

University of South China Quantum Mechanics Midterm Exam

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1. (20 points) If $[A, H] = 0$, $[B, H] = 0$, but $[A, B] \neq 0$, please prove that there is degeneracy happened in the system.

2. (40 points in total) Consider an electron in a box with the potential of the box to be defined as:

$$V(x, y, z) = \begin{cases} 0, & 0 \leq x \leq L, 0 \leq y \leq L, \text{ and } 0 \leq z \leq L, \\ \infty, & \text{otherwise.} \end{cases}$$

- (a) (10 points) Please write down the time-dependent Schrödinger equation of the system.
- (b) (10 points) Please find out the normalized general solution of the Schrödinger equation and energy spectrum.
- (c) (10 points) Please find out the commutation relation between the operators x, y, z, p_x, p_y , and p_z .
- (d) (10 points) By using the ground state solution, please find out the expectation values $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p_x \rangle$, and $\langle p_x^2 \rangle$, and check the uncertainty principle.
3. (40 points in total) Consider the following potential:

$$V(x) = 4x^4 - 2L^2x^2 + L^4$$

- (a) (10 points) In this potential, there are two minimums. Please sketch the potential and find out the position of the two minimums.
- (b) (10 points) Please write down the raising and lowering operators a_+ and a_- and work out the commutation relation $[a_+, a_-]$ for the two minimums individually.
- (c) (10 points) Please find out the unnormalized ground state solutions and the spectrum of the two minimums.
- (d) (10 points) Please find out the commutation relation between the raising and lowering operators and the position x .