$$\nabla^2 P - \frac{1}{v^2}\cdot\frac{\partial^2 P}{\partial t^2} = S.$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = S$$

$$\frac{\partial^2 P(x,y,z,t)}{\partial t^2} = \frac{1}{(\Delta t)^2}\left(P^{t-1}_{x,y,z} - 2P^{t}_{x,y,z} + P^{t+1}_{x,y,z}\right).$$

$$\frac{\partial^2 P}{\partial x^2} = \frac{1}{\Delta x^2}\sum_{i=-2}^{i=2} a_i \cdot P^{t}_{x+i,y,z}$$

$$P_{x} = P_x$$

$$P_{x+1} = P_x + \frac{\partial P_x}{\partial x}\cdot\Delta x + \frac{1}{2}\cdot\frac{\partial^2 P_x}{\partial x^2}\cdot(\Delta x)^2 + \frac{1}{6}\frac{\partial^3 P_x}{\partial x^3}(\Delta x)^3 + \frac{1}{24}\frac{\partial^4 P_x}{\partial x^4}(\Delta x)^4$$

$$P_{x-1} = P_x - \frac{\partial P_x}{\partial x}\Delta x + \frac{1}{2}\frac{\partial^2 P_x}{\partial x^2}(\Delta x)^2 - \frac{1}{6}\frac{\partial^3 P_x}{\partial x^3}(\Delta x)^3 + \frac{1}{24}\frac{\partial^4 P_x}{\partial x^4}(\Delta x)^4$$

$$P_{x+2} = P_x + 2\frac{\partial P_x}{\partial x}\cdot\Delta x + 2\frac{\partial^2 P_x}{\partial x^2}\cdot(\Delta x)^2 + \frac{4}{3}\cdot\frac{\partial^3 P_x}{\partial x^3}(\Delta x)^3 + \frac{2}{3}\frac{P_x}{\partial x^4}(\Delta x)^4$$

$$P_{x-2} = P_x - 2\frac{\partial P_x}{\partial x}\cdot\Delta x + 2\frac{\partial^2 P_x}{\partial x^2}\cdot(\Delta x)^2 - \frac{4}{3}\frac{\partial^3 P_x}{\partial x^3}(\Delta x)^3 + \frac{2}{3}\frac{P_x}{\partial x^4}(\Delta x)^4.$$

$$\begin{array}{ccccc} a_{-2}, & a_{-1}, & a_0, & a_1, & a_2 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \\ 2 & \frac{1}{2} & 0 & \frac{1}{2} & 2 \\ -\frac{4}{3} & -\frac{1}{6} & 0 & \frac{1}{6} & \frac{4}{3} \\ \frac{2}{3} & \frac{1}{24} & 0 & \frac{1}{24} & \frac{2}{3} \end{bmatrix} \cdot \begin{pmatrix} a_{-2} \\ a_{-1} \\ a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow a_2 = a_{-2} = -0.0833333$$

$$a_1 = a_{-1} = 1.3333333$$
$$a_0 = -2.5$$

由于空间结构的对称性，

对于 $\frac{\partial^2 P}{\partial y^2}$ 与 $\frac{\partial^2 P}{\partial z^2}$ 有相同的解.

因此：
$$\frac{\partial^2 P(x,y,z,t)}{\partial x^2} = \frac{1}{(\Delta x)^2}\cdot\sum_{i=-2}^{2} a_i \cdot P^t_{x+i,y,z}$$
$$\frac{\partial^2 P(x,y,z,t)}{\partial y^2} = \frac{1}{(\Delta y)^2}\cdot\sum_{i=-2}^{2} a_i P^t_{x,y+i,z}$$
$$\frac{\partial^2 P(x,y,z,t)}{\partial z^2} = \frac{1}{(\Delta z)^2}\cdot\sum_{i=-2}^{2} a_i P^t_{x,y,z+i}, \quad \text{where } \Delta x = \Delta y = \Delta z = h.$$

因此：
$$\nabla^2 P - \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = S$$

$$\Rightarrow \frac{1}{h^2}\cdot\sum_{i=-2}^{2} a_i \left(P^t_{x+i,y,z} + P^t_{x,y+i,z} + P^t_{x,y,z+i}\right) - \frac{1}{v^2}\cdot\frac{1}{(\Delta t)^2}\cdot\left(P^{t-1}_{x,y,z} - 2P^t_{x,y,z} + P^{t+1}_{x,y,z}\right) = S^t_{x,y,z}$$

$$\Rightarrow \frac{v^2\cdot(\Delta t)^2}{h^2}\cdot\sum_{i=-2}^{2} a_i \left(P^t_{x+i,y,z} + P^t_{x,y+i,z} + P^t_{x,y,z+i}\right) - \left(P^{t-1}_{x,y,z} - 2P^t_{x,y,z}\right) - (v\Delta t)^2\cdot S^t_{x,y,z} = P^{t+1}_{x,y,z}$$

$$\Rightarrow P^{t+1}_{x,y,z} = \frac{v^2\cdot(\Delta t)^2}{h^2}\cdot\sum_{\substack{i=-2\\ i\neq 0}}^{2} a_i \left(P^t_{x+i,y,z} + P^t_{x,y+i,z} + P^t_{x,y,z+i}\right) + \left(\frac{v^2\cdot(\Delta t)^2}{h^2}\cdot 3a_0 + 2\right)\cdot P^t_{x,y,z} - P^{t-1}_{x,y,z} - v^2\cdot\Delta t^2\cdot S^t_{x,y,z}$$