tf.distributions.StudentT

Contents

Class **StudentT**

Inherits From: `Distribution`

Aliases:

- Class `tf.contrib.distributions.StudentT`
- Class `tf.distributions.StudentT`

Defined in `tensorflow/python/ops/distributions/student_t.py`.

See the guide: Statistical Distributions (contrib) > Univariate (scalar) distributions

Student's t-distribution.

This distribution has parameters: degree of freedom `df`, location `loc`, and `scale`.

Mathematical details

The probability density function (pdf) is,

```
pdf(x; df, mu, sigma) = (1 + y**2 / df)**(-0.5 (df + 1)) / Z
where,
y = (x - mu) / sigma
Z = abs(sigma) sqrt(df pi) Gamma(0.5 df) / Gamma(0.5 (df + 1))
```

where: `loc = mu`, `scale = sigma`, and, `Z` *is the normalization constant, and,* `Gamma` is the gamma function.

The StudentT distribution is a member of the location-scale family, i.e., it can be constructed as,

```
X ~ StudentT(df, loc=0, scale=1)
Y = loc + scale * X
```

Notice that `scale` has semantics more similar to standard deviation than variance. However it is not actually the std. deviation; the Student's t-distribution std. dev. is `scale sqrt(df / (df - 2))` when `df > 2`.

Examples

Examples of initialization of one or a batch of distributions.

```
# Define a single scalar Student t distribution.
single_dist = tf.distributions.StudentT(df=3)

# Evaluate the pdf at 1, returning a scalar Tensor.
single_dist.prob(1.)

# Define a batch of two scalar valued Student t's.
# The first has degrees of freedom 2, mean 1, and scale 11.
# The second 3, 2 and 22.
multi_dist = tf.distributions.StudentT(df=[2, 3],
                                       loc=[1, 2.],
                                       scale=[11, 22.])

# Evaluate the pdf of the first distribution on 0, and the second on 1.5,
# returning a length two tensor.
multi_dist.prob([0, 1.5])

# Get 3 samples, returning a 3 x 2 tensor.
multi_dist.sample(3)
```

Arguments are broadcast when possible.

```
# Define a batch of two Student's t distributions.
# Both have df 2 and mean 1, but different scales.
dist = tf.distributions.StudentT(df=2, loc=1, scale=[11, 22.])

# Evaluate the pdf of both distributions on the same point, 3.0,
# returning a length 2 tensor.
dist.prob(3.0)
```

Properties

`allow_nan_stats`

Python `bool` describing behavior when a stat is undefined.

Stats return +/- infinity when it makes sense. E.g., the variance of a Cauchy distribution is infinity. However, sometimes the statistic is undefined, e.g., if a distribution's pdf does not achieve a maximum within the support of the distribution, the mode is undefined. If the mean is undefined, then by definition the variance is undefined. E.g. the mean for Student's T for df = 1 is undefined (no clear way to say it is either + or - infinity), so the variance = E[(X - mean)**2] is also undefined.

Returns:

- `allow_nan_stats`: Python `bool`.

`batch_shape`

Shape of a single sample from a single event index as a `TensorShape`.

May be partially defined or unknown.

The batch dimensions are indexes into independent, non-identical parameterizations of this distribution.

Returns:

- `batch_shape`: `TensorShape`, possibly unknown.

df

Degrees of freedom in these Student's t distribution(s).

dtype

The `DType` of `Tensor`s handled by this `Distribution`.

event_shape

Shape of a single sample from a single batch as a `TensorShape`.

May be partially defined or unknown.

Returns:

- `event_shape`: `TensorShape`, possibly unknown.

loc

Locations of these Student's t distribution(s).

name

Name prepended to all ops created by this `Distribution`.

parameters

Dictionary of parameters used to instantiate this `Distribution`.

reparameterization_type

Describes how samples from the distribution are reparameterized.

Currently this is one of the static instances `distributions.FULLY_REPARAMETERIZED` or `distributions.NOT_REPARAMETERIZED`.

Returns:

An instance of `ReparameterizationType`.

scale

Scaling factors of these Student's t distribution(s).

validate_args

Python `bool` indicating possibly expensive checks are enabled.

Methods

`__init__`

```
__init__(
    df,
    loc,
    scale,
    validate_args=False,
    allow_nan_stats=True,
    name='StudentT'
)
```

Construct Student's t distributions.

The distributions have degree of freedom `df`, mean `loc`, and scale `scale`.

The parameters `df`, `loc`, and `scale` must be shaped in a way that supports broadcasting (e.g. `df + loc + scale` is a valid operation).

Args:

- `df`: Floating-point `Tensor`. The degrees of freedom of the distribution(s). `df` must contain only positive values.
- `loc`: Floating-point `Tensor`. The mean(s) of the distribution(s).
- `scale`: Floating-point `Tensor`. The scaling factor(s) for the distribution(s). Note that `scale` is not technically the standard deviation of this distribution but has semantics more similar to standard deviation than variance.
- `validate_args`: Python `bool`, default `False`. When `True` distribution parameters are checked for validity despite possibly degrading runtime performance. When `False` invalid inputs may silently render incorrect outputs.
- `allow_nan_stats`: Python `bool`, default `True`. When `True`, statistics (e.g., mean, mode, variance) use the value "`NaN`" to indicate the result is undefined. When `False`, an exception is raised if one or more of the statistic's batch members are undefined.
- `name`: Python `str` name prefixed to Ops created by this class.

Raises:

- `TypeError`: if loc and scale are different dtypes.

`batch_shape_tensor`

```
batch_shape_tensor(name='batch_shape_tensor')
```

Shape of a single sample from a single event index as a 1-D `Tensor`.

The batch dimensions are indexes into independent, non-identical parameterizations of this distribution.

Args:

- `name`: name to give to the op

Returns:

- `batch_shape`: `Tensor`.

`cdf`

```
cdf(
    value,
    name='cdf'
)
```

Cumulative distribution function.

Given random variable `X`, the cumulative distribution function `cdf` is:

```
cdf(x) := P[X <= x]
```

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

- `cdf`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

copy

```
copy(**override_parameters_kwargs)
```

Creates a deep copy of the distribution.

> **Note:** the copy distribution may continue to depend on the original initialization arguments.

Args:

- `**override_parameters_kwargs`: String/value dictionary of initialization arguments to override with new values.

Returns:

- `distribution`: A new instance of `type(self)` initialized from the union of self.parameters and override_parameters_kwargs, i.e., `dict(self.parameters, **override_parameters_kwargs)`.

covariance

```
covariance(name='covariance')
```

Covariance.

Covariance is (possibly) defined only for non-scalar-event distributions.

For example, for a length-`k`, vector-valued distribution, it is calculated as,

```
Cov[i, j] = Covariance(X_i, X_j) = E[(X_i - E[X_i]) (X_j - E[X_j])]
```

where `Cov` is a (batch of) `k x k` matrix, `0 <= (i, j) < k`, and `E` denotes expectation.

Alternatively, for non-vector, multivariate distributions (e.g., matrix-valued, Wishart), `Covariance` shall return a (batch of)

matrices under some vectorization of the events, i.e.,

```
Cov[i, j] = Covariance(Vec(X)_i, Vec(X)_j) = [as above]
```

where `Cov` is a (batch of) `k' x k'` matrices, `0 <= (i, j) < k' = reduce_prod(event_shape)`, and `Vec` is some function mapping indices of this distribution's event dimensions to indices of a length-`k'` vector.

Args:

- `name`: The name to give this op.

Returns:

- `covariance`: Floating-point `Tensor` with shape `[B1, ..., Bn, k', k']` where the first `n` dimensions are batch coordinates and `k' = reduce_prod(self.event_shape)`.

`entropy`

```
entropy(name='entropy')
```

Shannon entropy in nats.

`event_shape_tensor`

```
event_shape_tensor(name='event_shape_tensor')
```

Shape of a single sample from a single batch as a 1-D int32 `Tensor`.

Args:

- `name`: name to give to the op

Returns:

- `event_shape`: `Tensor`.

`is_scalar_batch`

```
is_scalar_batch(name='is_scalar_batch')
```

Indicates that `batch_shape == []`.

Args:

- `name`: The name to give this op.

Returns:

- `is_scalar_batch`: `bool` scalar `Tensor`.

`is_scalar_event`

```
is_scalar_event(name='is_scalar_event')
```

Indicates that `event_shape == []`.

Args:

- `name`: The name to give this op.

Returns:

- `is_scalar_event`: `bool` scalar `Tensor`.

`log_cdf`

```
log_cdf(
    value,
    name='log_cdf'
)
```

Log cumulative distribution function.

Given random variable `X`, the cumulative distribution function `cdf` is:

```
log_cdf(x) := Log[ P[X <= x] ]
```

Often, a numerical approximation can be used for `log_cdf(x)` that yields a more accurate answer than simply taking the logarithm of the `cdf` when `x << -1`.

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

- `logcdf`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

`log_prob`

```
log_prob(
    value,
    name='log_prob'
)
```

Log probability density/mass function.

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

- `log_prob`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

`log_survival_function`

```
log_survival_function(
    value,
    name='log_survival_function'
)
```

Log survival function.

Given random variable `X`, the survival function is defined:

```
log_survival_function(x) = Log[ P[X > x] ]
                         = Log[ 1 - P[X <= x] ]
                         = Log[ 1 - cdf(x) ]
```

Typically, different numerical approximations can be used for the log survival function, which are more accurate than `1 - cdf(x)` when `x >> 1`.

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

`Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

`mean`

```
mean(name='mean')
```

Mean.

Additional documentation from `StudentT`:

The mean of Student's T equals `loc` if `df > 1`, otherwise it is `NaN`. If `self.allow_nan_stats=True`, then an exception will be raised rather than returning `NaN`.

`mode`

```
mode(name='mode')
```

Mode.

`param_shapes`

```
param_shapes(
    cls,
    sample_shape,
    name='DistributionParamShapes'
)
```

Shapes of parameters given the desired shape of a call to `sample()`.

This is a class method that describes what key/value arguments are required to instantiate the given `Distribution` so that a particular shape is returned for that instance's call to `sample()`.

Subclasses should override class method `_param_shapes`.

Args:

- `sample_shape`: `Tensor` or python list/tuple. Desired shape of a call to `sample()`.
- `name`: name to prepend ops with.

Returns:

`dict` of parameter name to `Tensor` shapes.

param_static_shapes

```
param_static_shapes(
    cls,
    sample_shape
)
```

param_shapes with static (i.e. `TensorShape`) shapes.

This is a class method that describes what key/value arguments are required to instantiate the given `Distribution` so that a particular shape is returned for that instance's call to `sample()`. Assumes that the sample's shape is known statically.

Subclasses should override class method `_param_shapes` to return constant-valued tensors when constant values are fed.

Args:

- `sample_shape`: `TensorShape` or python list/tuple. Desired shape of a call to `sample()`.

Returns:

`dict` of parameter name to `TensorShape`.

Raises:

- `ValueError`: if `sample_shape` is a `TensorShape` and is not fully defined.

prob

```
prob(
    value,
    name='prob'
)
```

Probability density/mass function.

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

- `prob`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

quantile

```
quantile(
    value,
    name='quantile'
)
```

Quantile function. Aka "inverse cdf" or "percent point function".

Given random variable `X` and `p in [0, 1]`, the `quantile` is:

```
quantile(p) := x such that P[X <= x] == p
```

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

- `quantile`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

sample

```
sample(
    sample_shape=(),
    seed=None,
    name='sample'
)
```

Generate samples of the specified shape.

Note that a call to `sample()` without arguments will generate a single sample.

Args:

- `sample_shape`: 0D or 1D `int32` `Tensor`. Shape of the generated samples.

- `seed`: Python integer seed for RNG
- `name`: name to give to the op.

Returns:

- `samples`: a `Tensor` with prepended dimensions `sample_shape`.

stddev

```
stddev(name='stddev')
```

Standard deviation.

Standard deviation is defined as,

```
stddev = E[(X - E[X])**2]**0.5
```

where `X` is the random variable associated with this distribution, `E` denotes expectation, and `stddev.shape = batch_shape + event_shape`.

Args:

- `name`: The name to give this op.

Returns:

- `stddev`: Floating-point `Tensor` with shape identical to `batch_shape + event_shape`, i.e., the same shape as `self.mean()`.

survival_function

```
survival_function(
    value,
    name='survival_function'
)
```

Survival function.

Given random variable `X`, the survival function is defined:

```
survival_function(x) = P[X > x]
                     = 1 - P[X <= x]
                     = 1 - cdf(x).
```

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

`Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

variance

```
variance(name='variance')
```

Variance.

Variance is defined as,

```
Var = E[(X - E[X])**2]
```

where `X` is the random variable associated with this distribution, `E` denotes expectation, and `Var.shape = batch_shape + event_shape`.

Additional documentation from `StudentT`:

The variance for Student's T equals

```
df / (df - 2), when df > 2
infinity, when 1 < df <= 2
NaN, when df <= 1
```

Args:

- `name`: The name to give this op.

Returns:

- `variance`: Floating-point `Tensor` with shape identical to `batch_shape + event_shape`, i.e., the same shape as `self.mean()`.
