(5) $y(1) = ?$

Usar Euler com $h = 0.1$

$$\begin{cases} y'' - 3y' + 2y = 0 \\ y'(0) = 0 \\ y(0) = -1 \end{cases}$$

Para resolver por Euler, precisamos transformar essa equação de 2ª ordem em 2 equações de primeira ordem e resolver o sistema. Fazendo:

$$w = y'$$

Temos:

$$\begin{cases} y' = w \\ w' - 3w + 2y = 0 \\ y(0) = -1 \\ w(0) = 0 \end{cases} \Rightarrow \begin{cases} y' = w \\ w' = -2y + 3w \end{cases}$$

$$\begin{cases} y(0) = -1 \\ w(0) = 0 \end{cases}$$

Que pode ser transformado na forma $Y' = AY + X$

$$\underbrace{\begin{pmatrix} y' \\ w' \end{pmatrix}}_{Y'} = \underbrace{\begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} y \\ w \end{pmatrix}}_{Y} + \underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{X}$$

$$\underbrace{\begin{pmatrix} y(0) \\ w(0) \end{pmatrix}}_{Y(0)} = \underbrace{\begin{pmatrix} -1 \\ 0 \end{pmatrix}}_{Y_0}$$

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Tendo:

$$\begin{cases} y' = Ay \\ y(0) = y_0 \end{cases}$$

Podemos aplicar o método de Euler cal que:

$$y_{n+1} = y_N + h A_n y_n$$

$$\begin{pmatrix} y_{n+1} \\ w_{n+1} \end{pmatrix} = \begin{pmatrix} y_n \\ w_n \end{pmatrix} + h \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} y_n \\ w_n \end{pmatrix}$$

$$\begin{pmatrix} y(x+h) \\ w(x+h) \end{pmatrix} = \begin{pmatrix} y(x) \\ w(x) \end{pmatrix} + h \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} y(x) \\ w(x) \end{pmatrix}$$

Começando:

$$\begin{pmatrix} y(0.1) \\ w(0.1) \end{pmatrix} = \begin{pmatrix} y(0) \\ w(0) \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} y(0) \\ w(0) \end{pmatrix}$$

$$\begin{pmatrix} y(0.1) \\ w(0.1) \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 0 \end{pmatrix} + 0.1 \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0.2 \end{pmatrix}$$

$$\begin{pmatrix} y(0.1) \\ w(0.1) \end{pmatrix} = \begin{pmatrix} -1 \\ 0.2 \end{pmatrix}$$

$$\begin{pmatrix} y(0.2) \\ w(0.2) \end{pmatrix} = \begin{pmatrix} -1 \\ 0.2 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -1 \\ 0.2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 0.2 \end{pmatrix} + 0.1 \begin{pmatrix} 0.2 \\ 2.6 \end{pmatrix} = \begin{pmatrix} -1 \\ 0.2 \end{pmatrix} + \begin{pmatrix} 0.02 \\ 0.26 \end{pmatrix} = \begin{pmatrix} -0.98 \\ 0.46 \end{pmatrix}$$

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$$\begin{pmatrix} y(0.3) \\ w(0.3) \end{pmatrix} = \begin{pmatrix} -0.98 \\ 0.46 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -0.98 \\ 0.46 \end{pmatrix}$$

$$= \begin{pmatrix} -0.98 \\ 0.46 \end{pmatrix} + 0.1 \begin{pmatrix} 0.46 \\ 3.34 \end{pmatrix} = \begin{pmatrix} -0.98 \\ 0.46 \end{pmatrix} + \begin{pmatrix} 0.046 \\ 0.334 \end{pmatrix}$$

$$= \begin{pmatrix} -0.934 \\ 0.794 \end{pmatrix}$$

$$\begin{pmatrix} y(0.4) \\ w(0.4) \end{pmatrix} = \begin{pmatrix} -0.934 \\ 0.794 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -0.934 \\ 0.794 \end{pmatrix} = \begin{pmatrix} -0.934 \\ 0.794 \end{pmatrix} + 0.1 \begin{pmatrix} 0.794 \\ 4.25 \end{pmatrix}$$

$$= \begin{pmatrix} -0.934 \\ 0.794 \end{pmatrix} + \begin{pmatrix} 0.0794 \\ 0.425 \end{pmatrix} = \begin{pmatrix} -0.8546 \\ 1.219 \end{pmatrix}$$

$$\begin{pmatrix} y(0.5) \\ w(0.5) \end{pmatrix} = \begin{pmatrix} -0.8546 \\ 1.219 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -0.8546 \\ 1.219 \end{pmatrix}$$

$$= \begin{pmatrix} -0.8546 \\ 1.219 \end{pmatrix} + 0.1 \begin{pmatrix} 1.219 \\ 5.3662 \end{pmatrix} = \begin{pmatrix} -0.7327 \\ 1.75562 \end{pmatrix}$$

$$\begin{pmatrix} y(0.6) \\ w(0.6) \end{pmatrix} = \begin{pmatrix} -0.7327 \\ 1.75562 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -0.7327 \\ 1.75562 \end{pmatrix}$$

$$= \begin{pmatrix} -0.7327 \\ 1.75562 \end{pmatrix} + 0.1 \begin{pmatrix} 1.75562 \\ 6.73226 \end{pmatrix} = \begin{pmatrix} -0.557138 \\ 2.428846 \end{pmatrix}$$

$$\begin{pmatrix} y(0.7) \\ w(0.7) \end{pmatrix} = \begin{pmatrix} -0.56 \\ 2.43 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -0.56 \\ 2.43 \end{pmatrix} = \begin{pmatrix} -0.56 \\ 2.43 \end{pmatrix} + 0.1 \begin{pmatrix} 2.43 \\ 8.42 \end{pmatrix}$$

$$= \begin{pmatrix} -0.32 \\ 3.27 \end{pmatrix}$$

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$$\begin{pmatrix} y(0.8) \\ w(0.8) \end{pmatrix} \approx \begin{pmatrix} -0.32 \\ 3.27 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -0.32 \\ 3.27 \end{pmatrix} = \begin{pmatrix} -0.32 \\ 3.27 \end{pmatrix} + 0.1 \begin{pmatrix} 3.27 \\ 10.45 \end{pmatrix}$$

$$= \begin{pmatrix} 0.01 \\ 4.32 \end{pmatrix}$$

$$\begin{pmatrix} y(0.9) \\ w(0.9) \end{pmatrix} \approx \begin{pmatrix} 0.01 \\ 4.32 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 0.01 \\ 4.32 \end{pmatrix} = \begin{pmatrix} 0.01 \\ 4.32 \end{pmatrix} + 0.1 \begin{pmatrix} 4.32 \\ 12.94 \end{pmatrix}$$

$$= \begin{pmatrix} 0.44 \\ 5.61 \end{pmatrix}$$

$$\begin{pmatrix} y(1) \\ w(1) \end{pmatrix} \approx \begin{pmatrix} 0.44 \\ 5.61 \end{pmatrix} + 0.1 \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 0.44 \\ 5.61 \end{pmatrix} = \begin{pmatrix} 0.44 \\ 5.61 \end{pmatrix} + 0.1 \begin{pmatrix} 5.61 \\ 15.95 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 7.21 \end{pmatrix}$$

Also, $\boxed{y(1) \approx 1}$

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