Problem Statement

- Numerically evaluate the magnetic field due to a coil using the Biot-Savart law, and compare with the analytical result
- Investigate the magnetic field generated by multiple coils e.g. in a Helmholtz configuration

Core Task 1

I broke down the challenge of calculating the magnetic field from a coil of wire by devising code modules that first generate the wire coil as `num_segment` straight line wire elements (`Coil.py`), and then calculating the magnetic field in the system by summing the contributions from each line element using the Biot-Savart law. This involved the use of numpy functions such as `np.linalg.norm` and `np.cross` to vectorise and simply the process. I decided to choose the length of the line elements such that they add up to the circumference of the coil under consideration; this choice originated by consideration of Ampère's Law:

$$\mathbf{B} = \mu_0 \oint I \mathrm{d}\mathbf{l} \tag{1}$$

For the evaluated magnetic field to be correct, the value of the line integral of the current with the broken up line elements must be equivalent to the analytical circular case - this can only be achieved if the sum of the lengths of the line elements were equivalent to the circumference of the coil.

For storing the values of the calculated magnetic field, I used a (`num_x` x `num_y`) numpy array where each element of the array was a 3-component array storing the (x,y,z) component values of the B-field. This matrix was generated using trick in numpy involving the use of `[np.newaxis,:]` in conjunction with a matrix multiplication.

To assess the validity of my solution, I compared my numerical results with the analytical solution on the axis of a single coil using $N = 1,000$ data points. From the resultant graphs, it can be seen that my implementation agrees extremely well (to within 10^{-16}, practically the limit of accuracy due to rounding errors in binary representation) for `num_segment`≥ 1.

To (qualitatively) assess the validity of my solution off-axis, I made a vector field plot in the $x-y$ plane of the system by generating a 2D grid of points using the numpy `mgrid` function, reshaping my B-field data matrix, and plotting using the `quiver` command from matplotlib. By inspection, a large number of line elements (≥ 16) is needed to fully capture the curvature of the B-field especially near the coil itself. Hence from this point onwards all plots will be generated with `num_segment`=32.

Core Task 2

To include the effects of multiple coils, I modified my code so that the magnetic field contribution from a coil at a given location is packaged into its own function (`Coil`) so that I can simply call `Coil` multiple times with different position arguments and conduct a vector sum of all of their contributions via the function `Place_Coils`. I firstly investigated the Helmholtz Coil configuration where two coils are placed with the distance between the centers being equal to their radii - this configuration is expected to generate a fairly uniform field close to the centre of the system so I evaluated the B field in a (10cm x 10cm) rectangular region around the centre. I made a vector field plot to qualitatively show the uniformity of the field there, as well as a contour plot that shows the percentage deviation of the field strength from the centre point. As expected, the field strength decreases away from the centre along the axis of the coils, while there is an increase in the diagonal directions. The numerical changes are very small in magnitude ($\sim 10^{-4}\%$ deviation) so this quantitatively confirms the uniformity of the field generated by a Helmholtz coil configuration.

I attempted to use `OOMFormatter` to format the colorbars of the contour plots to look a little nicer but it doesn't seem to work for older versions of `matplotlib` which seem to be installed on the MCS. It is used in generating the plots in this `.pdf` but in the actual `.py` file I have commented it out so that the program actually runs on the MCS.

Supplementary Task

As an extension, I investigated a system with N coaxial coils with uniform spacing such that the distance between the outermost coils was held constant at $D = 10R$ where R is the radii of the coils. Qualitatively it is expected that the field within the coils will become increasingly uniform as the number of coils increases, with the system converging towards a solenoidal configuration in the limit of $N = \infty$. This can be seen by visual inspection of the vector field as N increases (illustrated here with $N = 8$ and $N = 32$), as well as by numerical plot of the percentage difference of the calculated field on axis compared with the expected analytical value for a solenoid: $\mathbf{B} = \mu_0 \frac{N}{D} I$. As expected, the calculated value does not reach the theoretical value ($\sim 5\%$ less) because the system is not infinitely long and the difference becomes larger away from the centre of the system. The uniformity of the system can also be investigated via contour plot, where we see that for $N = 32$, there is a relatively large region ($\sim -3.5\text{m} \leq x \geq 3.5\text{m}$) where the deviation from the centre value is $\leq 6\%$ whereas for $N = 8$, the field deviates by up to 20% in an oscillatory manner with periodicity $\sim 1.5 textrmm$. Hence, qualitatively the behavior of the system is as expected.

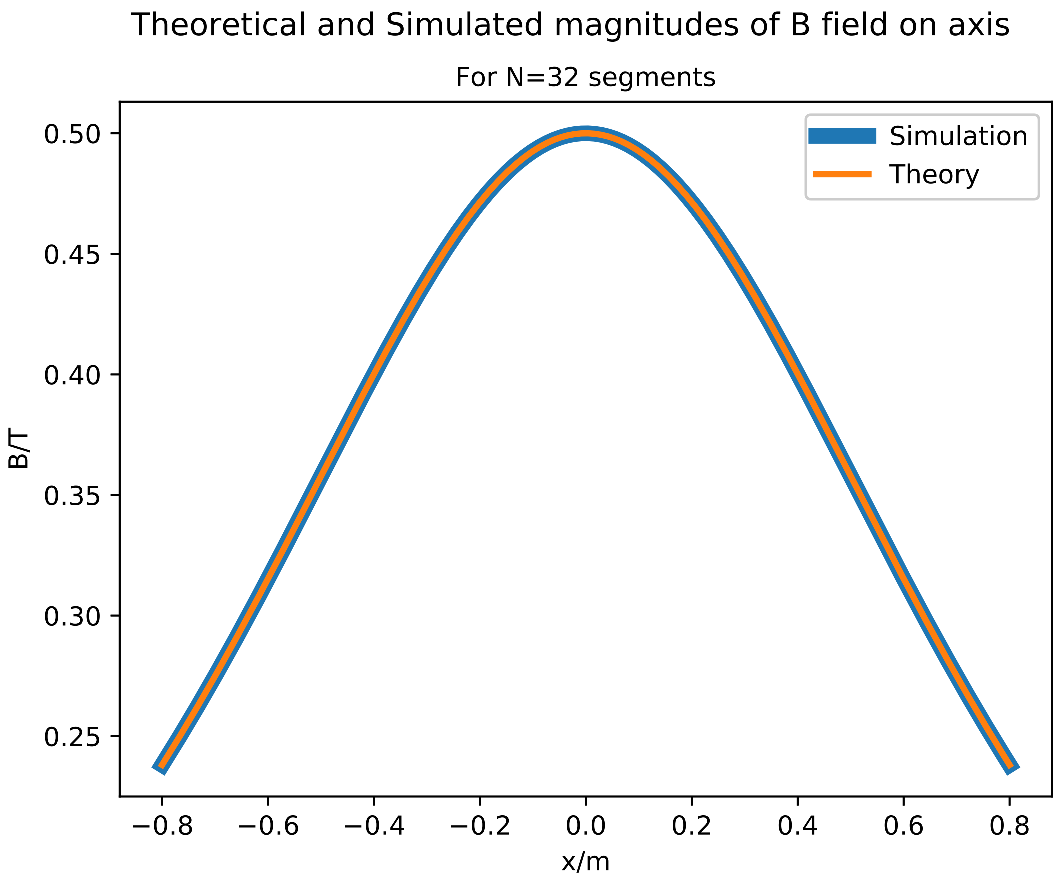

Figure 1: The magnitude of the analytical result for the magnetic field on axis for a single coil overlaid on top of the numerical solution using N=16 segments. There is an excellent agreement.

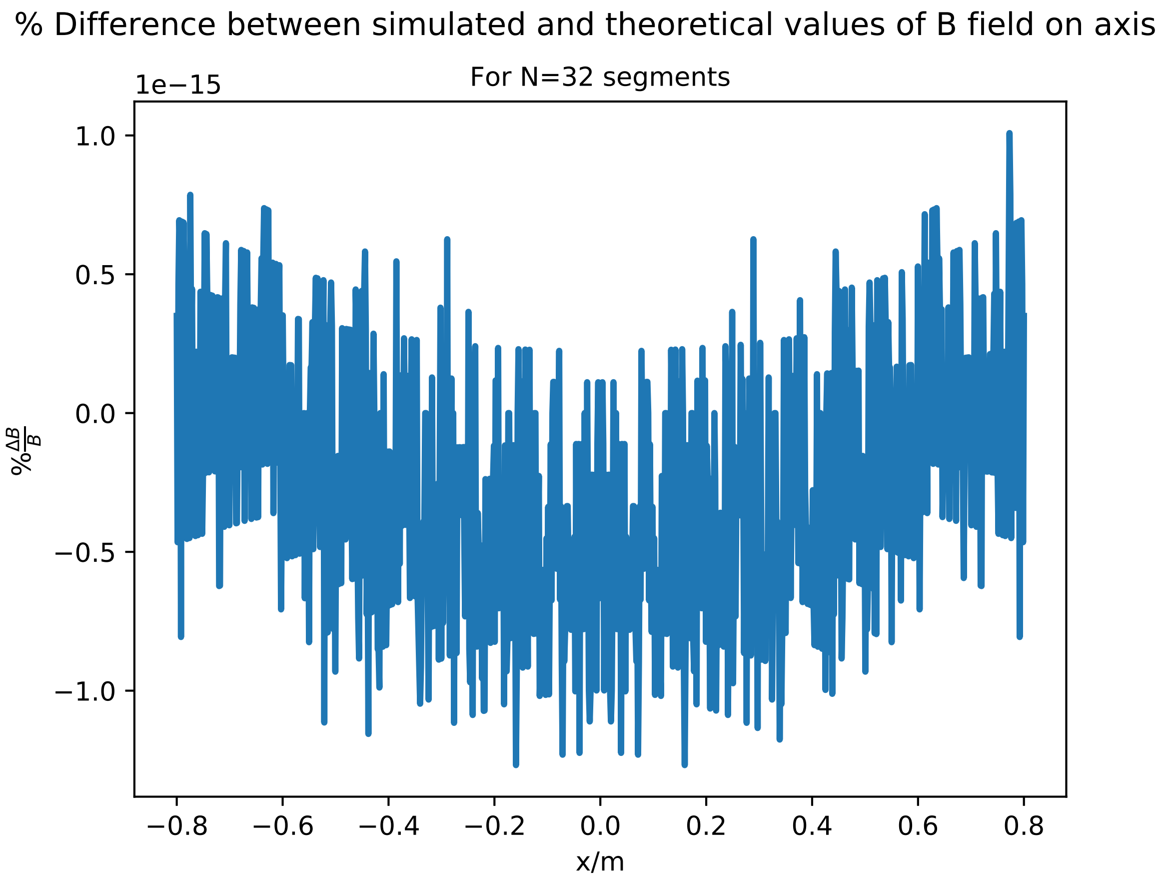

Figure 2: The percentage difference between the analytical and numerical results for the magnetic field on axis. The % difference is on the order of 10^{-16} and is effectively negligible.

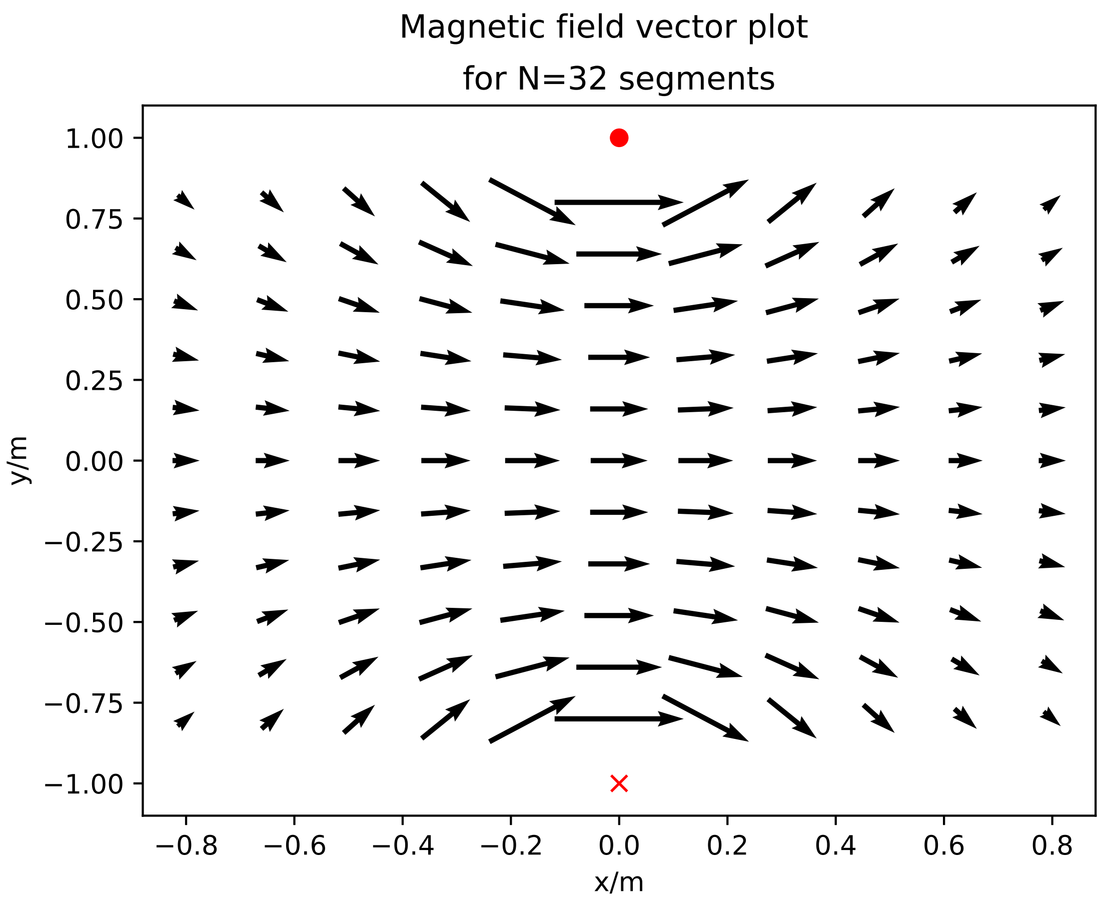

Figure 3: The magnitude and direction of the calculated magnetic field for a single coil is graphically plotted here with arrows. The position of the coil, with the associated current direction, is marked with a red circle and cross.

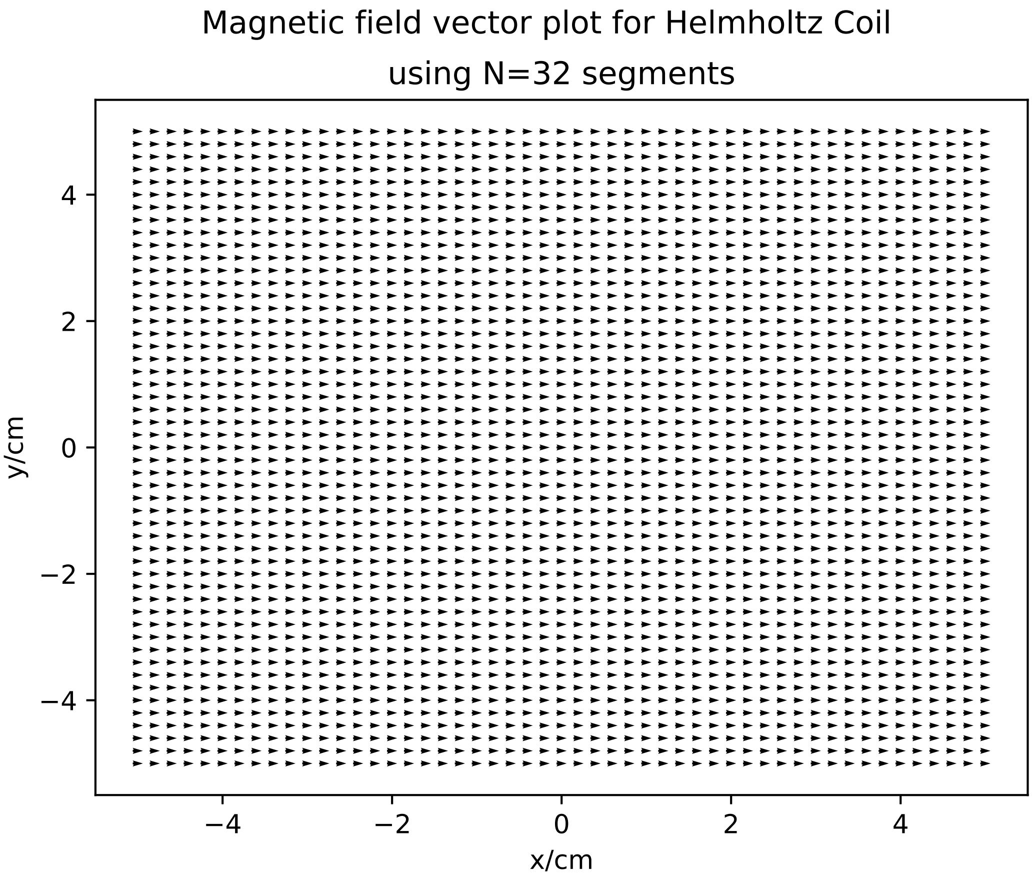

Figure 4: The magnitude and direction of the calculated magnetic field for a Helmholtz coil configuration is graphically plotted here with arrows in a small region around the centre. The position of the coils, with the associated current direction, are marked with a red circle and cross. The field is qualitatively seen to be extremely uniform.

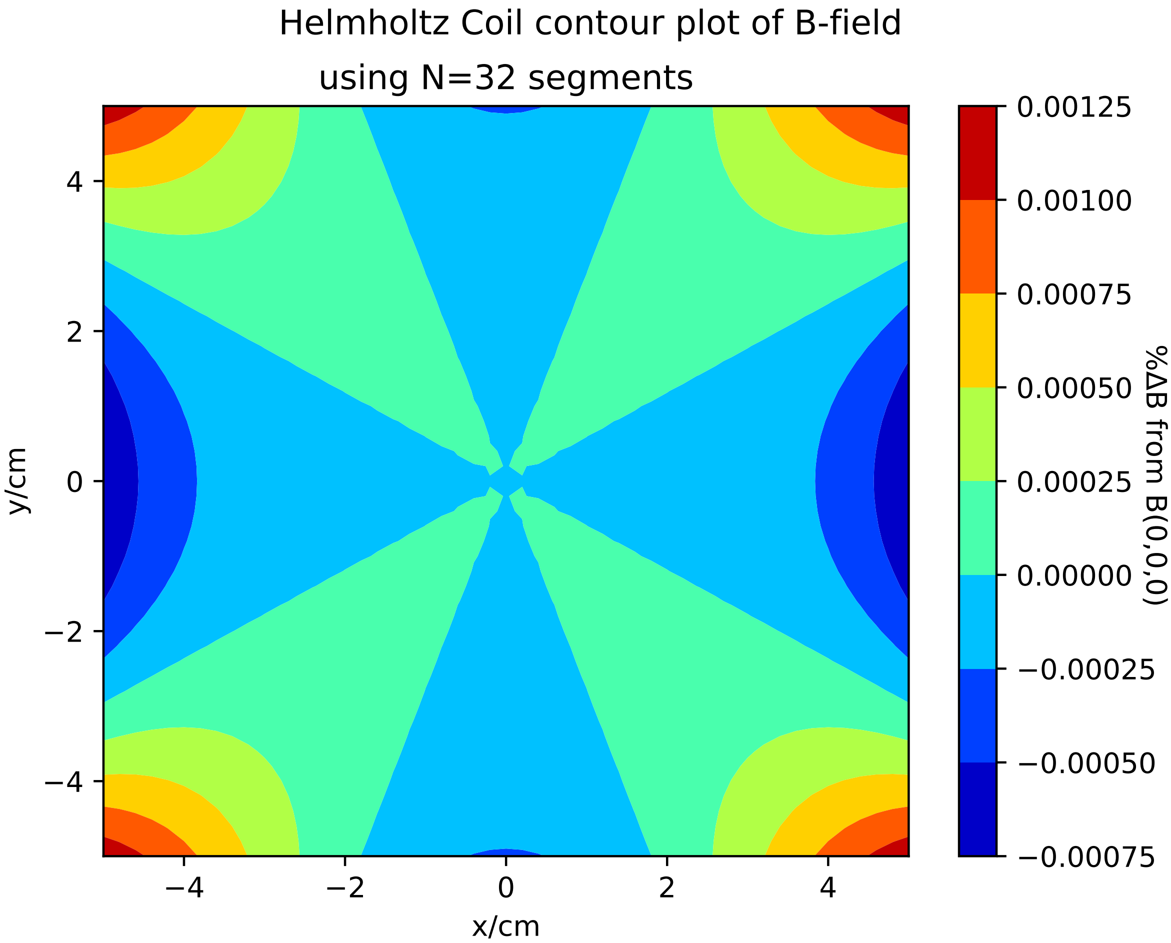

Figure 5: The percentage difference between the magnetic field strength and that at the centre is plotted with filled contours. The % deviations are of the order 10^{-4} and thus the system can be seen to be highly uniform.

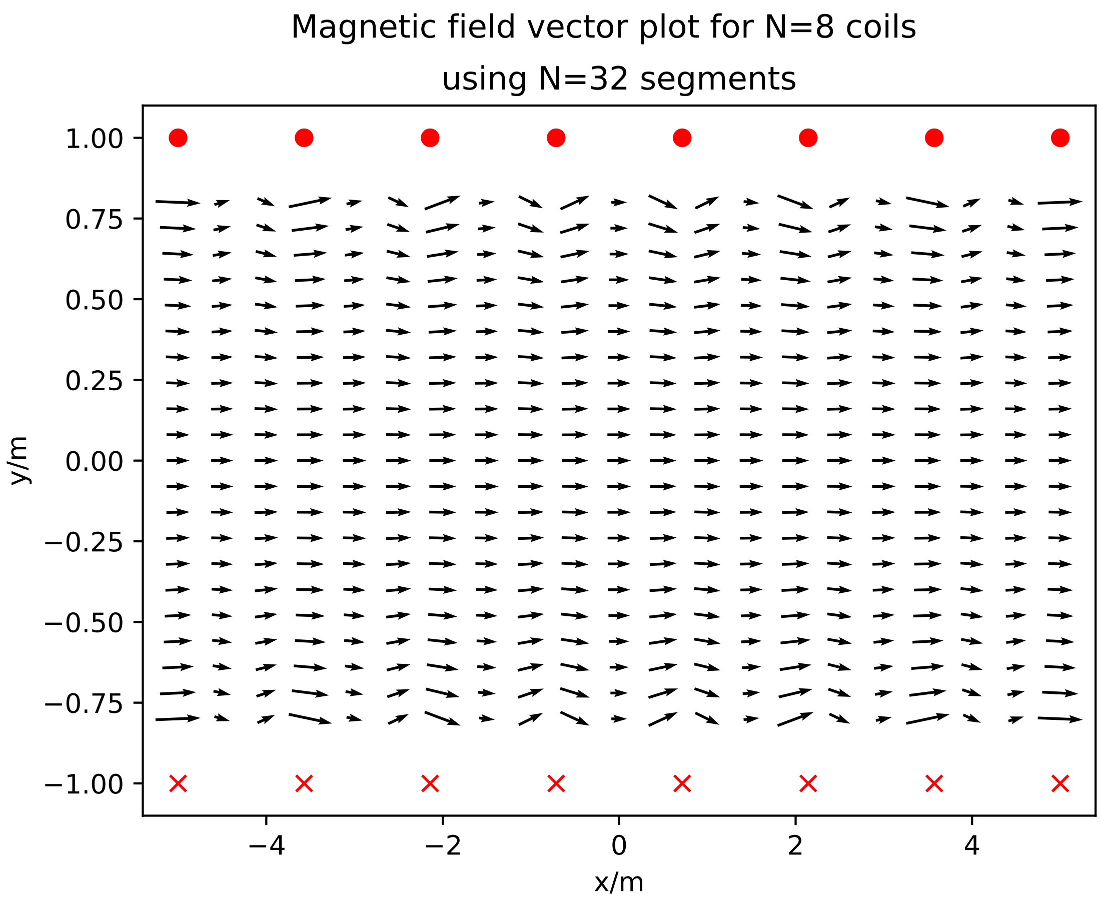

Figure 6: The magnitude and direction of the calculated magnetic field for N=8 coaxial coils graphically plotted here with arrows.The position of the coils, with the associated current direction, are marked with a red circle and cross.

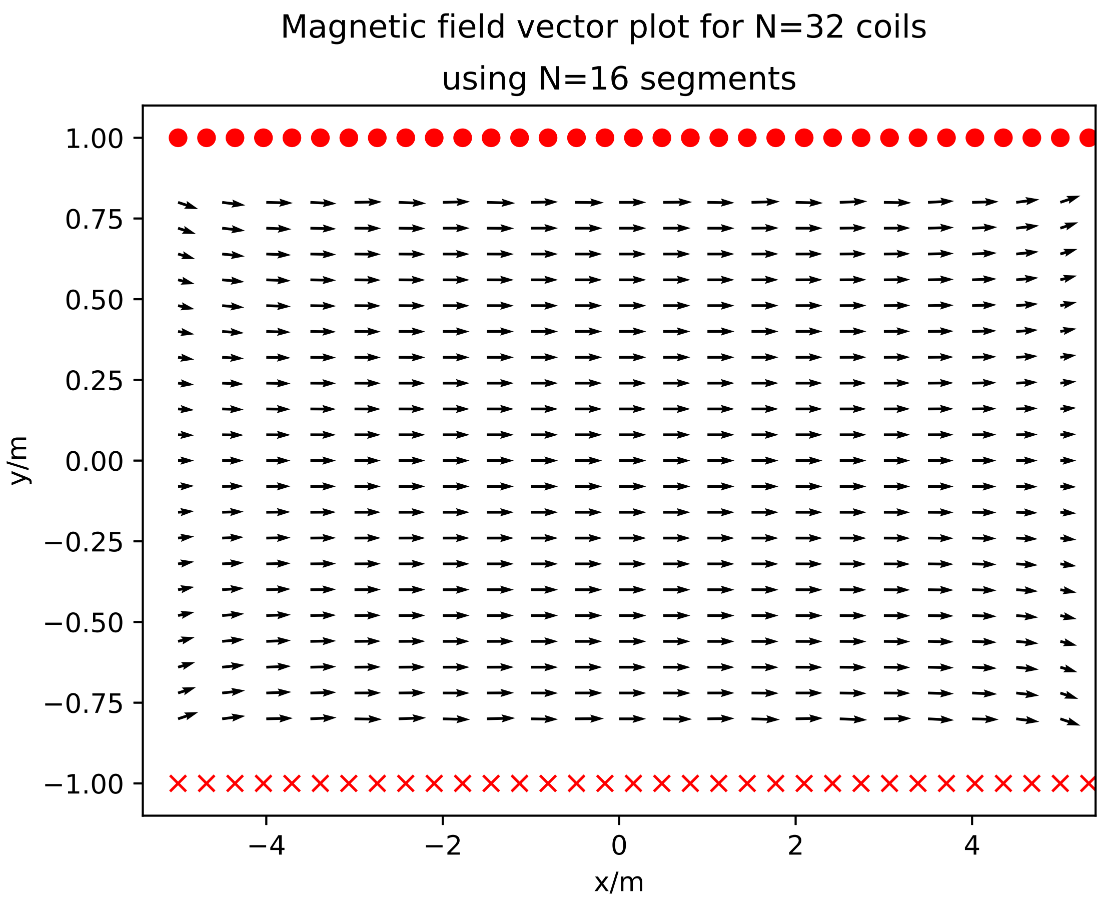

Figure 7: TThe magnitude and direction of the calculated magnetic field for N=32 coaxial coils graphically plotted here with arrows.The position of the coils, with the associated current direction, are marked with a red circle and cross.

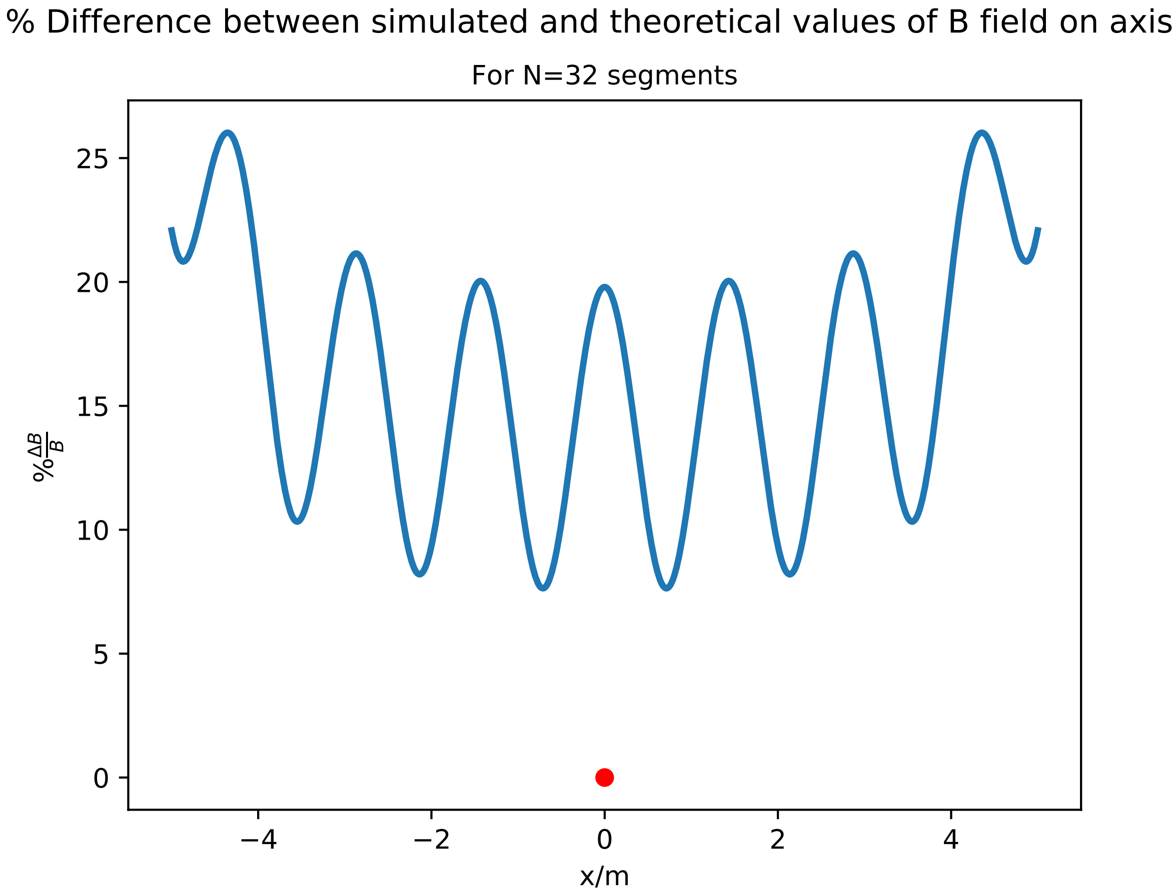

Figure 8: The percentage difference between the analytical magnetic field strength for a solenoid and the numerical result for N=8 coaxial coils is plotted as a function of distance away from the centre of the system. The deviations are highly non-uniform and tend to be $\geq 10\%$.

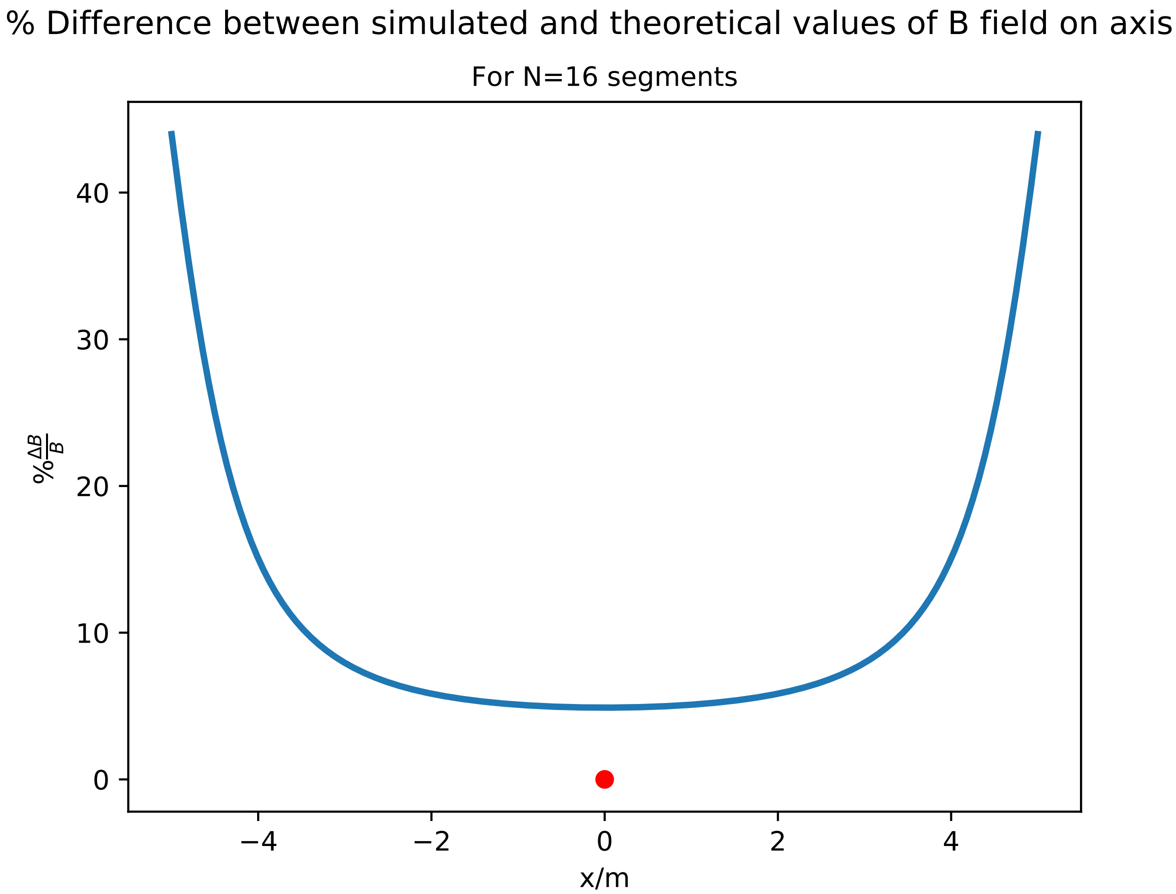

Figure 9: The percentage difference between the analytical magnetic field strength for a solenoid and the numerical result for N=32 coaxial coils is plotted as a function of distance away from the centre of the system. The deviations are highly much more uniform and tend to be $\leq 10\%$ for a relatively large region.

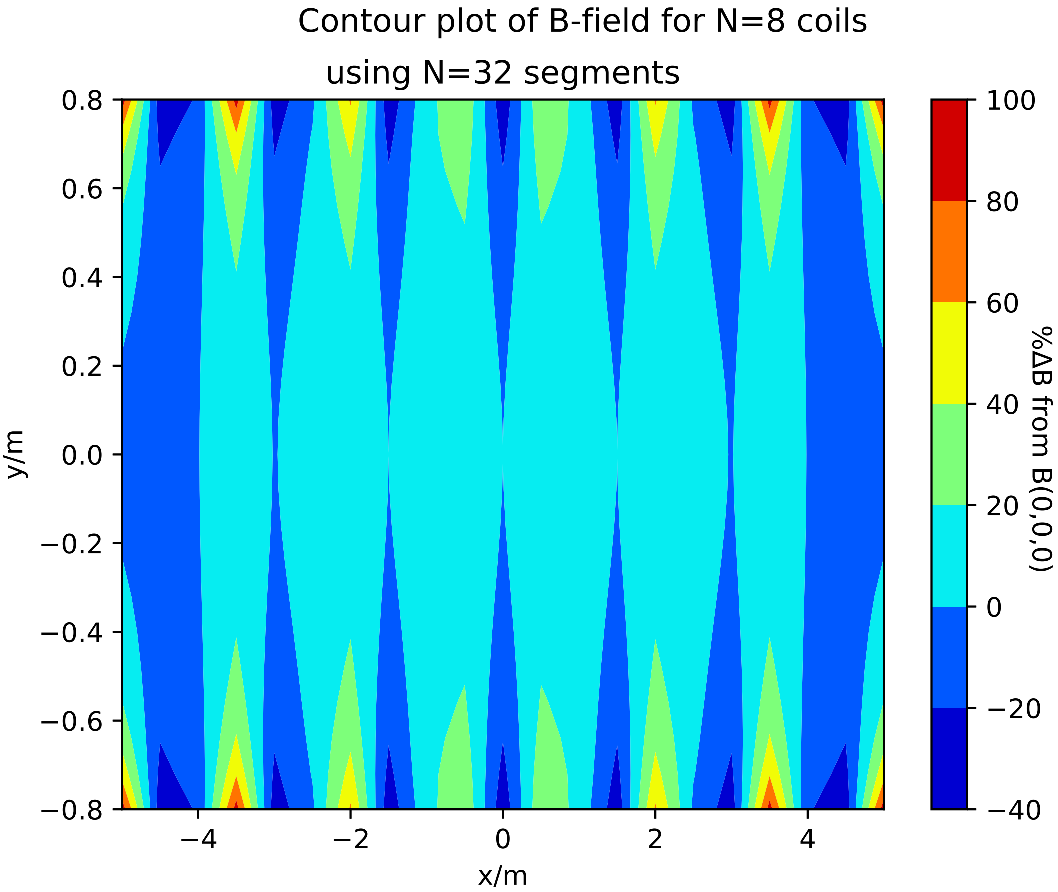

Figure 10: The percentage difference between the magnetic field strength and that at the centre is plotted with filled contours. The % deviations are often up to 20% with a periodicity of ~ 1.5m.

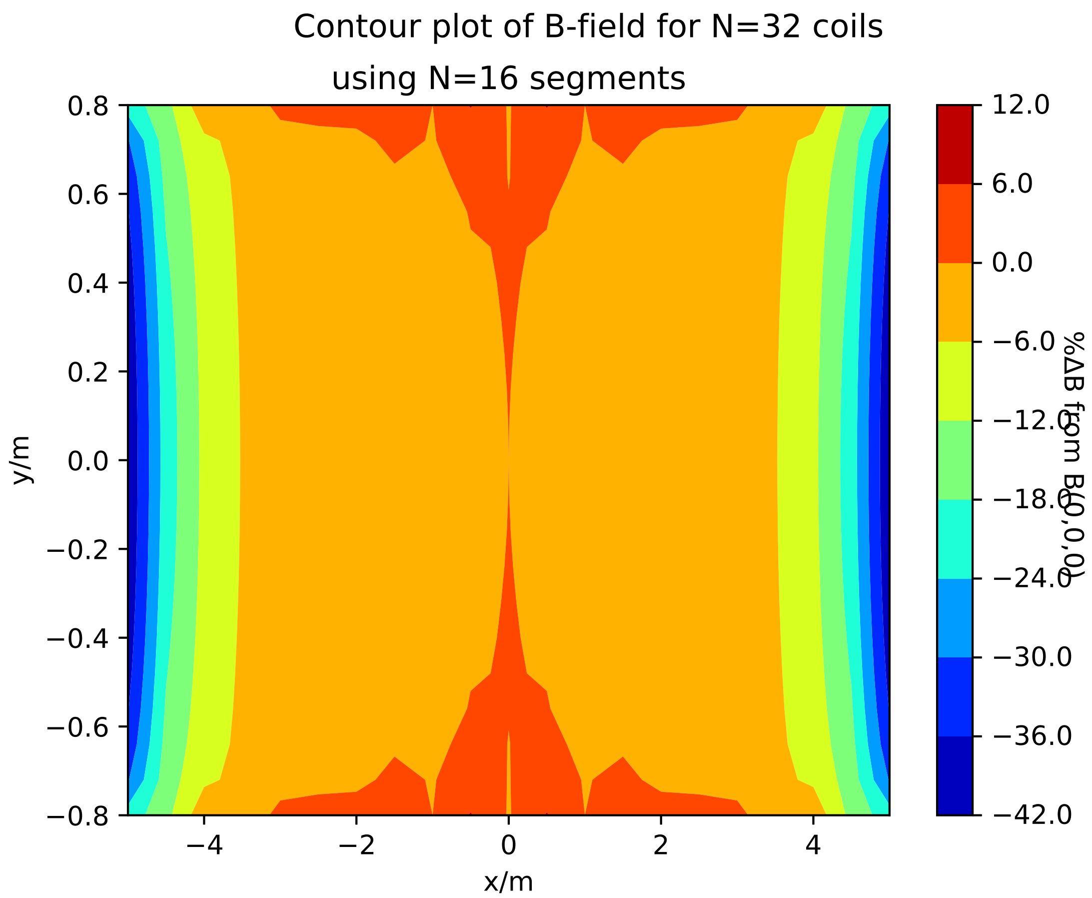

Figure 11: The percentage difference between the magnetic field strength and that at the centre is plotted with filled contours. The % deviations are of the order 6% for a significant region.