Ex 2

Epipolar Geometry

a). $\tilde{E} = [\bar{t}]_\times R$ $\quad \tilde{x}_1 = \bar{x}_1 \quad \tilde{x}_2 = \bar{x}_2$

$$\tilde{E}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}.$$

$$l_2 = \tilde{E}_1 \tilde{X}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ y \end{pmatrix}.$$

direction. $(0, -1)$.

$e_2^T l_2 = 0$.

epipole is ideal point.

$$\tilde{E}_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\tilde{l}_2 = \tilde{E}_2 \tilde{X}_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -x \end{pmatrix}.$$

direction $(1, 0)$

epipole is ideal point.

$$\tilde{E}_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{l}_2 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}.$$

epipole. $(0, 0, 1)$.

b) $\tilde{x}_1^T \tilde{E}^T \tilde{x}_2 = 0$

$\tilde{E}^T = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

$\tilde{l}_1 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} y' \\ -x' \\ 0 \end{pmatrix}$.

$\tilde{e}_1 = (0, 0, 1)$.

the base line is parallel with z-axis

c) $\tilde{x}_2^T \tilde{E} \tilde{x}_1 = 0$.

$\bar{x}_2 (K^{-1})^T \tilde{E} K^{-1} \bar{x}_1 = 0$.

$\tilde{F} = (K^{-1})^T \tilde{E} K^{-1}. = \tilde{E}$

$(K^{-1})^T \tilde{E} = \tilde{E} K$.

$\tilde{E} = K^T \tilde{E} K$.

obviously, when $K = I$.

triangulation.

$$P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

$$x_1^s = P_1 x_1^w \qquad x_1^s = \left(\tfrac{1}{4} \ \ \tfrac{1}{2} \ \ 1\right)$$

$$P_2 = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 & -3 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

$$x_2^s = P_2 x_2^w \qquad x_2^s = \left(-\tfrac{1}{5} \ \ \tfrac{1}{5} \ \ 1\right)$$

$$\begin{pmatrix} x_i^s \tilde{P}_{i3}^T - \tilde{P}_{i1}^T \\ y_i^s \tilde{P}_{i3}^T - \tilde{P}_{i2}^T \end{pmatrix} \tilde{x}_w = 0$$

$$\begin{pmatrix} -1 & 0 & \tfrac{1}{4} & 0 \\ 0 & -1 & \tfrac{1}{2} & 0 \\ 2 & 0 & -\tfrac{6}{5} & \tfrac{14}{5} \\ 0 & 2 & -\tfrac{4}{5} & -\tfrac{4}{5} \end{pmatrix} \tilde{x}_w = 0$$

$$\tilde{x}_w = \begin{pmatrix} 1 \\ 2 \\ 4 \\ 1 \end{pmatrix}.$$

stereo vision.

a). $$Z = \frac{fb}{d}$$

$$Z' = \frac{fb}{d - \Delta d}$$

$$\Delta Z = \frac{fb}{d - \Delta d} - \frac{fb}{d}$$

$$= \frac{\Delta d \cdot f \cdot b}{d^2 - d \cdot \Delta d}$$

$$\sim \frac{\Delta d \cdot f \cdot b}{d^2} = \frac{\Delta d \, Z^2}{f \cdot b}$$

b). increase focal length and distance between two cameras.

disadvantages: disobey the hypothesis of fronto-parallel.

Block Matching.

a) $$ZNCC(x,y,d) = \frac{(W_L(x,y) - \overline{W_L}(x,y))^T \cdot (W_R(x-d,y) - \overline{W_R}(x-d,y))}{\|W_L(x,y) - \overline{W_L}(x,y)\| \cdot \|W_R(x-d,y) - \overline{W_R}(x-d,y)\|}$$

$$W_i' = \alpha_i W_i + 1 \cdot \beta_i$$

$$ZNCC(x,y,d)' = \frac{\alpha_1 \alpha_2 \cdot (W_L(x,y) - \overline{W_L}(x,y))^T \cdot (W_R(x-d,y) - \overline{W_R}(x-d,y))}{\alpha_1 \|W_L(x,y) - \overline{W_L}(x,y)\| \cdot \alpha_2 \|W_R(x-d,y) - \overline{W_R}(x-d,y)\|}$$

$$= ZNCC(x,y,d)$$

b). $SSD(x,y,d) = \|W_L(x,y) - W_R(x-d,y)\|^2$

$d=0$

$$SSD(3,5,0) = \left\| \begin{bmatrix} 10 \\ 10 \\ 10 \\ 5 \\ 5 \\ 5 \\ 6 \\ 6 \\ 6 \end{bmatrix} - \begin{bmatrix} 5 \\ 5 \\ 5 \\ 6 \\ 6 \\ 6 \\ 7 \\ 7 \\ 7 \end{bmatrix} \right\| = 81$$

similarly.

$d=1$. $SSD(3,5,1) = 108$.

$d=2$ $SSD(3,5,2) = 6$.

according to SSD algorithm. $d=2$ is the optimal solution

c) they all pass the test

in the situation of red, it fails.