Ex 1.

Exercise 1. Image Formation

Homogeneous Coordinates

a) for point $\tilde{x}$ at $\tilde{l}_1$, there exists $\tilde{x} \perp \tilde{l}_1$

similarly. $\tilde{x} \perp \tilde{l}_2$

$$\begin{cases} \tilde{x} \perp \tilde{l}_1 \\ \tilde{x} \perp \tilde{l}_2 \end{cases} \Longleftrightarrow \tilde{x} = \tilde{l}_1 \times \tilde{l}_2$$

b) if $\tilde{x}_1$ is at $\tilde{l}$, the $\tilde{x}_1 \perp \tilde{l}$

similarly $\tilde{x}_2 \perp \tilde{l}$.

$$\begin{cases} \tilde{x}_1 \perp \tilde{l} \\ \tilde{x}_2 \perp \tilde{l} \end{cases} \Longleftrightarrow \tilde{l} = \tilde{x}_1 \times \tilde{x}_2$$

c) $$\begin{cases} x+y+3=0 \\ -x-2y+7=0 \end{cases} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -13 \\ 10 \end{pmatrix}$$

$$\tilde{l}_1 = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \quad \tilde{l}_2 = \begin{pmatrix} -1 \\ -2 \\ 7 \end{pmatrix}.$$

$$\tilde{l}_1 \times \tilde{l}_2 = \begin{pmatrix} 13 \\ -10 \\ -1 \end{pmatrix} = \begin{pmatrix} -13 \\ 10 \\ 1 \end{pmatrix} = \tilde{x}$$

remain the same.

d) $$\tilde{l} = \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 15 \end{pmatrix}$$

e) $$\tilde{l} = \begin{pmatrix} \frac{2}{\sqrt{29}} \\ \frac{5}{\sqrt{29}} \\ \frac{1}{5} \end{pmatrix} \quad \|d\| = \frac{1}{5}$$

Transformations

a). $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$$\tilde{H} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

b). $T = \begin{pmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \end{pmatrix}$

$$E(T) = \sum_{i=1}^{N} \| T x_i - y_i \|_2^2$$

$$= \sum_{i=1}^{N} \left\| \begin{pmatrix} x_1^i - y_1^i + t_1 \\ x_2^i - y_2^i + t_2 \end{pmatrix} \right\|_2^2$$

$$= \sum_{i=1}^{N} (x_1^i - y_1^i + t_1)^2 + (x_2^i - y_2^i + t_2)^2$$

$$\frac{\partial E}{\partial t_1} = \sum_{i=1}^{N} 2(x_1^i - y_1^i + t_1) = 0$$

$$t_1 = \frac{\sum_{i=1}^{N} y_1^i - x_1^i}{N}$$

similarly $t_2 = \dfrac{\sum_{i=1}^{N} y_2^i - x_2^i}{N}$

c) $t_1 = 2$

$t_2 = -4$

$$T = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \end{pmatrix}.$$

Camera projections

a) $$\tilde{P} = \begin{pmatrix} K & 0 \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix}.$$

$$= \begin{pmatrix} 100 & 0 & 25 & 0 \\ 0 & 100 & 25 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 100 & 25 & 0 & 150 \\ 0 & 25 & -100 & 50 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

b) $$\tilde{x}_s = \begin{pmatrix} 25 \\ 50 \\ 1 \\ 0.25 \end{pmatrix}$$

$$\tilde{P} x_w = \tilde{x}_s.$$

$$\tilde{x}_w = \begin{pmatrix} -0.25 \\ 0.5 \\ -0.25 \\ 0.25 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \\ 1 \end{pmatrix}. \qquad x_w = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

c) $K = \begin{pmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{pmatrix}$

center $(0, 0, 15)$. $s = 20$.

$A_1(10, 10, 25)$. $A_5(-10, 10, 25)$

$A_2(10, 10, 5)$ $A_6(-10, 10, 5)$

$A_3(10, -10, 25)$. $A_7(-10, -10, 25)$

$A_4(10, -10, 5)$ $A_8(-10, -10, 5)$

$\tilde{x}_s = \tilde{K}\,\tilde{x}_c$

$\begin{pmatrix} 5 & 0 & 10 & 0 \\ 0 & 5 & 10 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$A_1' = \begin{pmatrix} 300 \\ 300 \\ 25 \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \end{pmatrix}$

$A_2' = \begin{pmatrix} 100 \\ 100 \\ 5 \end{pmatrix} = \begin{pmatrix} 20 \\ 20 \end{pmatrix}$

$A_3' = \begin{pmatrix} 300 \\ 200 \\ 25 \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \end{pmatrix}$

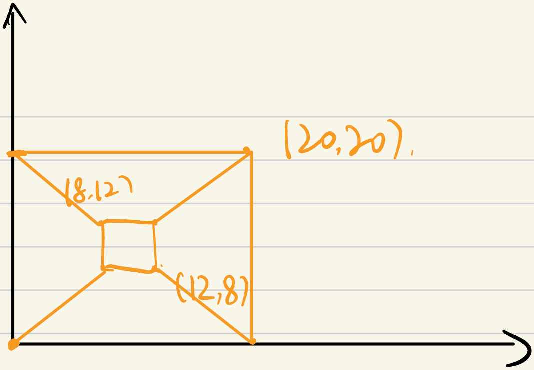
$A_4' = \begin{pmatrix} 100 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 20 \\ 0 \end{pmatrix}$

$A_5' = \begin{pmatrix} 200 \\ 300 \\ 25 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$

$A_6' = \begin{pmatrix} 0 \\ 100 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$

$A_7' = \begin{pmatrix} 200 \\ 200 \\ 25 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$

$A_8' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$



$$K = \begin{pmatrix} 90 & 0 & 10 & 0 \\ 0 & 90 & 10 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$(10, 0, 100)$

$s = 20$

$(10, 10, 100) \quad (-10, 10, 100)$

$(10, 10, 80) \quad (-10, 10, 80)$

$(10, -10, 100) \quad (-10, -10, 100)$

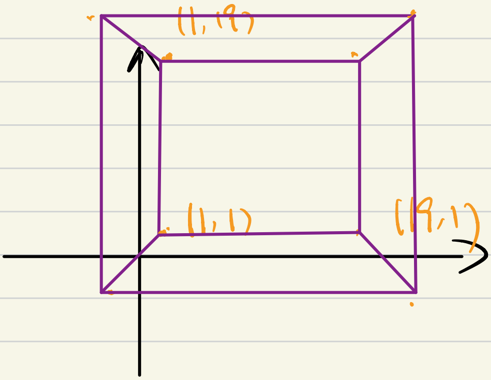
$(10, -10, 80) \quad (-10, -10, 80)$

$$\begin{pmatrix} 1900 \\ 1900 \\ 100 \end{pmatrix} = \begin{pmatrix} 19 \\ 19 \end{pmatrix} \qquad \begin{pmatrix} 100 \\ 1900 \\ 100 \end{pmatrix} = \begin{pmatrix} 1 \\ 19 \end{pmatrix}$$

$$\begin{pmatrix} 1700 \\ 1700 \\ 80 \end{pmatrix} = \begin{pmatrix} \frac{85}{4} \\ \frac{85}{4} \end{pmatrix} \qquad \begin{pmatrix} -100 \\ 1700 \\ 80 \end{pmatrix} = \begin{pmatrix} -\frac{25}{2} \\ \frac{85}{4} \end{pmatrix}$$

$$\begin{pmatrix} 1900 \\ 100 \\ 100 \end{pmatrix} = \begin{pmatrix} 19 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 100 \\ 100 \\ 100 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1700 \\ -100 \\ 80 \end{pmatrix} = \begin{pmatrix} \frac{85}{4} \\ -\frac{25}{2} \end{pmatrix} \qquad \begin{pmatrix} -100 \\ -100 \\ 80 \end{pmatrix} = \begin{pmatrix} -\frac{25}{2} \\ -\frac{25}{2} \end{pmatrix}$$

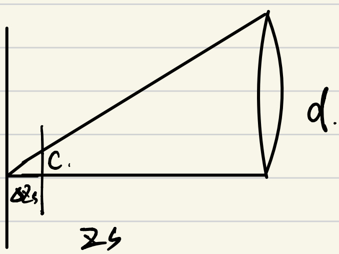


when f is bigger and distance is longer.

photometric image formation

$$\frac{1}{z_s} + \frac{1}{z_c} = \frac{1}{f}$$

$z_c = 5100$ mm



$$\frac{c}{d} = \frac{\Delta z_s}{z_s} \qquad N = \frac{f}{d}.$$

$$c = \frac{\Delta z_s}{z_s} d = \frac{\Delta z_s}{z_s} \frac{f}{N}.$$

$f = 35mm.$

$N = 1.4$

$z_s = 40 mm$

$\Delta z_s = 0.1mm \;/\; 0.03 mm$

$C = 0.0625mm \;/\; 0.01785mm$

$$l = \frac{\sqrt{64 mm^2}}{400} = \frac{8mm}{400} = 0.02 mm.$$

Δz_s is $0.1mm \Rightarrow$ blur.

Δz_s is $0.03mm \Rightarrow$ sharp.