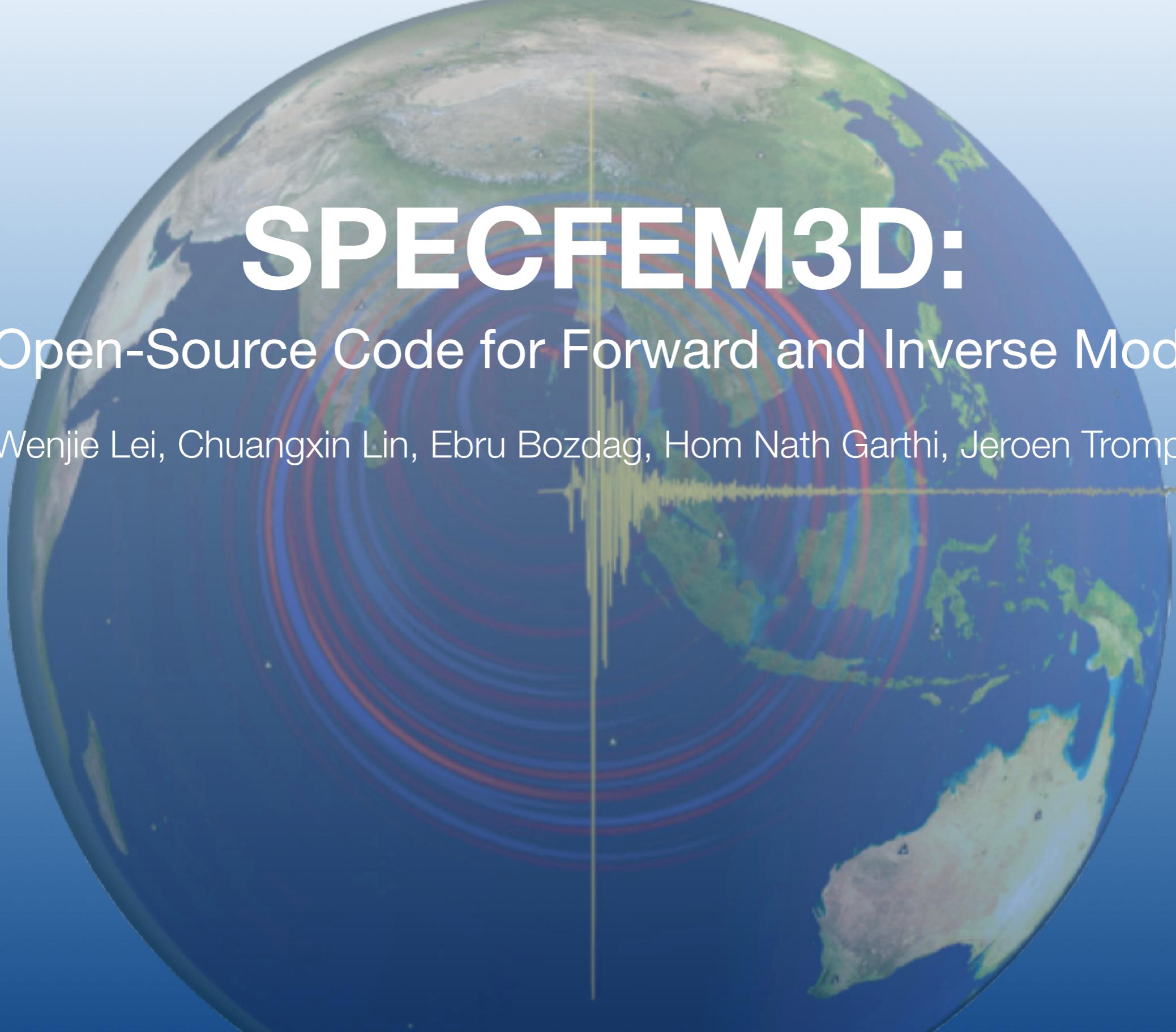


## Download material from github

Open a terminal on your desktop. Make a new directory and change to that directory. Then type in terminal:

```
git clone https://github.com/wjlei1990/USArray2015\_SPECFEM
```



# SPECFEM3D:

## An Open-Source Code for Forward and Inverse Modeling

Wenjie Lei, Chuangxin Lin, Ebru Bozdag, Hom Nath Garthi, Jeroen Tromp

USAarray Short Course, Bloomington, Aug 6



# Outline

1. Introduction

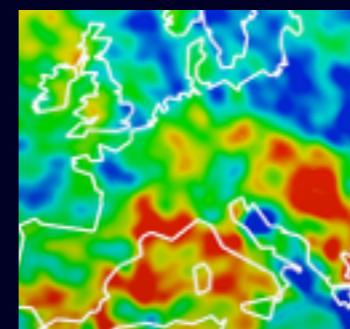
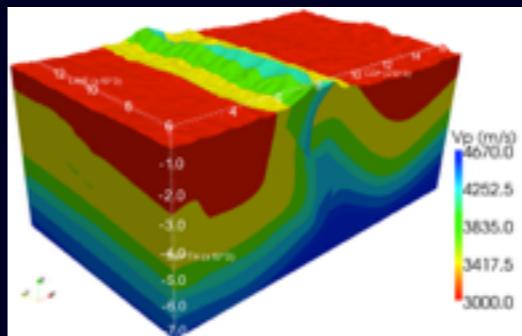
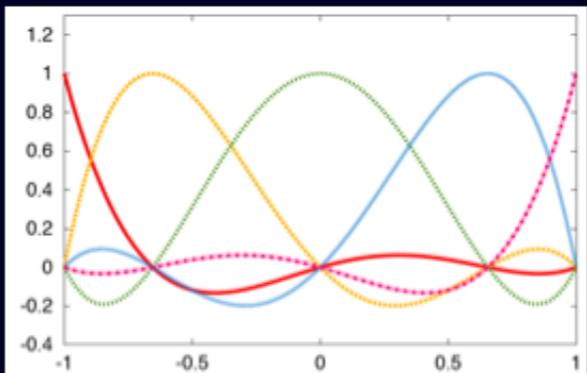
2. Basic Theory:  
Spectral Element Method

3. Forward Modeling:  
SPECFEM2D&3D

4. Adjoint Tomography

5. Summary

Degree 4 Lagrange polynomials:



# Outline

## 1. Introduction

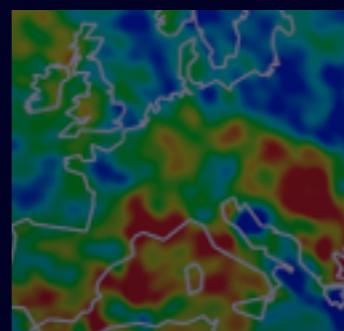
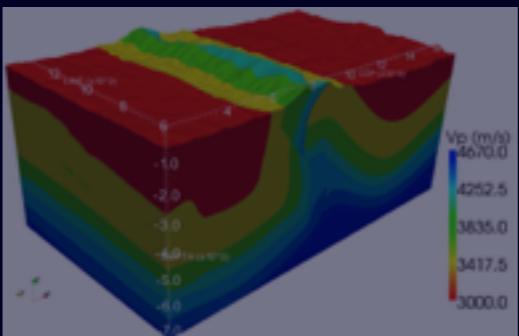
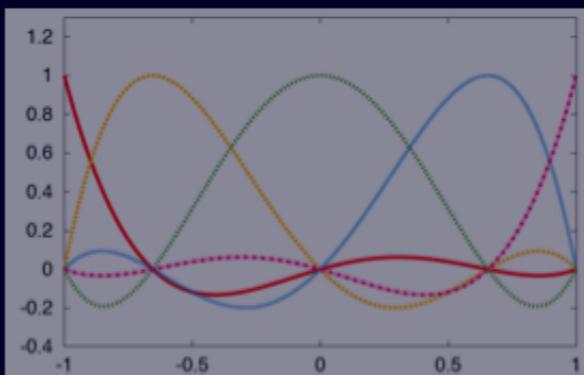
## 2. Basic Theory: Spectral Element Method

## 3. SPECFEM2D

## 4.

## 5. Summary

Degree 4 Lagrange polynomials:

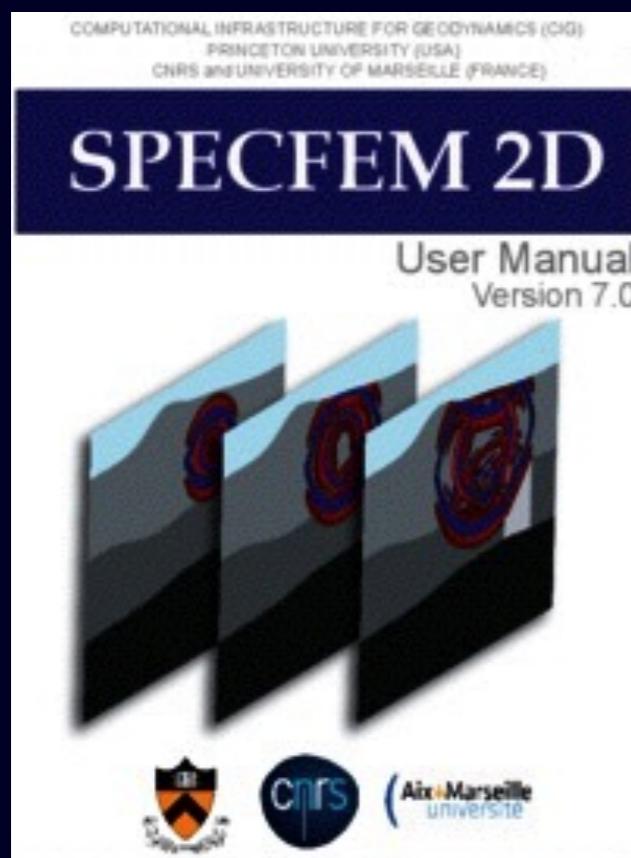


# SPECFEM: 2D, 3D\_Cartesian & 3D\_Globe

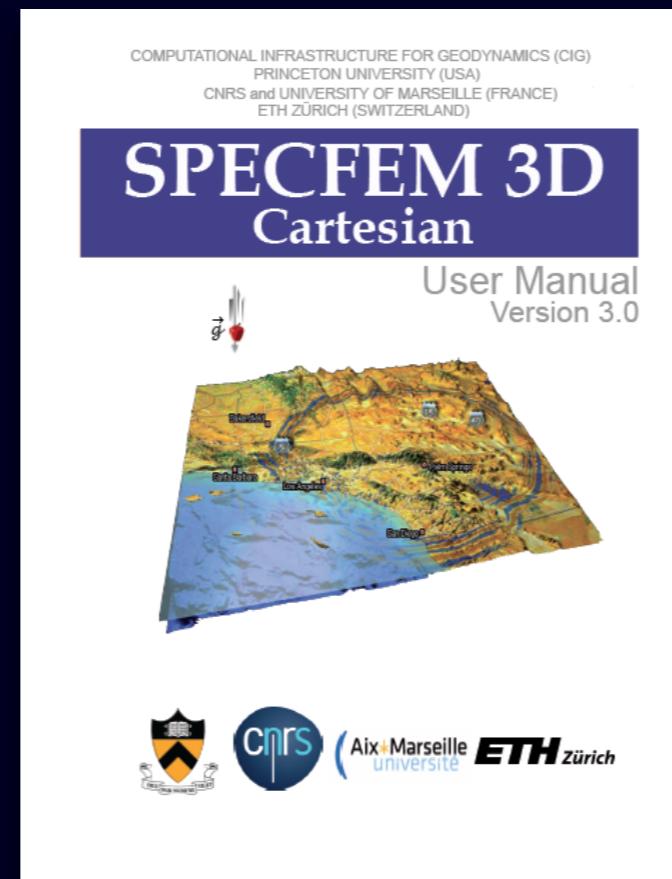


Komatitsch & Tromp 2002

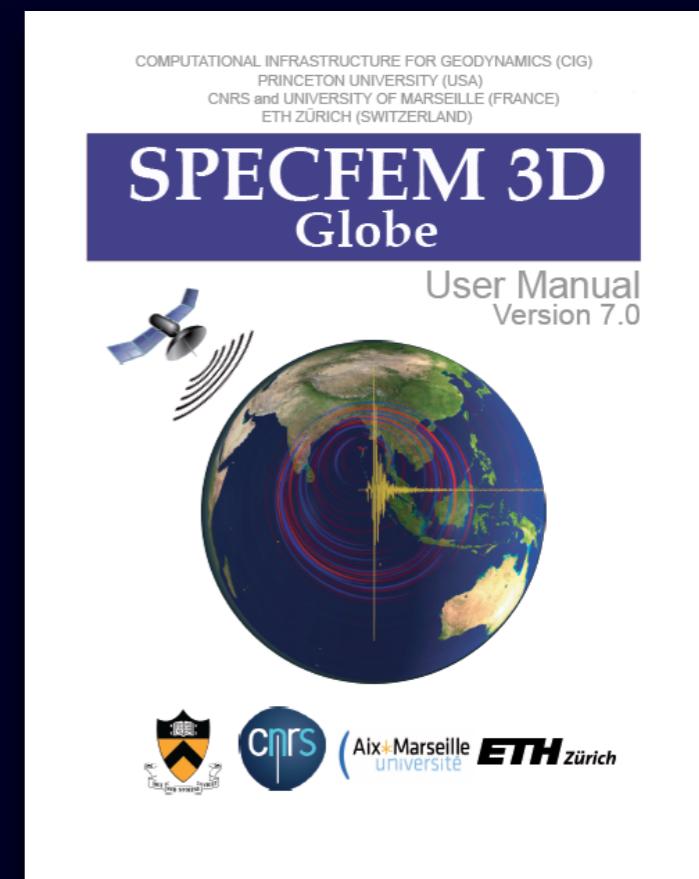
2D Version



3D Cartesian



3D Globe



## SPECFEM2D:

simulates forward and adjoint seismic wave propagation in **two-dimensional** acoustic, (an)elastic, poroelastic or coupled acoustic-(an)elastic-poroelastic media, with Convolution **PML** absorbing conditions.

## SPECFEM3D:

Cartesian simulates acoustic (fluid), elastic (solid), coupled acoustic/elastic, poroelastic or seismic wave propagation in any type of conforming mesh of **hexahedra**. It can, for instance, model seismic waves propagating in **sedimentary basins**.

## SPECFEM3D\_GLOBE:

simulates **global** and **regional (continental-scale)** seismic wave propagation.

# Outline

1. Introduction

**2. Basic Theory:**

Spectral Element Method

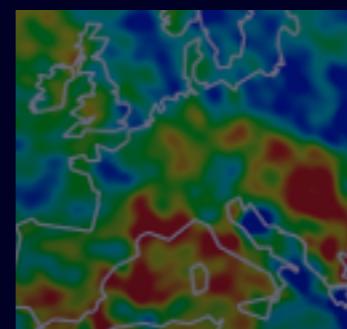
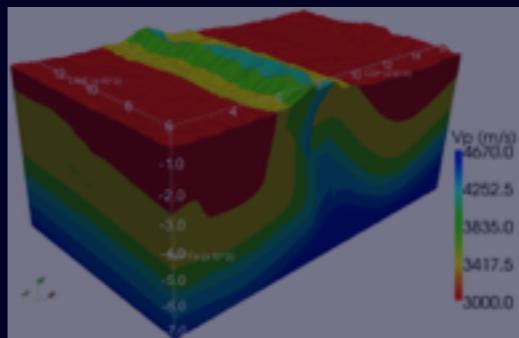
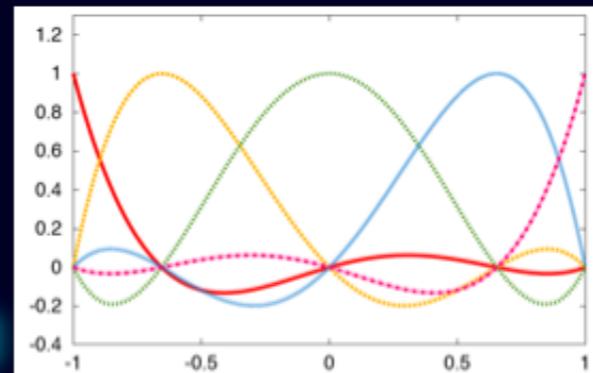
3. Forward Modeling:

SPECFEM2D&3D

4. Adjoint Tomography

5. Summary

Degree 4 Lagrange polynomials:



# Seismic Wave Propagation

Equation of motion:  $\rho \partial_t^2 \mathbf{s} - \nabla \cdot \mathbf{T} = \mathbf{f}$

Boundary condition:  $\hat{\mathbf{n}} \cdot \mathbf{T} = 0$

Hook's Law:  $\mathbf{T} = c : \boldsymbol{\varepsilon}$

Initial conditions:  $\mathbf{s}(\mathbf{x}, 0) = \mathbf{0}$   
 $\partial_t \mathbf{s}(\mathbf{x}, 0) = \mathbf{0}$

Earthquake source:

$$\mathbf{f} = -\mathbf{M} \cdot \nabla \delta(\mathbf{x} - \mathbf{x}_s) S(t)$$

# Numerical Method

## Strong Form

$$L(u) = 0$$

## Weak Form

$$\int w L(\tilde{u}) \, d\Omega = 0$$

## Finite Difference Method

$$\int w L(\tilde{u}) \, d\Omega = 0$$

$$\tilde{u} = u_\alpha \quad \text{and} \quad w = \delta_{\alpha\beta}$$

$$L(u_\beta) = 0$$

Solving strong form  
on discrete points

# Finite Element Method

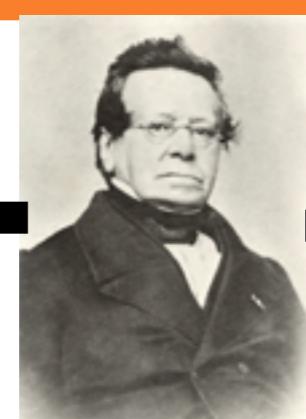
## FEM

$$\int w L(\tilde{u}) d\Omega = 0$$

$$\tilde{u} = \sum_{\alpha=1}^n N_\alpha u_\alpha \quad \text{and} \quad w = N_\alpha$$

$N_\alpha$  : Interpolation function

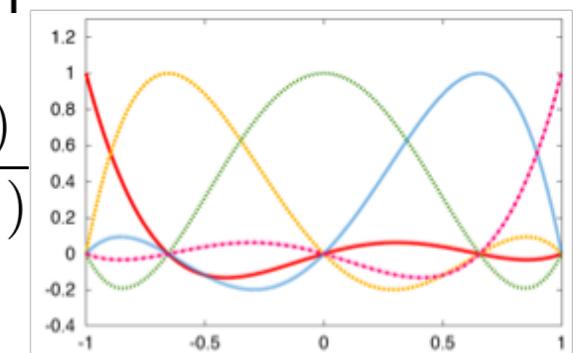
$$\int N_\alpha L \left( \sum_{\beta=1}^n N_\beta u_\beta \right) d\Omega = 0$$



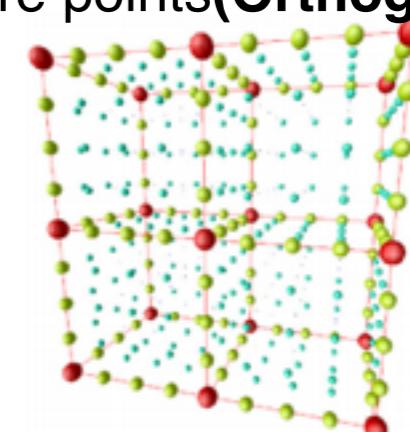
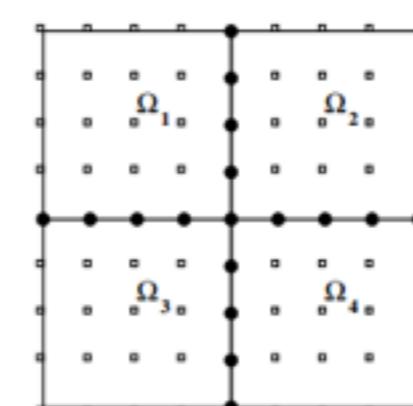
## Spectral FEM

1. use **Lagrange polynomials** as interpolation function

Degree 4 Lagrange polynomials

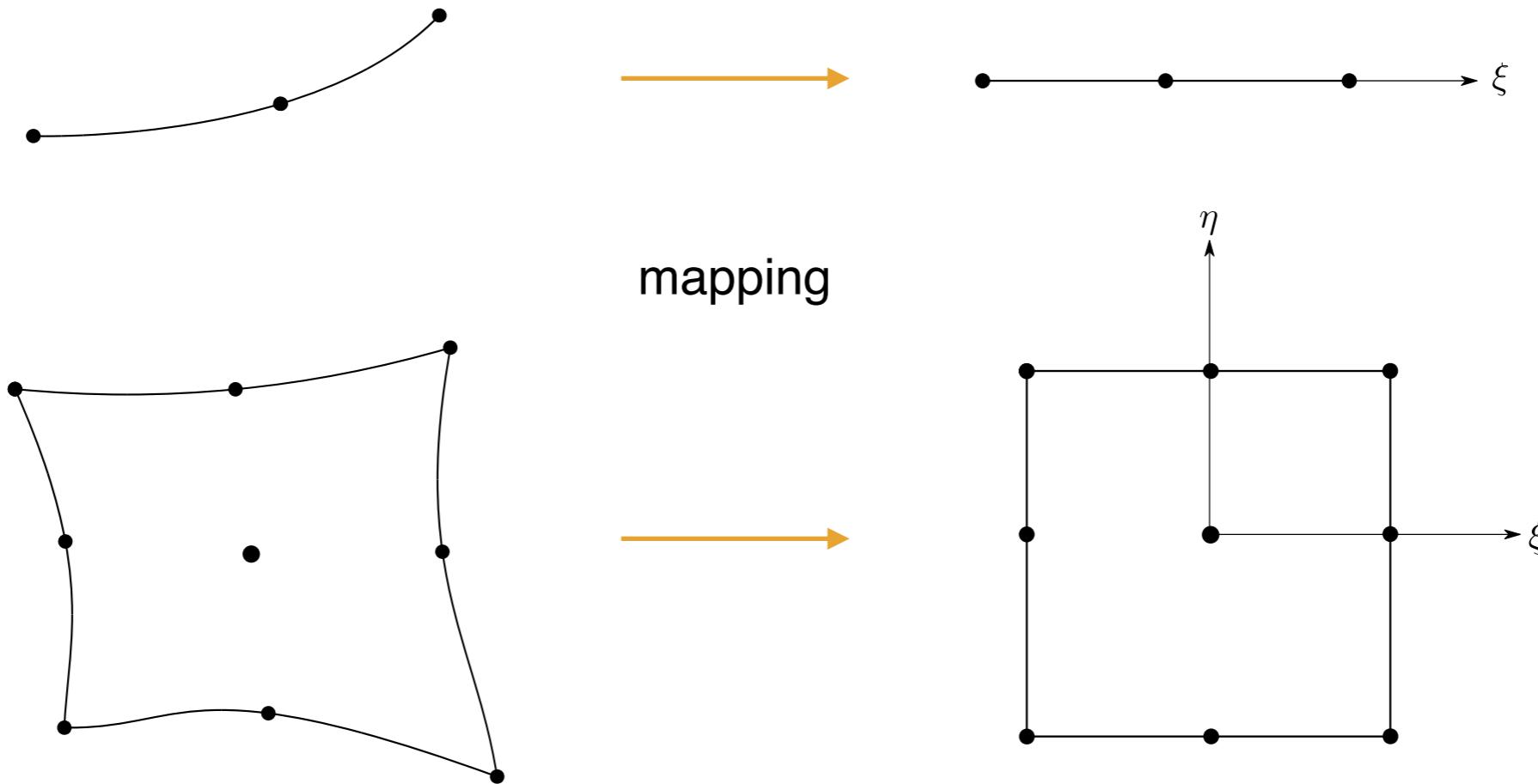


2. use nodes located at **Gauss-Lobatto-Legendre** quadrature points(**Orthogonality**)



# Finite Element Method

Integration on the original domain is often difficult, so....



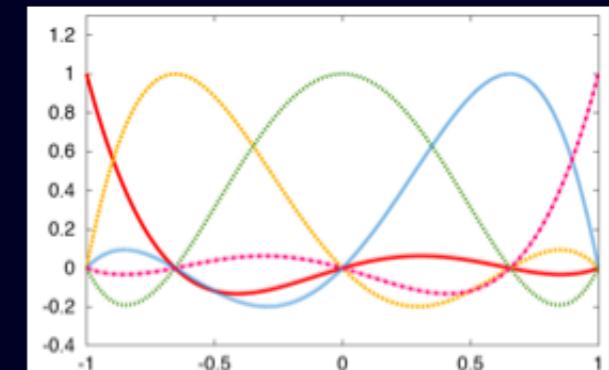
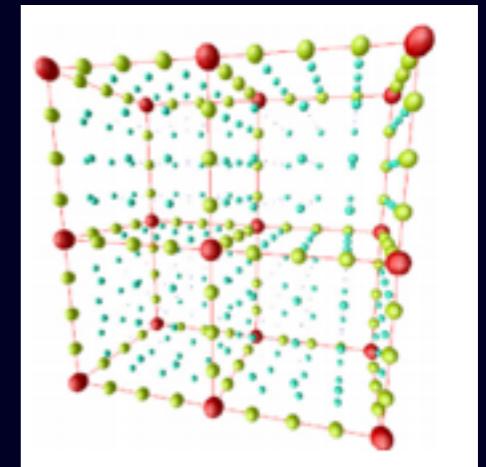
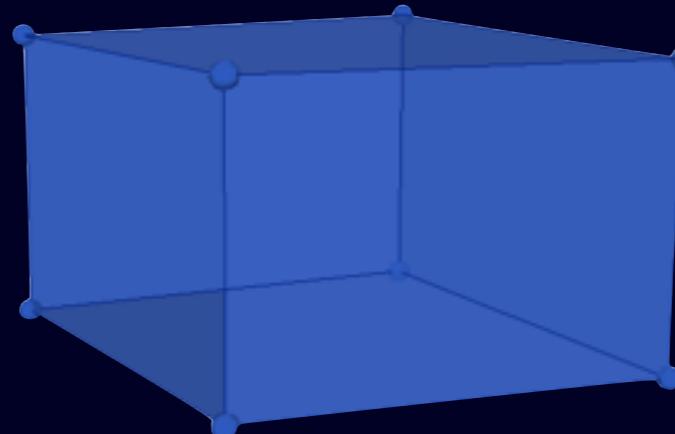
$$(x, y) \rightarrow (\xi, \eta)$$

$$dxdy = |J| d\xi d\eta$$

# Summary of Spectral-Element Method

Finite-elements:

- hexahedral elements
- Gauss-Lobatto-Legendre quadrature
- diagonal mass matrix
- explicit time-marching scheme



$$M\ddot{U} = -KU + F$$

# Outline

1. Introduction

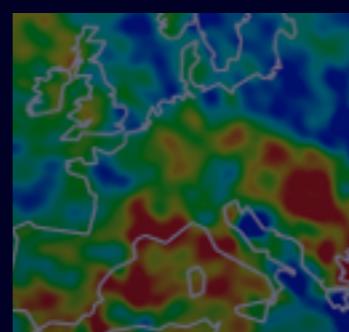
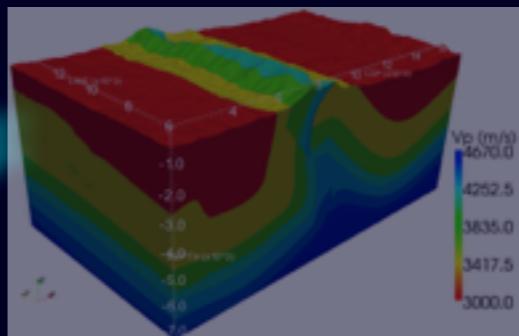
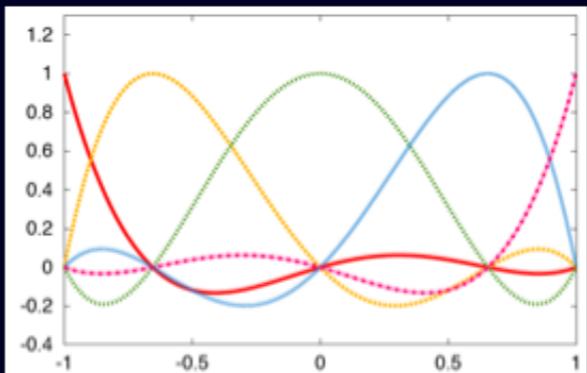
2. Basic Theory:  
Spectral Element Method

3. Forward Modeling:  
**SPECFEM2D&3D**

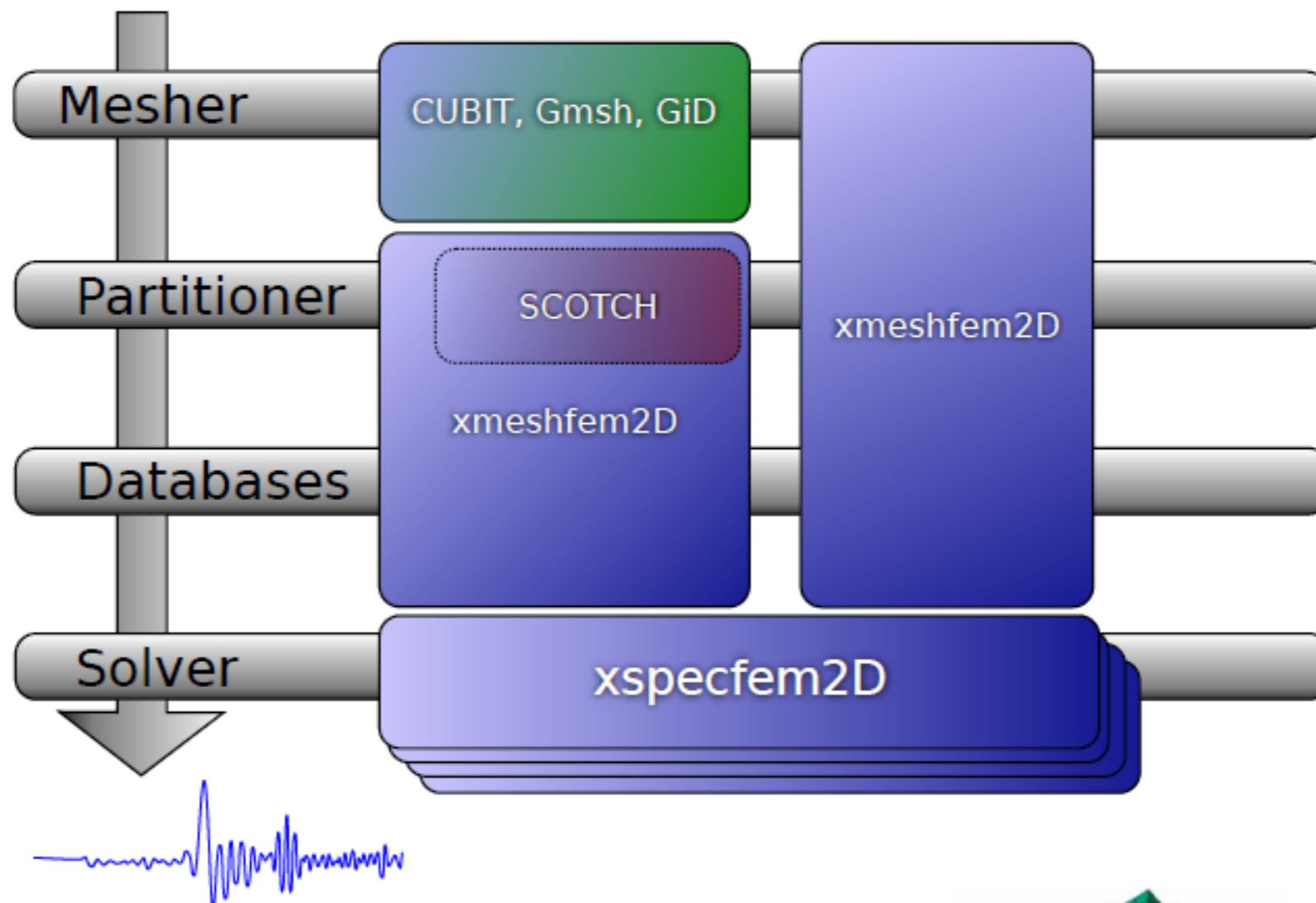
4. Adjoint Tomography

5. Summary

Degree 4 Lagrange polynomials:

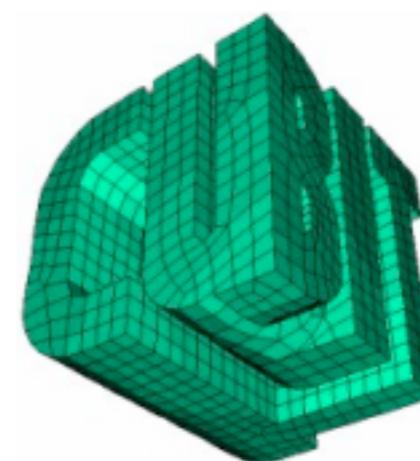


# Forward simulation Workflow

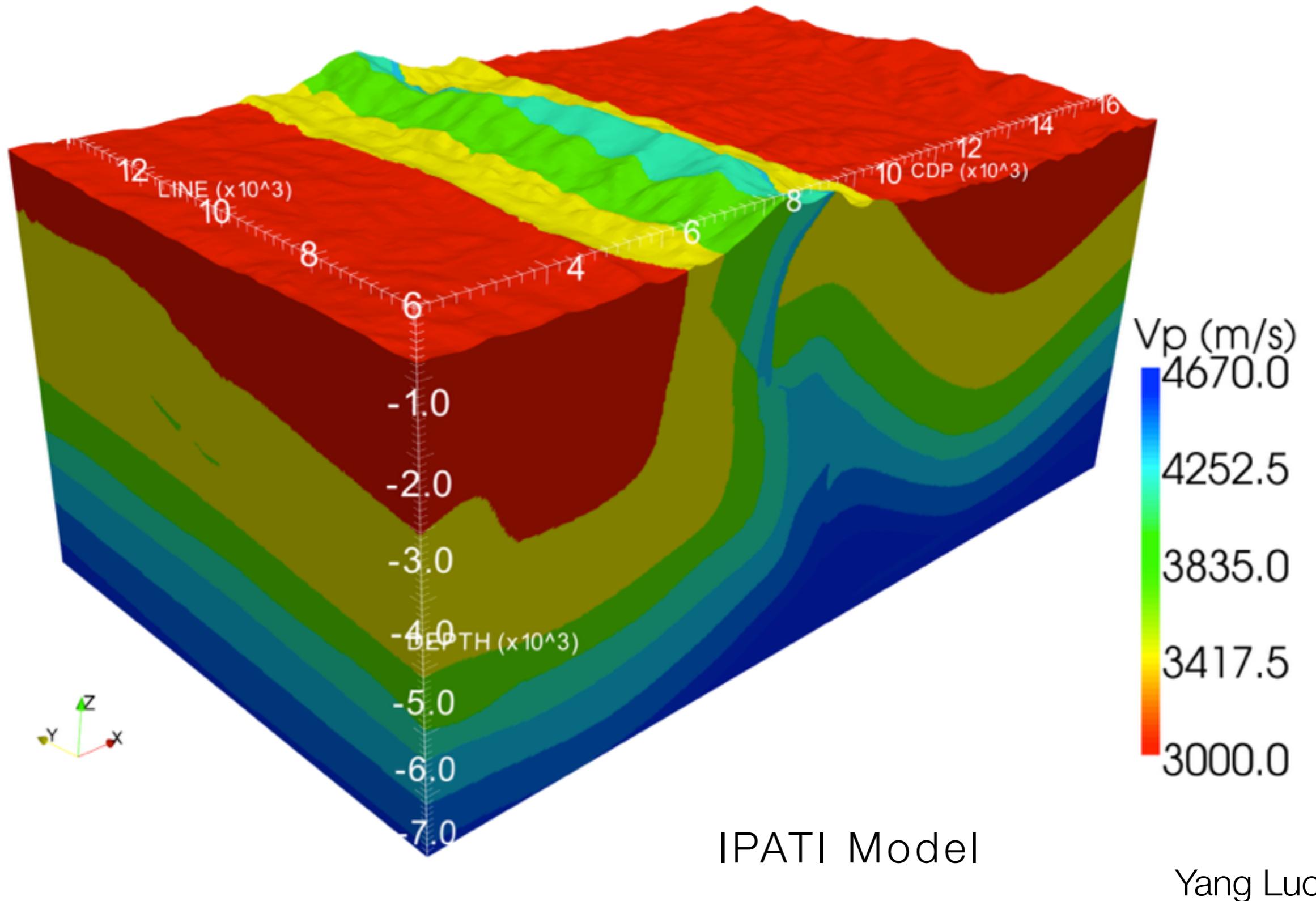


External Mesher: Cubit, Gmsh

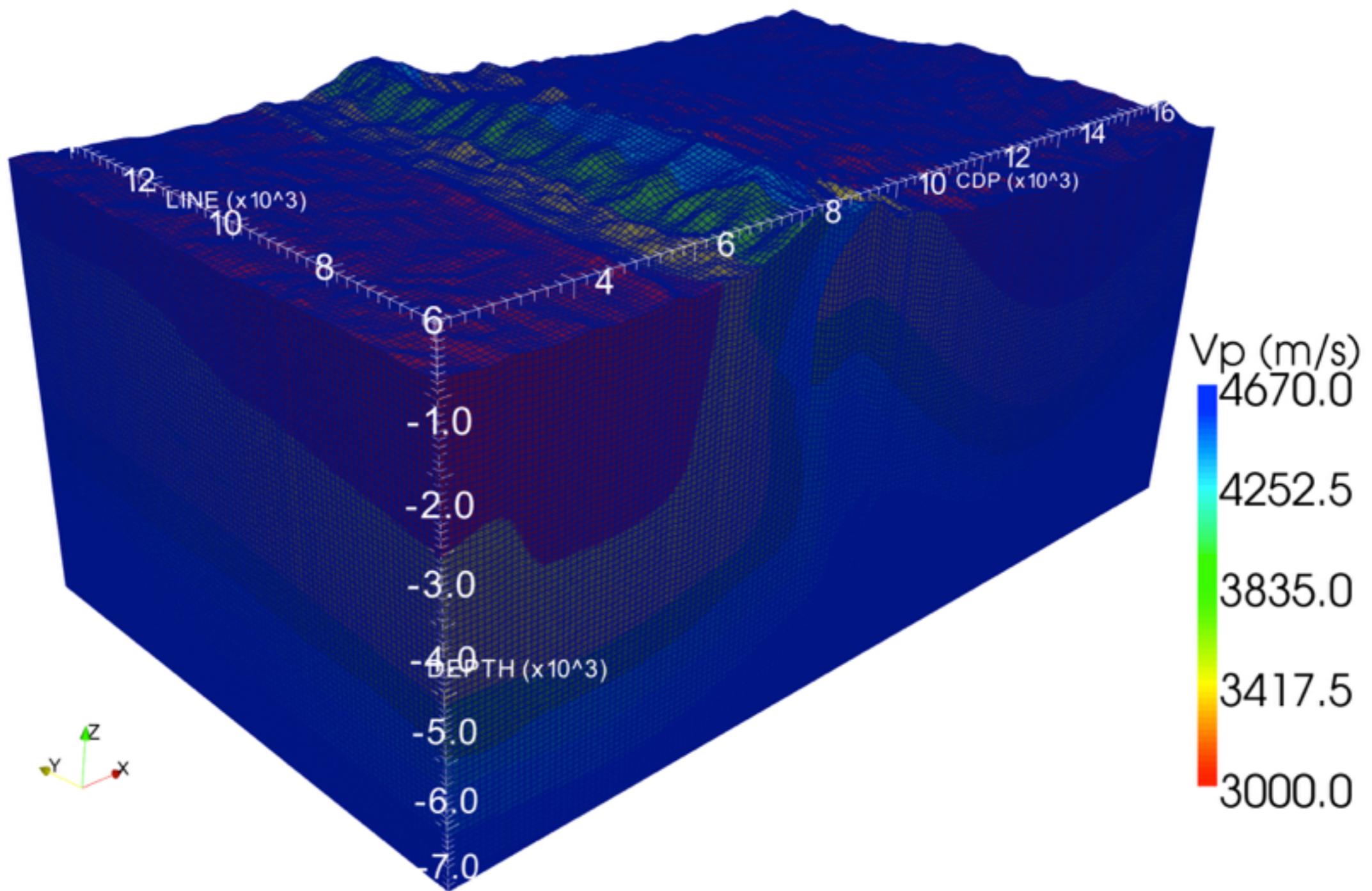
Internal Mesher: xmshfem2d



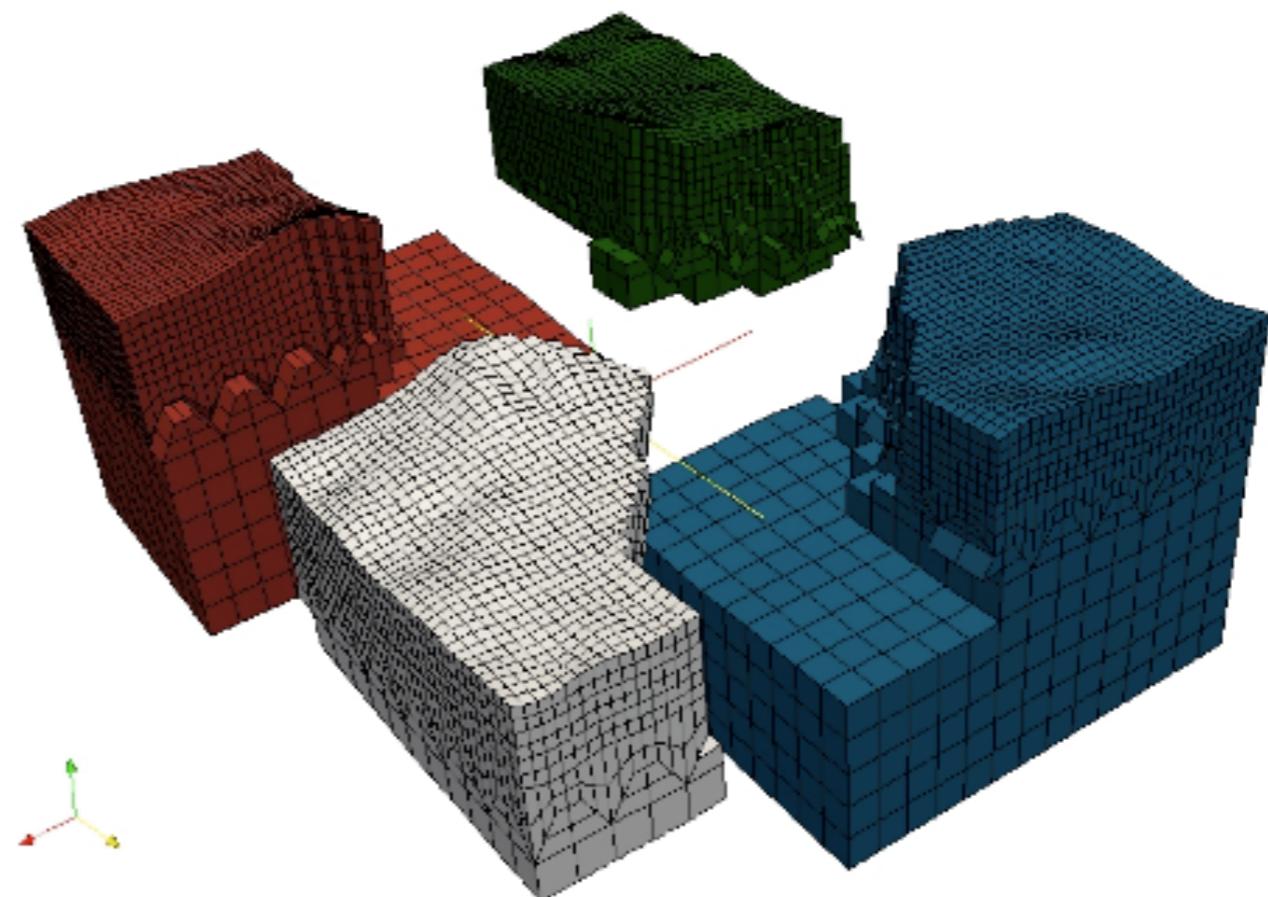
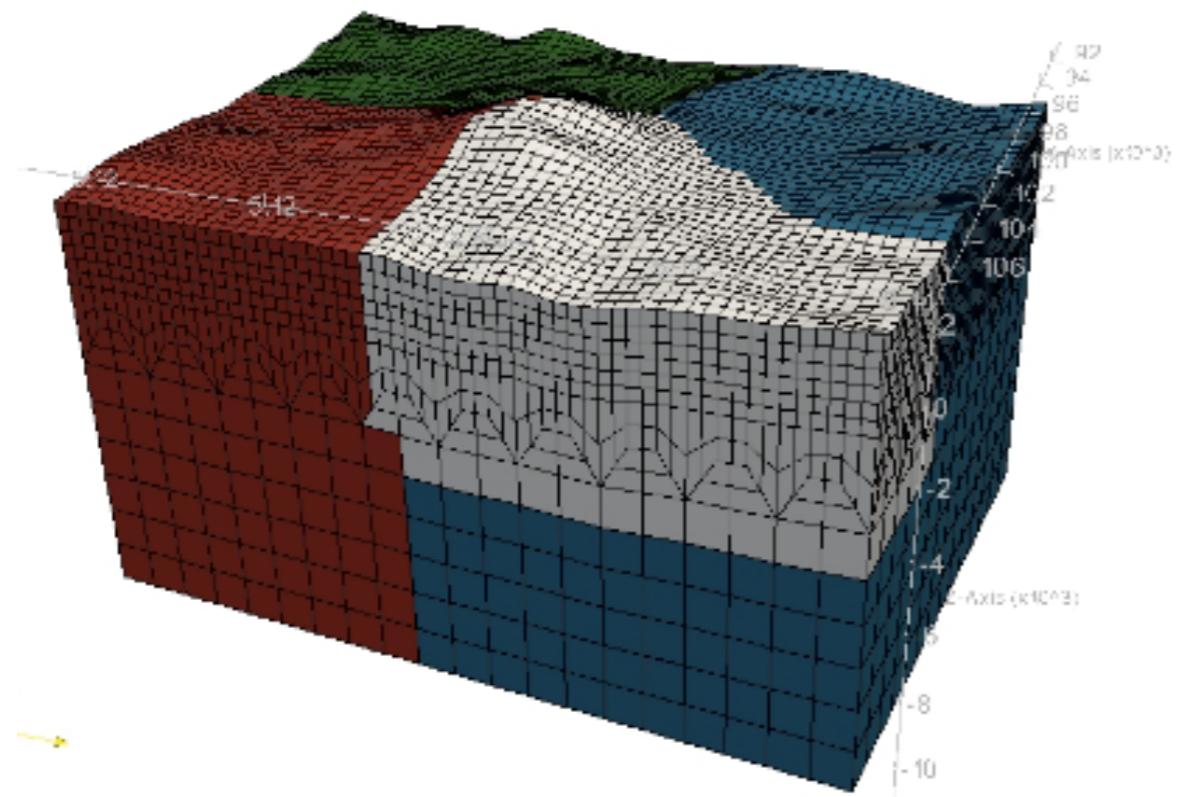
# Mesh: example



# Mesh example



# Mesh: mesh and Partition

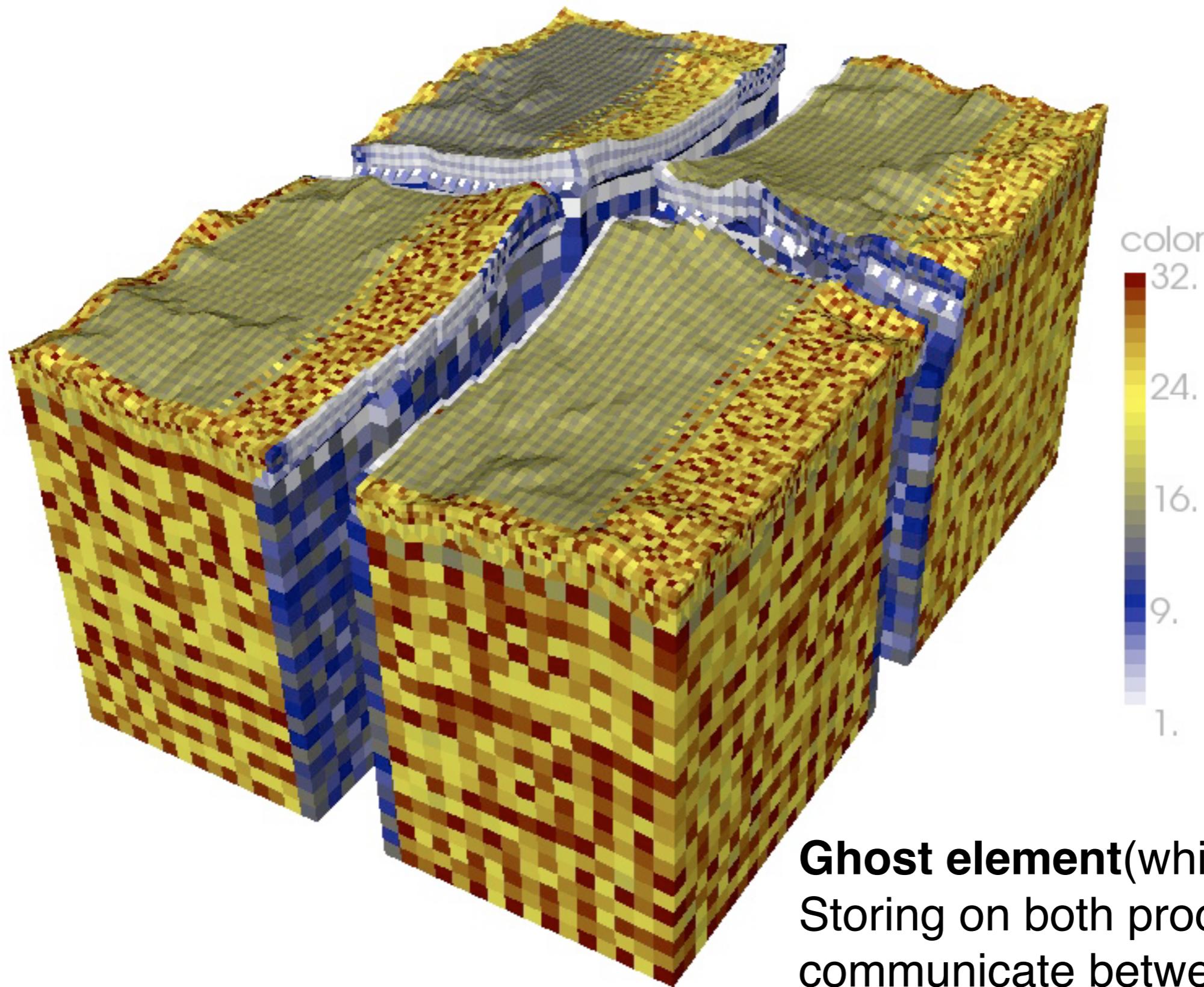


# Mesh: Partition and Parallel running(MPI buffer)



**Ghost element**(white color):  
Storing on both processors and  
communicate between each other

# Mesh: Partition and Parallel running(MPI buffer)



# Stability and accuracy condition

## Mesh size:

element size  $\leq$  minimum wave length / 2

(the minimum wavelength is usually decided by S wave if it is a elastic problem )

## Time marching step:

The parameter DT sets the length of each time step in seconds. The value of this parameter is crucial for the stability of the spectral-element simulation. Your time step DT will depend on the minimum ratio between the distance  $h$  of neighboring mesh points and the wave speeds  $v$  defined in your model. The condition for the time step  $\Delta t$  is:

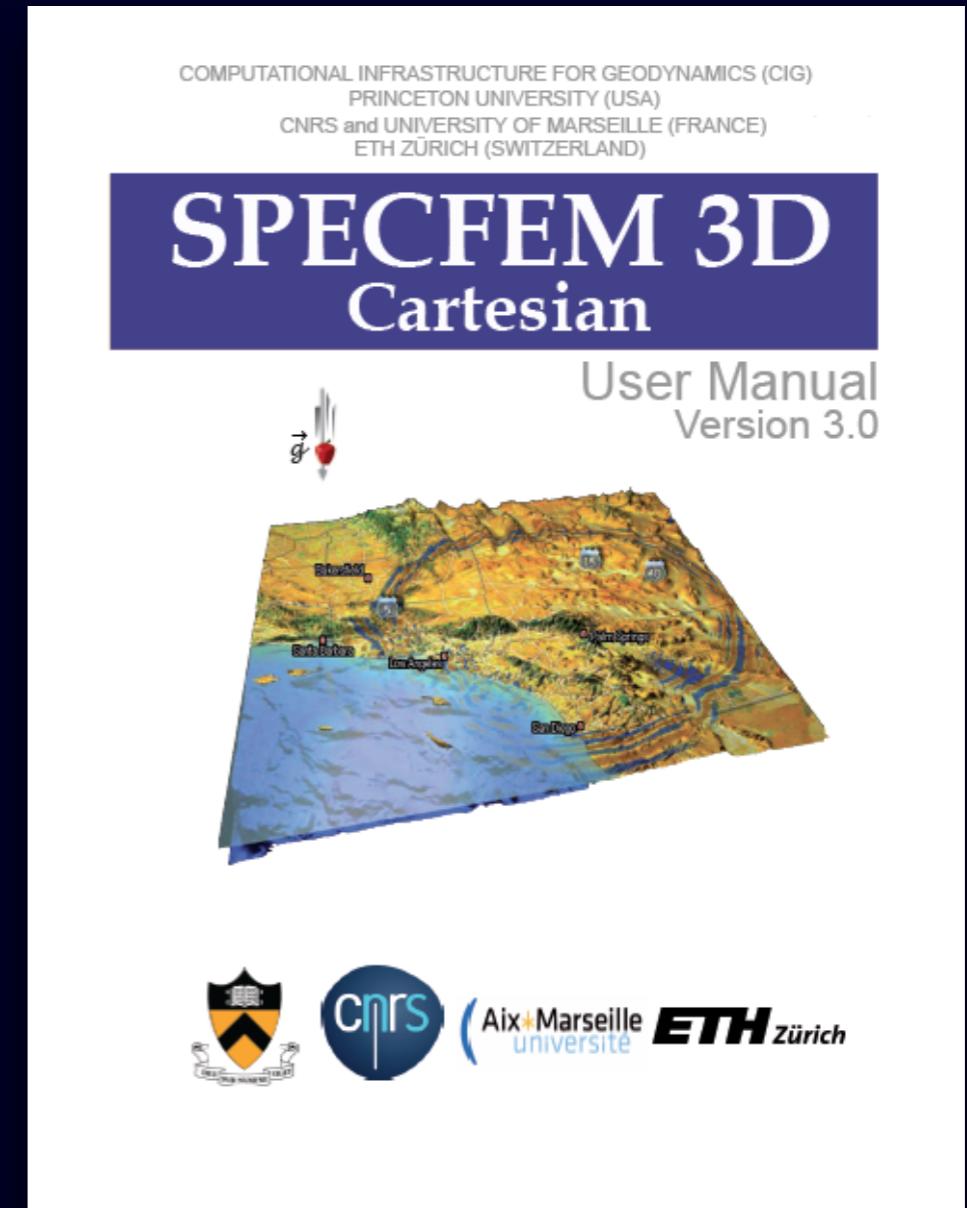
$$\Delta t < C \min_{\Omega} ( h/v )$$

where  $C$  is the so-called Courant number and  $\Omega$  denotes the model volume. The distance  $h$  depends on the mesh element size and the number of GLL points NGLL specified in the main constants file constants.h located in the src/shared/ subdirectory. The wave speed  $v$  is determined based on your model's P- (or S-) wave speed values.

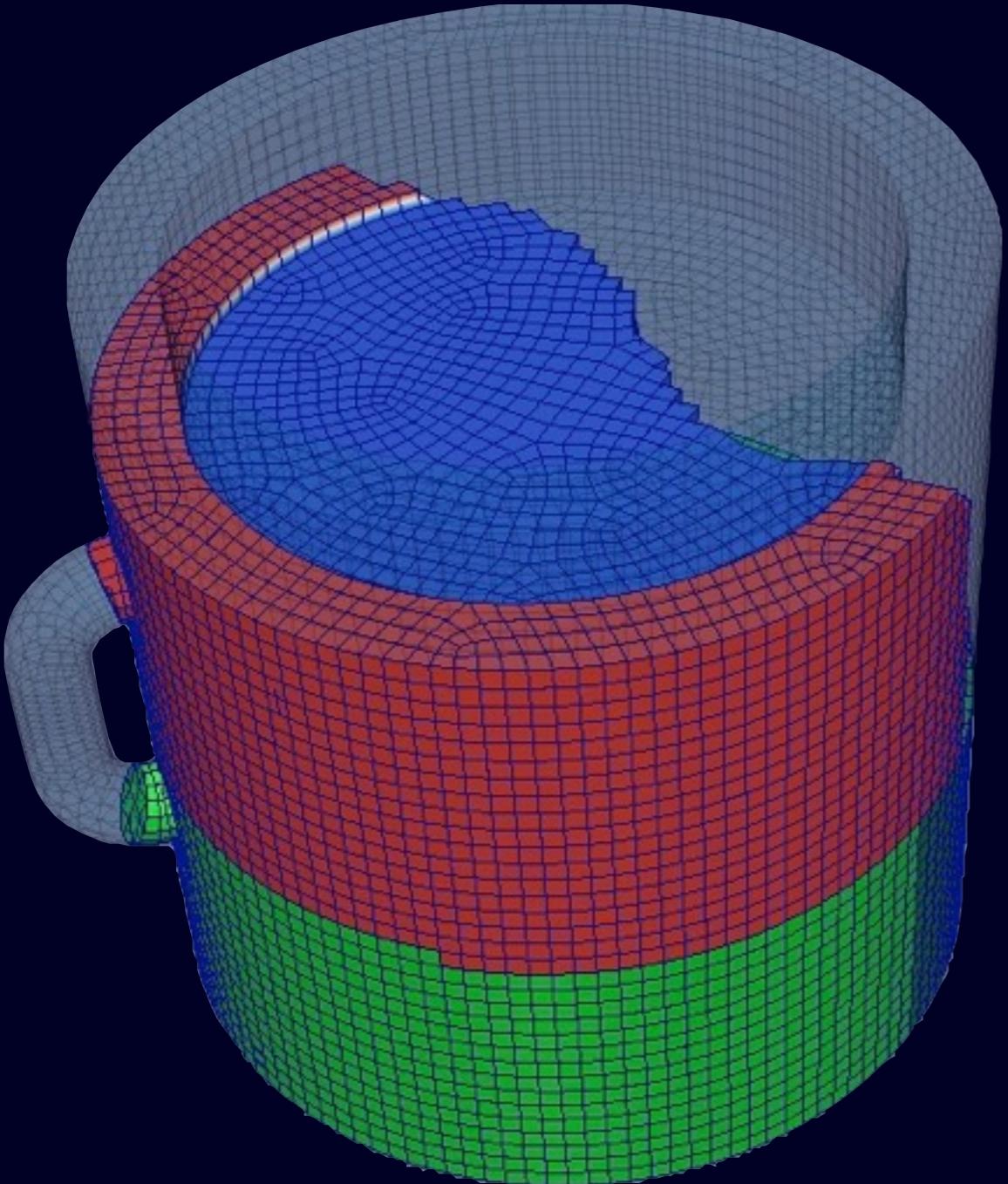
# Open Source Community Software

## SPECFEM3D

- Unstructured meshes (CUBIT)
- Load-balanced mesh partitioning
- Acoustic & elastic coupling, poroelasticity
- Anisotropy
- Attenuation
- Adjoint kernels
- GPU capable

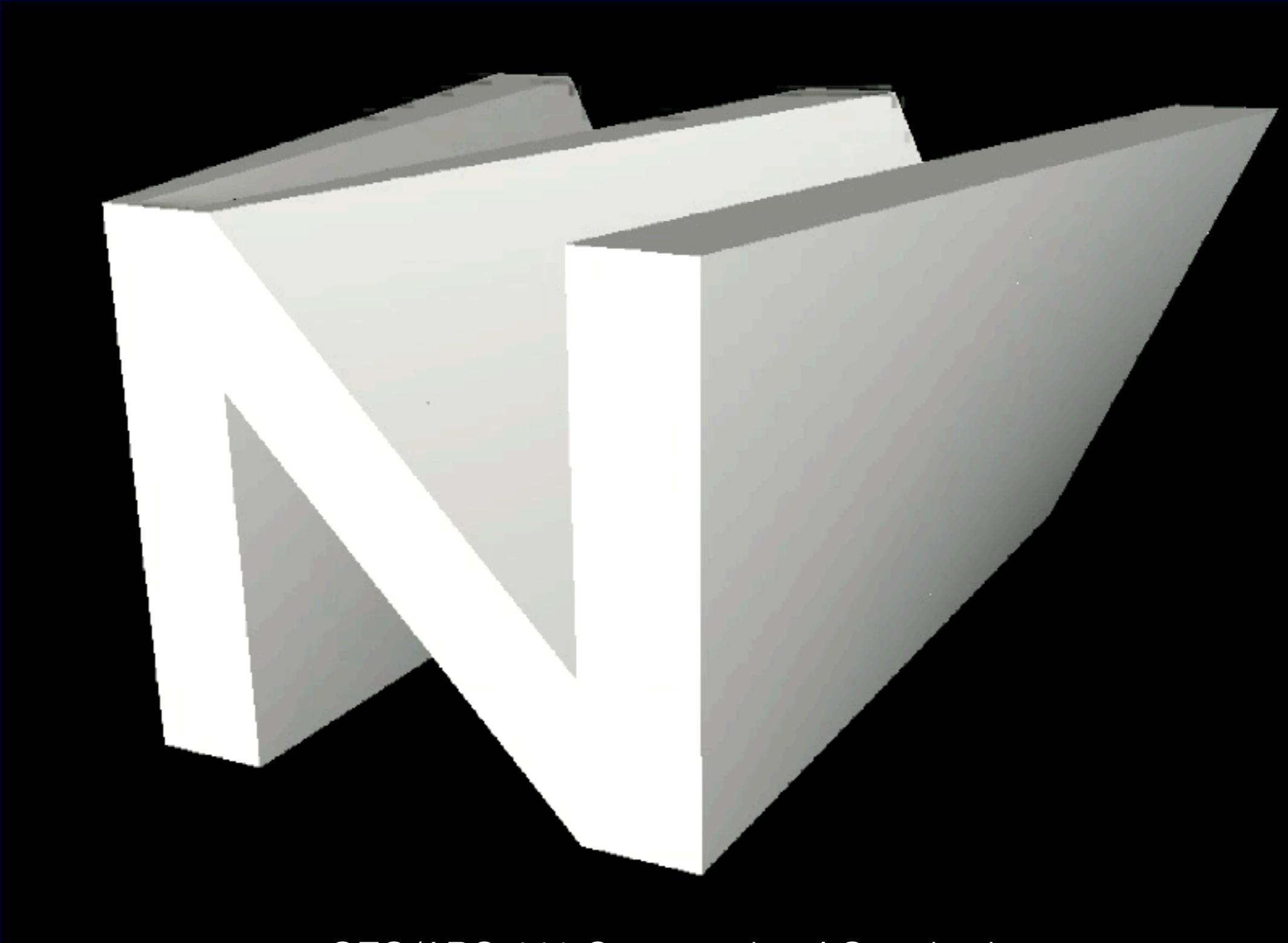


# Solver: Coffee cup simulation



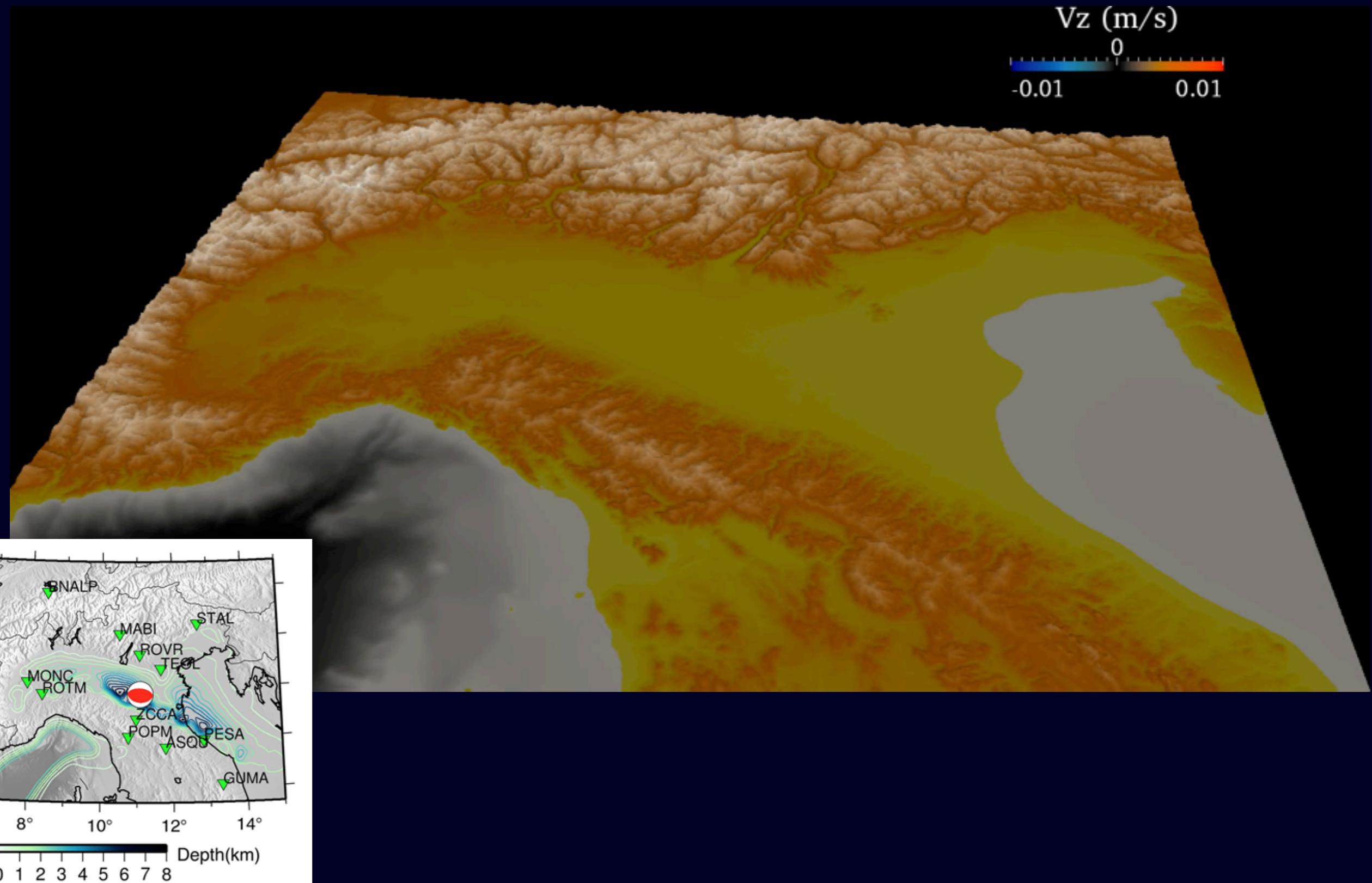
SPECFEM3D “Sesame”

# Solver: SPECFEM & Cubit in Education



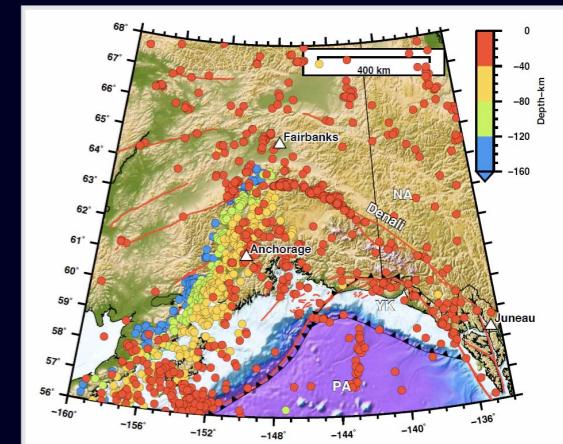
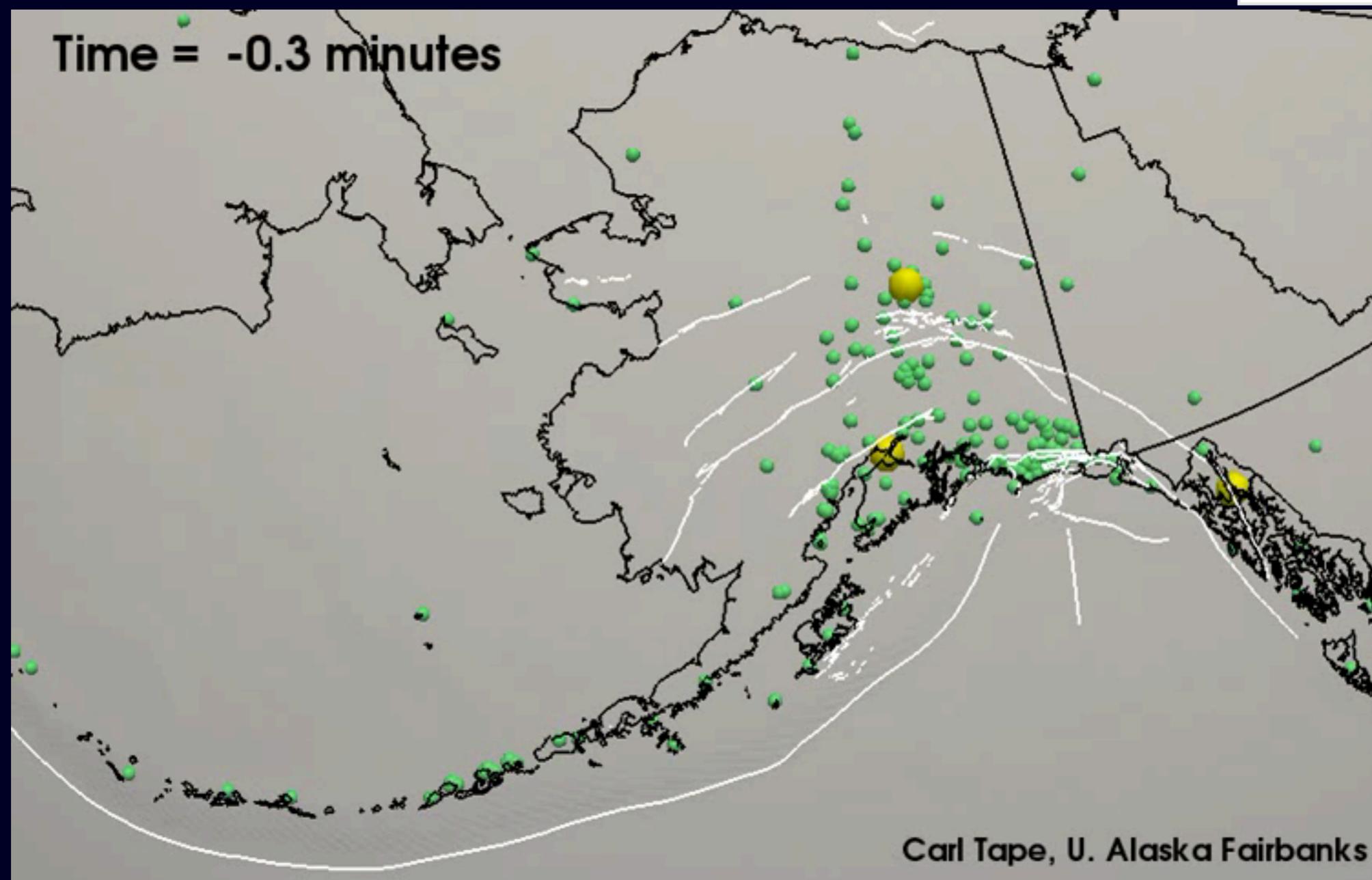
GEO/APC 441 Computational Geophysics

# Solver: Basin analysis: Northern Italy



# Solver: Finite-source simulations

## 1964 Alaska Earthquake



# Specfem2d running session(on your desktop)

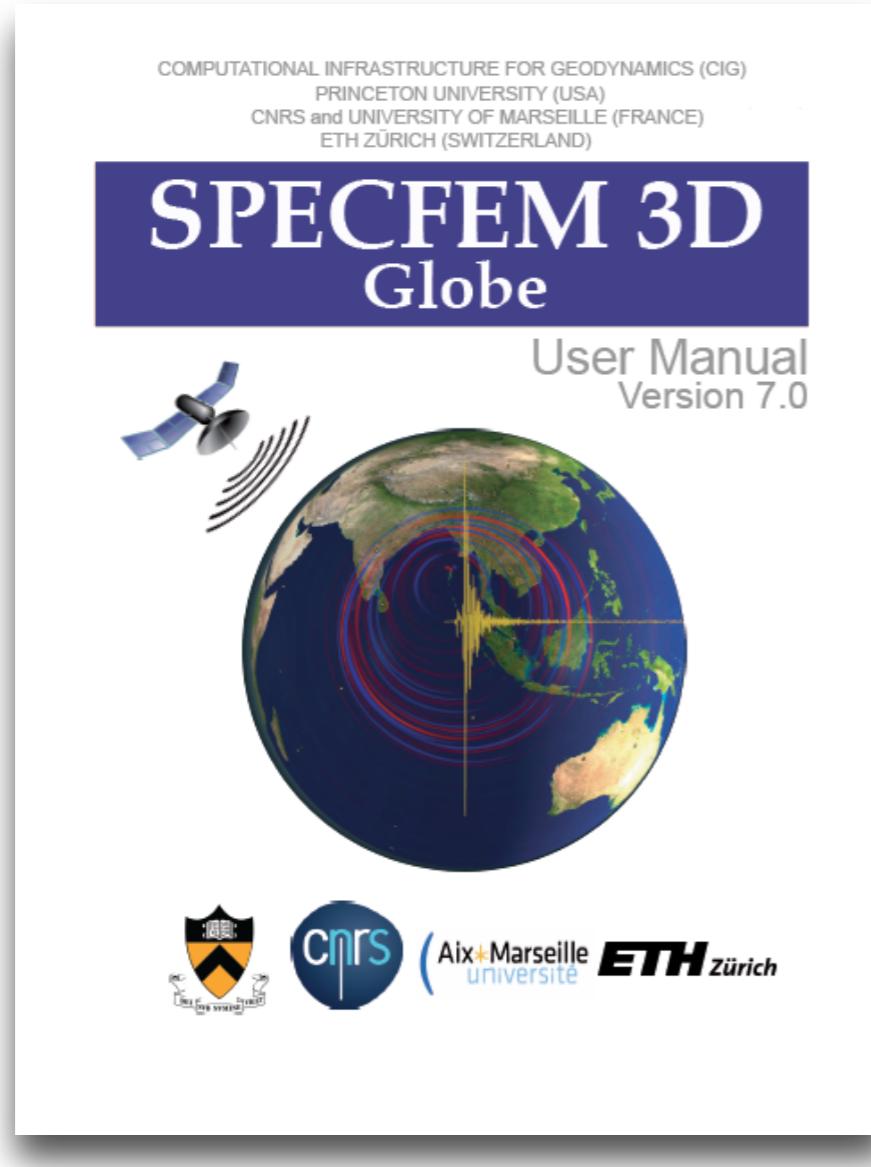
1. Open a terminal and make a **new directory** at your desktop where you want to put the specfem2d code;
2. Google “specfem2d” and find the CIG webpage
3. find the instructions to download the code;

The screenshot shows a web browser window with the URL <https://geodynamics.org/cig/software/specfem2d/>. The left sidebar contains links for Software Package List, SPECFEM2D (which is highlighted), Current Release, User Resources, Developer Resources, and User Map. The main content area describes SPECFEM2D as a package for acoustic, (an)elastic, poroelastic or coupled acoustic-(an)elastic-poroelastic media, with Convolution PML absorbing conditions. It mentions meshing tools like Gmsh, Cubit, and GID. Below this is a 'Current Release' section with a 'SOURCE PACKAGES' heading. A red box highlights the 'git clone --recursive https://github.com/geodynamics/specfem2d.git' command under 'Current Stable Release'. To the right, there's a 'User Manual Version 7.0' section with 3D visualization images, and a sidebar with status, code changes, contact, bug reports, and license information.

4. Change directory(command: cd) to the your directory(you just made) and type in the git command:

`git clone --recursive https://github.com/geodynamics/specfem2d.git`

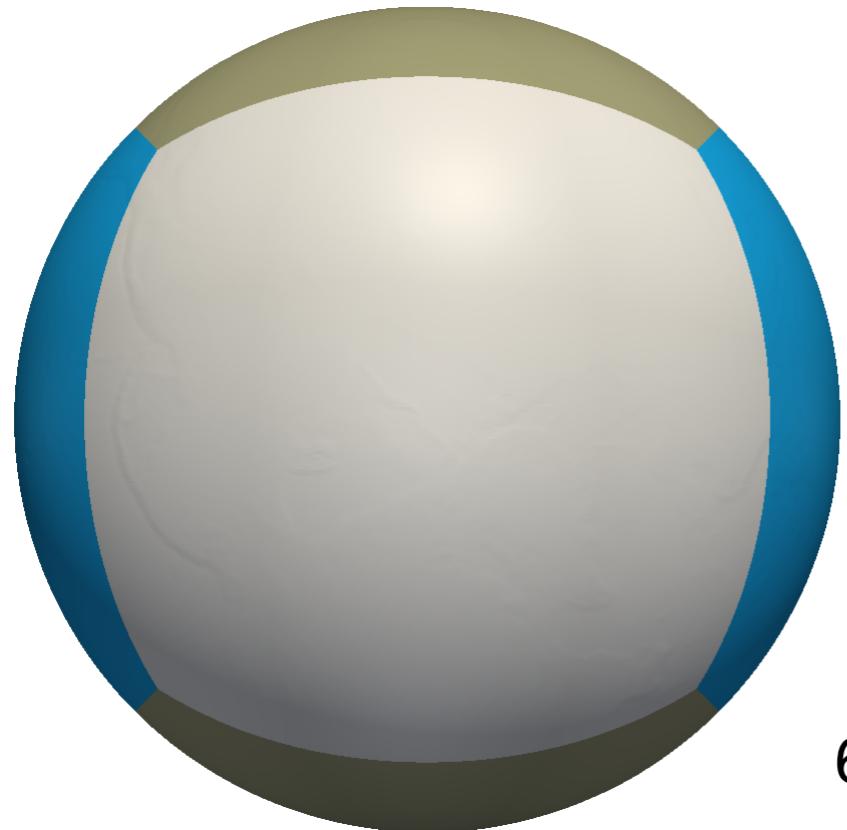
# Specfem3D Globe



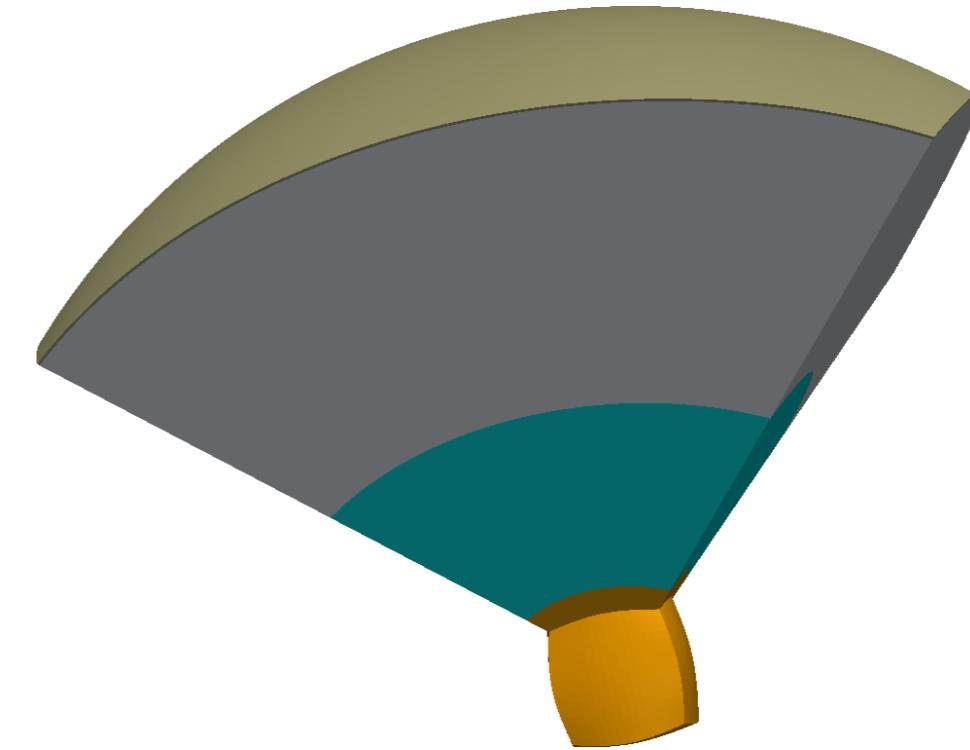
Continental/global simulation

- Attenuation
- Ellipticity
- Rotation
- Gravity
- Ocean
- Topography
- Anisotropy
- Adjoint kernels
- GPU capable

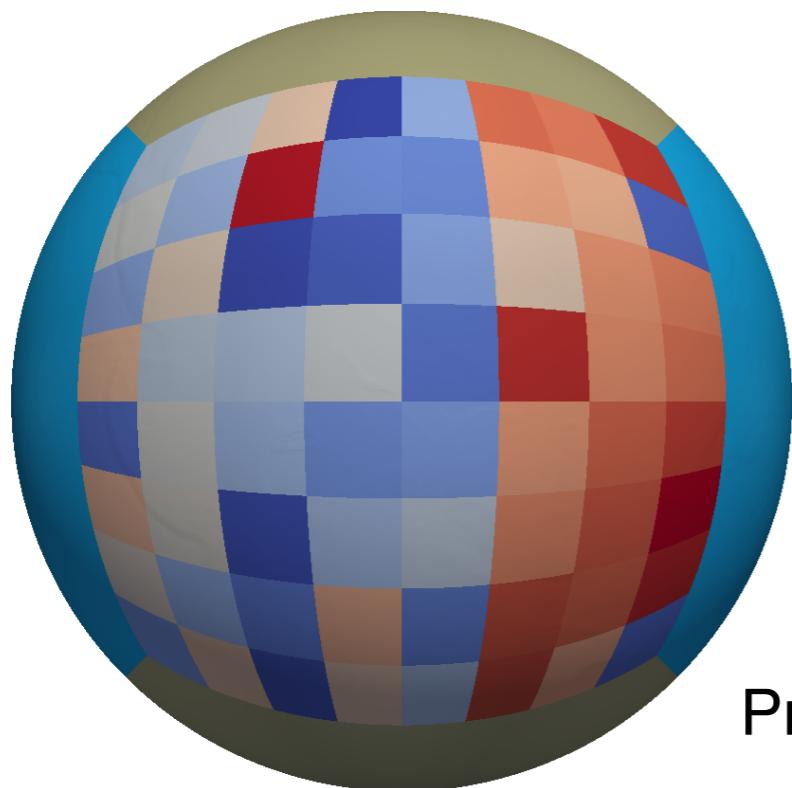
# Global Mesher: mesh & partition



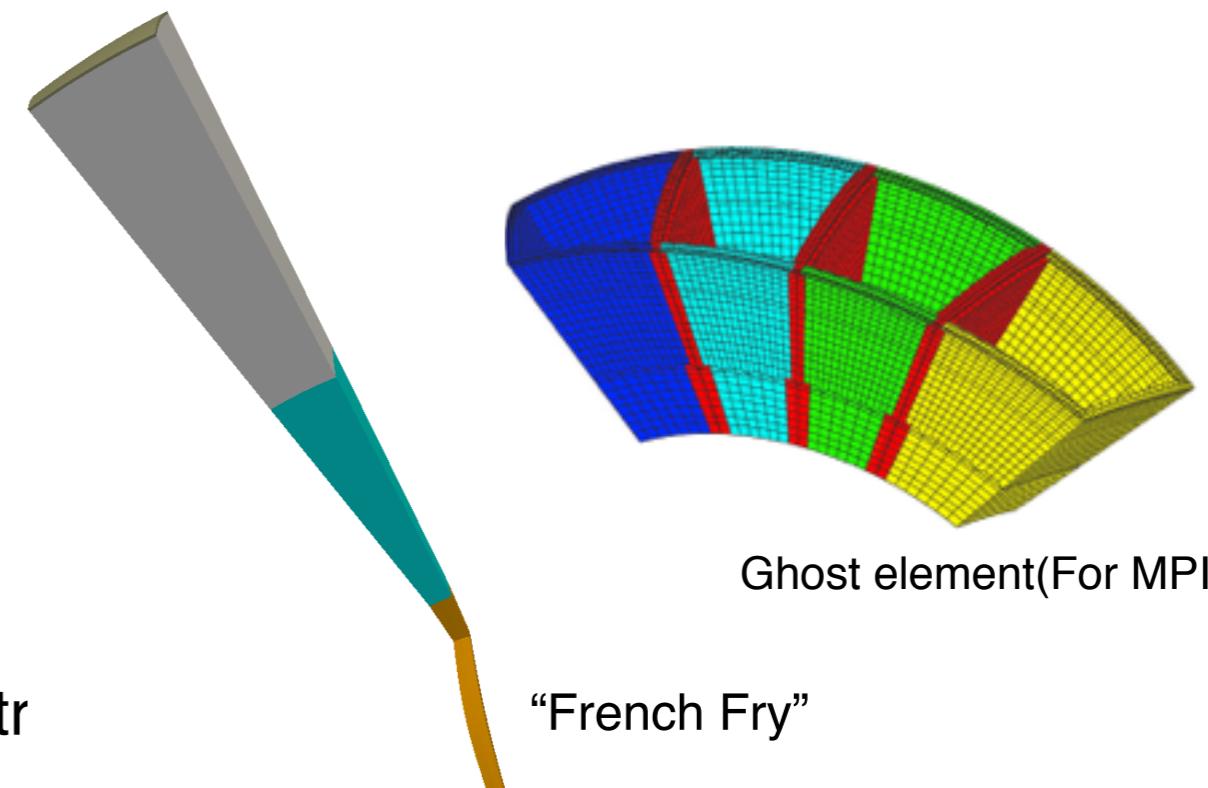
6 Chunks



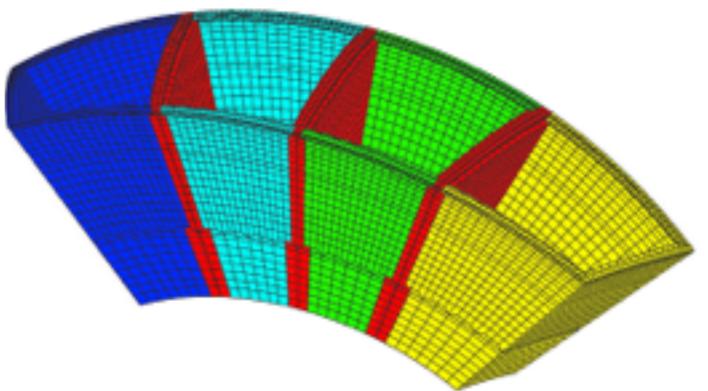
Chunks Partition



Processor distr

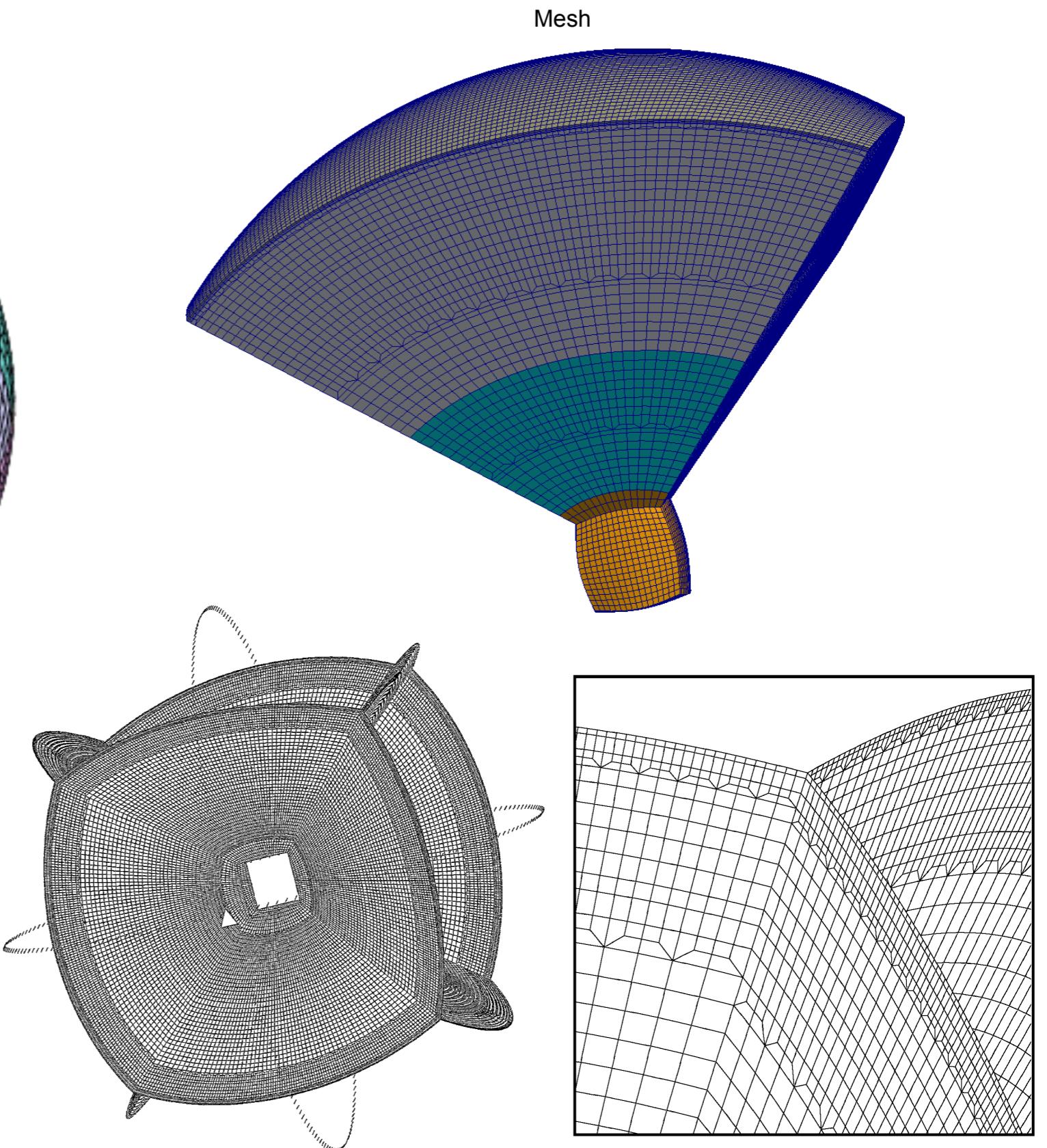
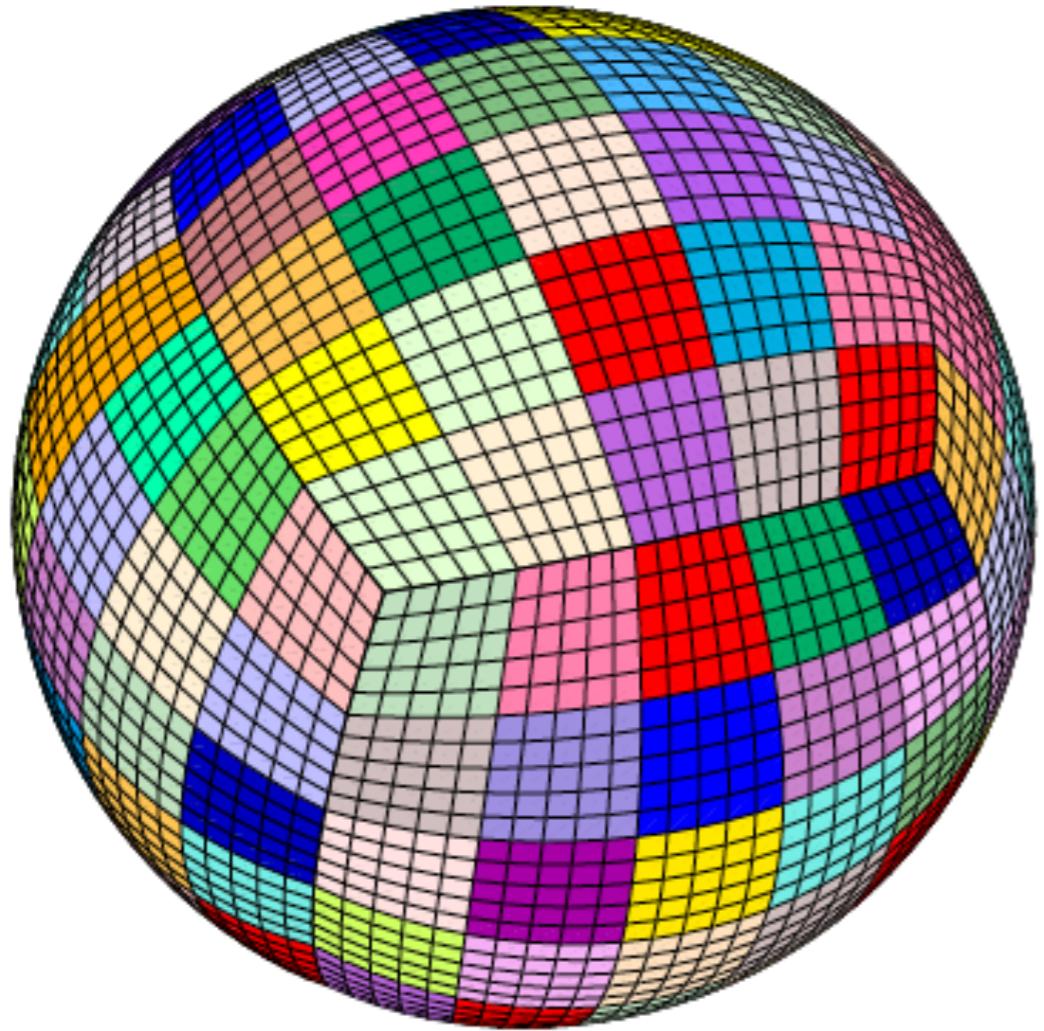


“French Fry”

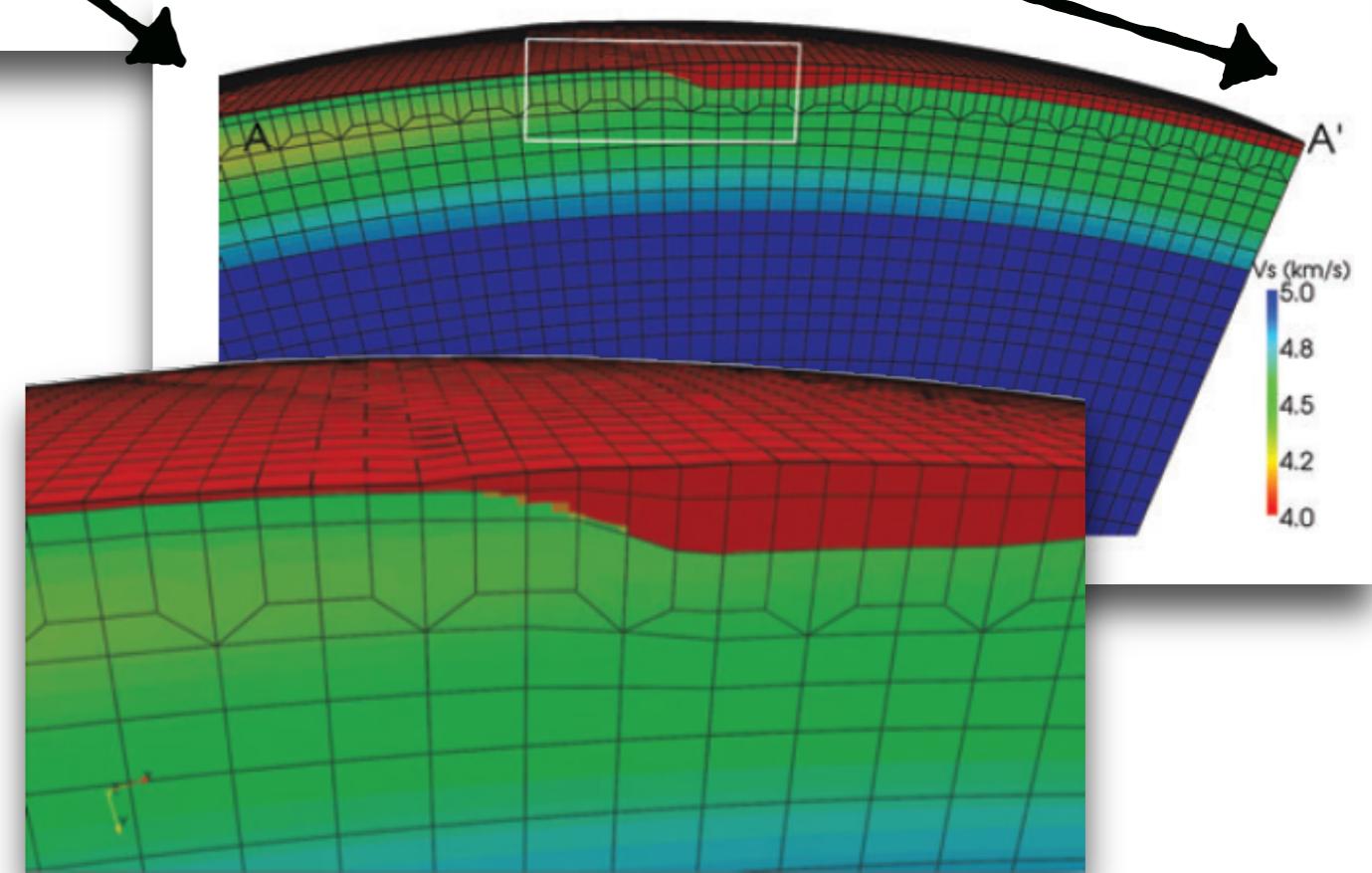
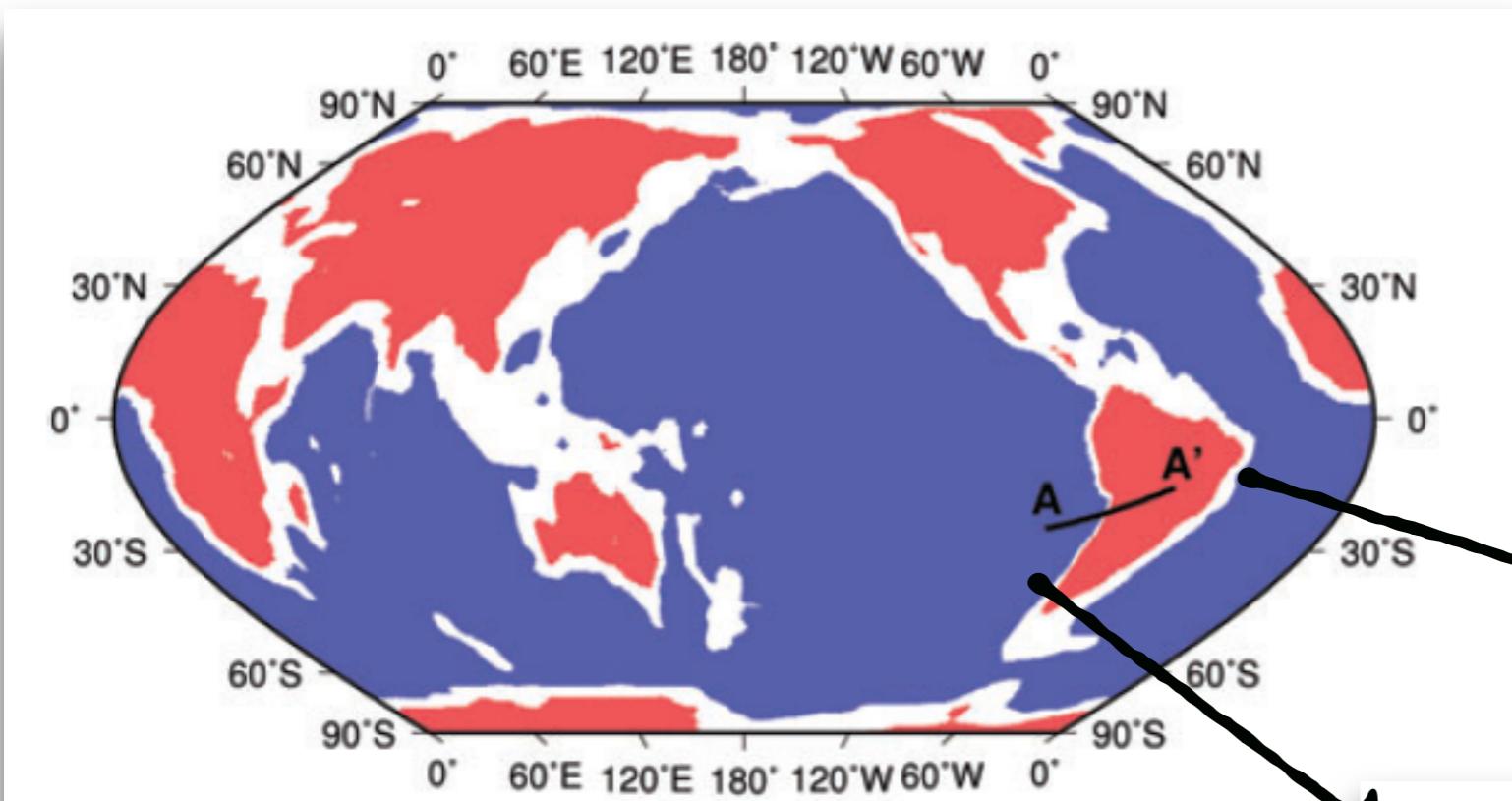


Ghost element(For MPI)

# Global Mesher: mesh & partition



# Global Mesher: Meshing the crust



- 1) Moho is honored;
- 2) Crust Option at setup/constant.h.in:  
Crust1.0/ Crust2.0

lithosphere & asthenosphere

# Global Seismic Wave Propagation

April 1, 2014, NEAR COAST OF NORTHERN CHILE  
Mw = 8.1 Depth = 21.9km

ShakeMovie  
GLOBAL

Princeton University's Near Real Time Global Seismicity Portal

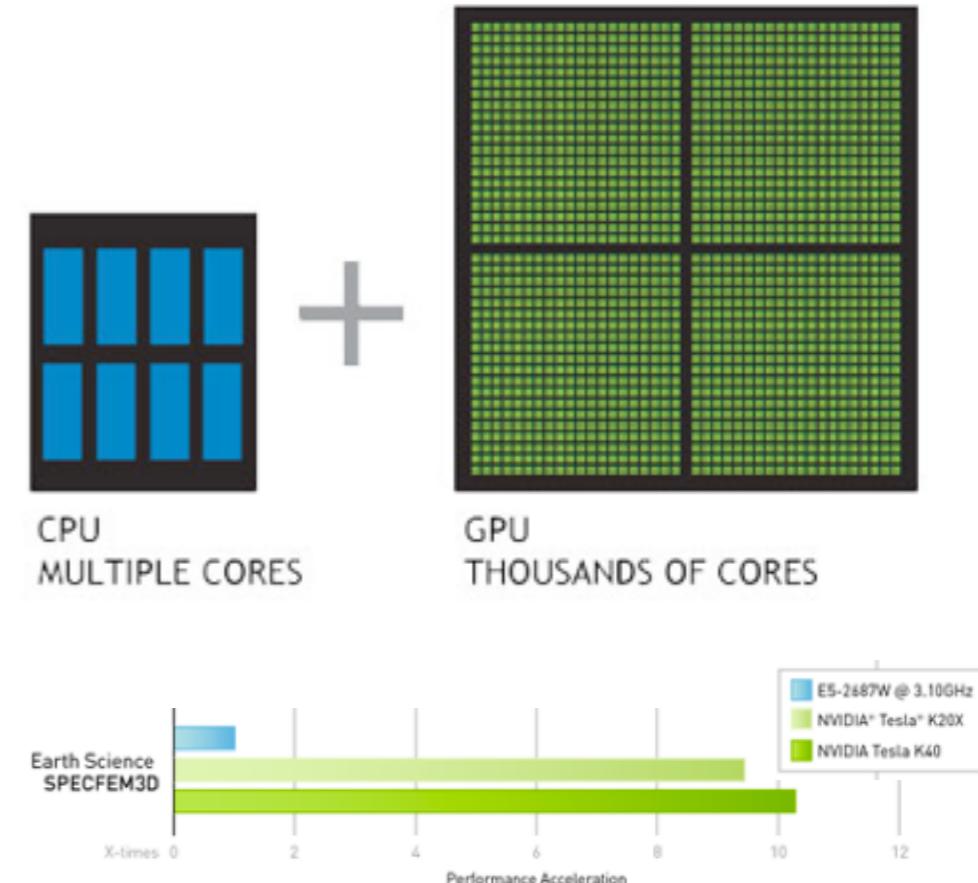
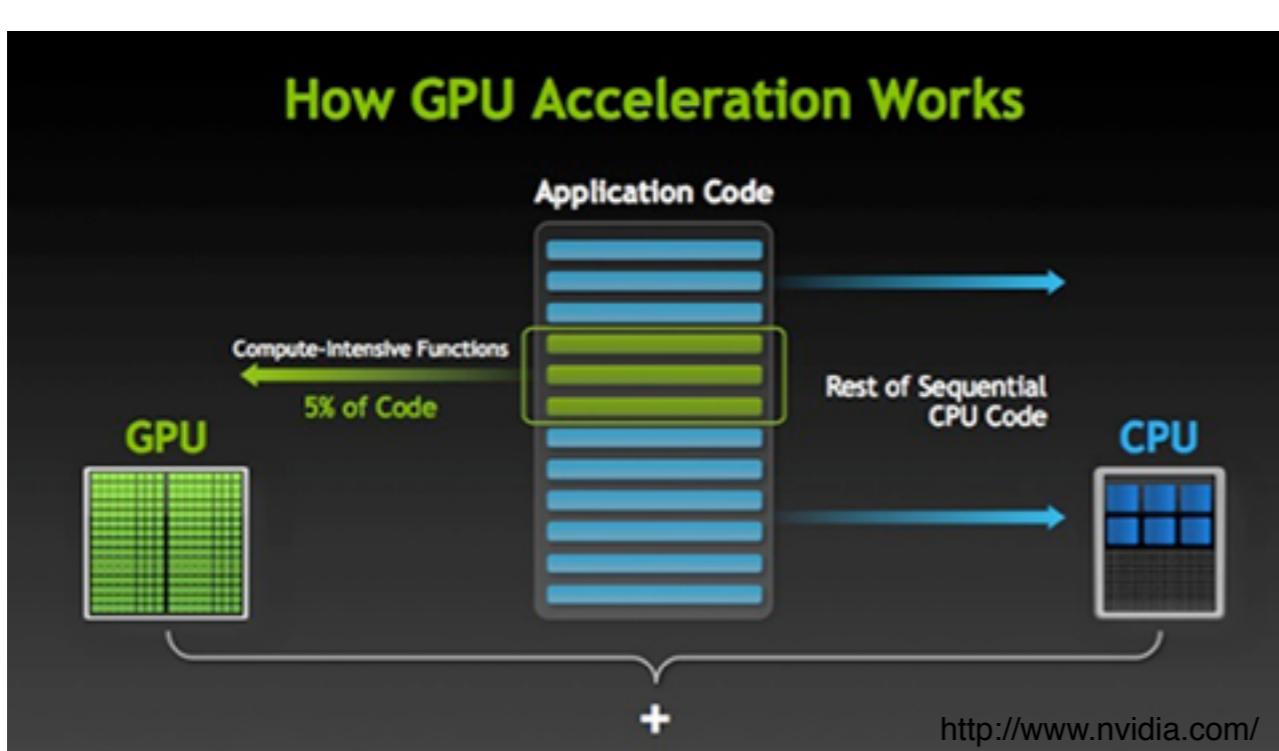
PRINCETON  
UNIVERSITY  
0:00:00



# Recent Development

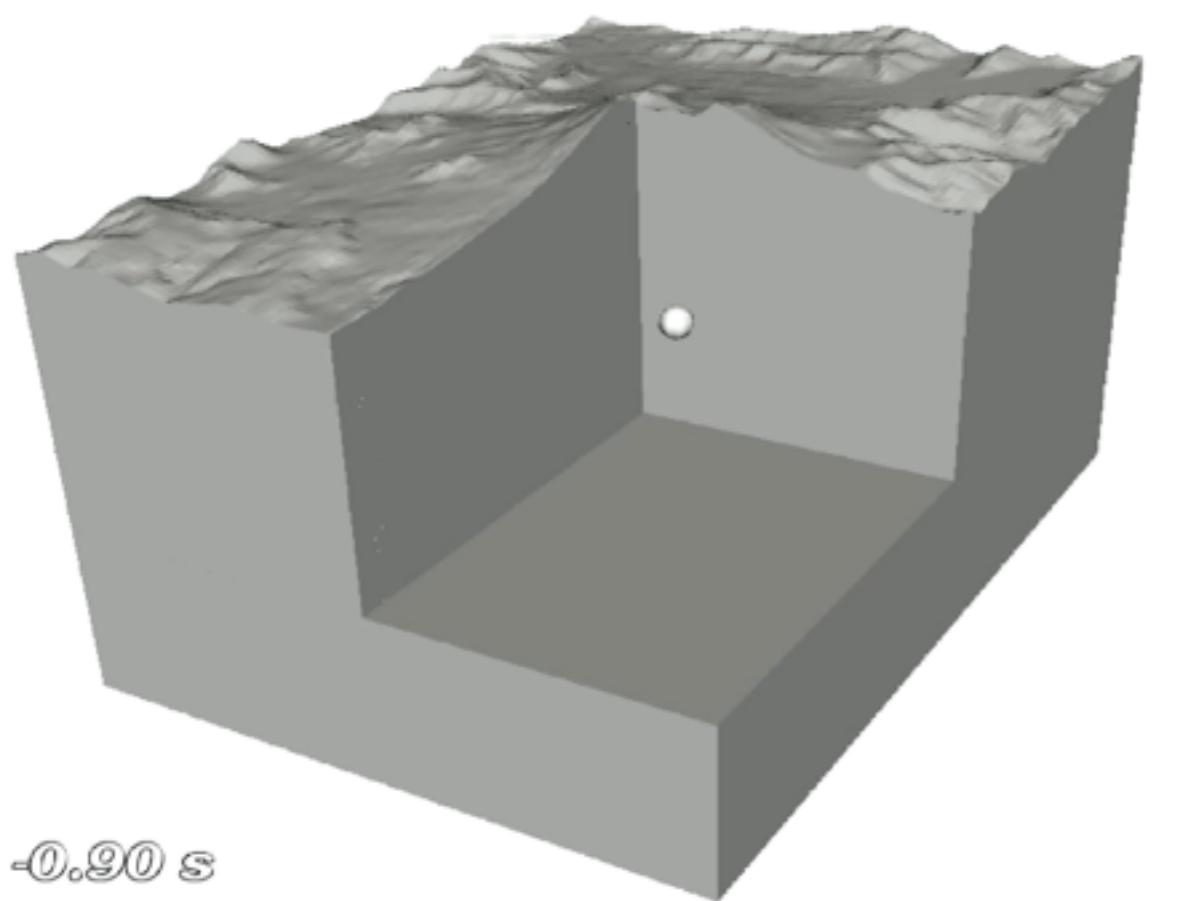
- GPU
- Adaptive Seismic Data Format(ASDF)
- Specfem3d\_Mars
- Undo attenuation(adjoint simulation)

# GPU Capability

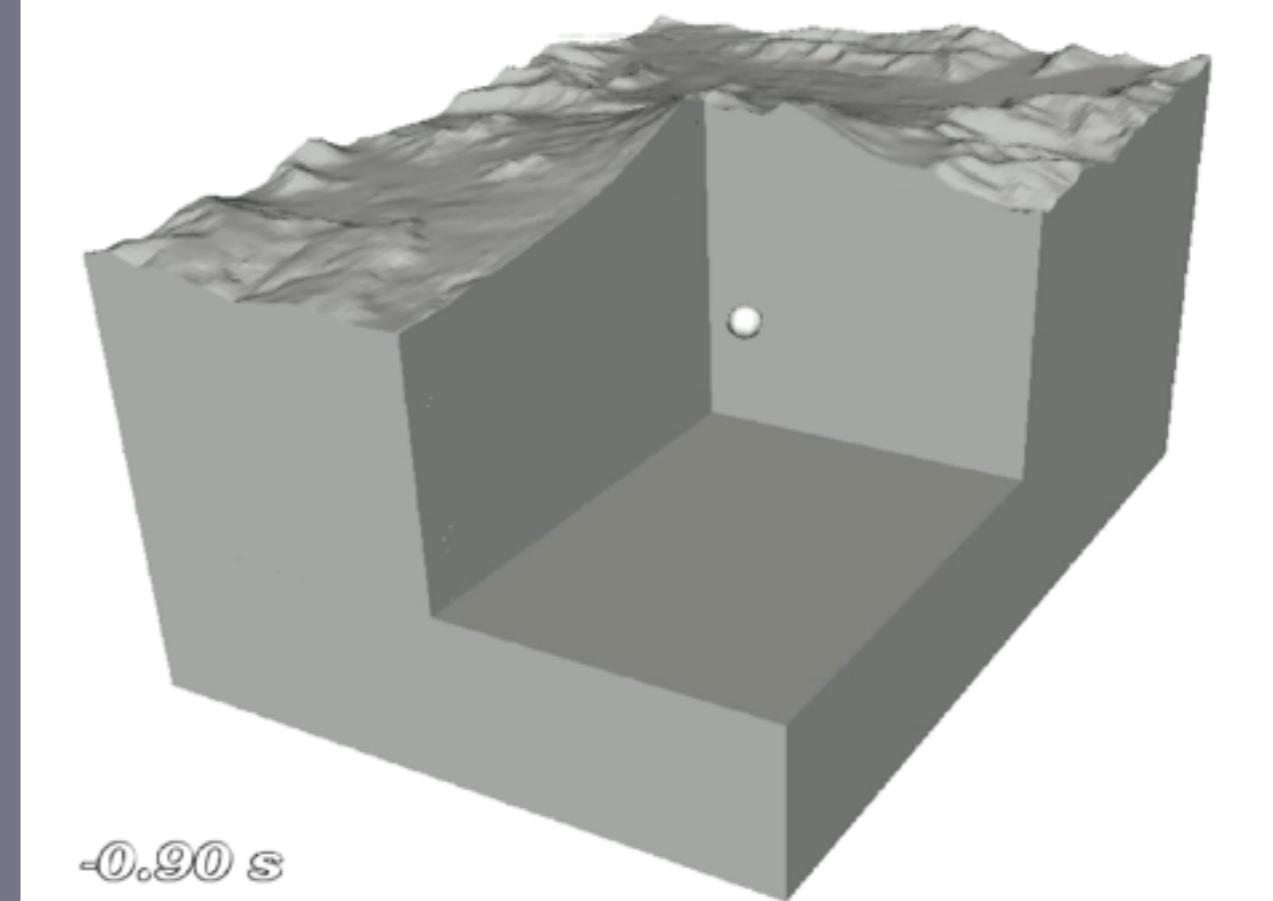


Architecture: Cray XK7  
Nodes: 18,688 AMD Opterons  
GPUs: 18,688 K20X Keplers

# GPU Capability



GPU



CPU

# Adaptive Seismic Data Format(ASDF)

- **To solve the I/O bottleneck**

4,200 events \* (12,000 observed sac + 12,000 synthetic sac)  
= 100,800,000 single sac files

or

4,200 events \* (1 observed asdf + 1 synthetic asdf)  
= 8,400 single asdf files

- **Parallel I/O**

MPI or multi-processing(internal support)

- **Obspy support**

working seamlessly with obspy

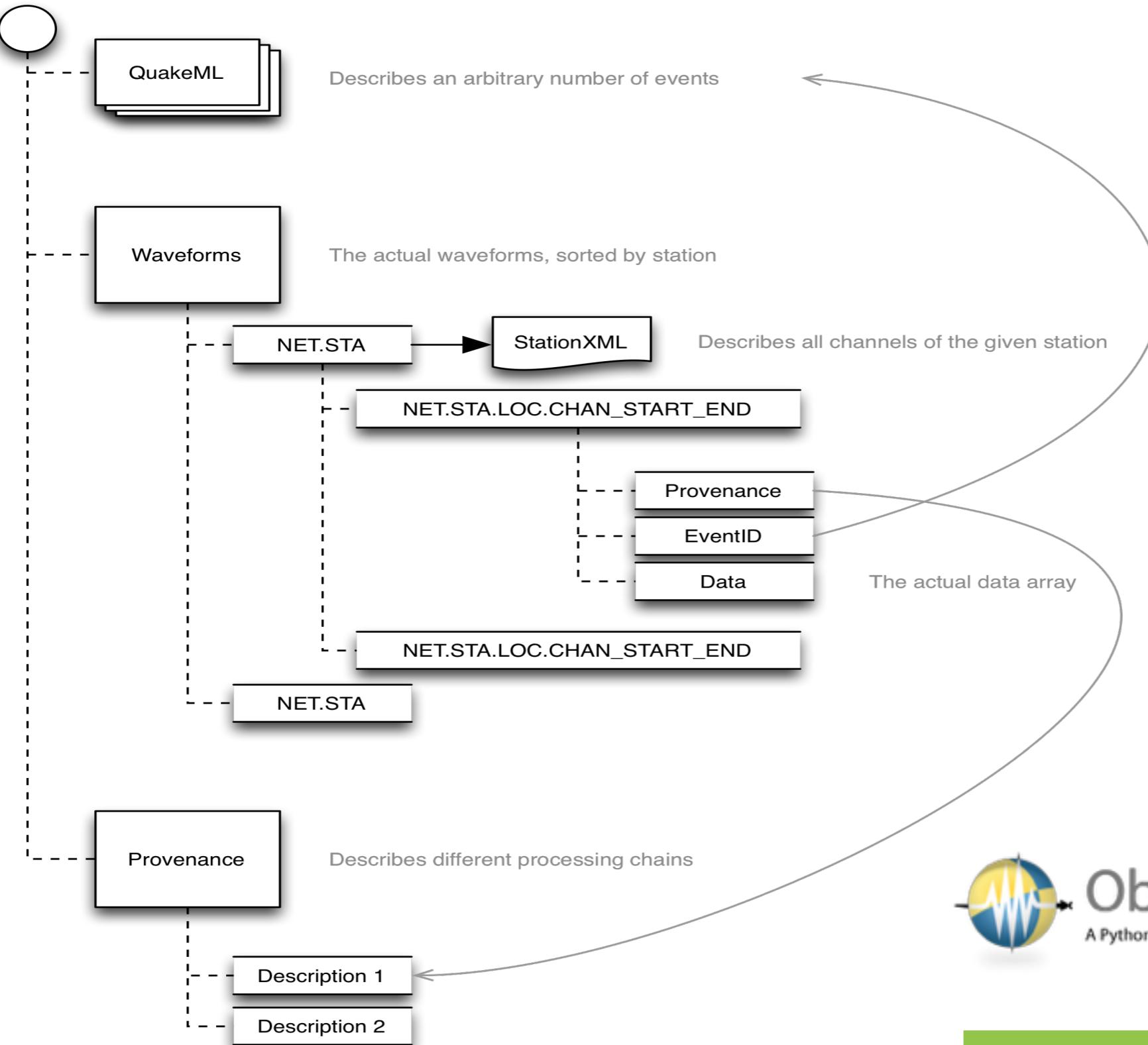
- **Multiple language interface**

python, fortran, C/C++

Hosted on github: <https://github.com/SeismicData>



# Adaptive Seismic Data Format(ASDF)



Documentation



ObsPy

A Python Framework for Seismology



Hosted on github: <https://github.com/SeismicData>

pyasdf 0.1.x



# Specfem3d\_Mars

- NEX=160 (160x160x6 surface elements)
- Tmin= $\sim$ 20 s
- modified from PREM mesh designed for global Earth simulations
- radius of the inner cube is  $\sim$ 90 km
- small inner core with a radius of  $\sim$ 110 km

CrMB

(s)  
6.809e+00

-2

0.000e+00

ICB

[Ebru, 2015]

# Specfem3d Globe running session(on the cluster)

Please follow the instructions on the quick guide(downloaded from github)

# Outline

1. Introduction

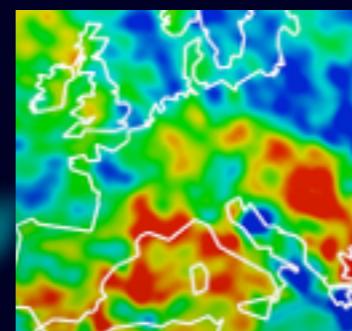
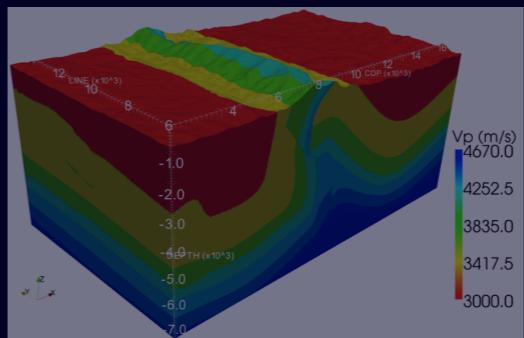
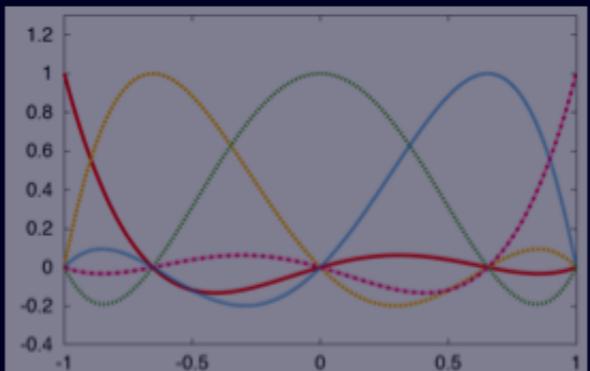
2. Basic Theory:  
Spectral Element Method

3.  
SPECFEM2D

4. Adjoint Tomography

5. Summary

Degree 4 Lagrange polynomials:



# Adjoint Method

$$\chi(\mathbf{m}) = \frac{1}{2} \sum_{r=1}^{N_r} \int_0^T w_r(t) \|\mathbf{s}(\mathbf{x}_r, t; \mathbf{m}) - \mathbf{d}(\mathbf{x}_r, t)\|^2 dt,$$

$$\delta\chi = \int_V [K_\rho(\mathbf{x}) \delta \ln \rho(\mathbf{x}) + K_\mu(\mathbf{x}) \delta \ln \mu(\mathbf{x}) + K_\kappa(\mathbf{x}) \delta \ln \kappa(\mathbf{x})] d^3\mathbf{x},$$

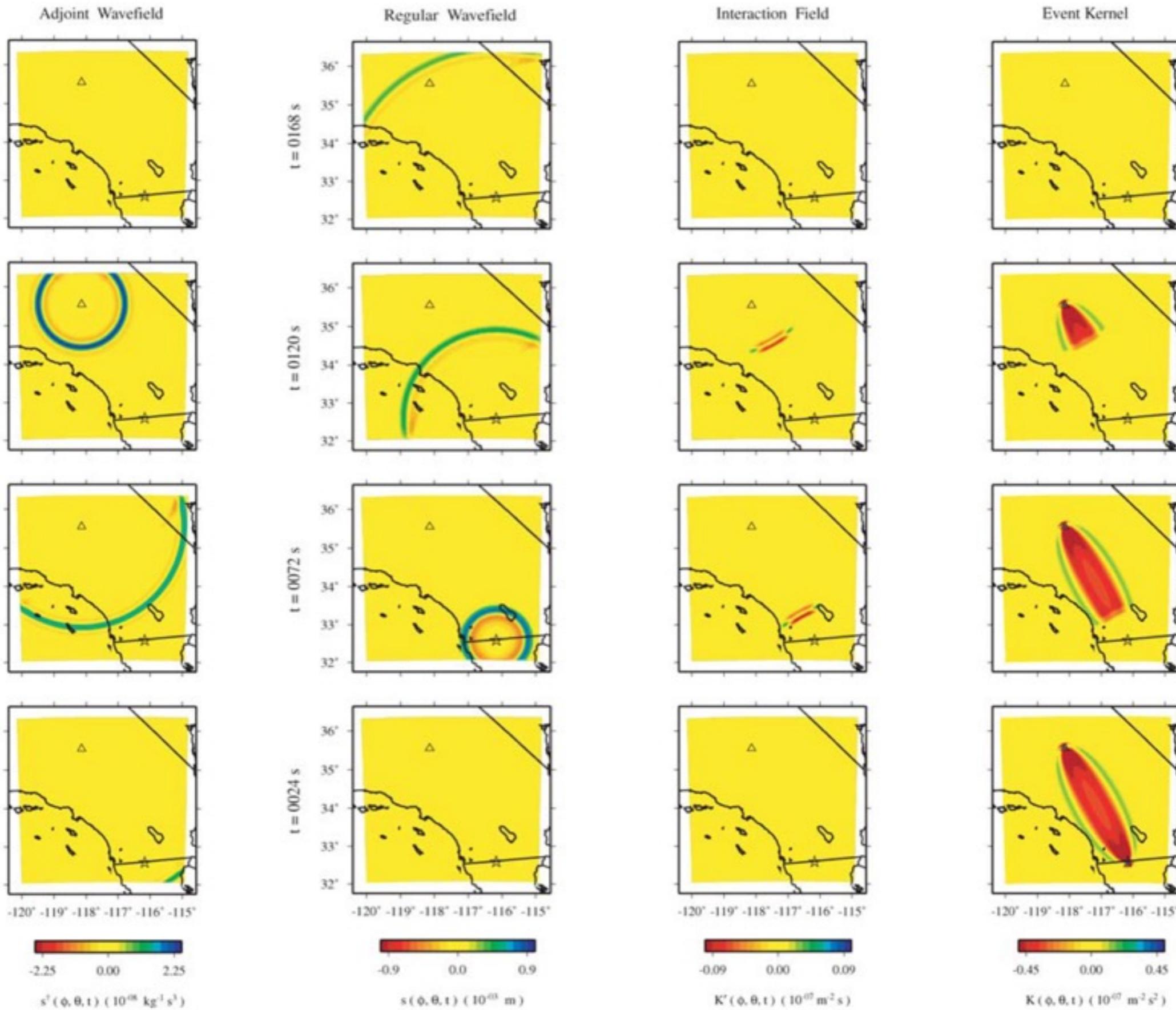
$$K_\rho(\mathbf{x}) = \int_0^T \rho(\mathbf{x}) \partial_t \mathbf{s}^\dagger(\mathbf{x}, T-t) \cdot \partial_t \mathbf{s}(\mathbf{x}, t) dt,$$

$$K_\mu(\mathbf{x}) = - \int_0^T 2\mu(\mathbf{x}) \mathbf{D}^\dagger(\mathbf{x}, T-t) : \mathbf{D}(\mathbf{x}, t) dt,$$

$$K_\kappa(\mathbf{x}) = - \int_0^T \kappa(\mathbf{x}) [\nabla \cdot \mathbf{s}^\dagger(\mathbf{x}, T-t)] [\nabla \cdot \mathbf{s}(\mathbf{x}, t)] dt,$$

$$\mathbf{f}_1^\dagger(\mathbf{x}, t) = \sum_{r=1}^{N_r} w_r(t) [\mathbf{s}(\mathbf{x}_r, T-t) - \mathbf{d}(\mathbf{x}_r, T-t)] \delta(\mathbf{x} - \mathbf{x}_r),$$

# Adjoint Simulation

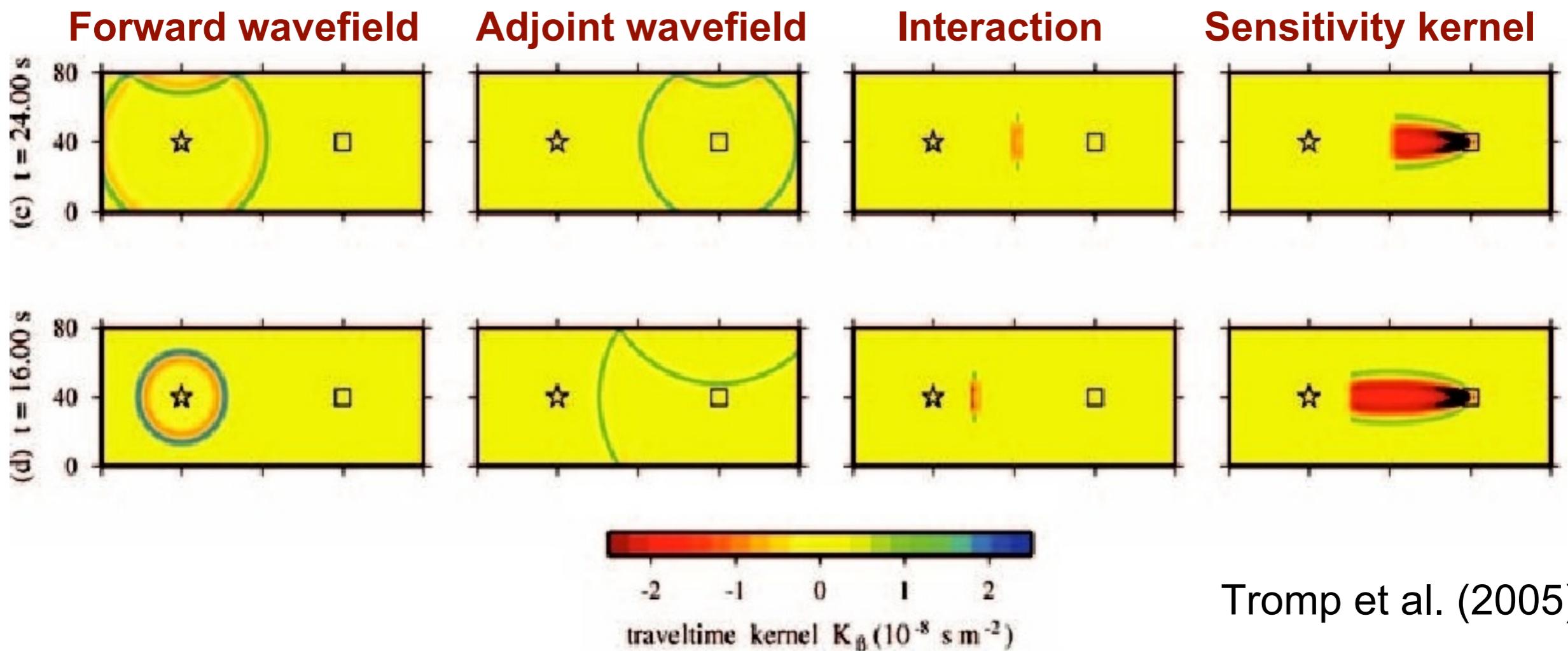


# Adjoint tomography

$$\delta\chi = \int_V [K_\rho(\mathbf{x})\delta \ln \rho(\mathbf{x}) + K_{c_{jklm}}(\mathbf{x})\delta \ln c_{jklm}(\mathbf{x})] d^3\mathbf{x}$$

$$K_\rho(\mathbf{x}) = - \int_0^T \rho(\mathbf{x}) \mathbf{s}^\dagger(\mathbf{x}, T-t) \cdot \partial_t^2 \mathbf{s}(\mathbf{x}, t) dt$$

$$K_{c_{jklm}}(\mathbf{x}) = - \int_0^T \epsilon_{jk}^\dagger(\mathbf{x}, T-t) c_{jklm}(\mathbf{x}) \epsilon_{lm}(\mathbf{x}, t) dt$$



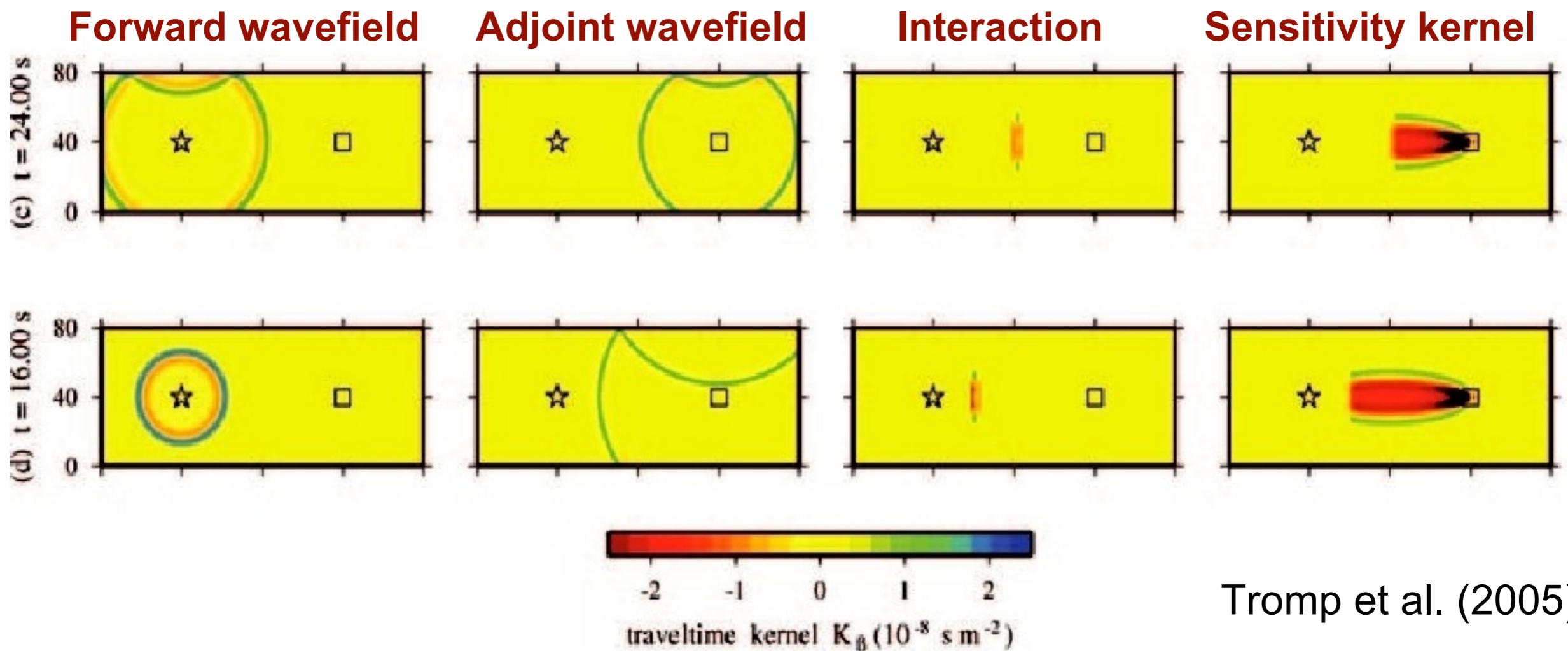
Tromp et al. (2005)

# Adjoint tomography

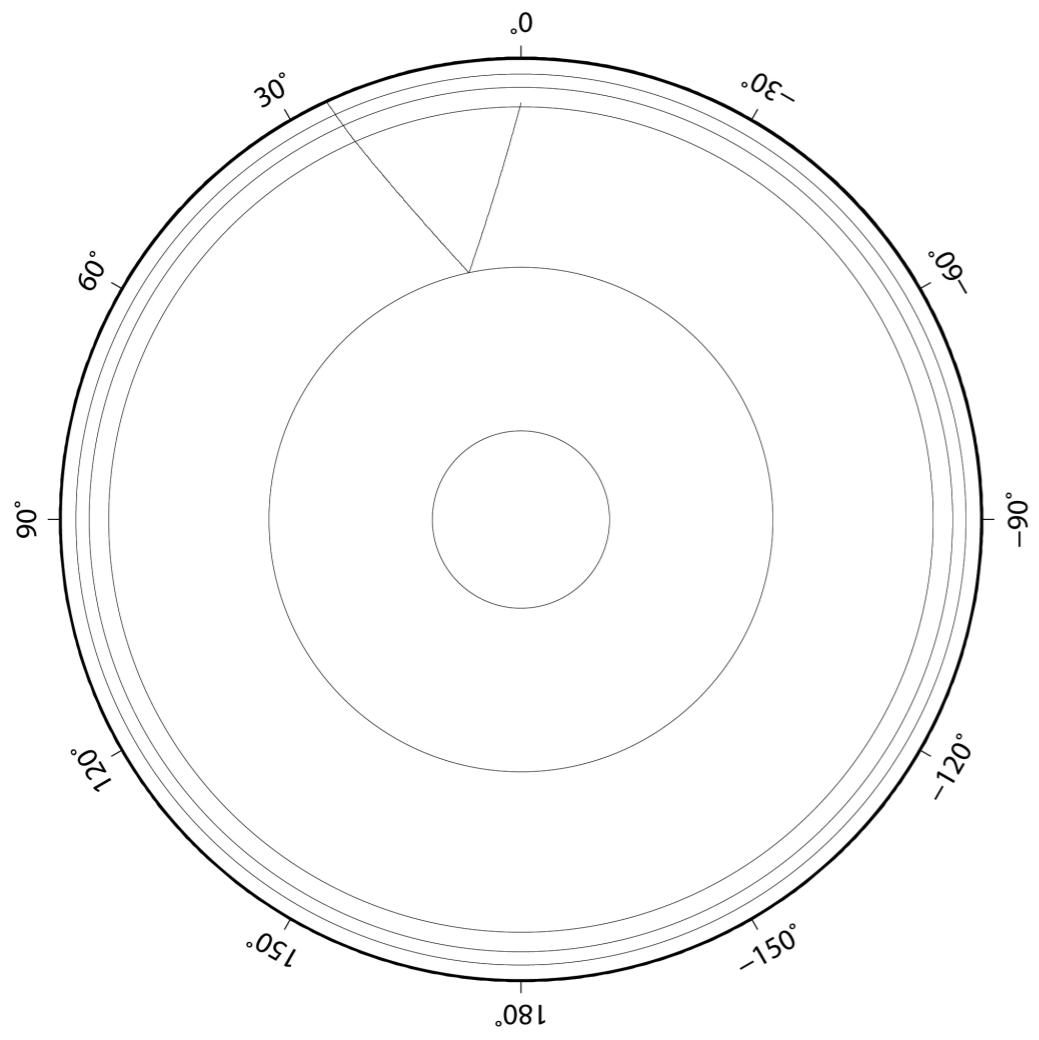
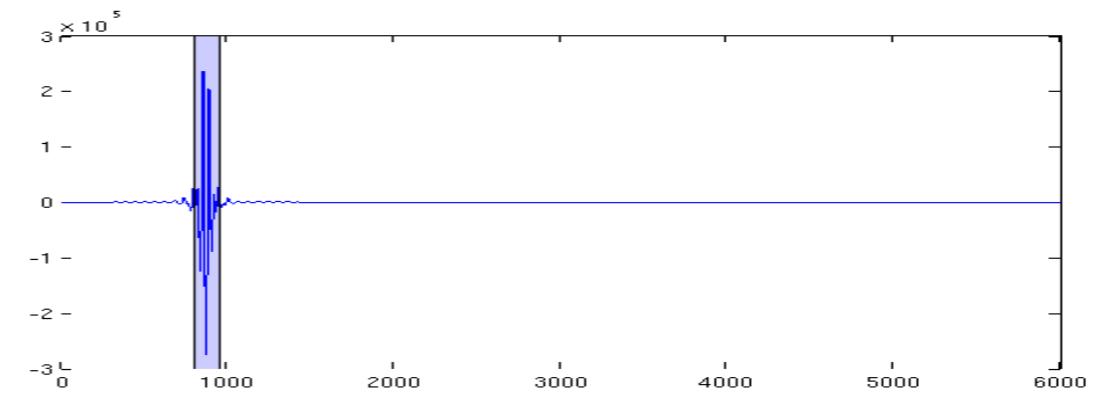
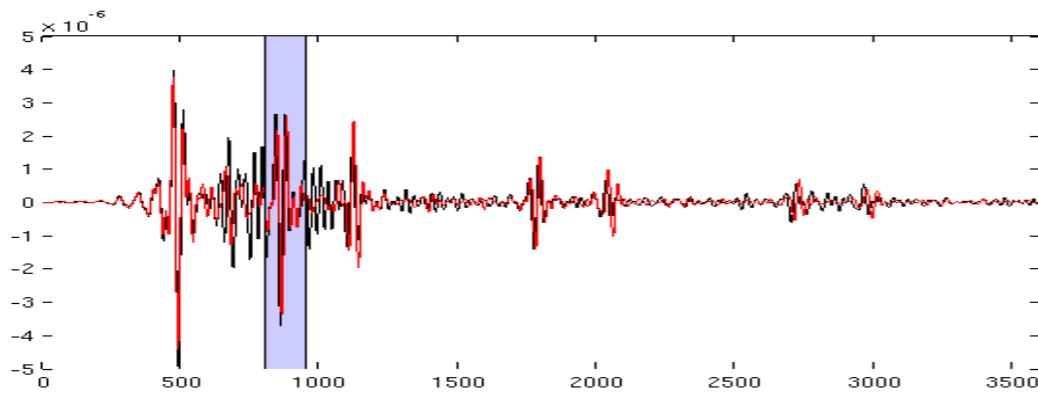
$$\delta\chi = \int_V [K_\rho(\mathbf{x})\delta \ln \rho(\mathbf{x}) + K_{c_{jklm}}(\mathbf{x})\delta \ln c_{jklm}(\mathbf{x})] d^3\mathbf{x}$$

What is undo-attenuation?

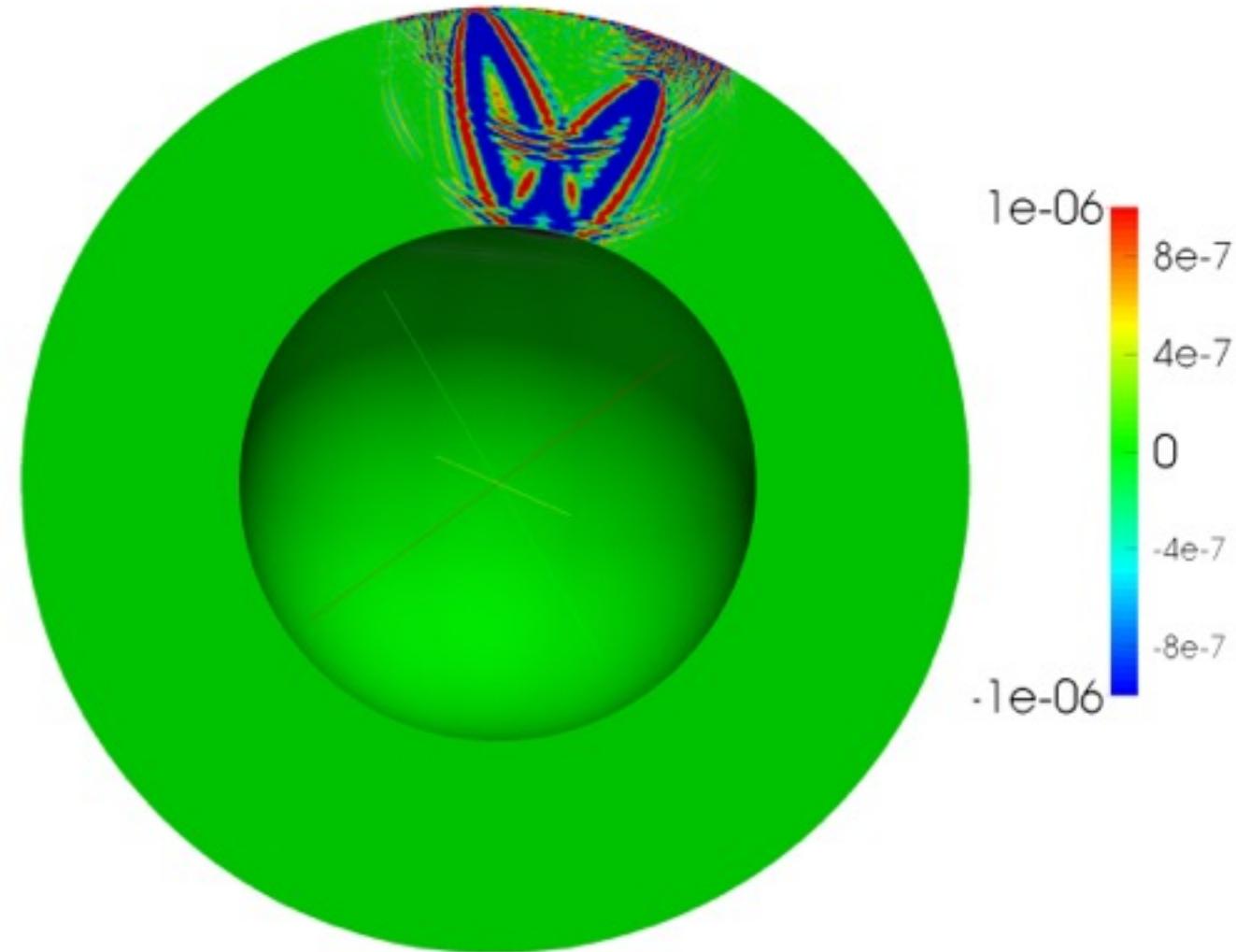
$$K_{c_{jklm}}(\mathbf{x}) = - \int_0^T \epsilon_{jk}^\dagger(\mathbf{x}, T-t) c_{jklm}(\mathbf{x}) \epsilon_{lm}(\mathbf{x}, t) dt$$



# Traveltime Kernel from ScS phase

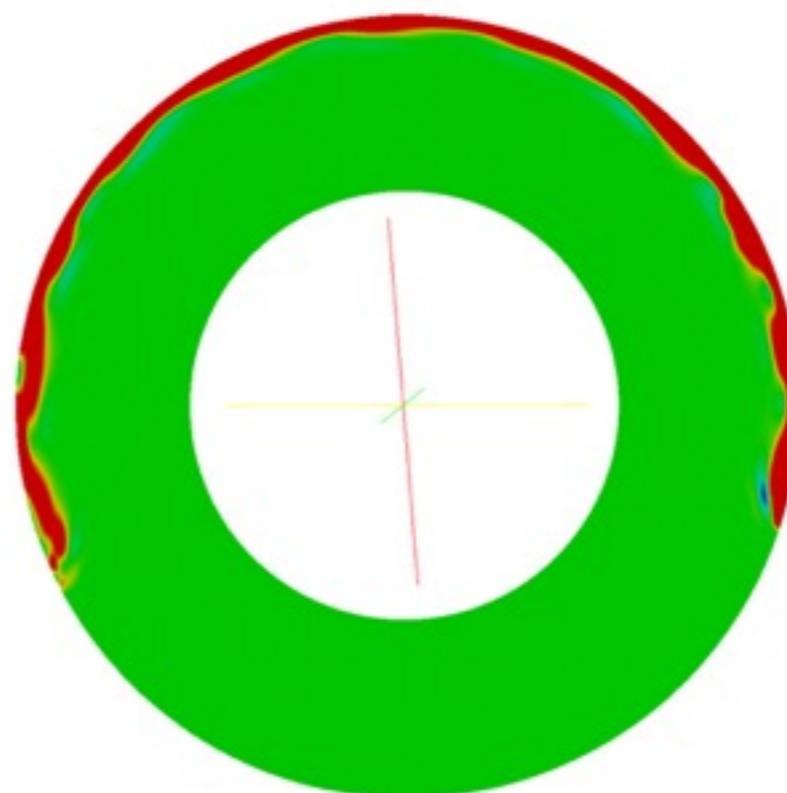
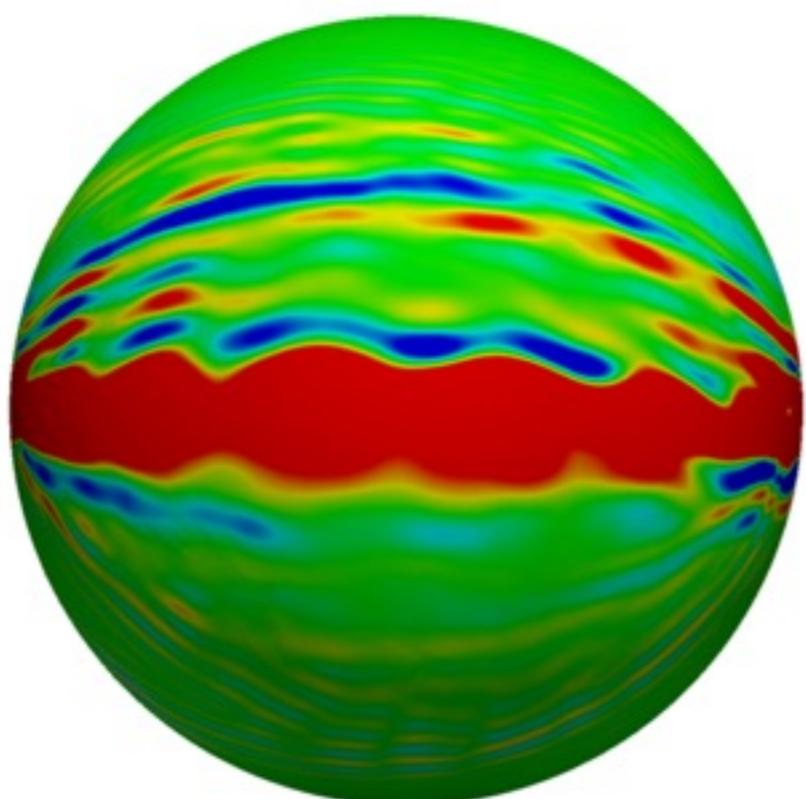
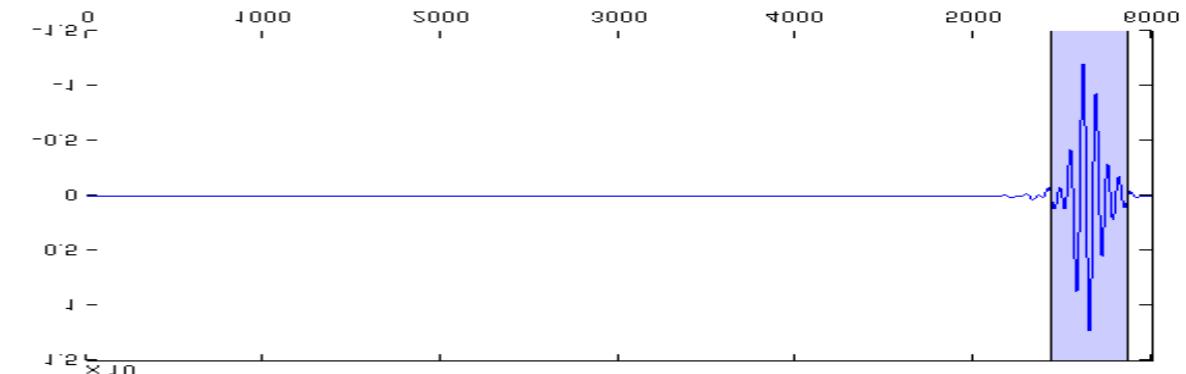
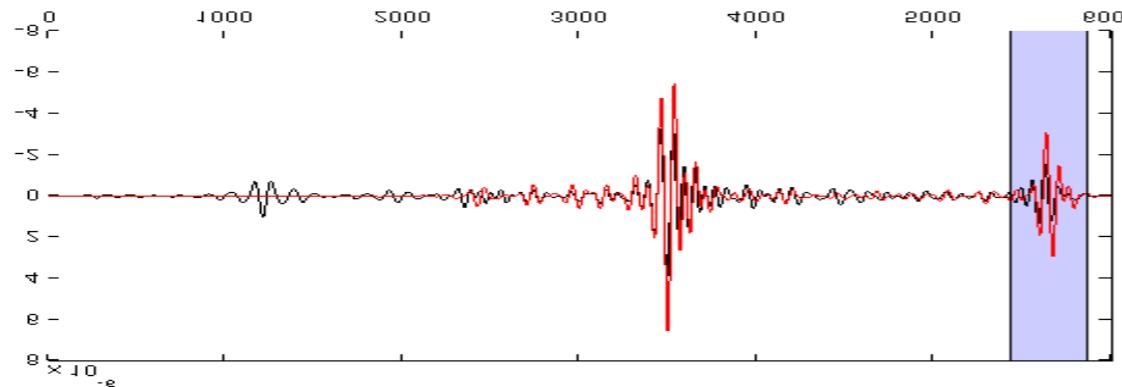


The raypath of the ScS phase



The  $\beta_v$  Kernel from the ScS phase

# Traveltime Kernels from Major-arc surface waves



The  $\beta_v$  Kernel from major-arc Love waves

# Summary and Questions