Exercise 3B

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Core Task 1 I chose to store the magnetic field vectors using a global variable field_list consisting of a 3D array of arrays. The 3D array corresponds to a set of uniformly spaced coordinates in 3-dimensions with chosen start and end points, which can be different for each dimension. The arrays contained within the 3D array would then represent the magnetic field vector at each set of coordinates. The magnetic field vectors are all initialised to 0.

The contribution to the magnetic fields from coils are added to field_list for every point as coils are added to the setup. Coils are added such that their centres lay on the x-axis, with a normal pointing along it. Each coil is divided into N line segments, with midpoints uniformly distributed around a circle. The length of each line element was determined such that the total length of all the line elements is equal to the circumference of the coil. This gives a measured field on the x-axis that has a tiny error when compared to the theoretical field (refer to handout for derivation), as illustrated in Figure 1. The resultant error of order 10^{-16} is probably due to floating point error.

I was able to obtain good results for $N > 1$, but that is because of the symmetry when looking on the axis; we would definitely need a higher N for accurate results off-axis, and I used $N = 32$ for all plots in this writeup, otherwise stated. With this choice, the quiver plot in Figure 2 shows that the measured field also resembles the theoretical predictions even off-axis.

There was another logical choice I could have made when deciding the length of the line element. This choice would be one where we deform a circle into a N-gon, but as a result not keep the perimeter of the shape constant. Moreover, the distance from the centre of the N-gon to midpoints on each edge should be kept equal to the radius of the coil. Such a choice might arise from the motivation to conserve current, but this choice produces the wrong result in that it calculates a field (along the x-axis) slightly higher than that expected from theory. We can make sense of this by noting that the perimeter of our constructed N-gon is higher than the circumference of our the initial loop.

Core Task 2 The magnetic field due to a pair of Helmholtz coils was studied by extending the method used in Core Task 1.

Figure 3(a) depicts the magnetic field caused by two coils. The field around each coil resembles that for the single coil, except that there is now a large area of uniformity in the region in between the coils (i.e. in a cylinder centered at the origin with radius 50cm and height 1m). The field in this region points towards the right of the figure as a result of symmetry; contributions to the field in the vertical direction from either coil cancel each other out. Moreover, the magnetic field in the region between the coils near the origin was observed to be uniform, as shown in Figure 3(b).

The uniformity of the field near the origin was then further analysed by plotting a contour plot showing the percentage difference in the field near the origin compared to the value on the origin. Figure 4 shows that the field in a cylinder of radius 10cm centred on the origin showed only a small maximum percentage deviation of $12.5 \times 10^{-4}\%$.

The field was then compared to that calculated by theory (refer to handout for exact expression). Likewise, the maximum percentage deviation in the field in the cylinder to that expected by theory was $12.5 \times 10^{-4}\%$, as depicted in Figure 5. This was because the field measure at the origin was only different from the theoretical value by a negligible value of the order of 10^{-16}, thus giving us effectively the same contour plot we saw in Figure 4.

I also experimented to see if I can reproduce such uniformity at low N. Similar levels of uniformity was achieved for $N \geq 4$, which can be seen in Figure 5. However, the field pattern measured was different to that for $N = 32$, despite having similar symmetry planes. Interestingly, the field pattern only started to resemble the pattern measured for $N = 32$ when $N \geq 5$. Hence, we conclude that we can still use reasonably low N to calculate the field very near the origin, but as mentioned earlier, we should expect this to break down as we move further away from it.

Supplementary Task I also studied the effects of N coaxial coils with uniform spacing carrying the same current I placed within a distance L. In the limit of large N, the setup can be approximated to be a solenoid. The measured field resembles that expected at the centre of a solenoid. In Figure 6, we see that the field at the centre of the coils was both uniform and pointed along the x-axis for $N = 20$. Moreover, when more coils were added, I observed that the field did not drop off as quickly when we move closer to the coils. Despite these observations, we note that the field at the origin differed to that calculated using the solenoid formula

$$B = \mu_0 \frac{N}{L} I$$

by ~3%. This shows that the solenoid approixmation is only roughly valid, since the coils do not stretch on to infinity. In light of that, we can deduce that the solenoid approximation is reasonably valid even for coils of finite length.

From Figure 7, we see that true uniformity is only achieved when N is high enough, for a given L. The field is uniform when $N = 20$ within a cylinder centred on the origin of radius 50cm and height 4m. In comparison, when $N = 5$, the field forms vertical "layers", in which the field oscillates in magnitude along the x-direction. In the same cylinder, the field magnitude deviates up to 60% relative to the field value at the origin.

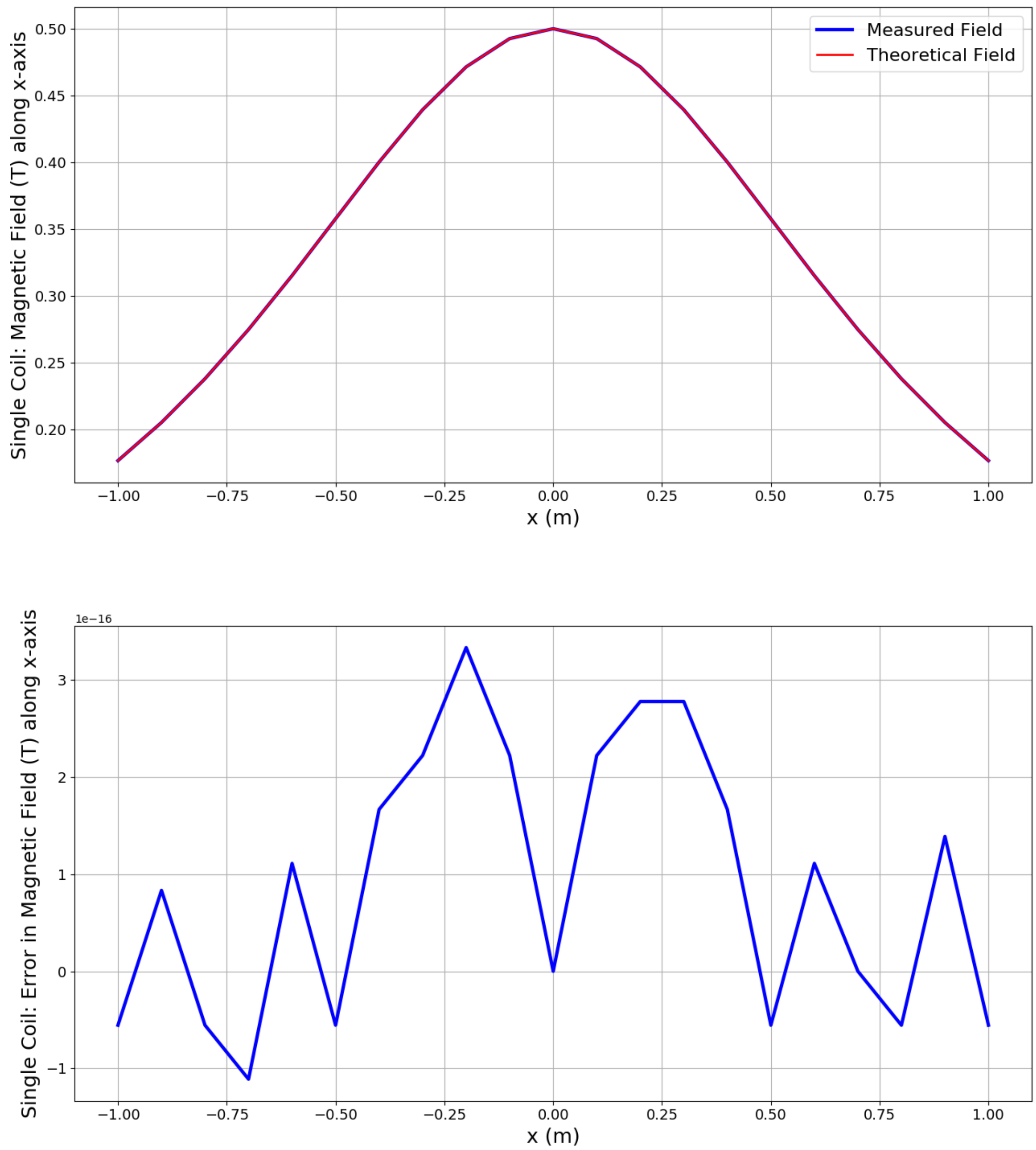


Figure 1: The magnetic field along the x-axis for a single coil was studied. (Above) A plot of theoretical and measured fields along the x-axis for a single coil. (Below) A plot of the difference between the measured and theoretical fields.

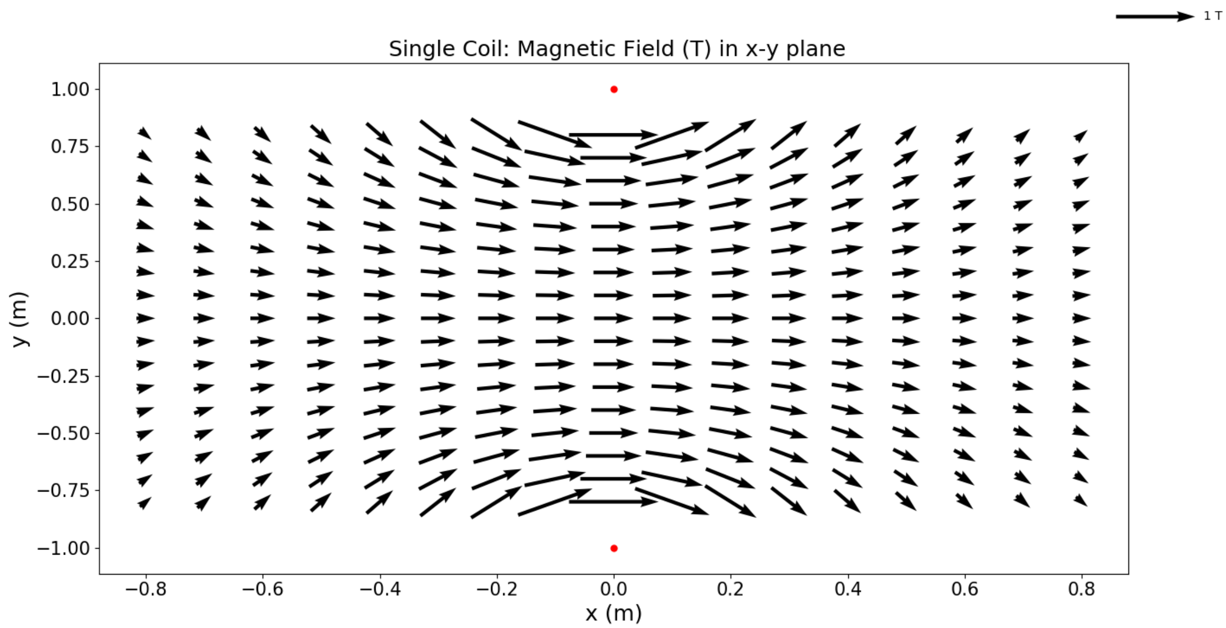


Figure 2: An arrow plot of magnetic field vectors caused by a single coil on the plane $z = 0$.

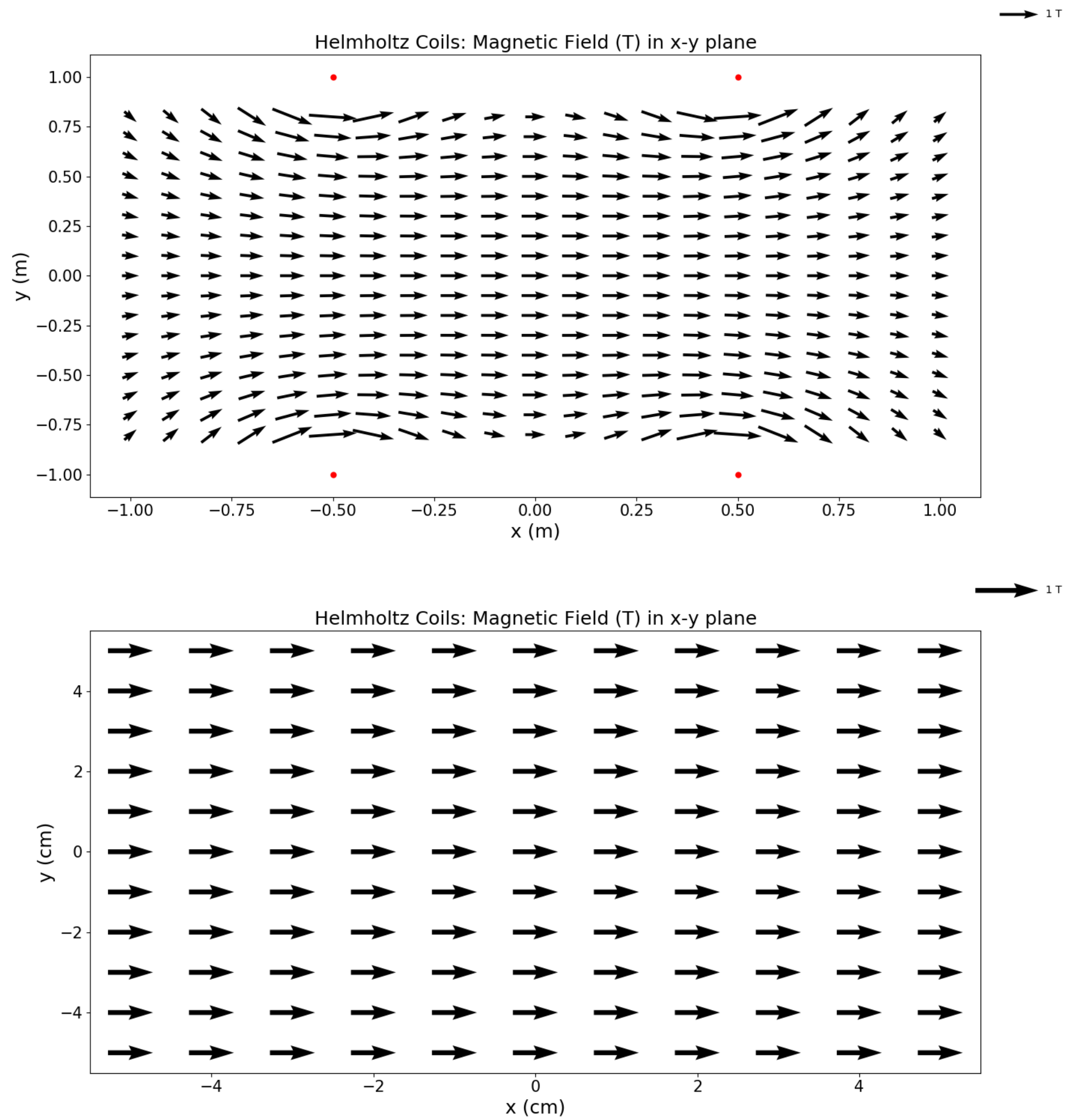


Figure 3: (a)-(b): Arrow plots of magnetic field vectors caused by a Helmholtz pair on the plane $z = 0$ in the region between (a) the coils (above) and (b) near the origin (below).

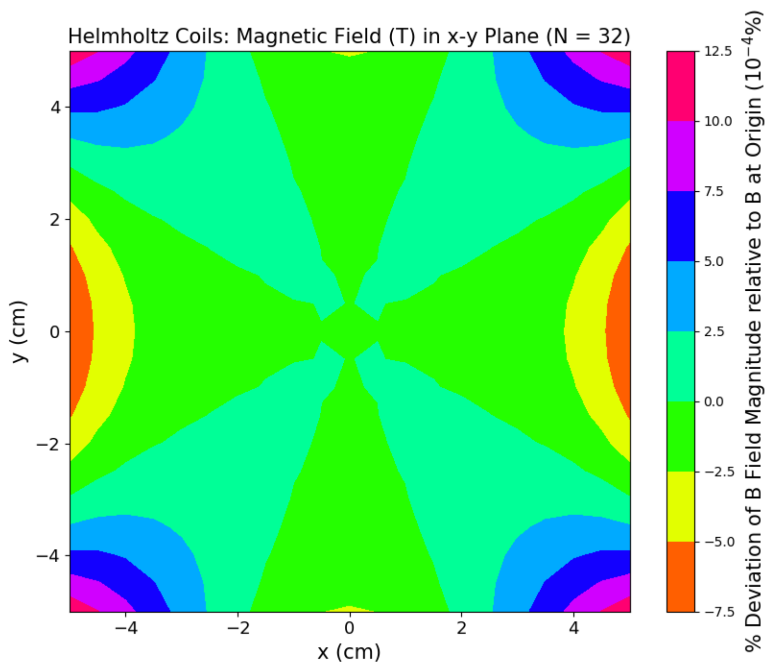


Figure 4: A contour plot showing the percentage deviation of the field magnitude relative to that at the origin, in the region near the origin.

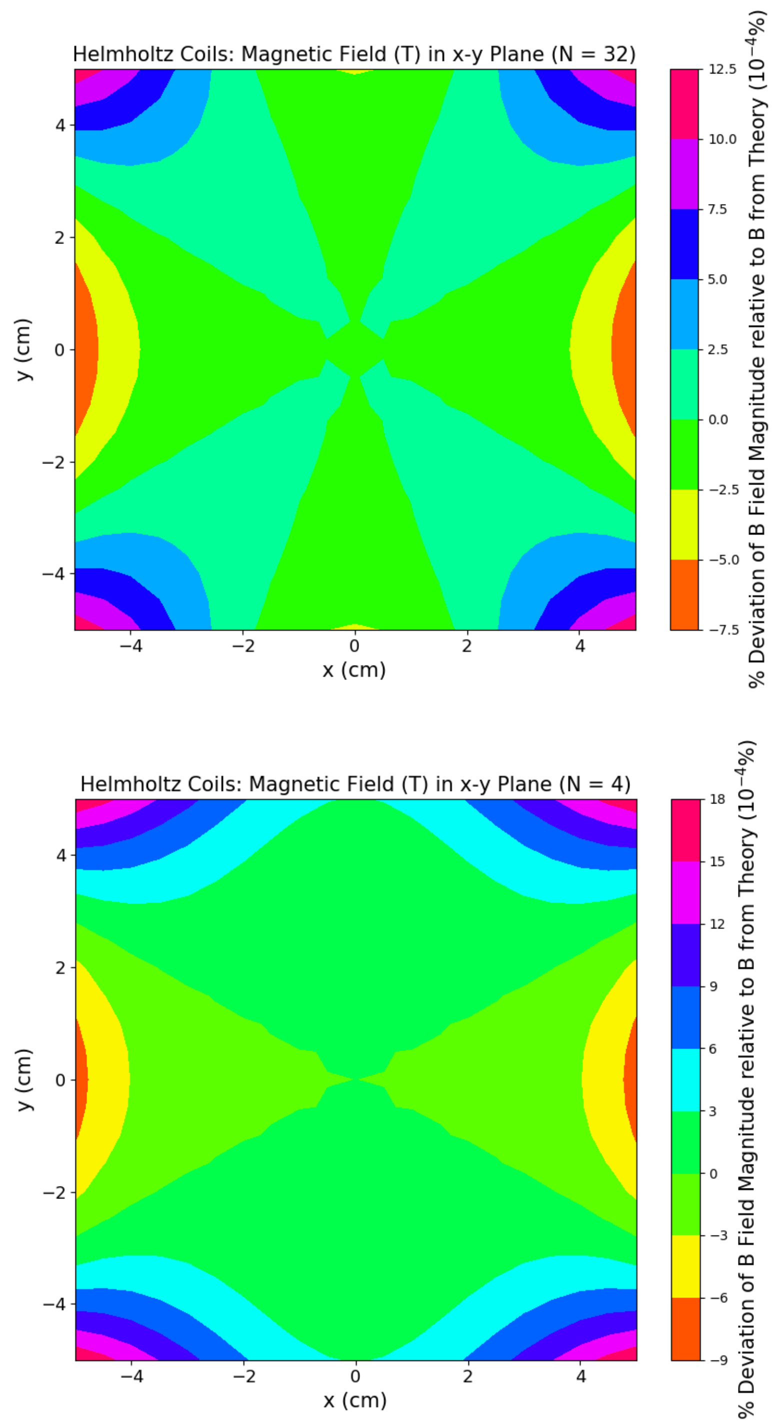


Figure 5: Contour plots showing the percentage deviation of the field magnitude relative to that calculated using theory, in the region near the origin, for $N = 32$ (above) and $N = 4$ (below).

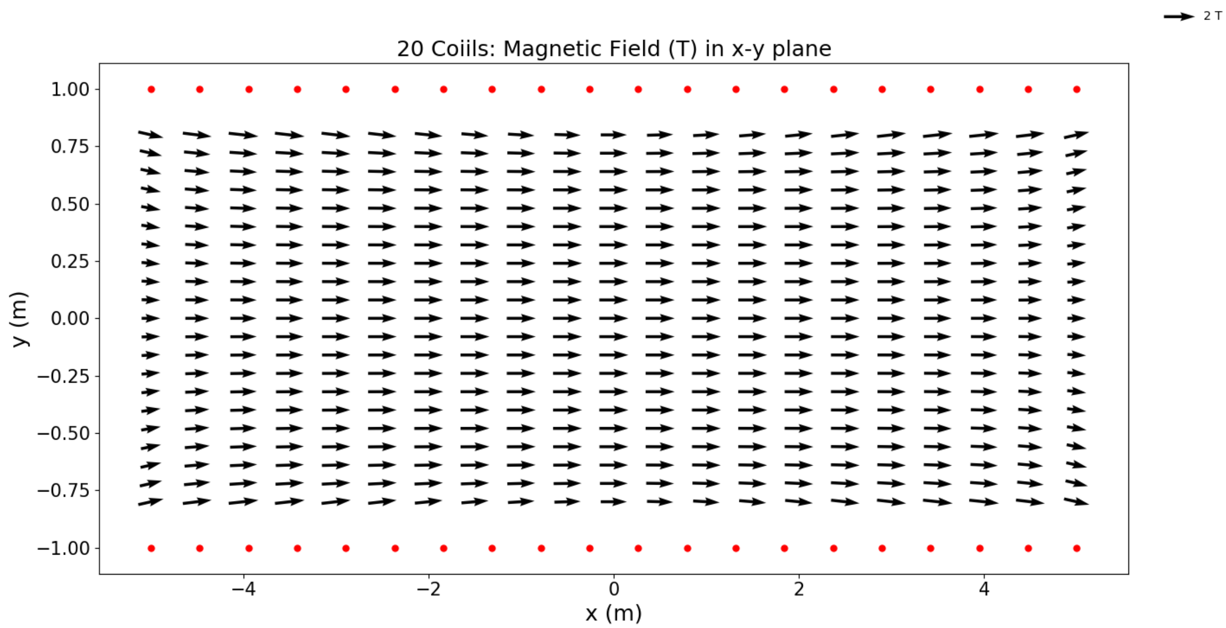
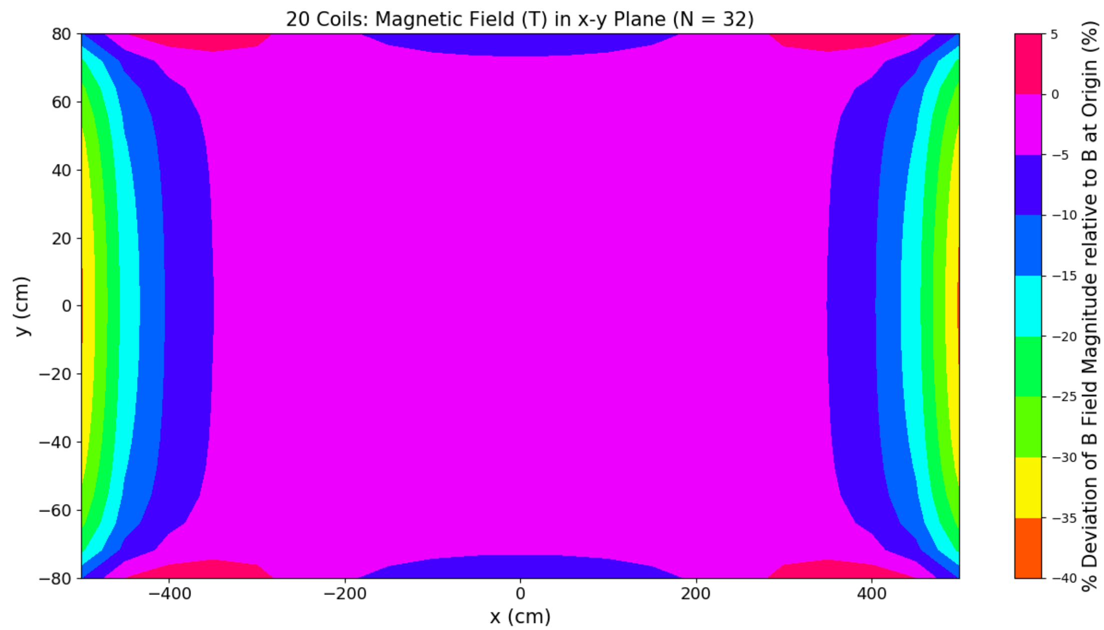


Figure 6: An arrow plot of magnetic field vectors caused by 20 coils on the plane $z = 0$.



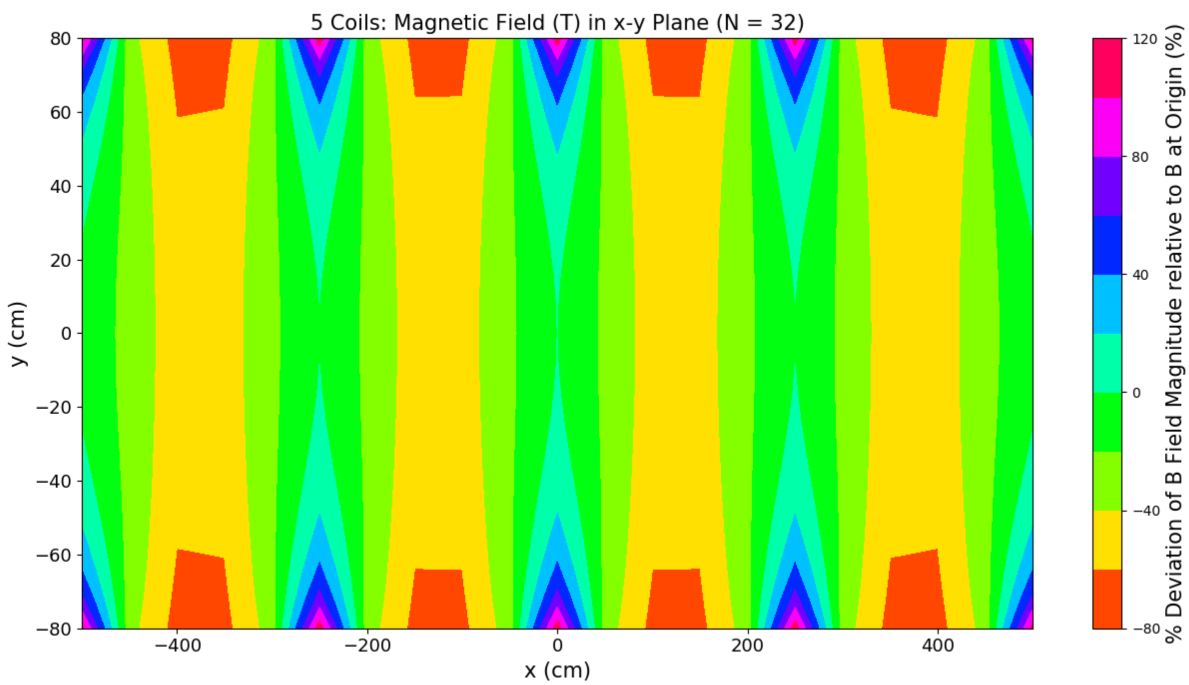


Figure 7: Contour plots showing the percentage deviation of the field magnitude relative to that at the origin, in the region near the origin, when the setup had 20 (above) and 5 (below) coils respectively.