_2^{TEMP} + (1 - t'_i)d_1^{RHS} + t'_i d_2^{RHS} \quad (102)$$

$$= y_i + (1 - t'_i)(d_1^{TEMP} + d_1^{RHS}) + t'_i(d_2^{TEMP} + d_2^{RHS}) \quad (103)$$

Here,

$$d_1^{TEMP} + d_1^{RHS} = d_1^{TEMP} + b_{3y}^{RHS} - y_1^{TEMP} \quad (104)$$

$$= d_1^{TEMP} + b_{3y}^{RHS} - (y_1 + d_1^{TEMP}) \quad (105)$$

$$= b_{3y}^{RHS} - y_1 \quad (106)$$

But

$$b_{3y}^{RHS} = bt \sin \mu_4(S, \rho_2 \rho_1 sl) + ab \sin \mu_3(S, \rho_2 \rho_1 sl) \quad (107)$$

$$= bt \sin \mu_4(S, \rho_2 \rho_1 sl) + ab \sin \mu_3(S, \rho_2 \rho_1 sl) \quad (108)$$

$$= b_{3y}^{LHS} \quad (109)$$

Therefore

$$d_1^{TEMP} + d_1^{RHS} = d_1^{LHS} \quad (110)$$

In a similar way we can prove

$$d_2^{TEMP} + d_2^{RHS} = d_2^{LHS} \quad (111)$$

which in turn proves

$$y_i^{LHS} = y_i^{RHS} \quad (112)$$

□

Corollary 1 For any two step length generalizations ϕ_{ρ_1} and ϕ_{ρ_2} , their composition is commutative.

$$\phi_{\rho_2} \circ \phi_{\rho_1} = \phi_{\rho_1} \circ \phi_{\rho_2} \quad (113)$$

□

Corollary 2 For any three step length generalizations ϕ_{ρ_1} , ϕ_{ρ_2} , and ϕ_{ρ_3} , their composition is transitive.

$$\phi_{\rho_1} \circ (\phi_{\rho_2} \circ \phi_{\rho_3}) = (\phi_{\rho_1} \circ \phi_{\rho_2}) \circ \phi_{\rho_3} \quad (114)$$

□

The types of generalization considered in this paper are (1) *SL* (step length), (2) *BC* (body condition), and (3) *BOTH*. One of these will be denoted by the type variable \mathcal{U} .

Definition 1 Let T be a walk data transformation on the type \mathcal{U} . For instance, ϕ_ρ is a walk data transformation on *SL*. $T|_{(p_1, p_2)}$ is a walk data transformation from p_1 to p_2 , for arbitrary parameters p_1 and p_2 within the type \mathcal{U} . For example, if T is a transformation on the step lengths, and sl_1 and sl_2 are step lengths, $T|_{(sl_1, sl_2)}$ is the transformation which tries to produce the walk data of step length sl_2 out of the walk data of step length sl_1 . □

Definition 2 Let W be a walk data. Let \mathcal{U} be one of the generalization types. $W|_{\mathcal{U}}$ is the value of \mathcal{U} of W . For example, $W|_{SL}$ is the step length of the walk data W . \square

Definition 3 Let T be a walk data transformation on \mathcal{U} . If T satisfies the following condition for an arbitrary walk data W with $W|_{\mathcal{U}} = p_1$, and arbitrary parameters p_2 and p_3 in \mathcal{U} , it is called transitive.

$$T|_{(p_2, p_3)} \circ T|_{(p_1, p_2)} = T|_{(p_1, p_3)} \quad (115)$$

\square

Suppose the original walk data W_1 is generalized into W_2 , and the generalized one W_2 can again be used as an original walk data to produce another one W_3 , and so on, until we get W_n . Specially if the result W_n of the serial generalization $(W_1, W_2, W_3, \dots, W_n)$ is identical with the result W_n of the direct generalization (W_1, W_n) , for any n , the original property of the walk data seems to be preserved in each generalization. This intuition leads to the following definition.

Definition 4 A walk data transformation is said to be characteristic preserving if the transformation is transitive. \square

Theorem 2 The walk data transformation ϕ on the step length defined in this section is characteristic preserving.

Proof 2 Let W_1 be a walk data with step length sl_1 . Let sl_2 and sl_3 be two arbitrary step lengths. The transformation from sl_1 to sl_2 is $T|_{(sl_1, sl_2)} = \phi_{\frac{sl_2}{sl_1}}$. The transformation from sl_2 to sl_3 is $T|_{(sl_2, sl_3)} = \phi_{\frac{sl_3}{sl_2}}$. The direct transformation from sl_1 to sl_3 is $T|_{(sl_1, sl_3)} = \phi_{\frac{sl_3}{sl_1}}$. By the theorem 1,

$$T|_{(sl_2, sl_3)} \circ T|_{(sl_1, sl_2)} = \phi_{\frac{sl_3}{sl_2}} \circ \phi_{\frac{sl_2}{sl_1}} \quad (116)$$

$$= \phi_{\frac{sl_3}{sl_2} \frac{sl_2}{sl_1}} \quad (117)$$

$$= \phi_{\frac{sl_3}{sl_1}} \quad (118)$$

$$= T|_{(sl_1, sl_3)} \quad (119)$$

\square

4 Generalization among Different Body Conditions

Let us imagine 2 human figures A and B. B's kinematic property is α times that of A's, in every aspect segmentwise. That is, $B = \alpha A$. In this situation, if B is walking at a step length that is α times as long as that of A's, what would be the joints angles of B compared to the corresponding joint angles of A? We assume that they are *the same*. Therefore if we have data for A at the step length sl , we can use it for B at the step length αsl . This will be called the *similarity assumption* later on.

The above is justifiable by Murray et al.'s experiment [8]. They divided 60 subjects into 3 groups (20 subjects in each group) according to their height: tall, medium, and short. Each subject was trained to walk freely as they usually do. A significant correlation between the height and

the stride length was found.

Group	Mean Height(in)	Stride Length(in)	Ratio(Stride/Height)
Tall	72.2	63.98	0.886
Medium	69.1	61.50	0.890
Short	66.0	59.37	0.899

The above table shows the stride length is linearly related with the height. In this experiment they observed that *there were no significant differences* in the major joint (hip, knee, ankle) angles among the groups, which advocates the above similarity assumption.

The flexion angle of the hip of the group Short was slightly bigger than the other two groups. That phenomenon can be explained by the slightly increasing ratio values in the above table. Because people live in a community, there tends to be a regression effect in walking. The shorter group's stride length relative to their height tends to be longer than that of the longer group. If the ratio value in the experiment was maintained constant, then the hip flexion of the Short might be more close to the other groups.

There have been many trials to find the relative size of the segments in human body. Since such kind of information depends on the individuals, the results depended on the sampled subjects from which the statistics were computed. The sampling may differ among the research groups. Therefore in using rotoscopy data for human walking animation in particular, the model used in the animation is more likely different from the subject on which the measurement was performed, not only in the total size but the ratio of the corresponding segments may not uniform. So the similarity assumption alone can not cover the variety of the locomotion phenomenon under the general human body conditions.

The walk data $W(\alpha S, \alpha sl)$ is simply derivable from $W(S, sl)$, based on the similarity assumption. Generally speaking, the Cartesian quantity is scaled by α and the angular quantity remains the same.

However, if the scale is not the same between the corresponding segments, the similarity assumption can not be applied directly. Let the walk data $W(S, sl)$ of the subject S be given at the step length sl . We want to derive the walk data of an arbitrary subject S' and step length αsl , where $S' \approx \alpha S$. As defined earlier, the total leg lengths $ll(S')$ and $ll(\alpha S)$ are the same, therefore the hip trajectories of S' and αS will be similar.

We assume that the trajectory of the hip of S' will be the same as αS except for the ankle height difference. That is, if ΔaN is the difference between the ankle heights of S' and αS (positive if S' is higher), and if the hip trajectory of S is H^1

$$H^1 = \{(t_i, x_i, y_i) \mid i = 1, \dots, n\} \quad (120)$$

then that of S' is defined by

$$H^2 = \{(t_i, x'_i, y'_i) \mid i = 1, \dots, n\} \quad (121)$$

where

$$x'_i = \alpha x_i \quad (122)$$

$$y'_i = \alpha y_i + \Delta aN \quad (123)$$

The foot angle trajectories and the meta angle trajectory of S' are the same as those of αS , which are in turn same as those of S . Note that the functions $\mu_i, i = 1, 2, 3, 4$, are invariant on the

body condition scaling. That is, for any subject S , and for any step length sl ,

$$\mu_i(\alpha S, \alpha sl) = \alpha_i \left(\frac{\alpha sl}{ll(\alpha S)} - sl^* \right) + \mu^* = \mu_i(S, sl) \quad (124)$$

Once the foot angle and the meta angle trajectories are available, ankle positions of both S and S' at TOM and HSM^+ can be computed. Let the coordinates of the ankle just before the TOM and right after the HSM^+ of the subject S with the step length sl be (b_{1x}, b_{1y}) and (b_{2x}, b_{2y}) , respectively. Likewise, those of the subject S' with step length αsl be (b_{3x}, b_{3y}) , and (b_{4x}, b_{4y}) , respectively. As in the step length generalization, b_{4x} is determined so that the resulting step length is αsl .

If the ankle trajectory of $W(S, sl)$ is given by

$$A^1 = \{(t_i, x_i, y_i) \mid i = 1, \dots, n_a\} \quad (125)$$

then that of $W(S', \alpha sl)$ is defined as a slight modification of the ankle trajectory in $W(\alpha S, \alpha sl)$,

$$A^2 = \{(t_i, x'_i, y'_i) \mid i = 1, \dots, n_a\} \quad (126)$$

where

$$x'_i = b_{3x} + \frac{b_{4x} - b_{3x}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (127)$$

and

$$y'_i = \alpha y_i + (1 - t'_i)d_1 + t'_i d_2 \quad (128)$$

with $d_1 = b_{3y} - \alpha y_1$, $d_2 = b_{4y} - \alpha y_{n_a}$, and $t'_i = \frac{t_i - TOM}{HSM^+ - TOM}$. The model discrepancy between αS and S' is absorbed at the knee angle, and somewhere at the foot.

The whole process above will be denoted by $\psi_{S_1}^{S_2}$, which maps the walk data $W(S_1, sl)$ to $W(S_2, \frac{ll(S_2)}{ll(S_1)}sl)$. I.e,

$$W(S_2, \frac{ll(S_2)}{ll(S_1)}sl) = \psi_{S_1}^{S_2}(W(S_1, sl)) \quad (129)$$

Lemma 3 For any subjects S_1 , S_2 , and for any step length sl ,

$$\psi_{S_1}^{S_2}(W(S_1, sl))|_{SL} = \frac{ll(S_2)}{ll(S_1)}sl \quad (130)$$

I.e., the step length is generalized correctly by the body condition generalization.

Proof 3 This is clear by the discussion above. \square

Theorem 3 For any subjects S , S_1 , and S_2 ,

$$\psi_{S_1}^{S_2} \circ \psi_S^{S_1} = \psi_S^{S_2} \quad (131)$$

Therefore the body condition generalization is characteristic preserving.

Proof 3 Let $\alpha_1 = \frac{S_1}{S}$ and $\alpha_2 = \frac{S_2}{S_1}$. Then $\frac{S_2}{S} = \alpha_1\alpha_2$.

(i) *Showing the equality of the x coordinates:* By $\psi_S^{S_1}$ x_i goes to α_1x_i , which in turn, by $\psi_S^{S_2}$ goes to $\alpha_2\alpha_1x_i$. But in the RHS, x_i directly goes to $\alpha_1\alpha_2x_i$ by $\psi_S^{S_2}$. *Showing the equality of the y coordinates:* Let the ankle heights of the subjects S , S_1 , and S_2 be aN , aN_1 , and aN_2 , respectively. Let

$$\Delta aN_1 = aN_1 - \alpha_1aN \quad (132)$$

$$\Delta aN_2 = aN_2 - \alpha_2aN_1 \quad (133)$$

By $\psi_S^{S_1}$, y_i goes to

$$y_i^{TEMP} = \alpha_1y_i + \Delta aN_1 \quad (134)$$

which again by $\psi_{S_1}^{S_2}$ goes to

$$y_i^{LHS} = \alpha_2y_i^{TEMP} + \Delta aN_2 \quad (135)$$

$$= \alpha_2(\alpha_1y_i + \Delta aN_1) + \Delta aN_2 \quad (136)$$

$$= \alpha_2\alpha_1y_i + \alpha_2\Delta aN_1 + \Delta aN_2 \quad (137)$$

But in the RHS, y_i directly goes to

$$y_i^{RHS} = \alpha_1\alpha_2y_i + \Delta aN_{12} \quad (138)$$

by $\psi_S^{S_2}$, where

$$\Delta aN_{12} = aN_2 - \alpha_1\alpha_2aN \quad (139)$$

But a simple manipulation of the equations 132 and 133 gives us

$$\alpha_2\Delta aN_1 + \Delta aN_2 = \Delta aN_{12} \quad (140)$$

and thus,

$$y_i^{LHS} = y_i^{RHS} \quad (141)$$

(ii)-(v) Because the foot angle and the meta angle trajectories remain the same during the body condition generalizations, the resulting angular trajectories in both sides will be identical to the original ones in $W(S, sl)$.

(vi) Let the ankle positions of S just before the TOM and right after the HSM^+ with the step length sl be (b_{1x}, b_{1y}) and (b_{2x}, b_{2y}) , respectively. Likewise, let those of the subject S_1 after applying $\psi_S^{S_1}$ be $(b_{3x}^{TEMP}, b_{3y}^{TEMP})$, and $(b_{4x}^{TEMP}, b_{4y}^{TEMP})$, respectively. Let the ankle positions of the subject S_2 by applying $\psi_{S_1}^{S_2}$ to the result of $\psi_S^{S_1}$ be $(b_{3x}^{LHS}, b_{3y}^{LHS})$ and $(b_{4x}^{LHS}, b_{4y}^{LHS})$, respectively. Let the ankle positions of the subject S_2 by the direct application of $\psi_S^{S_2}$ be $(b_{3x}^{RHS}, b_{3y}^{RHS})$ and $(b_{4x}^{RHS}, b_{4y}^{RHS})$, respectively.

By the lemma 3, the final step lengths of both the LHS and RHS are identical. And the foot angles and the meta angles of the LHS and RHS at the TOM and the HSM^+ are identical. Moreover, eventually at the end of both the applications, the subject S_2 is considered. Therefore

$$(b_{3x}^{LHS}, b_{3y}^{LHS}) = (b_{3x}^{RHS}, b_{3y}^{RHS}) \quad (142)$$

$$(b_{4x}^{LHS}, b_{4y}^{LHS}) = (b_{4x}^{RHS}, b_{4y}^{RHS}) \quad (143)$$

This can be also checked in the similar way as in the step length generalization. By $\psi_S^{S_1}$ x_i goes to

$$x_i^{TEMP} = b_{3x}^{TEMP} + \frac{b_{4x}^{TEMP} - b_{3x}^{TEMP}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (144)$$

which again by $\psi_S^{S_2}$ goes to

$$x_i^{LHS} = b_{3x}^{LHS} + \frac{b_{4x}^{LHS} - b_{3x}^{LHS}}{b_{4x}^{TEMP} - b_{3x}^{TEMP}}(x_i^{TEMP} - b_{3x}^{TEMP}) \quad (145)$$

$$= b_{3x}^{LHS} + \frac{b_{4x}^{LHS} - b_{3x}^{LHS}}{b_{4x}^{TEMP} - b_{3x}^{TEMP}}(b_{3x}^{TEMP} + \frac{b_{4x}^{TEMP} - b_{3x}^{TEMP}}{b_{2x} - b_{1x}}(x_i - b_{1x}) - b_{3x}^{TEMP}) \quad (146)$$

$$= b_{3x}^{LHS} + \frac{b_{4x}^{LHS} - b_{3x}^{LHS}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (147)$$

In the RHS, $\psi_S^{S_2}$ sends x_i directly to

$$x_i^{RHS} = b_{3x}^{RHS} + \frac{b_{4x}^{RHS} - b_{3x}^{RHS}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (148)$$

By the results 142 and 143, we conclude that

$$x_i^{LHS} = x_i^{RHS} \quad (149)$$

□

5 Combining the Two Types of Generalization

To have a full generalization, the two kinds of generalization, namely, the step length generalization and the body condition generalization, should be combined together. Let's suppose the original walk condition is (S_1, sl_1) , and the desired walk condition is (S_2, sl_2) . We can apply the body condition generalization first

$$W(S_2, \frac{S_2}{S_1}sl_1) = \psi_{S_1}^{S_2}(W(S_1, sl_1)) \quad (150)$$

and then apply the step length generalization to get the final result

$$W(S_2, sl_2) = \phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}}(W(S_2, \frac{S_2}{S_1}sl_1)) \quad (151)$$

Another way around is to apply step length generalization first

$$W(S_1, \frac{S_1}{S_2}sl_2) = \phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}}(W(S_1, sl_1)) \quad (152)$$

and then apply body condition generalization,

$$W(S_2, sl_2) = \psi_{S_1}^{S_2}(W(S_1, \frac{S_1}{S_2}sl_2)) \quad (153)$$

One obvious question here is which way is correct or better. It seems desirable that the order of the applications of the generalization does not affect the result. In fact, our generalization algorithm does have that property.

Theorem 4 In applying the generalizations ϕ and ψ defined in the previous sections, the order of the application does not affect the final result. I.e., for any walk data $W_1 = W(S_1, sl_1)$, and for any walk condition (S_2, sl_2) ,

$$\phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}}(\psi_{S_1}^{S_2}(W_1)) = \psi_{S_1}^{S_2}(\phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}}(W_1)) \quad (154)$$

or simply

$$\phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}} \circ \psi_{S_1}^{S_2} = \psi_{S_1}^{S_2} \circ \phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}} \quad (155)$$

Proof 4 We will use the superscript $LTEMP$ to denote the result right after the application of $\psi_{S_1}^{S_2}$ in the left hand side, LHS after the application of the whole thing of the left hand side $\phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}} \circ \psi_{S_1}^{S_2}$. $RTEMP$ and RHS are used in the similar way.

(i) Let sl_{before} be the step length of the previous step of S_1 . Then that of S_2 will be $\frac{S_2}{S_1}sl_{before}$. In the left hand side,

$$x_i^{LTEMP} = \frac{S_2}{S_1}x_i \quad (156)$$

$$x_i^{LHS} = \frac{\frac{S_2}{S_1}sl_{before} + sl_2}{\frac{S_2}{S_1}sl_{before} + \frac{S_2}{S_1}sl_1} \frac{S_2}{S_1}sl_1 x_i \quad (157)$$

In the right hand side,

$$x_i^{RTEMP} = \frac{sl_{before} + \frac{S_1}{S_2}sl_2}{sl_{before} + sl_1} x_i \quad (158)$$

$$x_i^{RHS} = \frac{S_2}{S_1}sl_1 \frac{sl_{before} + \frac{S_1}{S_2}sl_2}{sl_{before} + sl_1} x_i \quad (159)$$

But a simple manipulation of the equation 157 and 159 gives

$$x_i^{LHS} = x_i^{RHS} \quad (160)$$

Now, let's look at the y component of the hip trajectory.

$$y_i^{LTEMP} = \frac{S_2}{S_1}y_i + \Delta aN \quad (161)$$

$$y_i^{LHS} = y_i^{LTEMP} + (1 - t_i)(\tilde{y}_1^{LHS} - y_1^{LTEMP}) + t_i(\tilde{y}_n^{LHS} - y_n^{LTEMP}) \quad (162)$$

$$= \frac{S_2}{S_1}y_i + \Delta aN + (1 - t_i)(\tilde{y}_1^{LHS} - y_1^{LTEMP}) + t_i(\tilde{y}_n^{LHS} - y_n^{LTEMP}) \quad (163)$$

$$y_i^{RTEMP} = y_i + (1 - t_i)(\tilde{y}_1^{RTEMP} - y_1) + t_i(\tilde{y}_n^{RTEMP} - y_n) \quad (164)$$

$$y_i^{RHS} = \frac{S_2}{S_1}y_i^{RTEMP} + \Delta aN \quad (165)$$

$$= \frac{S_2}{S_1}y_i + (1 - t_i)(\frac{S_2}{S_1}\tilde{y}_1^{RTEMP} - \frac{S_2}{S_1}y_1) + t_i(\frac{S_2}{S_1}\tilde{y}_n^{RTEMP} - \frac{S_2}{S_1}y_n) + \Delta aN \quad (166)$$

We need to check the equalities of

$$\tilde{y}_1^{LHS} - y_1^{LTEMP} = \frac{S_2}{S_1} \tilde{y}_1^{RTEMP} - \frac{S_2}{S_1} y_1 \quad (167)$$

$$\tilde{y}_n^{LHS} - y_n^{LTEMP} = \frac{S_2}{S_1} \tilde{y}_n^{RTEMP} - \frac{S_2}{S_1} y_n \quad (168)$$

In the equation 161,

$$y_1^{LTEMP} = \frac{S_2}{S_1} y_1 + \Delta a N \quad (169)$$

Note that \tilde{y}_1^{RTEMP} is the height of the hip of S_1 just before the current step of step length $\frac{S_1}{S_2} sl_2$, and \tilde{y}_1^{LHS} is the height of the hip of S_2 just before the current step of step length sl_2 . Therefore they are related by the equation 123 as follows.

$$\tilde{y}_1^{LHS} = \frac{S_2}{S_1} \tilde{y}_1^{RTEMP} + \Delta a N \quad (170)$$

Replacing the results 169 and 170 into the equation 167 proves the equality. In a similar way we can show the equation 168 holds.

(ii)

$$f_{1i}^{LTEMP} = f_{1i} \quad (171)$$

$$f_{1i}^{LHS} = f_{1i} \frac{\mu_2(S_2, \frac{S_2}{S_1} sl_{before})}{f_{11}} \quad (172)$$

$$= f_{1i} \frac{\mu_2(S_1, sl_{before})}{f_{11}} \quad (173)$$

because

$$\mu_2(S_2, \frac{S_2}{S_1} sl_{before}) = \alpha_2 \left(\frac{\frac{S_2}{S_1} sl_{before}}{ll(S_2)} - sl^* \right) + \mu_2^* \quad (174)$$

$$= \alpha_2 \left(\frac{\frac{S_2}{S_1} sl_{before}}{\frac{S_2}{S_1} ll(S_1)} - sl^* \right) + \mu_2^* \quad (175)$$

$$= \alpha_2 \left(\frac{sl_{before}}{ll(S_1)} - sl^* \right) + \mu_2^* \quad (176)$$

$$= \mu_2(S_1, sl_{before}) \quad (177)$$

Meanwhile,

$$f_{1i}^{RHS} = f_{1i}^{RTEMP} = f_{1i} \frac{\mu_2(S_1, sl_{before})}{f_{11}} \quad (178)$$

$$= f_{1i}^{LHS} \quad (179)$$

(iii)-(v) Noting the fact that

$$\mu_i(S_2, sl_2) = \mu_i(S_1, \frac{S_1}{S_2} sl_2), \quad i = 1, 2, 3, 4 \quad (180)$$

we can show the equalities between the LHS and the RHS in a similar way as in (ii).

(vi) Let the ankle positions of S_1 just before the TOM and right after the HSM^+ with the step length sl be (b_{1x}, b_{1y}) and (b_{2x}, b_{2y}) , respectively. Likewise, let those of the subject S_2 after applying $\psi_{S_1}^{S_2}$ be $(b_{3x}^{LTEMP}, b_{3y}^{LTEMP})$, and $(b_{4x}^{LTEMP}, b_{4y}^{LTEMP})$, respectively. Let the ankle positions of the subject S_2 by applying $\phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}}$ to the result of $\psi_{S_1}^{S_2}$ be $(b_{3x}^{LHS}, b_{3y}^{LHS})$ and $(b_{4x}^{LHS}, b_{4y}^{LHS})$, respectively.

Let the ankle positions of the subject S_1 after the application of $\phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}}$ be $(b_{3x}^{RTEMP}, b_{3y}^{RTEMP})$ and $(b_{4x}^{RTEMP}, b_{4y}^{RTEMP})$, respectively. Let the ankle positions of the subject S_2 by applying $\psi_S^{S_2}$ to the result of $\phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}}$ be $(b_{3x}^{RHS}, b_{3y}^{RHS})$ and $(b_{4x}^{RHS}, b_{4y}^{RHS})$, respectively.

By the lemmas 2 and 3, the final step lengths of both the LHS and RHS are the same. And the corresponding foot angles and the meta angles of the LHS and RHS are identical as shown in (ii)-(v) above. Moreover, eventually at the end of both side the applications, the subject S_2 is considered. Therefore

$$(b_{3x}^{LHS}, b_{3y}^{LHS}) = (b_{3x}^{RHS}, b_{3y}^{RHS}) \quad (181)$$

$$(b_{4x}^{LHS}, b_{4y}^{LHS}) = (b_{4x}^{RHS}, b_{4y}^{RHS}) \quad (182)$$

In the LHS,

$$x_i^{TEMP} = b_{3x}^{TEMP} + \frac{b_{4x}^{TEMP} - b_{3x}^{TEMP}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (183)$$

$$x_i^{LHS} = b_{3x}^{LHS} + \frac{b_{4x}^{LHS} - b_{3x}^{LHS}}{b_{4x}^{LTEMP} - b_{3x}^{LTEMP}}(x_i^{LTEMP} - b_{3x}^{LTEMP}) \quad (184)$$

$$= b_{3x}^{LHS} + \quad (185)$$

$$\frac{b_{4x}^{LHS} - b_{3x}^{LHS}}{b_{4x}^{LTEMP} - b_{3x}^{LTEMP}}(b_{3x}^{LTEMP} + \frac{b_{4x}^{LTEMP} - b_{3x}^{LTEMP}}{b_{2x} - b_{1x}}(x_i - b_{1x}) - b_{3x}^{LTEMP}) \quad (186)$$

$$= b_{3x}^{LHS} + \frac{b_{4x}^{LHS} - b_{3x}^{LHS}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (187)$$

Similarly in the RHS,

$$x_i^{RHS} = b_{3x}^{RHS} + \frac{b_{4x}^{RHS} - b_{3x}^{RHS}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (188)$$

Using the facts 181 and 182, we can deduce

$$x_i^{LHS} = x_i^{RHS} \quad (189)$$

As for the y component,

$$y_i^{LTEMP} = \frac{S_2}{S_1}y_i + (1 - t'_i)d_1^{LTEMP} + t'_i d_2^{LTEMP} \quad (190)$$

$$y_i^{LHS} = y_i^{LTEMP} + (1 - t'_i)d_1^{LHS} + t'_i d_2^{LHS} \quad (191)$$

$$= \frac{S_2}{S_1}y_i + (1 - t'_i)(d_1^{LTEMP} + d_1^{LHS}) + t'_i(d_2^{LTEMP} + d_2^{LHS}) \quad (192)$$

$$y_i^{RTEMP} = y_i + (1 - t'_i)d_1^{RTEMP} + t'_i d_2^{RTEMP} \quad (193)$$

$$y_i^{RHS} = \frac{S_2}{S_1} y_i^{RTEMP} + (1 - t'_i) d_1^{RHS} + t'_i d_2^{RHS} \quad (194)$$

$$= \frac{S_2}{S_1} y_i + (1 - t'_i) (\frac{S_2}{S_1} d_1^{RTEMP} + d_1^{RHS}) + t'_i (\frac{S_2}{S_1} d_2^{RTEMP} + d_2^{RHS}) \quad (195)$$

But

$$d_1^{LHS} = b_{3y}^{LHS} - y_1^{LTEMP} \quad (196)$$

$$= b_{3y}^{LHS} - \frac{S_2}{S_1} y_1 - d_1^{LTEMP} \quad (197)$$

Therefore

$$d_1^{LTEMP} + d_1^{LHS} = b_{3y}^{LHS} - \frac{S_2}{S_1} y_1 \quad (198)$$

Similarly

$$d_1^{RHS} = b_{3y}^{RHS} - \frac{S_2}{S_1} y_1^{RTEMP} \quad (199)$$

$$= b_{3y}^{RHS} - \frac{S_2}{S_1} (y_1 + d_1^{RTEMP}) \quad (200)$$

Therefore

$$\frac{S_2}{S_1} d_1^{RTEMP} + d_1^{RHS} = b_{3y}^{RHS} - \frac{S_2}{S_1} y_1 \quad (201)$$

Again by the fact 181, the right hand sides of the equations 198 and 201 are the same, and we have

$$d_1^{LTEMP} + d_1^{LHS} = \frac{S_2}{S_1} d_1^{RTEMP} + d_1^{RHS} \quad (202)$$

Similarly we can derive the equality of

$$d_2^{LTEMP} + d_2^{LHS} = \frac{S_2}{S_1} d_2^{RTEMP} + d_2^{RHS} \quad (203)$$

Putting these results in the equations 192 and comparing it with the equation 195, we can conclude that

$$y_i^{LHS} = y_i^{RHS} \quad (204)$$

□

We have shown that (theorems 2 and 3) both types of generalization, i.e., the step length generalization ϕ and the body condition generalization ψ are characteristic preserving when they are applied *homogeneously*. One obvious question here is whether it is characteristic preserving even when they are mixed up together. In fact, our generalization scheme does have that property.

Theorem 5 Let $WC_1 = (S_1, sl_1)$, $WC_2 = (S_2, sl_2)$, and $WC_3 = (S_3, sl_3)$ be three arbitrary walk conditions. Let

$$\tau_{12} = \psi_{S_1}^{S_2} \phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}} \quad (205)$$

$$\tau_{23} = \psi_{S_2}^{S_3} \phi_{\frac{S_2}{S_3} \frac{sl_3}{sl_2}} \quad (206)$$

$$\tau_{13} = \psi_{S_1}^{S_3} \phi_{\frac{S_1}{S_3} \frac{sl_3}{sl_1}} \quad (207)$$

be the combined generalizations that try to transform WC_1 to WC_2 , WC_2 to WC_3 , and WC_1 to WC_3 , respectively. Then

$$\tau_{23} \circ \tau_{12} = \tau_{13} \quad (208)$$

Therefore the combined generalization is characteristic preserving.

Proof 5

$$\tau_{23} \circ \tau_{12} = \psi_{S_1}^{S_2} \phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}} \psi_S^{S_1} \phi_{\frac{S}{S_1} \frac{sl_1}{sl}} \quad (209)$$

$$\stackrel{thm4}{=} \psi_{S_1}^{S_2} \phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1}} \phi_{\frac{S}{S_1} \frac{sl_1}{sl}} \psi_S^{S_1} \quad (210)$$

$$\stackrel{thm1}{=} \psi_{S_1}^{S_2} \phi_{\frac{S_1}{S_2} \frac{sl_2}{sl_1} \frac{S}{S_1} \frac{sl_1}{sl}} \psi_S^{S_1} \quad (211)$$

$$= \psi_{S_1}^{S_2} \phi_{\frac{S}{S_2} \frac{sl_2}{sl}} \psi_S^{S_1} \quad (212)$$

$$\stackrel{thm4}{=} \psi_{S_1}^{S_2} \psi_S^{S_1} \phi_{\frac{S}{S_2} \frac{sl_2}{sl}} \quad (213)$$

$$\stackrel{thm3}{=} \psi_S^{S_2} \phi_{\frac{S}{S_2} \frac{sl_2}{sl}} \quad (214)$$

$$= \tau_{13} \quad (215)$$

□

6 Conclusion

The generalization algorithm was implemented in *Jack*TM [10]. In the implementation, the arm swing was done by a kinematic function that depends on the leg movement. The accompanying animation was based on the measured data included in David A. Winter's book, *Biomechanics and Motor Control of Human Movement* [12]. The following table shows the comparison between the subject measured and the figure animated, in their step lengths and body conditions. For all the big difference between the subject and the figure, our animation was quite successful.

	Subject (Measured)	Figure (Animated)	Ratio (Figure/Subject)
step length	70.69	variable	variable
ankle to heel	7.629	13.49	1.77
ankle to ball	12.20	17.55	1.44
heel to ball	15.89	15.19	0.96
ball to toe	7.095	7.200	1.01
ball angle	0.4842	0.8378	1.73
heel angle	0.8400	1.312	1.56
ankle angle	1.818	0.9916	0.55
heel to N	5.093	3.450	0.67
ankle to N	5.680	13.04	2.30
ball to N	10.80	11.74	1.09
shin	38.05	34.53	0.91
thigh	35.85	40.72	1.14
leg length	73.90	75.25	1.02

Even though the definition of the preservation was aimed for the animation, we should note the difference of its meaning, in the *mathematic space* and in the *animation space*. If the desired step is too different from the originally measured one, even though the characteristic is well preserved in mathematical way, it has less meaning in generating that step based on the normal one. For example, if the subject $2S$ tries to imitate the walk of (S, sl) in stepping $0.1sl$, the imitation and the achieving the given step length will be in a total conflict. In this case, to get a better animation we need a measurement of S at a smaller step.

In this work, we showed a new approach in generalizing the rotoscoping data for human walking animation, which promises good result by the nature of the method. From the measured data of *one step* of a particular subject, we can generate the steps of any step length and body condition, that resemble the original step. We can extend our system to simulate multiple walking style by acquiring other measurement. Also the characteristic preservation was suggested as a new criteria for determining the *quality of generalization*.

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