CHAPTER

Empirical perspectives on auctions

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Abstract

The empirical analysis of auction data has become a thriving field of research over the past thirty years. Relying on sophisticated models and advanced econometric methods, it addresses a wide range of policy questions for both public and private institutions. This chapter offers a guide to the literature by stressing how data features and policy questions have shaped research in the field. The chapter is organized by types of goods for sale and covers auctions of timber, construction and service procurements, oil and gas leases, online auctions, internet advertising, electricity, financial securities, spectrum as well as used goods. It discusses the idiosyncrasies of each good and the respective empirical findings.

Keywords

Empirical auctions, Natural resources, Procurement auctions, Online auctions, Financial securities, Electricity, Spectrum, Used goods

1 Introduction

The economic analysis of auction data has developed at a fast pace over the past thirty years. There are several reasons behind this tremendous expansion. First, a very large volume of goods and services in modern economies are either sold or allocated through auctions, ranging from procurement auctions and sales of government securities to spectrum rights or electricity wholesale markets. Moreover, auctions have become more widespread with the advent of the Internet, making auctions the central market/allocation mechanism to place advertisements on users' screens. Given the financial stakes at play, economists have played an active role both in the private and public sectors to address important questions such as market efficiency and revenue generation, and to check whether auctions ensure competition and minimize market distortions.

Second, behavior in auctions with well defined rules can be captured well by economic models emphasizing the role played by private information and strategic behavior. The study of auctions has emerged as one of the most successful applications of microeconomic theory. In this respect, it is quite striking that six Nobel Laureates in Economics awarded since 2001 have publications in auctions, including the 2020 Nobel laureates Paul Milgrom and Robert Wilson for their key contributions to auction theory and inventions of new auction formats used in the sale of spectrum rights. It is worth noting, however, that auction theory does not always provide definitive answers to policy questions such as the choice of auction mechanism. Empirical research plays a key role in filling these gaps through the use of counterfactual exercises. An example is whether central banks should use a discriminatory or uniform price auction in the sale of treasury bills. Empirical research can provide quantitative answers to policy questions such as whether changing auction rules will increase revenue or decrease procurement costs. The empirical analysis of auction data with the help of advanced econometric tools has become a complementary and essential apparatus to address policy questions.

Third, data from auctions are plentiful and quite often publicly available or accessible with relative ease. Auction data are typically high quality in the sense that the analyst typically has detailed information about the auction mechanism, and the data do not suffer from non-response or incomplete response as in surveys. This is attractive for empirical researchers in the field of industrial organization, as data access may constitute a barrier to entry. Frequently, firms are unlikely to reveal sensitive information that can be used by their competitors and/or are subject to confidentiality issues. However, since auctions are often used by public institutions for the sale and allocation of various goods and services, transparency laws and regulations often

require these institutions to make their data available to the public. Online auctions are another area where data have been relatively easy to collect. Many researchers have developed web scraping tools to collect data, whereas others collaborate with companies to gain data access.

This chapter focuses on the empirical auction literature. While we strive to be broad in our coverage, the literature is too extensive to provide a comprehensive review. We include papers adopting both reduced-form and structural approaches. There also exist several surveys with different focuses. McAfee and McMillan (1987), Wilson (1992), Klemperer (2004), Milgrom (2004), and Krishna (2010) offer overviews of the theory of auctions. Klemperer (2004) and Milgrom (2004) stand out as both make the link between auction theory and the practice of auctions. Most other surveys focus on structural econometric methods, with the exception of Laffont (1997), who surveys empirical studies adopting a reduced-form approach, i.e., looking at implications of equilibrium bidding on bid data.

The structural empirical analysis of auctions has generated several econometric challenges given the nonlinearity and complexity of auction models. It is then natural that papers contain a significant econometric component with the development of original estimators and testing procedures. Moreover, understanding the variations in data and other restrictions needed to recover the model, i.e., the identification of the model primitives, became a problem of interest. Hendricks and Paarsch (1995) and Perrigne and Vuong (1999) provide early surveys on the econometrics of auctions with a parametric focus for the former and a nonparametric focus for the latter, with nonparametric identification and estimation (see also Paarsch and Hong (2006)). Athey and Haile (2007) and Perrigne and Vuong (2021) have written the most extensive surveys on the econometrics of auctions (see also Hickman et al. (2012) and Gentry et al. (2018)). Hendricks and Porter (2007) in Volume 3 of this Handbook combine all the aspects of the analysis of auction data by stressing the link between auction theory, empirical practice and public policy questions. Other surveys are specific to some particular auction data such as Tadelis (2016) on e-commerce, Bajari and Hortaçsu (2004) on internet auctions and Hortaçsu and McAdams (2018) on multiple object auctions.

We choose here to adopt a format that is different from the existing surveys. Instead of adopting a classical format classified by theoretical features of auction models, we present the empirical literature on auctions by type of goods sold/acquired at auctions. In the practice of auctions, we first observe that the choice of mechanism used typically depends on the type of goods. For instance, electricity and securities are exchanged using share auctions, whereas internet auctions are organized through an open ascending format. Meanwhile, projects and services are allocated through first-price sealed-bid procurement auctions or scoring auctions. Second, institutional features motivating economic and/or policy questions often depend on the type of goods being sold. For example, electricity auctions are different from securities auctions because electricity generation involves start-up costs that create frictions to entry and exit in response to short-term fluctuations in demand. This results in different mechanism features with distinct policy questions. As another example, noting

that a small proportion of oil and gas tracts sold through auctions is actually drilled, some recent papers question whether the actual system of contingent payments with a royalty rate provides enough incentives to oil firms to explore and drill. Some questions of interest, however, are common across auctions: for example, the problem of entry in which bidders face a cost for the acquisition of information and/or bid preparation is as relevant in construction procurements and timber auctions as it is in takeover contests. Similarly, bid skewing is an issue faced in both timber auctions and construction projects in which bidders bid on estimated quantities for the various items but are rewarded ex post on actual quantities.

In addition to this specific format where papers are presented by types of goods and economic features, we have minimized mathematical displays and technicalities associated with models, identification, estimation, and testing to review instead the economic questions and empirical findings. The Handbook of Econometrics publishes in parallel a volume with an important chapter on the econometrics of auctions, constituting the most recent and updated survey on the identification and estimation of auction models. We invite our readers to consult the chapter by Isabelle Perrigne and Quang Vuong in the Handbook of Econometrics, Volume 7b if they want to learn more about methodologies associated with the empirical analysis of auction data. In this respect, these two chapters are complements and we hope that the combination of the two will offer a broad-ranging survey on the analysis of auction data ranging from methodological contributions to empirical and policy aspects. The empirical analysis of auctions is an endless source of inspiration for research. We hope that this chapter will provide useful information to anyone interested in undertaking research in this field.

The rest of this chapter is organized as follows. Section 2 surveys the empirical literature on timber auctions and questions related to reserve price and revenue optimization, bidders' entry, tests of common value and affiliation, bidders' risk aversion, collusion and market design insights. Section 3 covers the procurement auctions of construction projects and services, and questions related to bidders' asymmetries and unobservable heterogeneity, private vs. common values, entry and competition, collusion, cost synergies and combinatorial auctions, dynamic auctions, subcontracting, scaling and scoring rules, ex post uncertainty, incomplete contracts and renegotiation and bid preferences toward some businesses. Section 4 considers oil and gas lease auctions with topics on common value and winner's curse, bidding and drilling and some miscellaneous topics. Section 5 reviews online auctions (mostly eBay data) with topics related to dynamics and sniping, proxy auctions, online trust, and alternative selling formats such as fixed prices and penny auctions. Section 6 surveys internet advertising auctions and topics on modeling sponsored search auctions, empirical work and reserve prices in such auctions. Section 7 focuses on electricity auctions with models of bidding, tests of theory, generation costs, forward contracts and sequential markets and transmission constraints. Section 8 surveys auctions in financial markets with treasury bills, discrete bids, common value, municipal bonds, auctions in Central Bank operations, derivative markets and takeover auctions. Sec-

tion 9 covers spectrum auctions and incentive auctions, whereas Section 10 surveys auctions of used goods including used cars and collectibles. Section 11 concludes.

2 Timber auctions

Timber auctions have been both an important laboratory and policy application area for the empirical auction literature since the very beginning of this domain of inquiry. Timber, of course, is an essential input into several industries, including construction, furniture, and paper/pulp industries. The U.S. lumber industry (logging, sawmill, and panel) is among the top 10 manufacturing sector employers with half a million employees processing 30 billion board feet of timber (a cubic foot is about 12 board feet). It is also the largest producer of lumber in the world. Because close to 42% of U.S. forests is on public lands, public institutions are in charge of their management, making large data sets available for researchers. Since institutions such as the US Forest Service (USFS) are in charge of the public lands, auctions are the dominant mechanism of sale for the resources. Given the availability of auction data sets, a large number of papers analyzing timber auctions have used data from the USFS or State Departments of Natural Resources, along with a few using French and Canadian data.

Governments sell harvesting contracts on public lands using both first-price sealed-bid auctions and ascending price, open outcry auctions, and in each format, they set reserve prices that are often binding. This provides researchers a valuable opportunity to study these mechanisms comparatively.[1] License holders are required to cut the timber by the end of the contract term (or sell it to someone who will). Harvest contracts are short-term contracts, often only one year. Since tracts of forest are auctioned, there can be uncertainty regarding timber volume and composition. In order to remove some of this uncertainty, USFS posts estimates of timber volume of each species. Bidders typically also conduct their own surveys (called "cruises") to estimate species volume. The ability to conduct cruises further limits the presence of uncertainty about volume and species composition. Further, the USFS typically uses "scale auctions" where the bidder submits a vector of per-unit prices, one for each species on the tract. The USFS then calculates the inner product of this vector with its announced estimates of the relative volume of each species, and the winner is chosen based on the highest inner product. The payments by the winning bidder are based on the volumes harvested of each species, not the USFS estimates, shifting risk to the Forest Service. However, a large majority of empirical papers consider the submitted bids as the inner products thereby ignoring the scale bidding.

The availability of detailed data and specific features of the timber context provide a close to ideal environment to study important aspects of auction theory. We illustrate

[1] Another area where such comparative research has been conducted is oil lease auctions in the Permian Basin as surveyed in Section 4.3.

this by reviewing key papers that utilize timber auction data to focus on these important themes. A central question studied by auction theorists is how to optimize the revenue and/or allocation properties of an auction. Except in certain cases, revenue optimization depends on estimating the structural parameters of the game, especially the distribution of bidder values. We thus start with empirical papers that study the setting of reserve prices/revenue optimization, where timber auctions have been a very fruitful source of data. Another important aspect of real world auctions is that bidders' entry is endogenous, and thus attempts to optimize auction design should take the entry response of bidders into account. Indeed, participation varies greatly from auction to auction in the timber setting; this has led to a number of insightful papers that utilize timber auction data to study bidders' entry decisions and how these affect auction design. A feature of an auction environment that is crucial from the perspective of theoretical modeling and policy advice is determining whether an auction is characterized by common/interdependent vs. private values. Timber auctions provide data for pioneering empirical tests of the common/interdependent value assumption. Similarly, bidders' risk preferences, especially risk aversion, also plays an important role in auction theory. Timber auction data also allow researchers to test hypotheses about and to quantify risk preferences. Finally, the timber auction setting has proved to be a fertile ground to study questions regarding bidders' collusion as well as questions regarding the choice of auction mechanism, as reflected in the work we discuss below.

2.1 Reserve price and revenue optimization

In a private value setting in which bidders have a willingness-to-pay for the good and under additional assumptions such as bidders' risk neutrality, a reserve (minimum) price constitutes an important tool for revenue optimization. In a first-price sealed-bid auction, it induces bidders with a value above the reserve price to reduce their rents or profits and therefore to submit a higher bid. In an ascending auction, bidding his private value is still a dominant strategy and the outcome of the auction only differs when there is a single bidder with a value above the reserve price. It may also lead to a number of unsuccessful sales. Evaluating this tradeoff under independent private values, Myerson (1981) and Riley and Samuelson (1981) derive the optimal reserve price maximizing the seller's expected revenue: if V_0 is the seller's valuation for the good and $F(\cdot)$ the distribution of bidders' values, the optimal reserve price p_0^* solves $p_0^* = V_0 + (1 - F(p_0^*))/f(p_0^*)$. The optimal reserve price is independent of the number N of potential bidders. Implementing an optimal reserve price requires the knowledge of the distribution $F(\cdot)$, the estimation of which is the focus of the empirical work we describe below.

In one of the pioneering papers of the empirical auction literature, using data on ascending auctions of Canadian timber, Paarsch (1997) exploits order statistics to model the winning bid as the second-highest value as in the classical clock auction model. Noting that bidders bid per species based on estimated volumes of timber and pay on actual harvested volumes, the author rewrites the bidder's value in terms of

average harvesting cost and the winner becomes the one with the lowest cost. Using a parametric cost specification and a Weibull distribution, Paarsch (1997) estimates the cost and distribution parameters. Evaluating the optimal reserve price, he obtains an average optimal reserve price 3.4 to 4.2 times larger than the observed one when V_0 value ranges from zero to the appraisal value of timber.

Using USFS ascending auctions, Haile and Tamer (2003) develop an innovative and different approach to analyze ascending auctions. Although a weakly dominant strategy in this auction is for all bidders to bid until their values are reached, a wide variety of dynamic bidding strategies such as jump bidding are typically observed in the data. The authors resolve this through an incomplete auction model based on two simple and intuitive assumptions.[2] Namely, bidders do not bid more than they are willing to pay and bidders do not allow a competitor to win at a price that they can beat. This approach allows them to deviate from the dominant strategy and to account for the positive minimum bid increment. It also allows for jump bidding strategies where bidders do not incrementally bid until they reach their value. They derive an upper bound $F_U(\cdot)$ for $F(\cdot)$ using the first assumption based on the distributions of the order statistics $(i : N)$, denoting the ith-lowest order statistic out of N. The lower bound $F_L(\cdot)$ for $F(\cdot)$ is obtained from the distribution of the winning bid plus the observed minimum bid increment. These two bounds are equal in the button auction model where bidders drop out when their values are reached.

The authors estimate the bounds nonparametrically. When looking at the reserve price, the intuitive argument that consists in estimating bounds for the reserve price using $F_U(\cdot)$ and $F_L(\cdot)$ is misleading. The bounds on the reserve price depend on the shape of the seller's profit $(p - V_0)[1 - F_L(p)]$ and are the prices p_L and p_U along this profit curve at the maximum of profit $(p - V_0)[1 - F_U(p)]$. This may lead to larger bounds for the reserve price. Using data from the State of Oregon, the authors evaluate bounds on the optimal reserve price and the corresponding increase in profits. The results differ depending on the choice of V_0, varying from little gains to up to a 34% increase in seller's profits. The authors also evaluate the probability of unsuccessful sales which increases with V_0 to be above 90%.

Later papers relax the assumption of independent private values starting with Aradillas-Lopez et al. (2013), who consider correlated private values and a different setting. Since bidder values are correlated, the object of estimation is the joint distribution of bidders' values. While identifying the joint distribution is not a problem in the first-price sealed-bid setting, in ascending auctions, the highest bid is not observed. An important contribution of this paper is to show that one does not have to identify the entire joint distribution to compute the optimal reserve price if the number of bidders N is assumed to be exogenous. If this is the case, then the optimal reserve price can be computed using data on the transaction price only.

Specifically, the authors are interested in deriving bounds on seller's profit and bidders' surplus relying on the winning bid only (the second-highest value) and an

[2] Syrgkanis et al. (2021) adopt an alternative approach to derive bounds on value distribution or other functionals of interest under weak assumptions on the information structure.

arbitrary reserve price.[3] They derive expressions for the profit and surplus as functions of the order statistic distributions $F^{N-1:N}(\cdot)$ and $F^{N:N}(\cdot)$. The former can be directly estimated using the winning bids, while the latter can be lower bounded by $\phi^{N-1:N}[F^{N-1:N}(\cdot)]^N$, where $\phi^{N-1:N}(\cdot)$ is the well known monotonic transformation which allows one to obtain the underlying distribution from the second-highest order statistics distribution.[4] The upper bound of $F^{N:N}(\cdot)$ is simply $F^{N-1:N}(\cdot)$. The problem is that these bounds are quite wide. So, the authors exploit exogenous participation, i.e., $F(\cdot|N) = F(\cdot)$ to tighten these bounds and therefore tighten the bounds on seller's profit and bidders' surplus.[5] Using data from the State of Oregon, the nonparametrically estimated bounds on profit are smaller than those under independent values which also tend to overstate bidders' surplus. Also, independent values would lead to a too high reserve price, even lowering sellers' profit. The authors note that an optimal reserve price would violate the mandate of USFS of selling about 85% of offered tracts, which could explain their low reserve prices. In a similar setting, Coey et al. (2017) exploit observed bidders' asymmetries to tighten even further the bounds on seller's profit, bidders' surplus and optimal reserve price. Using data from the State of California and specifying bidders' asymmetry as loggers and mills following Athey et al. (2011), their empirical results display tighter bounds for seller's profit and a lower bound of similar magnitude for the optimal reserve price with an upper bound that is 10% to 15% lower.

In view of Aradillas-Lopez et al. (2013), Coey et al. (2019) propose a test of exogenous participation for conditionally independent private values, i.e., they are independent conditional upon some random variable whose value is unknown to bidders. Their test requires one to observe two order statistics and the number of potential bidders. The basic idea of their test relies on the bidder's exclusion effect. In simple terms, if a bidder is excluded randomly, her exclusion should lower the revenue if she is the highest or second highest bidder. The authors then establish a test statistic as the difference between the actual decrease in revenue between N and $N-1$ bidder auctions, i.e., $\mathrm{E}[B^{(N-1:N)}] - \mathrm{E}[B^{(N-2:N-1)}]$ and the bidder's exclusion effect as the expected fall in revenue from dropping a bidder at random, i.e., $[2/N]\mathrm{E}[B^{(N-1:N)} - B^{(N-2:N)}]$.[6] An interesting part of their result is that they are able to evaluate the gain of using an optimal reserve price without the need to compute it from bounds on revenue. Using data from the State of California, their empirical results show that the average increase in revenue from using an optimal reserve price

[3] Correlation cannot be identified using winning bids only in ascending auctions. See Athey and Haile (2002, 2007).

[4] The distribution of the i-th lowest-order statistic out of N i.i.d. draws from $F(v)$ is given by $F^{i:N}(v) = \frac{N!}{(N-i)!(i-1)!}\int_0^{F(v)} t^{i-1}(1-t)^{N-i}dt$. This function is monotonic, hence $F(v)$ can be inverted from the i-th order statistic distribution. The inversion function is $\phi^{i:N}(.)$.

[5] In the case of correlated values, this means that the m-dimensional joint value distribution with $m < N$ should be the same when considering two levels of competition with m values taken randomly.

[6] Interestingly, Athey and Haile (2002) show that a negative value of this test statistic is consistent with common value while a zero value is consistent with private value under exogenous participation.

is at most 13%. Regarding their test, overall exogenous participation is rejected with a tendency of positive selection in the sense that values tend to be larger in auctions with more participants.[7]

Lastly, Li and Perrigne (2003) analyze the use of secret reserve prices in French public timber first-price sealed-bid auctions. Using a random reserve price set at the seller's value V_0 and assuming independence of V_0 and bidders' values as well as independent bidders' values, the authors use the inversion method (or indirect approach) of Guerre et al. (2000) to estimate the value distribution and evaluate the optimal announced reserve price. Their empirical results show that the optimal reserve price is on average larger by 46% and tends to reduce the number of successful transactions. The total auction revenue slightly increases while the seller's net profit doubles. A random reserve price is also used for the sale of French private timber as studied by Elyakime et al. (1994). Based on a similar model, using an announced optimal reserve price would slightly increase the expected sales including the value of unsold tracts. The authors find that the expected sale amount excluding unsold tracts is 22% larger with a secret reserve price. Given that private owners delegate the sale process to an auctioneer who is paid as a fixed proportion of sales, the auctioneer may prefer to keep the reserve price secret. It appears that unsold tracts are proposed for sale right after the auction to the bidder with the highest bid. Elyakime et al. (1997) develop and estimate a bidding model accounting for bargaining in a second round. In particular, the seller sets the reserve price above his value. The use of a secret or random reserve price remains, however, a puzzle for economists, who justify it with bidders' risk aversion as Li and Tan (2017) or by favoring bidders' entry in a common value setting as Bajari and Hortaçsu (2003).

2.2 Bidders' entry

While much of the classical theory of auctions assumes that the number of bidders in an auction is exogenously set, in reality bidders decide whether or not to participate in an auction. Hence, e.g. revenue ranking results that imply differences across bidders' ex-ante payoffs across different mechanisms may need to be adjusted to allow for bidders' entry decisions. Entry decisions are an important feature of timber auctions, as bidding may require acquisition of information such as analysis of documents or site visits/cruises to estimate the value of the good so that the bidder can form a value and prepare a bid. This entails some costs and leads to an entry decision before the auction takes place which endogenizes the number of participating bidders. This calls for a consequent revision of policy questions. Indeed, the papers we discuss below show that accounting for entry leads to different policy conclusions.

Athey et al. (2011) combine sealed-bid and open-ascending auction data to study entry while considering asymmetry between loggers (L) and mills (M) that differ

[7] Aradillas-Lopez et al. (2016) fail to reject independence of values in N in an alternative test for ascending auctions. In the same vein, see also Liu and Luo (2017) who develop a test based on integrated quantile functions for first-price sealed-bid auctions.

through their manufacturing capacity. Using USFS data from Northern states (Idaho and Montana) and California, a detailed reduced-form analysis on bidders' participation and bidding shows that sealed-bid auctions attract 10% more loggers and generate 6% more revenue than open-ascending auctions. The authors consider an entry model in the spirit of Levin and Smith (1994) in which bidders learn their private value after entry, with bidders' asymmetries in entry costs κ_L, κ_M and (independent) private value distributions $F_L(\cdot)$, $F_M(\cdot)$ with the latter stochastically dominating the former. The entry game is a mixed strategy defining an entry probability p based on expected profit upon entry covering the entry cost and an uncertain number of bidders. Given that the number of potential bidders N (before entry) is unobserved, the authors adopt a data-driven estimate for it. The authors then use the inversion method of Guerre et al. (2000) with parametric specifications for the bid and private value distributions allowing for unobserved heterogeneity u to estimate $[F_L(\cdot|X,u,N), F_M(\cdot|X,u,N)]$, where X is a vector of observed heterogeneity. Using this information with an estimator for the entry probability, the entry expected profit condition estimates the entry costs. The entry costs are important, representing 6.4% and 11.2% of the median expected profit gross of entry cost in Northern and California forests, respectively. Accounting for endogenous entry explains the difference in the observed average sale price in favor of sealed-bid auctions. Indeed, under exogenous participation the auction format has little impact despite important asymmetries while under endogenous participation, sealed bidding favors loggers in both entry and allocation, and tends to increase expected revenue.

Li and Zheng (2012) extend the analysis of bidders' entry in first-price sealed-bid auctions under independent private values by considering the Samuelson (1985) entry model. In contrast to Levin and Smith (1994), bidders know their value before entry and entry is *selective* in the sense that the entry game defines a threshold value V^* above which bidders enter. The entry cost has a different interpretation. Under selective entry, κ is a cost associated with bid preparation while under nonselective entry, it is also a cost associated with information acquisition. Both models have different implications regarding the optimal reserve price. Under selective entry, it is given by the previous formula while under non selective entry it is just the seller's value V_0. The authors estimate parametrically the value distribution, accounting for unobserved heterogeneity. Using timber auction data from the Michigan Department of Natural Resources, the entry costs are about 6.8% of a bidder's private value on average under selective entry, while they are 9.5% under non selective entry. Overall, the estimated values and information rents are larger under selective entry. A natural question is which of the two models is the most appropriate to explain the data. Using a Bayesian factor as the ratio of the marginal likelihoods for the two estimated models, selective entry is favored.

Li and Zhang (2015) extend the analysis of bidders' entry by considering affiliated private values and asymmetry in both values and entry costs where asymmetry arises from the distance of the bidder to the timber tract. Because there is no closed-form equilibrium for asymmetric sealed-bid auctions, the authors rely on numerical meth-

ods and parameterize the joint distributions of values and entry costs with copulas.[8] Analyzing data from the Oregon Department of Forestry, they find a low degree of dependence among values but a larger degree among entry costs. The authors assess the impact of a hypothetical merger between two bidders in the auction. Ignoring entry, the welfare of merging bidders decreases and the payoff for non-merging bidders decreases, invariant to whether the merger happens between the two bidders with the largest or the shortest hauling distance. Accounting for entry, the welfare of merging bidders increases under the former scenario and slightly decreases under the latter scenario, whereas the effects are opposite for the payoff for non-merging bidders.

Roberts and Sweeting (2016) propose an empirical assessment of the effect of the U.S. government's firm bailout in 1984 on competition in USFS auctions. As a recap, the price of lumber increased in the late seventies (the so-called timber bubble) but decreased dramatically in the eighties with mortgage rates negatively impacting the demand for new lumber. Firms which bought lumber at a high price in the late seventies faced potential bankruptcy at lower prices in the eighties. To maintain competition in auctions, the Federal government offered these firms the chance to buy out their existing contracts. The authors consider a general model of selective entry that nests the Samuelson (1985) and Levin and Smith (1994) frameworks. In their model, every bidder receives a noisy informative signal S_i of his value V_i leading to the conditional distribution $F(V_i|S_i)$. The entry game defines a threshold signal S^* above which the bidders enter. In the bidding game, bidders learn their values and bid in a second-price auction.[9] To avoid the problem of multiple equilibria, the authors impose that the signal distributions and entry costs across mills and loggers are the same, whereas the value distributions differ only through their means across mills and loggers to allow for asymmetry.

Using parametric specifications for the underlying distributions, the empirical results confirm that entry is selective, and that mills have asymmetrically larger values. The entry cost represents 2.7% of the bid on average. The simulation of an auction without the bailout, i.e., without the insolvent firms, would lead to a decline of 11.1% of revenue. The authors explain this sharp decline by (i) the fact most that insolvent firms were mills which have larger values, (ii) the substantial value variability that tends to increase order statistics when adding one bidder and (iii) selective entry, though the authors find it at a moderate degree. Indeed, a model with no selective entry *a la* Levin and Smith (1994) would yield an increase in revenue with the removal of an entrant. Overall, they find that the bailout of insolvent mills increased competition as well as timber auction prices. Lastly, the authors also assess the implementation of an optimal reserve price and find that an additional bidder is more valuable than an optimal reserve price, thereby confirming Bulow and Klemperer's (1996) result.

[8] Gentry and Li (2014) provide conditions under which models of selective entry are identified.

[9] The Levin and Smith (1994) model corresponds to the case in which signals and values are independent, while the Samuelson (1985) model applies to the case in which signals and values are perfectly correlated.

2.3 Tests of common value and affiliation

The source of private information in auctions is a crucial modeling element in auction theory, as it determines the nature of strategic behavior. In a private values environment, bidders' valuations of the good being auctioned do not depend on the information their rivals may have. In an interdependent or common value environment, bidders face a common uncertainty about the value of the good, and hence care very much about how other bidders value the good, or what information other bidders have about the good. As shown in seminal theoretical work on auctions, e.g. Wilson (1977) and Milgrom and Weber (1982), the presence of a common value environment introduces an additional level of complexity to bidding strategies, and also has subtle implications regarding the choice and design of auction mechanism.

Timber auctions provide fertile ground for empirical researchers to develop tests of the common vs. private value paradigms. Since the value of the timber tract depends on the actual composition and volume of timber species, bidders' cruises of the tract are an important source of private information. To the extent that bidders may benefit from learning the information gathered by other bidders, a common value component may be present. Another source of common valuation is the common uncertainty faced by bidders regarding the sale of the timber.

Haile et al. (2006) propose a general test of common value in first-price sealed-bid auctions. Their test relies on equilibrium conditions and the well-known theoretical result that states that the winner's curse is more severe with more competition. In a common value setting, getting a large signal is considered optimistic, leading to the so-called winner's curse. Moreover, larger competition tends to render this curse even more severe. Bidders anticipate the winner's curse and revise their equilibrium bidding accordingly.[10] Using the inverse strategy of Guerre et al. (2000), Laffont and Vuong (1996) show that the inverse equilibrium strategy expressed in terms of bids gives the conditional expectation of the bidder's valuation upon his signal and on his equilibrium bid being pivotal. Considering the bid distribution of the maximum of other bids $B^{\max} = \max_{j \neq i} B_j$ conditional on his bid B, it gives

$$V(x, x, N) = B + [G_{B^{\max}|B}(B_i|B_i)/g_{B^{\max}|B}(B_i|B_i)],$$

where $V(x, x, N) = \mathrm{E}[V_i|S_i = x, \max_{j \neq i} S_j = x]$, V_i and S_i being the bidder's value and signal, respectively. In a private value setting, the left-hand side is equal to V_i. Under exogenous participation, the authors show that $V(x, x, N)$ is strictly decreasing in N under common value while it is invariant in N under private values. Their nonparametric test is based on the quantiles or mean of the distribution of $V(x, x, N)$ with N varying. They extend their test to endogenous participation under an exclusion restriction, with an instrumental variable affecting the number of bidders but not

[10] Looking at bids declining in some level of competition to test for the winner's curse is misleading (see Laffont (1997)). Pinkse and Tan (2005) show that affiliated private values can also lead to bids increasing or decreasing in the level of competition.

their value distribution.[11] Using USFS auctions in the western half of the U.S., the authors fail to find evidence of common value in scale auctions whereas it is mixed in lump sum sales where there was a stronger argument for common value. Compiani et al. (2020) provide a significant generalization of this testing strategy in the case of endogenous bidder participation and apply it to the setting of oil and gas lease auctions (see Section 4.1).

Another important feature of the informational environment of the auction is to determine whether bidders' private information is independent or correlated. While independence of private values is assumed in a large proportion of papers, there could be some positive dependence. The presence of affiliation among private values has an important effect on policy recommendations as it affects the revenue ranking of auction mechanisms and the reserve price. Li and Zhang (2010) propose a test of affiliation in a general setting of affiliated values *a la* Milgrom and Weber (1982) with endogenous entry. The affiliation of private information (signals and entry costs) among bidders leads to affiliation of entry decisions. The authors model the latter through a binary choice model with a reduced form entry model and test the positive correlation of the error terms. Applying their test to first-price sealed-bid auctions by the Oregon Department of Forestry, they find a significant positive correlation.

Aradillas-Lopez et al. (2016) propose a general testing procedure based on properties of order statistics under independence and correlation. They consider symmetric private value ascending auctions, in which the distribution of the transaction price is $F^{(N-1:N)}(v)$, that is, the $(N-1)$th order statistic among N bids. Recall that $\phi(\cdot)$ is the monotonic transformation from the order-statistics distribution to the underlying distribution. Under independent private values, at given N, $\phi^{k:N}[F^{(k:N)}(v)] = \phi^{N-1:N}[F^{(N-1:N)}(v)]$ for any $k \leq N-2$. However, if values are not independent, this holds with an inequality. They also extend their test using the distribution of the $(k:N)$th order statistics and the distribution of the winning bid plus the minimum bid increment. Assuming exogenous participation, they exploit variations in the level of competition since independence requires that the transformations must hold for different values of N. Applying their test to open auction data from the State of Oregon, they reject independence of values and find strong evidence of positive correlation.

2.4 Bidders' risk aversion

Departure from risk neutrality leads to deviations from celebrated theoretical results such as revenue equivalence. For example, while risk aversion does not affect weakly dominant bidding strategies in private value second-price auctions, bidding in a first-price auction depends on the level of risk aversion. In the timber context, risk aversion becomes a salient feature due to the institutional feature of scaled bidding – bidders make their payments based on actually harvested quantities while bids are based on

[11] To account for observed heterogeneity, this paper proposes a method to homogenize the bids. This method became very popular in the empirical auction literature using nonparametric estimators.

estimated quantities of timber species. In a pioneering paper, Athey and Levin (2001) report evidence of risk diversification across species in bidding behavior. This is consistent with bidders' risk aversion. Despite bids possibly being associated with firms or large institutions that are generally considered risk neutral, bids are submitted by individuals and risk aversion is a natural component of individual behavior.[12] Several papers assess the extent of risk aversion in timber auctions under independence of private values.

Campo et al. (2011) develop a semiparametric estimator based on the inversion method of Guerre et al. (2000). It is semiparametric since model identification requires a parameterization of the bidders' utility function and a parametric conditional quantile. This quantile restriction exploits variations in the observed characteristics X and the number of potential bidders N. Using the USFS sealed-bid auction data for the western half of the US, the authors find a constant relative risk aversion (CRRA) measure equal to 0.278, while the constant absolute risk aversion (CARA) measure is equal to $10^{-6} \times 3$. Under a CRRA specification, risk aversion acts as an increase in competition, leading to a decrease in bidders' information rent. The authors also simulate the average optimal reserve price, which is about 19% less than under risk neutrality. Given that bidding is more aggressive under risk aversion, the reserve price does not need to be as large since bidders naturally reduce their information rents. Lu and Perrigne (2008) exploit data from both sealed-bid and ascending auctions to estimate nonparametrically the bidders' utility function since risk aversion does not affect bidding in an ascending auction. Using the same data as in the previous paper, the authors' results display a concave utility function which is neither CRRA nor CARA, though CRRA provides a better fit than CARA. The results also show some dependence of the private values on N.

Li et al. (2015) study selective entry with risk averse bidders across sealed-bid and ascending auctions and conduct a reduced-form analysis of their model predictions on USFS auction data from all regions. Their model predictions state that under Decreasing ARA (DARA) preferences, ascending auctions enjoy a larger participation rate than sealed-bid auctions, whereas it is similar for CARA preferences and reverse for Increasing ARA (IARA) preferences. Using a parametric model, the empirical results suggest support for DARA preferences.

Given that Athey et al. (2011) and Roberts and Sweeting (2013, 2016) document unobserved heterogeneity in USFS auctions, Grundl and Zhu (2019) combine bidders' risk aversion and unobserved heterogeneity in analyzing USFS auction from Southern states.[13] Combining the methods of Krasnokutskaya (2011) for unobserved heterogeneity and the exclusion restriction of exogenous participation, i.e., $F(\cdot|N) = F(\cdot)$, of Guerre et al. (2009) for risk aversion, the authors find risk neutrality when accounting for unobserved heterogeneity. When the latter is ignored, they

[12] Using experimental data, Bajari and Hortaçsu (2005) exploit structural methods to estimate various models including learning, bidders' risk aversion and quantal response equilibrium to assess which model better fits the data. Their results point to bidders' risk aversion.

[13] The importance of unobserved heterogeneity in auction data is discussed in more detail in Section 3.1.

find CRRA measures ranging from 0.57 to 0.85. Intuitively, risk aversion tends to increase bid dispersion. If the bid distribution is not too sensitive to variations in N relative to bid dispersion, risk aversion is found in the absence of unobserved heterogeneity. With unobserved heterogeneity, the bid variation is attributed to dispersion in the bidders' private information. As a matter of fact, the overestimation/underestimation of risk aversion depends on the degree of correlation between unobserved heterogeneity and N. In the present case, it suggests a positive correlation between the two.

In 1976, the federal government recommended the use of first-price sealed-bid auctions over ascending auctions, mostly to reduce the incidence of collusion. In the northwestern states, this constituted a dramatic change as these states were mostly using ascending auctions. Aryal et al. (2018) argue that this change could introduce some ambiguity about the private value distribution and potential ambiguity aversion. Relying on maxmin expected utility, the authors find ambiguity in Northwestern states' data, while the ambiguity is minor in California which adopted first-price sealed-bid auctions well before 1976. Simulating the auction as if bidders knew the true value distribution would lead to a decrease of 3% to 4.5% of USFS revenues since ambiguity leads to aggressive bidding. The loss would be reduced to 2.5% to 3.5% if using an optimal reserve price. Given that CRRA is found at relatively large levels, the authors assess the roles played by risk aversion and ambiguity aversion in increased bids. Risk aversion is found to be the main determinant as it increases bids by 7.6% whereas ambiguity raises bids by only 2%.

2.5 Collusion

Collusion at US Forest Service sales was suspected in the seventies. Despite allegations and a congressional hearing in 1977, there have been few cases brought to prosecution. As detailed previously in this section, the USFS uses both open ascending and first-price sealed-bid auctions. The former mechanism could favor collusion because bidders are face-to-face and could react more easily to opponents' behavior. Moreover, if the collusive cartel allocates the good to the member who values it the most, then it is not in the interest of any of the other cartel members to deviate and participate in the auction, as the highest value cartel member can always outbid the deviating cartel member. For this reason, auction theorists recommend the use of first-price auctions. We review here five papers on collusion in timber sales. The first two papers study ascending auctions by the USFS whereas the three other papers focus on Canadian timber sales on public lands that adopt a first-price sealed-bid format. In this respect, Section 3.4 reviews a number of papers on collusion in construction procurements showing that first-price sealed-bid auctions are not exempt for the collusion problem.

Baldwin et al. (1997) note that the USFS offer of timber has dispersion across the years. A larger inventory of timber than usual could contribute to decreased bids following a simple oversupply argument. The authors consider several models based on order statistics of non cooperative and cooperative behaviors. For instance, in an

ascending auction, when a cartel member wins, the price paid is the largest value of the non-cartel members as opposed to the second largest value of all bidders. After controlling for timber supply conditions in the vector of covariates, the authors find that the low observed bids are better explained by collusive behavior. Athey et al. (2011) observe that timber prices are lower at open auctions than at first-price sealed-bid auctions. If the values are independent of the auction mechanism, the distribution of values that is estimated from the sealed-bid sales should predict the prices at open sales. If this is not the case, this could be due to collusion at open auctions. The authors consider a simple model of cooperation for mills which decreases the winning bid and increases their profits. They focus on open auctions with zero, one, and two and more participating mills. Under exogenous entry, prices are well predicted except when two mills or more are present at the auction. The results are confirmed under endogenous entry to a lesser extent. This suggests collusion of mills at open sales. To further assess the effect of collusion by mills, they conduct counterfactuals considering (i) competitive bidding in both auction formats, and (ii) moderate collusion in open auctions in the sense that mills would collude 18% of the time. Their results suggest that asymmetry among mills and loggers and endogenous entry contribute to only a small difference in prices across open and sealed-bid auctions. However, collusion could contribute to a difference in prices of around 10% on average. Their results also suggest different levels of competitiveness across regions.

List et al. (2007) and Price (2008) study potential collusion in first-price sealed-bid auctions in British Columbia. After noting several factors favoring collusion such as product homogeneity and local markets, the former paper finds inspiration in the treatment effect literature when the treatment assignment (collusive bidder) is not observed. They conduct a reduced-form bid analysis to assess conditional independence and exchangeability.[14] Their empirical results select 130 suspect bidders violating equality of bid distributions and exchangeability, i.e., equality of coefficients in bid regressions. The authors conclude to a mixed evidence of collusion. Price (2008) focuses on the spatial proximity of bidders as a factor favoring collusion. The author observes that firms located in the same town or close by tend to bid 10% to 13% less. Using a similar approach, exchangeability is rejected for firms in close proximity, suggesting cooperative behavior. Being aware of the potential value asymmetry of being close by due to, say transportation costs, the author provides discussions with the industry suggesting that distance is not causing asymmetry in values.

Analyzing the same data, Schurter (2020) adopts a structural approach and shows that a model with asymmetric independent private values and collusion is not identified if the identities of colluding bidders are unobserved. He shows that an exogenous change in the level of competition can help to achieve identification. In particular, an instrument that is independent of the values, conditional on auction characteristics such as the reserve price, restores identification. Based on this restriction, the author

[14] Bajari and Ye (2003) were the first to show that collusion leads to rejection of conditional independence and exchangeability of bids. We review this paper in Section 3.4 on collusion in construction procurements.

develops a test of collusive behavior by testing for each bidder whether his estimated values are independent of the reserve price. After accounting for censoring (because the reserve price is binding) and selecting a number of firms based on their participation, the test results suggest that seven firms could be colluding. After checking for their common participations in auctions, 4 firms end up not bidding competitively. A counterfactual shows that the revenue would increase by 3.2% if these 4 firms would behave competitively, which is a moderate cost of collusion compared to the results above.

2.6 Market design insights

In Section 2.1, we study the choice of a reserve price as a market design instrument to increase auction revenue. Alongside reserve prices, several other mechanism details can be tuned to achieve policy objectives. Timber auctions constitute an opportunity to study the effectiveness of these market design features, thanks to the presence of naturally occurring variations in the data. Some market design features that have been studied are set aside auctions, scale bidding, and whether to run auctions with simultaneous or sequential bid submissions. Another question that has been studied using timber data is how the presence of resale opportunities/secondary markets affects performance in the primary auction. We now survey work investigating these questions.

2.6.1 Set asides and subsidies

Set-aside auctions are reserved for small businesses that have less than 500 employees and self-manufacture the timber or resell to other small businesses. Athey et al. (2013) study the set-aside policies implemented by the USFS and question whether auctions with subsidies to targeted bidders would be a better mechanism in terms of revenue and efficiency. Because such a policy affects bidders' entry, their model includes entry as well as bidders' asymmetry following Athey et al. (2011). Their empirical analysis relies on California's open and sealed-bid auction data. Data show that set-aside auctions attract more small bidders relative to unrestricted sales and that prices tend to be lower in sealed-bid set-asides. Loggers are more likely to be small firms with lower values. The offered tracts differ in terms of logging costs since tracts with large logging costs are less likely to be sold in set-aside auctions. With estimated value distributions and entry costs for mills and loggers, the authors simulate opening up to large bidders. On average, two large businesses would participate, although there are fewer entrants overall. The proportion of sales won by small firms would decline sharply by 51% whereas revenue and surplus would increase by 6% and 21%, respectively. In view of Myerson's (1981) recommendation to appropriately handicap strong bidders to increase revenue, the authors simulate the outcome of a subsidy to small businesses that would discount their bids by a factor $1 + \alpha$ for values of α

ranging from 5% to 20%.[15] A low subsidy, i.e., 5 or 6%, would suffice to have small businesses winning a satisfactory proportion of sales and timber volume with a 4% increase in prices, a 2% efficiency gain and a slight decrease in large firms' profits. An alternative policy would be to select sales to be set-asides in which expected inefficiency would be small relative to subsidized auctions.

2.6.2 Scale bidding

As mentioned briefly above, a large proportion of sales, especially in the western half of the US, is under scale auctions in which bidders submit unit prices per species based on estimated volumes but pay on ex post harvested amounts. The idea behind this mechanism is to reduce the risk borne by the winner on the volume uncertainty. Athey and Levin (2001) consider a general model with affiliated values which include some common value element for two species. Bidders perform their own cruise and form their own estimates of volume. Then they choose their total bid and how to allocate that bid across the two species. Bidders tend to skew their bids onto the species they believe the USFS overestimated. The skewing phenomenon amplifies when the difference between the bidder's and USFS estimates is large. The authors test their model predictions on oral auction data for the states of Oregon and Washington and use ex post cutting data for the actual harvested amounts. Their reduced-form results show that skewing occurs in the right direction, is more important the more a species is overestimated by the USFS, and higher-ranked total bids tend to be more (and accurately) skewed. Empirical evidence also shows that bidders do not skew to the maximum possible extent, which is consistent with bidders' risk aversion (see also Section 3.8 on scaling rules in procurement auctions of road construction).

2.6.3 Sequential vs. simultaneous bid submission

On a different policy question, Roberts and Sweeting (2013) study whether sequential bidders' entry and bid submissions should be used instead of the simultaneous open auction as practiced by the USFS. In a simultaneous entry open auction, all bidders simultaneously decide to enter and the auction proceeds as in a button auction. Bulow and Klemperer (2009) develop a stylized mechanism in several rounds that Roberts and Sweeting (2013) extend to bidders' selective entry and asymmetry. In each round, the entering bidder observes the history of offers and attempts to outbid the current highest bidder. This new entrant can make a jump bid, which could deter later bidders from participating. This sequential mechanism could generate higher revenues for the seller and efficiency gains because later bidders could condition their participation decisions based on earlier bids. Bulow and Klemperer (2009) show that sellers prefer the simultaneous auction while buyers prefer the sequential format. Roberts and Sweeting (2013) show that with more selective entry and/or entry costs,

[15] Asymmetries complicate the computation of such counterfactuals. Instead of using numerical methods as in other papers, the authors discretize the bid space and use Athey's (2001) result which states that it converges to the equilibrium on the continuous bid space.

the difference in seller's revenues in favor of sequential auctions is increasing. Specifically, the authors find that under moderate entry selection and entry costs (at only 2% of the average winning bid), the sequential mechanism would lead to a revenue increase of 1.8% compared to a revenue increase of 0.2% with simultaneous auctions and an optimal reserve price. Their results also confirm an efficiency gain because of loggers' entry, who experience an important profit increase whereas mills' profits slightly decrease. They supplement their empirical analysis with a detailed simulation exercise with varying selection degree and entry costs which confirms the dominance of the sequential mechanism. It is unclear, however, whether these results hold in a more complex sequential auction in which bidders could buy several timber lots. Moreover, the sequential mechanism seems difficult to implement in practice because the sale process would require more time.

2.6.4 Resale

Many goods markets are characterized by a primary market in which the good is auctioned by a monopolist seller, followed by a secondary market in which a larger number of buyers and sellers re-trade the good. In the timber industry, there are active secondary markets where firms can resell their won contracts or buy timber. Haile (2001) studies how the possibility of resale after the auction affects bidding in open auctions within an independent private value setting. Specifically, a bidder's value is endogenously determined by the option value of buying/selling in secondary markets. The resale seller effect, i.e., the firm can resell its won contract, is positive, whereas the resale buyer effect, i.e., the firm can still buy timber afterward, is negative. A larger level of competition in the auction magnifies the seller effect since $N-1$ (losers at the auction) can be interpreted as a signal of the number of buyers in the secondary market. In the first-stage auction, bidders have a noisy signal S_i of their use value U_i, with S_i and U_i being affiliated but independent across bidders. The use value is revealed only after the auction. In the second-stage auction, the winner of the first-stage auction holds an ascending auction with M bidders, including the $N-1$ losers of the first auction. The conditional expectation $\mathrm{E}[U_i|S_i]$ can be interpreted as the bidder's value V_i without resale opportunities. The bidding equilibrium is in the spirit of Milgrom and Weber (1982) with a dramatically different prediction. The resale opportunity makes the values at which bidders drop increasing in N, while the effect is reverse under common values because of the winner's curse correction. Using oral auction data from Northern states and California, the author tests his model predictions using the highest recorded bids. Because N can capture unobserved heterogeneity and invalidate the model predictions, the author uses the number of sawmills in the county and vicinity as instrumental variables. Empirical evidence strongly supports the endogenization of resale opportunities, depending on whether or not bidders observe exits of previous bidders before their own withdrawal from the auction. As expected, resale opportunities affect revenue ranking and efficiency.

3 Procurement auctions of construction projects and services

Procurement auctions are widely used to award construction projects and various services in both the public and private sectors, and contribute to an important share of GDP (estimated at 10% in the U.S. for public procurement only). The importance and ubiquity of procurement auctions, along with the many context-driven idiosyncrasies and design challenges, have been a limitless source of research questions for economists. As a comprehensive survey of the entire literature on procurement auctions is not possible within the confines of this chapter, we focus on several important themes studied by researchers in this area.

Procurement auctions are typically *reverse auctions* in the sense that an institution or buyer is looking for businesses or sellers to perform a specific task at lowest cost. Most procurement auctions are conducted as sealed-bid first-price (lowest bid) auctions. Contracts being auctioned typically consist of a list of line items or tasks that the contractor is required to supply, along with the estimated quantities. As in scaled sales in the timber auction context, bids are submitted in the form of a vector of per-unit prices for the line items. The agency then calculates the lowest bid based on the estimated quantities. Payments are often based on the actual quantities used of the line items.

Firms bidding for procurement projects are often observably differentiated by their plant capacity and distance to project location. These factors affect cost, thus asymmetries across bidders are important to take into account, as we discuss below. The main source of uncertainty and private information is bidders' costs. Since procurement projects have specific needs and challenges, bidders' costs may have a common component due to project specificities that are not easily observable to the analyst studying bid data. Hence, allowing for project specific unobservables is an important part of the empirical exercise. The nature of uncertainty, e.g. whether a private vs. a common values environment best characterizes the environment, also plays a crucial role in determining bidders' strategic considerations.

As in timber auctions, bidders' entry and the potential for collusion are crucial determinants of the competitive structure of the auction. Additionally, projects in close proximity or scope are often auctioned separately. Hence accounting for cost synergies across projects is an important factor in studying bidding behavior. Capacity constraints may also play an important role and introduce a dynamic consideration into the bidding decision: Winning a project today affects the firm's ability to win a perhaps better contract tomorrow. The presence of complementarities across different projects also brings up the question of whether to auction projects separately or in a bundle, and how to auction multiple projects at the same time.

As noted above, procurement auctions are typically conducted as scale auctions. In some contexts, the auctioneer also implements a *scoring rule*, which allows departure from the lowest bid criterion by taking into account quality differences. Procurement auctions also bring up important issues at the interface of auctions and economics of contracts. Many projects are led by a general contractor who works with

a team of subcontractors who perform specific tasks. Understanding how subcontracting affects bidding behavior and auction outcomes therefore becomes important. Since cost overruns and project delays are a frequent occurrence, contract provisions regarding these ex post contingencies also affect bidding behavior and auction outcomes.

Given the sheer size and importance of procurement auctions, market/mechanism design questions have attracted a lot of attention from researchers in this area. In addition to classic auction format questions, market design considerations that respond to specific requirements of the procurement context have also generated a rich literature. An important strand of this literature is the study of bid-preferencing and affirmative action policies in the procurement context.

3.1 Asymmetries across bidders and unobservable heterogeneity

Asymmetry among bidders has been long acknowledged in empirical auctions. In his dissertation, Bajari (1997) was the first to show that the firm's distance to the contract location is an important source of cost differences among firms due to transportation costs, leading to asymmetric bidders. In the same spirit, Flambard and Perrigne (2006) study distance asymmetry in snow removal contracts in Montreal (Canada) by classifying firms depending on their location relative to the job location. Empirical evidence confirms different participation patterns, winning probabilities, and bid distributions. Using Guerre et al.'s (2000) inversion method, the difference in costs is around 6.5%, leading to an inefficient allocation in 24% of the procurement auctions. A subsidy to weak (high cost) firms to reestablish symmetry among businesses would increase the proportion of contracts allocated to those firms by 61%. Taking into account a potential cost of public funds, the overall cost would decrease by 1.34% under exogenous participation.

In the following papers, asymmetry arises from various sources. Relying on a reduced-form analysis, De Silva et al. (2003) and De Silva et al. (2009) show evidence of asymmetry between entrants and incumbents in Oklahoma road construction procurements, with entrants bidding more aggressively than incumbent businesses. Entrants seem to be less informed about the cost and/or less capable to evaluate the cost. In 2000, the State of Oklahoma eliminated the bidding differences between entrants and incumbents by releasing information on their own cost estimate. Campo (2012) highlights another source of asymmetry due to different attitudes toward risk. Indeed, construction is an activity subject to an important uncertainty as further reviewed in Section 3.9. Drawing from Campo et al. (2011), the author analyzes Los Angeles city procurement auctions combined with information on firms' foundation years and licenses. Under CRRA risk preferences, firms tend to be less risk averse with experience. In addition, firms tend to differ in costs based on their financial health, measured as the number of days it pays its creditors. These differences could be due to larger borrowing costs for firms in poor financial health.

The previous papers assume cost independence. Hubbard et al. (2012) analyze resurfacing contracts by the Michigan Department of Transportation. Using a copula

for the joint cost distribution, the authors assess the degree of positive dependence among firms' costs. In addition, low-cost bidders tend to be more aggressive than under independence because they are aware that other bidders are also likely to have low costs as well. Using the same data, de Castro and Paarsch (2010) perform a test of affiliation and find a strong affiliation in road resurfacing contracts. This strong affiliation which acts as fierce competition could explain the low participation observed in these auction data.

Krasnokutskaya (2011) provides an alternative justification to the positive dependence of bids. Construction projects are subject to a long and complex list of specifications with some even hard to quantify. These specifications are observed by bidders but not the analyst, leading to unobserved heterogeneity. The author models each bidder's cost C_i as the product of a common component Y, or unobserved heterogeneity, and an individual component, or private information σ_i, thereby creating some positive dependence. However, the model still has independent costs as the σ_is are independent across bidders. Because the equilibrium strategy is location-scale invariant, i.e., $B_i = Ys(\sigma_i) = YA_i$, the author interprets the bids as indicators in a measurement error model, assuming that Y and σ_i, and hence A_i, are independent. Using Kotlarski's (1967) result, she recovers and estimates the distributions of Y and A using characteristic functions and inverse Fourier transforms. Because dependence could be also due to affiliation with no unobserved heterogeneity, she derives restrictions imposed by independent and affiliated costs as well as unobserved heterogeneity to be able to distinguish these models. Analyzing Michigan highway road resurfacing procurements for regular and fringe bidders, empirical results show that only 34% of the bid variation is due to private information.[16] Fringe firms tend to have larger costs, but with a lower variance as well as a lower markup than regular firms. The asymmetry leads to 5% of inefficient allocation thereby increasing the overall cost by 2%. Using a misspecified model such as independent or affiliated costs without unobserved heterogeneity would increase estimated markups because it would lead to underestimated costs.

Following Krasnokutskaya (2011), Armstrong (2013) remarks that multiple bids are not always available whereas data on winning bids or from a single bidder only might be available. In this context, the author derives bounds on bidders' profits when ignoring unobserved heterogeneity that are expressed as function of the upper boundary of the bid distribution, and the expected bid in general or the expected bid order statistics, when considering the winning bid only. The author then uses these bounds to bound the efficiency loss of assigning the project randomly. Using Michigan Department of Transportation auction data, he compares the efficiency loss accounting for unobserved heterogeneity *a la* Krasnokutskaya with that from ignoring unobserved heterogeneity. Though he obtains conservative bounds for the latter, they are still informative.

[16] Regular bidders are those with a minimum of 100 employees and winning at least 10 million dollars of projects each year at these procurements.

Aryal et al. (2019) extend the analysis of bidder asymmetries in the presence of unobserved heterogeneity to estimate asymmetry in risk preferences using Russian procurements of services. Using CRRA preferences, their estimates show that risk aversion not only varies across bidders depending on their participation, but also on job types.

3.2 Private or common values?

Given the scope of the auctioned projects, especially in the construction industry, ex post uncertainty is an important component that firms integrate in their bidding. Uncertainty can be about the firms' ex post costs and payments. Regarding the latter, procurement of construction projects are often organized through scale auctions as in the timber context, i.e., bidders submit a price per unit on a number of items based on estimated quantities needed for every item, while their payment is based on actual quantities that are often revised after the auction.[17] We review below papers adopting a common value setting. Papers adapting private value models to entertain uncertainty on ex post payments and costs are reviewed in Sections 3.8 and 3.9.

Paarsch (1992) is one of the first papers adopting a structural approach in the empirical auction literature. Analyzing tree planting contracts in British Columbia, the author considers the two polar models of pure common value and independent private values. The author relies on specific parameterizations of the latent distributions so as to obtain simple equilibrium strategies which allow for a parametric likelihood approach to estimation. He then estimates both models and empirical results reject independence of costs. Ten years later, Hong and Shum (2002) quantify the winner's curse effect, considering a general model of affiliated values in the spirit of Milgrom and Weber (1982). The authors note that the level of competition has two effects in a common value setting: a negative competitive effect and a positive winner's curse effect. One can dominate the other, creating a frequently-observed U-shape curve between the bids and the level of competition. Based on the log-additive model of Wilson (1998) with a multiplicative decomposition of costs and signals, the relative importance of the standard deviations of common value, private component and noise signal sheds light on the importance of the private and common values. Using auction data by the New Jersey Department of Transportation, the authors consider three types of work (highway work, bridge construction and maintenance, and road paving), since road paving usually is a well-defined task subject to less uncertainty than, say, bridge construction. Estimating the model assuming that the underlying distributions depend on the level of competition, highway work is common value while the other two types of work seem to be subject to both private and common values. Accounting for unobserved heterogeneity, road paving switches to independent private costs, while highway work seems closer to affiliated private values, and bridge construction a mix of the two, though there is no clear cut for these latter two types of

[17] See Section 2.6 on how bidders exploit strategically this information through skewed bidding with Athey and Levin (2001).

work. The question is whether there is an "optimal" level of competition. For highway work, reducing the level of competition by half would reduce the procurement cost by 15%. For the other two types of work, a larger level of competition would reduce the procurement cost, suggesting a dominance of the competitive effect.

Somaini (2020) develops a novel identification and estimation strategy for interdependent/common value models and uses it to analyze auction data for paving projects of hot-mix asphalt by the Michigan Department of Transportation. These projects are subject to uncertainty regarding conditions at execution such as weather and traffic, and also on future prices of inputs. In order to identify common value models, one typically needs exclusion restrictions that affects the competitive structure of the auction. The author exploits variations induced by competing bidders' cost shifters that are arguably independent from the private information and valuation of the bidder. While the identification argument is nonparametric, he estimates the model using copulas in the bid distributions for regular and fringe firms, and finds that the correlation of signals varies from 0.11 to 0.29. He then performs several simulations to assess the impact of restricting participation of bidders given the importance of common value. By inviting seven bidders instead of two, the procurement cost declines by 5.5% under independent costs, 5% under affiliated private costs and 4.4% under common value. More generally, the counterfactual simulations show that the impact magnitude tends to decline as competition grows since less efficient bidders are invited. In addition, optimal reserve prices would yield negligible gains in this setting.

3.3 Entry and competition

As in timber auctions, bidders' entry plays an important and subtle role in the analysis and design of procurement auctions. Empirical work suggests the presence of significant entry costs into procurement auctions. These auctions usually require potential bidders to show their interest before the auction by asking for information on the auctioned contract. These bidders are called the planholders. Only planholders are qualified to participate in the auction. The number of planholders provides information on the number of potential entrants N.

For example, Li and Zheng (2009) study auctions of highway mowing by the Texas Department of Transportation. Data show that only 28% of planholders actually bid, suggesting the presence of entry costs. Relying on the entry models by Levin and Smith (1994) and Samuelson (1985), the non-selective entry model seems to fit the data better based on a mean squared error criterion for predictions. The entry cost represents on average 14% of the cost estimate or 8% of the winning bid. The authors then assess the (positive) entry effect and the (negative) competition effect on equilibrium bids by varying the number of planholders. The entry effect is slightly decreasing in N, while the competition effect increases with N to become almost flat after $N = 7$. Overall, the total effect is positive for large values of N and negative for low competition levels. Regarding the total procurement cost, the effect is positive since the competition effect dominates the entry effect, suggesting that encouraging more potential bidders is not a desirable policy.

Bhattacharya et al. (2014) study the efficiency of entry decisions and the issue of regulating bidders' participation. They estimate a first-price sealed-bid auction with selective entry and compare the outcome with a mechanism that regulates bidders' entry. To do so, they develop a model of an entry rights auction. This auction is conducted in two stages. In a first stage, bidders propose and pay a bid as a monotonic function of their cost signals. This stage, which resembles an all-pay auction, determines the bidders of the second-stage auction. In a second-stage auction, the buyer subsidizes the entry costs of the selected entrants who learn their costs. This entry rights auction tends to be more efficient relative to a first-price sealed-bid auction with free entry. Using bridge-building auction data from Oklahoma and Texas, the entry costs range from 1.5% to 1.9% of the cost estimate, and entry is moderately selective. Assuming that the number of entrants is chosen to minimize either the procurement cost or the social cost in the counterfactual entry rights auction, the latter would reduce the social costs by 2.49% and the total procurement cost by 2.5%. Regulating the number of bidders contributes to efficiency gains because it leads to less entry and a lower winner's cost. Note that the revenue from the first auction counterbalances the entry cost subsidy, but the implementation of this mechanism would require the buyer to know the firms' cost and entry cost densities.

Kang and Miller (2021) note that Federal procurements suffer from low competition/participation. Federal procurement data on Information Technology and Telecommunications products and services show that about 44% of auctions attract a single bid and that competition is low in general. The authors develop a Principal-Agent model that incorporates buyer's discretion on attracting competition, negotiation on payment and an ex post payment that can differ because of modifications and contract type. Indeed, the most observed mechanism includes negotiation with a subset of firms submitting a proposal. The model endogenizes the number of bidders that is chosen by the principal/buyer. Firms are either low-cost or high-cost. In equilibrium, the number of bidders depends on the proportion of low-cost firms, the difference between the low and high costs and how much rent the buyer can extract from the low-cost firms through negotiation. The latter implicitly depends on the solicitation costs and marginal search costs, i.e., buyer's costs to attract firms. The estimation results show that the large fraction of low-cost firms mostly explains the low participation. A counterfactual first-price sealed-bid auction would double the number of participating firms, but the current mechanism of providing a lot of discretion to procurement agencies seems to reduce procurement costs as agencies exploit their knowledge of firms.

3.4 Collusion

Collusion or bid rigging is a pervasive problem in auctions. Because procurement auctions mostly involve public institutions that are under close scrutiny, numerous prosecution cases have shown the extent of the problem and document how cartels are working. With this information in hand, economists have developed methods to detect collusive behavior. As a matter of fact, collusion is difficult to detect because

bidders' cartels use elaborate strategies to mimic competitive bidding. A large proportion of papers on collusion involve procurement auctions of construction and other services by public institutions. The number of papers is too large to review them all, although several document collusion schemes. We focus on those developing detection methods through some testing procedures and assessing the collusion damages, if any. Lastly, we review at the end a few papers on corruption, which is another common problem in public procurements. Corruption involves representatives of public institutions who favor some bidders in exchange for briberies or other forms of support.

Among the first papers proposing tests to detect collusion are Porter and Zona (1993, 1999). In the mid eighties, several firms in the road construction industry in Long Island (NY) were convicted for bid rigging either as main actors or co-conspirators. Based on this information, Porter and Zona (1993) remark that some market characteristics tend to facilitate collusion such as the geographic location, the timing of letting projects and some allowed joint bidding procedure, among others. The cartel selects a firm to submit a bid, while other cartel members are either absent from bidding (known as "phantom" bidding) or submitted "complementary" (larger) bids. Bidding data show the stability of market shares of cartel members over the years and also their concentrated bid distribution relative to non-cartel members. The first fact is related to the cartel sustainability, whereas the bid clustering's objective is more about influencing an engineer's estimate in future auctions. A reduced-form bid analysis for competitive and cartel firms displays striking differences, with some variables not having the expected sign for cartel members. Similar results are obtained using the bid ranks. During the same period, several states prosecuted firms for collusion in school milk delivery auctions. Using data from Ohio, Porter and Zona (1999) note again that some market characteristics favor collusion such as a homogeneous product, low demand elasticity, low competition because of transportation costs, the letting timing allowing cartels to adjust their strategies and trade associations facilitating communication among firms. Collusion was taking multiple forms such as complementary and phantom bidding but also assignment of territories. Competition is rather low with 45% of auctions with a single bid and 34% with two bidders with mostly firms that are close by. A reduced-form bid analysis shows that bids are decreasing in distance for collusive firms, while they are (correctly) increasing for competitive firms. Bid submissions and bids display a positive correlation as evidence of complementary bidding. The authors assess that bid rigging causes an increase in milk prices paid by schools by 6.5%.

In the same line, Pesendorfer (2000) studies school milk auctions in Florida and Texas, two other states among the fourteen under prosecution in the mid eighties. Market shares of cartel firms are quite constant over time in Texas but fluctuate in Florida, while the top firms maintain large market shares overall in both states. The absence of side payments in Texas justifies the observed constant shares for the cartel sustainability. Based on a model in which the cartel designates the firm with the lowest cost to bid, collusion creates asymmetry between this firm and non-cartel firms for a priori ex ante symmetric firms leading to a relationship of stochastic dominance for

their bid distributions. The author then tests stochastic dominance of non-cartel and cartel bid distributions. The former dominate the latter. A reduced-form bid analysis leads one to reject the equality of coefficients, confirming the presence of bid rigging in both states.

Bajari and Ye (2003) study potential bid rigging in seal coating procurement auctions in Minnesota, North Dakota, and South Dakota. They develop a model with asymmetric bidders under cost independence in which the cartel designates the firm with the lowest cost. They show that competitive bids satisfy two conditions of conditional independence and exchangeability.[18] Using distance to project, used capacity and other bidders' covariates, the authors implement their test of conditional independence by looking at the correlation of the residuals from bid regressions between pairs of bidders and their test of exchangeability by looking at the equality of coefficients between pairs of bidders. They find five pairs of firms acting suspiciously and retain two pairs when estimating the structural auction model. Given the absence of prosecution information, they complement their analysis with an additional test based on firms' markups. They obtain reliable information on competitive markups in the industry and compare the markup distribution quantiles since markups under collusion tend to be larger. This last test confirms that the two pairs collude.

Aryal and Gabrielli (2013) implement tests from the former paper to detect suspects in Caltrans procurement auctions. This gives a trio of firms and a pair of firms whose bids violate conditional independence and exchangeability. Using structural auction models of collusion and competitive bidding, the authors then recover the firms' costs under the two scenarios. Under collusion, the firm's cost should be lower at a given bid leading to stochastic dominance of cost distributions. However, the suspicious firms fail to display stochastic dominance. To conclude this part on testing collusion, it is interesting to note that these testing procedures might not be robust to unobserved heterogeneity, which is frequent in construction projects as discussed previously.

Collusion in Japanese procurements has been discussed and documented in several early papers that we do not review here. Two recent papers develop new tests of collusive behavior on Japanese data over 2001-2006. Kawai and Nakabayashi (2018) exploit a feature of the auction which can be in several rounds. If the lowest bid is above the secret reserve price, the public institution organizes a second round in which bidders are informed of the lowest bid at the first auction whereas the reserve price remains secret. A third and last round is possible if the second round fails. The authors' main idea is to look at the bids of the second auction, since 20% of procurements reach a second round. The probability that the second lowest bid of the first auction becomes the lowest at the second auction is only 1.6%, whereas this bidder could bid more aggressively at the second round to win the auction. This arises suspicion of collusion. Denoting $i(j)$ the bidder submitting the j lowest bid at the

[18] Bajari and Summers (2002) explain how the results of Porter and Zona (1993, 1999) and Pesendorfer (2000) are in agreement with violations of conditional independence and exchangeability.

first auction, the authors look at the bid difference at the second auction $B_{i(2)} - B_{i(1)}$ and they find that this distribution is almost nonexistent for negative values while it is symmetric for the difference $B_{i(3)} - B_{i(2)}$. Considering several scenarios such as potential bidding errors, bidder $i(2)$ would be able to increase his expected profit by lowering his second bid by a small margin except if $i(1)$ is the designated winner of the cartel. The different tests based on the former difference are in favor of collusion and the authors conclude that almost all auctions reaching the second round are uncompetitive with likely all inclusive rings.

Chassang et al. (2021) analyze two data sets of Japanese procurement auctions including the one from the previous paper. In one data set, the reserve price is announced. They observe that the distribution of the ratio of the difference between the bidder's bid and the minimum bid of his competitors by the reserve price has a striking missing mass at zero, in both data sets. The authors develop a model of dynamic auction under Markovian strategies and a general setting including common value, asymmetry among bidders and cost correlation. They define the residual demand as the probability that a bidder wins for any bid value given the game history. The elasticity of the bidder's residual demand should be less than -1 under competitive bidding. The observed striking mass in the data implies an elasticity close to zero in agreement with collusive behavior. In addition, from the counterfactual revenue, the authors estimate an upper bound for the share of competitive histories for each bidder with the firm's markup varying from 0.02 to 0.5. Looking at some types of work and before/after prosecution, the share of competitive histories can reach 100% after investigation, whereas in other activities the change is small, with an even lower share of competitive bidding after investigation. In addition to helping in detecting collusive firms, the results suggest that investigation of collusive behavior changes the firms' behavior but does not eradicate collusion.

Conley and Decarolis (2016) study the setting of the average bid auction (ABA) in which the bidder with the closest average bid wins the auction. This mechanism is rarely used in the U.S. but is used in a range of countries in Asia (China, Tawain, Japan), Latin America (Columbia, Peru), and Europe including Italy. The authors develop tests of collusive behavior for both entry and bidding and apply them to Italian data. Very high or very low bids are submitted by cartel members to pilot the average. In the area of Turin, eight cartels involving up to 95 firms have been detected. This information is used for test validation. The first part of the test is based on the distributions of the fractions of auctions in which the two groups of suspicious bidders participate. The second part of the test is based on the distributions of their bids. The authors find 42 clusters of coordination. The authors also note that the rivalry between a large number of cartels tends to act as competition by lowering the price.

Lastly, we discuss the problem of corruption. Corruption may have ties with political influence and organized crime but can also result from a few corrupt members of a local government or administration. Decarolis et al. (2019) study some important recent reforms in Italy and their impacts on corruption. Combining data on Italian public sector allocation of contracts and criminal investigation, the authors look at

whether the reform of increasing the limit from 300k to 1 million euros projects for discretionary contracts has also increased the level of corruption. Allowing discretionary criteria can increase efficiency because of contract incompleteness, but can also facilitate corruption as it gives more freedom to use ad hoc rules for a corrupt buyer to choose a designated firm. The authors investigate whether there is any link between the use of discretionary rules and the allocation of contracts to firms under investigation. Discretionary criteria include use of scoring rules but also negotiated auctions upon invitation only. The authors do not find any evidence that this reform increases corruption except for the negotiated auctions in which the buyer did not invite the minimum required number of bidders. In this case, it is more likely that the contract is won by a firm under investigation by 12%.

Andreyanov et al. (2018) study bid-leakage corruption in Russia. The auctioneer has a preferred bidder to whom he reveals information on competitors' bids. Observing the time at which sealed bids are submitted, the authors observe an abnormal correlation between winning and being the last firm submitting a bid. They estimate that the probability of winning while being the last bidder is close to 40%, while it is only 30% for others. The data also show an important bidding activity in the last 2 or 3 hours before the auction closing time. The authors develop a structural model for honest bidders, i.e., those not designated by the corrupt buyer. The model includes a probability of having a corrupt auction. Using the inversion method, the model shows that corruption has an indirect effect by affecting the bidding behavior of honest bidders. This indirect effect is positive as it forces honest bidders to bid more aggressively, whereas the direct effect is negative as corruption tends to increase the expected price. As shown in other papers such as Asker (2010), corruption does not have always negative impacts. The authors estimate that for a probability of a corrupt auction at 25%, the auction price increases by 7%, i.e., the direct effect dominates the indirect one. Eliminating corruption would decrease price by close to 3%, while the minimum price would be attained with a corruption probability at 5%.

3.5 Cost synergies and combinatorial auctions

When winning several auctions, bidders can enjoy scale or scope economies. This phenomenon is not exclusive to construction projects and services (see also Section 4.3). Auctions can be simultaneous or sequential or even under the form of combinatorial auctions in which bidders can bid on a package of goods in addition to single items composing the package. The so-called cost synergies can be positive as in the case of scale or scope economies but also negative as winning an additional contract can entail important additional costs.

Adopting a reduced-form approach, De Silva (2005) and De Silva et al. (2005) analyze procurement auction data from the Oklahoma Department of Transportation, in which there is a sequence of two auctions held the same day. Their empirical analysis shows that the winners of the first auction are more likely to participate to the second one; conditional on participation, they submit lower bids and hence are more likely to win the second auction, suggesting strong evidence of synergistic tasks

across auctioned projects. In addition, De Silva (2005) highlights that projects are spatially correlated with a correlation that tends to decrease as the distance between projects increases. His results suggest cost synergies in terms of distance between auctioned projects.[19]

In terms of structural analysis, two papers introduce a measure of cost synergies. Marshall et al. (2006) study milk procurement auction data by public schools in Georgia where auctions are sequential. Firms processing milk enjoy cost savings by delivering milk to adjacent districts. The authors consider Krishna and Rosenthal's (1996) model with a fixed cost synergy for winning two adjacent districts. Their model leads to an explicit solution for the equilibrium strategy for independent costs, with an additional term capturing the reduction in the firm's cost due to cost synergies. This model has the advantage of encompassing the case of no cost synergy. The authors find positive cost synergies representing between 15% to 25% of the processing and delivery costs, or 1.9% to 2.4% of the cost of processing milk. However, they do not account for (i) sequential auctions that could introduce asymmetries among bidders since those who won previously adjacent districts enjoy lower costs and (ii) the district size as larger amounts of milk could even further increase the cost synergy.

Gentry et al. (2020) consider simultaneous auctions with a complementarity or cost synergy as a deterministic function of bidders' and projects' characteristics. In addition, their model accounts for the fact that bidders bid simultaneously on several auctions. Bidders have independent standalone costs. Their cost for a given combination of projects is the sum of these costs in addition to a complementarity value that is specific to this combination through the characteristics associated with this combination. For instance, these characteristics could be the total value of the projects in the combination. Using the inversion method of Guerre et al. (2000), they recover the bidder's cost. To identify the complementarity function, they need to impose an exclusion restriction on the rivals' characteristics or other object characteristics. Analyzing auction data by the Michigan Department of Transportation, a letting of auctions contains on average 45 auctions, and more than half of the bidders submit on average close to 3 bids per letting. A preliminary bid analysis suggests that the size of the combined projects positively affects the bids. Empirical results show that the size of the combined projects and their overlapping time positively affect the complementarity value because of capacity constraints. The fact that projects are of similar types reduces the complementarity value. The complementarity value varies from -24% (positive cost synergy) to +11% (negative cost synergy) depending on projects' and bidders' characteristics.

Combinatorial auctions give bidders the possibility to place bids on a combination of projects in addition to stand alone bids with the restriction that the combinatorial bid is less than the sum of the stand alone bids. This practice is implemented by public institutions so that firms can express their cost synergies to reduce the total

[19] As noted by a referee, these empirical features could be also explained by firms' asymmetry due to their locations. Therefore, it would be interesting to control for firms' locations to assess cost synergies.

procurement cost. Using a reduced-form approach, Lunander and Lundberg (2013) analyze auction data of cleaning services in Sweden, exploiting a change of mechanism that switches from a standard first-price sealed-bid auction to a combinatorial auction. Combinatorial auctions may lower the procurement cost, though increasing the lot size may lower entry and hence reduce competition. Cantillon and Pesendorfer (2006a, 2006b) study auctions of bus routes in London (U.K.). On average an auction covers close to 4 routes. Bidders can submit bids on individual routes and/or (part of a) package. From a policy perspective, the main question is whether a combinatorial auction is better than a series of independent auctions. From the multi-product monopoly pricing literature, bidders may submit a lower bid on a combination not because of cost synergies but for strategic purposes just as in the case of strategic bundling (see Adams and Yellen (1976)). Cantillon and Pesendorfer (2006b) construct a simple example in which a combinatorial auction leads to a larger procurement cost with lower efficiency than a series of auctions. This is in agreement with Milgrom (2000). The response to the above question can only be addressed empirically by assessing whether cost synergies dominate the strategic bundling effect. In terms of the model, assuming that an auction proposes m routes, a bidder's cost/bid vector belongs to $\mathbb{R}^{2^m-1}$. The costs can be potentially correlated across bidders and contracts/routes. Optimal bids are characterized by a system of equations or in some instances satisfy inequalities only, making model identification challenging. Using an inversion approach in the spirit of Guerre et al. (2000), the authors estimate the costs and the synergy. On average, they find that for two-route auctions, the synergy is -0.11, indicating that the combined routes cost at least 11% more than for individual routes. For three-route auctions, it is even worse with a synergy of -0.24. A potential rationale for these negative cost synergies is the garage capacity issue since serving an additional route may require the acquisition of an additional garage. For two-route auctions, the efficiency loss has a median of 7.8% because of the combinatorial bidding, but also because of cost asymmetry among firms.

In the same spirit, Olivares et al. (2012) and Kim et al. (2014) study school meal combinatorial auctions in Chile. Adopting a reduced-form approach, Olivares et al. (2012) show evidence of both scale and density discounts. Firms tend to prefer larger and denser districts because they are cheaper to serve as well as closer districts for the same reason. The scale discount on bids is on average between 3% to 7% and tends to increase with size, while the density discount ranges from 0.8% to 1.8%. Relying on the multiproduct monopolist literature, the authors derive restrictions relating the discount magnitudes with the correlation and coefficient of variation of competitors' bid prices. They find support of strategic bundling, but the latter explains only a small portion of the observed discounts. Kim et al. (2014) propose a structural model using an indirect approach. They impose a restriction on the equilibrium strategy thereby simplifying the model relative to Cantillon and Pesendorfer (2006a,b) by reducing the dimensionality of the gradient of the probability of winning. This also makes the problem computationally simpler. Their empirical results show that the average mark-up is 4.4% of the average bid and the (positive) cost synergies range from 1.3% to 4.5%.

3.6 Dynamic auctions

When winning previous auctions affects the firm's future participation and cost, bidders are likely to account for the repeated nature of auction games. In the case of construction projects, firms commit for several months and possibly years before completing the won projects. During this time, they face so-called capacity constraints as their machinery and labor force are utilized. Thus, winning an additional project would entail additional costs including the recruitment of a labor force, etc. In some sense, it is similar to a negative synergy. In contrast, by winning more projects, firms can gain experience and become more cost-efficient, creating a positive synergy. There is an important difference in modeling dynamic auctions. Firms foresee at the time of the auction how winning present and future contracts affects their costs and potentially their auction participation in the future. For instance, the capacity constraints create an intertemporal linkage.

Jofre-Bonet and Pesendorfer (2003) model repeated auctions in the presence of capacity constraints. Analyzing California Department of Transportation (Caltrans) auction data which provide the number of days for project completion, the authors construct a backlog variable as the amount of work in value left from previously won Caltrans projects. Their regression analysis shows the negative effect of backlog on submitted bids and bid submissions. For instance, unconstrained bidders are twice more likely to submit a bid than their constrained counterparts. Only regular bidders are affected by dynamic bidding. The authors define a vector of state variables that include the bidder's backlog as well as its opponents' backlogs, introducing cost asymmetries among the regular bidders in addition to the cost asymmetry between regular and fringe bidders. Looking at the infinite horizon and considering the discounted sum of future expected payoffs for regular bidders, the authors consider Markovian strategies with independent costs. Relying on the indirect approach, the inverse bidding strategy contains two components: a first one that is similar to in a classical static asymmetric auction and a second one capturing the incremental effect on future discounted profits if the firm wins the project by considering an expectation on future contract characteristics. Rewriting the value function in terms of bids, and fixing the discount factor to 0.8, the authors estimate the model parameters and assess the negative impact of backlog on the value function by a reduction of 35% from the highest to the lowest normalized backlog value. The estimated mark-up is on average 40%, which is quite large, mostly explained by the loss in value of winning today versus winning future auctions. This mark-up increases with the discount factor. Based on simulations, the average efficiency loss is of the order of 13% with a probability of an inefficient allocation of 30%.

As shown in a reduced-form analysis, the intertemporal linkage can also affect firms' participation. Groeger (2014) extends the analysis of the previous paper by considering endogenous entry. Analyzing auction data from the Michigan Department of Transportation, the author finds a persistence in firms' entry probabilities for similar projects while bids do not show any dynamic linkage. This evidence suggests that firms enjoy a learning effect in the process of information acquisition and bid preparation by focusing on some type of project/work and thereby reduce their entry

costs. Considering non selective entry, the author extends Jofre-Bonet and Pesendorfer's (2003) approach to an auction with dynamic entry with the derivation of the value function for the entry stage. The estimation results show that firms can reduce their entry costs by half when focusing on specific tasks/contracts. In particular, the reduction is of the order of 46% for large firms and 64% for small firms among the regular bidders. Considering a discount factor at 0.99, large firms' mark-ups are small at 1.2% while they increase to 4.5% for fringe firms on average. The author then proposes a counterfactual with a resequencing of contracts by types of work so that firms can fully enjoy their entry cost savings. In turn, this would increase the participation of large firms by 50% and would double the participation of small firms.

Balat (2017) is interested in quantifying the impact on project costs of the American Recovery and Reinvestment Act (ARRA) that injected important amounts in public spending, creating an increase in demand for construction projects and therefore an increase in the number of auctions. A reduced-form analysis of bidders' participation in Caltrans auction data shows that backlog has a negative impact. Taking into account endogenous entry in a dynamic bidding model, the author finds that a variation in backlog from its lowest to highest value increases firms' costs by more than 7%. The counterfactual consists in eliminating the effect of the stimulus projects on firms' backlogs. The ARRA stimulus package increases the procurement cost by 6% while the prices of other projects, i.e., those not subject to ARRA funding, increase by close to 5%. In view of this, the author recommends to spreading out the injection of funds to smooth out the negative backlog effects.

In a different setting, Ji and Li (2008) consider a multiround procurement auction with a secret reserve price. Analyzing auction data for bridge projects by the Indiana Department of Transportation, close to 10% of auctions are unsuccessful as the lowest bid does not meet the engineering estimate, which acts as a secret reserve price. For those auctions, the institution reveals bid information and organizes a second auction. The observed bids at this second auction are lower despite a lower level of competition since only a subset of firms participates. Considering independence of costs, an update to the cost distribution given the release of the lowest bid and Markovian strategies, the authors show that the mean of bids tends to decrease over rounds. Estimation of the model under myopic behavior confirms some model predictions.

3.7 Subcontracting

Subcontracting is a specific feature of procurement auctions, especially in construction, where a general contractor manages the project, while distributing components of the project to often vertically separated subcontractors. The percent of the project that is subcontracted is important and can be rationalized by several factors. First, public institutions use subcontracting to implement preference policies toward disadvantaged business enterprises (DBEs) with some subcontracting goals that firms need to respect when submitting their bids. Second, subcontracting allows firms to reduce their costs and capacity constraints by outsourcing some tasks to specialized and more cost efficient firms.

Procurement auction data provide detailed information on subcontracting such as the name of the subcontracting firm, the amount and quantity to be subcontracted for every item of the project. This has generated an abundant reduced form literature that studies the effects of subcontracting on firms' bids, survival, and entry to name a few. De Silva et al. (2017) perform quantile regressions to assess the impact of subcontracting on entrants' success and survival. Texas Department of Transportation auction data show that besides firm's size, subcontracting has a positive effect on firms' survival, with more pronounced effects for DBEs which tend to be more experienced in specific tasks. Using the same data combined, with the Texas Census of Employment and Wages, De Silva et al. (2012b) study the effects of an early involvement as a government project subcontractor on the likelihood of firms' survival. Data display a larger exit rate for firms which never acted as contractor or subcontractor. Estimating models of exit probability and survival, the empirical results show that subcontracting experience has a negative impact on the former and a positive impact on the latter. Using an auction model and a similar data source from Texas, De Silva et al. (2012a) estimate the cost distributions for firms subject to DBE subcontracting goals and those that are not. They find little cost differences in contrast to Marion (2009b), who finds a decline in costs after the abolition of DBE subcontracting in California.

Using Caltrans data, Gil and Marion (2013) study how the relationships between contractors and subcontractors affect their bids and entry decisions. Bid regressions show that past interactions have a negative impact that is more accentuated when there is concentration on a single or few subcontractors. The rationalization is a reduction of coordination costs. Marion (2015) studies horizontal subcontracting when a subcontractor can also be a competitor as a main contractor on the same project. This may soften the subcontractor's bidding strategy since winning may entail losing subcontracting business. Using Caltrans data in which 50% of a project's value is subcontracted on average and 11% of auctions involve horizontal subcontracting, the factors explaining horizontal subcontracting are the experience in the subcontracted items and a close location. Overall there is no effect on the winning bid because bids are lower when the same bidder appears as a subcontractor, but firms tend to bid less aggressively as the opportunity cost of horizontal subcontracting grows. Branzoli and Decarolis (2015) study the effects of first-price and ABA auctions on entry, subcontracting and joint ventures. Using roadwork auction data in Italy, the authors exploit the government's recommendation to switch to first-price auctions in a difference-and-difference approach. Overall, there is a decline in both entry and subcontracting in first-price auctions compared to average bid auctions. Recalling that ABAs are similar to random lotteries, it is expected that in first-price auctions only the most cost efficient firms would enter. For instance, in Turin there were 60 bidders before the reform and only 8 on average after the requirement of first-price auctions, whereas the percent of subcontracting declined from 11% to 4%. Regarding joint ventures, they are more likely to win in first-price auctions for the same reason as for entry: they are expected to be more cost efficient.

There are few papers modeling subcontracting and adopting a structural approach. Jeziorski and Krasnokutskaya (2016) is the exception. The main idea is that subcontracting allows bidders to modify their cost realizations and to control for future costs by reducing the backlog accumulation. Based on Jofre-Bonet and Pesendorfer's (2003) model with Markov Perfect Equilibrium, the authors endogenize the firms' costs by choosing the percent of the project to be subcontracted. Subcontracting reduces current costs in case of unfavorable draws but also lowers future costs by controlling the accumulation of backlog. The model endogenizes entry, bidding, and subcontracting, where the latter two are functions of the firm's cost given the firm's backlog. The value function is based on the discounted value of actual and future contracts, taking into account the fact that the cost is partly composed of the "in house" cost and the subcontracting cost. Using Caltrans data in which 33% of the contract value is subcontracted on average, the authors calibrate key primitive parameters using observed data statistics. Eliminating subcontracting would raise procurement costs by 12% and contribute to a decline of completed projects in equilibrium by 20%. Lastly, Balat et al. (2017) observe in Caltrans auction data that prices charged by the same subcontractor vary across firms, suggesting some specific factors to the subcontractor-contractor matches. Their model is with two stages. In a first stage, there is an auction for subcontracting in which the lowest price does not necessarily win as reputation, quality and other factors are taken into account. The second stage corresponds to a procurement auction. The authors provide empirical evidence on subcontracting. Despite a promising model, the paper remains incomplete at this stage.

3.8 Scaling and scoring rules

Procurements often involve complex projects that include a large number of items. Public institutions often use itemized or, as in timber auctions, scale bidding. The total bid is then computed as the sum of the prices per unit times the corresponding estimated quantities, the lowest total bid wins the project. Realized quantities for every item define the ex post payment to bidder.

Bolotnyy and Vasserman (2020) study scale auction data by the Massachusetts Department of Transportation, which contain bidding information on all project items as well as the ex post realized quantities. Empirical evidence shows that bidders respond strategically to the uncertainty on quantities by skewing their bids, i.e., placing higher (lower) bids on items they believe will overrun (underrun) the quantity estimates. Their model assumes single dimension bidder's private information with an efficiency multiplier which differentiates costs, while ex post quantities are subject to additive normally distributed random shocks. Bidders are averse to such an uncertainty and their attitude toward risk is modeled as CARA. Bidders are ranked using a score as the sum of their item bids by the estimated quantities. Estimation results give a CARA measure of 0.046 and a mean cost efficiency of 0.975, suggesting that bidders are on average below the cost estimate. The median of their mark-ups is close to 6%. The counterfactuals assess the impact of eliminating the risk, or by compensating bidders proportionally for their borne risk up to the case in which the risk

is fully borne by the winning bidder through a lump-sum auction. The latter case would increase the procurement cost by 85%, while a 50-50 split would increase it by only 3.6%. Recall that in a scale auction the risk is borne by the public institution as bidders are fully compensated for the differences between actual and estimated quantities. Eliminating risk would lead to a cost saving of only 0.22%.

Luo and Takahashi (2019) also study scale auctions, this time by the Florida Department of Transportation. In their model, item quantities are subject to a multiplicative random shock composed of two elements: one is bidder's private information while the other is a demand shock. Thus, bidder's private information is multidimensional and concerns the over/underestimate of the item quantities. They assume that the cost of performing each task is common to all bidders. Their model encompasses both auction formats: fixed-price or lump-sum auction and unit-price or scale auction. Their counterfactuals show that lump-sum auctions perform well for projects with low risk whereas projects with important uncertainty should be allocated through scale auctions.

Public institutions, and more generally buyers of services, care not only about the price but also about the quality of the services they buy. A low price could be a signal of low quality and a poorly executed service could be the source of additional costs. Quality has several dimensions ranging from the quality of materials to environmental protection. Another example is completion time, as late projects may yield important costs to the society such as traffic congestion. To account for these factors, institutions design *scoring rules* that include the price and several dimensions of quality. Each bidder is assigned a score and the bidder with the lowest (or largest) score wins the project. Thus, the auction becomes multi-attribute and bidders' private information could be multidimensional, depending on whether the quality is endogenous.

Lewis and Bajari (2011) study Caltrans procurement auction data. Because completion time has important impacts on commuters, Caltrans experimented with scoring auctions in which firms could bid on price and a completion time. To be more effective and in the spirit of incentive contracts, the experiment also included a reward (penalty) for earlier (late) completion that was announced at the time of the auction.[20] The score takes the linear form $B + c_U d_B$, where c_U is a weight on completion time, called users' cost, and (B, d_B) is the bidder's bid, including the price B and completion time d_B, measured in days. The bidder with the lowest score wins the contract. The winner takes d_T days to complete the project. If $d_T < d_B$, the bidder gets a daily reward. In contrast, if he is late, he will be charged a daily penalty. The cost to perform the project is function of d_T and an efficiency type. Following Asker and Cantillon (2008), each firm has a pseudo cost as the sum of the cost to perform the project, the penalty/reward function and the number of submitted days times the weight in the score. The contract is awarded to the bidder with the lowest

[20] Besides this experiment, penalties for delays beyond the estimated completion time are rather low and Caltrans time estimates give generous deadlines.

pseudo cost. By comparing scoring auctions and standard auctions, projects allocated through scoring auctions are performed faster, within 60% of the time estimate with higher bids by 5.7% to 7.5%. The scoring auction model is estimated based on the accelerated days, measured as the estimated number of days minus the bidder's submitted number of days, and its supply curve, which is the marginal accelerated days cost. Time-efficient firms are also those bidding less, making the scoring auction attractive. When measuring the social cost based on the number of commuters, delay times, and time value, having all the procurements as scoring auctions would lead to a welfare gain of 22%. Because this gain would be accompanied by a larger cost as well, the authors suggest using smaller time incentives to achieve most of the gain without facing higher costs.

Andreyanov (2018) studies procurement auctions of services ranging from education programs to security and legal services in Russia under a linear scoring rule. The auction is organized under a maximum score that acts similarly as a reserve price. The score is a weighted average of bid and quality. The model, based on exogenous quality, includes the linear score as well as quasilinear and log linear scores with a maximum score above which bidders are rejected. The author shows that any log linear rule is dominated by a linear one. Estimating the model using the inverse approach, the counterfactuals confirm the superiority of the linear scoring rule.

Krasnokutskaya et al. (2020) analyze computer programming service online auctions. Online markets facilitate international firms' entry. Data show that 48% of submitted bids are from South and East Asia, 16% from North America, and 14% from East Europe, while 25% of projects are allocated in North America, 32% in South and East Asia, and 19% in East Europe. Buyers use a score to select the winner as suggested by the fact that 72% of allocated projects do not correspond to the lowest price. The weights are unknown and can vary across buyers. The (unobserved) exogenous quality is evaluated by buyers through exchanges of emails, sellers' past experience and qualifications. Quality of regular sellers is assumed to take discrete values. The authors develop a classification approach to recover these values. Their model combines discrete choice and bidding with entry. Assuming three quality groups (low, medium, and high), estimation results show that bidders of high quality tend to have larger costs than bidders of medium quality and that North American bidders of high quality tend to have larger costs than their foreign counterparts. The entry costs represent between 8% and 12% of the project costs on average. A counterfactual shuts down the international market for US buyers who would be more likely to choose the outside option (15% versus 3% in the open market), and their surplus would decline by 32% because of higher prices. In a related paper, Krasnokutskaya et al. (2018) analyze this online trade using a model of endogenous sorting between heterogeneous sellers in quality and cost and heterogeneous buyers in outside option and willingness-to-pay for quality. Empirical results show that these underlying distributions vary across countries for both sellers and buyers. Any trade policy such as the exclusion of low quality foreigners, a quota on foreign participation and preferential pricing for domestic providers would render the market less competitive, leading to higher prices by also affecting the composition of domestic sellers.

Takahashi (2018) analyzes procurement auctions by the Florida Department of Transportation, which uses a price per quality ratio as a scoring rule. Bidders submit a bid and "qualities" satisfying a number of criteria (construction methods, coordination, environmental protection, etc). External reviewers evaluate these qualities by giving a grade. The weighted average of these grades gives a quality grade for each reviewer. A variance decomposition of the reviewers' evaluations shows that 46% of the variance is within bidders between reviewers, 31% within auctions between bidders, and 23% between auctions. The first value suggests that reviewers evaluate the bidders' projects quite differently, introducing uncertainty for bidders. The author develops a model in which risk neutral bidders' private information is multidimensional and includes a variable cost for quality and a fixed cost for the project. The price per quality ratio is subject to randomness through a multiplicative noise on quality. The author solves the model using an aggregated (pseudo) type to reduce the two dimensions of private information into a single one. Considering unobserved heterogeneity, empirical results show that increased uncertainty tends to increase the dispersion in both submitted prices and qualities. In addition, high-cost bidders tend to enjoy higher payoffs since an increased uncertainty gives them a higher chance of winning the auction. When doubling the number of reviewers, the winning price would increase by 3.4% and the quality by 1.7%. It would be interesting to assess the effect of reviewers' uncertainty on risk averse bidders.

3.9 Ex post uncertainty, incomplete contracts, and renegotiation

As discussed briefly above, construction projects are subject to several sources of uncertainties that can affect firms' costs and abilities to perform the work on time or at all. This makes the design of procurement contracts very important, as the ex ante evaluation of ex post contingencies affects bidding in the auction. These issues have therefore inspired a literature at the intersection of auctions and contracts.

Fluctuations in input costs such as fuel and construction materials are often a risk that has to be borne by the contracting firm. Therefore, how such input cost fluctuations are handled within the contract has a direct effect on bidding. Using a reduced-form approach on auction data by the Oklahoma Department of Transportation, Kosmopoulou and Zhou (2014) exploit policy changes by the US Federal Highway Administration on price adjustment clauses to reduce risk faced by firms. The authors look at bids on asphalt items that are eligible for price adjustment. Indeed, asphalt and fuel prices experience important fluctuations. The empirical results show that winning bids on asphalt items decreased by 11.7% after the implementation of the policy in 2006, and average bids on eligible items decreased by 12.7% relative to non eligible ones. Results also show that bids become less disperse after the policy. These results suggest that reducing uncertainty renders bidding more aggressive.

The engineering literature documents the various risks faced by contractors. Risks can be technical (incomplete design, source of materials), logistical (insufficient availability of equipment and skilled labor), environmental (weather), and financial (delays in payment, uncertainty in material prices). Luo et al. (2020) rely on Luo et

al.'s (2018) model in which risk averse firms' ex post cost is subject to a random shock that can be idiosyncratic or common to all firms. Using auction data from the Department of Public Works by the City of Los Angeles, a reduced-form analysis of bids shows that firms' experience positively affects the bids. Considering firms' asymmetry in risk preferences based on their experience and hyperbolic absolute risk aversion (HARA), empirical results show that experienced firms are almost risk neutral, while less experienced firms display decreasing ARA. On average, the risk premium for the former is about 1% of the engineering estimate whereas it is 8% for the latter. In addition, the variance of the random shock is increasing in project size. This leads to a 3% inefficiency mostly caused by the risk premium with inefficiency affecting 27% of procurement auctions. Moreover, firms' costs are not subject to asymmetry. These results suggest that inadequate experience in logistics and coordination can represent an important source of uncertainty affecting firms' risk attitude. Moreover, the authors find that ex post uncertainty is the primary source of inefficiency.

While uncertainty in some input costs can be contracted upon ex ante, other contingencies may be difficult to contract on, and may require renegotiation or ex post adjustment of contract terms. Jung et al. (2019) analyze data from the Vermont Agency of Transportation in which 82% of projects are subject to some changes of order representing ex post 8% of the winning bid on average. The model involves a probability of renegotiation that bidders know from past auctions. Using an indirect approach, estimation results show that markups on renegotiated items are on average between 17% and 22%. Those values are much larger than the markups on non renegotiated items, which are in the 2%-6% range. In their counterfactual, a reduction of the probability of renegotiation by 5% would reduce the markups by 1% to 4% and the cost up to 6.3%.

Bajari et al. (2014) consider the problem of scale auctions through the lens of incomplete contracts. Indeed, most projects are subject to changes of orders by the public institution which lead to renegotiations of payments to the firm. Caltrans pavement auction data contain detailed information on changes of order and their related adjustment payment, extra work payment, and payment deduction. The authors model the firm's ex post revenue as the sum of the itemized bids by the estimated quantities plus the amounts of adjustment, extra work and deductions minus some adaptation costs, and a bid skewness function. The adaptation costs can be directly related to, say, work disruption or indirectly related to, say, renegotiation and dispute resolution. The model is solved using Asker and Cantillon's (2008) method. A reduced-form bid analysis highlights evidence of adaptation costs of 80 cents for every dollar spent in adjustment of compensation. The changes in ex post payments amount to 5.77% on average. Using a structural approach with bidders' asymmetry, the authors find larger values for adaptation costs. For every dollar of positive (negative) adjustment, the adaptation cost is equal \$2.08 (\$2.42). For extra work, it is estimated at \$1.23 (deduction at \$1.49). Bid skewing is found to be close to zero. Profit margins are about 3.8%, which are in line with the modest margins in the construction industry. The authors conclude that the incompleteness of project design and specifications entail important costs to the firms. It remains an open question whether

mechanism design could reduce adaptation costs while keeping bidding competitive. For instance, negotiated contracts as studied by Bajari et al. (2008) could reduce such costs but at the price of favoritism and potential corruption.

A number of reduced-form papers assess the impacts of other exogenous factors on contract renegotiation. Decarolis and Palumbo (2015) analyze cost and time renegotiations in Italian procurement data. They find that the types of work, auction procedures, and administration have an impact on renegotiations. For instance, both repair work and design build auctions have a positive (negative) impact on cost (time) renegotiations. Negotiated contracts in lieu of auctioned contracts are less (more) subject to cost (time) renegotiations. Result is similar for a local administration. De Silva et al. (2015) study the effect of budget reduction for changes of order in Texas. Before the implementation of this policy, the average ratio final payment by the winning bid is 7%, whereas this value decreases to 2% only after the policy. A structural analysis of procurement data confirms these values with a cost overrun of the order of 4 to 6% before the policy, reducing to a mere 0.5% after the budget reduction. In a reduced-form analysis, Decarolis et al. (2020) assess whether administration competence could explain time delays, cost overruns and renegotiations. Combining data from the US Federal Procurement Data System and the Federal Employee Viewpoint System, the authors find that administration competence has a positive impact on both cost and time performance and significantly reduces the number of negotiations on project cost and time completion. Competence includes several dimensions such as cooperation, skills or incentives, etc. It seems that public institutions with cooperative managerial practices are the most efficient.

Can mechanism design help mitigate the effect of ex post uncertainties? Decarolis (2018) compares two mechanisms that are used in procurement settings: first-price auctions with ex post screening of the bids for their reliability, and the ABA in which the bidder with the closest average bid wins the auction. As previously noted, the ABA is rarely used in the U.S. but used in a range of countries, including Italy, where the author's data come from. The idea behind these two mechanisms is to minimize ex post default of the winning firm. The revenue ranking of these two mechanisms is ambiguous. In terms of efficiency, the ABA is less efficient as it leads to a random allocation. Analyzing Italian first-price auction data with screening, the author estimates the underlying cost distribution with endogenous (but non selective) entry and unobserved heterogeneity. He then simulates the outcome of ABAs to assess their greater inefficiency relative to auctions with screening. The author points out the limitations of the screening procedure and recommends further effective methods to reduce default risk such as reputation, insurance policies, and higher penalties in case of default.

Lastly, Bajari et al. (2008) compare competitive bidding with direct negotiations for private nonresidential building construction projects in California. The public sector mainly uses auctions to favor competitiveness, equal opportunity, and corruption prevention. The situation is different in the private sector where both mechanisms are used. The authors take this opportunity to compare project characteristics for projects allocated through negotiations, auctions with invited bidders and competitive bidding

since the three are observed in their data. Their reduced-form analysis shows that larger projects in size and value or complex projects are more likely to be awarded using negotiations. Negotiations favor the exchange of information between the two parties, during which the buyer can utilize the contractor's expertise when designing the project. In the same line, they find that more experienced buyers are more likely to use competitive bidding as the exchange of information becomes less valuable to them.

3.10 Bidding preferences toward minority-owned and small businesses

Favoring some bidders is a standard practice in procuring construction contracts and services. This concerns minorities and women-owned businesses as part of affirmative action. At the state and city levels, it is frequent for public institutions to favor small businesses or businesses located in disadvantaged areas. The favor is under a bid preference. For instance, unfavored firms' bids are adjusted up by some value α when compared to the bids of favored firms. A favored firm wins if its bid is lower than the bids of other favored firms, as well as the minimum of the bids by unfavored firms adjusted by $\alpha > 1$. Marion (2007) studies the effect of such a policy toward small businesses in road projects in California where $\alpha = 1.05$.[21] In addition to this preference policy which introduces asymmetry among firms, the author also considers cost asymmetry as small businesses are expected to have larger costs on average. Analyzing Caltrans auction data for state funded projects and projects benefiting from federal aid (the latter are not subject to any preference policy), empirical evidence suggests that small businesses are more likely to participate and win in the former than in the latter. Using the inversion method of Guerre et al. (2000) with appropriate bid adjustments, the empirical results show that the procurement cost increases by 3.8% under a preference policy mainly because of the lower participation by low cost (large) firms. The efficiency loss is of the order of 0.1%, though this value increases to 3.6% when accounting for the reduced participation of large firms. At a fixed level of participation, the 5% preference seems to be close to its optimal level.

In a series of papers adopting a reduced-form analysis, the same author studies other aspects of preference policies related to affirmative action. Marion (2009a) studies the impact of Proposition 209 in California whose purpose is to eliminate race and gender considerations in the allocation of state funded projects. Empirical evidence suggests an important segregation in the sense that firms owned by minorities are usually located in areas with a higher concentration of own-race residents. A reduced-form analysis shows a reduction of the number of firms in African-American neighborhoods post Proposition 209. Overall, it tends to have a negative impact on firms' survival rates and size of minority-owned businesses. Marion (2009b) studies another aspect of the elimination of affirmative action on subcontracting goals with

[21] A small business should have less than 100 employees and less than 10 million dollars of annual revenue in addition to be based in California and being independently owned and operated.

DBEs. Comparing subcontracting levels before and after Proposition 209, the subcontracting goals sharply decrease from 13.2% to 2.98% on average in state funded contracts, whereas there is a slight decrease in federal aid projects. Given that the number of subcontractors slightly decreases, firms choose to perform more tasks by themselves. This contributes to lower procurement costs by 5.6%. This decline is not necessarily due to productivity differences between DBEs and non DBEs but rather to an improved mix of subcontractors. Marion (2011) exploits state level data across the US to explain utilization of DBEs under affirmative action. Empirical results show an overall increase in subcontracting with minorities-owned businesses whereas these effects are more accentuated in states with a stricter enforcement. Marion (2017) studies how exemptions to affirmative action impact efficiency procurement costs using data from Iowa. Until 2013, the State of Iowa allowed exemptions if the firm had a good history of DBE utilization. As a result, firms are acting strategically by building a history of DBE utilization during periods of low demand and exploiting exemptions during periods of high demand. Overall, it tends to reduce procurement costs. Fairlie and Marion (2012) study the effect of eliminating affirmative action in public procurements on employment rates by race in the states of California and Washington. Overall the effect is small, but self-employment tends to increase at various levels. For instance, women and minorities have increased self-employment relative to white men.

Krasnokutskaya and Seim (2011) evaluate Caltrans' preference policy while endogenizing entry. McAfee and McMillan (1989) argue in favor of bidding preferences because non-favored firms would bid more aggressively against the strengthened favored bids enhancing competition and thereby decreasing the procurement cost. This argument neglects the impact that preference bidding has on low-cost non-favored firms' participation. The authors estimate Levin and Smith's (1994) entry model with cost asymmetries and show that under exogenous participation the procurement cost does not vary much with the level of preference, whereas under endogenous participation the procurement cost varies and is minimized with a preference policy toward (low-cost) large firms. Indeed, favoring large firms would enhance their participation and therefore reduce the procurement cost. Their results show that favoring small firms does not respond to a cost minimization objective. The entry cost represents between 2.2% to 3.9% of the engineering cost estimate for small firms and between 3.1% to 3.3% for large firms. The cost asymmetry among large and small firms is important and also varies across project type. When implementing the preference policy, Caltrans has a goal of allocating 25% of procurement cost to small firms. This objective is not currently attained. Under fixed participation, achieving this goal would require a discount of 50% to small firms for a very modest increase in procurement cost, whereas under endogenous participation a bid discount of 20% would be needed with a total cost increase of 3.2%. Because of the variations in cost asymmetry across project types, the authors recommend tailoring the discount rate by project type to attain the 25% allocation objective at a minimal cost increase.

4 Oil and gas lease auctions

Oil and gas lease auctions have been the inspiration for early auction theory (e.g. Wilson (1967, 1969)) and the availability of auction data sets has enabled the development of a robust empirical literature. In these auctions, the government auctions the right to drill and extract resources from a plot of land. Oil and gas lease auctions have been thought of as the classic example of a common value auction, as the amount of oil or gas reserves is the same for every bidder, but bidders may have different signals about the amount of the deposits based on their individual analyses of the geological conditions. Given this, the winner of the auction may potential suffer a "winner's curse" if s/he does not take into account the fact that the winner of the auction has the highest signal regarding the deposit. The availability of detailed bidding data, along with information on ex post outcomes such as revenues and royalty payments, allows researchers to test the common value hypothesis and whether or not there is a winner's curse. While much of the earlier work has been surveyed by Hendricks and Porter (2007), we discuss some of the recent work, starting with a brief review of the earlier papers in Section 4.1.

Another important aspect of the oil and gas lease auction context is that the winner of the auction has the right or option, but not the obligation to drill and extract oil/gas, with the government getting a royalty payment out of extraction revenues. These drilling rights are given for a number of years with ownership reverting to the seller if the lease holder does not drill. In terms of modeling, these option contracts make the bidder's value endogenously determined by his decision on whether to drill. However, a large fraction of oil tracts in the U.S. are never drilled. This brings forth the question of whether the auction format or the terms and maturity of the royalty contract can be tailored to improve drilling rates and especially revenue for the government. In Section 4.2 we survey papers that study how drilling is connected to bidding.

Oil and gas lease are usually sold through first-price sealed-bid auctions. In Section 4.3, we survey work that studies alternative auction formats and other mechanism/market design issues in the oil and gas context.

4.1 Common value and winner's curse

The U.S. Outer Shelf Continental (OCS) data off the coasts of Louisiana and Texas over 1954-1979 have been extensively studied since the eighties. Hendricks et al. (1987) provide an extensive analysis of OCS data. There are roughly two kinds of tracts: "Wildcat" tracts are areas with no prior exploration and "drainage" tracts are areas with some prior development. Firms have a term of five years for exploration. Once drilling is performed, firms pay 1/6th of revenues as royalty to the government. If there is no drilling, the tract is lost after five years and reauctioned later. The reserve price is typically rather low, but the government has the right to reject the highest bid if found too low. In addition to bidding data, the authors collect information on ex post production as well as drilling costs from the American Petroleum Institute to compute discounted revenue and cost. This information is then used to compute the profit and the net profit.

The authors establish a comparison of bids, profits, and net profits across wildcat and drainage auctions. Though in both auctions, variables display important variability, bids, profits, and net profits per acre are much larger on drainage tracts than on wildcat tracts. They find a positive correlation between the highest bid per acre and the profit per acre on drainage tracts, whereas this correlation becomes negative on wildcat ones. It is worth noting that a large proportion of wildcat tracts remain undrilled. The authors also look at how many firms could have lowered or increased their bids to maximize profits. It appears that most big companies should have bid less as a result of overestimating the tract and not anticipating the winner's curse under common value. Regarding drilling, it seems that valuable tracts are drilled first and that firms delay drilling strategically. Regarding drainage tracts, they also find a clear advantage in terms of profits and net profits for firms owning an adjacent tract, suggesting that they possess better information on the potential oil deposit.

Hendricks and Porter (1988) develop a common value model with a random reserve price for drainage auctions in which a neighbor firm knows the tract value. Their model gives a number of predictions that they test on participation and bidding data. Specifically, neighbor firms win at least 50% of the tracts, and their profit upon winning is positive. However, non-neighbor firms have zero profit on average. Neighbor bids are independent of the level of participation whereas non neighbor bids are decreasing in competition. Overall, drainage data confirm the information advantage of neighbor firms over non neighbor ones. The latter seem to bid cautiously, accounting for the winner's curse. Hendricks et al. (1994) extend the model to a general setting of affiliated values with a random reserve price correlated with the tract value. Among the model predictions, uninformed bidders are less likely to participate, and if they do, they tend to bid high rather than low. In contrast, informed bidders tend to submit low bids. The informed bid distribution stochastically dominates the uninformed one within the support of the reserve price and is equal above. The bidding data overall validate the model predictions.

In the same line, Hendricks et al. (1989) confirm most of these results and focus on the rejection probabilities. Rejection of the highest bid by the government is more likely in drainage than in wildcat auctions, 18% versus 9%. In addition, the rejected highest bid is larger in drainage as well. The authors also discuss the quasi-absence of competition among neighbor firms despite a drainage tract being able to have up to eight potential neighbors. This point is also noted by Hendricks and Porter (1993) who question coordination of neighbor firms, as usually only one participates in drainage auctions.

Hendricks et al. (1993) develop an optimal mechanism in which the neighbor firm reveals information, pays zero, and never obtains the lease. Given that this is not implementable, they propose auctions in which neighbor firms would be excluded so that governments can extract more rents. In view of the government's concern for low drilling rates and revenues, possible policy instruments that have not been explored are the term and the royalty rate, though increasing the royalty rate can induce firms not to drill. Porter (1995) offers a complete overview of all these results while noting that the winning bid is increasing in the level of competition and that discounted

revenues also tend to be larger on tracts with more competition. This evidence agrees with an endogenous level of participation as more valuable tracts tend to attract more bidders, making it more difficult to derive testable restrictions and to find a valid instrument since any exogenous variable correlated with signals is also likely to affect bidding and participation.

In OCS auctions, some amount of coalition formation by bidders, called 'joint bidding' is allowed. Regarding joint bidding, Hendricks and Porter (1992) provide information on the evolution of joint bidding over time. For instance, in the seventies one observes more joint bidding with small or fringe firms.[22] Joint bids tend to be larger than solo ones and are observed more frequently on valuable tracts. Hendricks et al. (2008) study further the problem of joint bidding in the light of collusion models. Because of common value, a cartel gives the opportunity to its participants to pool information to make an efficient investment decision. The efficiency of a cartel works for bidders with low signals whereas firms with high signals might be better off not joining the cartel. Because of common value, a firm receiving a low signal does not have to worry about the winner's curse and can be more aggressive in requesting compensation for revealing its information/signal. This is not the case for a bidder receiving a high signal. These factors could explain the low incidence of joint bidding and the lack of concern for collusive agreements in joint bids.

More recent papers focus on the problem of testing for common value. Hendricks et al. (2003) exploit information on bidding and ex post estimated value to test equilibrium behavior. The authors propose a data-driven measure for the number of potential bidders as they note the low level of actual competition due to potential entry. Using the ex post estimated tract value, they estimate the expected value conditional on the level of competition, exogenous tract characteristics and a bid value, and then estimate the same conditional expectation with an additional conditioning on the maximum bid of other bidders who bid less than this bid value. The difference between these two expectations measures the extent of the winner's curse. The winner's curse is estimated to be around 107% of the average winning bid for a low level of competition, decreasing to 75% for a large level of competition. Their second test is based on the Nash equilibrium as they compare the second aforementioned expectation to the pivotal expected value which is obtained from the indirect approach. The latter is increasing in the bid, satisfying a condition of rationalization. Results show that bidders tend to overestimate the tract value for a low level of competition. The authors conclude that bidding is consistent with common value and that bidders account for the winner's curse by lowering their bids when receiving a high signal.

Compiani et al. (2020) consider a more general setting including affiliated values, endogenous entry, and unobserved heterogeneity to test for common value. Their entry model adopts a reduced-form approach with the number of bidders as a function of observed heterogeneity and nondecreasing in unobserved heterogeneity. The

[22] Because of competition concerns, by the mid-seventies, the federal government imposed some restrictions on joint bidding by banning the eight largest oil companies from forming a cartel.

authors make identifying assumptions on the model structure. They assume that bidders' private information is multiplicatively separable in a function of observed and unobserved heterogeneity and a base value that is idiosyncratic to bidders. This implies a multiplicative decomposition of the bids in the same function of observed and unobserved heterogeneity and the so-called "homogenized" bid that is a function of the number of bidders and their private signal. In addition, their model requires an instrument that affects entry but not private information and signals. In the application, the instrument is the number of neighbor firms. The authors propose a test statistic based on the joint distribution of signals and values in the spirit of Haile et al. (2006). Specifically, this joint distribution is independent of the number of bidders under private value while there is stochastic dominance under common value. They implement their test and conclude toward common value. It is worth noting that ignoring unobserved heterogeneity would lead to model misspecification as the stochastic ordering of the joint distributions is contrary to the common value model. In terms of policy, the seller would not gain from limiting competition to soften the winner's curse, and switching to a second-price auction to exploit the linkage principle associated with common value would generate a small increase of revenue for the seller.

In view of the above discussion regarding the low participation of neighbor firms, Aradillas-Lopez et al. (2018) develop a test of competitive bidding in drainage auctions within a common value setting. Their test focuses on neighbor firms because of their suspicious behavior. Upon recovering the pivotal expected values, their test is based on the stochastic ordering of their distributions in the number of neighbor firms. They also test for affiliation of participation decisions and bids. The authors conclude that neighbor bids appear to be competitive before the 1983 Area Wide Leasing (AWL) but not in deeper tracts.

4.2 Bidding and drilling

Oil lease auctions are auctions with contingent payments conditional on drilling. For this purpose, the winner is granted a period of time for exploration and drilling. Upon finding a deposit, a royalty on revenue is paid to the government. A striking feature of oil leases is that only a small proportion of tracts are effectively drilled and generate royalty revenues. A natural question is whether this is due to the auction mechanism and/or the term of the lease agreement.

Hendricks and Porter (1996) analyze drilling data of auctioned wildcat tracts. They observe that the number of drilled tracts and the hazard rate defined as the proportion of remaining tracts not yet explored follow a U-shape curve over the five year term. On the one hand, upon running a probit model for drilling decisions, they find that a high bid is a significant determinant of drilling early but becomes insignificant over time. On the other hand, firms rely on drilling activity in their geographic area to decide on drilling. Other evidence confirms that firms are unlikely to act cooperatively in their drilling decisions. Instead, the release of information on drilling activity creates a free-riding problem and a learning process in which firms update the beliefs based on their signals. Regarding the analysis of revenues, the authors run a Tobit

with the discounted revenue as the dependent variable. As for the drilling decision, a higher bid explains revenue, though drilling outcomes are more difficult to predict, suggesting the important uncertainty firms encounter in their drilling decisions.

Haile et al. (2010) study the changes in bidding pattern, participation, and exploration introduced by the AWL introduced in 1983 and the availability of new technologies such as 3D seismic surveys. Under AWL, most of offshore lands became available for sale, including deep water tracts. To encourage firms to bid, the term was increased to ten years and the royalty rate has been reduced to 1/8th on deep water with a possible relief. The authors compare a full set of variables before and after AWL. The offering of tracts was multiplied by thirty but only 7% were sold. Both participation and bid levels collapsed after 1983, with an average number of bidders at 3.2 falling to 1.3, whereas the average winning bid fell from fifteen millions to less than one million of dollars. Regarding drilling, the authors observe a similar declining trend over time though the fraction of productive tracts is similar. The increasing offering of tracts and the less favorable selection (under the pre-AWL system, firms had to nominate tracts to be on sale) have certainly contributed to lower tract values. A closer look at data shows that large oil firms have switched their exploratory efforts to deep water tracts that generate more revenue and profit. Moreover, 3D imaging, though more costly to perform, provides better information, making firms' signals more precise and reducing the uncertainty on the lease value.

Bhattacharya et al. (2018) combine models of bidding and drilling to assess how parameters of the auction mechanism such as the term and the royalty rate affect drilling decisions and activity. The authors model the tract value as an option depending on the expected quantity of oil, drilling costs, beliefs on future oil prices and the royalty rate. They adopt a pure common value framework in which the quantity of oil q on which bidders receive a signal is common to all bidders ex post the auction. The ex post revenue for the winner is $(1-\phi)Pq - c_i$, where P denotes the oil price, ϕ the fixed royalty rate, and c_i the firm's extraction costs. The value of the lease is defined as the expected value of the discounted revenue, taking account of the drilling decision. Indeed, the bidder needs to solve a stopping problem based on discounted revenue from tract drilling. This gives a time, i.e., a stopping time, at which the bidder drills. Assuming that the price of oil follows a Geometric Brownian motion, the model endogenizes the lease value that depends on the expected drilling cost and expected oil price, as well as the amount of oil and the royalty rate. In turn, the royalty rate and the term from the auction mechanism affect the drilling decision. Using various parameterizations for the model primitives, the authors estimate the model based on various bidding and drilling data moments on New Mexico data from the Permian Basin.

Using the estimated distribution of signals and common value, the authors conduct a number of counterfactuals, including an auction with an optimal royalty rate ϕ maximizing government's revenue, a royalty auction in which bidders would bid on ϕ instead of cash payments and a pure cash auction with no ex post contingent payment. In terms of total revenue, the former and the observed mechanism dominate all the other mechanisms. A pure cash auction would increase the drilling rate to 14%

from the observed one at 9%, and would reduce the delay in drilling, resulting in a larger amount of production. Overall, the empirical results display strong evidence of moral hazard in post-auction activity in the sense that a larger royalty rate induces less drilling. The authors conclude that the observed term and royalty rate seem to achieve a trade-off between maximizing revenue and drilling. Lastly, an analysis of oil price shocks shows that royalty auctions are more sensitive to ex post price shocks and that a higher oil price at the time of the auction tends to favor a pure cash auction.

Using similar data and modeling approach, Ordin (2019) interprets ϕ as a tax rate. Given that extraction costs are important and might prevent firms from drilling, he simulates a cost-side intervention by subsidizing such costs. The winning bids would remain unchanged, but drilling rates would be larger.

4.3 Miscellaneous

A common denominator of the papers surveyed in this subsection is that the authors adopt a private value paradigm. The latter gained in popularity with the structural approach since private value models are identified in general. Also, some of these papers analyze bidding data of tracts in areas with a long history of development, which could reduce the uncertainty on oil deposits that is inherent to common value models. Nevertheless, the adoption of alternative models gives a different empirical perspective. Private value models emphasize firms' productive and cost efficiencies and other idiosyncratic factors.

As discussed in Sections 4.1 and 4.2, auctions of wildcat tracts have been extensively analyzed through the lens of common value. Three papers from the early 2000s rely on the private value paradigm to analyze the wildcat auction data. Li et al. (2000) consider the conditionally private information model with signals S_i distributed independently, conditional on the common component C, and bidder's utility $U(S_i, C)$. Their model includes the pure common value model in which $U(S_i, C) = C$. Data are analyzed under (i) conditionally independent private values with $U(S_i, C) = S_i$, where $S_i = C\eta_i$ with η_i and C independent and (ii) pure common value with the pivotal expected value $V(s, s) = \mathrm{E}[C|S_i = s, Y_i = s]$ assumed to be log linear in $\log s$, Y_i being the largest signal of other bidders. The former model is identified noting that the recovered values are multiple indicators in a measurement error problem, where the second model is identified up to location and scale. The empirical results show a low degree of affiliation. In addition, the ratio of the variances of $\log C$ and $\log \eta$ is equal to 9.2%, suggesting that the variability of signals/values is mostly explained by bidders' idiosyncratic factors. Given that the reserve price is notoriously low in these auctions, Li et al. (2003) estimate optimal reserve prices under affiliated private values based on a rewriting of the expected seller's profit in terms of the (known) bid distribution instead of the (unknown) private value distribution. The estimated reserve price is close to thirty times the observed ones. It is worth noting that McAfee and Vincent (1992) find optimal reserve prices roughly one quarter less mainly due to the correction of the winner's curse. Campo et al. (2003) study joint bidding and how it introduces asymmetry in the auction. Their empirical results show a slight

degree of asymmetry. More importantly, they show that under a pure common value model with asymmetric bidders, the bid distribution should be quasi-symmetric, i.e., the likelihood that the joint bid is less than the solo bid is one half given that both are less than an arbitrary value. Bidding data show no evidence of quasi-asymmetry.

A series of papers study the lease auctions in the Permian Basin of New Mexico, which is the second most productive oil field in the world. The long history of development in the area as well as recent horizontal drilling reduce the uncertainty on oil volume. Kong (2017, 2020) analyze the effects of the uncertainty regarding the level of competition on auction revenue. These auctions are organized through both first-price sealed-bid and ascending oral auctions. The data show that the bids from the former do not display a mass point at the reserve price when a single bidder bids. The winning bid is roughly seven times the amount of the reserve price. This evidence suggests uncertainty on the level of competition, i.e., the number of bidders entering the auction. In addition, even after controlling for observed and unobserved heterogeneity, the sealed-bid auctions generate more revenue than the oral ones. The author studies the different factors that can explain such an important difference, namely the different entry rates, bidders' risk aversion, and bidders' asymmetry. Kong (2020) considers probabilities of entry for the types of bidders, while being agnostic on the nonselective entry model generating this probability. Asymmetry also affects their value distributions and their attitudes toward risk through their utility functions. The author exploits data from the two auction mechanisms as in Lu and Perrigne (2008). Conducting counterfactuals, the difference in auction revenues between the two mechanisms is due to the bidders' uncertainty about who will bid, combined with their risk aversion. Indeed, asymmetry causes a minor difference in revenues and the different entry rates would have a reverse effect on revenues. It is worth noting that the increase of auction revenue due to the combination of uncertainty and risk aversion partially compensates for the low level of competition, that is generally observed in oil lease auctions. In a previous version, Kong (2017) considers a model of selective entry with ex ante signals and entry costs. Selective entry defines a threshold level of signal above which bidders enter as reviewed previously. To identify the value distribution conditional on signal, the author needs to impose an exclusion restriction and a full support assumption in line with Gentry and Li (2014). Using a flexible HARA utility function, the empirical results show DARA preferences. In addition, entry costs are not negligible whereas the entry rates are quite similar across the two mechanisms. The analysis of factors explaining the difference in revenue between the two mechanisms reaches a similar conclusion as in Kong (2020), i.e., the risk aversion is the main contributor.

Kong (2021) notes that two leases from the same one-square mile are auctioned the same day: The first lease through a first-price sealed-bid auction denoted auction 1, and the second lease through an ascending auction denoted auction 2. Data show that more than 90% of auction 2 winners also participate in auction 1 and that auction 1 winners are more likely to win auction 2 as well. The author foresees two potential explanations: (i) synergies such as cost savings and horizontal drilling making the second tract more valuable to the winner of the first tract, and (ii) affiliation of

bidders' private values in the sense that if one tract is valued favorably, the adjacent tract with similar geology is also valued favorably. The author proposes a tractable model for sequential auctions in which bidders know their value for the first object but only know the distribution of their second value conditional on their value at the first auction. This is because intervening auctions and time are a source of uncertainty and bidders may adjust their valuations as the auction day proceeds. There are few restrictions on the distribution of value 2 conditional on value 1, allowing for affiliation, and synergy is a flexible function of the two values. The causal effect of winning from synergy is identified separately from affiliation by comparing the auction 2 behavior of bidders who just barely lost and just barely won auction 1. The empirical results confirm the presence of positive synergy, though affiliation among values is primarily responsible to explain the predominance of winning adjacent tracts. The nonparametric rank correlation measure, aka Kendall's tau, is 0.37. A related policy question to sequential auctions is whether bundling the goods would generate more revenue. Bundling the two tracts sold in one auction would increase the revenue by 7%. The simulation of bundling 3 and 4 leases relative to a sequence of 3 and 4 auctions shows even larger increase of revenues at 16% and 23%, respectively. These increases in revenue are nonetheless accompanied by decreases in allocative inefficiencies.

Kong et al. (2021) study multi-attribute auctions of contracts in which bidders bid on both cash payment and royalty rate. To explain the variability of observed submitted bids and royalty rates, the authors rely on multidimensional bidders' private information, i.e. their expected production volume and production cost, both being potentially dependent. In addition, this private information can be affiliated across bidders. Considering a private value paradigm, their model includes adverse selection through the government's payoff that depends on bidder's private information (production volume), and some form of moral hazard through the royalty payment, as a large royalty may induce the winner not to execute the contract. Borrowing from the option value literature, the authors model the contract value as an option value that accounts for oil price uncertainty with the oil price following a Geometric Brownian motion. The authors adopt a best response approach. Considering a flexible probability of winning that captures the (unknown) allocation rule and the government's preference for higher values of royalty and cash payment, the authors study the Louisiana auctions of oil leases. The empirical results display a strong correlation of private information, between production and cost but also across bidders. The estimate of the cost per barrel compared with revenue explains the low development rate. The predicted ex ante exercise probability is estimated at 0.44, which predicts well the ex post observed one at 0.42. Estimates of expected production volumes and costs are in line with the actual ex post production volumes in the area and cost estimates from Bhattacharya et al. (2018), though no drilling information is used by the authors. The comparison with standard fixed-royalty auctions shows no benefit of offering the possibility to bid on royalty rates. The cash-royalty auctions cause a drop in cash revenue that is not recouped by royalty revenue. In addition, they do not provide any gain either in the probability of exercise, which is a main concern for

governments. A comparison with a flexible scoring auction shows that a scoring auction would lead to less government revenue because it leaves more information rents to firms but would increase the exercise probability. These counterfactuals suggest that allocation mechanisms take complex forms in the presence of multidimensional private information. Two additional policies are examined. In contrast to popular belief, increasing the contract term would decrease the exercise probability but would increase revenue. Exploiting oil price fluctuations would increase both revenue and exercise probability. Overall, the cash-royalty auctions tend to exacerbate both adverse selection by giving more opportunities to bidders to take advantage of their private information and moral hazard.

The government of Brazil sells drilling rights through scoring auctions to account for firms' exploratory effort and use of local inputs. Sant'Anna (2018) studies these auctions for mature basins. Under independent private values, the author's model relies on a best response approach with a multidimensional source of private information that includes the bidder's value V_i and marginal cost of effort β_i, leading to a bidder's profit as $V_i - \beta_i e - B$, where (e, B) is the vector of submitted effort and bid. The author estimates the probability of winning nonparametrically, exploiting his knowledge of the scoring rule. The simulation of a standard first-price sealed-bid auction would lead to an increase of 10% in government revenue. The author also simulates first-price sealed-bid auctions with an exploratory effort fixed by the government. Results show that a scoring mechanism would lead to larger values of exploratory efforts, in line with the government's preferences.

5 Online auctions

The use of auctions to sell goods online started almost as early as the emergence of the World Wide Web, with Onsale (May 1995) and eBay (September 1995) as the two pioneers (Lucking-Reiley (2000)). eBay soon became the best-known online auction site, with 423 million items for sale in 18,000 categories in 2001 (Bajari and Hortaçsu (2003)). While the share of items sold through auctions vs. posted prices has declined even on eBay since the mid-2000s, auctions remain a popular method to sell one-of-a-kind goods such as collectibles, jewelry, and fan memorabilia on the Internet (Einav et al. (2018)).

A popular auction format for selling goods online is the *proxy auction* introduced by eBay. In this open ascending auction format, bidders declare a proxy bid to eBay, and the system automatically sets the going price to the second highest proxy bid plus a bid increment. Bidders can enter at will and can update their proxy bids. At the conclusion of the auction period, the highest bidder wins, but pays the second highest proxy bid plus a bid increment.

5.1 Bidding dynamics and sniping in online proxy auctions

An interesting feature of online proxy auctions is that many bids arrive at the last minutes of the auction, a phenomenon called sniping. A number of economic mechanisms have been proposed for the sniping phenomenon: Roth and Ockenfels (2002) argue that uncertainty in the ability to place a bid reliably (due to network congestion) at the very end of an auction can make last-minute bidding a (tacit) collusive equilibrium strategy in a private value second-price auction setting. Bajari and Hortaçsu (2003) show that the existence of a common value component can rationalize last minute bidding, since, under affiliation, early bids will be informative about the common value, potentially increasing competition for the item. Rasmusen (2006) and Hossain (2008) consider the case of an initially uninformed bidder, who learns her value gradually over time by observing the price. Informed bidders may thus respond by sniping to reduce value discovery by uninformed rivals.

While empirical evidence consistent with the above explanations has been provided by the literature, a single mechanism has not been singled out as the main driver of sniping behavior. Roth and Ockenfels (2002) compare proxy auctions across eBay, which have a pre-defined ending time, and auctions on Amazon, which had a "going-going-gone" period tha t extended with the arrival of new bids, and show that sniping does not occur in Amazon auctions. Roth and Ockenfels (2002) also compare auctions for computers vs. antiques on eBay, and find that sniping behavior is more prevalent in antique auctions, which they deem to have more of a common value component than computer auctions. In order to eliminate the common value explanation, Ariely et al. (2005) run a controlled laboratory experiment with an ascending auction and find evidence for sniping in this setting. However, their results reject a tacit collusion explanation, and appear to be driven by heterogeneous bidder behavior, in which some bidders bid incrementally, while others wait until the very end. This is in line with the theory that sophisticated bidders snipe as a best response to naive bidders who bid incrementally. Evidence corroborating this explanation is also given by the study of Ely and Hossain (2009) who conduct field experiments in which they place last minute bids in a randomly chosen set of DVD auctions against a control set of auctions (the authors bid according to a draw from an induced valuation distribution that is the same across the treatment vs. control auctions). However, Ely and Hossain (2009) show that the profitability of sniping is low, suggesting that free entry in the eBay setting eliminates profit gains from subtle strategic responses like sniping.

Alternatively, Hopenhayn and Saeedi (2016) develop a model to explain sniping and multiple bids by considering bidders' valuations and bidding opportunities as a joint Markov process that is independent across bidders. On the one hand, the arrival of new information during the auction such as new alternatives or preference shocks creates incentives for bidders to delay bidding since their estimate of their expected final values becomes more precise. On the other hand, the random arrival of bidding opportunities creates frictions for bidders' information acquisition, providing incentives for multiple bids along the auction. Indeed, their model shows that early bids are shaded more than in a static game and that bids tend to increase toward the end. They estimate their model on sales of tablets, iPads, concert tickets, and artworks.

5.2 Structural econometric models of proxy auctions

The presence of sniping in proxy auctions has led some modelers to use the sealed-bid second-price model to approximate behavior on eBay, instead of an open-ascending auction model. For example, Bapna et al. (2008) use an independent private value model to estimate bidders' valuations and to quantify consumer surplus in eBay auctions. Bajari and Hortaçsu (2003) use a common value second-price auction model with stochastic entry for their structural econometric model of eBay coin auctions. They use their estimates of the information game structure to quantify the revenue implications of different reserve price policies, specifically the use of public vs. secret reserve prices, on which theory does not provide a clear-cut analytical answer. The authors find that while secret reserve prices have a significant deterrent effect on bidder entry, and impose an extra cost on the seller due to an extra fee charged by the platform. It may be profitable especially for higher value items.

Zeithammer and Adams (2010) and Hickman (2010) point out the limitations of the model behavior in the proxy auction. In particular, Hickman (2010) finds that a subtle aspect of eBay's pricing rule differentiates their auction format from a second-price auction. The difference is that when the top two bids in an eBay auction are within a bid increment of each other, the top bid becomes the price, not the second-highest bid. In such situations, the auction is a first-price auction, not a second-price auction, and since such situations have a non-zero probability of occurring, the eBay auction format is no longer strategically equivalent to a second-price auction, and bidding one's value truthfully is no longer a dominant strategy. The author then characterizes equilibrium strategies in the eBay format, and obtains the nice result that the equilibrium bid functions lie between equilibrium bids in first price and second price auctions. Hickman et al. (2017) develop a method to structurally estimate bidders' values under the equilibrium strategies derived by Hickman (2010).

In much of the empirical auction literature, the analysis takes each auction in the data as being an independent game. On an auction platform like eBay, however, there can be many parallel auctions of the same or similar goods. Thus, even if each auction is a private value second price auction, bidders may not bid their true willingness-to-pay; they instead bid their outside options. Zeithammer (2006) investigates how the presence of parallel or subsequent auctions affects bidder behavior on eBay. Backus and Lewis (2016) embed the bidders' problem in an environment where bidders utilize Markovian strategies conditioned on a relatively coarse set of observable state variables. They also assume that the number of bidders is large enough to ignore the effects of their own actions on the evolution of state variables. They then utilize an argument similar to Bajari et al. (2007) to show that bidders' valuations can be estimated from the bidding data if the econometrician has information on the discount rate, the set of variables considered by each bidder when forming his strategy, and data on state transitions on a large enough support of the state space. They then conduct a Monte Carlo study in which they first generate data from a computed equilibrium model, and recover bidders' valuations.

Adachi (2017) and Bodoh-Creed et al. (2021) also develop and estimate dynamic models of bidding by considering a series of auctions of identical items with unit

demand. Losing bidders remain in the pool of potential bidders whereas winners exit the auction platform. Bodoh-Creed et al.'s (2021) model also includes endogenous entry. An important implication of these models is that the latent value distribution is that of new and returning buyers, not only the value distribution of new buyers as in a static model. In terms of policy, both papers address the interesting question of whether these large auction platforms reduce frictions to reach the outcome of perfect competition. Analyzing sales of iPods, Adachi (2017) finds that distortions on prices and quantities relative to perfect competition are small overall but that there is a significant distortion on the consumer surplus because of dynamic misallocation of items to the bidders. Analyzing sales of Kindle tablets, Bodoh-Creed et al. (2021) find that one third of auctions lead to an inefficient allocation, i.e., the item is sold to low private value bidders, and the deadweight loss is of the order of 14% of the market surplus. The authors explain their result by the important selection of bidders. They simulate multi-unit auctions ranging from 2 to 4 units for sale in each auction and find that they would lead to an efficiency gain, but at the cost of lowering the price and revenues for eBay.

On eBay, some auctions feature a buy-it-now feature, which allows anyone to purchase the item at a posted price before bidding begins. This allows risk averse or impatient buyers to pay, thereby avoiding the uncertainty and time involved with the standard auction format, potentially increasing seller revenue. Ackerberg et al. (2006, 2017) present a structural model of these auctions and estimate it using data from a subset of eBay auctions. They assume that bidders arrive in continuous time according to a Poisson point process. Bidders decide whether to participate, and conditional on participation, whether to buy it now or bid. A complication arises with bidding strategies because eBay auctions present the not well-explained phenomenon of re-bidding. In what is essentially a second-price auction, we observe agents revising their bids after being outbid. This implies they are not bidding their valuations. This requires the authors to specify a partial likelihood for the arrival and buy-it-now data. They combine this with simulated moment conditions using bid amount data. They are able to identify arrival rates, coefficients of risk aversion, and parametrically identify the valuation distribution. It is difficult or impossible to identify risk preferences in traditional auction data, making this one their main contribution.[23] Ackerberg et al. (2017) show that the bidders' value distribution, the Poisson process rate, bidders' utility functions, and their preferences are nonparametrically identified, exploiting observations on the reserve price, the buy-it-now price, the bidding length, the buying length, the outcome (buy-it-now option chosen), and the support condition.

Marra (2021) studies data from a wine auction site in U.K. and models endogenous entry for both buyers and sellers. Her focus is on the two-sided market effects on this auction site. The auction site becomes more valuable to buyers when more sellers enter and vice versa. Auction sites earn revenues from commissions on sale amount

[23] Ackerberg et al. (2017) provide a nonparametric identification result for the time preference function, risk aversion, and the value distribution based on a large support condition.

(possibly for both seller and buyer), listing fees (for sellers), and even entry fees (for both seller and buyer). Endogenizing sellers' entry allows the author to assess the sellers' selection effect when increasing listing fees. Her counterfactuals show that increasing the listing fees would cause sellers with high private values/marginal costs not to enter. This would make the auction site more attractive to buyers, resulting in an increase in transaction prices and in revenues for the auctioneer.

5.3 Online trust

The advent of e-commerce and online auctions brought the important question of whether one could trust online sellers to actually delivered on their promises. E-commerce and online auction sites responded to this challenge by implementing feedback and reputation mechanisms. For example, on eBay, buyers and sellers can rate their counterparty with a positive, negative, or neutral rating, along with a short textual comment (buyers can also leave detailed ratings about various aspects of the purchase). The feedback ratings and comments are publicly displayed for each user, along with summary statistics showing the percentage of positive ratings and the sum of ratings received to date.

The public availability of ratings and textual comments data along with the availability of data on sales outcomes (e.g. whether the item was sold and its eventual price) allow a large number of researchers to study whether prices/sales outcomes reflect the information in the ratings and comments. We do not cover this large literature in much detail here as it has been covered in past surveys of the literature (see Resnick and Zeckhauser (2002), Dellarocas (2003), Bajari and Hortaçsu (2004), Tadelis (2016)). Since the early work in this area, which focused on the association between sales outcomes and summary statistics of feedback ratings, observational studies utilize more of the textual/image content (e.g. Chevalier and Mayzlin (2006), Pavlou and Dimoka (2006), Lewis (2011)), and some studies have also pursued a field experimental approach to overcome the issue of omitted variable bias (e.g. Resnick and Zeckhauser (2002), Bolton et al. (2004), Jin and Kato (2006)).

A more recent line of research strives to understand the sources of information asymmetry/quality uncertainty and to design appropriate mechanisms to alleviate this asymmetry. Several studies use aspects of the reputation mechanism and changes in the mechanism to test whether the source of information asymmetry is due to adverse selection vs. moral hazard. For example, Cabral and Hortaçsu (2010) find evidence that the process with which eBay sellers receive feedback is non-stationary; the rate at which negative feedback arrives accelerates right after the seller receives her first negative feedback, and right before the seller exits the market. This suggests moral hazard, as opposed to adverse selection. Klein et al. (2016) and Hui et al. (2018) study the effect of a change in eBay policy that prevents sellers from leaving negative feedback on buyers as retaliation for negative reviews. As argued previously by Klein et al. (2006) and Bolton et al. (2013), the ability of sellers to retaliate lowers the cost of leaving negative feedback. Consistent with this hypothesis, both sets of authors find that removing retaliation leads to higher quality transactions after the

policy change. Klein et al. (2006) report not finding significant change in sellers' exit, suggesting that moral hazard plays a major role in quality uncertainty. Hui et al. (2018) argue that the sample analyzed by Klein et al. (2016) focuses on seasonal sellers who are less likely to exit the market, and report substantial exit from the marketplace, suggesting that adverse selection induced exit plays an important role in the observed quality improvement.

Nosko and Tadelis (2015) dig deeper into the problem of biased feedback caused by the relative costliness of leaving negative vs. positive feedback on marketplaces like eBay, also noted by Dellarocas and Wood (2008). Using internal data from eBay, they show that there are three times more consumer complaints to eBay than publicly reported negative feedback, and that many buyers appear to not leave any feedback instead of leaving negative feedback. Nosko and Tadelis (2015) then propose an alternative feedback score that penalizes sellers who have the same number of positive feedback ratings but have more transactions, i.e. sellers with more buyers who chose to leave no feedback. They then implement a field experiment in which eBay's search algorithm favors sellers with this new feedback score and show that the intervention leads to higher customer retention (in terms of buyers returning to eBay to purchase goods) compared to the control group.

Recent research also studies how more traditional trust mechanisms such as warranties and certification perform in online marketplaces. For example, Roberts (2011) finds that the introduction of a money-back warranty by an auction marketplace selling tractors does not appear to affect the reputation premium significantly, and thus concludes that warranties do not substitute for reputations. Hui et al. (2016) study a wider array of products for which eBay instituted a buyer protection policy, and find that the buyer protection program reduces the reputation premium, and increases prices, with higher exit rates for lower quality sellers.

We believe this remains a fertile area of research as online trust remains an open challenge affecting the many millions of consumers and sellers who participate in online transactions every day.

5.4 Auctions vs. fixed prices

As noted by Einav et al. (2018), fixed prices have emerged as the dominant method of selling physical goods online as opposed to auctions (the same conclusion, however, does not apply to the sale of Internet advertisement slots, the topic of the next section). This is even true for eBay, the pioneering online auction site. Einav et al. (2018) investigate the reasons for this shift using data from eBay. They provide a simple model of the posted price and auction mechanisms with homogeneous consumers. Consumers have a common valuation, a common reservation utility (which captures competition from other e-commerce retailers), and a common convenience cost of participating in an auction, due to, for example, having to wait for the auction to end. The main trade-off the model is intended to capture is that auctions allow for price discovery and buyer competition but are less convenient for buyers. The authors use data of eBay listings from 2003 to 2009, focusing on matched listings, where the

same seller lists the same item multiple times using each mechanism at least once. They find that changes in item or seller composition do not explain the shift towards posted prices. They estimate the change in hassle cost and reservation utilities from 2003 to 2009 and find that they both have increased. They estimate that changes in the convenience cost of auctions had more than three times the effect on the posted price profit differential over auctions than did the change in reservation utilities. The authors' conjecture for the source of this shift is that online auctions began losing their attraction as a novel form of entertainment as the Internet evolved and many other forms of online entertainment began to command users' attention.

5.5 Beyond proxy auctions

Another popular auction format to sell goods on the Internet is the "penny auction," studied by Augenblick (2016). In each period of a penny auction, bidders can pay a nonrefundable bid cost to become the leader of the auction. Each time the leader changes, the bid amount increases. The auction ends when no one is willing to pay the bid cost to become the leader. The leader then receives the object and pays the bid amount. The author restricts attention to a perfect information common value setting.

Augenblick (2016) describes two equilibrium concepts for this auction. One is a simple refinement of the Markov perfect equilibrium. The other is a naive sunk cost modification of the first in which agents value the good more than sunk costs increase but they do not anticipate this for themselves. The main difference in prediction from these two models is that with sunk costs, profits are positive and the probability a particular agent bids in a period increases with time (as sunk costs accumulate), rather than decreasing. Turning to data gathered from a penny auction website, the author observes the patterns predicted by the sunk cost model, even when controlling for risk-seeking or joy-of-winning preferences.

6 Internet advertising auctions

Perhaps one of the most important recent use of auctions in recent years is in the area of Internet advertising, which, as of 2019, has become a $124 billion business (Pricewaterhouse Coopers (2019)). Search engines such as Google and Bing and social media sites like Facebook utilize auction mechanisms to allocate space on their webpages for advertisers to promote their products. Not surprisingly, this application of auctions has become a very fertile area of research that combines theory and empirics. Empirical work is aided by the availability of large amounts of data, and the ability to run field experiments in collaboration with auctioneers or advertisers. As this is a large and fast-growing area of research, our aim here is to provide only an overview of some of the research strands. For other surveys of this literature see e.g. Choi et al. (2020) or Hortaçsu and McAdams (2018).

6.1 Modeling sponsored search auctions

Some of the early ad auctions on the Internet, for example, by paid search pioneer Overture.com were conducted as first-price auctions (Edelman and Ostrovsky (2007)). The system allowed bidders to update their bids over time, and this led to bid instability, since bidders had the incentive to explore where their competitors' bids were, and to overbid by just enough to avoid paying too much. Using data from Overture.com, Edelman and Ostrovsky (2007) document "sawtooth" bidding patterns consistent with this kind of behavior.

The later generation of sponsored search auctions tends to use a variant of the second-price auction called the "generalized second-price auction". Varian (2007) and Edelman et al. (2007) provide seminal models of these auctions. These models consider a set of advertising slots on a search page associated with a search keyword. The slots are ordered according to their expected click-through rate. Advertisers have heterogeneous values for clicks and bid a payment-per-click rather than for specific slots. Advertisers are assigned to slots in the order of their bids, with the highest bidder getting the slot with the highest expected click-through rate. Each advertiser pays per-click the next highest bidder's bid so that the advertiser's total expenditure is the product of their assigned slot's click-through rate and the next-highest bidder's bid.

Varian (2007) characterizes the Nash equilibria of this auction. In particular, he finds a system of inequalities that must be satisfied in any Nash equilibrium and that can be used to bound valuations. Edelman et al. (2007) independently find similar theoretical results. They describe a similar equilibrium concept to Varian's symmetric Nash equilibrium and find that the worst case revenue for this equilibrium concept is equal to the revenue from the Vickrey-Clarke-Groves (VCG) auction. They find that, unlike the second-price or the VCG auction, there does not exist a dominant strategy equilibrium for the position auction. However, in an associated English auction there is a unique perfect Bayesian equilibrium. It has the same outcome and payments as a VCG auction. Furthermore, it is an ex post equilibrium – that is, given full information, advertisers would not want to deviate from their strategies.

Athey and Ellison (2011) integrate a model of consumer search into the model of position auctions considered in the previous papers. Advertisers are assumed to have heterogeneous probabilities q_i to satisfy consumer needs. Consumers pay a search cost to click on an advertisement and their need is satisfied with probability q_i. If bidding strategies are restricted to be monotone in q_i then the position of an advertisement signals quality to consumers. They find that reserve prices can increase both search engine revenues and consumer surplus.

Many sponsored search auctioneers use click-through weighting to place advertisers into slots. That is, the winners of a position auction are determined by the product of the bids with a bidder-specific quality score that is estimated from click-throughs (the sponsored search site typically reserves a number of impressions to show advertisers in randomized positions, and uses these experiments to estimate an advertiser specific click-through rate). These are intended to prevent advertisers with high valuations but very low click probability from winning the best positions. Athey and

Ellison (2011) find that while click-weighting is efficient as search costs become negligible, positions will be less informative about advertiser quality and click-weighting is not necessarily efficient with search costs. Finally, they find that click-weighting can potentially give incentives for advertisers to make their ads less informative.

6.2 Empirical work

As noted above, Varian (2007) characterizes a system of inequalities that must be satisfied in any Nash equilibrium. In particular, this system of inequalities can be used to bound valuations. These inequalities are functions of observed click-through rates and bids and can therefore be used as an empirical test of the Nash equilibrium prediction. Varian (2007) applies this idea to data from search auctions and finds that only small perturbations to measured quantities are need to satisfy these inequalities.

Börgers et al. (2013) generalize the framework of Varian (2007) to allow for variation in advertisers' valuations per click across positions and differences in click rates between advertisers. Assuming bidders are using Nash equilibrium strategies, the authors identify a set of bounds for valuations. These bounds are a function of the allocation, bids, and click-through rates. Using data from Overture (in this later period, Overture uses a generalized second-price auction, not a first-price auction as in Edelman and Ostrovsky (2007)), they test whether there is a nonempty set of valuation profiles satisfying these bounds for observed bidding data. They find that bidders' behavior can only be rationalized over relatively short time periods of time, after which they must assume an unexplained structural break in preferences. They then test the assumption of constant valuations across positions implicit in Varian (2007) and Edelman et al. (2007). They reject the constant valuation hypothesis and find that monotone valuations, with clicks from higher positions being valued more than those from lower positions, are consistent with the data.

Ghose and Yang (2009) estimate a simultaneous equation model of sponsored search advertising. They use data from a single advertiser bidding for ad space on Google. They find that the monetary value of a click is not uniform across positions because conversion rates, i.e. the probability of a purchase conditional on a click, decrease with rank. This anticipates the result in Börgers et al. (2013). While it is known that Google uses prior click-through rates in deciding quality score, they find that the current bid has the largest effect on realized ad rank. Interestingly, despite having higher click-through and conversion rates, the highest ranked positions are not the most profitable. Instead, the cost per click declines fast enough that profit is often maximized in the middle slots. Keywords with retailer specific terms are associated with higher click-through and conversion rates and at lower cost per click and rank. Keywords with product brand specific terms see lower click-through and conversion rates and lower rank but a higher cost per click.

Hsieh et al. (2018) estimate a structural model of an un-scored position auction and consider the counterfactual of implementing a scoring rule. Their data is from an online marketplace rather than a search engine. In this environment, assuming monotonically increasing click-through rates with position rank is no longer an intuitive

assumption nor observed. They also do not assume multiplicatively separable preferences into bidder- and position-specific effects. They estimate the model by transforming it into a Shapley-Shubik assignment game and utilizing techniques similar to those used in the empirical matching literature (Fox (2018)). Their results indicate horizontal differentiation between keywords, with high and low quality bidders sorting between informative and vague keywords. In the score-weighting counterfactual, they predict a steepening in the price-by-position curve. While scoring should provide discounts to high quality advertisers, they find that increased competition leads to higher prices for top positions. Overall, they do not find large effects on platform revenue and sorting patterns from shifting to a scoring rule.

Yao and Mela (2011) obtain bidder, product, and consumer data from a software shopping search engine. They use a multistage dynamic discrete choice model for both consumers and bidders. Consumers choose whether to search, whether to sort or filter the results, whether to download software, and if so, which software to download. Bidders dynamically bid in a score-weighted position auction where the score is the previous period downloads. They find considerable consumer heterogeneity, with frequent link clickers being more sensitive to ad position. The authors use their structural model to conduct several policy counterfactuals. They consider eliminating the sort/filter option for consumers to find that this reduces search engine revenue, consumer welfare, and advertiser profits. They also consider a consumer segmented auction where bidders bid for positions for a specific keyword-segment pair rather than just for a keyword. This decreases competition but increases efficiency. Overall it increases consumer welfare, search engine revenue, and advertiser profits. Finally, they compare first-price versus second-price auctions and find approximate revenue equivalence.

6.3 Reserve prices in sponsored search auctions

Auctions are typically designed for settings with multiple buyers, but the online advertising setting provides a great backdrop in which, for example, the particular content/viewer-demographic pairing will have a high value for only one of the potential advertisers. When a monopolist faces a single buyer for the good, the standard approach is to offer a take-it-or-leave-it price optimized against the distribution of the buyer's willingnesses-to-pay. If there are two competing buyers, however, it is often better to run an auction between them. While the auctioneer per se does not require knowledge of the valuation distribution (which may not be necessary to beat the take-it-or-leave-it price according to Bulow and Klemperer (1996)), the monopolist, under "regular" conditions, can further improve her profits by setting a reserve price to insert a wedge between the first and second highest bid. The design of this reserve price, though, depends once again on the knowledge of the distribution of buyer valuations.

Ostrovsky and Schwarz (2016) investigate the role of reserve prices in a field experiment conducted on Yahoo! search auctions for over 400,000 keywords. They estimate the distribution of bidders' values based on the generalized second-price

auction model of Edelman et al. (2007) and Varian (2007), and compute the optimal reserve price for each auction using a generalization of the Myerson (1981) optimal reserve price formula. They find a median optimal reserve price of 20 cents, which is substantially higher than the 10 cent reserve price that was uniformly assigned by Yahoo! at the time. Indeed, the estimated optimal reserve price was higher than 10 cents for 90% of the search keywords they had in their sample. The authors then run a field experiment with about 95% of their keyword sample, keeping 5% of the keywords as a control group. In the implementation of their experiment, to comply with the company's policy, they set advertiser specific reserve prices weighted by advertiser specific ad-quality scores. This adjustment, they argue, may lead to lower revenues for the subset of keywords whose reserve prices are close to the 10 cent default. Their empirical results appear to corroborate this prediction: For keywords for which the estimated reserve price is high, they find a significant increase in revenue per search, while revenues decline for keywords with low estimated reserve prices.

Celis et al. (2014) focus on the case where there are, effectively, one and two buyers for a good. The online advertising auction setting provides a great backdrop for the model: For many ad serving instances, the particular content-viewer demographic pairing has a high value for only one of the potential advertisers. Other advertisers likely have much lower values for serving an ad to this particular viewer on this particular site. This would likely lead to a valuation distribution that violates the increasing virtual valuation condition that is often assumed in the auction literature. In such situations, an optimal mechanism that involves ironing can be pursued. However, this requires a lot of knowledge of the distribution of buyer valuations, and spelling out the mechanism to bidders (who are supposed to realize that it is incentive compatible and bid their true values) may not straightforward.

Celis et al. (2014) then propose a practical mechanism, BIN-TAC, that is a hybrid between take-it-or-leave-it pricing and running an auction: First a "buy it now" (BIN) price p is offered, which becomes the price of the good if a single bidder accepts this price. If multiple bidders accept the BIN price, then an auction is run with a reserve price equal to p. If no bidder meets the reserve price, again an auction is run, but with the lower reserve price r. Ties in the second round auction are broken randomly; in particular, one of the top d bidders is randomly chosen, and if that bid is above r, the winner pays the $d+1$st price. Thus, this is a mechanism with 3 parameters to optimize over, namely (p, r, d). The authors illustrate the benefits of this mechanism. Essentially, this is a "sequential screening" mechanism, where the BIN price can be tuned to screen out high value bidders from low value bidders. There is, however, randomization in the take-a-chance stage, which is what essentially happens in ironing as well. Indeed, the authors show that the BIN-TAC mechanism achieves revenue optimality in the two-type case. To generalize their result, the authors follow two strategies: One is to run a number of simulation exercises over classes of distributions, suggesting that BIN-TAC delivers revenues close to the optimal screening mechanism, and is certainly better than running an auction with a reserve price in non-regular cases. The second exercise is to run an empirical calibration using data from Microsoft's ad exchange, which uses second-price auctions. The authors esti-

mate valuation distributions, and show that the BIN-TAC mechanism performs well empirically against the optimal mechanism.

7 Electricity auctions

Electricity has traditionally been thought as a classic example of a natural monopoly-produced commodity, where regulation is needed to balance consumer interests with that of the incentives of the monopolist to invest and assure uninterrupted supply. However, in the last three decades, many countries and regions have unbundled the transmission component from the generation and retail distribution components of the electricity supply chain, with the thought that generation and retail distribution may sustain competition while relying on the (regulated) use of transmission resources. Auctions have become particularly prevalent especially in the restructured wholesale electricity markets around the world, where generators connected to the transmission network bid for the right to inject (and sometimes not to inject) electricity into the grid (Wilson (2002)). These auctions occur throughout the day not only to balance real-time supply and demand, but also as a chain of forward markets, generating a path of market expectations to aid the considerable production planning effort that goes into ensuring seamless market clearing. Along with the spot and forward market clearing auctions, a plethora of auxiliary auctions or auction-like mechanisms are utilized to procure and allocate, for example, reserve generation that may be needed to balance out short-term supply or demand shocks, or transmission rights, which help reveal the shadow value of transmission resources and hence guide transmission investment decisions.

7.1 Models of bidding in electricity markets

There is, by now, a very robust theoretical and empirical literature on the design and functioning of electricity markets, even when confined to the use of auctions in these complex and multi-layered ecosystems. Our discussion therefore will not aim to be comprehensive. That said, a common approach towards understanding these auctions is through the lens of multi-unit or divisible good auctions, in which bidders submit, essentially, supply or demand schedules to be aggregated by the auctioneer. The theoretical study of such auctions started with the seminal work of Vickrey (1961), who derived the concept of Vickrey auctions and truthful revelation. Subsequent seminal contributions were made, especially towards the understanding of strategic behavior in non-dominant strategy solvable auctions, by Wilson (1979), Kyle (1989), Klemperer and Meyer (1989), and Ausubel et al. (2014). A key feature of these theoretical analyses is to characterize the best response of bidders to their beliefs about the distribution of the residual demand curve (total demand minus the supply by other bidders) and search for a set of mutual best responses consistent with the beliefs.

We now discuss a model that is based on the uniform price share auction setup of Wilson (1979). The cost of generation (suppressing the time subscript t) of the N

firms in this market is $\{C_i(q), i = 1, .., N\}$. Total demand $\tilde{D}(p) = D(p) + \varepsilon$ is the sum of a deterministic price-elastic component and a stochastic constant term. Prior to the auction, each firm has signed forward contracts to deliver certain quantities of power each hour, given by QC_i, at a fixed price PC_i. QC_i and PC_i are variables on their own that denote contract quantity and contract price, not Q and P (standard quantity and price notation) multiplied by cost C_i. Section 7.4 discusses the interaction between the forward contracting market and the spot market in more detail.

For a given time period, each firm submits a supply schedule, $S_i(p, QC_i)$. The auctioneer adds up the supply schedules and computes the market clearing price p^c satisfying the market clearing condition $\sum_{i=1}^{N} S_i(p^c, QC_i) = \tilde{D}(p^c)$.

Under the uniform price rule, firms get paid the market clearing price for their infra marginal units, giving the ex post profit expression:

$$\pi_{it} = S_i(p^c, QC_i)p^c - C_i(S_i(p^c)) - (p^c - PC_i)QC_i$$

where the last term is due to the contract position: The firm has to guarantee its contract customers the contract price PC_i (Wolak (2000)).

The market clearing price p^c, however, is uncertain. A first source of uncertainty is the market demand, $\tilde{D}$. Firms may also be uncertain about each others' costs, and especially their rivals' contract positions. Most of the literature on electricity assumes that firms bidding into the market know their rivals' total cost functions. In contrast, contract positions are more difficult to observe since these are negotiated in over-the-counter trades. Hence, we assume contract positions are private information.

Bidder i's expected profit is given by

$$\max_{\hat{S}_i(p)} \int_{\underline{p}}^{\bar{p}} p\hat{S}_i(p) - C_i(\hat{S}(p)) - (p - PC_i)QC_i dH_i(p, \hat{S}_i(p); QC_i)$$

where

$$H_i(p, \hat{S}_i(p); QC_i) \equiv \Pr(p^c \leq p | QC_i, \hat{S}_i(p))$$

is the probability distribution over the market clearing price p^c.

Wilson (1979) shows that the Euler-Lagrange condition for the optimal supply schedule $S_(^* p)$ is (as derived in Hortaçsu and Puller (2005)):

$$p - C_i'(S_i^*(p)) = \left(S_i^*(p) - QC_i\right) \frac{H_S(p, S_i^*(p); QC_i)}{H_p(p, S_i^*(p); QC_i)} \tag{1}$$

where

$$H_p(p, S_i^*(p); QC_i) = \frac{\partial}{\partial p} \Pr(p^c \leq p | QC_i, S_i^*(p))$$

$$H_S(p, S_i^*(p); QC_i) = \frac{\partial}{\partial S} \Pr(p^c \leq p | QC_i, S_i^*(p))$$

The equation can be seen as a "markup" expression, where the markup in price above the marginal cost depends on the density of the market clearing price ($H_p(.)$ term in the denominator), and the ability of the firm to shift the market clearing price distribution by increasing its supply ($H_S(.)$ in the numerator). Note that as $H_S \to 0$, the firm has no market power, hence price approaches marginal cost.

As noted by Wolak (2000), the contract quantity QC_i plays a very important role in the markup expression. When $S_i^*(p) > QC_i$, the firm is a net seller of electricity, and hence the markup is positive, as would be expected from an oligopolist. However, when $S_i^*(p) < QC_i$, the firm is a net buyer of electricity, since its own supply does not cover the contracted quantity. In this situation, the firm is an *oligopsonist* and hence will seek a lower price. Therefore, the markup is negative; the firm will bid below its marginal cost in the part of its supply bid that is less than QC_i.

The set of first-order conditions (1) for each firm characterizes the equilibrium strategies $S_i(p, QC_i)$ for the participating firms, given their cost curves $C_i(q), i = 1, .., N$, the joint distribution of contract quantities and the distribution of the uncertain demand component. A number of papers in the literature propose simplifications that render the solution more tractable. For example, Klemperer and Meyer (1989) assume that the only source of uncertainty is the demand noise, in which case the set of first-order conditions (1) becomes a system of differential equations. In the symmetric case, Klemperer and Meyer (1989) show that there exist multiple solutions to the differential equation, depending on the support of the demand noise. Green and Newbery (1992) explore asymmetric solutions to the Klemperer and Meyer model. A popular approach is to use the method of undetermined coefficients: Assume that the equilibrium strategies satisfy a particular functional form (e.g. linear), and show that, under certain assumptions on the distribution of uncertainty and the profit/cost function, the best response to the conjectured strategy also satisfies the posited functional form. In a common value version of the model, Wilson (1979) and Kyle (1989) solve symmetric versions of the model that admits an analytical solution. Boyarchenko et al. (2021) generalize the Kyle (1989) model to allow for asymmetries.

7.2 Tests of theory

The availability of detailed and abundant bid data, and the ability to measure marginal generation costs with relative accuracy, is a feature of electricity markets that has made them fertile ground for researchers interested in studying auctions and imperfect competition in general. Green and Newbery (1992) pioneer the application of the multi-unit auction framework, especially the supply function equilibrium model of Klemperer and Meyer (1989), to understand competition in U.K.'s then newly restructured electricity market. They calibrate cost functions for the effective duopolists in the market, along with the demand, and solve for the supply function equilibria (SFE). There are multiple SFE that range between marginal cost pricing and one that implements Cournot quantities and prices, hence they report a range of outcomes. They find that the highest price/lowest quantity equilibrium implies large markups above cost, though the prices they observe do not appear to reflect such high markups,

which they suggest may be due to the fact that the generator has already sold significant portions of its capacity in the forward market. In order to investigate long run outcomes, they use their model to study entry incentives and outcomes, and find that entry incentives are highly dependent on demand elasticity and the pricing strategies followed by the incumbents.

Subsequent work by Wolfram (1998, 1999) implements empirical tests of strategic behavior in this market. Wolfram (1999) utilizes marginal costs of production and documents the margin of difference in supply functions and the measured marginal costs. A second set of tests utilizes responses by bidders to demand-weighted price caps. Controlling for cost factors, bidders do appear to respond to demand, suggesting strategic behavior. Wolfram (1998) delves further into the multi-unit aspect of bids, and shows evidence consistent with strategic withholding of supply.

Borenstein et al. (2002) utilize detailed measurements of marginal costs to quantify inefficiencies generated by strategic behavior/exercise of market power in California's power market. They find that market power was responsible for the majority of the dramatic price increases during the summer of 2000. Puller (2007) uses similar data to measure firm conduct to examine hypotheses about dynamic imperfectly competitive behavior. Hortaçsu and Puller (2008) utilize measured marginal cost data to examine whether generation firms in the Texas electricity market exhibit static best response behavior in their submitted bids.[24] They document heterogeneity in the ability of the static best response model to describe behavior. While larger firms appear to behave closer to the static best response benchmark, smaller firms appear to bid supply curves that are too inelastic. The latter finding is somewhat counterintuitive, since smaller firms possess less market power, and hence would be expected to bid more elastic supply curves closer to marginal cost. The authors also show that most productive inefficiencies in this market arise not from the strategic behavior of firms following static best response, but from the inelastic supply response of firms that do not appear to follow static best response.

Hortaçsu et al. (2019) rationalize these findings through the non-equilibrium "k-level" model of Camerer et al. (2004). The k-level model posits that some participants (called "level 0" players) are non-strategic, while others ("level 1" players) assume that everybody but themselves are non-strategic (and act to optimize against level 0s). Some more sophisticated players ("level 2") may realize that there exist both level 1 and 0 players in the population, and maximize against that, and so on. The authors show that the k-level model with its hierarchy of strategic sophistication fits the data from power generators' bidding behavior extremely well. Indeed, the levels of strategic sophistication estimated for the firms seem to coincide well with observable measures of firm/managerial experience in strategic environments. The k-level model of boundedly rational behavior is also analytically and computationally more tractable than the full rational model of Nash equilibrium behavior, and allows the

[24] To test theory, one must have data not only on generation costs, but also on contract positions. Hortaçsu and Puller (2008) show that in Eq. (1), where $S_i^*(p) - QC_i = 0$, $p = C_i'(S_i^*(p))$. Thus, the quantity at which a firm's marginal cost curve crosses its supply curve identifies the contract position.

authors to compute societal gains/losses from mergers. Such merger calculations are more difficult to conduct using the (Bayesian) Nash equilibrium model, which is computationally involved.

7.3 Estimating generation costs

Another approach in this literature is to treat production costs as primitives to be estimated from the data. As in the literature on single unit auctions, the set of first-order conditions (1) is helpful towards econometric inference on unobserved model primitives. As in Guerre et al. (2000), if one can obtain estimates of the market clearing price distribution, $H_i(p, S_i^*(p); QC_i)$ and its derivatives, one can estimate the markup, up to the contract position. (The Guerre et al. (2000) insight was generalized to the setting of multi-unit auctions by Hortaçsu (2000) to study Treasury securities auctions.) In a pioneering paper, Wolak (2003) utilizes best-response behavior and data on firms' contract quantities to estimate marginal costs of production of power generators in Australia's electricity market.

Along with marginal costs, fossil fuel-based electricity generators have non-trivial start-up costs, which creates frictions to entry and exit in response to short-term fluctuations in demand. Reguant (2014) seeks to recover generators' economic startup costs. The Spanish electricity market she studies provides a unique opportunity to recover these costs. In this market, firms have to submit (and commit to) bids for the next 24 hours, and their bids can specify minimum daily revenue requirements that need to be satisfied for certain units to be turned on. Thus, she models the optimal declaration of minimum revenue requirements, which leads to a transparent way to identify the units' startup costs, under the assumption that bidders have (rational) expectations over residual demand realizations (which is a reasonable assumption given that the auctions take place every day). The results lead to revised estimates of market power in this market (market power screens based on such bid data are routinely used in these markets), with firms appearing to have larger market power than they do when startup costs are not taken into account.

7.4 Forward contracts and sequential markets

In most restructured wholesale electricity markets, operational/regulatory constraints call for forecasted demand to be allocated in a forward market, with unforecasted demand adjustments cleared in the spot market. Wolak (2007) uses estimated generation costs to argue that forward contracts allow generation firms to smooth out the ramp up/ramp down costs, leading to more efficient production. Beyond operational benefits, forward contracts may also have a competitive benefit, as pointed out by the seminal theoretical work of Allaz and Vila (1993). The authors characterize a multi-stage game in which firms can make forward sales until a final production period. An interesting insight of the model is that in equilibrium, forward sales lower price. As the number of forward trading periods increases, price gets closer to marginal cost.

As pointed out above, contract positions play an important role in a generator's optimal bidding decision (Wolak (2000)). Often, forward contracts are operational-

ized through vertical integration. Bushnell et al. (2008) show that accounting for the vertical integration structure of electricity generators and retailers is essential to understanding market outcomes across three U.S. electricity markets (New England, PJM, and California). Borenstein et al. (2008) study the interrelationship between forward and spot market prices in California's electricity exchange. They find persistent price differentials, despite the profit opportunities from arbitrage. Ito and Reguant (2016) study the Spanish electricity market. They argue that given the relative intertemporal unresponsiveness of demand, electricity generators with market power can try to price discriminate between the forward and spot markets to increase profits. The authors show evidence consistent with this hypothesis: a persistent and predictable forward premium as in Borenstein et al. (2008). The authors show that the main source of the price differential is wind generators who try to take advantage of the forward premium by selling above their forecasted production in the forward market, and covering their shorts at a profit in the spot market.

Many restructured markets allow the participation of purely financial bidders as well as generators. Jha and Wolak (2021) show that the presence of arbitrageurs closes the forward-spot price differential in the California energy market. Birge et al. (2018) show that the ability of arbitrageurs to close price gaps is limited by their access to capital and transaction costs. They show that the forward-spot price differential on MISO (Midcontinent Independent System Operator) is responsive to the cost of capital and regulatory constraints imposed on financial traders by the market operator.

7.5 Transmission constraints

Physical constraints often bind on the transmission network, making it difficult to balance supply and demand through a single market-wide price. Some electricity markets thus follow a zonal price model, where the overall market is divided into sub-markets that clear separately in the case of congestion, or a nodal price model, in which every supply or demand node is assigned a different price resulting from the solution of a network-wide optimization problem. In the presence of transmission constraints, it becomes difficult to assess how competitors' supply or demand bids may affect the price affecting a particular bidder, since the relevant zonal/nodal price depends not on the entire set of bids, but on the set of bids that are feasible for that zone/node. That is, competition becomes a complicated function of the profile of bids that are submitted and physical network constraints. Moreover, congestion can easily generate local market power, especially in networks where the concentration of supply vs. demand is separated in space.

Ryan (2021) studies the electricity market in India, where transmission constraints create considerable price differentials across different regions. Accounting for transmission constraints is crucial for understanding market structure; the effective concentration in congested periods is much higher than in uncongested periods. The author estimates the marginal costs rationalizing observed bids, and computes counterfactual equilibrium outcomes (under a Cournot model) when new transmission capacity is added. He finds that most of the surplus gains are realized by sellers

in exporting regions, who are able to sell more electricity at higher prices. In importing regions, increases in buyers' surplus are offset by the decrease in the surplus of sellers, who now face more competition.

Mercadal (2021) studies MISO, which uses a nodal price system allowing every node where a seller/buyer injects/draws electricity to have a separate price. In such markets, defining the set of competing bidders is difficult, since it is a complicated function of submitted bids and network characteristics. The author utilizes a clustering algorithm that groups together nodes that appear to co-move together, and thus bases her market definition on these clusters.

Transmission constraints also create the interesting problem of allocating transmission rights (Joskow and Tirole (2000)). Birge et al. (2018) show that financial transmission rights on the MISO grid interact with bidding on the forward and spot markets. Along with its impact on the forward and spot markets, the design of transmission rights allocation systems also affects incentives for investment into new transmission capacity, which we believe is a relatively understudied question.

8 Auctions in financial markets

The value of financial securities depends on expectations about the future, which can change rapidly in response to news and information flows. This makes auctions natural mechanisms to buy and sell financial securities. We discuss here only a small set of papers that have studied auctions of financial securities. Importantly, we do not cover auctions or auction-like mechanisms used in important swathes of securities markets, such as equities or currency markets. For book-length coverage on the topic of market microstructure in finance, we refer the reader to the excellent texts of O'Hara (1997), Hasbrouck (2007), Brunnermeier (2001), and Vives (2010). Our focus here is on the study of auctions employed in financial transactions. Chapter 15 of this Handbook provides a detailed account of auctions of securities and wholesale funding.

8.1 Treasury auctions

Auctions of treasury bills and securities are used to finance national and local government debt around the world. As in electricity markets, most of these auctions utilize a multi-unit auction format, following one of two main mechanisms: the *discriminatory auction* mechanism, also known as the "pay-as-bid" auction, and the *uniform price auction*. In both formats, bidders submit a vector of price-quantity pairs as their bids, defining *bid functions*. The seller computes the sum of the bid functions, and calculates the market clearing price. In the discriminatory/pay-as-bid auction, bidders pay the price they bid for their infra-marginal units. In the uniform price auction, bidders pay the market clearing price for their infra-marginal units. The choice of auction format has been an active question of market design since Friedman (1960). Unfortunately, the revenue equivalence theorem does not apply in the multi-unit auc-

tion setting where bidders have multi-unit demands (Ausubel et al. (2014)), hence the relative revenue or efficiency performance of pay-as-bid or uniform-price auction formats is theoretically ambiguous.

Hortaçsu (2000) and Hortaçsu and McAdams (2010) investigate the question of revenue performance empirically, utilizing the Wilson (1979) "share auction" model described above as the basis of their structural econometric model. The model is applied to data from Turkish treasury auctions, which utilizes a discriminatory/pay-as-bid format. In the Treasury auction setting, each bidder i has a marginal valuation function $v_i(q, s_i)$, where s_i is the bidder's private information (we will discuss the private vs. common values assumption below). The vector $(s_1, ..., s_N)$ is drawn from a known joint distribution $f_s(.)$. Bidders' strategies are bid functions, $y_i(p, s_i)$. The profile of bid functions generated by the realization of bidder signals defines a residual supply function $RS_i(p) = Q - \sum_{j \neq i} y_j(p, s_j)$, faced by each bidder, whose intersection with bidder i's bid function, $y(p, s_i)$, determines the market clearing price p^c.

As in the (uniform price) electricity auction setting, one can derive Euler equations characterizing the optimal $y_i(p, s_i)$ for the discriminatory auction:

$$v_i(y_i(p), s_i) = \underbrace{p}_{\text{bid for } y_i(p) \text{ units}} + \underbrace{\frac{H(p, y_i(p), s_i)}{\frac{\partial H(p, y_i(p))}{\partial p}}}_{\text{"shading" factor}}, \tag{2}$$

where $H(.)$ is the probability distribution of the market clearing price conditional on bidder i's bid and signal s_i. As in Guerre et al. (2000), one can use data from past auctions to estimate $H(.)$ and its derivatives in order to estimate the marginal valuations of bidders. Given estimates of marginal valuations, one can assess, for example, whether the observed auction (which can be either uniform price or discriminatory) is *efficient*, by checking whether the bidders with the highest marginal values were awarded the securities. Hortaçsu and McAdams (2010), for example, find very little efficiency loss in their sample of Turkish treasury auctions, which are conducted using discriminatory auctions. Kastl (2011) finds similarly low efficiency losses in his study of Czech treasury auctions, which are conducted using the uniform price auctions. Kang and Puller (2008), in their study of Korean treasury auctions conducted using both discriminatory and uniform price formats, echo very similar results regarding the small loss of efficiency, though they find the discriminatory auction to have better allocational properties.

8.1.1 Discrete bids

In the real world, bidders submit discrete step functions instead of the continuous, differentiable bid functions characterized by the Wilson (1979) model. This discreteness may play an important role in the empirical analysis. McAdams (2008) and Hortaçsu and McAdams (2010) show that when bids are in the form of discrete step functions, bidders' underlying marginal valuations are only *partially identified*, i.e.,

one can only characterize upper and lower bounds on the underlying marginal valuations. Hortaçsu and McAdams (2010) show that the upper and lower bounds are indeed empirically distinct, making the partial identification problem empirically relevant in this setting. McAdams (2008) provides theoretical characterizations of sharp bounds on the underlying marginal valuations.

As described in more detail in Chapter 15 of this Handbook, the discreteness of bids can have important consequences for strategic behavior. Kastl (2011) shows that when there are constraints on the number of price-quantity bids a bidder can place in a given auction, it might be optimal for bids to exceed marginal valuations in a uniform price auction. Kastl (2011) finds that bidding above one's marginal valuation is quantitatively important in the Czech auction context.

8.1.2 Private vs. common values and the role of dealers

Along with Wilson (1979), most theoretical analyses of share auctions (e.g. Kyle (1989)), especially those applied to securities auctions, adopt a common values framework. Février et al. (2002) and Armantier and Sbaï (2006) take the common value approach in their econometric approach. Février et al. (2002) use the Euler equations characterized by Wilson (1979) in the common value case to pursue an estimation strategy analogous to the private values case considered above, whereas Armantier and Sbaï (2006) pursue a parametric estimation strategy, minimizing the distance between approximate Bayesian-Nash equilibria implied by parameters and observed bid strategies.

Hortaçsu and Kastl (2012) utilize data from Canadian treasury auctions to test for common values. In Canadian treasury auctions, primary dealers are allowed to observe the bids of their non-dealer customers. In a private value second-price auction, this allows for an empirical test of the private value hypothesis, as observing the bid of another bidder should not lead to a revision on one's own bid, especially if one's own bid prior to seeing the competing bids was higher. However, in a common value second-price auction, observing another's bid leads to a revision of one's expected value of winning the auction, and thus to a revision of one's bid, even if it originally exceeds the observed bid.

Hortaçsu and Kastl's (2012) data set allows for the observation of dealer bids before and after the observation of customer bids, thus making the above testing strategy feasible. An important complication that one needs to deal with when implementing this test, however, is the fact that the auction uses the discriminatory format. Even in a private values setting, observation of a competitor's bid can lead to a revision in one's own bid, since the competitor's bid resolves some of the uncertainty regarding the competition one faces in the auction. Hortaçsu and Kastl (2012) thus propose a method to account for this "strategic" source of revisions in dealers' bids. Their statistical tests, after accounting for "strategic" bid revisions, fail to reject the null hypothesis of private values in their sample of 3-month and 12-month Canadian treasury bills. The private value environment also allows them to estimate the surplus that dealers derive from observing customer bids in this auction. They calculate that

close to a third of dealers' profits in these auctions can be attributed to their ability to observe customer information.

Hortaçsu et al. (2018) study US Treasury bill auction data and assess the market power of primary dealers relative to direct and indirect bidders (indirect bidders need to route their bids through primary dealers as in the Canadian market; see Hortaçsu and Kastl (2012) and the discussion above). Because primary dealers possess more information on the residual supply they face than their opponents, they are in the position of extracting more surplus. Considering asymmetry among the three types of bidders, the authors' empirical results confirm that primary dealers enjoy more market power by bidding higher yields. This asymmetry, however, introduces modest efficiency losses.

8.2 Municipal bond auctions

The municipal bond market traditionally uses a first-price, sealed-bid auction format. Considering interdependent values, Milgrom and Weber's (1982) seminal paper ranks the revenue of the English auction above that of the second-price auction and the revenue of the second-price auction above that of the first-price auction. Shneyerov (2006) quantifies the magnitude of these differences for the municipal bond market. The author is able to nonparametrically identify the counterfactual revenue of the second-price auction and a bound for the revenue of the English auction by inverting the bidding strategy in the first-price auction. In particular, the difference in revenue between the first and second-price auction is estimated to be between .10-.16% of the par value of the bond and 9-13% of the gross underwriting spread. The difference between the bound of the English auction and the first-price auction is between .14-.23% of the par value of the bond and 11-19% of the gross spread. Thus, a majority of the gains from the English auction could be achieved with just a second-price auction.

Tang (2011) derives nonparametric bounds on revenue distributions from bid distributions to assess the impact of reserve prices on maximizing revenue. Using U.S. municipal bond auction data, his empirical results show that the use of optimal reserve prices would lead to little gains in first-price and second-price auctions relative to the observed first-price auction format.

Garrett et al. (2017) study the special tax exemptions that municipal bonds carry. These tax exemptions are intended to lower the borrowing costs of state and local governments. However, this tax advantage mostly benefits top income earners, which has made it a target for political debate. Utilizing auction data on municipal bonds, the authors provide evidence that these tax subsidies are an efficient way to reduce borrowing costs. Their reduced form analysis uses within-state variations in tax rates to estimate that a 1 percentage-point increase in the tax advantage leads to a decrease in borrowing costs of 6.5-7 basis points. This implies a passthrough elasticity of the borrowing rate to the tax advantage of 1.7-1.9.

The authors then estimate a structural model to recover markups, which average 17 basis points and are smaller for state issuers than cities, counties and school districts. They find that as the tax advantage increases, more bidders participate in the

auction and their valuations are more compressed. This increases the competitiveness of the auctions and leads to a passthrough elasticity greater than one. They conduct a counterfactual evaluation of the Obama administration's proposal of limiting the tax exemption and estimate that it would lead to an increase in markups of about 185%, and in borrowing rates of 31%. Compared to the reduction in the federal tax expenditure, the increase in borrowing costs is 2.8 times as large. They also evaluate the Tax Cuts and Jobs Act of 2017, finding that it would lead to a 2.5% reduction in borrowing costs and a 9.5% reduction in markups.

8.3 Auctions in central bank operations

Auctions play an important role in the implementation of Central bank operations. For example, the European Central Bank's (ECB) primary mode of monetary operations is through weekly collateralized loans to the banking sector, called main refinancing operations (MROs). Bindseil et al. (2009) provide a descriptive study of bidding in these auctions in the early 2000s. Cassola et al. (2013) focus on bidders' behavior during the 2007 financial market crisis that led to the Great Recession. The auctions studied by Bindseil et al. (2009) and Cassola et al. (2013) are conducted in the pay-as-bid format. Cassola et al. (2013) document a sudden and dramatic increase in the bids for ECB loans following August 2007, as reflective of funding shortages in the inter-bank market. However, along with the increase in the bid levels, the dispersion also increases, suggesting heterogeneity in funding problems across banks. While some of the observed bid increases may reflect a true and sudden shift in the underlying willingness-to-pay for short-term funding, some banks may have started bidding higher just to remain competitive in the auctions. Thus, the strategic nature of bids may have masked the true heterogeneity of funding troubles across Eurozone banks. Assuming optimal bidding, the authors quantify the strategic component of bids to isolate banks' underlying willingness-to-pay (WTP) for ECB loans. One interesting result of this exercise is that the estimated bidders' WTPs are much better predictors of balance sheet troubles at the end of 2007 than the bids are. Another practical offshoot of the exercise is that the bidding data, after accounting for strategic behavior, can provide high-frequency snapshots of the short-term funding rates faced by individual banks in the Eurozone. Since interbank markets are over-the-counter and thus not very transparent, and published rates based on surveys like the LIBOR and EURIBOR are based on self-declarations of panelist banks, banks' willingness-to-pay for short term funding rates may provide better measurement of funding troubles in money markets.

Using a similar data source on ECB liquidity auctions, Bonaldi et al. (2015b) estimate the network links among banks from their estimated funding costs by looking at the covariation of a given bank's funding cost with other banks' funding costs. The authors derive a measure of systemic risk as the externality a bank would impose on the funding costs of all other banks. The authors' empirical results show that the links are weak overall among banks and that a few banks play a central role in the market. They also find that vulnerable financial institutions subject to spillover effects from other banks are more likely to be bailed out.

The Federal Reserve (hereafter the Fed) also uses auctions to acquire US Treasury securities from dealers as part of its quantitative easing (QE) policy. For each purchase operation, the Fed announces a range of purchase quantity and a maturity bucket of the treasuries to be purchased. The dealers then provide offers for eligible securities. The Fed chooses the most attractive offers using secondary market prices and internally computed spline-based prices as benchmarks.

Song and Zhu (2018) provide a collection of empirical observations about the outcomes of these auctions, the most important of which are the following: The Fed tends to exclude from QE auctions the most liquid treasuries and securities with heightened specialness in repo markets.[25] Among Treasury securities included in QE auctions, the Fed's acquisitions tend towards the more liquid ones. Relative to secondary market ask quotes at the end of the auction, the Fed only pays an additional 0.7 cents per $100 par value on average, indicating low average costs. Dealers' profits are concentrated among the top five dealers, who account for 96% of all dollar profits. Illiquid bonds are more profitable for dealers to sell. QE auctions do not seem to have an economically significant effect on bond liquidity.

Bonaldi et al. (2015a) study auctions conducted by the Federal Reserve in the second half of 2014 to purchase agency-backed mortgage backed securities. Along with the auctions to purchase treasury securities, this was also part of the quantitative easing policy. These auctions are conducted using a multi-unit discriminatory format. Utilizing methods similar to Hortaçsu and McAdams (2010) and Cassola et al. (2013), they find that cost-savings from switching to a uniform price or Vickrey auction would have been small. However, some efficiency gains may result from switching to a Vickrey auction.

8.4 Derivatives markets

Auctions are used in important derivatives markets as well. For example, Chernov et al. (2013) study auctions used to settle credit default swap (CDS) contracts, which are important derivative contracts typically used to insure against bond defaults. When a credit default swap (CDS) is settled, the buyer and seller of default insurance can decide whether they prefer cash or physical settlement. The market for the underlying bonds is often illiquid, so the industry has set up an auction mechanism to discover cash settlement prices. In the first stage, the parties on either side of each CDS independently decide whether to settle physically or with cash. This determines the net open interest (NOI) for bonds. Bond dealers provide bids and ask prices which are

[25] A repo is a repurchase agreement of a cash loan which is collateralized by a safe, liquid security such as a government bond. A security can have a "special repo rate" when there is a premium for the security over other securities with the same payoff structure. For example, suppose we have a 30-year US Treasury bond with 10 years until maturity, and a 10-year US Treasury bond that was just sold on auction. Both have the same structure, i.e. the same present discounted value. However, on the repo market, the 10-year bond would be easier to sell if the counterparty were to default. Hence, people would rather write collateralized loan contracts on the 10-year bond, meaning it would trade at a slight premium compared to the 30-year bond, meaning it would have "specialness".

used to generate a price ceiling for the second stage. In the second stage, depending on whether the NOI is negative or positive, participants are asked to provide a supply or demand schedule for bonds. These schedules are used to find market clearing prices.

Holding CDS positions distorts the incentives of participants in the second stage. For example, the sellers of CDS contracts would like the price to be higher. In a theoretical model, Chernov et al. (2013) show that overpricing and underpricing are possible in equilibrium; both are observed in the data. They find that, empirically, underpricing is more common and that the amount of underpricing increases with the NOI, as predicted by theory. They suggest a change to the allocation rule when the NOI cannot be satisfied and propose a conditional price cap to reduce mispricing.

8.5 Takeover auctions

The more than $3 trillion per year global market for corporate mergers and acquisitions plays an important role for allocating resources for productive uses. Many takeovers and acquisitions are conducted in a competitive setting with multiple bidders, and hence can be considered as an auction (Boone and Mulherin (2007)). Gorbenko and Malenko (2014) and Gorbenko (2019) study data on corporate takeovers with multiple bidders. These latter two papers model the takeover auctions as having two types of buyers: the first are "strategic" buyers, which are companies in the same/similar/complementary industries who have a clear synergistic or strategic use for the productive assets of the firm being sold. The second type consists of "financial" buyers. These are investment firms that focus on buying companies, restructuring them, and then selling them for a profit.

The extant literature on the market for firm acquisitions lays out interesting stylized facts about these two types of buyers. For example, Bargeron et al. (2008) find that when a company is bought by a strategic buyer, the takeover premium paid by the buyer above the pre-acquisition share price is typically higher than the premium paid by a financial buyer. This finding corroborates intuition, given that one may expect the financial buyer to "flip" the acquired company for a profit, while the strategic buyer typically holds on to it as a productive asset.

However, the winning bid of a takeover contest is typically the maximum bid among several submitted bids. Therefore, we cannot conclude, based on an observation of a maximum, that the bid contains information about the average valuation of a strategic vs. financial bidder. Indeed, Gorbenko (2019) shows that along with the mean of valuations, the dispersion of valuations is also responsible for determining the maximum bid. While the mean valuations of strategic vs. financial bidders may indeed be different, it is the difference in valuation dispersions that is captured the most in the apparent difference in takeover premia.

How can we learn about the distribution of valuations? For this, one needs data on non-winning bids. However, even if all bids are observed, bids are strategic objects that reflect the competition a bidder faces, and the rules of the bidding game, and do not necessarily reflect the valuations of bidders. To invert observed bids into

the underlying valuations, Gorbenko and Malenko (2014) use the approach of Haile and Tamer (2003): Instead of specifying exactly what the rules of the takeover contest might be, the authors make a number of reasonable behavioral restrictions that a rational bidder may obey, and show that these restrictions provide information bounds on the distribution of valuations across the two types of bidders.[26] Once the distributions of valuations of financial and strategic bidders are recovered, one can explore the factors determining the observed difference in takeover premia (Gorbenko (2019)). One can also document the existence of distinct segments in the market for acquisitions by showing the presence of a substantial subset of firms that are more attractive, on average, to financial vs. strategic bidders. Gorbenko and Malenko (2014) also show that financial bidders' demand for acquiring companies is more sensitive to macroeconomic factors than that of strategic bidders, a factor that may serve as an amplification mechanism for negative shocks.

8.5.1 Auctions of insolvent banks

Another important use of auctions is in the context of bankruptcy resolution; the sale of insolvent firms. For example, the FDIC uses an auction mechanism to sell insolvent banks. In these auctions, bids are multidimensional as in scoring auctions, but the scoring rule used to evaluate bids is proprietary. Allen et al. (2019) report that the uncertainty in the scoring rule leads banks to submit multiple differentiated bids. They then propose a method to structurally estimate the valuations of bidders that takes into account the uncertainty in scoring rules. The authors then show in counterfactual simulations that removing the uncertainty in scoring rules can result in substantial revenue gains for the FDIC.

9 Spectrum auctions

A very important application of auctions in recent years is the sale of radio spectrum rights, which has been a very important step in the development of wireless communication industries. In 1994, the Federal Communications Commission introduced the first radio spectrum auction in the U.S., and since then, many such auctions have been conducted across the globe to sell radio spectrum rights to telecommunications companies.

A large theoretical literature covers the design of spectrum auctions, typically combinatorial auctions, in which multiple dissimilar objects are sold simultaneously. A popular format is the simultaneous ascending auction, used for multiple items in which bidding occurs in rounds. In each round, bidders simultaneously make sealed

[26] Gorbenko and Malenko (2014) assume that bidders have independent private values. The private value assumption may describe the valuations of strategic buyers well, as the complementarities with the target firm are likely bidder-specific. Financial bidders may care about a common value element, as their primary motive is resale. However, many financial bidders, too, bring their own management teams and structures to acquired firms. Hence a private value assumption may be reasonable for them as well.

bids for any items they desire. At the end of each round the currently winning bids and winning bidders are revealed. There are different possible rules for ending the auction: usually either the whole auction ends when no new bids for any item are submitted in a round, or bidding ends item-wise when each item receives no new bids in a round. The closing rule can affect bidders' abilities to collude; for example, item-wise closing can prevent bidders from making the threat to bid on others' items if they bid on his desired item.

As discussed e.g. in Milgrom (2000), when bidders bid straightforwardly (that is, reveal their demand over the licenses truthfully) and items are mutual substitutes, the auction's outcome is the competitive (and efficient) allocation and final bids are close to competitive personalized prices. If valuations exhibit any complementarity, existence of a competitive equilibrium is not guaranteed and straightforward bidding can result in inefficient outcomes. Activity rules govern bidders' ability to participate in the bidding for particular items. These rules restrict bidders from having the option of skipping rounds of bidding and participating later on. This improves the pacing of auctions and can rule out certain types of strategic manipulation.

9.1 Empirical work on spectrum auctions

Despite the importance of spectrum auctions or combinatorial auctions in general, there has been relatively little empirical work studying these auctions (some of this work is covered in Hortaçsu and McAdams (2018)). Fox and Bajari (2013) structurally estimate bidders' valuations in the 1995-1996 FCC C-Block spectrum auction. They argue that the outcome of this auction should be pairwise stable in matches, meaning no two bidders should want to make a one-for-one exchange of licenses. This is implied by efficiency which occurs if bidders bid straightforwardly and there are no complementarities as in Milgrom (2000). However, they show that this stability condition is also satisfied in collusive equilibria found in Brusco and Lopomo (2002) and Engelbrecht-Wiggans and Kahn (2005). Also supporting their claim is experimental evidence and the lack of observed trading after the FCC auction. The pairwise stability condition gives rise to a set of inequalities that the valuations of the final allocation must satisfy. They decompose valuations into a deterministic component capturing complementarity and bidder characteristics, a fixed effect for each license, and an idiosyncratic component. Their estimator maximizes the number of the inequalities the deterministic component of valuations satisfies. They find that complementarities account for 24% of the total package value. They also estimate that combining licenses into 4 large regions would increase efficiency over the realized allocation by 48%.

Xiao and Yuan (2020) seek to quantify the complementarity of spectrum licenses and the effect of the "exposure problem" in the FCC auctions: Bidders who would like to assemble a collection of complementary licenses face the risk of only winning some of those licenses. Once the allocation is realized, bidders may regret acquiring the licenses they won or regret not bidding on complementary licenses. This may lead to departures from straightforward bidding and may give rise to sophisticated strategic behavior, as shown by Szentes and Rosenthal (2003) and Bulow et al. (2009).

A proposed solution to the exposure problem is to allow bidders to place package bids, i.e., bidders are able to place separate bids for different license packages, allowing them to express package level preferences including preferences for license complementarities. Prior work by Cantillon and Pesendorfer (2006b) that study auctions for London bus route operation licenses and Kim et al. (2014) on auctions to procure Chilean school meals study similar questions about the prospective benefits from package bidding (see also Section 3.5).

In the FCC auctions, for each license and round there is a minimum acceptable bid that is an increment above the current winning bid. In each round, most bidders bid this amount if they bid. This allows Xiao and Yuan (2020) to model the auction as a discrete choice entry/exit game. Bidders' beliefs about their winning probabilities change over the course of the auction, changing the expected marginal contribution of complementary licenses. This leads bidders to enter and exit bidding for those licenses. These changes in bidding behavior allow the authors to identify and estimate the complementarities between licenses. They conduct a counterfactual exercise where package bidding is added to a particular FCC auction. They find mixed results: Package bidding alleviates the exposure problem and increases social surplus but benefits larger bidders at the expense of medium and small bidders.

9.2 Incentive auctions

Recently, the FCC has also conducted auctions to reallocate TV broadcast licenses to be resold to mobile networks. These "incentive auctions" consist of a reverse auction in which TV broadcasters declare bids to sell their licenses, and a forward auction where wireless carriers bid to purchase spectrum. The process is complicated by the fact that not all TV broadcasters find it profitable to sell their spectrum rights, thus requiring a repackaging of TV spectrum in order to create broad enough spectrum bands for wireless carriers, and to prevent interference between remaining TV broadcast stations.

While, in principle, a Vickrey auction may provide an incentive compatible and efficient solution, Milgrom and Segal (2020) argue that the re-packing problem makes the computation of Vickrey prices practically impossible. As a computationally feasible solution, Milgrom and Segal (2020) discuss the deferred acceptance clock procurement auction mechanism for the reverse auction. This is a dynamic mechanism that, in a sequence of rounds, presents a weakly decreasing sequence of prices to each bidder. Each bidder whose price is reduced in a round may choose to exit or continue. When the auction ends the remaining bidders are paid their last presented price. Different pricing functions result in different auctions.

In order to study the properties of these auctions, Milgrom and Segal (2020) assume that bidders are "single-minded," that is, they only have one object to consider selling, as opposed to multi-minded bidders who have multiple objects to potentially sell. When bidders are single-minded, auctions in this class have four properties that are not shared by Vickrey auctions: (i) each is strategy-proof (indeed "obviously strategy proof" as in Li (2017)) and group strategy-proof, (ii) set auction prices that are

competitive equilibrium prices and Nash equilibrium winning bids in the related first-price auction, (iii) preserves winner's privacy about values, and (iv) can be extended to satisfy a budget constraint. In the FCC spectrum auction context, computation of efficient or even feasible allocations is often impractical. Whereas the Vickrey auction is sensitive to approximate computations, in simulations, clock auctions are able to achieve quick computation, high efficiency, and low procurement prices. The budget constrained extension is also important to the FCC context when conducting double auctions to repackage spectrum. The government would like the revenue from the forward auction to pay for the costs of the reverse auction.

Doraszelski et al. (2019) estimate the consequences of departures from the single-mindedness assumption in incentive auctions. When some bidders own multiple broadcast licenses, they may have an incentive to withhold some licenses in order to boost prices received on other licenses. They may also accept lower prices on some licenses so that more of their licenses can be feasibly sold. In order to quantify the effects of this kind of behavior, the authors estimate the license-holders' reservation prices then simulate the auction both with and without strategic bidding. The effect of strategic bidding depends on the demand of the FCC. In the initial auction where the FCC wanted to clear 126 MHz of spectrum, supply reduction increased payouts by 26%. In the final auction with an 84 MHz target, the increase was just 7%. Single-license holders experience an increase in payouts almost as much as multi-license holders. Strategic bidding does not significantly affect the set of licenses that are ultimately acquired. The authors propose a simple rule for restricting multi-license holders' ability to withhold licenses. They then estimate that this rule would reduce the payout increase by between 62% and 80%, depending on the clearing target. Finally, they show that relaxing the interference constraints can significantly decrease the effects of strategic bidding and overall payouts.

10 Auctions of used goods

Another important use of auctions is in the sale of used goods. Certain used goods, such as collectibles, can be rare or hard to find, with dispersion in potential buyers' willingnesses-to-pay. Thus, an auction can serve as an effective price discovery method. Other used goods, such as used vehicles or capital equipment, may be differentiated due to their differential wear and tear, again creating dispersion in buyers' valuations.

10.1 Used car auctions

New car dealers typically do not hold on to trade-ins, and sell them at auctions to used car dealers. Similarly, rental car companies or companies with fleets of vehicles typically liquidate their portfolios through wholesale auto auctioneers. The National Auto Auction Association, whose members comprise of wholesale auto auctioneers,

estimates that close to 10 million used cars were sold in auctions in 2016 (Manheim (2017)).

Roberts (2013) studies used car auctions at a wholesale auctioneer in South Korea. The auctions were conducted as button auctions, of the form modeled in Milgrom and Weber (1982): an ascending price auction where bidders have to press a button in order to stay active. The button auction format allows the researcher to map bids into valuations for a given auction, but Roberts (2013) points out that pooling data across auctions to get precise inference on how car attributes may affect valuations is made difficult by the presence of unobserved heterogeneity. The author proposes an alternative to deconvolution-based approaches (such as Li and Vuong (1998) and Krasnokutskaya (2011)) that is applicable even when one observes a single bid (most likely the transaction price) for each auction, and when the separability of the unobserved component of valuations might be too restrictive an assumption. The approach utilizes the additional information contained in the reserve prices set by the seller. It is likely that sellers observe the unobserved factors shifting buyers' valuations, and take this into account when setting reserve prices. The reserve price might not be the only source of additional information regarding the unobserved factor: Other (continuous) mechanism design decisions by the seller, such as how long to run the auction (an important factor on eBay), how much to spend on advertising the auction, etc., can also be quite informative.

A crucial assumption that the author requires for his method is that the reserve price (or another observed decision by the seller) is monotonic in the unobserved factor. The other assumption is that there is at least one observable covariate underlying bidder valuations that is independent of the unobserved factor. This allows him to utilize techniques similar to Matzkin (2003) to identify, in a symmetric independent private value second-price or ascending auction, the distribution of bidders' valuations conditional on the common unobserved factor.

Another interesting study of used auto auctions is by Larsen (2021), who analyzes some interesting data from a wholesale auto auctioneer who uses a hybrid mechanism. In the mechanism, auto dealers first participate in an auction with a secret reserve price. If the auction price exceeds the secret reserve price, the car is sold. If not, the high dealer/bidder enters into an alternating offer bargaining stage with the seller. The dataset records bids in the auction, and, importantly, the alternating offers. The author's objective is to assess how surplus is actually allocated. The data has only information about price offers and the realized (when there is agreement) bargaining price, so the author utilizes a structural model of the auction and bargaining games played by the dealers. Assuming (best response) optimal behavior in these games, he estimates the underlying buyers' and sellers' valuations that rationalize observed bids/offers and bargaining outcomes. The headline empirical finding of the paper is that, based on the estimated valuations, the realized bargaining outcomes are inefficient. However, this efficiency loss is not due to the gap between the second-best and first-best efficient outcomes; indeed, at the estimated valuations, the second-best and first-best surpluses are quantitatively quite close. This leaves an important residual loss of surplus to explain for this important market, which is left for future research.

10.2 Auctions of collectible goods: a case study in collusion

Although a large number of papers have been written on the topic of collusion by firms, there has been very little in the way of direct evidence as to how firms actually collude. Using data that emerged from a bid-rigging investigation in the collectible stamps market, Asker (2010) analyzes a particularly interesting form of collusion, one with side-payments and without resort to repeated game/dynamic incentives as its primary mode of enforcement. The prosecutor's data allows the author to observe the exact mechanism (called a "knockout auction") that is used to allocate side-payments and to coordinate the bids of the ring members. In the knockout auction, each member places a sealed bid, and the highest bid is taken as the ring's bid in the "target" stamp auction, which is conducted as an ascending price auction. The losing bidders in the knockout refrain from participating in the target auction. If the winner of the knockout auction also wins the target auction, she allocates some of her surplus, which is calculated as the difference between her knockout bid and the closing price of the real auction, to bidders in the knockout who also submitted bids that could have won the real auction. Ring members get compensated for not competing with each other if their bids would have made a difference in the target auction.

This is not a straightforward mechanism, but it serves the purpose of limiting competition in the target auction by providing enough surplus to losing knockout members so that they would not be tempted to bid in the real auction themselves. There is also the added complication of avoiding free-riders: Some of the ring members may inflate their bids in the knockout in order to carve away surplus from the winner. Indeed, the author shows that the incentive to inflate one's bids in the knockout auction is endemic: The equilibrium bids in the knockout auction necessarily exceed ring members' private valuations for the stamps. This surprising result opens up the counterintuitive possibility that the price in the target auction may turn out to be even higher than it would have been without the bidding ring in action.

The auctioneer does not necessarily benefit from the ring being in operation, as even if the ring submits an inflated bid, only one bid is submitted by the ring, and the auction price suffers from this restraint on competition (the starkest case is when all bidders are members of the ring; in this case the ring wins the auction at the auctioneer's reserve price). However, the presence of inflated bids in the knockout auction makes the calculation of damages to the auctioneer very difficult, since one has to estimate the real valuations attached to the stamps by the ring members (these valuations are below their bids) to calculate what would have happened if ring members had bid honestly at the target auctions. Therefore, the author estimates the valuations that rationalize observed knockout bids, and uses the estimated valuations to compute damages. The estimation approach controls for sources of observed and unobserved heterogeneity, which is a first-order confounding factor in these types of analyses. He concludes that the damages suffered by non-ring bidders are similar in magnitude to the damages suffered by the stamp sellers.

11 Concluding remarks

This chapter shows how the analysis of auction data successfully provided quantitative empirical assessments of policy questions that are pertinent to both the public and private sectors. Soon after this field of study began, Laffont (1997) stated that the methodological progress in the econometrics of auctions will be an indicator of what game theory can bring to empirical industrial organization. Not only has the field produced a large number of methodological developments as surveyed by Perrigne and Vuong (2021), but it appears that the combination of data, methodologies, and auction models has been fruitful in addressing important policy questions, as surveyed in this chapter. In addition, the empirical auction literature has provided quantitative answers to questions about mechanism choice that were left unanswered by the theoretical and experimental literature. Along this line, we often observe that projects in empirical auctions are conducted by economists combining diverse skills in theory, econometrics and empirical research.

This chapter also illustrates that different auction data have raised different questions and empirical contributions. There are large swathes of auction data and auction problems that have not yet been analyzed, and we expect numerous important developments in the coming years. We also observe that research in this field has become impactful beyond auctions. This chapter provides two interesting examples with auctions of treasury bills, bonds and securities as well as takeover auctions building bridges with finance and corporate finance, respectively. Since an auction is an allocation mechanism among a finite number of competing agents resulting in a price, a number of markets can be viewed through the lens of auctions though there has not been a formal auction *per se*. For instance, Bodoh-Creed and Hickman (2019) consider a matching problem in which a continuum of college applicants with private learning costs compete for admission. Students offer their academic achievements to colleges as a function of their unobserved (potential asymmetric) learning costs. The authors model this part as an auction and then use the recovered learning costs in an assortative matching model to analyze the effects of affirmative action on students, such as on their human capital investment, minority enrollment, and graduation probability, to name a few. Salz (2021) studies businesses looking for trash collection service with a private search cost interpreted as the marginal cost for an additional price inquiry. Each business chooses the carter that quotes the lowest price, but carters do not know the level of competition they face as the latter is a function of the business' search costs. This problem relates to a procurement auction with an unknown number of competitors. Drawing from the auction literature, the author analyzes the role played by intermediaries in the trade-waste market and whether their presence improves welfare. As a last example, Pehlivan and Vuong (2016) model international trade through a model of competition in supply functions among countries with different productivity distributions. The major difference with, say, treasury bill auctions is that they only observe the outcome of the equilibrium, i.e. total revenue and quantity for each exporting country. These recent examples suggest that there is room for more empirical research on auctions and applications of auction models and methodologies to new domains.

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