tf.contrib.linalg.LinearOperatorDiag

Contents

Class `LinearOperatorDiag`

Inherits From: `LinearOperator`

Defined in `tensorflow/contrib/linalg/python/ops/linear_operator_diag.py`.

See the guide: Linear Algebra (contrib) > `LinearOperator`

`LinearOperator` acting like a [batch] square diagonal matrix.

This operator acts like a [batch] diagonal matrix `A` with shape `[B1,...,Bb, N, N]` for some `b >= 0`. The first `b` indices index a batch member. For every batch index `(i1,...,ib)`, `A[i1,...,ib, : :]` is an `N x N` matrix. This matrix `A` is not materialized, but for purposes of broadcasting this shape will be relevant.

`LinearOperatorDiag` is initialized with a (batch) vector.

```
# Create a 2 x 2 diagonal linear operator.
diag = [1., -1.]
operator = LinearOperatorDiag(diag)

operator.to_dense()
==> [[1.,  0.]
     [0., -1.]]

operator.shape
==> [2, 2]

operator.log_abs_determinant()
==> scalar Tensor

x = ... Shape [2, 4] Tensor
operator.matmul(x)
==> Shape [2, 4] Tensor

# Create a [2, 3] batch of 4 x 4 linear operators.
diag = tf.random_normal(shape=[2, 3, 4])
operator = LinearOperatorDiag(diag)

# Create a shape [2, 1, 4, 2] vector.  Note that this shape is compatible
# since the batch dimensions, [2, 1], are brodcast to
# operator.batch_shape = [2, 3].
y = tf.random_normal(shape=[2, 1, 4, 2])
x = operator.solve(y)
==> operator.matmul(x) = y
```

Shape compatibility

This operator acts on [batch] matrix with compatible shape. `x` is a batch matrix with compatible shape for `matmul` and `solve` if

```
operator.shape = [B1,...,Bb] + [N, N],  with b >= 0
x.shape =   [C1,...,Cc] + [N, R],
and [C1,...,Cc] broadcasts with [B1,...,Bb] to [D1,...,Dd]
```

Performance

Suppose `operator` is a `LinearOperatorDiag` of shape `[N, N]`, and `x.shape = [N, R]`. Then

- `operator.matmul(x)` involves `N * R` multiplications.
- `operator.solve(x)` involves `N` divisions and `N * R` multiplications.
- `operator.determinant()` involves a size `N` `reduce_prod`.

If instead `operator` and `x` have shape `[B1,...,Bb, N, N]` and `[B1,...,Bb, N, R]`, every operation increases in complexity by `B1*...*Bb`.

Matrix property hints

This `LinearOperator` is initialized with boolean flags of the form `is_X`, for `X = non_singular, self_adjoint, positive_definite, square`. These have the following meaning:

- If `is_X == True`, callers should expect the operator to have the property `X`. This is a promise that should be fulfilled, but is *not* a runtime assert. For example, finite floating point precision may result in these promises being violated.
- If `is_X == False`, callers should expect the operator to not have `X`.
- If `is_X == None` (the default), callers should have no expectation either way.

Properties

`batch_shape`

`TensorShape` of batch dimensions of this `LinearOperator`.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `TensorShape([B1,...,Bb])`, equivalent to `A.get_shape()[:-2]`

Returns:

`TensorShape`, statically determined, may be undefined.

`diag`

`domain_dimension`

Dimension (in the sense of vector spaces) of the domain of this operator.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `N`.

Returns:

`Dimension` object.

`dtype`

The `DType` of `Tensor`s handled by this `LinearOperator`.

`graph_parents`

List of graph dependencies of this `LinearOperator`.

`is_non_singular`

`is_positive_definite`

`is_self_adjoint`

`is_square`

Return `True/False` depending on if this operator is square.

`name`

Name prepended to all ops created by this `LinearOperator`.

`range_dimension`

Dimension (in the sense of vector spaces) of the range of this operator.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `M`.

Returns:

`Dimension` object.

`shape`

`TensorShape` of this `LinearOperator`.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `TensorShape([B1,...,Bb, M, N])`, equivalent to `A.get_shape()`.

Returns:

`TensorShape`, statically determined, may be undefined.

`tensor_rank`

Rank (in the sense of tensors) of matrix corresponding to this operator.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `b + 2`.

Args:

- `name`: A name for this `Op.

Returns:

Python integer, or None if the tensor rank is undefined.

Methods

`__init__`

```
__init__(
    diag,
    is_non_singular=None,
    is_self_adjoint=None,
    is_positive_definite=None,
    is_square=None,
    name='LinearOperatorDiag'
)
```

Initialize a `LinearOperatorDiag`.

Args:

- `diag`: Shape `[B1,...,Bb, N]` `Tensor` with `b >= 0` `N >= 0`. The diagonal of the operator. Allowed dtypes: `float32`, `float64`, `complex64`, `complex128`.
- `is_non_singular`: Expect that this operator is non-singular.
- `is_self_adjoint`: Expect that this operator is equal to its hermitian transpose. If `diag.dtype` is real, this is auto-set to `True`.
- `is_positive_definite`: Expect that this operator is positive definite, meaning the quadratic form `x^H A x` has positive real part for all nonzero `x`. Note that we do not require the operator to be self-adjoint to be positive-definite. See: https://en.wikipedia.org/wiki/Positive-definite_matrix\ #Extension_for_non_symmetric_matrices
- `is_square`: Expect that this operator acts like square [batch] matrices.
- `name`: A name for this `LinearOperator`.

Raises:

- `TypeError`: If `diag.dtype` is not an allowed type.
- `ValueError`: If `diag.dtype` is real, and `is_self_adjoint` is not `True`.

`add_to_tensor`

```
add_to_tensor(
    x,
    name='add_to_tensor'
)
```

Add matrix represented by this operator to `x`. Equivalent to `A + x`.

Args:

- `x`: `Tensor` with same `dtype` and shape broadcastable to `self.shape`.
- `name`: A name to give this `Op`.

Returns:

A `Tensor` with broadcast shape and same `dtype` as `self`.

`assert_non_singular`

```
assert_non_singular(name='assert_non_singular')
```

Returns an `Op` that asserts this operator is non singular.

This operator is considered non-singular if

```
ConditionNumber < max{100, range_dimension, domain_dimension} * eps,
eps := np.finfo(self.dtype.as_numpy_dtype).eps
```

Args:

- `name`: A string name to prepend to created ops.

Returns:

An `Assert` `Op`, that, when run, will raise an `InvalidArgumentError` if the operator is singular.

`assert_positive_definite`

```
assert_positive_definite(name='assert_positive_definite')
```

Returns an `Op` that asserts this operator is positive definite.

Here, positive definite means that the quadratic form `x^H A x` has positive real part for all nonzero `x`. Note that we do not require the operator to be self-adjoint to be positive definite.

Args:

- `name`: A name to give this `Op`.

Returns:

An `Assert` `Op`, that, when run, will raise an `InvalidArgumentError` if the operator is not positive definite.

`assert_self_adjoint`

```
assert_self_adjoint(name='assert_self_adjoint')
```

Returns an `Op` that asserts this operator is self-adjoint.

Here we check that this operator is *exactly* equal to its hermitian transpose.

Args:

- `name`: A string name to prepend to created ops.

Returns:

An `Assert` `Op`, that, when run, will raise an `InvalidArgumentError` if the operator is not self-adjoint.

`batch_shape_tensor`

```
batch_shape_tensor(name='batch_shape_tensor')
```

Shape of batch dimensions of this operator, determined at runtime.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns a `Tensor` holding `[B1,...,Bb]`.

Args:

- `name`: A name for this `Op.

Returns:

`int32` `Tensor`

`determinant`

```
determinant(name='det')
```

Determinant for every batch member.

Args:

- `name`: A name for this `Op.

Returns:

`Tensor` with shape `self.batch_shape` and same `dtype` as `self`.

Raises:

- `NotImplementedError`: If `self.is_square` is `False`.

`diag_part`

```
diag_part(name='diag_part')
```

Efficiently get the [batch] diagonal part of this operator.

If this operator has shape `[B1,...,Bb, M, N]`, this returns a `Tensor` `diagonal`, of shape `[B1,...,Bb, min(M, N)]`, where `diagonal[b1,...,bb, i] = self.to_dense()[b1,...,bb, i, i]`.

```
my_operator = LinearOperatorDiag([1., 2.])

# Efficiently get the diagonal
my_operator.diag_part()
==> [1., 2.]

# Equivalent, but inefficient method
tf.matrix_diag_part(my_operator.to_dense())
==> [1., 2.]
```

Args:

- `name`: A name for this `Op`.

Returns:

- `diag_part`: A `Tensor` of same `dtype` as self.

`domain_dimension_tensor`

```
domain_dimension_tensor(name='domain_dimension_tensor')
```

Dimension (in the sense of vector spaces) of the domain of this operator.

Determined at runtime.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `N`.

Args:

- `name`: A name for this `Op`.

Returns:

`int32` `Tensor`

`log_abs_determinant`

```
log_abs_determinant(name='log_abs_det')
```

Log absolute value of determinant for every batch member.

Args:

- `name`: A name for this `Op.

Returns:

`Tensor` with shape `self.batch_shape` and same `dtype` as `self`.

Raises:

- `NotImplementedError`: If `self.is_square` is `False`.

`matmul`

```
matmul(
    x,
    adjoint=False,
    adjoint_arg=False,
    name='matmul'
)
```

Transform [batch] matrix `x` with left multiplication: `x --> Ax`.

```
# Make an operator acting like batch matrix A.  Assume A.shape = [..., M, N]
operator = LinearOperator(...)
operator.shape = [..., M, N]

X = ... # shape [..., N, R], batch matrix, R > 0.

Y = operator.matmul(X)
Y.shape
==> [..., M, R]

Y[..., :, r] = sum_j A[..., :, j] X[j, r]
```

Args:

- `x`: `Tensor` with compatible shape and same `dtype` as `self`. See class docstring for definition of compatibility.
- `adjoint`: Python `bool`. If `True`, left multiply by the adjoint: `A^H x`.
- `adjoint_arg`: Python `bool`. If `True`, compute `A x^H` where `x^H` is the hermitian transpose (transposition and complex conjugation).
- `name`: A name for this `Op.

Returns:

A `Tensor` with shape `[..., M, R]` and same `dtype` as `self`.

`matvec`

```
matvec(
    x,
    adjoint=False,
    name='matvec'
)
```

Transform [batch] vector `x` with left multiplication: `x --> Ax`.

```
# Make an operator acting like batch matric A.  Assume A.shape = [..., M, N]
operator = LinearOperator(...)

X = ... # shape [..., N], batch vector

Y = operator.matvec(X)
Y.shape
==> [..., M]

Y[..., :] = sum_j A[..., :, j] X[..., j]
```

Args:

- `x`: `Tensor` with compatible shape and same `dtype` as `self`. `x` is treated as a [batch] vector meaning for every set of leading dimensions, the last dimension defines a vector. See class docstring for definition of compatibility.
- `adjoint`: Python `bool`. If `True`, left multiply by the adjoint: `A^H x`.
- `name`: A name for this `Op.

Returns:

A `Tensor` with shape `[..., M]` and same `dtype` as `self`.

`range_dimension_tensor`

```
range_dimension_tensor(name='range_dimension_tensor')
```

Dimension (in the sense of vector spaces) of the range of this operator.

Determined at runtime.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `M`.

Args:

- `name`: A name for this `Op`.

Returns:

`int32` `Tensor`

`shape_tensor`

```
shape_tensor(name='shape_tensor')
```

Shape of this `LinearOperator`, determined at runtime.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns a `Tensor` holding `[B1,...,Bb, M, N]`, equivalent to `tf.shape(A)`.

Args:

- `name`: A name for this `Op.

Returns:

`int32` `Tensor`

solve

```
solve(
    rhs,
    adjoint=False,
    adjoint_arg=False,
    name='solve'
)
```

Solve (exact or approx) `R` (batch) systems of equations: `A X = rhs`.

The returned `Tensor` will be close to an exact solution if `A` is well conditioned. Otherwise closeness will vary. See class docstring for details.

Examples:

```
# Make an operator acting like batch matrix A.  Assume A.shape = [..., M, N]
operator = LinearOperator(...)
operator.shape = [..., M, N]

# Solve R > 0 linear systems for every member of the batch.
RHS = ... # shape [..., M, R]

X = operator.solve(RHS)
# X[..., :, r] is the solution to the r'th linear system
# sum_j A[..., :, j] X[..., j, r] = RHS[..., :, r]

operator.matmul(X)
==> RHS
```

Args:

- `rhs`: `Tensor` with same `dtype` as this operator and compatible shape. `rhs` is treated like a [batch] matrix meaning for every set of leading dimensions, the last two dimensions defines a matrix. See class docstring for definition of compatibility.
- `adjoint`: Python `bool`. If `True`, solve the system involving the adjoint of this `LinearOperator`: `A^H X = rhs`.
- `adjoint_arg`: Python `bool`. If `True`, solve `A X = rhs^H` where `rhs^H` is the hermitian transpose (transposition and complex conjugation).
- `name`: A name scope to use for ops added by this method.

Returns:

`Tensor` with shape `[...,N, R]` and same `dtype` as `rhs`.

Raises:

- `NotImplementedError`: If `self.is_non_singular` or `is_square` is False.

solvevec

```
solvevec(
    rhs,
    adjoint=False,
    name='solve'
)
```

Solve single equation with best effort: `A X = rhs`.

The returned `Tensor` will be close to an exact solution if `A` is well conditioned. Otherwise closeness will vary. See class docstring for details.

Examples:

```
# Make an operator acting like batch matrix A.  Assume A.shape = [..., M, N]
operator = LinearOperator(...)
operator.shape = [..., M, N]

# Solve one linear system for every member of the batch.
RHS = ... # shape [..., M]

X = operator.solvevec(RHS)
# X is the solution to the linear system
# sum_j A[..., :, j] X[..., j] = RHS[..., :]

operator.matvec(X)
==> RHS
```

Args:

- `rhs`: `Tensor` with same `dtype` as this operator. `rhs` is treated like a [batch] vector meaning for every set of leading dimensions, the last dimension defines a vector. See class docstring for definition of compatibility regarding batch dimensions.
- `adjoint`: Python `bool`. If `True`, solve the system involving the adjoint of this `LinearOperator`: `A^H X = rhs`.
- `name`: A name scope to use for ops added by this method.

Returns:

`Tensor` with shape `[...,N]` and same `dtype` as `rhs`.

Raises:

- `NotImplementedError`: If `self.is_non_singular` or `is_square` is False.

tensor_rank_tensor

```
tensor_rank_tensor(name='tensor_rank_tensor')
```

Rank (in the sense of tensors) of matrix corresponding to this operator.

If this operator acts like the batch matrix `A` with `A.shape = [B1,...,Bb, M, N]`, then this returns `b + 2`.

Args:

- `name`: A name for this `Op.

Returns:

`int32` `Tensor`, determined at runtime.

`to_dense`

```
to_dense(name='to_dense')
```

Return a dense (batch) matrix representing this operator.

`trace`

```
trace(name='trace')
```

Trace of the linear operator, equal to sum of `self.diag_part()`.

If the operator is square, this is also the sum of the eigenvalues.

Args:

- `name`: A name for this `Op`.

Returns:

Shape `[B1,...,Bb]` `Tensor` of same `dtype` as `self`.
