

# Time-varying Correlations Between Sectors of the All Share Index

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## 1. Introduction

Diversification is the cornerstone of modern day finance, thus understanding the co-movements of assets is essential for any diversification or hedging strategy. Katzke (2013) highlights that investors have a “home-bias”, where local assets are favoured over international assets. This can lead to risks of underdiversification when there is homogeneity in return movements between local sectors and assets. This project, therefore, aims to identify and investigate the co-movements of three of the largest sectors in South Africa.

The use of Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MV-GARCH) to obtain dynamic conditional correlations has become more prevalent in finance and portfolio theory literature. It has been used for many different purposes. Ho & Tsui (2004) & Katzke (2013) employ MV-GARCH models to investigate diversification opportunities and threats through inter-sector correlations. Fakhfekh, Jeribi, Ghorbel & Hachicha (2021) & Ali, Raza, Vo & Le (2022) instead look at hedging strategies utilising a vast array of different financial assets and instruments.

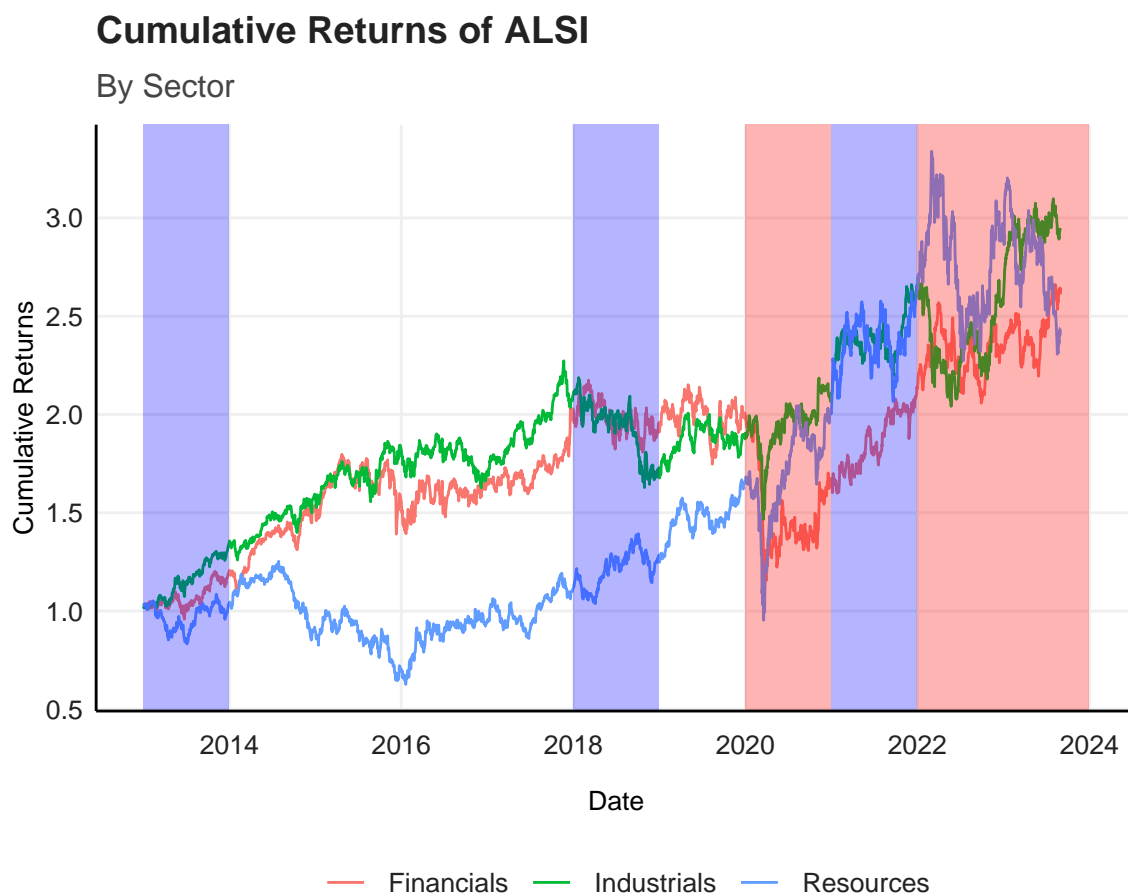
In this paper I extract three sectors’ returns from the All Share J203 in order to derive the time-varying correlations between the three sectors and the United States Dollar to South African Rand exchange rate. The rest of the paper is structured as follows, section two describes the data, section three describes the methods and statistical tests used, section four presents the final results and lastly, section five concludes.

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## 2. Data

The data used to get the sector returns is the All Share (ALSI) J203 index. This index represents 99% of the full market cap value of all eligible securities listed on the Main Board of the Johannesburg Stock Exchange (JSE) ([JSE, 2024](#)). Sector returns are then created for the financial, industrial and resource sectors. All stocks categorised in each of these sectors is reweighted in order to create a index for each sector. The cumulative returns are presented in figure 2.1.

Other data that was used is the United States Dollar (USD) to South African Rand (ZAR) exchange rate as the depreciation or appreciation of the domestic currency compared to the Dollar is an indicator of changes in the global economy. Lastly the historical repurchase rate (REPO) was obtained from the South African Reserve Bank (SARB).



Note: Blue (red) shaded areas represent years of low (high) volatility of the REPO rate.

Figure 2.1: Cumulative Returns of ALSI

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### 3. Methodology

I begin by conducting multiple tests for autoregressive conditional heteroskedasticity (ARCH) effects. I do this through graphing the returns, absolute returns and squared returns which can be found in figures 6.1, 6.2 and 6.3. These figures highlights that the return series of all three sectors show strong first order persistence and potential periods of second order persistence. Lastly, it appears that the series has a long memory in the second order process. To check for this I conduct more formal tests for ARCH effects.

#### 3.1. Tests for Autoregressive Conditional Heteroskedasticity (ARCH) effects.

The first formal test that I conduct is the plotting of the autocorrelation functions of the returns, absolute returns and squared returns for all sectors. Figures 6.4, 6.5 and 6.6 can be found in the appendix. These figures provide more evidence of conditional heteroskedasticity and long memory for all of the sectors. The last test is a formal Box-Ljung test where the null hypothesis is that there are no ARCH effects. Table 3.1 presents the results of the test and it is clear that the null hypothesis of no ARCH effects can be rejected at the 1% confidence level. This means that the conditional heteroskedasticity needs to be controlled for.

Table 3.1: Ljung-Box Test Results

	TestStatistic	PValue	Lag
Financials	2667.7971	0	12
Resources	1431.3349	0	12
Industrials	948.8227	0	12

#### 3.2. Univariate Model Selection

A univariate GARCH specification is necessary for the multivariate specification. There are a vast number of extentions to the original GARCH formulation, therefore to obtain the model that best fits the data I employ multiple information criteria to assess the potential model fits in the univariate case. Table 3.2 presents the results of the information criteria and the best model for each sector is different but on average the best fitting model across all sectors is eGARCH so that is the specification that will be used to estimate the dynamic conditional correlations.

Table 3.2: GARCH Model Comparison Results

	Sector	sGARCH	gjrGARCH	eGARCH	apARCH
Akaike	Financials	-5.872282	-5.881912	-5.881702	-5.884481
Bayes	Financials	-5.861235	-5.868656	-5.868446	-5.869016
Shibata	Financials	-5.872289	-5.881922	-5.881712	-5.884495
Hannan-Quinn	Financials	-5.868285	-5.877115	-5.876905	-5.878885
Akaike1	Resources	-5.466560	-5.479735	-5.475142	-5.479025
Bayes1	Resources	-5.455513	-5.466479	-5.461886	-5.463559
Shibata1	Resources	-5.466567	-5.479745	-5.475153	-5.479038
Hannan-Quinn1	Resources	-5.462562	-5.474938	-5.470345	-5.473428
Akaike2	Industrials	-6.246073	-6.267709	-6.270655	-6.261725
Bayes2	Industrials	-6.235026	-6.254453	-6.257399	-6.246260
Shibata2	Industrials	-6.246080	-6.267719	-6.270665	-6.261739
Hannan-Quinn2	Industrials	-6.242075	-6.262912	-6.265858	-6.256129

### 3.3. Multivariate Models

Now that I have the univariate model specifications I can now fit the multivariate model. I fit three different multivariate GARCH models: DCC, aDCC and GO-GARCH. This follows the literature as the comparison between these two or three models is common Ali *et al.* (2022). After fitting the DCC model I extract the model diagnostics that present multiple Portmanteau and rank based tests that assess whether there is serial correlation. Tsay (2013) provides an in depth discussion about these tests. The null hypothesis for all the tests are that there is no serial correlation and hence no conditional heteroskedasticity. Tsay (2013: 403) notes that  $Q_k(m)$  statistic works well when the distribution of innovations are normal but struggles when fatter tails are present. The robust statistic employs 5% trimming as a means to get a more robust statistic. The results presented in table 3.3 show that for all tests except for the robust version ( $Q_r^k(m)$ ) rejects the null hypothesis of no conditional heteroskedasticity whereas the robust version fails to reject.

Table 3.3: Portmanteau tests

$Q(m)$	<i>Rank – based test</i>	$Q_k(m)$	$Q_r^k(m)$
51.32 (0.000)	18.63 (0.045)	255.96 (0.000)	173.18 (0.225)
Note: P-values given in brackets.			

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## 4. Results

Figures 4.1, 4.2 and 4.3 present the time varying conditional correlations between the three sectors of interest as well as the USD/ZAR exchange rate. Figure 4.1 presents the standard dynamic conditional correlations (DCC). 4.2 presents the asymmetric dynamic conditional correlations (aDCC), which accounts for the asymmetry of positive and negative shocks. Lastly, figure 4.3 presents the time varying correlations from a Generalized Orthogonal GARCH (GO-GARCH) specification. I also employed stratification methods to get the most and least volatile periods of the REPO rate.

Across all of the specifications it is clear to see that during high periods of volatility in the REPO rate the correlations between the sectors is also more volatile. All specifications also highlight that there is large heterogeneity in the correlations over time. This serves as another motivating factor to not use time-invariant estimations of the correlations. The inter-sector correlations tend to move around 0.5 for both the DCC and aDCC with fluctuations within a band of 0.8 and 0. The correlation between the financial and industrial sectors is the highest followed by resources and industrials and then resources and financials. The correlations between the sectors and the exchange rate clearly show the financial sector is consistently negatively correlated with the exchange rate. For the other two sectors a positive correlation tends to be present during low volatility periods of the REPO and negative during high volatility periods.

There appears to be little difference between the DCC and aDCC specifications. When looking at GO-GARCH however, the correlations instead have much smaller fluctuations. Boswijk & Weide (2006: 21) state that the smaller fluctuations can be seen as positive or a negative depending on the case. In this case it helps to provide a more consistent measure. The most important finding is that there does not seem to be a trend in any of the correlations.

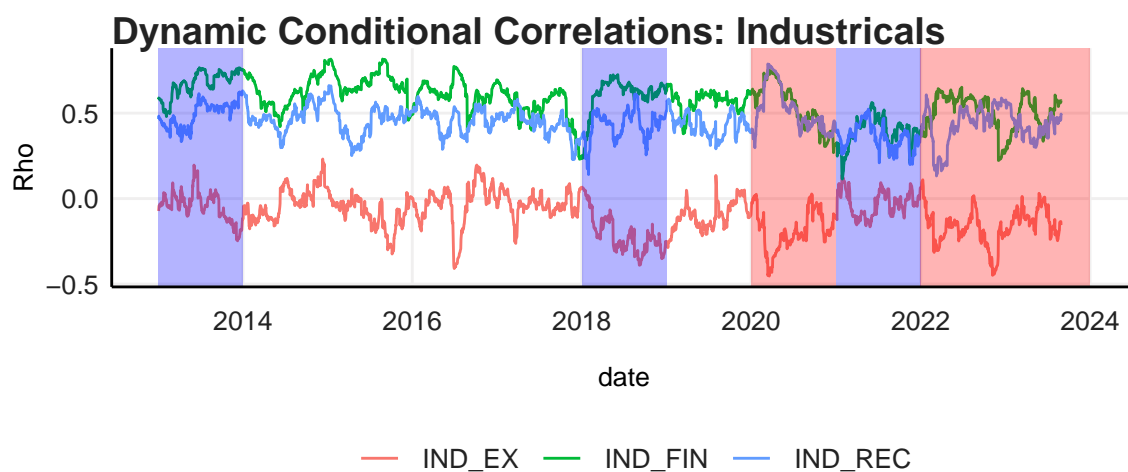
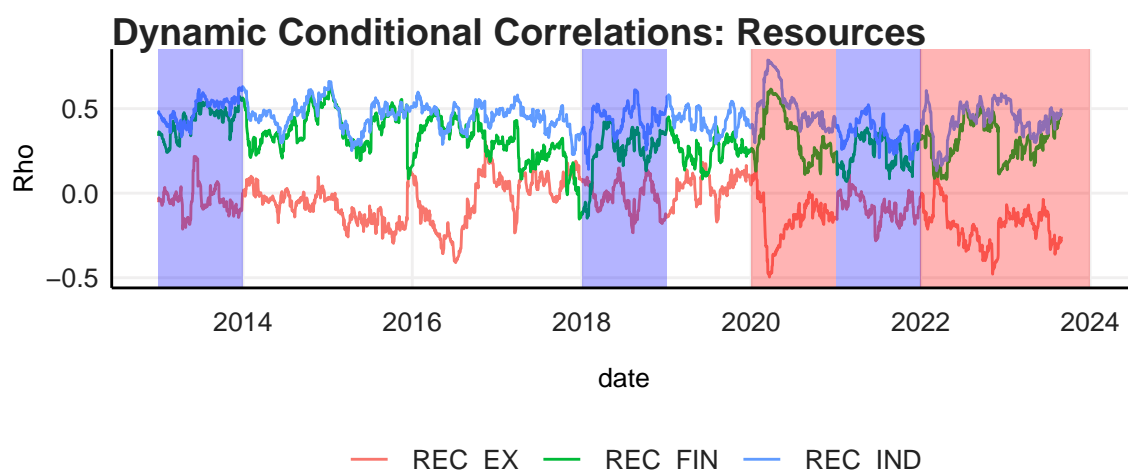
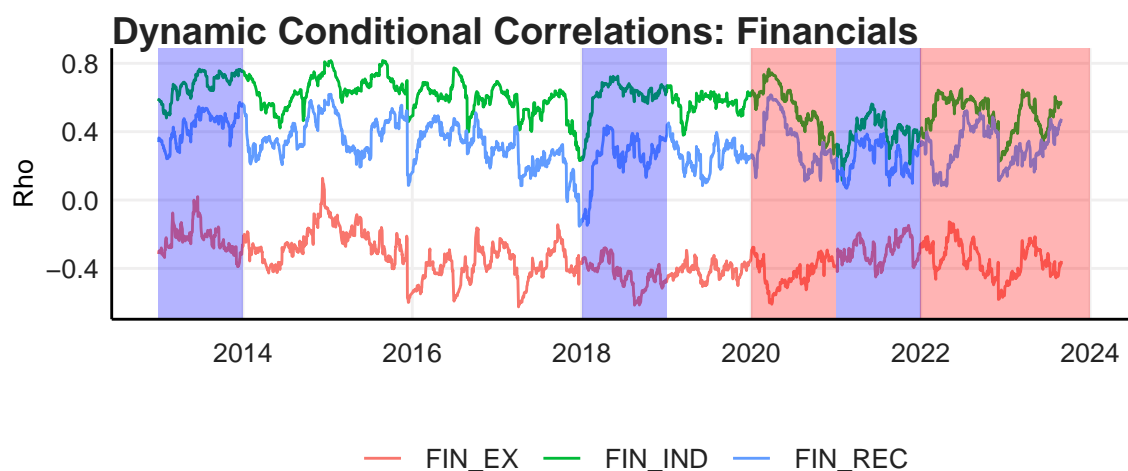


Figure 4.1: DCC

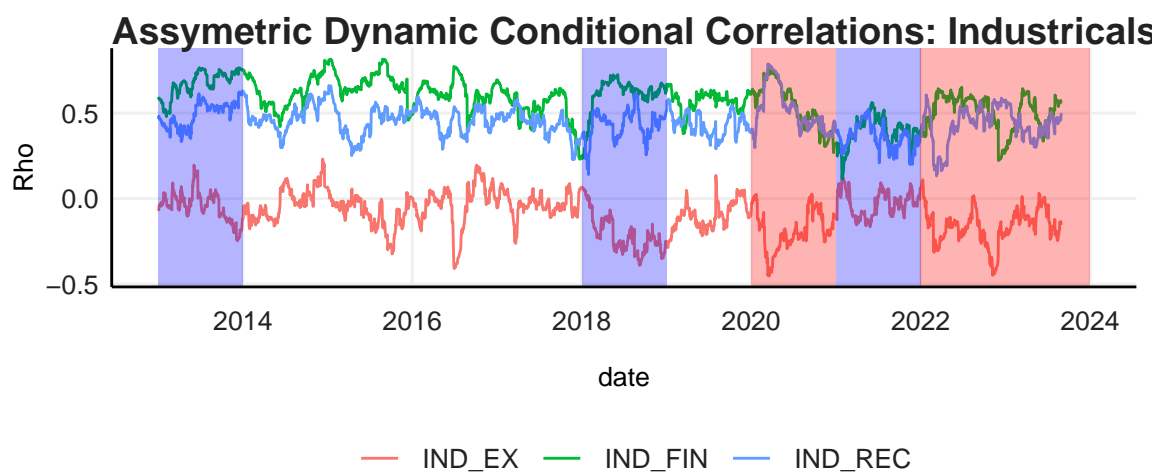
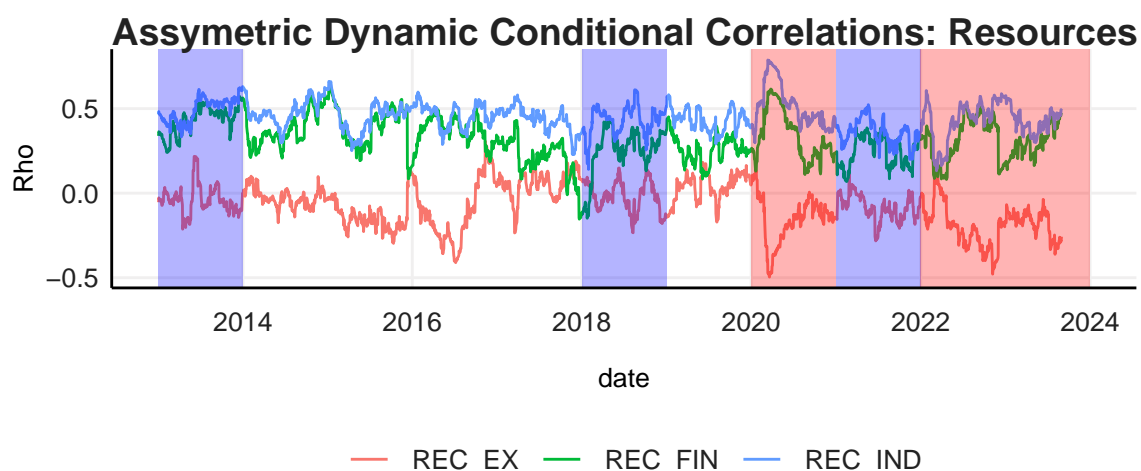
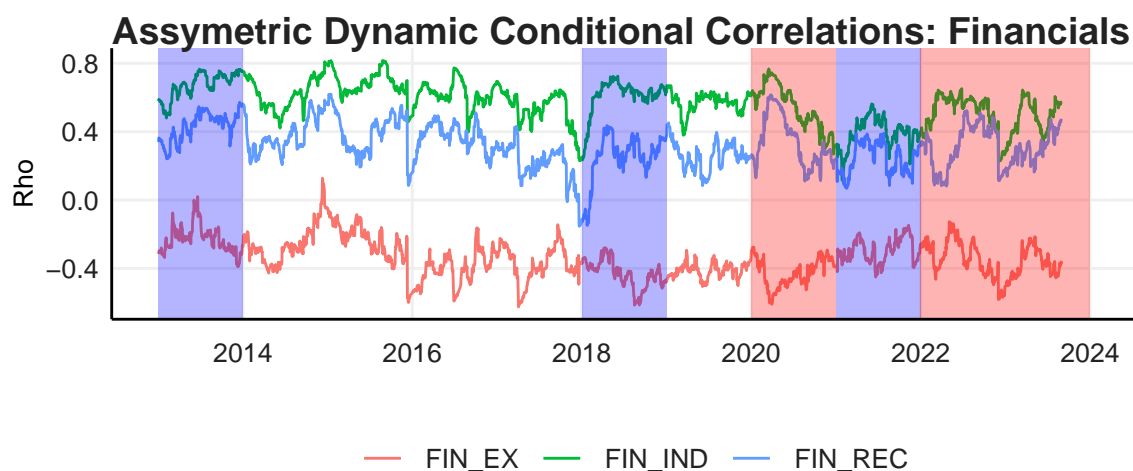


Figure 4.2: aDCC

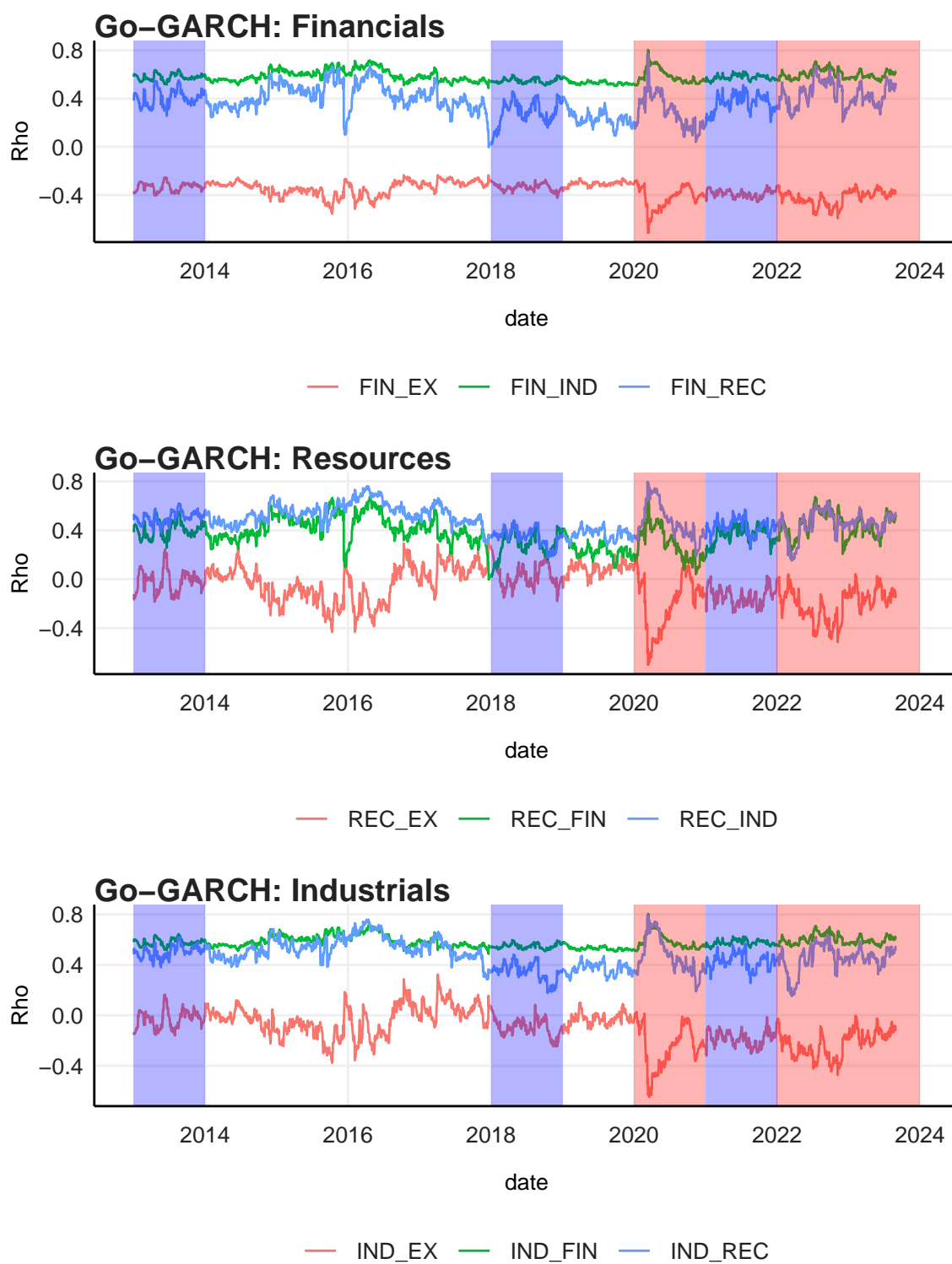


Figure 4.3: GO-GARCH



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## 5. Conclusion

This paper set out to estimate time-varying correlations of three sectors of the All Share index J203 as well as the USD/ZAR exchange rate. This was done by first fitting the multiple univariate GARCH models before evaluating the fit with multiple information criteria. Once the best univariate fit was selected it was used to fit the multivariate GARCH. The results show that the use of time-varying correlations are justified and if time-invariant correlations are used it will lead to an inefficiently diversified portfolio. Secondly, I found that the financial sector is the most susceptible to Rand depreciation while the other sectors correlations fluctuate around 0.

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## References

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## 6. Appendix

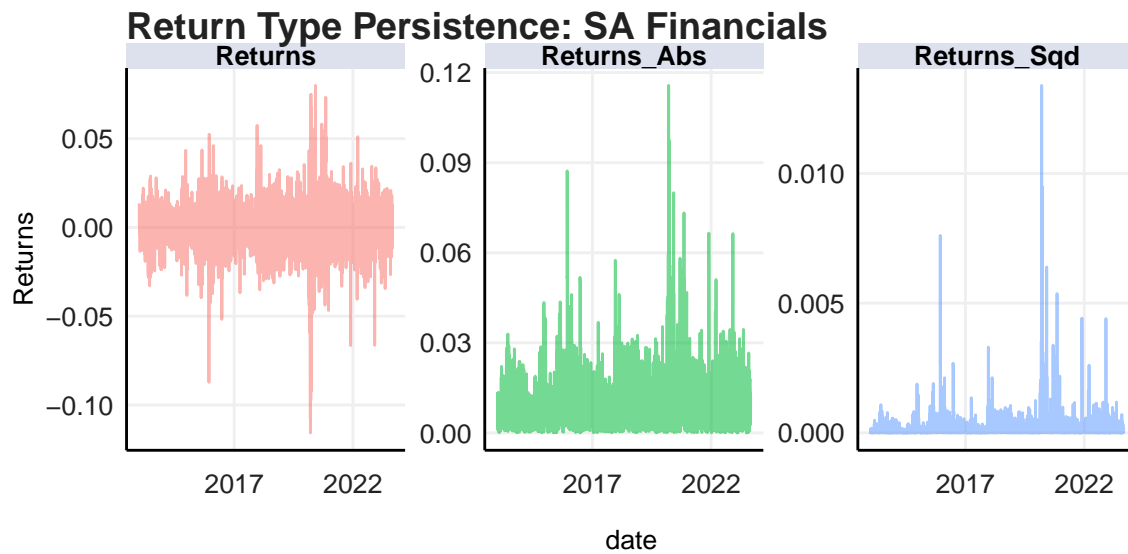


Figure 6.1: Return Persistence: Financials

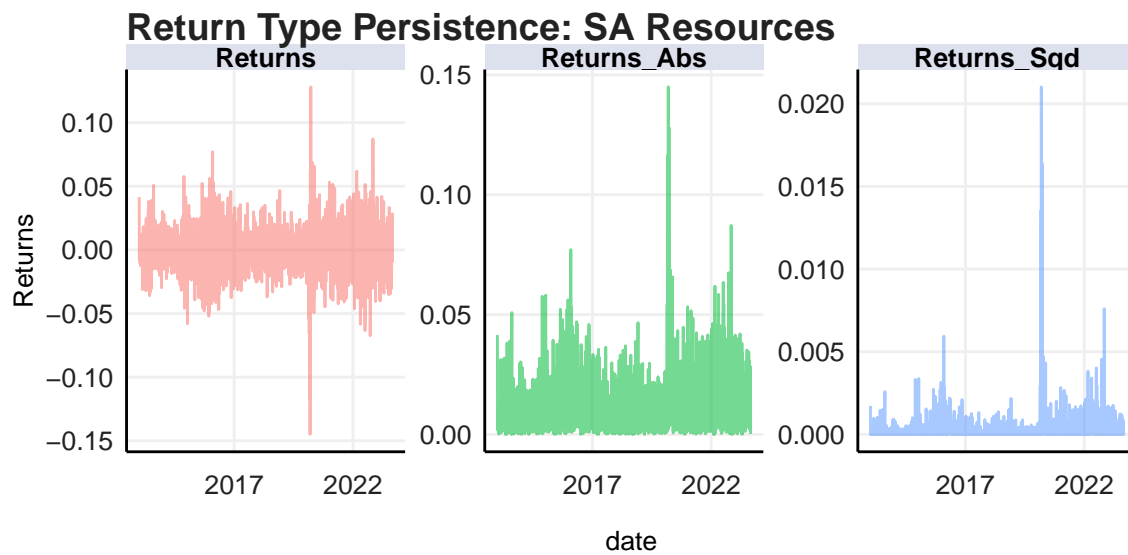


Figure 6.2: Return Persistence: Resources

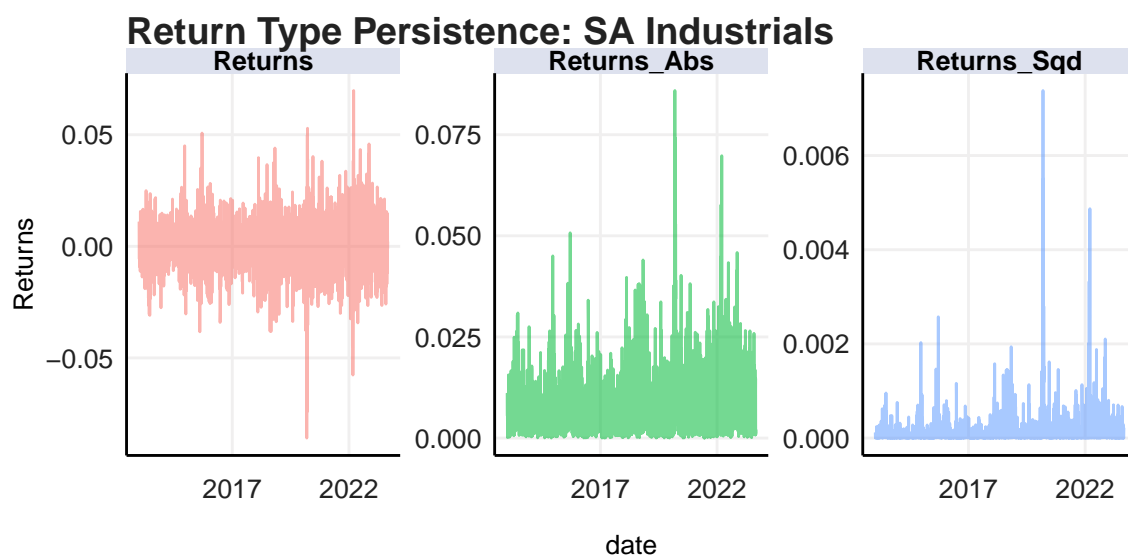


Figure 6.3: Return Persistence: Industrials

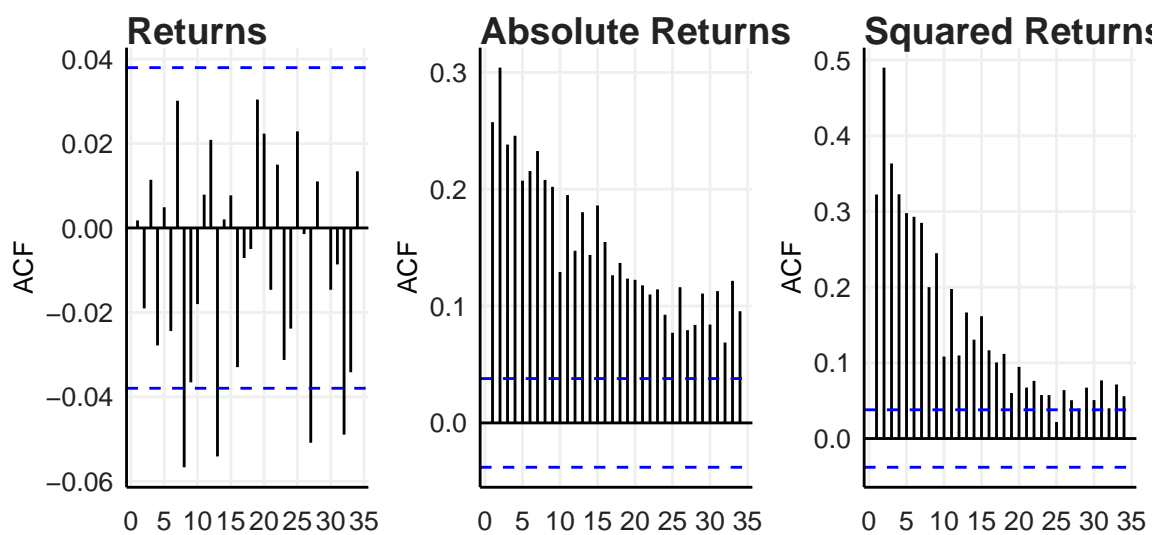


Figure 6.4: Autocorrelation Functions: Financials

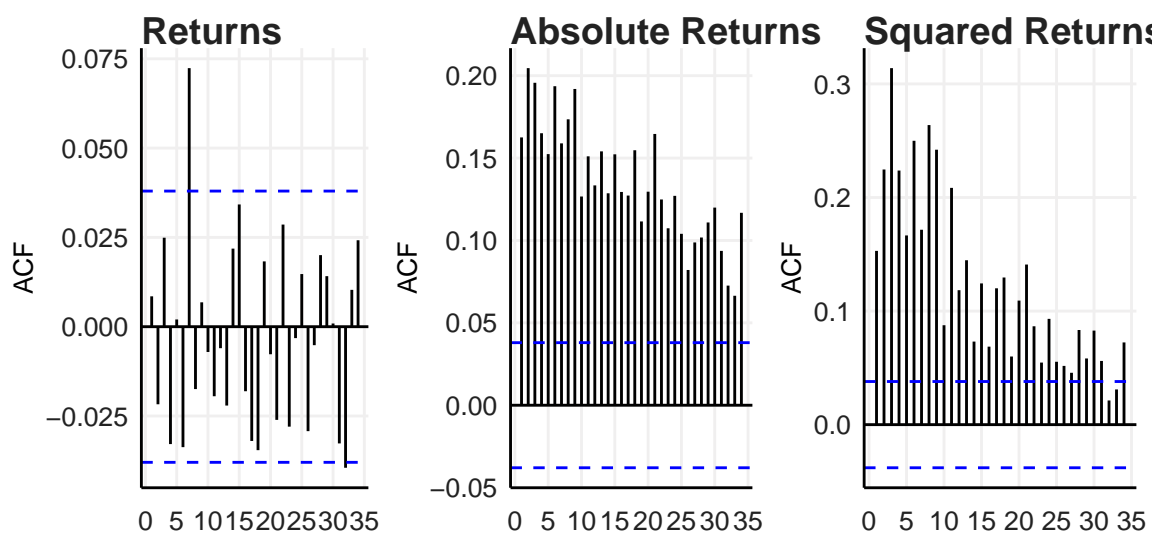


Figure 6.5: Autocorrelation Functions: Resources

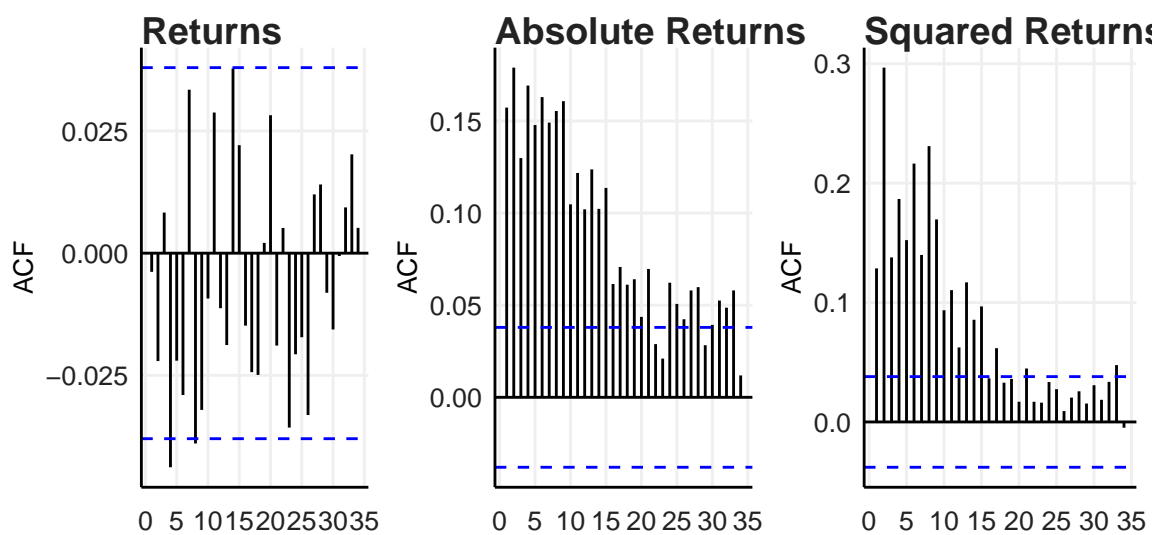


Figure 6.6: Autocorrelation Functions: Industrials