Chapter 10

PRICE DISCRIMINATION

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Contents

1. Introduction

Price discrimination is one of the most prevalent forms of marketing practices. One may occasionally doubt whether firms really engage in some of the kinds of sophisticated strategic reasoning economists are fond of examining, but there can be no doubt that firms are well aware of the benefits of price discrimination.

Consider, for example, the following passage taken from a brochure published by the Boston Consulting Group:

A key step is to avoid average pricing. Pricing to specific customer groups should reflect the true competitive value of what is being provided. When this is achieved, no money is left on the table unnecessarily on the one hand, while no opportunities are opened for competitors though inadvertent overpricing on the other. Pricing is an accurate and confident action that takes full advantage of the combination of customers' price sensitivity and alternative suppliers they have or could have [Miles (1986)].

Although an economist might have used somewhat more technical terminology, the central ideas of price discrimination are quite apparent in this passage.

Every undergraduate microeconomics textbook contains a list of examples of price discrimination; the most popular illustrations seem to be those of student discounts, Senior Citizen's discounts, and the like. Given the prevalence of price discrimination as an economic phenomenon, it is surprisingly difficult to come up with an entirely satisfactory definition.

The conventional definition is that price discrimination is present when the same commodity is sold at different prices to different consumers. However, this definition fails on two counts: different prices charged to different consumers could simply reflect transportation costs, or similar costs of selling the good; and price discrimination could be present even when all consumers are charged the same price – consider the case of a uniform delivered price.[1]

We prefer Stigler's (1987) definition: price discrimination is present when two or more similar goods are sold at prices that are in different ratios to marginal costs. As an illustration, Stigler uses the example of a book that sells in hard cover for $15 and in paperback for $5. Here, he argues, there is a presumption of

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[1]For further discussion, see Phlips (1983, pp. 5–7).

discrimination, since the binding costs are not sufficient to explain the difference in price. Of course, this definition still leaves open the precise meaning of "similar", but the definition will be useful for our purposes.

Three conditions are necessary in order for price discrimination to be a viable solution to a firm's pricing problem. First, the firm must have some market power. Second, the firm must have the ability to sort customers. And third, the firm must be able to prevent resale. We will briefly discuss each of these points, and develop them in much greater detail in the course of the chapter.

We turn first to the issue of market power. Price discrimination arises naturally in the theory of monopoly and oligopoly. Whenever a good is sold at a price in excess of its marginal cost, there is an incentive to engage in price discrimination. For to say that price is in excess of marginal cost is to say that there is someone who is willing to pay more than the cost of production for an extra unit of the good. Lowering the price to all consumers may well be unprofitable, but lowering the price to the marginal consumer alone will likely be profitable.

In order to lower the price only to the marginal consumer, or more generally to some specific class of consumers, the firm must have a way to sort consumers. The easiest case is where the firm can explicitly sort consumers with respect to some exogenous category such as age. A more complex analysis is necessary when the firm must price discriminate on the basis of some *endogenous* category such as time of purchase. In this case the monopolist faces the problem of structuring his pricing so that consumers "self-select" into appropriate categories.

Finally, if the firm is to sell at different prices to different consumers, the firm must have a way to prevent consumers who purchase at a discount price from reselling to other consumers. Carlton and Perloff (forthcoming) discuss several mechanisms that can be used to prevent resale:

- Some goods such as services, electric power, etc. are difficult to resell because of the nature of the good.
- Tariffs, taxes and transportation costs can impose barriers to resale. For example, it is common for publishers to sell books at different prices in different countries and rely on transportation costs or tariffs to restrict resale.
- A firm may legally restrict resale. For example, computer manufacturers often offer educational discounts along with a contractual provision that restricts resale.
- A firm can modify its product. For example, some firms sell student editions of software that has more limited capabilities than the standard versions.

The economic analyst is, of course, interested in having a detailed and accurate model of the firm's behavior. But in addition, the economist wants to be able to pass judgment on that behavior. To what degree does price discrimination of various types promote economic welfare? What types of discrimination should be encouraged and what types discouraged? Price discrimination is illegal only

insofar as it "substantially lessens competition". How are we to interpret this phrase? These are some of the issues we will examine in this survey.

2. Theory

2.1. Types of price discrimination

The traditional classification of the forms of price discrimination is due to Pigou (1920).

First-degree, or *perfect* price discrimination involves the seller charging a different price for each unit of the good in such a way that the price charged for each unit is equal to the maximum willingness to pay for that unit.

Second-degree price discrimination, or *nonlinear* pricing, occurs when prices differ depending on the number of units of the good bought, but not across consumers. That is, each consumer faces the same price schedule, but the schedule involves different prices for different amounts of the good purchased. Quantity discounts or premia are the obvious examples.

Third-degree price discrimination means that different purchasers are charged different prices, but each purchaser pays a constant amount for each unit of the good bought. This is perhaps the most common form of price discrimination; examples are student discounts, or charging different prices on different days of the week.

We will follow Pigou's classification in this survey, discussing the forms of price discrimination in the order in which he suggested them. Subsequently, we will take up some more specialized topics that do not seem to fit conveniently in this classification scheme.

We have had the benefit of a number of other surveys of the topic of price discrimination and have not hesitated to draw heavily from those works. Some of these works are published and some are not, and it seems appropriate to briefly survey the surveys before launching in to our own.

First, we must mention Phlips' (1983) extensive book, *The Economics of Price Discrimination*, which contains a broad survey of the area and many intriguing examples. Next, we have found Tirole's (1988) chapter on price discrimination to be very useful, especially in its description of issues involving nonlinear pricing. Robert Wilson's class notes (1985) provided us with an extensive bibliography and discussion of many aspects of product marketing, of which price discrimination is only a part. Finally, Carlton and Perloff (forthcoming) give a very nice overview of the issues and a detailed treatment of several interesting sub-topics. We are especially grateful to these authors for providing us with their unpublished work.

2.2. First-degree price discrimination

First-degree price discrimination, or perfect price discrimination, means that the seller sells each unit of the good at the maximum price that anyone is willing to pay for that unit of the good. Alternatively, perfect price discrimination is sometimes defined as occurring when the seller makes a single take-it-or-leave-it offer to each consumer that extracts the maximum amount possible from the market.

Although the equivalence of these two definitions has been long asserted – Pigou mentions it in his discussion of first-degree price discrimination – it is not entirely clear just how generally the two definitions coincide. Is the equivalence true only in the case of quasilinear utility, or does it hold true more generally? As it turns out, the proposition is valid in quite general circumstances.

To see this, consider a simple model with two goods, x and y and a single consumer. We choose y as the numeraire good, and normalize its price to one. (Think of the y-good as being money.) The consumer is initially consuming 0 units of the x-good, and the monopolist wishes to sell x^* units for the largest possible amount of the y-good. Let y^* be the amount of the y-good that the consumer has after making this payment; then y^* is the solution to the equation

$$u(x^*, y^*) = u(0, y), \tag{1}$$

and the payment is simply $y - y^*$. This is clearly the largest possible amount of the y-good that the consumer would pay on a take-it-or-leave-it basis to consume x^* units of the x-good.

Suppose instead that the monopolist breaks up x^* into n pieces of size Δx and sells each piece to the consumer at the maximum price the consumer would be willing to pay for that piece. Let (x_i, y_i) be the amount the consumer has at the ith stage of this process, so that thus $y_{i-1} - y_i$ is the amount paid for the ith unit of the x-good. Since utility remains constant during this process we have

$$u(x_1, y_1) - u(0, y) = 0,$$

$$u(x_2, y_2) - u(x_1, y_1) = 0,$$

$$\vdots$$

$$u(x^*, y_n) - u(x_{n-1}, y_{n-1}) = 0. \tag{2}$$

We want to show that y_n, the total amount held of the y-good after this process is completed, is equal to y^*, the amount paid by the take-it-or-leave-it offer described above.

But this is easy; just add up the equations in (2) to find

$$u(x^*, y_n) - u(0, y) = 0.$$

Examining (1) we see that $y_n = y^*$, as was to be shown.

2.2.1. Welfare and output effects

It is well known that a perfectly discriminating monopolist produces a Pareto efficient amount of output, but a formal proof of this proposition may be instructive. Let $u(x, y)$ be the utility function of the consumer, as before, and for simplicity suppose that the monopolist cares only about his consumption of the y-good. (Again, it is convenient to think of the y-good as being money.) The monopolist is endowed with a technology that allows him to produce x units of the x-good by using $c(x)$ units of the y-good. The initial endowment of the consumer is denoted by (x_c, y_c), and by assumption the monopolist has an initial endowment of zero of each good.

The monopolist wants to choose a (positive) production level x and a (negative) payment y of the y-good that maximizes his utility subject to the constraint that the consumer actually purchases the x-good from the monopolist. Thus, the maximization problem becomes:

$$\max_{x, y} y - c(x)$$

$$\text{s.t. } u(x_c + x, y_c - y) \geq u(x_c, y_c).$$

But this problem simply asks us to find a feasible allocation that maximizes the utility of one party, the monopolist, subject to the constraint that the other party, the consumer, has some given level of utility. This is the definition of a Pareto efficient allocation. Hence, a perfectly discriminating monopolist will choose a Pareto efficient level of output.

By the Second Welfare Theorem, and the appropriate convexity conditions, this Pareto efficient level of output is a competitive equilibrium for some endowments. In order to see this directly, denote the solution to the monopolist's maximization problem by (x^*, y^*). This solution must satisfy the first-order conditions:

$$1 - \lambda \frac{\partial u(x_c + x^*, y_c - y^*)}{\partial y} = 0,$$

$$-c'(x^*) + \lambda \frac{\partial u(x_c + x^*, y_c - y^*)}{\partial x} = 0. \tag{3}$$

Dividing the second equation by the first and rearranging gives us:

$$\frac{\partial u(x_c + x^*, y_c - y^*)/\partial x}{\partial u(x_c + x^*, y_c - y^*)/\partial y} = c'(x^*).$$

If the consumer has an endowment of $(x_c + x^*, y_c + y^*)$ and the firm faces a parametric price set at

$$p^* = \frac{\partial u(x_c + x^*, y_c - y^*)/\partial x}{\partial u(x_c + x^*, y_c - y^*)/\partial y},$$

the firm's profit maximization problem will take the form:

$$\max_x \; p^* x - c(x).$$

In this case it is clear that the firm will optimally choose to produce x^* units of output, as required.

Of course, the proof that the output level of a perfectly discriminating monopolist is the same as that of a competitive firm only holds if the appropriate reassignment of initial endowments is made. However, if we are willing to rule out income effects, this caveat can be eliminated.

To see this, let us now assume that the utility function for the consumer takes the quasilinear form $u(x) + y$. In this case $\partial u/\partial y \equiv 1$ so the first-order conditions given in (3) reduce to

$$\frac{\partial u(x_c + x^*)}{\partial x} = c'(x^*).$$

This shows that the Pareto efficient level of output produced by the perfectly discriminating monopolist is independent of the endowment of y, which is what we require. Clearly the amount of the x-good produced is the same as that of a competitive firm that faces a parametric price given by $p^* = \partial u(x_c + x^*)/\partial x$.

2.2.2. Prevalence of first-degree price discrimination

Take-it-or-leave-it offers are not terribly common forms of negotiation for two reasons. First, the "leave-it" threat lacks credibility: typically a seller has no way to commit to breaking off negotiations if an offer is rejected. And once an initial offer has been rejected, it is generally rational for the seller to continue to bargain.

Second, even if the seller had a way to commit to ending negotiations, he typically lacks full information about the buyers' preferences. Thus, the seller cannot determine for certain whether his offer will actually be accepted, and must trade off the costs of rejection with the benefits of additional profits.

If the seller was able to precommit to take-it-or-leave-it, and he had perfect information about buyers' preferences, one would expect to see transactions made according to this mechanism. After all, it does afford the seller the most possible profits.

However, vestiges of the attempt to first-degree price discrimination can still be detected in some marketing arrangements. Some kinds of goods – ranging from aircraft on the one hand to refrigerators and stereos on the other – are still sold by haggling. Certainly this must be due to an attempt to price discriminate among prospective customers.

To the extent that such haggling is successful in extracting the full surplus from consumers, it tends to encourage the production of an efficient amount of output. But of course the haggling itself incurs costs. A full welfare analysis of attempts to engage in this kind of price discrimination cannot neglect the transactions costs involved in the negotiation itself.

2.2.3. Two-part tariffs

Two-part tariffs are pricing schemes that involve a fixed fee which must be paid to consume any amount of good, and then a variable fee based on usage. The classic example is pricing an amusement park: one price must be paid to enter the park, and then further fees must be paid for each of the rides. Other examples include products such as cameras and film, or telephone service which requires a fixed monthly fee plus an additional charge based on usage.

The classic exposition of two-part tariffs in a profit maximization setting is Oi (1971). A more extended treatment may be found in Ng and Weisser (1974) and Schmalensee (1981a). Two-part tariffs have also been applied to problems of social welfare maximization by Feldstein (1972), Littlechild (1975), Leland and Meyer (1976), and Auerbach and Pellechio (1978). In this survey we will focus on the profit maximization problem.

Let the indirect utility function of a consumer of type t facing a price p and having income y be denoted by $v(p, t) + y$. If the price of the good is so high that the consumer does not choose to consume it, let $v(p, t) = 0$. The prices of all other goods are assumed to be constant. Note that we have built in the assumption that preferences are quasilinear; i.e. that there are no income effects. This allows for the cleanest analysis of the problem.[2]

Let $x(p, t)$ be the demand for the good by a consumer of type t when the price is p. By Roy's Identity, $x(p, t) = -\partial v(p, t)/\partial p$. Suppose that the good can be produced at a constant marginal cost of c per unit.

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[2] The literature on two-part tariffs seems a bit confused about this point; some authors explicitly examine cases involving income effects, but then use consumer's surplus as a welfare measure. Since consumers' surplus is an exact measure of welfare only in the case where income effects are absent, this procedure is a dubious practice at best.

Initially, we consider the situation where all consumers are the same type. If the firm charges an entrance fee e, the consumer will choose to enter if

$$v(p, t) + y - e \geq y,$$

so that the *maximum* entrance fee the park can charge is

$$e = v(p, t).$$

The profit maximization problem is then

$$\max_{e, p} e + (p - c)x(p, t)$$

$$\text{s.t. } e = v(p, t).$$

Incorporating the constraint into the objective function and differentiating with respect to p yields:

$$\frac{\partial v(p, t)}{\partial p} + (p - c)\frac{\partial x(p, t)}{\partial p} + x(p, t) = 0.$$

Substituting from Roy's Identity yields:

$$-x(p, t) + (p - c)\frac{\partial x(p, t)}{\partial p} + x(p, t) = 0,$$

which in turn simplifies to

$$(p - c)\frac{\partial x(p, t)}{\partial p} = 0.$$

It follows that if all the consumers are the same type, the profit maximizing policy is to set price equal to marginal cost and set an entrance fee that extracts all of the consumers' surplus. That is, the optimal policy is to engage in first-degree price discrimination. This is, of course, a Pareto efficient pricing scheme.

Things become more interesting when there is a distribution of types. Let $F(t)$ denote the cumulative distribution function and $f(t)$ the density of types. Suppose that $\partial v(p, t)/\partial t < 0$, so that the value of the good is decreasing in t. Then if the firm charges an entrance fee e and a price p, a consumer of type t will choose to enter if

$$v(p, t) + y - e \geq y.$$

Thus, for each p there will be a marginal consumer T such that

$$e = v(p, T).$$

Given p, the monopolist's choice of an entrance fee is equivalent to the choice of the marginal consumer T. For any T we can denote the aggregate demand of the admitted consumers as

$$X(p, T) = \int_0^T x(p, t) f(t) \, dt.$$

Note carefully the notational distinction between the demand of the admitted consumers, $X(p, T)$, and the demand of the marginal consumer, $x(p, T)$.

The profit maximization problem of the monopolist is

$$\max_{T, p} v(p, T) F(T) + (p - c) X(p, T),$$

which has first-order conditions:

$$\frac{\partial v}{\partial p} F(T) + (p - c) \frac{\partial X}{\partial p} + X(p, T) = 0,$$

$$\frac{\partial v}{\partial T} F(T) + v(p, T) F'(T) + (p - c) \frac{\partial X}{\partial T} = 0.$$

Using Roy's Identity and the fact that $F'(T) = f(T)$, these conditions become:

$$-x(p, T) F(T) + (p - c) \frac{\partial X}{\partial p} + X(p, T) = 0, \tag{4}$$

$$\frac{\partial v}{\partial T} F(T) + v(p, T) f(T) + (p - c) \frac{\partial X}{\partial T} = 0. \tag{5}$$

Multiply and divide the middle term of the first equation by X/p to find:

$$-x(p, T) F(T) + X(p, T) \frac{p - c}{p} \frac{\partial X}{\partial p} \frac{p}{X} + X(p, T) = 0. \tag{6}$$

Let ε denote the elasticity of aggregate demand of the admitted consumers so that

$$\varepsilon = \frac{\partial X(p, T)}{\partial p} \frac{p}{X(p, T)}.$$

Using this notation, we can manipulate (6) to find:

$$\frac{p-c}{p}|\varepsilon| = 1 - \frac{x(p,T)}{X(p,T)/F(T)}. \tag{7}$$

Expression (7) tells us quite a bit about two-part tariffs. First, note that the expression is a simple transformation of the ordinary monopoly pricing rule:

$$\frac{p-c}{p}|\varepsilon| = 1.$$

When a two-part tariff is possible, the right-hand side is adjusted down by a term which can be interpreted as a ratio of the demand of the marginal consumer to the demand of the average admitted consumer. When the marginal consumer has the same demand as the average consumer – as in the case where all consumers are identical – the optimal two-part tariff involves setting price equal to marginal cost, as we established earlier.

We would normally expect that the marginal consumer would demand less than the average consumer; in this case, the price charged in the two-part tariff would be greater than marginal cost. However, it is possible that the marginal consumer would demand more than the average consumer. Imagine a situation where the marginal consumer does not value the good very highly, but wants to consume a larger than average amount, i.e. $v(p,T)$ is small, but $|\partial v(p,T)/\partial p|$ is large. In this case, the optimal price in the two-part tariff would be less than marginal cost, but the monopolist makes up for it through the entrance fees. (This is an example of the famous auto salesman claim – where the firm loses money on every sale but makes up for it in volume – due to the entrance fee!)

For more on the analytics of two-part tariffs, see the definitive treatment by Schmalensee (1981a). Schmalensee points out that two-part tariffs are essentially a pricing problem involving two especially complementary goods – entrance to the amusement park and the rides themselves. Viewed from this perspective it is easy to see why one of the goods may be sold below marginal cost, or why p is typically less than the monopoly price. Schmalensee also considers much more general technologies and analyzes several important subcases such as the case where the customers are downstream firms.

2.2.4. Welfare effects of the two-part tariff

We have already indicated that a two-part tariff with identical consumers leads to a full welfare optimum. What happens with nonidentical consumers?

Welfare is given by consumers' surplus plus profits:

$$W(p,T) = \int_0^T v(p,t)f(t)\mathrm{d}t + (p-c)X(p,T).$$

Differentiating with respect to p and T and using Roy's Identity, yields:

$$\frac{\partial W(p, T)}{\partial p} = -\int_0^T x(p, t) f(t) dt + (p - c) \frac{\partial X}{\partial p} + X(p, T),$$

$$\frac{\partial W(p, T)}{\partial T} = [v(p, T) + (p - c)x(p, T)] f(T).$$

Evaluating these derivatives at the profit maximizing two-part tariff (p^*, T^*) given in (4) and (5) we have

$$\frac{\partial W(p^*, T^*)}{\partial p} = (p^* - c) \frac{\partial X}{\partial p} = x(p^*, T^*) F(T^*) - X(p^*, T^*),$$

$$\frac{\partial W(p^*, T^*)}{\partial T} = -\frac{\partial v(p^*, T^*)}{\partial T} F(T^*) > 0.$$

The first equation will be positive or negative as the demand of the marginal consumer is greater or less than the demand of the average consumer. If all consumers have the same tastes, the marginal consumer and the average consumer coincide so that price will be equal to marginal cost and therefore optimal from a social viewpoint. Normally, we would expect the marginal consumer to have a lower demand than the average consumer, which implies that price is greater than marginal cost. Hence, it is too high from a social viewpoint, and welfare would increase by lowering it. However, as we have seen above, if the marginal consumer has a higher demand than the average consumer, price will be set below marginal cost, which implies that the monopolist *under* prices his output.

The second equation shows that the monopolist always serves too small a market, since $\partial v / \partial T < 0$ and $F(T) > 0$ by assumption. This holds regardless of whether price is greater or less than marginal cost.

Although the primary focus of this essay is on the profit maximizing case, we can briefly explore the welfare maximization problem. Setting the derivatives of welfare equal to zero and rearranging, we have

$$\frac{\partial W(p, T)}{\partial p} = (p - c) \frac{\partial X}{\partial p} = 0,$$

$$\frac{\partial W(p, T)}{\partial T} = [v(p, T) + (p - c)x(p, T)] f(T) = 0.$$

The first equation requires that price be equal to marginal cost; using this fact the second equation can be rewritten as

$$\frac{\partial W(p, T)}{\partial T} = v(p, T) f(T) = 0.$$

This equation implies that consumers be admitted until the marginal valuation is reduced to zero (or as low as it can go and remain non-negative.) This simply says that anyone who is willing to purchase the good at marginal cost should be allowed to do so, hardly a surprising result.

A more interesting expression results if we require that the firm must cover some fixed costs of providing the good. This adds a constraint to the problem of the form:

$$v(p, T) F(T) + (p - c) X(p, T) = K,$$

where K is the fixed cost that must be covered. The first term on the left-hand side is the total entrance fees collected, and the second term is the profit earned on the sales of the good.

Here it is natural to take the objective function as being the consumers' welfare alone, rather than the consumers' plus the producer's surplus. This leads to a maximization problem of the form:

$$\max_{p, T} \int_0^T v(p, t) f(t) \mathrm{d}t - v(p, T) F(T)$$

$$\text{s.t. } v(p, T) F(T) + (p - c) X(p, T) = K.$$

The first-order conditions for this problem are

$$-X(p, T) + x(p, T) F(T) - \lambda\left[-x(p, T) F(T)\right.$$

$$\left. + (p - c)\frac{\partial X}{\partial p} + X(p, T)\right] = 0,$$

$$-\frac{\partial v}{\partial T} F(T) - \lambda\left[\frac{\partial v}{\partial T} F(T) + v(p, T) f(T)\right.$$

$$\left. + (p - c) x(p, T) f(T)\right] = 0.$$

It is of interest to ask when the constrained maximization problem involves

setting price equal to marginal cost. Rearranging the first equation, we have

$$(1 + \lambda)[x(p, T)F(T) - X(p, T)] = \lambda(p - c)\frac{\partial X}{\partial p}.$$

From this expression it is easy to see that if price equals marginal cost we must have either

$$x(p, T) = \frac{X(p, T)}{F(T)},$$

which means that the average demand equals the marginal demand, or

$$\lambda = -1.$$

We have encountered the first condition several times already, and it needs no further discussion. Turning to the second condition, we note that if $\lambda = -1$, the second first-order condition implies that $v(p, T) = 0$, i.e. that the entrance fee is zero. This can only happen when the fixed costs are zero, so we conclude that essentially the only case in which price will be equal to marginal cost is when the average demand and the marginal demand coincide.

2.2.5. Pareto improving two-part tariffs

Willig (1978) has observed that there will typically exist pricing schemes involving two-part tariffs that Pareto dominate nondiscriminatory monopoly pricing. This is a much stronger result than the welfare domination discussed above: Willig shows that all consumers *and* the producer are at least as well off with a particular pricing scheme than with flat rate pricing.

To illustrate the Willig result in its simplest case, suppose that there are two consumers with demand functions $D_1(p)$ and $D_2(p)$ with $D_2(p) > D_1(p)$ for all p. For simplicity we take fixed costs to be zero. Let p_f be any price in excess of marginal cost, c. In order to construct the Pareto dominating two-part tariff, choose p_t to be any price between p_f and c and choose the lump-sum entry fee e to satisfy:

$$e = (p_f - p_t)D_2(p_f).$$

Now consider a pricing scheme where all consumers are offered the choice between purchasing at the flat rate p_f or choosing the two-part tariff. If neither consumer chooses the two-part tariff, the situation is unchanged and uninteresting. If consumer 2 chooses the two-part tariff, then it must make him better off.

The revenue received by the firm from consumer 2 will be

$$(p_f - p_t)D_2(p_f) + p_t D_2(p_t) = p_f D_2(p_f) + p_t(D_2(p_t) - D_2(p_f))$$
$$> p_f D_2(p_t).$$

The inequality follows from the fact that demand curves slope down and $p_t < p_f$. Hence, revenue from consumer 2 has increased, so the firm is better off. We only have to examine the case of consumer 1. If consumer 1 stays with the flat rate, we are done. Otherwise, consumer 1 chooses the two-part tariff and the revenue received from him will be

$$(p_f - p_t)D_2(p_f) + p_t D_1(p_t) \geq (p_f - p_t)D_1(p_f) + p_t D_1(p_t)$$
$$= p_f D_1(p_f) + p_t(D_1(p_t) - D_1(p_f))$$
$$\geq p_f D_1(p_f).$$

The first inequality follows from the assumption that $D_2(p_f) > D_1(p_f)$ and the second from the fact that demand curves slope down. The conclusion is that the firm makes at least as much from consumer 1 under the two-part tariff as under the flat rate. Hence, offering the choice between the two pricing systems yields a Pareto improvement. As long as consumer 2 strictly prefers the two-part tariff – the likely case – this will be a strict Pareto improvement.

Of course this argument only shows that it is *possible* for a flat rate plus a two-part tariff to Pareto dominate a pure flat rate scheme. It does not imply that moving from flat rates to such a scheme will necessarily result in a Pareto improvement, since a profit maximizing firm will not necessarily choose the correct two-part tariff. Thus the result is more appropriate in the context of public utility pricing rather than profit maximization. See Srinagesh (1985) for a detailed study of the profit maximization case and Ordover and Panzar (1980) for an examination of the Willig model when demands are interdependent.

2.3. Second-degree price discrimination

Second-degree price discrimination, or nonlinear price discrimination, occurs when individuals face nonlinear price schedules, i.e. the price paid depends on the quantity bought. The standard example of this form of price discrimination is quantity discounts.

Curiously, the determination of optimal nonlinear prices was not carefully examined until Spence (1976). Since then there have been a number of contributions in this area; see the literature survey in Brown and Sibley (1986). Much of his work uses techniques originally developed by Mirrlees (1971, 1976), Roberts

(1979) and others for the purpose of analyzing problems in optimal taxation. Much of the work described by Brown and Sibley is motivated by public utility pricing. Here the appropriate objective is welfare maximization rather than profit maximization. Since this literature is already discussed in another contribution to this Handbook, we will focus only on the profit maximization problem.

Our treatment follows the excellent discussion in Tirole (1988) which in turn is based on Maskin and Riley (1984). However, we conduct the main derivation using a general utility structure and resort to the special case considered by these authors only when needed.

2.3.1. Two types of consumers

It is useful to begin by considering a situation where there are only two types of consumers, a fraction f_1 of type t_1 and a fraction f_2 of type t_2. The monopolist wants to sell x_1 to the type 1 consumers and amount x_2 to the type 2 consumers, collecting total payments of r_1 and r_2 from each type.

The utility functions of the consumers are of the quasilinear form $u(x_i, t_i) + y_i$, where y_i is the consumption of the numeraire good. For convenience, we take the endowment of the numeraire good to be zero. We also assume that $u(x, t_2) > u(x, t_1)$ and that $\partial u(x, t_2)/\partial x > \partial u(x, t_1)/\partial x$. These assumptions imply that not only is consumer 2 willing to pay more than consumer 1 for a given amount of the good, but also that consumer 2's *marginal* willingness-to-pay exceeds consumer 1's. We will refer to consumer 2 as the *high-demand* consumer and consumer 1 as the *low-demand* consumer. The assumptions imply that the demand function for the high-demand consumer is always greater than the demand function for the low-demand consumer, a property sometimes known as the *noncrossing condition*.

The demand constraints facing the monopolist are as follows. First, each consumer must want to consume the amount x_i and be willing to pay the price r_i:

$$u(x_1, t_1) - r_1 \geq 0,$$

$$u(x_2, t_2) - r_2 \geq 0.$$

This is simply defining the domain of the problem we will analyze. Second, each consumer must prefer this consumption to the consumption of the other consumer:

$$u(x_1, t_1) - r_1 \geq u(x_2, t_1) - r_2,$$

$$u(x_2, t_2) - r_2 \geq u(x_1, t_2) - r_1.$$

These are the so-called *self-selection constraints*. If the plan (x_1, x_2) is to be feasible in the sense that it will be voluntarily chosen by the consumers, then each consumer must prefer consuming the bundle intended for him as compared to consuming the other person's bundle.

Our assumptions about the utility functions and the fact that the monopolist wants the prices to be as high as possible imply that two of the above four inequalities will be binding constraints. Specifically the low-demand consumer will be charged his maximum willingness-to-pay, and the high-demand consumer will be charged the highest price that will just induce him to consume x_2 rather than x_1. Solving for r_1 and r_2 gives:

$$r_1 = u(x_1, t_1)$$

and

$$r_2 = u(x_2, t_2) - u(x_1, t_2) + u(x_1, t_1).$$

The profit function of the monopolist is

$$\pi = [r_1 - cx_1]f_1 + [r_2 - cx_2]f_2,$$

which, upon substitution for r_1 and r_2 becomes:

$$\pi = [u(x_1, t_1) - cx_1]f_1 + [u(x_2, t_2) - u(x_1, t_2) + u(x_1, t_1) - cx_2]f_2.$$

This expression is to be maximized with respect to x_1 and x_2. Differentiating, we have

$$\left[\frac{\partial u(x_1, t_1)}{\partial x_1} - c\right]f_1 + \left[\frac{\partial u(x_1, t_1)}{\partial x_1} - \frac{\partial u(x_1, t_2)}{\partial x_1}\right]f_2 = 0, \tag{8}$$

$$\frac{\partial u(x_2, t_2)}{\partial x_2} - c = 0. \tag{9}$$

Equation (8) can be rearranged to give:

$$\frac{\partial u(x_1, t_1)}{\partial x_1} = c + \left[\frac{\partial u(x_1, t_2)}{\partial x_1} - \frac{\partial u(x_1, t_1)}{\partial x_1}\right]\frac{f_2}{f_1}, \tag{10}$$

which means that the low-demand consumer has a (marginal) value for the good that exceeds marginal cost. Hence, he consumes an inefficiently small amount of the good. Equation (9) says that at the optimal nonlinear prices, the high-demand

consumer has a marginal willingness-to-pay which is equal to marginal cost. Thus, he consumes the socially correct amount.

The result that the consumer with the highest demand faces a marginal price equal to marginal cost is very general. If the consumer with the highest demand faced a marginal price in excess of marginal cost, the monopolist could lower the marginal price charged to the largest consumer by a small amount, inducing him to buy more. Since marginal price still exceeds marginal cost, the monopolist would make a profit on these sales. Furthermore, such a policy would not affect the monopolist's profits from any other consumers, since they are all optimized at lower values of consumption.

In order to get more explicit results about the optimal pricing scheme, it is necessary to make more explicit assumptions about tastes. For example, it is common to observe price discounts in certain types of goods – high-demand consumers pay a lower per-unit cost than low-demand consumers. Maskin and Riley (1984) show that if preferences take the specific form $u(x, t) + y = tv(x) + y$, then the optimal pricing policy will exhibit quantity discounts in this sense.

2.3.2. A continuum of types

Suppose now that there are a continuum of types, and let $f(t)$ be the density of consumers of type t. For convenience, let the types range from 0 to T. Let the utility function of a consumer of type t be given by $u(x, t) + y$, and let $r(x)$ be the revenue collected from a consumer who chooses to consume x units of the good. Again, we assume that increasing t increases both the total and the marginal willingness-to-pay, which in this context means that

$$\frac{\partial u(x, t)}{\partial t} > 0$$

and

$$\frac{\partial^2 u(x(t), t)}{\partial t \partial x} > 0.$$

Let $x(t)$ be the optimal consumption of a consumer of type t when facing a revenue function $r(\cdot)$. The self-selection constraints imply that a consumer of type t prefers his consumption to a consumer of type s, which means

$$u(x(t), t) - r(x(t)) \geq u(x(s), t) - r(x(s)).$$

Consider the function $g(s)$ defined by

$$g(s) = [u(x(t), t) - r(x(t))] - [u(x(s), t) - r(x(s))].$$

We have just seen that $g(s) \geq 0$ and of course $g(t) = 0$. It follows that $g(s)$ reaches its minimum value when $s = t$. Hence, the derivative of g with respect to

s must vanish at $s = t$, which implies:

$$\left(\frac{\partial u(x(t), t)}{\partial x} - \frac{\partial r(x(t))}{\partial x} \right) \frac{dx(t)}{ds} = 0,$$

which in turn implies that

$$\frac{\partial u(x(t), t)}{\partial x} - \frac{\partial r(x(t))}{\partial x} = 0. \tag{11}$$

This is the analog of the self-selection constraint given above.[3]

Let $V(t)$ be the maximized utility of an agent of type t when facing a pricing schedule $r(\cdot)$. That is,

$$V(t) \equiv u(x(t), t) - r(x(t)). \tag{12}$$

We will have occasion to use the derivative of $V(t)$. Using (11) we calculate

$$V'(t) = \left(\frac{\partial u(x(t), t)}{\partial x} - \frac{\partial r(x(t))}{\partial x} \right) \frac{dx}{dt} + \frac{\partial u(x(t), t)}{\partial t}$$

$$= \frac{\partial u(x(t), t)}{\partial t}.$$

This is simply the envelope theorem – the total derivative of utility reduces to the partial derivative after substituting in the first-order conditions for maximization.

The monopolist wants to choose $x(t)$ so as to maximize profits subject to the self-selection constraints. Profits are given by

$$\pi = \int_0^T [r(x(t)) - cx(t)] f(t) dt.$$

The trick is to build the self-selection constraints into the objective function in a useful way. Using (12) we can rewrite profits as

$$\pi = \int_0^T [u(x(t), t) - cx(t)] f(t) dt - \int_0^T V(t) f(t) dt. \tag{13}$$

Integrating the last term by parts, we have

$$\int_0^T V(t) f(t) dt = V(t)(F(t) - 1) \Big|_0^T - \int_0^T V'(t)[F(t) - 1] dt.$$

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[3] The self-selection constraint can also be thought of as no-envy constraint. The calculation given here was used in examining envy-free allocations in Varian (1976).

[Here we have used $F(t) - 1$ as the integral of $f(t)$.] The utility of type 0 is normalized to be 0, and $F(T) = 1$; hence, the first term on the right-hand side of this expression vanishes. Substituting from (12) leaves us with

$$\int_0^T V(t) f(t) dt = - \int_0^T \frac{\partial u(x(t), t)}{\partial t} [F(t) - 1] dt.$$

Substituting this back into the objective function, equation (13), gives us the final form of the profit function:

$$\pi = \int_0^T \left\{ [u(x(t), t) - cx(t)] f(t) - \frac{\partial u(x(t), t)}{\partial t} [1 - F(t)] \right\} dt.$$

Along the optimal path, the derivative of the integrand with respect to each $x(t)$ should vanish. This gives us the first-order condition:

$$\left[\frac{\partial u(x(t), t)}{\partial x} - c \right] f(t) - \frac{\partial^2 u(x(t), t)}{\partial t \partial x} [1 - F(t)] = 0.$$

Solving for $\partial u / \partial x$ yields:

$$\frac{\partial u(x(t), t)}{\partial x} = c + \frac{\partial^2 u(x(t), t)}{\partial t \partial x} \left[\frac{1 - F(t)}{f(t)} \right]. \tag{14}$$

It is instructive to compare this expression to the formula for the optimal marginal price in the case of two consumers given in (10). Note the close analogy. As in the case with two consumers, all consumers pay a price in excess of marginal cost except for the consumer with the highest willingness-to-pay, consumer T.

In order to derive further results about the shape of the optimal policy, it is necessary to make more detailed assumptions about preferences and the distribution of tastes. For example, suppose that we adopt the form of preferences used by Maskin and Riley (1984), $tv(x) + y$. Let $p(x) = r'(x)$ be the marginal price when purchases are x. Equation (11) then implies that $r'(x(t)) = tv'(x(t))$ is the optimal solution. Substituting this into (14) and rearranging yields:

$$\frac{p(x(t)) - c}{p(x(t))} = \frac{1 - F(t)}{tf(t)}.$$

The expression $(1 - F(t))/f(t)$ is known as the *hazard rate*. For a wide variety of distributions, including the uniform, the normal, and the exponential, the

hazard rate increases with t. Assuming this, and using the concavity of $v(x)$, it is not hard to show that $x(t)$ increases with t and the marginal price $p(x(t))$ decreases with $x(t)$. Thus, this form of preference leads to quantity discounts, a result first proved by Maskin and Riley (1984).

The reader should be warned that our derivation of the profit maximizing nonlinear price was rather cavalier. We assumed differentiability as needed as well as assuming that various second-order conditions would be satisfied. Unfortunately, these assumptions are not innocuous. Optimal pricing policies can easily exhibit kinks so that consumers of different types end up bunching at common quantities, or gaps, so that some consumer types end up not being served. For a detailed and lucid taxonomy of what can go wrong, as well as some illustrative examples, see the discussion in Brown and Sibley (1986, pp. 208–215). See Ordover and Panzar (1982) for a treatment of the nonlinear pricing of inputs.

2.3.3. Welfare and output effects

Katz (1983) has examined the welfare and output effects of nonlinear pricing. He shows that in general the monopolist may produce too little or too much output as compared with the social optimum, but when the noncrossing condition is satisfied, the monopolist will typically restrict total output. In general, total welfare will be positively associated with total output so that changes in output may serve as appropriate indicators for changes in welfare.

2.4. Third-degree price discrimination

Third-degree price discrimination occurs when consumers are charged different prices but each consumer faces a constant price for all units of output purchased. This is probably the most common form of price discrimination.

The textbook case is where there are two separate markets, where the firm can easily enforce the division. An example would be discrimination by age, such as youth discounts at the movies. If we let $p_i(x_i)$ be the inverse demand function for group i, and suppose that there are two groups, then the monopolist's profit maximization problem is

$$\max_{x_1, x_2} p_1(x_1)x_1 + p_2(x_2)x_2 - cx_1 - cx_2.$$

The first-order conditions for this problem are

$$p_1(x_1) + p_1'(x_1)x_1 = c,$$

$$p_2(x_2) + p_2'(x_2)x_2 = c.$$

Let ε_i be the elasticity of demand in market i, we can write these expressions as

$$p_1(x_1)\left[1 - \frac{1}{|\varepsilon_1|}\right] = c,$$

$$p_2(x_2)\left[1 - \frac{1}{|\varepsilon_2|}\right] = c.$$

It follows that $p_1(x_1) > p_2(x_2)$ if and only if $|\varepsilon_1| < |\varepsilon_2|$. Hence, the market with the more elastic demand – the market that is more price sensitive – is charged the lower price.

Suppose now that the monopolist is unable to separate the markets as cleanly as assumed, so that the price charged in one market influences the demand in another market. For example, consider a theater that has a bargain night on Monday; the lower price on Monday would presumably influence demand on Tuesday to some degree.

In this case the profit maximization problem of the firm is

$$\max_{x_1, x_2} p_1(x_1, x_2)x_1 + p_2(x_1, x_2)x_2 - cx_1 - cx_2,$$

and the first-order conditions become:

$$p_1 + \frac{\partial p_1}{\partial x_1}x_1 + \frac{\partial p_2}{\partial x_1}x_2 = c,$$

$$p_2 + \frac{\partial p_2}{\partial x_2}x_2 + \frac{\partial p_1}{\partial x_2}x_1 = c.$$

We can rearrange these conditions to give:

$$p_1[1 - 1/|\varepsilon_1|] + \frac{\partial p_2}{\partial x_1}x_2 = c,$$

$$p_2[1 - 1/|\varepsilon_2|] + \frac{\partial p_1}{\partial x_2}x_1 = c.$$

It is not easy to say anything very interesting about these equations, but we will try.

One simplification we can make is to assume there are no income effects so that $\partial p_1/\partial x_2 = \partial p_2/\partial x_1$, i.e. the cross price effects are symmetric.[4] Subtracting the second equation from the first and rearranging, we have

$$p_1\left[1 - \frac{1}{|\varepsilon_1|}\right] - p_2\left[1 - \frac{1}{|\varepsilon_2|}\right] = [x_1 - x_2]\frac{\partial p_2}{\partial x_1}.$$

It is natural to suppose that the two goods are substitutes – after all, they are the same good being sold to different groups – so that $\partial p_2/\partial x_1 > 0$. Without loss of generality, assume that $x_1 > x_2$, which, by the equation immediately above, implies that

$$p_1\left[1 - \frac{1}{|\varepsilon_1|}\right] - p_2\left[1 - \frac{1}{|\varepsilon_2|}\right] > 0.$$

Rearranging, we have

$$\frac{p_1}{p_2} > \frac{1 - 1/|\varepsilon_2|}{1 - 1/|\varepsilon_1|}.$$

It follows from this expression that if $|\varepsilon_2| > |\varepsilon_1|$, we must have $p_1 > p_2$. That is, if the *smaller* market has the more elastic demand, it must have the lower price. Thus, the intuition of the separate markets carries over to the more general case under these additional assumptions.

2.4.1. Welfare effects

Much of the discussion about third-degree price discrimination has to do with the welfare effects of allowing this form of price discrimination. Would we generally expect consumer plus producer surplus to be higher or lower when third-degree price discrimination is present than when it is not?

Since Robinson (1933) first raised this question, it has been the subject of a number of investigations, including Battalio and Ekelund (1972), Holahan (1975), Hsu (1983), Ippolito (1980), Kwoka (1984), Yamey (1974), Hausman and MacKie-Mason (1986), Schmalensee (1981b), and Varian (1985). Varian's results are the most general and serve as the focus of our discussion.

We begin with formulating a general test for welfare improvement. Suppose for simplicity that there are only two groups and start with an aggregate utility

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[4]Actually assuming no income effects is a stronger assumption than we need. Willig (1976) and Varian (1978) have shown that all that is necessary is that the income elasticities of goods 1 and 2 are locally constant over some region in price–income space.

function of the form $u(x_1, x_2) + y$. Here x_1 and x_2 are the consumptions of the two groups and y is money to be spent on other consumption goods. The inverse demand functions for the two goods are given by

$$p_1(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1},$$

$$p_2(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_2}.$$

We assume that $u(x_1, x_2)$ is concave and differentiable, though this is slightly stronger than needed.

Let $c(x_1, x_2)$ be the cost of providing x_1 and x_2, so that social welfare is measured by

$$W(x_1, x_2) = u(x_1, x_2) - c(x_1, x_2).$$

Now consider two configurations of output, (x_1^0, x_2^0) and (x_1', x_2'), with associated prices (p_1^0, p_2^0) and (p_1', p_2'). By the concavity of $u(x_1, x_2)$, we have

$$u(x_1', x_2') \le u(x_1^0, x_2^0) + \frac{\partial u(x_1^0, x_2^0)}{\partial x_1}(x_1' - x_1^0) + \frac{\partial u(x_1^0, x_2^0)}{\partial x_2}(x_2' - x_2^0).$$

Rearranging and using the definition of the inverse demand functions we have

$$\Delta u \le p_1^0 \Delta x_1 + p_2^0 \Delta x_2.$$

By an analogous argument we have

$$\Delta u \ge p_1' \Delta x_1 + p_2' \Delta x_2.$$

Since $\Delta W = \Delta u - \Delta c$, we have our final result:

$$p_1^0 \Delta x_1 + p_2^0 \Delta x_2 - \Delta c \ge \Delta W \ge p_1' \Delta x_1 + p_2' \Delta x_2 - \Delta c. \tag{15}$$

In the special case of constant marginal cost, $\Delta c = c\Delta x_1 + c\Delta x_2$, so the inequality becomes:

$$\left(p_1^0 - c\right)\Delta x_1 + \left(p_2^0 - c\right)\Delta x_2 \ge \Delta W \ge \left(p_1' - c\right)\Delta x_1 + \left(p_2' - c\right)\Delta x_2. \tag{16}$$

Note that these welfare bounds are perfectly general, based only on the concavity of the utility function, which is, in turn, basically the requirement that

demand curves slope down. Varian (1985) derived the inequalities using the indirect utility function, which is slightly more general.

In order to apply these inequalities to the question of price discrimination, let the initial set of prices be the constant monopoly prices so that $p_1^0 = p_2^0 = p^0$, and let (p_1', p_2') be the discriminatory prices. Then the bounds in (16) become:

$$(p^0 - c)(\Delta x_1 + \Delta x_2) \geq \Delta W \geq (p_1' - c)\Delta x_1 + (p_2' - c)\Delta x_2. \qquad (17)$$

The upper bound implies that *a necessary condition for welfare to increase is that total output increase*. Suppose to the contrary that total output decreased so that $\Delta x_1 + \Delta x_2 < 0$. Since $p^0 - c > 0$, (17) implies that $\Delta W < 0$. The lower bound gives a sufficient condition for welfare to increase under price discrimination, namely that the sum of the *weighted output* changes is positive, with the weights being given by price minus marginal cost.

The simple geometry of the bounds is shown in Figure 10.1. The welfare gain ΔW is the indicated trapezoid. The area of this trapezoid is clearly bounded above and below by the area of the two rectangles.

As a simple application of the welfare bounds, let us consider the case of two markets with linear demand curves:

$$x_1 = a_1 - b_1 p_1; \qquad x_2 = a_2 - b_2 p_2.$$

For simplicity set marginal costs equal to zero. Then if the monopolist engages in price discrimination, he will maximize revenue by selling halfway down each demand curve, so that $x_1 = a_1/2$ and $x_2 = a_2/2$.

Now suppose that the monopolist sells at a single price to both markets. The total demand curve will be
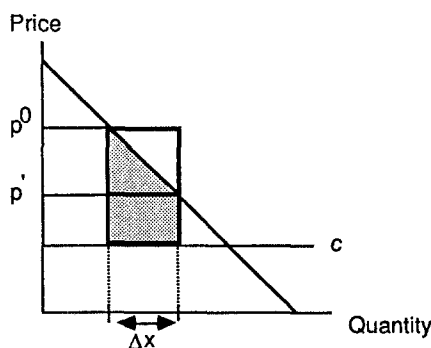
$$x_1 + x_2 = a_1 + a_2 - (b_1 + b_2)p.$$

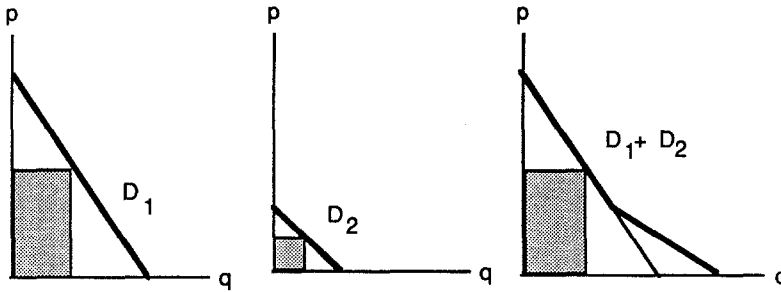Figure 10.1. Illustration of the welfare bounds.

Figure 10.2. An example where the monopolist chooses not to serve one of the markets.

To maximize revenue the monopolist will operate halfway down the demand curve which means that

$$x_1 + x_2 = \frac{a_1 + a_2}{2}.$$

Hence, with linear demand curves the total output is the same under price discrimination as under ordinary monopoly. The bound given in (17) then implies that welfare must decrease under price discrimination.

However, this result relies on the assumption that both markets are served under the ordinary monopoly. Suppose that market 2 is very small, so that the profit maximizing firm sells zero to this market if price discrimination is not allowed, as illustrated in Figure 10.2.

In the case allowing price discrimination results in $\Delta x_1 = 0$ and $\Delta x_2 > 0$, providing an unambiguous welfare gain by (17). Of course, this is not only a welfare gain, but is in fact a Pareto improvement.

This example is quite robust. If a new market is opened up because of price discrimination – a market that was not previously being served under the ordinary monopoly – then we will typically have a Pareto improving welfare enhancement. This case is emphasized by Hausman and MacKie-Mason (1986) with respect to new patents. On the other hand, if linearity of demand is not a bad first approximation, and output does not change too drastically in response to price discrimination, we might well expect that the net impact on welfare is negative.

2.4.2. Output effect of price discrimination

Since the change in output provides some clue as to the welfare change, it is worthwhile considering conditions under which it has a determinate sign. Robinson (1933), Battalio and Ekelund (1972), Edwards (1950), Smith and

Formby (1981), Finn (1974), Greenhut and Ohta (1976), Löfgren (1971), Silberberg (1970), Schmalensee (1981b) and others have contributed to this question. For some general results and a good summary of the literature, see Shih and Mai (1986), whose treatment we follow here.

Let x_i' be the output in the ith market under discrimination and x_i^0 be the output under uniform pricing. By manipulating the first-order conditions for profit maximization and using the mean-value theorem, it can be shown that the difference between the total output under uniform and discriminatory pricing is given by

$$X^0 - X' = \frac{1}{2} \sum_{i=1}^{n} \left(x_i^0 - x_i' \right) \left[p_i(\hat{x}_i) - c \right] \frac{p_i''(\hat{x}_i)}{p_i'(\hat{x}_i)^2}. \tag{18}$$

In this expression, $\hat{x}_i$ is an output level between x_i' and x_i^0 (this is where the mean value theorem is used). Following Robinson, we call a market *strong* if $x_i' < x_i^0$ and *weak* if $x_i' > x_i^0$. It follows from equation (18) that if all weak markets have strictly convex demands and all strong markets have concave or linear demands, then total output will be greater under discrimination than under uniform pricing and vice versa.[5] Of course if all markets have linear demands, total output is constant, as we have observed earlier.

2.4.3. Intermediate goods markets

Katz (1987) has examined the welfare effect of price discrimination in intermediate goods markets. This phenomenon is of some interest since the Robinson–Patman Act (discussed below) was explicitly concerned with this form of price discrimination. There are two important differences in the analysis of intermediate goods markets as compared to final goods markets. The first is that the buyers' demands for the product are interdependent: the profits and factor demands by a downstream firm depend on the factor demands of its competitors. The second is that the buyers in intermediate goods markets often have the possibility of integrating upstream and produce the intermediate good themselves.

The first effect means that the welfare analysis must take into account the induced changes in the degree of competition engendered by different policies. The second effect means that the welfare analysis must take into account

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[5]Note that Robinson, and several other subsequent authors, reverse the sense of concave and convex when discussing these results.

inefficiencies in production decisions caused by the different policies. Accordingly, there are two components to the change in welfare when price discrimination is allowed in intermediate goods markets. The first is the standard effect on the output of the final good which we have discussed above. The second is the decision of whether or not to integrate and the resulting impact on the costs of production.

These two components interact in complex ways. Katz shows that if there is no integration under either regime, total output and total welfare are both lower when price discrimination is allowed than when it is forbidden. In fact, under reasonable conditions price discrimination in intermediate goods markets can lead to higher prices being charged to *all* buyers of the good, a result that cannot arise in final goods markets.

Katz shows that integration can only occur when price discrimination is banned, not when it is allowed. If there is increasing returns in the intermediate good production, this means that price discrimination may serve to prevent socially inefficient integration.

2.5. Defining the market

In most of the literature on third-degree price discrimination, the determination of the different groups of consumers is taken as exogenous to the model. The monopolist has already decided to charge one price to people over 18 and another price to people under 18, or one price to customers who purchase drinks between 5 and 7 p.m. and another price to those who purchase drinks at other hours. The only issue is what the prices should be.

However, it is clear that the choice of how to divide the market is a very important consideration for the monopolist. In this subsection we will briefly examine this decision in a highly specialized framework.

We will conduct our discussion in terms of pricing beverages by time of day, but a variety of other interpretations are possible. We assume that the demand for drinks at an arbitrary time t depends only on the time and the price charged at that time, so we write $x(p_t, t)$. The assumption that demands are independent across the times of day is admittedly a drastic simplification. We assume that $0 \leq t \leq 1$.

Then if the monopolist charges p_1 before time T and p_2 after time T, the total amount that he sells in each time period will be given by

$$X_1(p_1, T) = \int_0^T x(p_1, t) \, dt,$$

$$X_2(p_2, T) = \int_T^1 x(p_2, t) \, dt.$$

The profit maximization problem of the monopolist can then be written as

$$\max_{p_1, p_2, T} (p_1 - c) X_1(p_1, T) + (p_2 - c) X_2(p_2, T).$$

The first-order conditions for this problem are

$$(p_1 - c)\frac{\partial X_1}{\partial p_1} + X_1(p_1, T) = 0,$$

$$(p_2 - c)\frac{\partial X_2}{\partial p_2} + X_2(p_2, T) = 0,$$

$$(p_1 - c)x_1(p_1, T) - (p_2 - c)x_2(p_2, T) = 0.$$

The first two equations are the standard marginal revenue equals marginal cost conditions. They can be transformed into the standard elasticity form and can be interpreted in exactly the same way. Thus, if the elasticity of demand increases with the time of day, consumers after the breakpoint will pay a price lower than those before the breakpoint.

The third equation is new; it indicates how the monopolist determines the optimal breakpoint. The interpretation of this condition is straightforward: when the monopolist chooses the optimal breakpoint T, the profits earned from charging marginal consumer the higher price must equal the profits earned from charging the lower price.

What about the welfare effects of the choice of breakpoint? Is the monopolist choosing the correct breakpoint given his pricing decision? Or will there be a systematic distortion?

As it turns out there is a very general result available here: social welfare will *always* increase by shifting the breakpoint in the direction of the lower prices. In our context of pricing beverages by time of day we can refer to this as the *Happy Hours Theorem*: Happy Hours are always too short.

The proof is easy. Consider moving the breakpoint a small amount in the direction of lower prices. This certainly makes the consumers better off. But since the breakpoint was the profit maximizing choice by the monopolist, changing the breakpoint slightly must have a zero first-order effect on profits. Hence, consumer plus producer surplus must necessarily increase.

In the context of our independent demand example, if the elasticity of demand is increasing with the time of day, then the monopolist always sets the breakpoint too early in the day, while the reverse is true if the elasticity of demand decreases with the time of day.

Note that the Happy Hours Theorem itself does not rely on the assumption of independent demands; it is true in complete generality. In other contexts it can

be interpreted as saying that airlines always have too severe requirements for the special fares, and that the definition of Senior Citizens used by price discrimination is always too stringent.

2.6. Bundling

Bundling refers to the practice of selling two or more goods in a package. *Pure bundling* means that the goods are only available in the package, while *mixed bundling* means that the goods are available either individually or bundled together in a package. Examples of bundling include *prix fixe* menus, mandatory service contracts, season tickets, and so on. Mixed bundling includes such practices as roundtrip air fares, all inclusive vacation plans, and different sizes of packaged goods.

The earliest reference to the phenomenon of bundling appears to be that of Burnstein (1960), followed by Stigler (1963). Adams and Yellen (1976) provide a clear discussion with numerous examples.

Despite the prevalence of bundling as a marketing phenomenon, there is only a small theoretical literature concerning this topic, probably due to the difficulty of getting analytic results. Schmalensee (1984) conducts extensive numerical simulations to address some of the questions surrounding the bundling literature. We will begin by summarizing the framework of Adams and Yellen (1976).

The simplest case of bundling is that involving quantity bundling. Formally, this is a case of second-degree price discrimination, i.e. nonlinear pricing. The simplest example occurs when there are two types of consumers, a high-demand and a low-demand type, but the firm cannot explicitly discriminate between the two types.

Suppose that the high-demand consumer is willing to pay $10 for one unit of the good or $11 for two units, while the low-demand consumer is willing to pay $1 for one unit or $2 for two units. Suppose for simplicity that marginal costs are zero. Then in this case an effective strategy for the firm is to sell the good only in pairs, allowing the monopolist to price discriminate between the high-demand and low-demand consumers. Here quantity bundling serves as a way to satisfy the self-selection constraints and allows the monopolist to engage in price discrimination.

Another example of bundling is discussed in Stigler (1963), who describes the common practice of "block booking" movies. This practice required that theaters bought films in packages, or bundles, rather than buying the individual films separately.

Suppose that there are two theaters, A and B. A is willing to pay $9000 for film 1, $3000 for film 2, and $12 000 for the package. B is willing to pay $10 000 for film 1, $2000 for film 2, and $12 000 for the package. Notice that the value of the

bundle to each theater is simply the sum of the values of the two films; there are no "interaction effects" in the consumption of the two goods.

Suppose that costs are zero, so that the movie rental company is only interested in maximizing revenue. If the rental company rents each film individually, profit maximization requires that it rents film 1 for $9000 and film 2 for $2000 making a total of $11 000 from each theater. But if it rents only the bundled package it makes $12 000 from each theater. Effectively the rental company has managed to price discriminate between the two theaters; it is renting film 1 to theater A for $9000 and to firm B for $10 000, and similarly for film 2.

This example illustrates an important point: bundling is most effective when there is a negative correlation between the consumers' valuations of the goods. In the case illustrated, theater A's value for film 1 is less than theater B's, but theater A's value for film 2 is higher than theater B's.

Adams and Yellen (1976) present a series of useful diagrams to analyze the effects of bundling. Consider a model in which each consumer wants at most one unit of each of two goods, which are produced costlessly by a monopolist. The reservation prices of the two goods will be denoted by r_1 and r_2. For simplicity, we suppose that each consumer's value of the bundle is simply the sum of his or her reservation prices r_1 and r_2. Let $f(r_1, r_2)$ denote the density function for consumers who have reservation prices (r_1, r_2).

Figure 10.3(a) depicts the outcome under separate, nonbundled sales. The firm picks prices (p_1, p_2), and sells to all consumers in the shaded area, N. Figure 10.3(b) shows the outcome of pure bundling. The monopolist sells only the package at some price p_b and all consumers in the shaded area B are purchasers.

Finally, Figure 10.4 illustrates the effect of mixed bundling. Here we suppose that the monopolist will sell the items separately at prices p_1 and p_2, or together
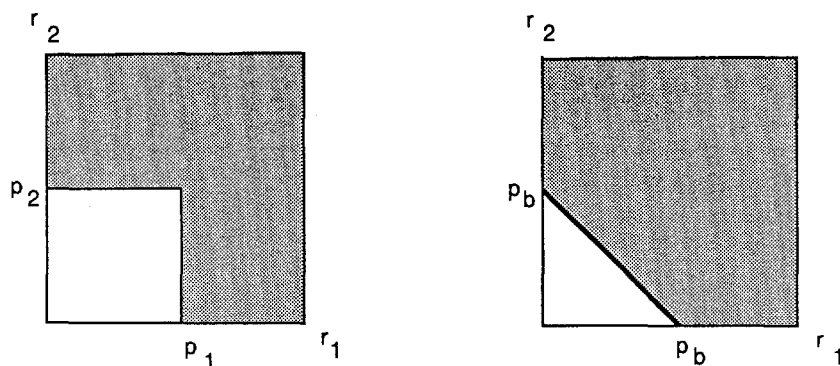

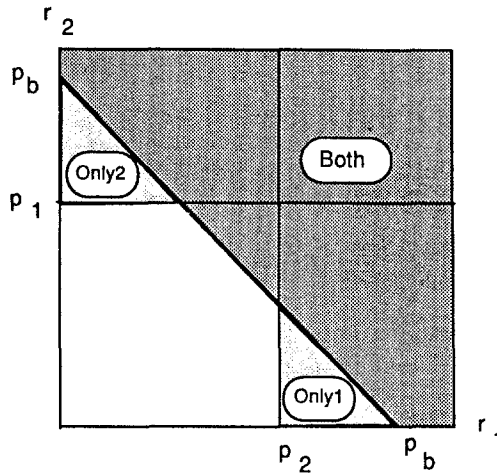Figure 10.3. Illustration of nonbundled and bundled strategies.

Figure 10.4. Illustration of mixed bundling strategy.

in a bundle at price p_b. The areas Only 1, Only 2, and Both indicate the goods purchased by the consumers with various combinations of reservation prices.

Adams and Yellen (1976) show using examples in this diagrammatic framework that nearly anything can happen with bundling: bundling may be more or less profitable than nonbundling, and consumers' surplus and total welfare may be higher or lower.

Since analytic results in the general model are so sparse, it is sensible to look for plausible restrictions that yield results. For example, Schmalensee (1982) shows that if the monopolist can only bundle its product with another good that is competitively produced, it is never better off bundling than simply selling its product separately. However, if there is a negative correlation among buyers' reservation prices, it may pay the monopolist to engage in mixed bundling.

Adams and Yellen (1976) show that mixed bundling always dominates pure bundling. McAfee, McMillan and Winston (1987) have derived a condition on the distribution of reservation price which guarantees that mixed bundling will dominate unbundled sales. In particular, they show that if the buyers' reservation prices are independently distributed, then mixed bundling will *always* dominate unbundled sales. Furthermore, when purchases can be monitored, then bundling is preferred to unbundled sales for virtually all distributions of reservations prices.

Schmalensee (1984) describes a detailed set of simulations using Gaussian distributions for the reservation prices of consumers. One of the most interesting results of Schmalensee's simulations is the role played by the distribution of buyers' valuations of the two goods. Intuition suggests that bundling is most

effective when buyers' valuations are negatively correlated – buyers who value good 1 highly place a small value on good 2 and vice versa. However, Schmalensee suggests that the real role of bundling is to reduce the heterogeneity of buyers' valuations. He demonstrates that, in the Gaussian case, the standard deviation of the valuation of the bundles is always less than the sum of the standard deviations for the individual components. By reducing the dispersion of buyers' valuations, the monopolist is better able to extract the surplus from the population.

Although Schmalensee's simulations provide significant insight into bundling, it would still be helpful to have some analytic results. One plausible research strategy is to examine restrictions on preferences under which analytic results might be feasible. Suppose, for example, that we consider the case of the representative consumer – where the aggregate demand behavior of the population behaves like that of a single representative consumer.

Let $u(x_1, x_2) + y$ be the utility function of this representative consumer for goods x_1 and x_2, which may be bundled, and y, which represents all other goods. The inverse demand functions for the two goods purchased separately are given by

$$p_1(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1},$$

$$p_2(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_2}.$$

Suppose that the bundle consists of 1 unit of good 1 and k units of good 2 and that it sells for a price of p_b. If we let x_b be the number of units of the bundle purchased, the utility maximization problem of the representative consumer can be written as

$$\max_{x_b} u(x_b, kx_b) + y$$

$$\text{s.t. } p_b x_b + y = m.$$

Substituting from the constraint and differentiating, we see that the inverse demand function for the bundle is given by

$$p_b(x_b, kx_b) = \frac{\partial u(x_b, kx_b)}{\partial x_1} + k\frac{\partial u(x_b, kx_b)}{\partial x_2}. \tag{19}$$

This equation says that the marginal willingness-to-pay for the bundle is simply

sum of the willingnesses-to-pay for the individual components, which is quite plausible.

Let the constant marginal costs of the two goods be c_1 and c_2. The profit maximization problem for the monopolist who sells the unbundled goods is

$$\max_{x_1, x_2} \big(p_1(x_1, x_2) - c_1 \big) x_1 + \big(p_2(x_1, x_2) - c_2 \big) x_2, \tag{20}$$

while the profit maximization problem for the bundled monopolist is

$$\max_{x_b, k} \big(p_b(x_b, kx_b) - c_1 - kc_2 \big) x_b. \tag{21}$$

We can now state our main result: in the context of the above model, the monopolist can achieve exactly the same profits whether he bundles or not – there is no inherent advantage to either strategy.

In order to prove this, simply substitute the expression for the inverse demand function given in (19) into (21) and rearrange to get:

$$\max_{x_b, k} \big[p_1(x_b, kx_b) - c_1 \big] x_b + \big[p_2(x_b, kx_b) - c_2 \big] kx_b. \tag{22}$$

Comparing (20) and (22) we see that they describe exactly the same maximization problem: for any pair (x_1^*, x_2^*) that yields a given profit level in (20) we can set $x_b^* = x_1^*$ and $k = x_2^*/x_1^*$ to achieve exactly the same profit level in (22), and vice versa. Hence, in the context of a representative consumer model, bundling does no better than not bundling.

Upon reflection this is not terribly surprising. Bundling is inherently a strategy that exploits differences in willingness-to-pay across people, and bundling with a single consumer – or an economy that acts like a single consumer – cannot be expected to be of much interest. However, the representative consumer model can be used to generate some other interesting questions. For example, for what distributions of willingness-to-pay does the reservation price model of Adams and Yellen reduce to the representative consumer model? The answer to this question would give us more of handle on determining for what distributions bundling is profitable. Will mixed bundling generally dominate separate sales in the reservation price model? Since mixed bundling dominates pure bundling, one being a special case of the other, we expect that the answer to this question is yes.

3. Applications

As we saw in the introductory remarks, price discrimination is a very commonly used marketing tactic. In the following subsections, we will discuss a number of ways that firms can use to price discriminate among their customers.

3.1. Spatial price discrimination

Firms often use delivery charges – or the absence of delivery charges – to price discriminate among customers. For example, cement in Belgium is sold at a uniform delivered price throughout the country; while plasterboard is sold at a uniform delivered price in the United Kingdom. See Phlips (1983, pp. 23–30) for a detailed discussion of such pricing policies in Europe.

Even though delivery costs of these goods can be a significant fraction of their value, firms may find it profitable to charge one price for all areas. In many cases, FOB pricing – in which the customers provide the transportation and pays one price for "freight on board" at a central warehouse – is explicitly excluded by the providers of the good. Instead, the producers offer only a price which includes delivery charges.

In order to model the effect of transportation costs on the monopolist's pricing policy, let x be the quantity sold at a particular location, let the inverse demand function for the delivered good be denoted by $p(x)$ and let the total transportation costs to the given location be denoted by tx. The net demand function facing the firm is then $p(x) - t$. Assuming constant marginal costs, the profit maximization problem for the firm becomes:

$$\max_{x} \left[p(x) - t - c \right] x,$$

which has the standard first-order conditions:

$$p(x) + p'(x)x = c + t. \tag{23}$$

How will the net price to the consumer change as the transportation costs change? The above formulation makes it clear that this is the same as asking how the price to consumers changes as we change an excise tax on a monopolist.

Implicitly differentiating (23) with respect to t and solving for dx/dt, we have

$$\frac{dx}{dt} = \frac{1}{2p'(x) + p''(x)x}.$$

Hence,

$$\frac{dp}{dt} = \frac{dp}{dx}\frac{dx}{dt} = \frac{1}{2 + p''(x)x/p'(x)}.$$

Thus, the amount of the transportation costs that are passed along depends on the second derivative of the inverse demand function, $p''(x)$. We can examine a few special cases. If the inverse demand curve is linear, $p''(x) = 0$ and therefore

half of the transportation costs will be paid by the consumers. In this case the monopolist practices freight absorption and discriminates against customers with lower transport costs. This is similar to the examples of cement and plasterboard described above: the firm effectively absorbs part or all of the transportation costs.

If the inverse demand curve has a constant elasticity of ε, then the monopolist charges a constant markup on marginal cost, so that the customers at each distance will effectively pay more for delivery than the actual delivery costs. However, this form of price discrimination is especially sensitive to consumer arbitrage – consumers with lower transportation costs can transship to those with higher transportation costs, essentially undermining the monopolist's position.

Delivered pricing systems have sometimes been attacked as anticompetitive under the Robinson–Patman Act; for a discussion of some of the issues involved and citations to a number of relevant cases, see Neale and Goyder (1980, pp. 245–248).

3.2. Intertemporal discrimination

Often new products are introduced at a high price which later declines. For example, books are typically introduced in expensive hardcover editions and only later published as less expensive paperbacks. Such a policy appears to be a form of intertemporal price discrimination: the monopolist attempts to first extract the surplus from the high-demand consumers and only later sells to the low-demand consumers.

This kind of intertemporal price discrimination was first analyzed in detail by Stokey (1979). She considered a model with a continuum of consumers and times in which the consumers and the firm have the same discount rate. In the context of this model, Stokey proved a very surprising result: the profit maximizing policy of the firm is to charge a uniform price, *not* to engage in price discrimination.

We will investigate Stokey's result in the context of a much simpler model based on the discussion in Subsection 2.5 concerning self-selection constraints. Suppose that there are two consumers with reservation prices r_1 and r_2 and a common discount factor of $0 < \alpha < 1$. Without loss of generality, suppose that $r_1 > r_2$. The monopolist, who also has a discount rate of α, can costlessly supply up to two units of the good.

We now consider whether it is optimal to price discriminate in this model. Suppose that the monopolist sets prices p_1 and p_2 that succeed in inducing consumer 1 to consume in the first period and consumer 2 to consume in the second period. Then each consumer must prefer the discounted net value of his purchase to that of the other consumer; this gives us the self-selection con-

straints:

$$r_1 - p_1 \geq \alpha(r_1 - p_2), \tag{24}$$

$$\alpha(r_2 - p_2) \geq r_2 - p_1. \tag{25}$$

The first inequality says that the period 1 purchaser prefers his choice to buying in period 2, while the second inequality states the analogous condition for consumer 2. If we multiply these two inequalities together and cancel the α from each side, we have

$$(r_1 - p_1)(r_2 - p_2) \geq (r_1 - p_2)(r_2 - p_1).$$

After some manipulation, this inequality becomes:

$$(p_2 - p_1)(r_2 - r_1) \geq 0.$$

Since $r_1 > r_2$ by assumption, we must have $p_1 \geq p_2$, which simply means that the price in the first period must be higher than the price in the second, hardly a surprising result.

The problem faced by the price discriminating monopolist can now be written as

$$\max_{p_1, p_2} p_1 + \alpha p_2$$

$$\text{s.t. } r_1 - p_1 \geq \alpha(r_1 - p_2), \tag{26}$$

$$\alpha(r_2 - p_2) \geq r_2 - p_1, \tag{27}$$

$$r_1 \geq p_1, \tag{28}$$

$$r_2 \geq p_2. \tag{29}$$

Although there are four constraints in this linear program, they are not all independent. Rearranging (26) we have

$$r_1 \geq \frac{1}{1 - \alpha}(p_1 - \alpha p_2).$$

Subtracting p_1 from each side gives:

$$r_1 - p_1 \geq \frac{1}{1 - \alpha}(p_1 - \alpha p_2) - p_1$$

$$= \frac{\alpha}{1 - \alpha}(p_1 - p_2) \geq 0.$$

It follows that (26) implies (28), so we drop (28) from the set of constraints.

At the solution to this linear programming problem, at least two of the constraints will be binding. Eliminating the trivial cases, this leaves us with three possibilities.

Case 1. $r_1 - p_1 = \alpha(r_1 - p_2)$ and $\alpha(r_2 - p_2) = r_2 - p_1$. Manipulating these inequalities gives us $(1 - \alpha)r_1 = p_1 - \alpha p_2 = (1 - \alpha)r_2$, a contradiction.

Case 2. $r_2 = p_2$ and $\alpha(r_2 - p_2) = r_2 - p_1$. These two equations imply that $p_1 = p_2$, so uniform pricing is optimal.

Case 3. $r_2 = p_2$ and $r_1 - p_1 = \alpha(r_1 - p_2)$. Solving for p_1 and substituting into the profit function shows that the profits of the firm are

$$\pi = r_1 + \alpha(2r_2 - r_1). \tag{30}$$

If these are the maximum profits the firm can make, they must dominate the policy of selling only in period 1 at a price of either r_1 and only satisfying the high-demand customers or selling at r_2 and satisfying both customers. Hence we must satisfy the following two inequalities:

$$r_1 + \alpha(2r_2 - r_1) \geq r_1,$$

$$r_1 + \alpha(2r_2 - r_1) \geq 2r_2.$$

However, it is easily checked that these inequalities imply $r_1 = 2r_2$. Substituting back into the profit function (30) we find that $\pi = r_1 = 2r_2$, so that the profits from price discrimination are equal to those from uniform pricing.

In summary, the optimal policy of the firm either involves charging a constant price in each period or selling at a price of r_1 or r_2 in the first period, and not selling at all in the second period. Neither alternative involves intertemporal price discrimination.

Salant (1987) asks why this extreme solution arises in the case of intertemporal price discrimination, but not in the general case of nonlinear pricing. He points out that the general analysis of nonlinear pricing typically *assumes* appropriately curved objective functions and constraints and limits itself to examining only interior solutions. In the intertemporal case, linearity of the objective function and constraints is a very natural assumption, and we should not be surprised that boundary solutions may be optimal.

However, the result that intertemporal price discrimination is not profit maximizing is a disturbing one, since firms seem to engage in such behavior. There are several ways to relax the assumptions of the model to allow for intertemporal price discrimination. For example, if the discount rates differ across consumers, price discrimination can easily be optimal. Similarly, if the discount rates of the consumers differ from that of the monopolist, price discrimination may be optimal.

It is easy to see when this may occur in our model. The only case involving the profit function of the firm is Case 3. Letting β be the discount factor of the monopolist, a sufficient condition for price discrimination to be optimal is that the discounted profits from price discrimination dominate selling only in the first period:

$$(1 - \alpha)r_1 + (\alpha + \beta)r_2 \geq \max\{r_1, 2r_2\}.$$

It follows that intertemporal price discrimination can be optimal only when $\beta > \alpha$, i.e. when the monopolist is less impatient than the consumers. Landsberger and Meilijson (1985) examine the profitability of price discrimination when discount rates differ and derive a similar condition using Stokey's original continuous-time formulation.

We have also assumed that the good was costlessly produced. It is trivial to generalize our argument to the case of constant marginal costs, but if marginal costs are increasing the argument may fail. In the case of increasing marginal costs, it may be profitable to spread out the sales over time so as to keep production costs down.

One assumption implicit in our analysis is that the firm can *credibly precommit* to charging a constant price every period. To see that this can be a problem, let us consider the case where it is profitable to serve only the first-period consumers. In this case the prices are set at $p_1 = p_2 = r_1$, but no consumers purchase in the second period.

However, once the monopolist has satisfied the needs of the high-demand, first-period consumers, he is left with only the low-demand consumers. In this subgame, the optimal policy of the monopolist is to charge the low-demand consumers their reservation price r_2 in the second period. But the first-period consumers should be able to realize that the monopolist will charge this lower price second period – and therefore refuse to purchase in the first period!

The problem is that the solution of charging a constant price each period is not *subgame perfect* – the behavior of the monopolist is not optimal for each subgame in which he may find himself. Without the ability to precommit to the constant price schedule, the monopolist may be unable to enforce the constant-price, no-discrimination solution.

In our model, the only subgame perfect equilibrium is to charge $p_1 = r_2$ and sell to both groups of consumers in the first period, *regardless* of the size of the two groups of consumers. Any policy in which firm sells only to the high-demand consumers is not credible in that the firm will always be tempted to sell at a lower price later on. Note that this result follows no matter how small r_2 is, or no matter how many period or groups of consumers are involved. The inability to precommit has essentially eroded the monopoly power of the firm!

This possibility was first pointed out by Coase (1972). He argued that as long as there were no constraints on the rate of sales, all units of the good would be

sold in the first period at marginal cost. (Coase implicitly assumed that there were consumers with arbitrarily low reservation prices, so that the price would be pushed as low as possible. In our model, the price is pushed down to the lowest reservation price.)

This "Coase conjecture" has been analyzed formally by Stokey (1981), Bulow (1982), Kahn (1986), Gul, Sonnenschein and Wilson (1986), and Bagnoli, Salant and Swierzbinski (1987). The analysis is somewhat delicate, as it depends on a sensitive limiting argument about how often the monopolist is allowed to adjust prices. Stokey (1981) shows that if the monopolist can only adjust prices at discrete points in time, then the further apart these points are, the higher are the monopoly profits. Essentially the requirement that prices can only be adjusted at discrete times makes it credible that the monopolist will charge a particular price at least until the next opportunity to adjust it. This tends to make the precommitment constraint on the monopolist less binding, and therefore leads to higher profits.

3.3. Vertical integration and price discrimination

To understand how vertical integration can help enforce price discrimination, let us consider a model where a producer of a primary product, such as aluminum, sells to two competitive industries that product distinct final goods. To take an extreme case, suppose that each of the final goods producers uses one unit of the input to produce one unit of their output. Let $p_1(x_1)$ and $p_2(x_2)$ be the inverse demand functions for the outputs of the two industries; for simplicity suppose that these demand functions have constant elasticities of ε_1 and ε_2, respectively.

In this case, it is easy to see that it will typically pay the producer of the primary product to price discriminate in its provision to the two industries. Effectively the primary producer controls the output in each industry and will set the price of the primary product to maximize its profits. This leads to the conventional solution:

$$p_1^* \left[1 - \frac{1}{|\varepsilon_1|} \right] = c,$$

$$p_2^* \left[1 - \frac{1}{|\varepsilon_2|} \right] = c.$$

The solution is only viable if the monopolist is able to prevent arbitrage. If the industry that receives the lower price can resell to the high price industry, the monopolist's price discrimination policy cannot be implemented. However, there is a strategy that can accomplish much the same thing. Suppose that industry 2

has the larger elasticity of demand and therefore the lower price. Then the monopolist can integrate forward by operating a firm in industry 2, selling its output at price p_2^* and sell the rest of the primary product at a uniform higher price of p_1^*.

In this case the firms in industry 1 will still be willing to pay the higher price, but the firms in industry 2 will be squeezed by the monopolist's transfer pricing to its subsidiary. If nothing is done, the firms in industry 2 will be driven out of business, and the monopolist will be able to enforce the price discrimination outcome through the vertical integration mechanism.

There have been a number of attempts to address the problem of how to detect subsidized transfer pricing of this sort. See Perry (1978, 1980, and Chapter 4 in this Handbook) and Joskow (1985) for a more detailed discussion of price discrimination as a motive for vertical integration.

3.4. Imperfect information

There are conflicting intuitions about the effect of imperfect information in monopolized markets. One the one hand, search activities by consumers represent a price paid by consumers that is not captured by the firm, so it is in the interest of the firm to "internalize" that cost by eliminating price dispersion. On the other hand, if consumers *differ* in their costs of search, price dispersion may be an effective means of sorting consumers and dividing the market, thereby allow for price discrimination in equilibrium.

Sorting consumers based on the basis of their cost of search is especially convenient since it is natural to suppose that consumers who are well informed about prices being offered elsewhere have more elastic demands than consumers who are poorly informed. Or, more generally, consumers with low costs of search will have more elastic demands than consumers with high costs of search. This observation suggests that it may be profitable for stores to use "noisy prices" as a selection device to discriminate among consumers.

This phenomenon was first examined by Salop (1977) and later extended by Berninghaus and Ramser (1980) and Wiesmeth (1982). In Salop's model there is a single monopolist with several outlets; the consumers know the distribution of prices charged at the various outlets, but do not know precisely which stores charge which prices. Hence, they engage in costly search before purchasing the good. Consumers have different search costs and different reservation prices for the good. Salop (1977) and Wiesmeth (1982) show that, under certain assumptions about the joint distribution of search costs and reservation prices, the monopolist can use price dispersion to sort consumers in a way that increases his profits.

Sorting by information costs in an important idea, but the particular case examined by Salop does not seem terribly plausible. Causal empiricism suggests that chain stores, such as McDonald's, typically try to charge uniform prices across their outlets rather than randomizing their prices. It seems that the first intuition – that of minimizing consumer search costs – is a more important consideration for chain stores and franchises than price discrimination.

However, instead of price dispersion across space, we can consider price discrimination across time. Suppose we consider a model where there are several imperfectly competitive firms, each with one outlet, that randomly charges different prices in different weeks. By randomizing their prices, the stores are able to compete for the price-sensitive consumers when their prices are low, but still charge high prices to price-insensitive consumers on the average. One can interpret this behavior as stores engaging in random sales. Varian (1981) has constructed a formal model of this behavior to determine the equilibrium pattern of sales. Here we briefly consider Varian's model.

Suppose that there are n stores selling an identical product which have identical, strictly decreasing average cost curves. Each store chooses the frequency $f(p)$ with which it advertises each price. Some fraction of the consumers read the ads and learn the entire distribution of prices; they therefore only shop at the lowest price store. The rest of the consumers shop at random. Each consumer purchases at most one unit of the good.

We seek to characterize a symmetric Nash equilibrium in this model. The first observation is that since average costs are always declining, there can be no pure strategy equilibrium in which all firms charge a single price. The only possible equilibrium therefore involves a mixed strategy.

It turns out that one can show that the equilibrium frequency distribution must be *atomless* – that is, there can be no prices that are charged with strictly positive probability. The intuition is not difficult: suppose that there were such an atom – some price that all stores charged with positive probability. Then there would be a positive probability of a tie at such a price, so some number of stores would split the informed consumers. By choosing a frequency distribution that charged a slightly lower price with positive probability, a store would capture the entire market of informed consumers in the event that the other stores tied, and only make slightly smaller profits in the other events. Hence, charging one price with positive probability cannot be a profit maximizing symmetric Nash equilibrium.

Given this observation, it is not difficult to calculate the expected profits of a firm. If $F(p)$ is the equilibrium price distribution function, then exactly two events are relevant. Either the firm in question is charging the lowest price, an event which happens with probability $(1 - F(p))^{n-1}$, or it does not have the lowest price, an event which has the complementary probability. If it has the

lowest price, it gets $I + U$ customers, where I is the total number of informed customers and U is the number of uninformed customers per store.

Suppose for simplicity that the firm has constant marginal costs of zero and fixed costs of k. Then the expected profits of a representative firm are

$$
\pi = \int_0^\infty \left\{ (1 - F(p))^{n-1}[pI + pU - k] \right.
$$

$$
\left. + \left[1 - (1 - F(p))^{n-1} \right][pU - k] \right\} f(p) \mathrm{d}p.
$$

If the store is choosing the optimal density function $f(\cdot)$, then the integrand must be constant – if expected profits were higher at some price than at some other price, it would pay the store to charge the more profitable price more frequently. Assuming free entry, this constant level of profits must be zero. This gives us the equilibrium condition:

$$
(1 - F(p))^{n-1}[pI + pU - k] + \left[1 - (1 - F(p))^{n-1} \right][pU - k] = 0.
$$

Solving for $F(p)$ gives us

$$
F(p) = 1 - \left(\frac{k - pU}{pI} \right)^{1/(n-1)}
$$

This distribution function is the unique symmetric Nash equilibrium pricing pattern in this model. The associated density function, $f(p)$, is simply the derivative of this distribution function.

Rather than having the normal bell shape that we expect from a probability distribution, the equilibrium density $f(p)$ has a U-shape – that is, each store charges high and low prices more often than intermediate prices. This seems quite intuitive: a store wants to charge high prices to exploit the uninformed and low prices to compete for the informed. Intermediate prices serve neither goal and so are charged less frequently. However, they are still charged sometimes, since if no one ever charged an intermediate price, it would pay some store to do so.

For more on sales and related marketing techniques, see Conlisk, Gerstner and Sobel (1984), Sobel (1984), Gerstner (1985), Png and Hirschleifer (1986), and Raju (1986).

3.5. Quality differences

It has long been recognized that a monopolist may use quality differences to discriminate among consumers. Witness, for example, Dupuit's insightful remarks:

It is not because of the few thousand francs which would have to be spent to put a roof over the third-class carriages or to upholser the third-class seats that some company or other has open carriages with wooden benches...What the company is trying to do is prevent the passengers who can pay the second class fare from traveling third-class; it hits the poor, not because it wants to hurt them, but to frighten the rich...And it is again for the same reason that the companies, having proved almost cruel to third-class passengers and mean to second-class ones, become lavish in dealing with first-class passengers. Having refused the poor what is necessary, they give the rich what is superfluous. [Quoted by Ekelund (1970).]

This passage clearly states the considerations facing the monopolist: by exaggerating the quality difference in the classes of service, he can effectively price discriminate between customers with different willingnesses-to-pay for the basic transportation service.

This phenomenon has been modeled by Mussa and Rosen (1978), Maskin and Riley (1984) and several others. At a formal level quality difference can be analyzed using techniques of nonlinear pricing. For example, consider the model of nonlinear pricing described in Subsection 2.3. In this model we used $u(x, t)$ to represent the utility of a consumer of type t who consumed a quantity x of the good in question, and used $c(x)$ to denote the cost of producing the quantity x. The pricing function, $r(\cdot)$, measured the cost of purchasing a quantity x.

But suppose instead we let x be the *quality* of a good, $c(x)$ the cost of producing it, and $r(x)$ the price of purchasing one unit of quality level x. Given these substitutions, the quantity–pricing problem considered in Subsection 2.5 is isomorphic to the quality–pricing problem of a monopolist. All of the analysis and results go through virtually unchanged.

The fundamental constraint in the quality–pricing problem is the same as that in the quantity–pricing, namely the self-selection constraint: choosing a pricing scheme that induces consumers of each quality level to prefer their own quality to any other quality. This is the emphasis of the Dupuit passage quoted above: the nature of the quality choices made are exaggerated so as to satisfy the self-selection constraints. We would generally expect that the monopolist would widen the quality choice spectrum in order to more effectively discriminate among the consumers it faces.

The major result of the nonlinear pricing model is that the largest consumer faces a price equal to marginal cost; here the analogous result is that the

monopolist sells the highest quality item at its marginal cost of production. Purchasers of lower quality items in general pay more than the marginal cost of the quality they choose. For more on quality choice see Mussa and Rosen (1978), Maskin and Riley (1984), Milgrom (1987), Gabszewicz, Shaked, Sutton and Thisse (1986), and Oren, Smith and Wilson (1982).

3.6. Monopolistic competition

Although we generally describe price discrimination in terms of pure monopoly behavior, many of the most common real life examples of price discrimination occur in markets with free entry. For example, magazine subscriptions and movies are sold at student discounts; drug stores provide Senior Citizen discounts; and airlines sell trips for different durations at different prices. Certainly none of these industries could be thought of as pure monopolies. Instead, they would probably be characterized by the presence of significant product differentiation and by the availability of relatively free entry and zero long-run profits. In short, most economists would think of these industries as monopolistically competitive.

Of course, the effects we have described in the pure monopoly context are still relevant in a monopolistically competitive environment. We have taken the demand curve facing the monopolist as exogenously given. In equilibrium the demand facing a particular firm depends on the other firms' behavior, but under the common Nash–Cournot assumption, each firm will take those other firms actions as given. Insofar as we only wish to describe a *single* firm's behavior, the previous discussion has been adequate.

However, when we turn our attention to a model with multiple firms, several new effects can arise. In particular, we can ask how price discrimination tends to affect the number of firms in equilibrium and the variety of products they offer. Does price discrimination tend to increase or decrease welfare in a monopolistically competitive environment? What is the most effective form of discrimination when there are heterogeneous products?

There have been a few attempts to address these questions in a monopolistically competitive framework. In general the authors have taken one of the standard models of monopolistic competition, allowed the firms to price discriminate, and then investigated the effect of this price discrimination on the equilibrium behavior of the firms.

Katz (1984b) applied such a research strategy using the Salop and Stiglitz (1977) "bargains and ripoffs" model. In this model, there are two types of consumers, the informed and uninformed. The informed consumers know the prices charged by all stores and the uninformed know none of the prices being

charged. Each type of consumer has a reservation price, L-shaped demand, but the informed consumers wish to purchase a larger amount than the uninformed.

In the Salop and Stiglitz model, in which stores could only charge a uniform price, there could exist equilibria involving price dispersion. They showed that for some parameter configurations, there would be exactly two prices charged in equilibrium: low-price stores competed for the informed consumers and high-price stores exploited the uninformed consumers.

Katz (1984b) asked what would happen if the stores were allowed to price discriminate. He showed that in this case there would exist a unique, symmetric equilibrium where all stores charged the same price schedule. The price schedule exhibits quantity discounting and thereby allows the stores to discriminate between the high-demand, informed consumers and the low-demand uninformed consumers.

In the equilibrium involving price discrimination, the price paid by the informed consumers is lower and the price paid by the uninformed consumers is higher than in the uniform pricing equilibrium. There are more firms in the price discrimination equilibrium than the uniform pricing equilibrium. If uninformed consumers are a small part of the market, then uniform pricing is better in terms of total surplus, but this result is reversed if the uninformed consumers dominate the market.

Borenstein (1985) and Holmes (1986) have investigated a different set of issues in the context of the location model of Lerner and Singer (1937) and Salop (1979). Here the emphasis is much more on the product heterogeneity aspect of price discrimination and on the form that price discrimination can take in such an environment.

In the classic monopolistically competitive location model described by Lerner and Singer (1937), consumers are located around a circle. Each consumer's location indicates his most preferred brand preference, and each consumer faces some travel cost if he consumes a more distant brand. In most treatments of this model, consumers have identical reservation prices and travel costs; Borenstein (1985) relaxes this assumption and allows consumers to differ in both characteristics. Differences in reservation price are interpreted as the usual differences in tastes; differences in travel costs are interpreted as the strength of brand preference.

Borenstein assumes that firms observe some characteristic of consumers that allows them to sort on one or the other of these two dimensions, and charge different prices accordingly. He uses numerical simulation to determine the effectiveness of the two forms of price discrimination. Borenstein finds that the effectiveness of the two forms of price discrimination depends on the type of equilibrium exhibited by the model.

If the monopolistically competitive equilibrium is highly competitive, with firms packed close together in the product space, then sorting by reservation price

does not have much effect on the total size of the market – consumers simply switch form one brand to another. In this case, price discrimination on the basis of reservation price simply redistributes consumers from one brand to another and does little in the way of enhancing total sales or overall welfare. In this sort of equilibrium sorting on the base of strength of brand preference is the more effective form of price discrimination yielding larger price differentials, larger profits in the short run, and more firms in the long-run equilibrium.

On the other hand, if the monopolistically competitive equilibrium is more "monopolized" than "competitive", the strength of preference criterion is not as effective a means of sorting consumers as is the standard reservation price method. This equilibrium is much like the standard story of monopoly price discrimination: output and welfare can either increase or decrease under discrimination as compared with uniform pricing. However, Borenstein shows that in this case welfare is more likely to increase when price discrimination is allowed than in the standard monopoly case. This follows from free entry: allowing price discrimination will increase the profits of the existing firms and thereby induce entry. But with more firms in the market, the market will be more competitive which will be better for the consumers.

The Katz model and the Borenstein model emphasize different aspects of price discrimination under monopolistic competition: Katz is really concerned with the equilibrium form of the optimal second-degree price discrimination rule, while Borenstein is concerned with the sorting criterion involved with using third-degree price discrimination. The questions they investigate are similar, however: How does this new dimension affect the industry performance in a monopolistically competitive environment? It appears that there are a variety of other models of monopolistic competition that could benefit from similar analyses.

3.7. Legal aspects of price discrimination

Price discrimination has long been regarded as a dubious practice from the legal viewpoint, though the complaints about the practice voiced by legislators are typically not those voiced by economists. In this subsection we will briefly review the history and current legal thought on price discrimination. The major source for this material, and an excellent guide to antitrust law in general, is Neale and Goyder (1980).

The Clayton Act of 1914 was the first attempt to make price discrimination illegal. The intent of Congress appeared to have been to restrict the practice of "predatory pricing" rather than to restrict price discrimination per se. The focus of the law involved situations where a supplier provided a good to retailers in one region at a price below cost in order to drive out the competition; the law was intended to protect small businesses from such competition, not to protect the end customers.

The law recognized that not every difference in pricing should be construed as price discrimination, but the attempts to define exactly what was and was not legal were not very successful. The original section 2 of the Clayton Act read:

> That it shall be unlawful for any person engaged in commerce...to discriminate in price between different purchasers of commodities...where the effect of such discrimination may be to substantially lessen competition or tend to create monopoly in any line of commerce; *Provided*, that nothing herein contained shall prevent discrimination in price between purchasers of commodities on account of differences in the grade, quality, or quantity of the commodity sold, or that makes only due allowance for differences in the cost of selling or transportation, or discrimination in price in the same or different commodities made in good faith to meet competition ... .

The astute reader of this survey will note that the proviso allows for many of the forms of price discrimination we have discussed. The ambiguity of the definition of price discrimination resulted in very few cases being successfully brought to trial under this section.

In the early 1930s the spread of chain stores, in particular groceries, brought pressure to bear for strengthening the law against price discrimination, and in 1936 Congress passed the Robinson–Patman Act. As with the Clayton Act, the Robinson–Patman Act was primarily designed to protect the small independent from the large chain, rather than to protect the end users. However, the Robinson–Patman Act had a different focus. Rather than focusing on the powerful supplier who used local price cutting to drive out competitors, the Act was designed to control the large, powerful *buyers* who could use their size to negotiate more favorable terms than their competition.

Section 2 of the Robinson–Patman Act goes as follows:

> That it shall be unlawful for any person engaged in commerce, in the course of such commerce, either directly or indirectly, to discriminate in price between different purchasers of commodities of like grade and quality ... where the effect of such discrimination may be substantially to lessen competition or tend to create a monopoly in any line of commerce, or to injure, destroy, or prevent competition with any person who either grants or knowingly receives the benefit of such discrimination, or with customers of either of them.

As Neale and Goyder (1980, p. 215) put it:

> The Robinson–Patman revision of section 2 of the Clayton Act, whatever its other failing, has at least given rise to plenty of cases; but since many of these have been disposed of by consent order and other informal processes, the legal principles now regarded as applicable have been established in relatively few litigated cases.

Given the focus of the law on protecting small businesses from large chains, it is not surprising that many of the first cases brought to trial under the Robinson–Patman Act were cases of *second-degree* price discrimination – i.e. nonlinear pricing. A notable example was the *Federal Trade Commission v. Morton Salt* (Supreme Court, 1948). Morton Salt charged different prices for different quantities of table salt; the discounts ranged up to 15 percent, but only five large chain stores had ever qualified for this magnitude of discount. It was claimed that this form of price discrimination injured competition at the retail store level. Much of the judicial debate centered on the issue of whether the magnitude of the discounts was substantial enough to create serious injury to competition.

In other cases, such as *American Can Company v. Bruce's Juices*, the debate was centered on the issue of whether the cost reductions associated with larger customers justified the price differences charged. In this case, it was found that discounted schedule was "tainted with the inherent vice of too broad averaging" since 98 percent of the customers involved failed to qualify for the discounts offered. However, in *American Can Company v. Russelville Canning* (1951) the Eight Circuit Court of Appeals decided that a pricing schedule did not have to be precisely related to costs at each level of sales, but only needed to have been adopted in good faith after some reasonable study of costs.

The other main line of defense against the Robinson–Patman Act is that the low prices were charged only to meet "in good faith" the prices charged by the competition. *Standard Oil Company (Indiana) v. Federal Trade Commission* (Supreme Court, 1951) is an important case in point. Standard Oil sold gasoline to larger jobbers who supplied their own stations at prices of 1.5 cents per gallon less than the prices at which Standard directly supplied individual retailers. The Federal Trade Commission argued that this differential resulted in "injuring, destroying and preventing competition between said favored dealers and retailer dealers".

Standard first attempted to show that the difference in price could be accounted for by differences in costs of supply, but this defense was not successful. The second line of defense was to show that other refiners were attempting to get the business of the four large jobbers by offering them equally low prices. Although rejected by the FTC, this defense was accepted by the Supreme Court. Subsequently, the Supreme Court has attempted to clarify exactly what evidence should be brought to bear in adopting a defense of meeting the competitors' prices. [See Neale and Goyder (1980, pp. 224–228).]

Essentially, the burden of proof should be on the alleged price discriminator to show that it was meeting lawful prices of its competition, rather than simply copying the pricing strategy of other discriminators. However, subsequent cases have weakened the nature of the proof required in these cases: typically the defense of meeting the competition is only required to "embody the standard of

the prudent businessman responding fairly to what he reasonably believes is a situation of competitive necessity". *Continental Banking Company* (1963).

Economic aspects of the Robinson–Patman Act

In reading the legal discussion of the Robinson–Patman Act, one is struck by the difference between the legal concerns and the concerns of economists. The legal issues surrounding the issue of price discrimination and the Robinson–Patman Act are those of unfair competition, predatory pricing, and the like. The issues of concern to ecomomists are those of efficient pricing.

As we saw in the discussion of the welfare effects of third-degree price discrimination, we can expect that allowing price discrimination will typically enhance welfare if it provides a means of serving markets that the monopolist would otherwise not serve. Conversely, if the size of the market does not increase under price discrimination, there can be no net increase in consumers' plus producers' surplus. Thus, it would seem that an economically sound discussion of whether price discrimination is in the social interest should focus on the output effects. However, as we have seen above, this consideration has not played much of a role in the legal discussion of price discrimination.

4. Summary

As we indicated at the beginning of this chapter, price discrimination is a ubiquitous phenomenon. Nearly all firms with market power attempt to engage in some type of price discrimination. Thus, the analysis of the forms that price discrimination can take and the effects of price discrimination on economic welfare are a very important aspect of the study of industrial organization.

In this survey we have seen some of the insights offered by the economic theory of price discrimination. However, much work remains to be done. For example, the study of marketing behavior at the retail level is still in its infancy. Retail firms use a variety of marketing devices – sales, coupons, matching offers, price promotions, and so on – that apparently enhance sales. The marketing literature has examined individual firm choices of such promotional tools. But what is the ultimate effect of such promotions on the structure and performance of market equilibrium? What kinds of marketing devices serve to enhance economic welfare and what kinds represent deadweight loss?

One particularly interesting set of questions in this area that has received little attention concerns the computational costs involved in using complex forms of price discrimination. In the post-deregulation airline industry of the United States, airlines have taken to using very involved pricing schemes. Finding the most inexpensive feasible fight may involve a considerable expenditure of time

and effort. What are the welfare consequences of this sort of price discrimination? Do firms appropriately take into account the computational externality imposed on their customers?

Even in more prosaic case of public utilities, pricing schedules have become so complex that households often make the "wrong" choice of telephone service or electricity use. Questions of simplicity and ease-of-use have not hitherto played a role in the positive and normative analysis of price discrimination. Perhaps this will serve as a fruitful area of investigation in future studies of price discrimination.

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