Algebra Final

1.
a) $5 - 2 \cdot 4 = 5 - 8 = \boxed{-3}$ 3 pts each

b) $(1-3)^4 + 5 \cdot 3 =$
$= (-2)^4 + 15$
$= 16 + 15$
$\boxed{31}$

c) $2^4 + 5$
$= 16 + 5$
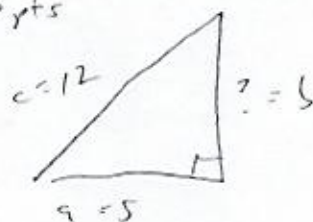$= \boxed{21}$

d) $\sqrt{36} = \boxed{6}$

e) $\sqrt{50} = \sqrt{25} \cdot \sqrt{2}$
$= \boxed{5\sqrt{2}}$

f) $\dfrac{45}{-10} = \dfrac{9 \cdot 5}{-2 \cdot 5} = \boxed{-\dfrac{9}{2}}$

2. 10 pts

$a^2 + b^2 = c^2$ 3 pts
$5^2 + b^2 = 12^2$ 3 pts
$25 + b^2 = 144$ 2 pts
$\quad -25 \qquad\qquad -25$
$\overline{\qquad\qquad\qquad}$
$b^2 = 119$ 2 pts
$\boxed{b = \sqrt{119}}$

3. $2x + 10 = 25$
6 pts
$\quad -10 \quad\quad -10$ 3 pts
$\overline{\qquad\qquad}$
$\dfrac{2x}{2} = \dfrac{15}{2}$ 3 pts
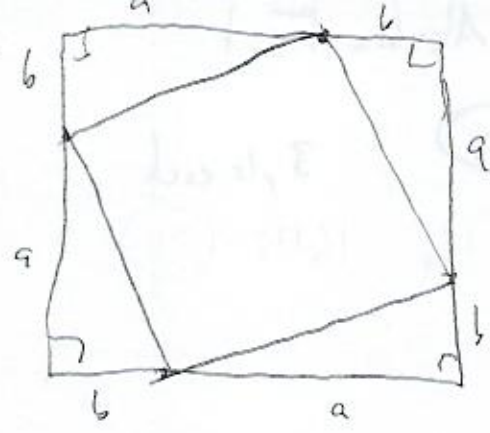$\boxed{x = {}^{15}\!/_{2}}$

4.
15 pts

$$a^2 + b^2 = c^2$$

5 pts

Pf:
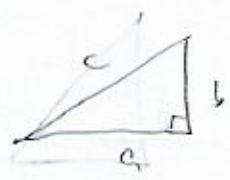
$$(a+b)^2 = 4\left(\tfrac{1}{2}ab\right) + c^2 \qquad 3$$

$$a^2 + 2ab + b^2 = 2ab + c^2 \qquad 1$$

$$\underline{\quad -2ab \qquad\qquad -2ab \quad} \qquad 1$$

$$a^2 + b^2 = c^2 \qquad \boxed{} \qquad 1$$

5 pts

10 pts

5. Mathematics is the art and science of the discovery and use of patterns.

10 pts

6. a) $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ 2 pts each

b) $\mathbb{R}$

c) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$

d) $\mathbb{Q}, \mathbb{R}$

e) $\mathbb{Q}, \mathbb{R}$

9 pts

7. $4x - 5 = x - 23$

$\underline{\;\; -x + 5 \quad -x + 5 \;\;}$ 3

             3

$\dfrac{3x}{3} = \dfrac{-18}{3}$ 3

$\boxed{x = -6}$

6 pts

8. $3x + 1 = 15$

$\underline{\quad -1 \qquad -1 \quad}$ 3

$\dfrac{3x}{3} = \dfrac{14}{3}$ 3

$\boxed{x = 14/3}$

10 ptr

9. 5a) $f(3) = 3 \cdot 3 - 5 = \boxed{4}$

5b) $g(-2) = 2(-2)^2 - (-2) + 3$

$= 8 + 2 + 13$

$= \boxed{13}$

10.
10 pts.

4 pts
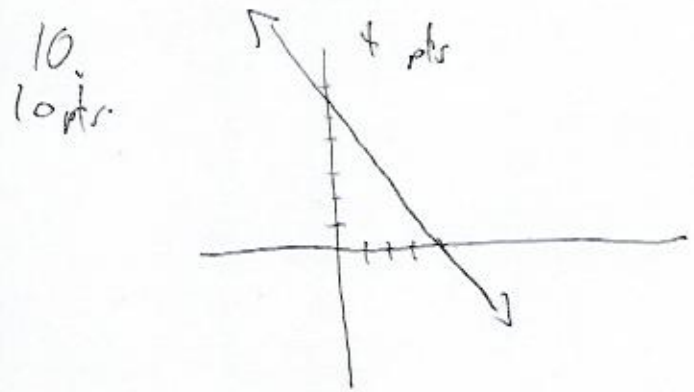
$3x + 2y = 12$

x-int; $3x + 2 \cdot 0 = 12$

$\boxed{x = 4}$ 3 pts

y-int; $3 \cdot 0 + 2y = 16$

$\boxed{y = 6}$ 3 pts

11. $3x + 2y = 12$
10 pts

$\underline{-3x \qquad\qquad 3x}$ 3 pt

$\dfrac{2y}{2} = \dfrac{-3x + 12}{2}$ 3 pts

$y = -\dfrac{3}{2}x + 6$ 3 pts

$\boxed{m = -\dfrac{3}{2}}$ 1 pt

12. $\boxed{m_\perp = \dfrac{2}{3}}$ 6 pts
6 pts

13. $x^2 - 7x + 12 = 0$
11 pts

$(x - 4)(x - 3) = 0$ 5

$x - 4 = 0$ or $x - 3 = 0$

$\boxed{x = 4}$ or $\boxed{x = 3}$
3   3

14. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
5 pts

10 pts/5. $1x^2 - 7x + 12 = 0$

3    $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3    $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(12)}}{2 \cdot 1}$

2. $x = \dfrac{7 \pm \sqrt{49 - 48}}{2}$

1    $x = \dfrac{7 \pm 1}{2} = \dfrac{8}{2}$ or $\dfrac{6}{2}$

1    $\boxed{x = 4 \text{ or } 3}$

16. 3 facts

9 pts.   3 pts per fact