Shape from X.

shape from shading 3-D重建

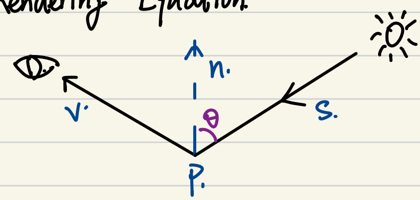
// 可以单张图实现3-D重建

intensity 和 shape 的关系？

光强.

但是一张图可能有多种解释

Rendering Equation



$$L_{out}(p,v,\lambda) = L_{emit}(p,v,\lambda) + \int_\Omega BRDF(p,s,v,\lambda)\cdot L_{in}(p,s,\lambda)\overset{\cos\theta.}{\underline{(-n^T s)}}\,ds$$

对半球面上任意来源的光进行积分.

直射 $\theta=0$. 反射的光最多

$\cos\theta$ 的原因. 能源.

假设入射光通量 Φ.

垂直入射. Φ/A.

倾斜入射 $\Phi/(A/\cos\theta)$ 同样的光通量分布在更大面积

实际单位面积接收能量 $L_{in}\cdot\cos\theta$.

diffuse + specular.

只考虑单一光源 s.

$L_{out}(v) = BRDF(s,v) \cdot L_{in}(-n^T s)$.

当 BRDF = ρ 是常数.

$L_{out} = \rho \cdot L_{in}(n^T s)$ → 颠倒 s 的方向，消去负号

ρ → purely diffuse 完全漫反射

$L_{out} = R(n)$

// $L_{out\_total} = \int_s R(n) \cdot ds$.

对整个曲面积分

假设.

1 材料漫反射. BRDF 是常数 入

2 光源无穷远，各点入射光的方向一致.

正交投影在相机平面

$$\vec{n} = \frac{\left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\right)}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$$

$(p, q) = \left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}\right)$

$\rho \cdot L_{in} = 1$.

$$L_{out} = \frac{-\frac{\partial z}{\partial x} S_x - \frac{\partial z}{\partial y} S_y + S_z}{\sqrt{p^2 + q^2 + 1}} = R(p, q)$$

$(p_s, q_s, 1)$. 光线与平面交点

$R = n^T S = \cos\theta$. 圆锥.

同一个 shade. /(L_{out}) 的 p, q 不唯一.

对. p, q. 做一定的变换 ⇒ f. g. $f(x,y)$ $g(x,y)$

p. q. 是梯度, 当平面垂直, $p, q \to +\infty$.

$L_{out} = R(f, g)$

f. g 在 $R=2$ 的圆内

$$E_{smooth} = \iint f_x^2 + f_y^2 + g_x^2 + g_y^2 \, dx dy.$$

Occluding boundaries.

分割后. 边界处法线 ⊥ 观察方向

⊥ 切线

$n = e \times v$.

$$E(f,g) = E_{image}(f,g) + \lambda E_{smooth}(f,g).$$

从偏导推断出表面.

假设平滑.

$$E(z) = \iint \left(\frac{\partial z}{\partial x} + p\right)^2 + \left(\frac{\partial z}{\partial y} + q\right)^2 dx dy$$ FFT.

photometric stereo.

有 $k \geq 3$ 张图像．视角相同．光源不同

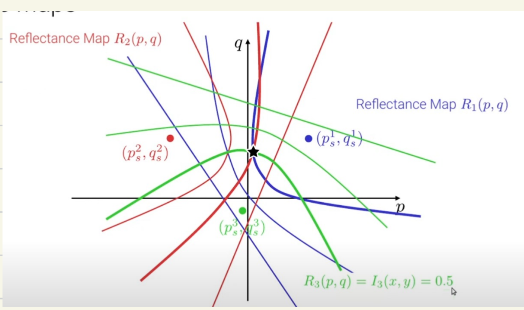
// 可以不用平滑假设.

可确认反射率 albedo. ，法线

1 dof　　2 dof.

far light source. // 但可以扩展.

camera.



对于已知方向的光源，接收光强已知的情况下可确定 gradient (p,q) 在圆锥曲线上.

$$\underbrace{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}}_{I} = \underbrace{\begin{pmatrix} S_1^T \\ S_2^T \\ S_3^T \end{pmatrix}}_{S} \cdot \underbrace{\rho n}_{\tilde{n}}$$

$$\tilde{n} = S^{-1} I. \quad \rho = \|\tilde{n}\|_2. \quad n = \frac{\tilde{n}}{\rho}.$$

I是接收到的光强. S是光源方向

not work.

if. S non-singular.

(s_1 , s_2 . s_3不能线性相关)

I 可能有noise

$$\begin{pmatrix} I_1 \\ \vdots \\ I_n \end{pmatrix} = \begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} \rho n.$$

$I \qquad S.$

$$S^T I = S^T S \tilde{n}$$

$$\tilde{n} = (S^T S)^{-1} S^T I$$

3-D 重建回顾.

1 epipolar geometry / 双目视觉.

找对应的patch

得到深度图.

2. multi view.

CNN.

3. shape from shading

4. shape from texture. 纹理

5 structured light

主动照明 投射 texture.

6 Monocular Depth Estimation

CNN.

volumetric fusion.

1 Depth-to-SDF

2. Volumetric Fusion

3. Mesh. Extraction.

1. distance to surface ≈ distance along ray.

// assumption.

2. average. SDF. 加权平均

改变了隐式表面.

SDF: 各个体素离最近表面距离

SDF Fusion calculates the **weighted average** per voxel:

$$D(\mathbf{x}) = \frac{\sum w_i(\mathbf{x})\, d_i(\mathbf{x})}{\sum w_i(\mathbf{x})}$$

$$W(\mathbf{x}) = \sum w_i(\mathbf{x})$$

- $w_i(\mathbf{x}), d_i(\mathbf{x})$: weight and distance along ray at voxel $\mathbf{x}$ for camera i
- $W(\mathbf{x}), D(\mathbf{x})$: fused weight and distance at voxel $\mathbf{x}$

This can be conveniently expressed with an **incremental update rule:** (ex

$$D_{i+1}(\mathbf{x}) = \frac{W_i(\mathbf{x})\, D_i(\mathbf{x}) + w_{i+1}(\mathbf{x})\, d_{i+1}(\mathbf{x})}{W_i(\mathbf{x}) + w_{i+1}(\mathbf{x})}$$

$$W_{i+1}(\mathbf{x}) = W_i(\mathbf{x}) + w_{i+1}(\mathbf{x})$$

weight：表面背后的位置 weight↓. 递减

原因：表面后的位置只是推测

噪声随距离增加，会更大

less certain ray 权重小

$$D(x) = \frac{\sum w_i(x)\, d_i(x)}{\sum w_i(x)}.$$

D实际上满足.

$$D^* = \underset{D}{\arg\min} \sum w_i (d_i - D)^2$$

3. Marching cubes.

extension.

Dynamic Fusion.