tf.contrib.linalg.LinearOperatorUDVHUpdate

Contents

Class `LinearOperatorUDVHUpdate`

Inherits From: `LinearOperator`

Defined in `tensorflow/contrib/linalg/python/ops/linear_operator_udvh_update.py`.

See the guide: Linear Algebra (contrib) > `LinearOperator`

Perturb a `LinearOperator` with a rank `K` update.

This operator acts like a [batch] matrix `A` with shape `[B1,...,Bb, M, N]` for some `b >= 0`. The first `b` indices index a batch member. For every batch index `(i1,...,ib)`, `A[i1,...,ib, : :]` is an `M x N` matrix.

`LinearOperatorUDVHUpdate` represents `A = L + U D V^H`, where

```
L, is a LinearOperator representing [batch] M x N matrices
U, is a [batch] M x K matrix.  Typically K << M.
D, is a [batch] K x K matrix.
V, is a [batch] N x K matrix.  Typically K << N.
V^H is the Hermitian transpose (adjoint) of V.
```

If `M = N`, determinants and solves are done using the matrix determinant lemma and Woodbury identities, and thus require L and D to be non-singular.

Solves and determinants will be attempted unless the "is_non_singular" property of L and D is False.

In the event that L and D are positive-definite, and U = V, solves and determinants can be done using a Cholesky factorization.

```
# Create a 3 x 3 diagonal linear operator.
diag_operator = LinearOperatorDiag(
    diag_update=[1., 2., 3.], is_non_singular=True, is_self_adjoint=True,
    is_positive_definite=True)

# Perturb with a rank 2 perturbation
operator = LinearOperatorUDVHUpdate(
    operator=diag_operator,
    u=[[1., 2.], [-1., 3.], [0., 0.]],
    diag_update=[11., 12.],
    v=[[1., 2.], [-1., 3.], [10., 10.]])

operator.shape
==> [3, 3]

operator.log_abs_determinant()
==> scalar Tensor

x = ... Shape [3, 4] Tensor
operator.matmul(x)
==> Shape [3, 4] Tensor
```

Shape compatibility

This operator acts on [batch] matrix with compatible shape. `x` is a batch matrix with compatible shape for `matmul` and `solve` if

```
operator.shape = [B1,...,Bb] + [M, N],  with b >= 0
x.shape =        [B1,...,Bb] + [N, R],  with R >= 0.
```

Performance

Suppose `operator` is a `LinearOperatorUDVHUpdate` of shape `[M, N]`, made from a rank `K` update of `base_operator` which performs `.matmul(x)` on `x` having `x.shape = [N, R]` with `O(L_matmul*N*R)` complexity (and similarly for `solve`, `determinant`. Then, if `x.shape = [N, R]`,

- `operator.matmul(x)` is `O(L_matmul*N*R + K*N*R)`

and if `M = N`,

- `operator.solve(x)` is `O(L_matmul*N*R + N*K*R + K^2*R + K^3)`
- `operator.determinant()` is `O(L_determinant + L_solve*N*K + K^2*N + K^3)`

If instead `operator` and `x` have shape `[B1,...,Bb, M, N]` and `[B1,...,Bb, N, R]`, every operation increases in complexity by `B1*...*Bb`.

Matrix property hints

This `LinearOperator` is initialized with boolean flags of the form `is_X`, for `X = non_singular`, `self_adjoint`, `positive_definite`, `diag_update_positive` and `square`. These have the following meaning:

- If `is_X == True`, callers should expect the operator to have the property `X`. This is a promise that should be fulfilled, but is *not* a runtime assert. For example, finite floating point precision may result in these promises being violated.
- If `is_X == False`, callers should expect the operator to not have `X`.
- If `is_X == None` (the default), callers should have no expectation either way.

Properties

`base_operator`

If this operator is `A = L + U D V^H`, this is the `L`.

`batch_shape`

`TensorShape` of batch dimensions of this `LinearOperator`.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `TensorShape([B1,...,Bb])`, equivalent to `A.get_shape()[:-2]`

Returns:

`TensorShape`, statically determined, may be undefined.

`diag_operator`

If this operator is `A = L + U D V^H`, this is `D`.

`diag_update`

If this operator is `A = L + U D V^H`, this is the diagonal of `D`.

`domain_dimension`

Dimension (in the sense of vector spaces) of the domain of this operator.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `N`.

Returns:

`Dimension` object.

`dtype`

The `DType` of `Tensor`s handled by this `LinearOperator`.

`graph_parents`

List of graph dependencies of this `LinearOperator`.

`is_diag_update_positive`

If this operator is `A = L + U D V^H`, this hints `D > 0` elementwise.

`is_non_singular`

`is_positive_definite`

`is_self_adjoint`

`is_square`

Return `True/False` depending on if this operator is square.

`name`

Name prepended to all ops created by this `LinearOperator`.

`range_dimension`

Dimension (in the sense of vector spaces) of the range of this operator.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `M`.

Returns:

`Dimension` object.

`shape`

`TensorShape` of this `LinearOperator`.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `TensorShape([B1,...,Bb, M, N])`, equivalent to `A.get_shape()`.

Returns:

`TensorShape`, statically determined, may be undefined.

`tensor_rank`

Rank (in the sense of tensors) of matrix corresponding to this operator.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `b + 2`.

Args:

- `name`: A name for this `Op.

Returns:

Python integer, or None if the tensor rank is undefined.

`u`

If this operator is `A = L + U D V^H`, this is the `U`.

`v`

If this operator is `A = L + U D V^H`, this is the `V`.

Methods

`__init__`

```
__init__(
    base_operator,
    u,
    diag_update=None,
    v=None,
    is_diag_update_positive=None,
    is_non_singular=None,
    is_self_adjoint=None,
    is_positive_definite=None,
    is_square=None,
    name='LinearOperatorUDVHUpdate'
)
```

Initialize a `LinearOperatorUDVHUpdate`.

This creates a `LinearOperator` of the form `A = L + U D V^H`, with `L` a `LinearOperator`, `U, V` both [batch] matrices, and `D` a [batch] diagonal matrix.

If `L` is non-singular, solves and determinants are available. Solves/determinants both involve a solve/determinant of a `K x K` system. In the event that L and D are self-adjoint positive-definite, and U = V, this can be done using a Cholesky factorization. The user should set the `is_X` matrix property hints, which will trigger the appropriate code path.

Args:

- `base_operator`: Shape `[B1,...,Bb, M, N]` real `float32` or `float64` `LinearOperator`. This is `L` above.
- `u`: Shape `[B1,...,Bb, M, K]` `Tensor` of same `dtype` as `base_operator`. This is `U` above.
- `diag_update`: Optional shape `[B1,...,Bb, K]` `Tensor` with same `dtype` as `base_operator`. This is the diagonal of `D` above. Defaults to `D` being the identity operator.
- `v`: Optional `Tensor` of same `dtype` as `u` and shape `[B1,...,Bb, N, K]` Defaults to `v = u`, in which case the perturbation is symmetric. If `M != N`, then `v` must be set since the perturbation is not square.
- `is_diag_update_positive`: Python `bool`. If `True`, expect `diag_update > 0`.
- `is_non_singular`: Expect that this operator is non-singular. Default is `None`, unless `is_positive_definite` is auto-set to be `True` (see below).
- `is_self_adjoint`: Expect that this operator is equal to its hermitian transpose. Default is `None`, unless `base_operator` is self-adjoint and `v = None` (meaning `u=v`), in which case this defaults to `True`.
- `is_positive_definite`: Expect that this operator is positive definite. Default is `None`, unless `base_operator` is positive-definite `v = None` (meaning `u=v`), and `is_diag_update_positive`, in which case this defaults to `True`. Note that we say an operator is positive definite when the quadratic form `x^H A x` has positive real part for all nonzero `x`.
- `is_square`: Expect that this operator acts like square [batch] matrices.
- `name`: A name for this `LinearOperator`.

Raises:

- `ValueError`: If `is_X` flags are set in an inconsistent way.

`add_to_tensor`

```
add_to_tensor(
    x,
    name='add_to_tensor'
)
```

Add matrix represented by this operator to `x`. Equivalent to `A + x`.

Args:

- `x`: `Tensor` with same `dtype` and shape broadcastable to `self.shape`.
- `name`: A name to give this `Op`.

Returns:

A `Tensor` with broadcast shape and same `dtype` as `self`.

`assert_non_singular`

```
assert_non_singular(name='assert_non_singular')
```

Returns an `Op` that asserts this operator is non singular.

This operator is considered non-singular if

```
ConditionNumber < max{100, range_dimension, domain_dimension} * eps,
eps := np.finfo(self.dtype.as_numpy_dtype).eps
```

Args:

- `name`: A string name to prepend to created ops.

Returns:

An `Assert` `Op`, that, when run, will raise an `InvalidArgumentError` if the operator is singular.

`assert_positive_definite`

```
assert_positive_definite(name='assert_positive_definite')
```

Returns an `Op` that asserts this operator is positive definite.

Here, positive definite means that the quadratic form `x^H A x` has positive real part for all nonzero `x`. Note that we do not require the operator to be self-adjoint to be positive definite.

Args:

- `name`: A name to give this `Op`.

Returns:

An `Assert` `Op`, that, when run, will raise an `InvalidArgumentError` if the operator is not positive definite.

`assert_self_adjoint`

```
assert_self_adjoint(name='assert_self_adjoint')
```

Returns an `Op` that asserts this operator is self-adjoint.

Here we check that this operator is *exactly* equal to its hermitian transpose.

Args:

- `name`: A string name to prepend to created ops.

Returns:

An `Assert` `Op`, that, when run, will raise an `InvalidArgumentError` if the operator is not self-adjoint.

`batch_shape_tensor`

```
batch_shape_tensor(name='batch_shape_tensor')
```

Shape of batch dimensions of this operator, determined at runtime.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns a `Tensor` holding `[B1,...,Bb]`.

Args:

- `name`: A name for this `Op.

Returns:

`int32` `Tensor`

`determinant`

```
determinant(name='det')
```

Determinant for every batch member.

Args:

- `name`: A name for this `Op.

Returns:

`Tensor` with shape `self.batch_shape` and same `dtype` as `self`.

Raises:

- `NotImplementedError`: If `self.is_square` is `False`.

diag_part

```
diag_part(name='diag_part')
```

Efficiently get the [batch] diagonal part of this operator.

If this operator has shape `[B1,...,Bb, M, N]`, this returns a `Tensor` `diagonal`, of shape `[B1,...,Bb, min(M, N)]`, where `diagonal[b1,...,bb, i] = self.to_dense()[b1,...,bb, i, i]`.

```
my_operator = LinearOperatorDiag([1., 2.])

# Efficiently get the diagonal
my_operator.diag_part()
==> [1., 2.]

# Equivalent, but inefficient method
tf.matrix_diag_part(my_operator.to_dense())
==> [1., 2.]
```

Args:

- `name`: A name for this `Op`.

Returns:

- `diag_part`: A `Tensor` of same `dtype` as self.

domain_dimension_tensor

```
domain_dimension_tensor(name='domain_dimension_tensor')
```

Dimension (in the sense of vector spaces) of the domain of this operator.

Determined at runtime.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `N`.

Args:

- `name`: A name for this `Op`.

Returns:

`int32` `Tensor`

log_abs_determinant

```
log_abs_determinant(name='log_abs_det')
```

Log absolute value of determinant for every batch member.

Args:

- `name`: A name for this `Op.

Returns:

`Tensor` with shape `self.batch_shape` and same `dtype` as `self`.

Raises:

- `NotImplementedError`: If `self.is_square` is `False`.

`matmul`

```
matmul(
    x,
    adjoint=False,
    adjoint_arg=False,
    name='matmul'
)
```

Transform [batch] matrix `x` with left multiplication: `x --> Ax`.

```
# Make an operator acting like batch matrix A.  Assume A.shape = [..., M, N]
operator = LinearOperator(...)
operator.shape = [..., M, N]

X = ... # shape [..., N, R], batch matrix, R > 0.

Y = operator.matmul(X)
Y.shape
==> [..., M, R]

Y[..., :, r] = sum_j A[..., :, j] X[j, r]
```

Args:

- `x`: `Tensor` with compatible shape and same `dtype` as `self`. See class docstring for definition of compatibility.
- `adjoint`: Python `bool`. If `True`, left multiply by the adjoint: `A^H x`.
- `adjoint_arg`: Python `bool`. If `True`, compute `A x^H` where `x^H` is the hermitian transpose (transposition and complex conjugation).
- `name`: A name for this `Op.

Returns:

A `Tensor` with shape `[..., M, R]` and same `dtype` as `self`.

`matvec`

```
matvec(
    x,
    adjoint=False,
    name='matvec'
)
```

Transform [batch] vector `x` with left multiplication: `x --> Ax`.

```
# Make an operator acting like batch matric A.  Assume A.shape = [..., M, N]
operator = LinearOperator(...)

X = ... # shape [..., N], batch vector

Y = operator.matvec(X)
Y.shape
==> [..., M]

Y[..., :] = sum_j A[..., :, j] X[..., j]
```

Args:

- `x`: `Tensor` with compatible shape and same `dtype` as `self`. `x` is treated as a [batch] vector meaning for every set of leading dimensions, the last dimension defines a vector. See class docstring for definition of compatibility.
- `adjoint`: Python `bool`. If `True`, left multiply by the adjoint: `A^H x`.
- `name`: A name for this `Op.

Returns:

A `Tensor` with shape `[..., M]` and same `dtype` as `self`.

`range_dimension_tensor`

```
range_dimension_tensor(name='range_dimension_tensor')
```

Dimension (in the sense of vector spaces) of the range of this operator.

Determined at runtime.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `M`.

Args:

- `name`: A name for this `Op`.

Returns:

`int32` `Tensor`

`shape_tensor`

```
shape_tensor(name='shape_tensor')
```

Shape of this `LinearOperator`, determined at runtime.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns a `Tensor` holding `[B1,...,Bb, M, N]`, equivalent to `tf.shape(A)`.

Args:

- `name`: A name for this `Op.

Returns:

`int32` `Tensor`

`solve`

```
solve(
    rhs,
    adjoint=False,
    adjoint_arg=False,
    name='solve'
)
```

Solve (exact or approx) `R` (batch) systems of equations: `A X = rhs`.

The returned `Tensor` will be close to an exact solution if `A` is well conditioned. Otherwise closeness will vary. See class docstring for details.

Examples:

```
# Make an operator acting like batch matrix A.  Assume A.shape = [..., M, N]
operator = LinearOperator(...)
operator.shape = [..., M, N]

# Solve R > 0 linear systems for every member of the batch.
RHS = ... # shape [..., M, R]

X = operator.solve(RHS)
# X[..., :, r] is the solution to the r'th linear system
# sum_j A[..., :, j] X[..., j, r] = RHS[..., :, r]

operator.matmul(X)
==> RHS
```

Args:

- `rhs`: `Tensor` with same `dtype` as this operator and compatible shape. `rhs` is treated like a [batch] matrix meaning for every set of leading dimensions, the last two dimensions defines a matrix. See class docstring for definition of compatibility.
- `adjoint`: Python `bool`. If `True`, solve the system involving the adjoint of this `LinearOperator`: `A^H X = rhs`.
- `adjoint_arg`: Python `bool`. If `True`, solve `A X = rhs^H` where `rhs^H` is the hermitian transpose (transposition and complex conjugation).
- `name`: A name scope to use for ops added by this method.

Returns:

`Tensor` with shape `[...,N, R]` and same `dtype` as `rhs`.

Raises:

- `NotImplementedError`: If `self.is_non_singular` or `is_square` is False.

`solvevec`

```
solvevec(
    rhs,
    adjoint=False,
    name='solve'
)
```

Solve single equation with best effort: `A X = rhs`.

The returned `Tensor` will be close to an exact solution if `A` is well conditioned. Otherwise closeness will vary. See class docstring for details.

Examples:

```
# Make an operator acting like batch matrix A.  Assume A.shape = [..., M, N]
operator = LinearOperator(...)
operator.shape = [..., M, N]

# Solve one linear system for every member of the batch.
RHS = ... # shape [..., M]

X = operator.solvevec(RHS)
# X is the solution to the linear system
# sum_j A[..., :, j] X[..., j] = RHS[..., :]

operator.matvec(X)
==> RHS
```

Args:

- `rhs`: `Tensor` with same `dtype` as this operator. `rhs` is treated like a [batch] vector meaning for every set of leading dimensions, the last dimension defines a vector. See class docstring for definition of compatibility regarding batch dimensions.
- `adjoint`: Python `bool`. If `True`, solve the system involving the adjoint of this `LinearOperator`: `A^H X = rhs`.
- `name`: A name scope to use for ops added by this method.

Returns:

`Tensor` with shape `[...,N]` and same `dtype` as `rhs`.

Raises:

- `NotImplementedError`: If `self.is_non_singular` or `is_square` is False.

`tensor_rank_tensor`

```
tensor_rank_tensor(name='tensor_rank_tensor')
```

Rank (in the sense of tensors) of matrix corresponding to this operator.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `b + 2`.

Args:

- `name`: A name for this `Op.

Returns:

`int32` `Tensor`, determined at runtime.

`to_dense`

```
to_dense(name='to_dense')
```

Return a dense (batch) matrix representing this operator.

`trace`

```
trace(name='trace')
```

Trace of the linear operator, equal to sum of `self.diag_part()`.

If the operator is square, this is also the sum of the eigenvalues.

Args:

- `name`: A name for this `Op`.

Returns:

Shape `[B1,...,Bb]` `Tensor` of same `dtype` as `self`.

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