Note of Group Theory

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摘要

这一篇笔记是本科阶段学习群论的笔记。

参考文献

[1] Joseph J. Rotman, An Introduction to the Theory of Groups

[2] 周彬, Lie 群与 Lie 代数

[3] Pierre Ramond, Group Theory：A Physicist's Survey

[4] 崔建伟, 群论讲义

[5] A. Das and S. Okubo, Lie Groups and Lie Algebras for Physicists

目录

1 有限群 (Finite Group)

1.1 群与群乘法

废话也就不多说，我们直接给出群的定义，并在之后讲解一下为什么要这样去定义

定义 1 (Definition of a group).

Suppose we give a set $\mathbb{G} = \{g_0, g_1, g_2, \cdots\}$ and an operator $\cdot$, if they satisfy the following properties

1. ***Closure*** *For every ordered pair of elements, g_i and g_j, there exists a unique element*

$$a_i \cdot a_j = a_k \tag{1}$$

for any three i, j, k.

2. ***Associativity*** *The $\cdot$ operation is associative*

$$g_i \cdot (g_j \cdot g_k) = (g_i \cdot g_j) \cdot g_k \tag{2}$$

3. ***Unit element*** *The set G contains a unique element e(here we will use g_0 to symbol it) such that*

$$g_0 \cdot g_i = g_i \cdot g_0 = g_i \tag{3}$$

for all i. In particular, this means that

$$g_0 \cdot g_0 = g_0.$$

4. ***Inverse element*** *Corresponding to every element g_i, there exists a unique element of G, the inverse $(g_i)^{-1}$ such that*

$$g_i \cdot (g_i)^{-1} = (g_i)^{-1} \cdot g_i = g_0 \tag{4}$$

In particular,this means that

$$g_0 = (g_0)^{-1}$$

then we called the set $\mathbb{G}$ a Group.

同时，我们根据集合 $\mathbb{G}$ 的元素个数

1.2 群同态 (Group Homomorphisms) 与群的直乘

1.3 群的线性表示理论

2 表示论

3 李群 (Lie Group)

4 李代数 (Lie Algebra)