tf.contrib.distributions.WishartCholesky

Contents

Class `WishartCholesky`

Defined in `tensorflow/contrib/distributions/python/ops/wishart.py`.

See the guide: Statistical Distributions (contrib) > Multivariate distributions

The matrix Wishart distribution on positive definite matrices.

This distribution is defined by a scalar degrees of freedom `df` and a lower, triangular Cholesky factor which characterizes the scale matrix.

Using WishartCholesky is a constant-time improvement over WishartFull. It saves an O(nbk^3) operation, i.e., a matrix-product operation for sampling and a Cholesky factorization in log_prob. For most use-cases it often saves another O(nbk^3) operation since most uses of Wishart will also use the Cholesky factorization.

Mathematical Details

The probability density function (pdf) is,

```
pdf(X; df, scale) = det(X)**(0.5 (df-k-1)) exp(-0.5 tr[inv(scale) X]) / Z
Z = 2**(0.5 df k) |det(scale)|**(0.5 df) Gamma_k(0.5 df)
```

where: `df >= k` *denotes the degrees of freedom,* `scale` is a symmetric, positive definite, `k x k` matrix, `Z` *is the normalizing constant, and,* `Gamma_k` is the multivariate Gamma function.

Examples

```
# Initialize a single 3x3 Wishart with Cholesky factored scale matrix and 5
# degrees-of-freedom.(*)
df = 5
chol_scale = tf.cholesky(...)  # Shape is [3, 3].
dist = tf.contrib.distributions.WishartCholesky(df=df, scale=chol_scale)

# Evaluate this on an observation in R^3, returning a scalar.
x = ...  # A 3x3 positive definite matrix.
dist.prob(x)  # Shape is [], a scalar.

# Evaluate this on a two observations, each in R^{3x3}, returning a length two
# Tensor.
x = [x0, x1]  # Shape is [2, 3, 3].
dist.prob(x)  # Shape is [2].

# Initialize two 3x3 Wisharts with Cholesky factored scale matrices.
df = [5, 4]
chol_scale = tf.cholesky(...)  # Shape is [2, 3, 3].
dist = tf.contrib.distributions.WishartCholesky(df=df, scale=chol_scale)

# Evaluate this on four observations.
x = [[x0, x1], [x2, x3]]  # Shape is [2, 2, 3, 3].
dist.prob(x)  # Shape is [2, 2].

# (*) - To efficiently create a trainable covariance matrix, see the example
#   in tf.contrib.distributions.matrix_diag_transform.
```

Properties

`allow_nan_stats`

Python `bool` describing behavior when a stat is undefined.

Stats return +/- infinity when it makes sense. E.g., the variance of a Cauchy distribution is infinity. However, sometimes the statistic is undefined, e.g., if a distribution's pdf does not achieve a maximum within the support of the distribution, the mode is undefined. If the mean is undefined, then by definition the variance is undefined. E.g. the mean for Student's T for df = 1 is undefined (no clear way to say it is either + or - infinity), so the variance = E[(X - mean)**2] is also undefined.

Returns:

- `allow_nan_stats`: Python `bool`.

`batch_shape`

Shape of a single sample from a single event index as a `TensorShape`.

May be partially defined or unknown.

The batch dimensions are indexes into independent, non-identical parameterizations of this distribution.

Returns:

- `batch_shape`: `TensorShape`, possibly unknown.

`cholesky_input_output_matrices`

Boolean indicating if `Tensor` input/outputs are Cholesky factorized.

`df`

Wishart distribution degree(s) of freedom.

`dimension`

Dimension of underlying vector space. The `p` in `R^(p*p)`.

`dtype`

The `DType` of `Tensor`s handled by this `Distribution`.

`event_shape`

Shape of a single sample from a single batch as a `TensorShape`.

May be partially defined or unknown.

Returns:

- `event_shape`: `TensorShape`, possibly unknown.

`name`

Name prepended to all ops created by this `Distribution`.

`parameters`

Dictionary of parameters used to instantiate this `Distribution`.

`reparameterization_type`

Describes how samples from the distribution are reparameterized.

Currently this is one of the static instances `distributions.FULLY_REPARAMETERIZED` or `distributions.NOT_REPARAMETERIZED`.

Returns:

An instance of `ReparameterizationType`.

`scale_operator`

Wishart distribution scale matrix as an Linear Operator.

`validate_args`

Python `bool` indicating possibly expensive checks are enabled.

Methods

`__init__`

```
__init__(
    df,
    scale,
    cholesky_input_output_matrices=False,
    validate_args=False,
    allow_nan_stats=True,
    name='WishartCholesky'
)
```

Construct Wishart distributions.

Args:

- `df`: `float` or `double` `Tensor`. Degrees of freedom, must be greater than or equal to dimension of the scale matrix.
- `scale`: `float` or `double` `Tensor`. The Cholesky factorization of the symmetric positive definite scale matrix of the distribution.
- `cholesky_input_output_matrices`: Python `bool`. Any function which whose input or output is a matrix assumes the input is Cholesky and returns a Cholesky factored matrix. Example `log_prob` input takes a Cholesky and `sample_n` returns a Cholesky when `cholesky_input_output_matrices=True`.
- `validate_args`: Python `bool`, default `False`. When `True` distribution parameters are checked for validity despite possibly degrading runtime performance. When `False` invalid inputs may silently render incorrect outputs.
- `allow_nan_stats`: Python `bool`, default `True`. When `True`, statistics (e.g., mean, mode, variance) use the value "`NaN`" to indicate the result is undefined. When `False`, an exception is raised if one or more of the statistic's batch members are undefined.
- `name`: Python `str` name prefixed to Ops created by this class.

`batch_shape_tensor`

```
batch_shape_tensor(name='batch_shape_tensor')
```

Shape of a single sample from a single event index as a 1-D `Tensor`.

The batch dimensions are indexes into independent, non-identical parameterizations of this distribution.

Args:

- `name`: name to give to the op

Returns:

- `batch_shape`: `Tensor`.

`cdf`

```
cdf(
    value,
    name='cdf'
)
```

Cumulative distribution function.

Given random variable `X`, the cumulative distribution function `cdf` is:

```
cdf(x) := P[X <= x]
```

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

- `cdf`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

copy

```
copy(**override_parameters_kwargs)
```

Creates a deep copy of the distribution.

Note: the copy distribution may continue to depend on the original initialization arguments.

Args:

- `**override_parameters_kwargs`: String/value dictionary of initialization arguments to override with new values.

Returns:

- `distribution`: A new instance of `type(self)` initialized from the union of self.parameters and override_parameters_kwargs, i.e., `dict(self.parameters, **override_parameters_kwargs)`.

covariance

```
covariance(name='covariance')
```

Covariance.

Covariance is (possibly) defined only for non-scalar-event distributions.

For example, for a length-`k`, vector-valued distribution, it is calculated as,

```
Cov[i, j] = Covariance(X_i, X_j) = E[(X_i - E[X_i]) (X_j - E[X_j])]
```

where `Cov` is a (batch of) `k x k` matrix, `0 <= (i, j) < k`, and `E` denotes expectation.

Alternatively, for non-vector, multivariate distributions (e.g., matrix-valued, Wishart), `Covariance` shall return a (batch of) matrices under some vectorization of the events, i.e.,

```
Cov[i, j] = Covariance(Vec(X)_i, Vec(X)_j) = [as above]
```

where `Cov` is a (batch of) `k' x k'` matrices, `0 <= (i, j) < k' = reduce_prod(event_shape)`, and `Vec` is some function mapping indices of this distribution's event dimensions to indices of a length-`k'` vector.

Args:

- `name`: The name to give this op.

Returns:

- `covariance`: Floating-point `Tensor` with shape `[B1, ..., Bn, k', k']` where the first `n` dimensions are batch coordinates and `k' = reduce_prod(self.event_shape)`.

`entropy`

```
entropy(name='entropy')
```

Shannon entropy in nats.

`event_shape_tensor`

```
event_shape_tensor(name='event_shape_tensor')
```

Shape of a single sample from a single batch as a 1-D int32 `Tensor`.

Args:

- `name`: name to give to the op

Returns:

- `event_shape`: `Tensor`.

`is_scalar_batch`

```
is_scalar_batch(name='is_scalar_batch')
```

Indicates that `batch_shape == []`.

Args:

- `name`: The name to give this op.

Returns:

- `is_scalar_batch`: `bool` scalar `Tensor`.

`is_scalar_event`

```
is_scalar_event(name='is_scalar_event')
```

Indicates that `event_shape == []`.

Args:

- `name`: The name to give this op.

Returns:

- `is_scalar_event`: `bool` scalar `Tensor`.

`log_cdf`

```
log_cdf(
    value,
    name='log_cdf'
)
```

Log cumulative distribution function.

Given random variable `X`, the cumulative distribution function `cdf` is:

```
log_cdf(x) := Log[ P[X <= x] ]
```

Often, a numerical approximation can be used for `log_cdf(x)` that yields a more accurate answer than simply taking the logarithm of the `cdf` when `x << -1`.

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

- `logcdf`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

`log_normalization`

```
log_normalization(name='log_normalization')
```

Computes the log normalizing constant, log(Z).

`log_prob`

```
log_prob(
    value,
    name='log_prob'
)
```

Log probability density/mass function.

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

* `log_prob`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

`log_survival_function`

```
log_survival_function(
    value,
    name='log_survival_function'
)
```

Log survival function.

Given random variable `X`, the survival function is defined:

```
log_survival_function(x) = Log[ P[X > x] ]
                         = Log[ 1 - P[X <= x] ]
                         = Log[ 1 - cdf(x) ]
```

Typically, different numerical approximations can be used for the log survival function, which are more accurate than `1 - cdf(x)` when `x >> 1`.

Args:

* `value`: `float` or `double` `Tensor`.
* `name`: The name to give this op.

Returns:

`Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

`mean`

```
mean(name='mean')
```

Mean.

`mean_log_det`

```
mean_log_det(name='mean_log_det')
```

Computes E[log(det(X))] under this Wishart distribution.

`mode`

```
mode(name='mode')
```

Mode.

`param_shapes`

```
param_shapes(
    cls,
    sample_shape,
    name='DistributionParamShapes'
)
```

Shapes of parameters given the desired shape of a call to `sample()`.

This is a class method that describes what key/value arguments are required to instantiate the given `Distribution` so that a particular shape is returned for that instance's call to `sample()`.

Subclasses should override class method `_param_shapes`.

Args:

- `sample_shape`: `Tensor` or python list/tuple. Desired shape of a call to `sample()`.
- `name`: name to prepend ops with.

Returns:

`dict` of parameter name to `Tensor` shapes.

`param_static_shapes`

```
param_static_shapes(
    cls,
    sample_shape
)
```

param_shapes with static (i.e. `TensorShape`) shapes.

This is a class method that describes what key/value arguments are required to instantiate the given `Distribution` so that a particular shape is returned for that instance's call to `sample()`. Assumes that the sample's shape is known statically.

Subclasses should override class method `_param_shapes` to return constant-valued tensors when constant values are fed.

Args:

- `sample_shape`: `TensorShape` or python list/tuple. Desired shape of a call to `sample()`.

Returns:

`dict` of parameter name to `TensorShape`.

Raises:

- `ValueError`: if `sample_shape` is a `TensorShape` and is not fully defined.

`prob`

```
prob(
    value,
    name='prob'
)
```

Probability density/mass function.

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

- `prob`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

`quantile`

```
quantile(
    value,
    name='quantile'
)
```

Quantile function. Aka "inverse cdf" or "percent point function".

Given random variable `X` and `p in [0, 1]`, the `quantile` is:

```
quantile(p) := x such that P[X <= x] == p
```

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

- `quantile`: a `Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

`sample`

```
sample(
    sample_shape=(),
    seed=None,
    name='sample'
)
```

Generate samples of the specified shape.

Note that a call to `sample()` without arguments will generate a single sample.

Args:

- `sample_shape`: 0D or 1D `int32` `Tensor`. Shape of the generated samples.

- `seed`: Python integer seed for RNG
- `name`: name to give to the op.

Returns:

- `samples`: a `Tensor` with prepended dimensions `sample_shape`.

`scale`

```
scale()
```

Wishart distribution scale matrix.

`stddev`

```
stddev(name='stddev')
```

Standard deviation.

Standard deviation is defined as,

```
stddev = E[(X - E[X])**2]**0.5
```

where `X` is the random variable associated with this distribution, `E` denotes expectation, and `stddev.shape = batch_shape + event_shape`.

Args:

- `name`: The name to give this op.

Returns:

- `stddev`: Floating-point `Tensor` with shape identical to `batch_shape + event_shape`, i.e., the same shape as `self.mean()`.

`survival_function`

```
survival_function(
    value,
    name='survival_function'
)
```

Survival function.

Given random variable `X`, the survival function is defined:

```
survival_function(x) = P[X > x]
                     = 1 - P[X <= x]
                     = 1 - cdf(x).
```

Args:

- `value`: `float` or `double` `Tensor`.
- `name`: The name to give this op.

Returns:

`Tensor` of shape `sample_shape(x) + self.batch_shape` with values of type `self.dtype`.

`variance`

```
variance(name='variance')
```

Variance.

Variance is defined as,

```
Var = E[(X - E[X])**2]
```

where `X` is the random variable associated with this distribution, `E` denotes expectation, and `Var.shape = batch_shape + event_shape`.

Args:

- `name`: The name to give this op.

Returns:

- `variance`: Floating-point `Tensor` with shape identical to `batch_shape + event_shape`, i.e., the same shape as `self.mean()`.

Last updated November 2, 2017.