## Problem Statement

- Numerically evaluate an 8-dimensional integral using Monte-Carlo methods, and discuss the errors.
- Find the relative amplitude and phase due to Fresnel diffraction from a narrow slit via use of the Cornu Spiral.

## Core + Supplementary Task 1

Using the formulae described in the class manual, I wrote a Monte-Carlo integration procedure to evaluate the 8-dimensional integral given. Computationally, I broke up the task into several functions dealing with specific problems, such as `Monte_Carlo` for evaluating the value and theoretical error for a given run with $N$ points, and `Generate_Estimates` for looping the Monte-Carlo function call over the required number of points to generate the data of interest. In addition, I used `numpy` functions such as `np.sum` & `np.multiply` to vectorise my code and keep the running time of the simulation reasonable.

I chose a 25 points equally log-spaced starting from $N = 10,000$ to $N = 1,000,000$ so that there would be enough data points to clearly see the relationship between the error and $N$ at large values of $N$. For each value of $N$, 25 Monte-Carlo simulations were conducted with the mean value, theoretical error, calculated standard deviation of the mean, as well as the exact error using the mean value evaluated. For the highest value of $N = 1,000,000$, I found the (mean) value of the integral to be 537.172779426; for reasons given below, I have chosen to use the standard deviation of the mean as the error estimate, with a value of 0.007862485.

The graph of the various errors ($\sigma$ of the Monte-Carlo mean, the theoretical error from the class manual, and the exact error of the mean from the exact value 537.1873411) as well as a straight line fit is given below.

It is immediately seen that the theoretical error is a vast overestimate of the true error, while the standard deviation of the Monte-Carlo mean gives a better estimate of the true error. From the straight-line fit which has a gradient of -0.502552, we can also see that the errors fall as $N^{-\frac{1}{2}}$ at these large values of $N$.

## Core + Supplementary Task 2

Following the notes and instructions in the class manual, I wrote a routine (`Fresnel.py`) to plot the Cornu Spiral by numerically integrating the Fresnel integrals using the `scipy.quad` function, as well as writing a routine to calculate and plot the relative amplitude and phase of the diffraction pattern from a narrow slit of width $d = 10$cm with light of wavelength $\lambda = 1$cm by finding the length and phase of the phasor along the Cornu spiral, as per the theory of Fresnel diffraction. My results are plotted for 25cm either side of the origin for the distances $D = 30$cm,50cm,100cm from the screen below:

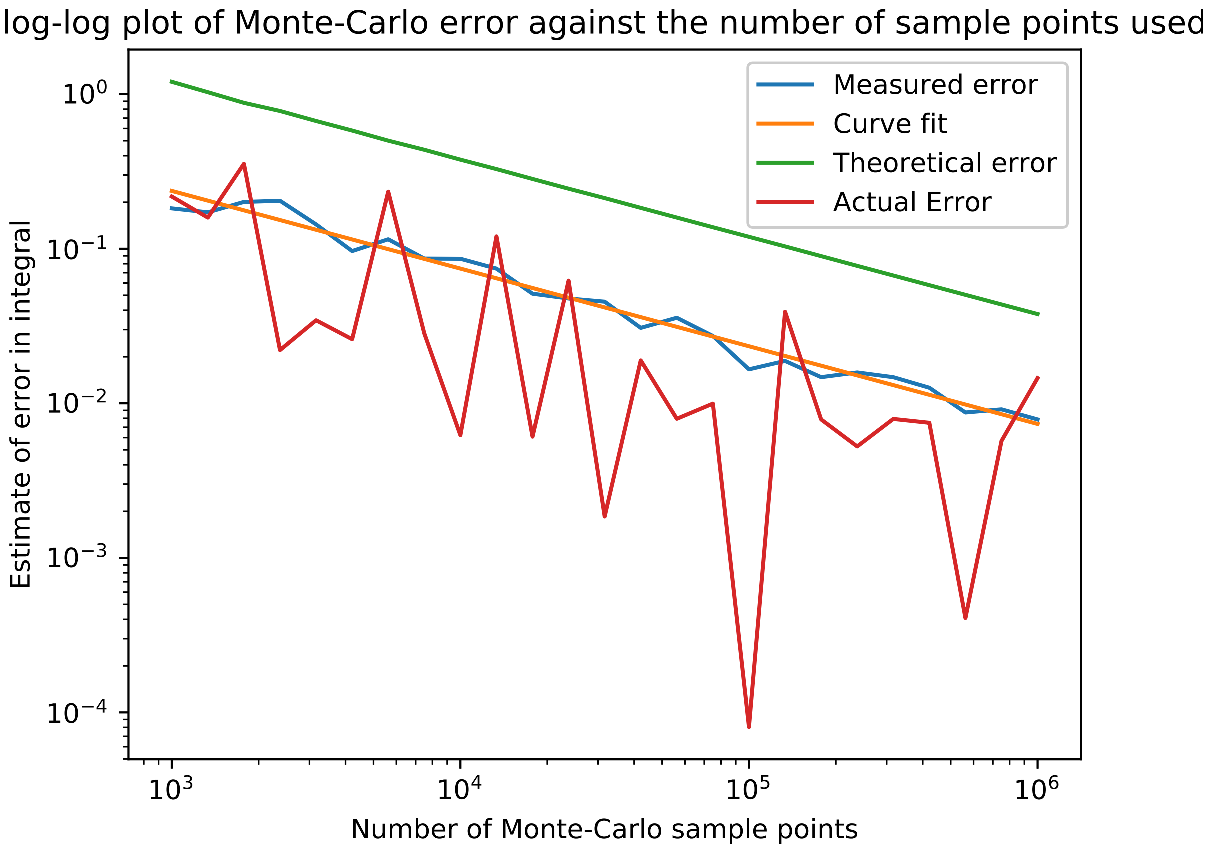


Figure 1: The standard deviation of the Monte-Carlo mean values, a straight line fit, the theoretical error estimate, and the actual error are plotted as a function of the number of points $N$.

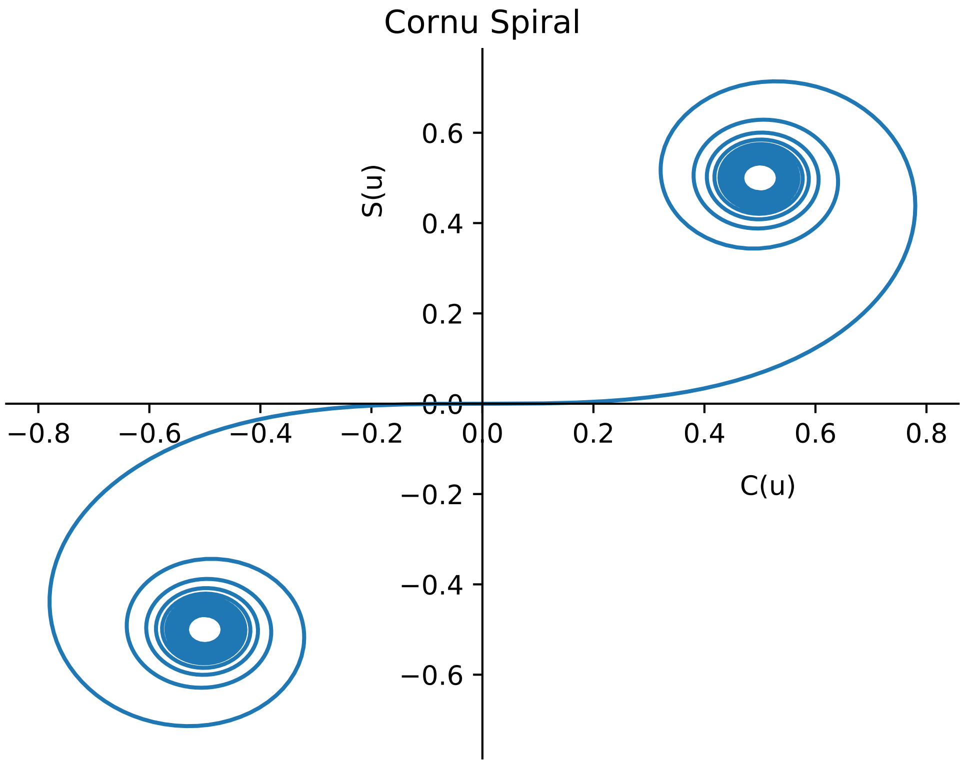


Figure 2: This is the Cornu spiral.

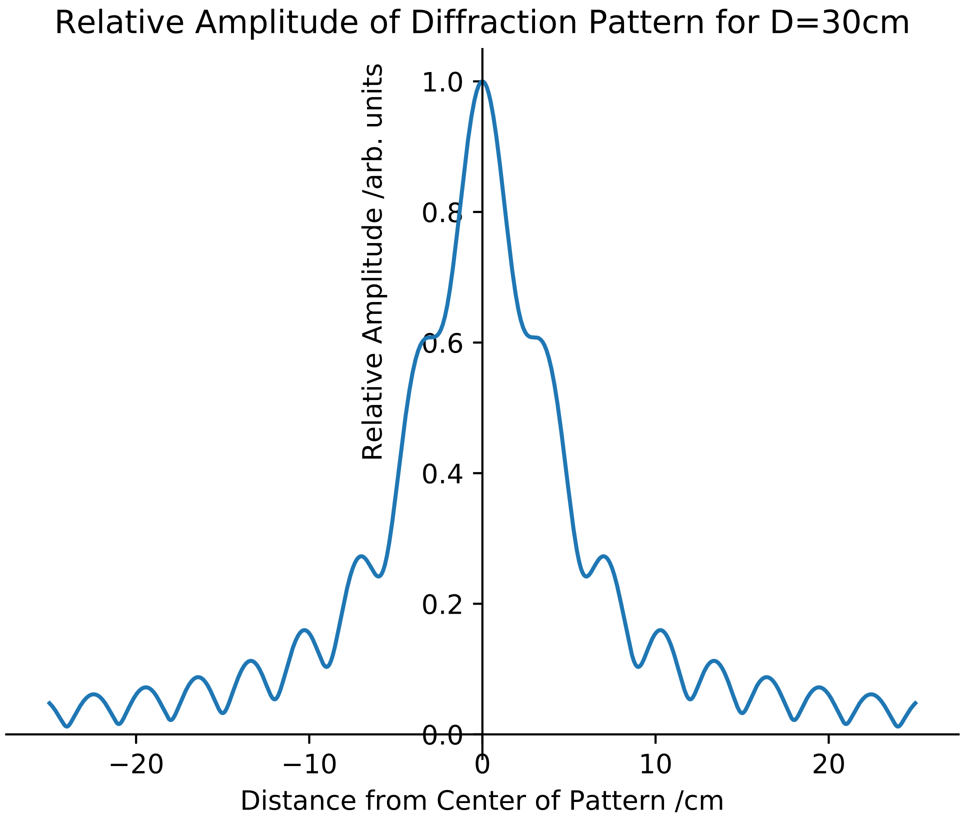


Figure 3: This is the relative diffraction amplitude for $D = 30\text{cm}$.

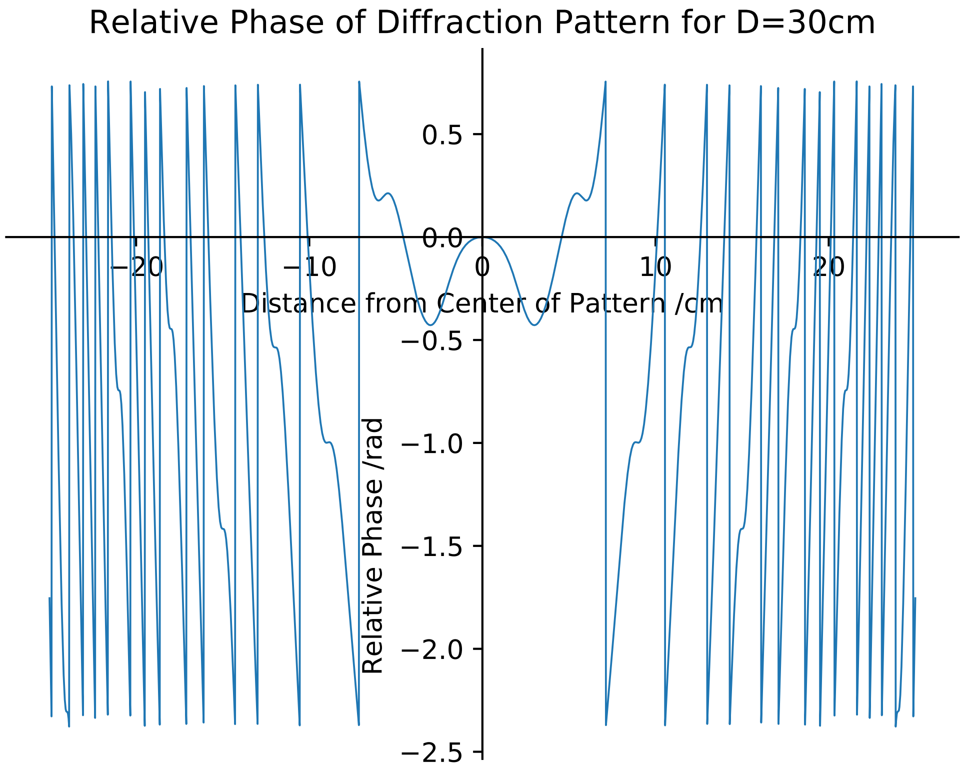


Figure 4: This is the relative diffraction phase for $D = 30\text{cm}$.

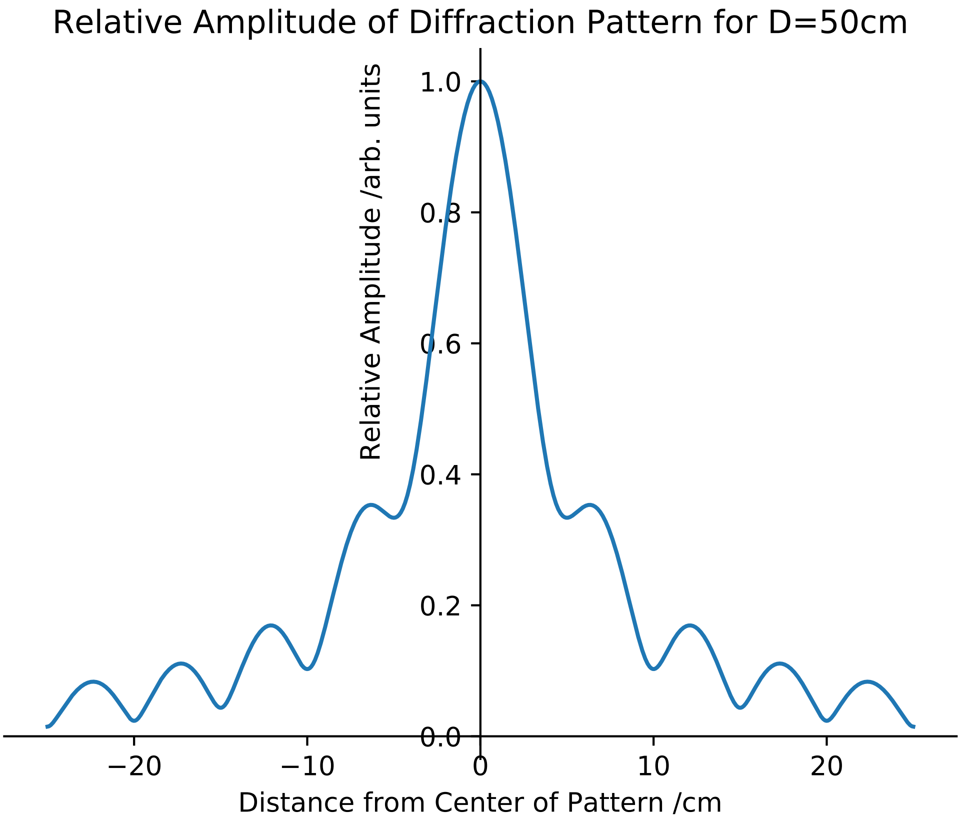


Figure 5: This is the relative diffraction amplitude for $D = 50\text{cm}$.

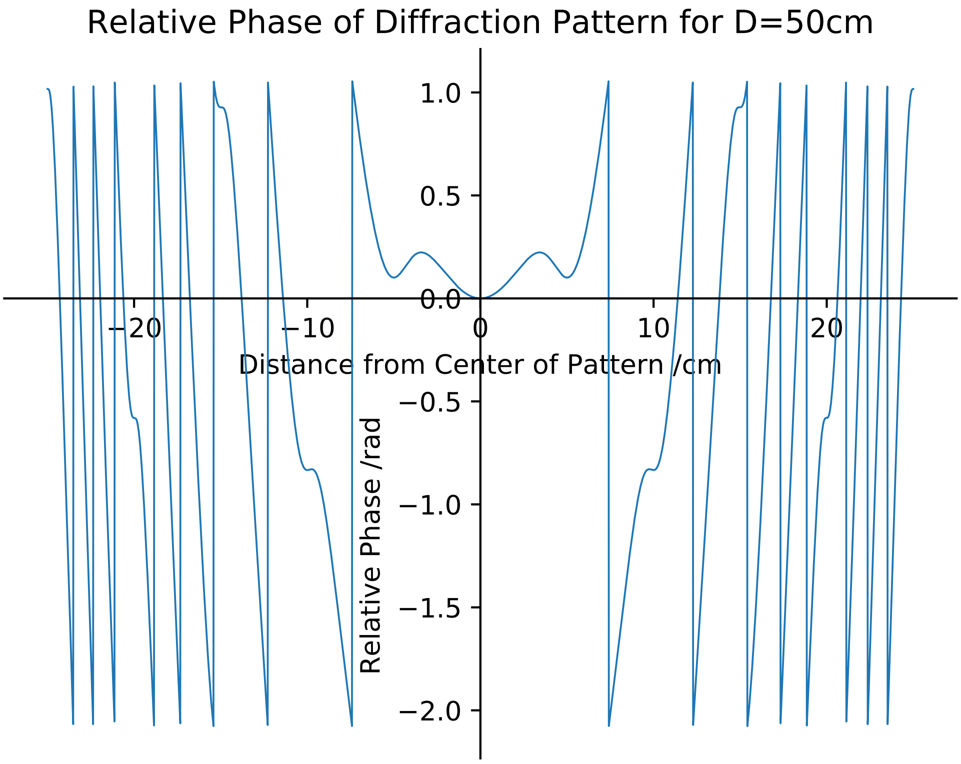


Figure 6: This is the relative diffraction phase for $D = 50\text{cm}$.

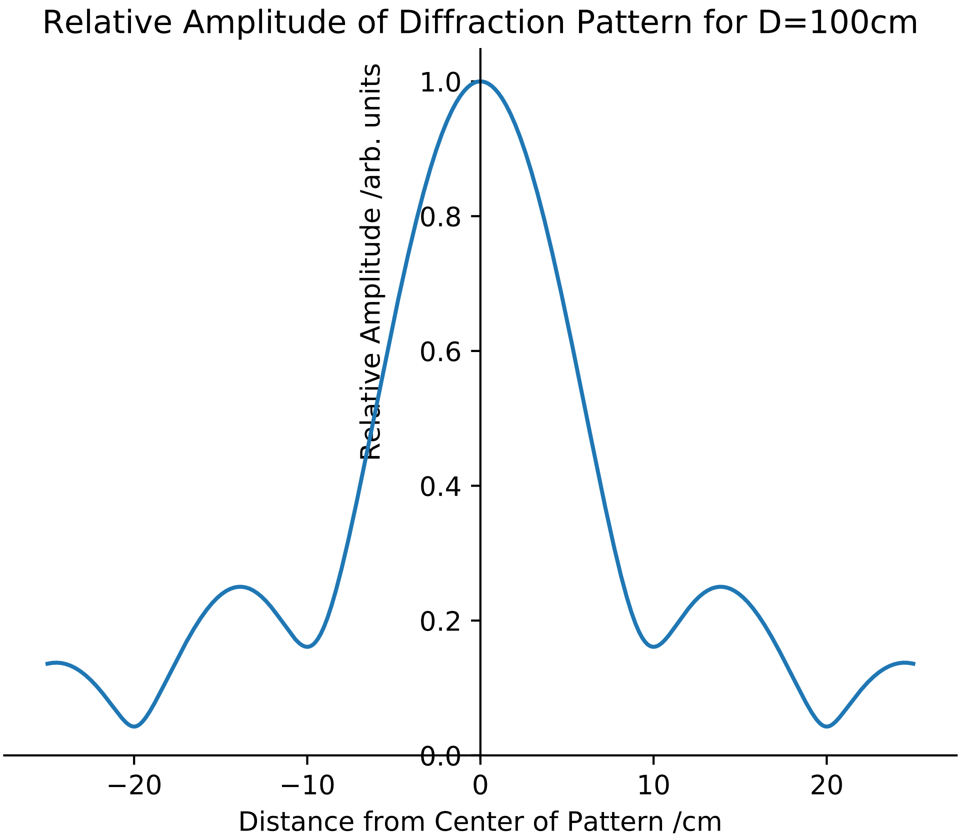


Figure 7: This is the relative diffraction amplitude for $D = 10\text{cm}$.

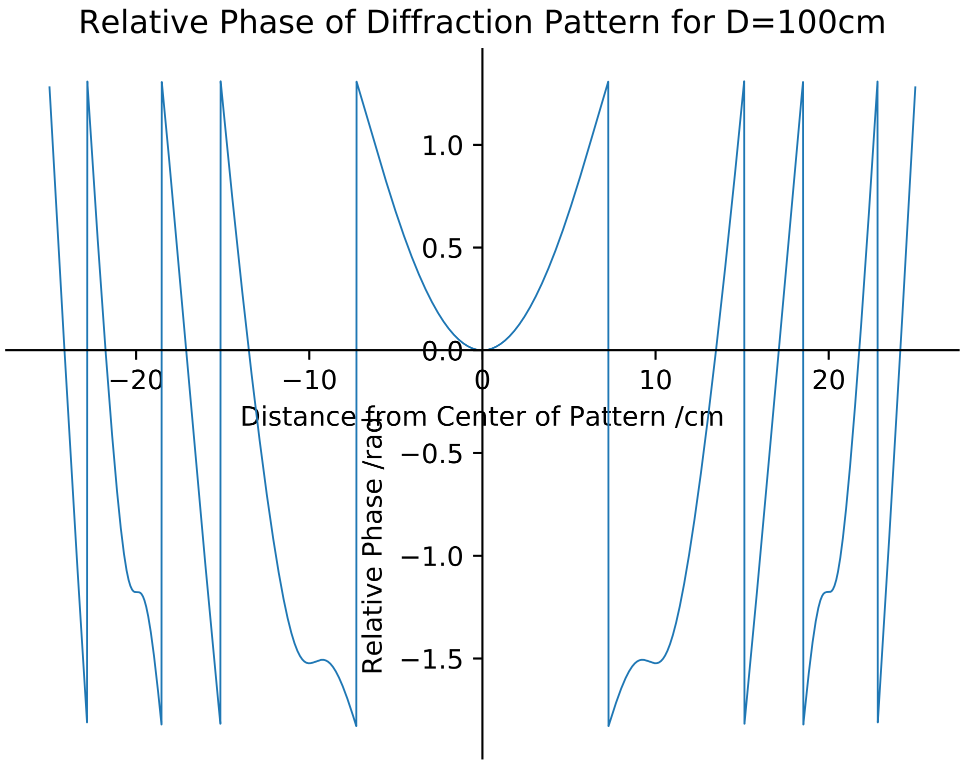


Figure 8: This is the relative diffraction phase for $D = 100\text{cm}$.