#### TopcorFlow

TensorFlow API r1.4

# tf.contrib.distributions.WishartFull

Contents
Class WishartFull
Properties
allow\_nan\_stats
batch\_shape

## Class WishartFull

Defined in tensorflow/contrib/distributions/python/ops/wishart.py.

See the guide: Statistical Distributions (contrib) > Multivariate distributions

The matrix Wishart distribution on positive definite matrices.

This distribution is defined by a scalar degrees of freedom df and a symmetric, positive definite scale matrix.

Evaluation of the pdf, determinant, and sampling are all 0(k^3) operations where (k, k) is the event space shape.

### Mathematical Details

The probability density function (pdf) is,

```
pdf(X; df, scale) = det(X)**(0.5 (df-k-1)) exp(-0.5 tr[inv(scale) X]) / Z

Z = 2**(0.5 df k) |det(scale)|**(0.5 df) Gamma_k(0.5 df)
```

where: df >= k denotes the degrees of freedom, scale is a symmetric, positive definite,  $k \times k$  matrix,  $k \times k$  matrix, k

## Examples

```
# Initialize a single 3x3 Wishart with Full factored scale matrix and 5
# degrees-of-freedom.(*)
df = 5
scale = ... # Shape is [3, 3]; positive definite.
dist = tf.contrib.distributions.WishartFull(df=df, scale=scale)
# Evaluate this on an observation in R^3, returning a scalar.
x = \dots \# A 3x3 positive definite matrix.
dist.prob(x) # Shape is [], a scalar.
# Evaluate this on a two observations, each in R^{3x3}, returning a length two
x = [x0, x1] # Shape is [2, 3, 3].
dist.prob(x) # Shape is [2].
# Initialize two 3x3 Wisharts with Full factored scale matrices.
df = [5, 4]
scale = ... # Shape is [2, 3, 3].
dist = tf.contrib.distributions.WishartFull(df=df, scale=scale)
# Evaluate this on four observations.
x = [[x0, x1], [x2, x3]] # Shape is [2, 2, 3, 3]; xi is positive definite.
dist.prob(x) # Shape is [2, 2].
# (*) - To efficiently create a trainable covariance matrix, see the example
   in tf.contrib.distributions.matrix_diag_transform.
```

# **Properties**

### allow\_nan\_stats

Python **bool** describing behavior when a stat is undefined.

Stats return +/- infinity when it makes sense. E.g., the variance of a Cauchy distribution is infinity. However, sometimes the statistic is undefined, e.g., if a distribution's pdf does not achieve a maximum within the support of the distribution, the mode is undefined. If the mean is undefined, then by definition the variance is undefined. E.g. the mean for Student's T for df = 1 is undefined (no clear way to say it is either + or - infinity), so the variance = E[(X - mean)\*\*2] is also undefined.

#### Returns:

• allow\_nan\_stats: Python bool.

## batch\_shape

Shape of a single sample from a single event index as a TensorShape.

May be partially defined or unknown.

The batch dimensions are indexes into independent, non-identical parameterizations of this distribution.

## Returns:

batch\_shape: TensorShape, possibly unknown.

#### cholesky\_input\_output\_matrices

Boolean indicating if  ${\bf Tensor}\$  input/outputs are Cholesky factorized.

### df

Wishart distribution degree(s) of freedom.

#### dimension

Dimension of underlying vector space. The p in  $R^{(p*p)}$ .

## dtype

The DType of Tensor's handled by this Distribution.

## event\_shape

Shape of a single sample from a single batch as a TensorShape.

May be partially defined or unknown.

#### Returns:

• event\_shape: TensorShape, possibly unknown.

#### name

Name prepended to all ops created by this **Distribution**.

### parameters

Dictionary of parameters used to instantiate this **Distribution**.

### reparameterization\_type

Describes how samples from the distribution are reparameterized.

Currently this is one of the static instances **distributions.FULLY\_REPARAMETERIZED** or **distributions.NOT\_REPARAMETERIZED**.

### Returns:

An instance of **ReparameterizationType**.

## scale\_operator

Wishart distribution scale matrix as an Linear Operator.

## validate\_args

Python **bool** indicating possibly expensive checks are enabled.

### \_\_init\_\_

```
__init__(
    df,
    scale,
    cholesky_input_output_matrices=False,
    validate_args=False,
    allow_nan_stats=True,
    name='WishartFull'
)
```

Construct Wishart distributions.

## Args:

- df: float or double Tensor. Degrees of freedom, must be greater than or equal to dimension of the scale matrix.
- scale: float or double Tensor. The symmetric positive definite scale matrix of the distribution.
- cholesky\_input\_output\_matrices: Python bool. Any function which whose input or output is a matrix assumes the
  input is Cholesky and returns a Cholesky factored matrix. Example log\_prob input takes a Cholesky and sample\_n
  returns a Cholesky when cholesky\_input\_output\_matrices=True.
- validate\_args: Python **bool**, default **False**. When **True** distribution parameters are checked for validity despite possibly degrading runtime performance. When **False** invalid inputs may silently render incorrect outputs.
- allow\_nan\_stats: Python bool, default True. When True, statistics (e.g., mean, mode, variance) use the value
   "NaN" to indicate the result is undefined. When False, an exception is raised if one or more of the statistic's batch members are undefined.
- name: Python str name prefixed to Ops created by this class.

## batch\_shape\_tensor

```
batch_shape_tensor(name='batch_shape_tensor')
```

Shape of a single sample from a single event index as a 1-D Tensor.

The batch dimensions are indexes into independent, non-identical parameterizations of this distribution.

## Args:

name: name to give to the op

### Returns:

batch\_shape: Tensor.

## cdf

```
cdf(
   value,
   name='cdf'
)
```

Cumulative distribution function.

Given random variable X, the cumulative distribution function cdf is:

```
cdf(x) := P[X \le x]
```

## Args:

- value: float or double Tensor.
- name: The name to give this op.

#### Returns:

• cdf:a Tensor of shape sample\_shape(x) + self.batch\_shape with values of type self.dtype.

## copy

```
copy(**override_parameters_kwargs)
```

Creates a deep copy of the distribution.



Note: the copy distribution may continue to depend on the original initialization arguments.

#### Args:

\*\*override\_parameters\_kwargs: String/value dictionary of initialization arguments to override with new values.

## Returns:

 distribution: A new instance of type(self) initialized from the union of self.parameters and override\_parameters\_kwargs, i.e., dict(self.parameters, \*\*override\_parameters\_kwargs).

## covariance

```
covariance(name='covariance')
```

Covariance.

Covariance is (possibly) defined only for non-scalar-event distributions.

For example, for a length-k, vector-valued distribution, it is calculated as,

```
Cov[i, j] = Covariance(X_i, X_j) = E[(X_i - E[X_i]) (X_j - E[X_j])]
```

where Cov is a (batch of)  $k \times k$  matrix,  $0 \leftarrow (i, j) \leftarrow k$ , and E denotes expectation.

Alternatively, for non-vector, multivariate distributions (e.g., matrix-valued, Wishart), **Covariance** shall return a (batch of) matrices under some vectorization of the events, i.e.,

```
Cov[i, j] = Covariance(Vec(X)_i, Vec(X)_j) = [as above]
```

where Cov is a (batch of)  $k' \times k'$  matrices,  $0 \le (i, j) \le k' = reduce\_prod(event\_shape)$ , and Vec is some

function mapping indices of this distribution's event dimensions to indices of a length- k' vector.

## Args:

• name: The name to give this op.

#### Returns:

covariance: Floating-point Tensor with shape [B1, ..., Bn, k', k'] where the first n dimensions are batch coordinates and k' = reduce\_prod(self.event\_shape).

# entropy

```
entropy(name='entropy')
```

Shannon entropy in nats.

## event\_shape\_tensor

```
event_shape_tensor(name='event_shape_tensor')
```

Shape of a single sample from a single batch as a 1-D int32 Tensor.

### Args:

• name: name to give to the op

### Returns:

• event\_shape: Tensor.

## is\_scalar\_batch

```
is_scalar_batch(name='is_scalar_batch')
```

Indicates that batch\_shape == [].

## Args:

• name: The name to give this op.

## Returns:

• is\_scalar\_batch: bool scalar Tensor.

## is\_scalar\_event

```
is_scalar_event(name='is_scalar_event')
```

Indicates that event\_shape == [].

## Args:

• name: The name to give this op.

#### Returns:

• is\_scalar\_event: bool scalar Tensor.

## log\_cdf

```
log_cdf(
    value,
    name='log_cdf'
)
```

Log cumulative distribution function.

Given random variable X, the cumulative distribution function cdf is:

```
log\_cdf(x) := Log[P[X \le x]]
```

Often, a numerical approximation can be used for  $log_cdf(x)$  that yields a more accurate answer than simply taking the logarithm of the cdf when x << -1.

## Args:

- value: float or double Tensor.
- name: The name to give this op.

#### Returns:

• logcdf: a Tensor of shape sample\_shape(x) + self.batch\_shape with values of type self.dtype.

## log\_normalization

```
log_normalization(name='log_normalization')
```

Computes the log normalizing constant, log(Z).

## log\_prob

```
log_prob(
    value,
    name='log_prob'
)
```

Log probability density/mass function.

### Args:

- value: float or double Tensor.
- name: The name to give this op.

## Returns:

log\_prob: a Tensor of shape sample\_shape(x) + self.batch\_shape with values of type self.dtype.

## log\_survival\_function

```
log_survival_function(
    value,
    name='log_survival_function'
)
```

Log survival function.

Given random variable X, the survival function is defined:

```
log\_survival\_function(x) = Log[ P[X > x] ]
= Log[ 1 - P[X <= x] ]
= Log[ 1 - cdf(x) ]
```

Typically, different numerical approximations can be used for the log survival function, which are more accurate than 1 - cdf(x) when x >> 1.

## Args:

- value: float or double Tensor.
- name: The name to give this op.

### Returns:

Tensor of shape sample\_shape(x) + self.batch\_shape with values of type self.dtype.

## mean

```
mean(name='mean')
```

Mean.

### mean\_log\_det

```
mean_log_det(name='mean_log_det')
```

Computes E[log(det(X))] under this Wishart distribution.

## mode

```
mode(name='mode')
```

Mode.

### param\_shapes

```
param_shapes(
    cls,
    sample_shape,
    name='DistributionParamShapes'
)
```

Shapes of parameters given the desired shape of a call to sample().

This is a class method that describes what key/value arguments are required to instantiate the given **Distribution** so that a particular shape is returned for that instance's call to **sample()**.

Subclasses should override class method \_param\_shapes .

## Args:

- sample\_shape: Tensor or python list/tuple. Desired shape of a call to sample().
- name: name to prepend ops with.

#### Returns:

dict of parameter name to Tensor shapes.

## param\_static\_shapes

```
param_static_shapes(
    cls,
    sample_shape
)
```

param\_shapes with static (i.e. TensorShape ) shapes.

This is a class method that describes what key/value arguments are required to instantiate the given **Distribution** so that a particular shape is returned for that instance's call to **sample()**. Assumes that the sample's shape is known statically.

Subclasses should override class method \_param\_shapes to return constant-valued tensors when constant values are fed.

## Args:

• sample\_shape: TensorShape or python list/tuple. Desired shape of a call to sample().

#### Returns:

dict of parameter name to TensorShape.

#### Raises:

• ValueError: if sample\_shape is a TensorShape and is not fully defined.

## prob

```
prob(
   value,
   name='prob'
)
```

Probability density/mass function.

## Args:

- value: float or double Tensor.
- name: The name to give this op.

#### Returns:

• prob: a Tensor of shape sample\_shape(x) + self.batch\_shape with values of type self.dtype.

## quantile

```
quantile(
   value,
   name='quantile'
)
```

Quantile function. Aka "inverse cdf" or "percent point function".

Given random variable X and p in [0, 1], the quantile is:

```
quantile(p) := x such that P[X \leftarrow= x] == p
```

### Args:

- value: float or double Tensor.
- name: The name to give this op.

#### Returns:

• quantile: a Tensor of shape sample\_shape(x) + self.batch\_shape with values of type self.dtype.

## sample

```
sample(
    sample_shape=(),
    seed=None,
    name='sample'
)
```

Generate samples of the specified shape.

Note that a call to sample() without arguments will generate a single sample.

## Args:

• sample\_shape: 0D or 1D int32 Tensor. Shape of the generated samples.

- seed: Python integer seed for RNG
- name: name to give to the op.

### Returns:

• samples: a **Tensor** with prepended dimensions **sample\_shape**.

### scale

```
scale()
```

Wishart distribution scale matrix.

### stddev

```
stddev(name='stddev')
```

Standard deviation.

Standard deviation is defined as,

```
stddev = E[(X - E[X])**2]**0.5
```

where X is the random variable associated with this distribution, E denotes expectation, and stddev.shape = batch\_shape + event\_shape .

### Args:

• name: The name to give this op.

### Returns:

• stddev: Floating-point **Tensor** with shape identical to **batch\_shape + event\_shape**, i.e., the same shape as **self.mean()**.

## survival\_function

```
survival_function(
   value,
   name='survival_function'
)
```

Survival function.

Given random variable X, the survival function is defined:

```
survival\_function(x) = P[X > x]
= 1 - P[X \le x]
= 1 - cdf(x).
```

Args:

- value: float or double Tensor.
- name: The name to give this op.

### Returns:

Tensor of shape sample\_shape(x) + self.batch\_shape with values of type self.dtype.

## variance

variance(name='variance')

Variance.

Variance is defined as,

$$Var = E[(X - E[X])**2]$$

where X is the random variable associated with this distribution, E denotes expectation, and Var.shape = batch\_shape + event\_shape.

## Args:

• name: The name to give this op.

#### Returns:

variance: Floating-point Tensor with shape identical to batch\_shape + event\_shape , i.e., the same shape as self.mean().

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