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Hedging stock market prices with WTI, Gold, VIX and cryptocurrencies: a comparison between DCC, ADCC and GO-GARCH models

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Abstract

Purpose – In a first place, the present paper is designed to examine the dynamic correlations persistent between five cryptocurrencies, WTI, Gold, VIX and four stock markets (SP500, FTSE, NIKKEI and MSCIEM). In a second place, it investigates the relevant optimal hedging strategy.

Design/methodology/approach – Empirically, the authors examine how WTI, Gold, VIX and five cryptocurrencies can be applicable to hedge the four stock markets. Three variants of multivariate GARCH models (DCC, ADCC and GO-GARCH) are implemented to estimate dynamic optimal hedge ratios.

Findings – The reached findings prove that both of the Bitcoin and Gold turn out to display remarkable hedging commodity features, while the other assets appear to demonstrate a rather noticeable disposition to act as diversifiers. Moreover, the results show that the VIX turns out to stand as the most effectively appropriate instrument, fit for hedging the stock market indices various related refits. Furthermore, the results prove that the hedging strategy instrument was indifferent for FTSE and NIKKEI stock while for the American and emerging markets, the hedging strategy was reversed from the pre-cryptocurrency crash to the during cryptocurrency crash period.

Originality/value – The first paper's empirical contribution lies in analyzing emerging cross-hedge ratios with financial assets and compare hedging effectiveness within the period of crash and the period before Bitcoin crash as well as the sensitivity of results to refits choose to compare between short term hedging strategy and long-term one.

Keywords Cryptocurrencies, Stock market indices, Commodities, VIX, Multivariate GARCH models, Optimal hedge ratios, Hedging effectiveness

Paper type Research paper

1. Introduction

Ever since the breakdown of Bretton Woods, gold no longer appears to demonstrate the same prominence on the international monetary system. Nevertheless, it continues to draw considerable attention from the part of investors, media and researchers. By virtue of the increasing financial markets associated uncertainty, the portfolio diversification procedure, as undertaken through the hedging process, turns out to gain an increasingly greater prominence. More particularly, ever since the global economic and financial crisis, relevant to

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the United States subprime mortgage market, which triggered in 2007, gold prices have begun to record intense increases while other assets appeared to display noticeable losses (Beckmann *et al.*, 2015). More recently, significant correlations among most of the assets types have increased remarkably. However, gold continues to be recognized as a zero-beta asset (McCown and Zimmerman, 2006), and has still been frequently considered uncorrelated with other assets. In this regard, Bentes (2016) indicated that gold returns shifted from long-range dependence in the pre-crisis period to a short-memory one throughout the crisis period. She also outlined that gold was used to act as a hedge at times of financial turbulence.

As for Dyhberg (2016), he stated that the global uncertainty surrounding the 2007 global financial crisis enhanced the emergence of the first decentralized cryptocurrency based on the block-chain technology dubbed Bitcoin and strengthened its popularity. Initially devised by Nakamoto (2008), the Bitcoin facilitated electronic payments among individuals without the need of having to go through a third party. Ever since its inception and launch in the different markets, the Bitcoin has been subject to intense challenges and opportunities for policymakers, consumers, entrepreneurs and economists alike. It is considered to differ from any other asset on the financial market, by creating new possibilities for stakeholders relevant to portfolio analysis, risk management and consumer sentiment analysis (Dyhberg, 2016). More often, the Bitcoin is compared to gold owing mainly to the diversity of similarities they have in common. Neither of them has a nationality, nor is it government controlled. They are mined by several independent operators and companies. Even though Gold bears some intrinsic value, it does not necessarily justify its current market value (Dyhrberg, 2016).

In most cases, the Bitcoin generally defined as a highly volatile asset (Selmi *et al.*, 2018). It has drawn considerable attention and constitutes a major issue or subject of discussion in the financial press and academia. Indeed, owing to its wide range acceptance as an investment device and the increasing importance it acquired, modeling the Bitcoin associated price volatility turns out to be highly important for effective investment decisions and risk management to take place (Katsiampa, 2017). In this respect, most of the studies conducted appeared to implement the GARCH-family of models as a backbone for modeling the Bitcoin related volatility (Bouri *et al.*, 2017; Guesmi *et al.*, 2019; Fakhfekh and Jeribi, 2020). In parallel, another line of research as focused on highlighting the diversification ability of the Bitcoin through studying the correlation persistent between the conventional asset classes and the Bitcoin. In this respect, various studies appear to apply a variety of methods, concluding that the Bitcoin turns out to be very weakly correlated with such conventional assets as bonds, commodities and equities (e.g., Bouri *et al.*, 2017; Gajardo *et al.*, 2018; Klein *et al.*, 2018; Charfeddine *et al.*, 2020; Jeribi and Ghorbel, 2021). Additionally, Bouri *et al.* (2017), Kajtazi and Moro (2018), Guesmi *et al.* (2019), Charfeddine *et al.* (2020), Jeribi and Ghorbel (2021), Ghorbel and Jeribi (2021) and Jeribi and Fakhfekh (2021) outline that significant benefits, relating mainly to portfolio diversification and risk management, are yielded on introducing the Bitcoin to the scene.

Most of the volatility dynamics conducted research works, dealing with the correlations and hedge ratios between cryptocurrencies and other assets, appear to apply such multivariate GARCH models as the BEKK (Klein *et al.*, 2018; Ghorbel and Jeribi, 2021), the DCC (Bouri *et al.*, 2017; Ghorbel and Jeribi, 2021) or the ADCC (Gajardo *et al.*, 2018) to assess such relationships. Nevertheless, estimating GARCH models on large data sets is not void of any challenges. For instance, implementing the BEKK model or VARMA-GARCH models may well reflect a poorly behaved likelihood function, due to the presence of a vast number of free parameters, likely to render estimation difficult for models involving more than two variables. In this respect, conditional correlation models such as the constant conditional correlation (CCC), the dynamic conditional correlation (DCC) or the asymmetric DCC (ADCC) turn out to be rather robust in dealing with such estimation issues, allowing for more variables to be incorporated into the model. Similarly, conditional correlation models are designed to address some of the problems associated with the BEKK and VECH model types. Not only are they easy to estimate, but they also retain analytical tractability for large data sets, which makes them very widely applicable for hedge ratios estimating purposes.

While several existing studies appeal to the DCC and ADCC models to estimate optimal hedge ratios (Guesmi *et al.*, 2019), these approaches are applied in the present study along with the generalized orthogonal GARCH (GO-GARCH) model to calculate optimal hedge ratios. In this work, a comparison is established between the DCC and ADCC drawn optimal hedge ratios and the GO-GARCH attained ones. Hedge ratios are computed between three stock market prices (namely, S&P500, FTSE and NIKKEI) and five cryptocurrencies in relation to gold. In addition, the possibility of cross-hedging stock market indices with the WTI and VIX volatility indexes is also investigated.

Actually, the GO-GARCH model has given rise to factor GARCH models (Engle *et al.*, 1990). Unlike the DCC and ADCC, commonly applied to account for the shocks and volatilities related transmission factors, the GO-GARCH is generally used to explore the volatility spillover effects from one market to another, which may well stand as an important consideration on calculating the hedge ratios. The latter are usually computed by means of a fixed width rolling window approach, often used to reduce the changing dynamics, structural change and parameter heterogeneity related effects.

In this paper, we extend the literature on hedging in several ways. First we consider four stock markets indices: emerging indices MSCIEM and three international indices to compare hedging effectiveness of different alternative assets in one side and to study whether investor in emerging markets can proceed in the same way as an investor in others stock markets or not. This study examines the possibilities of hedging an investment in emerging stocks markets with oil, gold, VIX and cryptocurrencies. Second, dynamic conditional correlation (DCC) and generalized orthogonal GARCH (GO-GARCH) are used to calculate optimal hedge ratios. Third, we divided sample period in two sub period: during bubble crash period and before crash period. This is the first study that analyses emerging cross-hedge ratios with financial assets and compare hedging effectiveness within the period of crash and the period before crash. Our analysis reveals a number of important results that are of interest to investors and others interested in international stock market and especially in emerging stock markets. Fourth, optimal hedge ratios are calculated using a fixed with rolling window approach. This approach is used to mitigate the effects of changing dynamics, parameter heterogeneity and structural change. We discuss the sensitivity of results to refits choose to compare between short term hedging strategy and long-term one.

The remainder of the paper is organized as follows. Section 2 provides details of the data associated methodological specifications. Section 3 provides details about the preliminary analysis related data. Section 4 is devoted to discuss the empirical results, while Section 5 depicts the major relating discussions and concluding remarks.

2. Econometric methodology

To reach our targeted objective of minimizing a portfolio risk involving stock market indices and cryptocurrencies, we consider it necessary to compute the relevant hedging ratio. Yet, calculating these ratios necessitates the estimation of conditional variances. To this end, we undertake to model both of the conditional variance and correlation by means of three MGARCH models (DCC, ADCC, GO-GARCH). Finally, through implementation of the hedging effectiveness, we consider establishing a comparison between the implemented models, to demonstrate the extent to which hedge ratios differ across GARCH models.

2.1 The DCC representation model

Initially devised by Engle (2002), the dynamic conditional correlation (DCC) model serves to enable the conditional correlation matrix to vary over time. This particular modeling specific estimation procedure is usually undertaken in conformity to a two-step process. In the first step, we start with estimating the GARCH model associated parameters, and in a second stage, we go on with estimating the time varying correlation. Accordingly, the DCC model turns out to allow for the formulation of two equations, namely:

The return equation:

$$r_{i,t} = \mu_{i,t} + ar_{t-1} + \varepsilon_{i,t} \tag{1}$$

The conditional variance equation with $p = q = 1$, such as:

$$h_{iit} = \omega_i + \alpha_i \varepsilon^2_{i,t-1} + \beta_i h_{ii,t-1} \tag{2}$$

$$\omega > 0,\ \alpha > 0 \ et \ \beta > 0$$

The DCC-GARCH model is defined as follows:

$$H_t = D_t R_t D_t \tag{3}$$

where: H_t is the 2×2 conditional covariance matrix, R_t stands for the conditional correlation matrix and D_t denotes a diagonal matrix with time-varying standard deviations.

$$D_t = \operatorname{diag}\left(\sqrt{h_{11}},\ \sqrt{h_{22}}\right) \tag{4}$$

$$D_t = \begin{pmatrix} h^{\frac{1}{2}}_{1,t} & 0 & 0 & \cdots & 0 \\ 0 & h^{\frac{1}{2}}_{2,t} & 0 & \cdots & 0 \\ 0 & 0 & h^{\frac{1}{2}}_{3,t} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & h^{\frac{1}{2}}_{N,t} \end{pmatrix}$$

and:

$$R_t = \operatorname{diag}\left((Q)^{-\frac{1}{2}}\right) Q_t \operatorname{diag}\left((Q)^{-\frac{1}{2}}\right) \tag{5}$$

where: Q_t is a (2×2) symmetric positive definite matrix $Q_t = q^{ij}_t$, and is given as:

$$Q_t = (1 - \theta_1 - \theta_2)\overline{Q} + \theta_1 \varepsilon_{t-1}\varepsilon'_{t-1} + \theta_2 Q_{t-1} \tag{6}$$

where: $\overline{Q}$ is a (2×2) matrix of the unconditional correlation of standardized residuals; θ_1 and θ_2 are non-negative scalars, assuming that $\theta_1 + \theta_2 < 1$. The correlation estimates are provided by:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{ii,t} \cdot q_{jj,t}}} \tag{7}$$

2.2 ADCC representation model

Relying on the DCC model and the asymmetric GARCH model of Glosten *et al.* (1993) and Cappiello *et al.* (2006) have further extended this model by incorporating an asymmetric term, thus, establishing the Asymmetric DCC (ADCC) model such as:

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1} + d_i \varepsilon_{i,t-1}^2 I(\varepsilon_{i,t-1}) \tag{8}$$

$$\omega > 0 \;,\; \alpha > 0 \;\text{ and } \beta > 0$$

where: h_t represents the conditional variance; ω_i is a constant; α_i and β_i are the parameters enabling to capture the persistence of short-term and long-term volatilities, respectively; while d_i designates the asymmetric parameter.

The indicator function $I(\varepsilon_{i,t-1})$ is equal to one if $\varepsilon_{i,t-1} < 0$, and to 0 otherwise. In terms of this specification, a positive value of d should denote that it is actually the negative residuals, rather than the positive ones, which tend to increase the variance. The asymmetric effect is designed to capture an often observed feature of financial assets, namely, that an unexpected drop in asset prices tends to increase volatility more than an unexpected increase in asset prices of the same magnitude. This fact implies well that bad news tend to contribute in increasing volatility more than the good news do.

In regard to the ADCC model, the Q dynamics are provided by:

$$Q_t = \left(\overline{Q} - A'\overline{Q}A - B'\overline{Q}B - G'\overline{Q}^{-}G\right) + A'z_{t-1}z_{t-1}'A + B'Q_{t-1}B + G'^{z_t^- z_t'^-}G \tag{9}$$

In the above equation, A, B and G are $n \times n$ parameter matrices, and z_t^- are zero-threshold standardized errors that are equal to z_t once discovered to be inferior to zero, and zero otherwise. $\overline{Q}$ and $\overline{Q}^-$ are the unconditional matrices of z_t and z_t^-, respectively.

2.3 GO-GARCH representation model

Concerning the generalized orthogonal GARCH model, Van der Weide (2002) stipulated the asset returns r_t to be as follows:

$$r_t = n_t + \varepsilon_t \tag{10}$$

where: n_t designates the conditional mean, and ε_t represents the error term.

The GO-GARCH model incorporates $r_t - n_t$ on a set of unobserved exogenous factors, as follows:

$$\varepsilon_t = Bf_t \tag{11}$$

where: B represents a mixing matrix that is disintegrated in an orthogonal matrix R and an unconditional covariance matrix Π, such as:

$$B = \Pi^{1/2}R \tag{12}$$

While the rows in the mixing matrix B denote the assets, the columns involve factors which are represented as follows:

$$f_t = G^{1/2}z_t \tag{13}$$

where: the random variable z_t is characterized as $E(z_{it}) = 0$ and $E(z_{it}^2) = 1$.

The factor conditional variances can also be specified through a GARCH model. Hence, the combination of Equations (10), (11) and (14) provides the following:

$$r_t = n_t + BG_t^{1/2} z_t \tag{14}$$

The conditional covariance matrix of asset returns, $r_t - n_t$, is specified through:

$$\Pi_t = BG_t B' \tag{15}$$

At this level, it is important to note that the GO-GARCH model rests on two main assumptions, namely, that B is time invariant, and that the G_t matrix is diagonal. In the study conducted by Van der Weide (2002), a single-step maximum likelihood method was employed to simultaneously estimate the orthogonal matrix along with the relevant dynamics. Yet, this method turns out to be difficult to apply for the case of prevalent multiple assets. More recently, the orthogonal matrix R has been proposed to fit for executing estimations through accomplishing independent component analysis. A similar approach has been implemented in the present work. Unlike the OGARCH model in which B is restricted to be orthogonal, the GOGARCH model uses a one-step maximum likelihood to jointly estimate the rotation matrix R and other dynamics. Matrix R can be estimated using NLS (Nonlinear Least Square), MM (Method of Moments) and even by ICA (Independent Component Analysis).

2.4 Hedge ratio

On setting up the hedging process, we need to consider estimating the optimal hedge ratio. At this level, the conditional variance and covariance estimates can be applied to calculate the optimal hedge ratio, which highly depend on minimizing the portfolio return variance (Kroner and Sultan, 1993). The risk-minimizing hedge ratio is rendered through:

$$\beta_{ij,t} = \frac{h_{ij,\,t}}{h_{jj,t}} \tag{16}$$

where: $h_{ij,\,t}$ designates the conditional covariance between asset i and j at time t, while $h_{jj,t}$ stands for the conditional variance of asset j at time t. It is worth noting, in this regard, that a long position in one Dollar in asset i can be hedged by a short position in $\beta_{ij,t}$ Dollars of asset j.

2.5 Optimal portfolio weights and hedging effectiveness

Let us suppose that an investor, who is holding asset i, wants to hedge against his exposure to unfavorable movements in asset j. Following Kroner and Ng (1998), the optimal portfolio weights can be constructed by minimizing the portfolio associated risk, without impacting the expected return, such as:

$$\omega_{ij,t} = \frac{h_{jj,t} - h_{ij,\,t}}{h_{ii,t} - 2h_{ij,t} + h_{jj,t}} \tag{17}$$

$$\omega_{ij,t} = \begin{cases} 0, & \text{if } \omega_{ij,t} < 0 \\ \omega_{ij,t}, & \text{if } 0 \le \omega_{ij,t} \le 1 \\ 1, & \text{if } \omega_{ij,t} > 1 \end{cases} \tag{18}$$

where: $\omega_{ij,t}$ is the weight on the first asset in a one Dollar portfolio of two assets (assets i and j) at time t. The weight on the second asset is in the form of $(1 - \omega_{ij,t})$.

The hedging effectiveness (HE) across the proposed portfolios can be determined by analyzing the achieved hedging errors, as suggested by Ku *et al.* (2007), as provided by:

$$HE = \frac{Var_{unhedged} - Var_{hedged}}{Var_{unhedged}} \tag{20}$$

where: Var_{hedged} designates the returns' variance of the stock-cryptocurrency portfolio, and $Var_{unhedged}$ indicates the variance of the stocks' portfolio returns. A higher HE of a given portfolio indicates a greater portfolio risk reduction, which implies that the underlying investment strategy is deemed as a rather effective hedging strategy.

3. Data and preliminary analysis

Concerning the present study conducted analysis, the daily data applied involve those relating to gold, WTI crude oil and VIX prices as well as the five most popular cryptocurrencies (Bitcoin, Dash, Ethereum, Monero and Ripple). These data were collected from the site: https://coindesk.org. The covered periods span from January 1, 2016 to September 10, 2019. Regarding the three most frequently applied stock market indices, we have made use of the S&P500, FTSE100, NEIKKEI 225 and MSCI Emerging market indices "MSCIEM." These three variables associated data were extracted from the Data Stream database, The entirety of the returns' series are computed on a continuous compound basis: $r_{i,t} = 100 * \log(P_{i,t}/P_{i,t-1})$, where $P_{i,t}$ designates the closing price of asset i at time t.

We begin our preliminary analysis of the cryptocurrencies and conventional financial assets by graphically examining their return evolution as reported on Figure 1. In addition, volatility clustering is visible with respect to the majority of cryptocurrencies, essentially depicting the crash period (2017–2018). Furthermore, volatility clustering turns out to be rather pronounced concerning the Bitcoin along with the Ethereum.

Table 1 reports the daily returns' descriptive statistics as based on the entirety of indices prices. Accordingly, the Ethereum appears to display the highest daily mean return, while the Bitcoin proves to exhibit the lowest mean return in relation to the other variables. Noteworthy, however, is that both of the cryptocurrency markets and the WTI crude oil as well as the MSCI emerging market index seem to be rather too volatile relevant to the other indices. Except for the Bitcoin and Dash related stock market indices, all the remaining index returns turn out to be positively skewed. Moreover, except for both FTSE and MSCI Emerging market indices and the Gold price returns, the entirety of the other indices appears to display a significant leptokurtic behavior. As for the Jarque-Bera statistic, it proves to confirm well the persistence of significant non-normality associated with all the series. Besides, the ADF test proves to testify that all series turn out to be stationary. The Box-Pierce Q-test appears to reject strongly the presence of non-significant autocorrelations within the first 20 lags in the entirety of the series, except for the MSCI emerging market index and the cyptocurrency markets, while the Box-Pierce squared Q-test proves the presence of non-significant autocorrelations among the entirety of the series. As regards the ARCH-LM test, it appears to provide evidence confirming the presence of conditional heteroscedasticity within the examined return series.

Unconditional correlations among raw returns are depicted on Table 2. The highest correlation is noticed to occur between Bitcoin and Ethereum (with a rate of 0.4812). The unconditional correlations between cryptocurrency-cryptocurrency and cryptocurrency-Gold are discovered to be positive, except for Dash-Gold. The lowest and negative correlation has been recorded between VIX and the S&P500 index (with a rate of −0.39).

4. Results and discussion

We initiated our analysis by estimating the three versions of the GARCH models (DCC-GARCH, ADCC as well as the GO-GARCH) with an AR (1) term in mean equation to take into

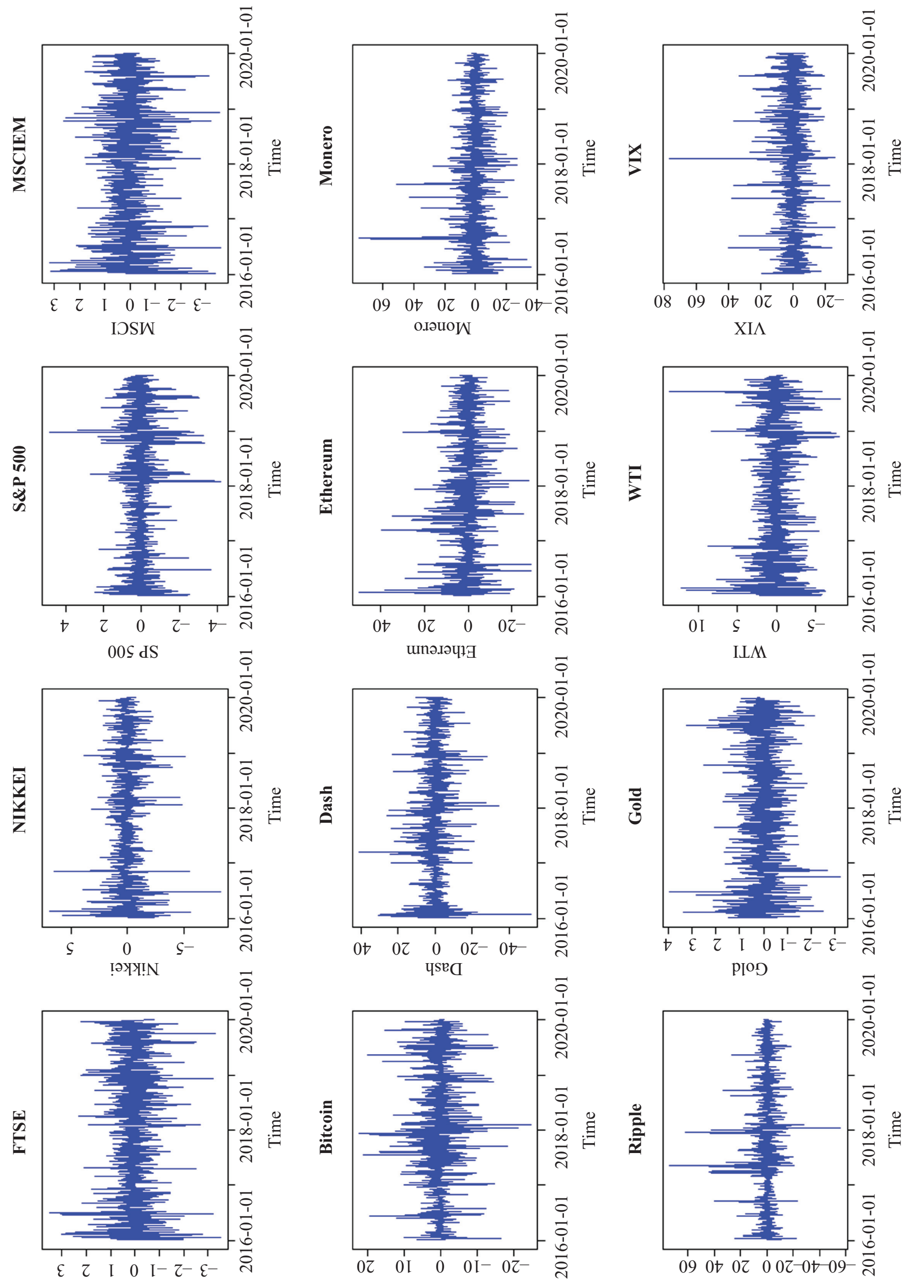

Figure 1.
Daily returns of sample variables

Table 1.
Preliminary statistics relevant to the raw return series

	FTSE	NIKKEI	S&P	MSCIEM	Bitcoin	Dash	Euthereum	Monero	Ripple	Gold	WTI	VIX
Mean	0.0193	0.0155	0.0414	0.0251	−24.7405	0.3484	0.5574	0.5207	0.4061	0.0356	0.0579	−0.0341
Std. dev.	0.8002	1.1843	0.8178	5.2044	4.7035	7.1418	7.5978	8.6027	8.4602	0.7565	2.1412	8.1205
Min.	−3.5192	−8.2529	−4.1842	−3.6024	−1.2623	−51.5940	−28.5582	−35.9127	−56.3335	−3.2436	−7.4355	−29.9831
Max.	3.5149	6.9113	4.8403	3.1991	22.1747	40.7275	49.7580	75.0506	74.0686	3.9434	12.1342	76.8245
Skewness	−0.0899	−0.4408	−0.6237	−0.4128	−0.1229	−0.1076	0.7150	1.5731	1.9279	0.2987	0.2166	1.5552
Kurtosis	2.2456	7.1777	4.7186	1.5806	3.9203	6.3687	5.0439	12.3509	15.0577	2.0684	3.0904	11.1168
JB test	202.65***	2080.2***	948.23***	127.04***	614.67***	1615.7***	1093.9***	6456.3***	9600.7***	185.09***	388.25***	5296.7***
ADF Test	−10.185***	−10.946***	−10.48***	−9.912***	−9.005***	−9.09***	−8.201***	−9.132***	−8.4808***	−9.957***	−9.816***	−11.96***
Q (20)	15.819	28.963	28.963	45.107***	23.44***	45.654***	40.316***	33.255***	42.985***	24.246	21.486	22.01
Q^2 (20)	466.26***	155.69***	328.16***	151.62***	109.07***	184.65***	116.17***	214.14***	152.98***	73.768***	410.79***	36.278***
ARCH (12)	167.47***	72.02***	123.84***	78.696***	45.112***	83.667***	50.909***	203.96***	94.181***	41.287***	134.46***	27.997***
N obs	950	950	950	950	950	950	950	950	950	950	950	950

Note(s): ***, ** and* denote significant at 1%, 5% and 10% significance levels, respectively. Where: Std. dev represents the standard deviation of returns; ARCH-LM indicates the Lagrange multiplier test for conditional heteroskedasticity with 10 lags; JB Test indicates the Jarque-Berastatistics, and Q (20) statistic is the LjungeBox test up to 20 lags

Table 2.
Spearmen Rho for raw returns

	FTSE	NIKKEI	S&P	MSCI	Bitcoin	Dash	Euthereum	Monero	Ripple	Gold	WTI	VIX
FTSE	1.0000	0.1369***	0.3833***	0.3689***	−0.0205	0.0214	−0.0061	0.0524	0.0709**	−0.0909***	0.1966***	−0.2857***
NIKKEI		1.0000	0.1049***	0.1950***	−0.0204	−0.0411	−0.0502	0.0195	−0.0259	−0.1112***	0.0154	−0.0630*
S&P			1.0000	0.2213***	−0.0465	0.0179	−0.0246	0.0068	−0.0541*	−0.1109***	0.2061***	−0.4542***
MSCIEM				1.0000	−0.0201	0.0180	0.0020	0.0543*	0.0510	0.0529	0.2412***	−0.3153***
Bitcoin					1.0000	0.3685***	0.4986***	0.4148***	0.3488***	0.0683**	0.0002	0.0044
Dash						1.0000	0.4612***	0.4937***	0.4336***	0.0020	0.0453	−0.0480
Euthereum							1.0000	0.4373***	0.3781***	0.0313	−0.0020	−0.0132
Monero								1.0000	0.4709***	0.0254	0.0689**	−0.0358
Ripple									1.0000	0.0760**	0.0034	−0.0176
Gold										1.0000	0.0174	0.1196***
WTI											1.0000	−0.2676***
VIX												1.0000

Note(s): ***, ** and* denote significant at 1%, 5% and 10% significance levels, respectively account potential autocorrelation in raw returns. We assume that innovations follow the multivariate t distribution in the case of DCC and ADCC versions (see Table 3), and multivariate affine negative inverse Gaussian (MANIG) distribution in the case of GOGARCH model.

4.1 Regression results

Table 3 reports the parameter estimates relevant to both of the DCC-GARCH and ADCC-GARCH models. The coefficient corresponding to a (i.e., the AR(1) term) in the mean equation is discovered to be significant at the 10% level, with a negative value being associated only with the SP500 and all cryptocurrencies. Concerning the two estimated models (DCC and ADCC), still, the reached results prove that the short-term persistence (i.e., the alpha1 "α" term) turns out to be significant at the 1% level and lower than the long-term persistence value (i.e., beta1 "β" term). Long-term persistence has also been evident, as the long-term corresponding coefficient has also been statistically significant. Actually, the statistical significance of both α and β lends support to the persistence of volatility clustering.

According to Table 3,the shape parameter (λ) is estimated to reach a maximum respectively for the Gold, WTI crude oil, MSCI emerging market and FTSE indices (over 5,4), denoting that the remainder of the variables related return series' distributions appear to bear heavier tails relative to the Gold, WTI, MSCIEM and FTSE associated returns' distribution. In addition, the ADCC model related asymmetry (eta11 "γ" term) estimated coefficient, relevant to the four stock market indices along with the crude oil, is discovered to be significant and to bear a positive value. Such a finding indicates well that negative shocks contribute in increasing conditional volatility (variance) in such a way that its value turns out to exceed the positive movement of a similar extent. Inversely, however, the corresponding asymmetric term is estimated to be negative and significant with respect to the entirety of the five cryptocurrencies along with the Gold price and VIX index. This finding implies well that as far as these series are concerned, the negative shocks seem to participate in reducing conditional volatility. Such results appear to be consistent with those published by Pal and Mitra (2019).

As regards the DCC model, both of the dcca1 (θ_1) and dccb1 (θ_2) corresponding coefficients are discovered to be positive and statistically significant at the threshold of 1%. The sum of both (θ_1) and (θ_2) proves not to exceed the value 1, which highlights and testifies well the mean reverting nature of the DCC model. Similarly, the ADCC-GARCH associated outcome is also discovered to be mean reverting in character.

Wald test and likelihood ratio test indicate the no acceptation of null hypothesis at 1% significance level. These results show that DCC version is better than CCC version and so it should be chosen to model data. A test of non-constant correlation based on Engle and Sheppard (2001) show the reject of null hypothesis and indicates that correlation is not constant over time. Table 4, below, depicts the estimates' results relevant to the GO-GARCH model. As the estimates indicate, the GO-GARCH factors appear to demonstrate that no standard errors seem to persist. The first table panel illustrates the rotation matrix R, while the second table panel displays the mixing matrix B, and the third table panel depicts the estimated parameters' attained results concerning the GO-GARCH model. As already cited, the rotation matrices turn out to be orthogonal. For each factor, the short-term parameters "alpha1" appear to bear lower values than those associated with the long-term parameters "beta1," highlighting short-term persistence over the long-term. The sum of short (α) and long (β) terms persistence parameters is less than 1, implying the volatility process is mean-reverting. Similarly, these results prove also to confirm the results displayed by both of the DCC and ADCC models. The mixing matrix B is not orthogonal as the null hypothesis BB' is close to unit is rejected at 1% confidence level. This confirms that the orthogonality assumption of O-GARCH is statistically too restrictive and

Parameters	DCC Estimate	ADCC Estimate
[FTSE].mu	0.049147*	0.023722
[FTSE].ar1	0.003453	0.024031
[FTSE].omega	0.042124*	0.035304**
[FTSE].alpha1	0.169257***	0.106192***
[FTSE].beta1	0.772391***	0.873996***
[FTSE].eta11		0.866658***
[FTSE].shape	5.767981***	6.508212***
[Nikkei].mu	0.071353**	0.049335
[Nikkei].ar1	0.043584	0.056479
[Nikkei].omega	0.072830*	0.066572***
[Nikkei].alpha1	0.146773**	0.127395***
[Nikkei].beta1	0.821029***	0.848490***
[Nikkei].eta11		1.000000***
[Nikkei].shape	3.582347***	4.167834***
[SP500].mu	0.064465***	0.052338***
[SP500].ar1	−0.101730**	−0.106552**
[SP500].omega	0.021119*	0.028881***
[SP500].alpha1	0.192214***	0.142889***
[SP500].beta1	0.800082***	0.861089***
[SP500].eta11		0.801841***
[SP500].shape	3.585159***	3.751202***
[MSCIEM].mu	0.091471**	0.061160**
[MSCIEM].ar1	0.196194***	0.204068***
[MSCIEM].omega	0.018879	0.024857
[MSCIEM].alpha	0.077238**	0.071730***
[MSCIEM].beta1	0.895773***	0.914184***
[MSCIEM].eta11		0.693701**
[MSCIEM].shape	9.933181***	8.709445***
[Bitcoin].mu	0.357045***	0.350010***
[Bitcoin].ar1	−0.044052	−0.075232**
[Bitcoin].omega	0.204299	0.054315
[Bitcoin].alpha1	0.150989***	0.239481**
[Bitcoin].beta1	0.848011***	0.885277***
[Bitcoin].eta11		−0.374273**
[Bitcoin].shape	3.210503***	2.332127***
[Dash].mu	0.154195	0.106342
[Dash].ar1	−0.015131	−0.018859
[Dash].omega	1.402345*	0.271869**
[Dash].alpha1	0.165241***	0.183498***
[Dash].beta1	0.833758***	0.835885***
[Dash].eta11		−0.174548
[Dash].shape	3.631768***	3.619561***
[Ethereum].mu	0.218865	0.198771
[Ethereum].ar1	−0.000631	−0.017436
[Ethereum].omega	6.061125	0.725356
[Ethereum].alpha1	0.273099	0.272028**
[Ethereum].beta1	0.725901**	0.771073***
[Ethereum].eta11		−0.024347
[Ethereum].shape	2.992506***	2.718620***
[Monero].mu	0.089706	0.155666
[Monero].ar1	−0.080334**	−0.076006**
[Monero].omega	13.345117*	1.528079**
[Monero].alpha1	0.210679**	0.186937***

(*continued*)

Table 3.
The DCC and ADCC parameters' estimates

Parameters	DCC Estimate	ADCC Estimate
[Monero].beta1	0.703123***	0.719407***
[Monero].eta11		−0.346134**
[Monero].shape	3.099934***	3.119282***
[Ripple].mu	−0.36877***	−0.38410***
[Ripple].ar1	−0.12959***	−0.14192***
[Ripple].omega	1.194575	0.247086
[Ripple].alpha1	0.122497**	0.204273**
[Ripple].beta1	0.876503***	0.868597***
[Ripple].eta11		−0.067390
[Ripple].shape	2.928138***	2.481378***
[Gold].mu	0.013810	0.022538
[Gold].ar1	−0.027925	−0.017311
[Gold].omega	0.003673*	0.013760*
[Gold].alpha1	0.019620***	0.037092***
[Gold].beta1	0.972151***	0.952975***
[Gold].eta11		−0.777065**
[Gold].shape	8.061849***	6.934664***
[WTI].mu	0.130486**	0.077026
[WTI].ar1	−0.013572	−0.010671
[WTI].omega	0.066752	0.031337**
[WTI].alpha1	0.066576**	0.061558***
[WTI].beta1	0.916796***	0.936833***
[WTI].eta11		1.000000***
[WTI].shape	9.465446***	12.061923***
[VIX].mu	−0.321589	−0.117805
[VIX].ar1	−0.029190	−0.040933
[VIX].omega	11.541494*	0.487707**
[VIX].alpha1	0.160246**	0.115057***
[VIX].beta1	0.682934***	0.856439***
[VIX].eta11		−1.000000***
[VIX].shape	3.233107***	3.538618***
[Joint]dcca1	0.015670***	0.014545***
[Joint]dccb1	0.974755***	0.976756***
[Joint]dccg1		0.000952
[Joint]mshape	7.418555***	6.720363***
Akaike	53.137	53.373
Bayes	54.108	54.434
Shibata	53.063	53.288
Hannan-Quinn	53.514	53.785
Wald Test	96617***	193925***
LL ratio test	0.5091***	0.4803***
Engle and Sheppard Test	197.8769***	

Table 3.

Note(s): ***, ** and* denote significant at 1%, 5% and 10% significance levels, respectively

that it might exclude many linkages existing in financial markets which justify the use of GOGARCH version.

4.2 Rolling window and dynamic correlation analysis

A commonly maintained hypothesis stipulates that the parameters be considered constant over time between these time series' models. Actually, such a hypothesis overlooks any new information and economic change considerations, hence the need for a technique allowing to change the model's parameters so as to be aligned with recent economic changes. In practical

Table 4. GOGARCH parameters' estimates

The rotation matrix R

	U(1)	U(2)	U(3)	U(4)	U(5)	U(6)	U(7)	U(8)	U(9)	U(10)	U(11)	U(12)
U(1)	−0.0336	−0.0002	0.4340	0.6992	0.0780	0.1776	−0.3683	−0.1630	0.3236	−0.0414	−0.0998	−0.0732
U(2)	0.0360	−0.0848	0.1510	0.1480	0.0398	0.1306	−0.2481	−0.0079	−0.9233	0.0013	−0.1185	−0.0039
U(3)	−0.0296	0.3897	−0.7093	0.2969	0.2075	0.0208	−0.2081	−0.0008	−0.0076	0.3347	−0.2320	0.0574
U(4)	−0.0509	−0.1133	−0.0683	−0.3679	0.0652	−0.0792	−0.8295	0.2902	0.1353	−0.1682	0.1133	0.0484
U(5)	−0.0944	0.0184	−0.1282	0.2368	0.1453	−0.0311	0.0000	−0.0767	−0.1061	0.0422	0.9362	0.0265
U(6)	0.1013	−0.1906	−0.1709	−0.2290	0.0409	0.0570	−0.1692	−0.9114	0.0229	−0.0801	−0.0244	−0.0034
U(7)	0.3098	0.1464	0.1530	0.1128	−0.0024	−0.9058	−0.0791	−0.1022	−0.0608	0.0699	−0.0272	−0.0043
U(8)	0.3252	0.8136	0.2454	−0.2366	−0.0659	0.2601	−0.0723	−0.0874	−0.0218	−0.1152	0.1258	−0.0408
U(9)	0.1779	−0.0210	−0.2658	0.1319	0.2435	−0.0348	0.0964	0.0916	−0.0278	−0.6516	−0.0393	−0.6122
U(10)	0.8170	−0.3103	−0.1546	0.1138	−0.2738	0.2132	−0.0500	0.1480	0.0714	0.1988	0.0990	0.0117
U(11)	0.2716	−0.0882	0.1468	−0.0914	0.8395	0.0658	0.1336	0.0645	0.0425	−0.0269	−0.0554	0.3897
U(12)	0.0188	0.0599	−0.1891	0.2231	−0.2808	−0.0376	0.0410	−0.0027	−0.0277	−0.6056	−0.0345	0.6781

The mixing matrix B

	A(1)	A(2)	A(3)	A(4)	A(5)	A(6)	A(7)	A(8)	A(9)	A(10)	A(11)	A(12)
A(1)	−0.0005	−0.3926	−0.1620	0.1304	−0.1339	0.0845	0.0084	0.0962	0.0269	−0.6418	0.0507	−0.1315
A(2)	−0.5513	−1.0206	−0.1389	0.1272	−0.0717	−0.1427	−0.1120	0.0661	0.0408	0.2361	−0.0637	0.2076
A(3)	0.0375	−0.3139	0.0730	−0.0379	−0.1571	0.0938	−0.0374	0.0384	−0.0136	−0.0829	0.1118	−0.6094
A(4)	−0.2495	−0.4213	0.1298	−0.0839	0.5365	0.0459	0.0738	0.0253	0.0204	−0.3563	−0.0272	−0.2059
A(5)	−0.4717	0.7619	1.8823	3.1174	0.0701	0.3654	0.0401	2.9016	0.3390	0.3124	−0.1484	−0.2149
A(6)	−0.3834	−0.1933	2.2979	5.1797	0.8567	0.9650	−0.1035	−2.3668	−0.3916	0.0633	4.1141	−0.3769
A(7)	0.4308	0.6049	2.3928	5.2464	0.0077	1.8217	1.1213	−2.5727	−0.5337	0.7848	−4.5094	−0.6471
A(8)	−0.3459	−0.0844	2.0406	4.3754	1.3537	1.4351	−7.8335	−0.2279	0.4122	−0.4971	−1.4764	−0.2441
A(9)	−0.3872	0.3627	1.8064	2.7492	0.0524	0.1374	−0.5621	−1.0389	8.6028	−0.4654	−0.0631	−0.4335
A(10)	−0.5844	0.4256	0.0189	−0.0512	−0.0502	−0.0900	0.0064	−0.1336	−0.0166	−0.1311	−0.0911	−0.0988
A(11)	−0.5902	−0.4216	0.1674	−0.3919	−0.1009	1.9320	0.1000	0.1798	0.0774	−0.3161	0.4256	0.0311
A(12)	−0.3486	3.1559	−6.5743	1.3451	1.5367	−0.1879	−0.0169	0.1213	0.3353	2.7905	−0.4174	0.5560

(continued)

Table 4.

GO-GARCH parameters

Coef	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12
Omega	0.0078	0.0438	0.1139	0.0026	0.0193	0.0182	0.0760	0.0199	0.0321	0.0412	0.0110	0.0198
alpha1	0.0257	0.1090	0.1645	0.0233	0.0275	0.0559	0.1518	0.1188	0.1641	0.0825	0.1117	0.1051
beta1	0.9651	0.8391	0.7027	0.9757	0.9521	0.9234	0.7703	0.8668	0.8080	0.8770	0.8796	0.8714
Skew	0.0891	0.2213	−0.1352	0.0129	−0.0961	−0.0445	−0.3288	−0.0058	0.3684	0.0991	0.0175	0.1337
Shape	2.6006	1.1839	1.2150	0.9336	2.1902	3.0832	0.5705	1.0019	0.6488	1.2970	0.8134	1.5905

terms, however, a day-to-day change does not prove to be too considerable. Hence, the appearance of a common rolling window technique, which emerged to help evolve the model's constant parameters by dividing the relevant data into a sample of estimates and predictions. Following Basher and Sadorsky (2016) as well as Ahmad *et al.* (2018), we opt for applying a rolling window analysis in a bid to construct one-step ahead dynamic conditional correlations.

We consider fixing the estimation window at 650, with the aim of generating 300 one-step-ahead dynamic conditional correlations. Accordingly, we undertake to refit each GARCH model to suite for every 20, then for every 40, then for every 60 daily observations. The objective lying behind these different window estimations (i.e., 20, 40 and 60 daily observations) consists in determining the different temporal horizons associated correlations and hedging strategies (i.e., short time, medium time and long time). Figure 2 [1] exhibits the one-period-ahead dynamic conditional correlations estimated in accordance to the three considered MGARCH models on a 20 [2] daily observation basis.

The different correlations between each of the stock markets subject of study (i.e., the S&P 500, FTSE and Nikkei) and each of the cryptocurrencies, Gold, VIX and WTI reveal well that the time-varying conditional correlations, estimated via both of the DCC and ADCC models, turn out to display similar patterns. As for the GO-GARCH model based conditional correlations, they appear to exhibit greater peaks and troughs, possibly because the GO-GARCH proves to incorporate volatility spillovers, which the DCC and ADCC are lacking. Among all the sample variables under review, both of the Bitcoin and Ethereum have been discovered to exhibit negative dynamic dependences with the FTSE and Nikkei indices. This finding appears to corroborate well those documented by Dyhberg (2016) as well as Guesmi *et al.* (2019), while remaining inconsistent with those released by Bouri *et al.* (2017) as well as Kajtazi and Moro (2018). Indeed, the correlation between the Nikkei index and each of VIX and WTI indices, as well as between the S&P 500 and each of the Bitcoin, Ripple, Gold and VIX indices turns out to be negative. These results suggest well that these assets could well provide rather effective hedging opportunities. Considering the S&P500 as a representative index of the USA market, one may well state that such a negative association with gold does not constitute a novel phenomenon, as several studies have reported an inverse relationship between equities and gold as a hedging commodity (Shahzad *et al.*, 2017; Klein *et al.*, 2018). The correlations between the FTSE and Ethereum as well as between the S&P500 and Bitcoin appear to exhibit an asymmetric dependence structure as far as the GO-GARCH extent is concerned. Such findings indicate well that the risks persistent between the FTSE and S&P500 can be hedged by both of the Ethereum and Bitcoin.

4.3 Hedging effectiveness analysis

At this level, we consider calculating the out-of-sample hedge ratios, based on a rolling window analysis. Concerning the period t, we first undertake to forecast the one-step-ahead conditional volatility and, in a second step, we use these forecasts to establish the one-step-ahead hedge ratios. We then reuse these forecast hedge ratios to formulate our hedging strategies. We fix the rolling window size at 650 observations, for the purpose of obtaining 300 one-step-ahead hedging ratios. For comparison purposes, we consider estimating the hedge ratios relevant to the different MGARCH (DCC, ADCC and GO-GARCH) models.

Figure 3 [3] illustrates the optimal hedge ratios relevant to the three stock market indices, as calculated on the basis of the cited GARCH models. The entirety of the three models derived hedge ratios turn out to demonstrate high variability in relation to the GO-GARCH estimation model. Indeed, the DCC associated hedge ratios appear to coincide with the ADCC derived ones, reflecting equitable stability in most cases. In effect, both of the DCC and ADCC models' drawn hedge ratios are discovered to be inferior to those obtained via the GO-GARCH model. Actually, the hedge ratios are predominantly distinguished to reflect lower values.

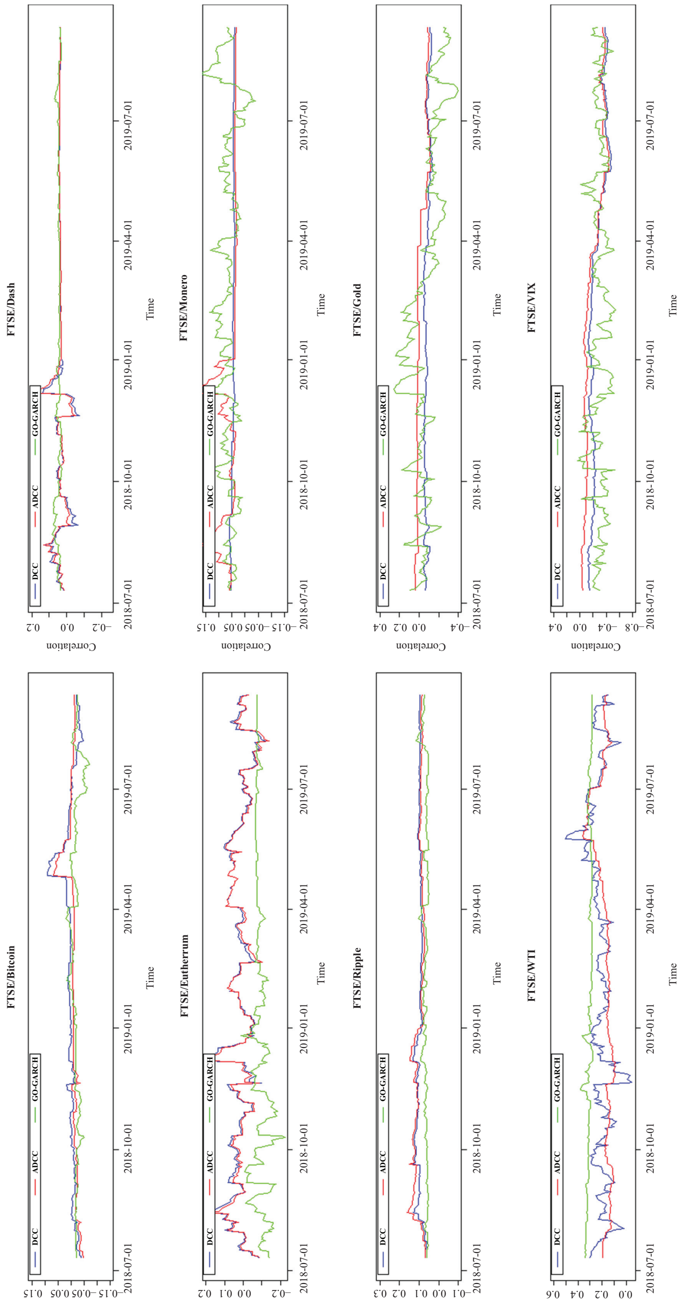

Figure 2.
Rolling one-step-ahead dynamic conditional correlations as reflected through the various MGARCH versions (refit = 20) relevant to the FTSE 100 index

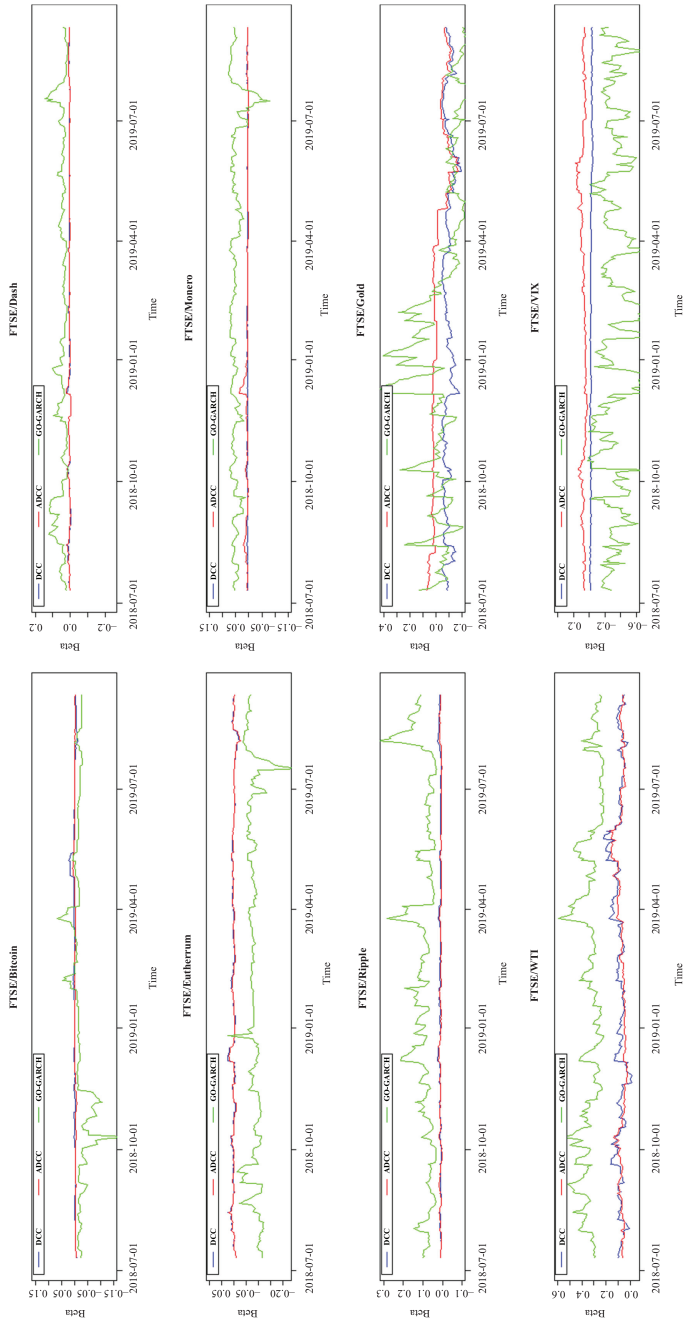

Figure 3.
Rolling one-step-ahead dynamic optimal hedge ratios drawn from the various MGARCH-model versions (refit = 20) relevant to the FTSE 100 index

Table 5 reports summary statistics for optimal weights, hedge ratios and hedging effectiveness from three variants of MGARCH models and from model refits every 20, 40 and 60 days.

Based on Table 5 reported results, one could well note that both of the DCC and ADCC model versions tend to yield similar results. On comparing the three MGARCH model versions' effectiveness, it can be noticed that the GO-GARCH model proves to score the highest (HE) value rates with respect to the entirety of the refits. Indeed, it can be observed that, overall, the GO-GARCH model version estimated portfolio turns out to help in reducing risk more effectively as compared to the DCC and ADCC model versions estimated portfolios.

The comparison established between the different MGARCH-model versions recorded estimates, relevant to the different refits (20, 40 and 60 days), appears to reveal that no significant difference proves to persist in regard to the hedging effectiveness coefficients (HE). Additionally, Table 5 depicted results, relevant to the various refits, also indicate that the best hedging instrument fit for the FTSE stock market index turns out to be the VIX volatility index, followed by WTI and Ripple. As for the NIKKEI stock-market index case, GOLD proves to stand as the most effective hedging instrument, followed by the VIX and the WTI. Gold, VIX and Bitcoin were the best effective hedging instruments for the MSCI emerging market index. Concerning the SP500 stock market index, VIX, WTI, Gold and Dash are discovered to represent the most effective hedging instruments, respectively. Such findings prove to corroborate those documented by Hood and Malik (2013) as well as Ahmad *et al.* (2018), highlighting that VIX proves to represent a rather effective hedging instrument in regard to the US equities than gold. It is also worth noting that some hedging effectiveness values appear to bear negative signs, reflecting that the hedged portfolios turn out to be even worse than the unhedged ones.

Among these results, all hedging instruments tend to exhibit negative average hedging ratio values. This is due mainly to the negative conditional correlations, which indicates that, as far as these asset pairs are concerned, long-term positions should be considered with respect to each single asset. The mean hedge-ratio value recorded between the FTSE and VIX indexes is of the rate of −0.02391 with respect to the DCC model (concerning the 20-day refit). These values may well be interpreted differently, namely, that a $1 short/long position in the FTSE index may, on average, be hedged for by 2.39 cents with a short/long position in VIX.

Based on these results, one could well note that except concerning the pairs (Nikkei-Bitcoin), (FTSE-WTI), (SP500-WTI), (Nikkei-WTI), (FTSE-Gold), (SP500-Gold), (Nikkei-Gold) and all pairs with MSCIEM, the optimal hedging-instrument weights in the investor portfolio prove to range from 10.26 to 99.36%. As for the remaining pairs, the optimal hedging-instrument weights appear to range constantly below the 8.56% rate, regardless of whether the DCC or ADCC method is being used. Overall, one may well deduce that for the sake of minimizing risk, while preserving the same expected returns of the digital-conventional financial asset portfolio, the investor is well recommended to hold more conventional financial assets rather than digital ones. Such an empirical finding can have its explanation in the low dependence between the digital asset returns and the conventional ones, on the one hand, and in the high volatility characterizing the digital asset prices, on the other hand. Such findings denote well that incorporating a very small proportion of digital assets into a diversified portfolio of conventional financial assets would substantially reduce its overall risk for a given level of expected return.

4.4 Hedging effectiveness strategies during the crypto-currency crash

In order to compare the effectiveness hedging strategies for the stock market indices during the crypto-currency market crash (2017–2018), we calculate the different HE coefficients during the period before and during crash.

Table 5. Optimal weights, Hedge ratios and hedging effectiveness for a 20, 40, 60-day refits pan relevant to the MGARCH-model versions

	Coef.	Refit: 20	Refit: 40 ω	Refit: 60	Refit: 20		Refit: 40 B		Refit: 60	
	Model	Mean	Mean	Mean	Mean	HE	Mean	HE	Mean	HE
FTSE/Bitcoin	DCC	0.0504	0.0501	0.0509	1.33×10^{-5}	0.0005	$-1.48*10^{-5}$	0.0004	−0.00033	0.0002
	ADCC	0.0287	0.0291	0.0308	−0.00160	0.0004	−0.00167	0.0004	−0.00188	0.0002
	GOGARCH	0.5387	0.5374	0.5386	0.46127	0.0006	−0.01927	0.0005	−0.01902	0.0005
FTSE/Dash	DCC	0.0140	0.0140	0.0142	0.004991	0.0020	0.00500	0.0021	0.00501	0.0021
	ADCC	0.0159	0.0161	0.0165	0.00526	0.0021	0.00506	0.0018	0.00504	0.0018
	GOGARCH	0.4422	0.5576	0.5604	0.04519	0.0022	0.05131	0.0021	0.05334	0.0023
FTSE/Ether.	DCC	0.0105	0.0105	0.0108	0.002057	0.0033	0.00212	0.0035	0.00216	0.0035
	ADCC	0.0097	0.0096	0.0099	0.001229	0.0028	0.00129	0.0029	0.00133	0.0029
	GOGARCH	0.5178	0.5152	0.4425	−0.10114	0.0084	−0.1040	0.0090	−0.03894	0.0152
FTSE/Monero	DCC	0.0064	0.0063	0.0063	0.00465	0.0021	0.00466	0.0021	0.00473	0.0022
	ADCC	0.0066	0.0065	0.0064	0.00573	0.0039	0.00575	0.0039	0.00588	0.0041
	GOGARCH	0.4387	0.5588	0.5608	0.05049	0.0051	0.08175	0.0051	0.08385	0.0053
FTSE/Ripple	DCC	0.0057	0.0056	0.0059	0.01185	0.0096	0.011792	0.0095	0.01198	0.0096
	ADCC	0.0030	0.0030	0.0032	0.00964	0.0099	0.00957	0.0097	0.00983	0.0100
	GOGARCH	0.3842	0.3844	0.3814	0.05267	0.0045	0.05212	0.0044	0.05209	0.0045
FTSE/Gold	DCC	0.5526	0.5543	0.5527	−0.09062	0.0068	−0.09001	0.0066	−0.09032	0.0065
	ADCC	0.5435	0.5452	0.5485	−0.01571	0.0029	−0.00756	0.0027	0	0.0019
	GOGARCH	0.6840	0.4620	0.4507	0.00450	0.0236	−0.10813	0.0261	−0.09147	0.0231
FTSE/WTI	DCC	0.0855	0.0843	0.0851	0.08799	0.0526	0.08760	0.0521	0.08693	0.0509
	ADCC	0.1106	0.1100	0.1101	0.07289	0.0348	0.07257	0.0341	0.07300	0.0339
	GOGARCH	0.4601	0.4623	0.5361	0.29374	0.0916	0.29673	0.0929	0.33496	0.0960
FTSE/VIX	DCC	0.0315	0.0314	0.0314	−0.02391	0.0775	−0.02385	0.0758	−0.02371	0.0738
	ADCC	0.0270	0.0267	0.0262	−0.01814	0.0569	−0.01782	0.0539	−0.01726	0.0508
	GOGARCH	0.5059	0.4934	0.4937	−0.27936	0.1019	−0.32778	0.1020	−0.32965	0.1029
NIKKEI/Bitcoin	DCC	0.1036	0.1030	0.1026	−0.00289	0.0002	−0.00270	0.0002	−0.00265	0.0002
	ADCC	0.0574	0.0584	0.0608	−0.00096	0.0001	−0.00089	0.0001	−0.00092	0.0001
	GOGARCH	0.4869	0.4853	0.4847	−0.07622	0.0040	−0.07438	0.0040	−0.07167	0.0037

(continued)

Table 5.

	Coef. Model	Refit: 20 ω Mean	Refit: 40 ω Mean	Refit: 60 ω Mean	Refit: 20 B Mean	Refit: 20 B HE	Refit: 40 B Mean	Refit: 40 B HE	Refit: 60 B Mean	Refit: 60 B HE
NIKKEI/Dash	DCC	0.0323	0.0322	0.0323	0.00765	0.0017	0.00765	0.0017	0.00764	0.0017
	ADCC	0.0369	0.0370	0.0374	0.00705	0.0014	0.00699	0.0014	0.00707	0.0014
	GOGARCH	0.5252	0.5252	0.5258	0.04351	0.0035	0.04302	0.0034	0.04372	0.0036
NIKKEI/Ether.	DCC	0.0347	0.0343	0.0349	−0.00893	0.0052	−0.00885	0.0053	−0.00902	0.0054
	ADCC	0.0274	0.0264	0.0276	−0.00555	0.0033	−0.00471	0.0031	−0.00507	0.0031
	GOGARCH	0.4826	0.4817	0.5175	−0.08363	0.0061	−0.08282	0.0060	−0.07907	0.0059
NIKKEI/Monero	DCC	0.0148	0.0144	0.0143	0.00909	0.0038	0.00907	0.0039	0.00922	0.0039
	ADCC	0.0160	0.0159	0.0157	0.00980	0.0043	0.00978	0.0044	0.00994	0.0045
	GOGARCH	0.4812	0.4835	0.4837	0.08788	0.0073	0.08942	0.0075	0.09048	0.0077
NIKKEI/Ripple	DCC	0.0363	0.0357	0.0363	−0.00211	0.0004	−0.00191	0.0004	−0.00180	0.0004
	ADCC	0.0240	0.0237	0.0244	−0.00136	0.0005	−0.00129	0.0005	−0.00124	0.0006
	GOGARCH	0.4338	0.5661	0.5670	0.00530	0.0001	0.00570	0.0001	0.00811	0.0001
NIKKEI/Gold	DCC	0.6873	0.6890	0.6894	−0.23183	0.0187	−0.23205	0.0187	−0.23106	0.0186
	ADCC	0.6601	0.6611	0.6617	−0.15299	0.0134	−0.15555	0.0139	−0.15787	0.0132
	GOGARCH	0.5273	0.4736	0.5260	−0.11382	0.0393	−0.16018	0.0397	−0.11456	0.0387
NIKKEI/WTI	DCC	0.2514	0.2499	0.2486	0.02298	0.0026	0.02412	0.0025	0.02179	0.0026
	ADCC	0.2463	0.2458	0.2451	0.02169	0.0032	0.02211	0.0029	0.01920	0.0031
	GOGARCH	0.4848	0.4828	0.5204	0.11068	0.0161	0.11037	0.0163	0.05858	0.0154
NIKKEI/VIX	DCC	0.0355	0.0354	0.0354	0.01612	0.0143	−0.01611	0.0143	−0.01607	0.0143
	ADCC	0.0350	0.0351	0.0351	−0.01523	0.0130	−0.01530	0.0131	−0.01522	0.0131
	GOGARCH	0.5561	0.5568	0.4410	−0.06445	0.0169	−0.06388	0.0170	−0.10449	0.0167
S&P/Bitcoin	DCC	0.0744	0.0743	0.0735	−0.00410	0.0004	−0.00404	0.0004	−0.00402	0.0004
	ADCC	0.0436	0.0445	0.0457	−0.00201	0.0002	−0.00201	0.0002	−0.00201	0.0002
	GOGARCH	0.4321	0.4324	0.5691	−0.01391	0.0063	−0.01113	0.0065	−0.13136	0.0073
S&P/Dash	DCC	0.0225	0.0225	0.0225	0.00707	0.0038	0.00716	0.0039	0.00716	0.0039
	ADCC	0.0244	0.0243	0.0247	0.00832	0.0042	0.00873	0.0045	0.00847	0.0041
	GOGARCH	0.4099	0.4089	0.5948	0.05136	0.0092	0.05170	0.0092	−0.01923	0.0095

(*continued*)

	Coef. Model	Refit: 20 Mean	Refit: 40 ω Mean	Refit: 60 Mean	Refit: 20 Mean	Refit: 20 HE	Refit: 40 B Mean	Refit: 40 HE	Refit: 60 Mean	Refit: 60 HE
S&P/Ether.	DCC	0.0177	0.0177	0.0178	0.00116	0.0025	0.00108	0.0027	0.00105	0.0028
	ADCC	0.0146	0.0145	0.0149	0.00199	0.0029	0.00197	0.0030	0.00191	0.0031
	GOGARCH	0.5760	0.5762	0.4199	−0.01801	0.0035	−0.01845	0.0035	−0.05243	0.0038
S&P/Monero	DCC	0.0138	0.0137	0.0136	0.00342	0.0024	0.00340	0.0026	0.00335	0.0025
	ADCC	0.0133	0.0135	0.0133	0.00511	0.0035	0.00484	0.0035	0.00475	0.0036
	GOGARCH	0.6025	0.6013	0.3955	0.00032	0.0004	0.00062	0.0004	0.01111	0.0005
S&P/Ripple	DCC	0.0270	0.0270	0.0271	−0.00175	0.0032	−0.00194	0.0028	−0.00190	0.0026
	ADCC	0.0180	0.0180	0.0183	−0.00189	0.0038	−0.00196	0.0039	−0.00196	0.0040
	GOGARCH	0.6420	0.3574	0.6463	−0.01052	0.0002	−0.00303	0.0003	−0.00391	0.0004
S&P/Gold	DCC	0.5846	0.5864	0.5864	−0.13350	0.0180	−0.1355	0.0183	−0.13788	0.0186
	ADCC	0.5582	0.5594	0.5614	−0.05540	0.0136	0.05549	0.0138	−0.05286	0.0141
	GOGARCH	0.5797	0.4195	0.4222	−0.03581	0.0268	−0.12407	0.0258	−0.13144	0.0221
S&P/WTI	DCC	0.1349	0.1347	0.1329	0.08005	0.0411	0.07927	0.0407	0.07879	0.0402
	ADCC	0.1293	0.1293	0.1284	0.08374	0.0425	0.08355	0.0423	0.08246	0.0417
	GOGARCH	0.3980	0.3992	0.6048	0.23889	0.0731	0.23929	0.0730	0.35061	0.0754
S&P/VIX	DCC	0.0501	0.0500	0.0502	−0.04315	0.2742	−0.04296	0.2723	−0.04326	0.2747
	ADCC	0.0461	0.0460	0.0462	−0.03849	0.2589	−0.03840	0.2572	−0.03868	0.2598
	GOGARCH	0.4723	0.5282	0.4730	−0.36190	0.1472	−0.46144	0.1474	−0.36186	0.1509
MSCIEM/Bitcoin	DCC	0.9315	0.9320	0.9326	−0.00186	0.0728	−0.00136	0.0708	−0.00099	0.0690
	ADCC	0.9621	0.9619	0.9603	0.000071	0.0397	0.00027	0.0395	−0.00012	0.0406
	GOGARCH	0.1482	0.4658	0.4659	−0.10572	0.0101	−0.10537	0.0107	−0.10753	0.0115
MSCIEM/Dash	DCC	0.9822	0.9821	0.9821	0.00686	0.0133	0.00686	0.0133	0.00687	0.0133
	ADCC	0.9828	0.9828	0.9826	0.01122	0.0115	0.01126	0.0115	0.01119	0.0117
	GOGARCH	0.0821	0.5935	0.4031	0.0891	0.0045	0.05174	0.0046	0.09565	0.0050
MSCIEM/Ether.	DCC	0.9885	0.9889	0.9888	0.00722	0.0197	0.00696	0.0178	0.00802	0.0175
	ADCC	0.9908	0.9914	0.9915	0.00938	0.0163	0.00924	0.0139	0.01120	0.0133
	GOGARCH	0.5487	0.5476	0.5495	−0.0227	0.0025	−0.02326	0.0029	−0.02567	0.0036

(continued)

Table 5.

	Coef.	ω			B					
		Refit: 20	Refit: 40	Refit: 60	Refit: 20		Refit: 40		Refit: 60	
	Model	Mean	Mean	Mean	Mean	HE	Mean	HE	Mean	HE
MSCIEM/Monero	DCC	0.9929	0.9931	0.9932	0.00707	0.0037	0.00708	0.0036	0.00711	0.0035
	ADCC	0.9933	0.9934	0.9936	0.00875	0.0033	0.00877	0.0032	0.00877	0.0031
	GOGARCH	0.5991	0.5970	0.4003	0.07539	0.0115	0.07561	0.0113	0.13650	0.0122
MSCIEM/Ripple	DCC	0.9873	0.9876	0.9876	0.00817	0.0082	0.00836	0.0079	0.00871	0.0077
	ADCC	0.9937	0.9938	0.9936	0.00822	0.0033	0.00823	0.0033	0.00839	0.0033
	GOGARCH	0.3546	0.3546	0.6496	0.07312	0.0029	0.07215	0.0028	0.04245	0.0031
MSCIEM/Gold	DCC	0.3682	0.3662	0.3668	0.06102	0.6064	0.06133	0.6086	0.05613	0.6096
	ADCC	0.3732	0.3716	0.3722	0.12126	0.5832	0.12645	0.5847	0.11774	0.5871
	GOGARCH	0.5465	0.5460	0.4544	0.00829	0.0492	0.01069	0.0472	0.11116	0.0471
MSCIEM/WTI	DCC	0.9016	0.9011	0.9035	0.11047	0.0548	0.11052	0.0551	0.11083	0.0532
	ADCC	0.8974	0.8966	0.8976	0.11602	0.0571	0.11595	0.0577	0.11521	0.0572
	GOGARCH	0.5445	0.4556	0.4564	0.28368	0.0889	0.33929	0.0896	0.33608	0.0885
MSCIEM/VIX	DCC	0.9469	0.9471	0.9475	−0.04645	0.2564	−0.04622	0.2545	−0.04572	0.2514
	ADCC	0.9480	0.9483	0.9488	−0.04369	0.2239	−0.04338	0.2215	−0.04276	0.2179
	GOGARCH	0.4782	0.4778	0.4776	−0.41427	0.1495	−0.41298	0.1483	−0.40659	0.1446

Table 5.

Tables 6 and 7 summarize the results of the hedge and optimal weighted ratios for the period before and during cryptocurrencies crash, respectively. For the pre-cryptocurrency crash period, the optimal weight range was 84.5% for NIKKEI with Bitcoin under DCC model estimation to 99.9% for FTSE, SP500 and MSCIEM with Ripple under ADCC model estimation. These results indicate that the optimal weight of the Bitcoin holdings was 84.5% for NIKKEI index. Indeed, the optimal weight for Bitcoin holdings in $1 NIKKEI–Bitcoin portfolios should be 84.5 cents in Bitcoin and 15.5 cents in NIKKEI. This table shows that investors should hold more cryptocurrencies than stock market indices to minimize risk while maintaining unchanged expected portfolio returns. A comparison of the optimal weight values before and during the cryptocurrency crash indicates an insignificant difference in cryptocurrency investments but there is a significant decline in commodity investments. Before the cryptocurrency crash, investors needed to invest 52.7% of their budgets in GOLD and 47.3% in FTSE. During the cryptocurrency crisis, only 44.8% needed to be invested in Gold and 55.2% in FTSE, thereby indicating an 18% decrease in Gold investments.

For the two periods, the mean values of the hedge ratio were low under the three GARCH model estimations. This suggests a high effective hedge in stock market indices. For example and during the pre-cryptocurrency crash, a hedge ratio of 0.06 (0.15) implies that a $1 long in Bitcoin assets should be shorted by approximately 6 (15) cents of the FTSE (NIKKEI) under DCC model estimation.

Additionally, Tables 6 and 7 depicted results, relevant to the various GARCH model estimations, also indicate that the best hedging instrument fit for the FTSE stock market index turns out to be the Gold, followed by VIX during the two periods indicating that no difference in the hedging strategy before and during the cryptocurrency crash. As for the NIKKEI stock-market index case, the same result was detected during the two periods. Indeed, GOLD and WTI prove to stand as the most effective hedging instrument. Concerning the American and emerging markets, the hedging strategy was reversed between the two periods. As for MSCI Emerging stock market, the VIX and the Bitcoin were the appropriate effective hedging instruments during the pre-cryptocurrency crash while the Gold and VIX are the best instruments during the cryptocurrency crash.

5. Conclusions

In the present paper, an attempt is made to examine the usefulness extent of applying gold, WTI oil, VIX and five cryptocurrencies' prices to hedge four stock market indices (FTSE, SP500, NIKKEI and MSCIEM) related investments, via three different variants of MGARCH models. A major distinctive feature of this study lies in the comparison it establishes among the dynamic conditional correlations estimated through the DCC, ADCC and GO-GARCH models. The study is then redesigned to take a different perspective, by using a rolling-window estimation to calculate the one-step-ahead volatility forecasts, followed by a comparison of the most optimally achieved hedge ratios.

As a matter of fact, four major conclusions can be drawn following conduction of this study. In a first place, and on applying three MGARCH model variants, it has been discovered that the correlations between each of the stock market indices and each of the cryptocurrencies, Gold, VIX and WTI appear to indicate that the time-varying conditional correlations estimated via both of the DCC and ADCC model versions turn out to display similar patterns. As regards the GO-GARCH based conditional correlations, they tend to exhibit greater peaks and troughs, possibly due to the fact that the GO-GARCH modeling framework turns out to incorporate noticeable volatility spillovers. In a second place, it has been revealed that both of the Bitcoin and Ethereum appear to demonstrate negative dynamic dependence with the FTSE and Nikkei associated indices. Indeed, the correlations between the Nikkei and each of the VIX and WTI, on the one hand, and between the SP500 index and each of the Bitcoin, Ripple, Gold and VIX

Table 6. Optimal hedge ratio, optimal weight (in mean) and hedging effectiveness for period before cryptocurrency bubble

Instrument	Model	FTSE ω	FTSE β	FTSE HE	NIKKEI ω	NIKKEI B	NIKKEI HE	S&P 500 ω	S&P 500 β	S&P 500 HE	MSCIEM Ω	MSCIEM β	MSCIEM HE
Bitcoin	DCC	0.932	0.0074	0.0629	0.845	−0.0009	0.1554	0.942	−0.0046	0.0624	0.912	−0.0134	0.1019
	ADCC	0.985	0.0071	0.0114	0.914	0.0002	0.0858	0.961	−0.0052	0.0426	0.945	−0.0094	0.0654
	GOGARCH	0.499	0.0545	0.0145	0.487	0.0193	0.0134	0.485	−0.0931	0.0084	0.507	−0.0279	0.0197
Dash	DCC	0.984	0.0072	0.0123	0.955	0.0044	0.0415	0.994	0.0112	0.0033	0.982	0.0045	0.0147
	ADCC	0.991	−0.0005	0.0166	0.955	0.0033	0.0423	0.993	0.0116	0.0040	0.983	0.0102	0.0121
	GOGARCH	0.503	0.0736	0.0112	0.519	−0.0474	0.0134	0.494	0.1660	0.0281	0.535	0.0530	0.0056
Euther.	DCC	0.987	−0.0016	0.0236	0.966	−0.0111	0.0536	0.992	0.0005	0.0149	0.984	−0.0051	0.0357
	ADCC	0.991	−0.0005	0.0165	0.983	−0.0003	0.0185	0.994	0.0003	0.0136	0.991	0.0005	0.0191
	GOGARCH	0.427	−0.0364	0.0216	0.560	−0.0942	0.0082	0.475	−0.1419	0.0143	0.462	−0.0809	0.0097
Monero	DCC	0.997	0.0054	0.0012	0.990	0.0072	0.0066	0.998	0.0041	0.0008	0.998	0.0054	0.0009
	ADCC	0.999	0.0087	0.0004	0.989	0.0075	0.0079	0.999	0.0065	0.0003	0.999	0.0074	0.0005
	GOGARCH	0.473	0.0471	0.0071	0.446	0.1668	0.0157	0.524	0.0538	0.0026	0.502	0.0958	0.0102
Ripple	DCC	0.990	0.0081	0.0081	0.955	−0.0015	0.0475	0.986	−0.0018	0.0161	0.987	0.0059	0.0094
	ADCC	0.997	0.0092	0.0018	0.976	0.0003	0.0254	0.992	−0.0005	0.0107	0.994	0.0064	0.0032
	GOGARCH	0.471	0.0705	0.0034	0.490	−0.0125	0.0003	0.467	−0.0238	0.0004	0.542	0.0746	0.0033
Gold	DCC	0.527	−0.1447	0.5442	0.358	−0.2945	0.7314	0.598	−0.1831	0.5032	0.501	−0.0142	0.0033
	ADCC	0.541	−0.1105	0.5162	0.368	−0.1841	0.6930	0.625	−0.0773	0.4278	0.503	0.0655	0.4688
	GOGARCH	0.497	−0.1934	0.0604	0.450	−0.1201	0.0508	0.435	−0.1436	0.0707	0.451	0.0033	0.0356
WTI	DCC	0.940	0.0862	0.0380	0.796	0.0488	0.1719	0.969	0.0799	0.0168	0.944	0.0882	0.0290
	ADCC	0.940	0.0819	0.0402	0.783	0.0513	0.1847	0.972	0.0803	0.0165	0.930	0.0880	0.0395
	GOGARCH	0.521	0.3633	0.1253	0.524	0.1434	0.0229	0.499	0.3935	0.1481	0.515	0.2995	0.0920
VIX	DCC	0.944	−0.0483	0.2499	0.944	−0.0285	0.1063	0.931	−0.0706	0.5724	0.949	−0.0415	0.1959
	ADCC	0.951	−0.0404	0.2025	0.949	−0.0218	0.0878	0.939	−0.0617	0.5219	0.956	−0.0321	0.1421
	GOGARCH	0.514	−0.4774	0.2342	0.434	−0.0799	0.0478	0.518	−0.7147	0.5506	0.496	−0.3292	0.1425

Table 7. Optimal hedge ratio, optimal weight (in mean) and hedging effectiveness for period during cryptocurrency bubble

Instrument	Model	FTSE ω	FTSE β	FTSE HE	NIKKEI ω	NIKKEI β	NIKKEI HE	S&P 500 ω	S&P 500 β	S&P 500 HE	MSCIEM Ω	MSCIEM β	MSCIEM HE
Bitcoin	DCC	0.960	−0.0069	0.0477	0.923	−0.0181	0.0974	0.945	−0.0107	0.0671	0.960	0.0036	0.0406
	ADCC	0.972	−0.0055	0.0345	0.951	−0.0113	0.0631	0.967	−0.0031	0.0365	0.977	0.0062	0.0205
	GOGARCH	0.530	−0.0937	0.0236	0.494	−0.1372	0.0146	0.443	−0.0655	0.0052	0.479	0.0549	0.0172
Dash	DCC	0.983	−0.0018	0.0186	0.973	0.0022	0.0249	0.972	−0.0076	0.0424	0.986	0.0042	0.0109
	ADCC	0.984	−0.0013	0.0177	0.977	0.0053	0.0191	0.974	−0.0052	0.0367	0.987	0.0049	0.0099
	GOGARCH	0.498	−0.0233	0.0139	0.528	−0.0275	0.0095	0.555	−0.0955	0.0083	0.496	0.0334	0.0112
Euther.	DCC	0.989	0.0002	0.0113	0.967	−0.0109	0.0487	0.981	−0.0026	0.0219	0.991	0.0079	0.0133
	ADCC	0.988	−0.0004	0.0121	0.970	−0.0104	0.0460	0.984	−0.0001	0.0162	0.992	0.0083	0.0123
	GOGARCH	0.490	0.0064	0.0089	0.529	−0.0676	0.0070	0.421	0.0597	0.0091	0.514	0.0746	0.0064
Monero	DCC	0.981	−0.0020	0.0211	0.968	−0.0005	0.0327	0.971	−0.0058	0.0361	0.982	0.0022	0.0163
	ADCC	0.982	−0.0018	0.0201	0.974	0.0029	0.0240	0.974	−0.0039	0.0327	0.985	0.0040	0.0124
	GOGARCH	0.511	−0.0473	0.0187	0.480	−0.0347	0.0010	0.446	−0.0547	0.0023	0.489	0.0333	0.0373
Ripple	DCC	0.993	0.0088	0.0036	0.967	−0.0057	0.0399	0.976	−0.0031	0.0299	0.990	0.0083	0.0063
	ADCC	0.995	0.0074	0.0021	0.977	−0.0041	0.0282	0.985	0.0002	0.0201	0.994	0.0078	0.0040
	GOGARCH	0.441	0.0782	0.0040	0.467	−0.0007	0.0010	0.493	0.0174	0.0019	0.558	0.0567	0.0051
Gold	DCC	0.448	−0.0634	0.5802	0.344	−0.1547	0.7014	0.420	−0.0590	0.6098	0.384	0.0954	0.5783
	ADCC	0.449	−0.0640	0.5807	0.374	−0.1341	0.6665	0.440	−0.0404	0.5841	0.389	0.1193	0.5691
	GOGARCH	0.495	−0.0818	0.0066	0.521	−0.1043	0.0114	0.449	−0.1525	0.0142	0.529	0.0343	0.0343
WTI	DCC	0.901	0.0388	0.0820	0.783	−0.0130	0.2272	0.865	0.0370	0.1180	0.919	0.1042	0.0427
	ADCC	0.904	0.0621	0.0694	0.783	−0.0150	0.2285	0.869	0.0479	0.1083	0.908	0.1114	0.0488
	GOGARCH	0.491	0.1163	0.0135	0.474	−0.0381	0.0020	0.433	0.0791	0.0048	0.505	0.2553	0.0677
VIX	DCC	0.969	−0.0242	0.1324	0.974	−0.0086	0.0403	0.958	−0.0351	0.3091	0.949	−0.0450	0.2611
	ADCC	0.969	−0.0232	0.1269	0.976	−0.0075	0.0384	0.961	−0.0318	0.2967	0.949	−0.0452	0.2556
	GOGARCH	0.458	−0.3972	0.1167	0.512	−0.0335	0.0136	0.519	−0.2571	0.0643	0.530	−0.4290	0.2013

indices turn out to be negative. These results testify well that these assets may well serve to provide rather effective diversification opportunities. The correlations between FTSE and Ethereum, as well as between the SP500 index and the Bitcoin appear to exhibit a noticeable asymmetric dependence structure as far as the GO-GARCH modeling extent is concerned. In a third place, our empirical results tend to highlight well that the digital assets related hedging capabilities turn out to be rather fragile and weak, in terms of the maximum values of the hedging effectiveness. In addition, both of the DCC and ADCC models appear to yield similar results while the GO-GARCH model tends to record the highest (HE) values with respect to the entirety of the considered refits. Indeed, one can notice that, overall, the GO-GARCH model estimated portfolio proves to stand as the most effective framework in terms of risk reduction, as compared to the DCC and ADCC models' estimated portfolios.

Moreover, the results achieved appear to reveal well that no significant difference seems to persist between the attained hedging effectiveness coefficients (HE), no matter of which MGARCH model variant is being, and in regard to the different considered refits (20, 40 and 60 days). Concerning these refits, it has been discovered that the most appropriate instrument fit to hedge the FTSE stock market index turns out to be the VIX volatility index, respectively followed by the WTI and Ripple. As for the NIKKEI stock market index case, GOLD proves to represent the most effectively suitable hedging instrument, respectively followed by the VIX and the WTI. Finally, regarding the SP500 stock market index, VIX, WTI, Gold and Dash turn out to represent the most appropriately efficient hedging instruments, respectively.

According to these reached results, the entirety of these hedging instruments tends to exhibit negative average hedge-ratio values. This finding is justified by the persistence of negative conditional correlations, which indicates that for each of these asset pairs, long positions should be retained and considered with respect to each asset. Worth highlighting, also, is that except for the pairs (Nikkei-Bitcoin), (FTSE-WTI), (SP500-WTI), (Nikkei-WTI), (FTSE-Gold), (SP500-Gold) and (Nikkei-Gold), regarding which the optimal hedging-instrument weights, associated with the investor portfolio, tend to range from 10.36 to 68.73%. As to the remaining pairs, the optimal hedging instruments' weights tend to remain persistently below rate of 8.56%, regardless of whether it is the DCC or ADCC model version that is being used.

Finally, the results show that the hedging strategy instrument was indifferent for FTSE and NIKKEI stock markets during the two periods while for the American and emerging markets, the hedging strategy was reversed from the pre-cryptocurrency crash to the during cryptocurrency crash period.

As far as investors and market participants are concerned, a direct implication of this result is the recommendation to follow the evolution of the different hedging instruments. Indeed, as already stated, the evolutionary prices of gold, crude oil, VIX and cryptocurrencies turn out to be largely fueled by the legal uncertainties surrounding these markets. A clear legal framework should certainly culminate in the diversification of the different behaviors of these instruments, which would, therefore, affect their diversification, hedging capabilities as well as their relationships with the existing asset classes. For policymakers, our modest analysis offers a reliable clue effectively useful to resolve the query as to whether institutional and traditional investors in commodity and cryptocurrency markets should be held responsible for the rise noticeable in the stock market indices.

Notes

1. The rest of Time varying correlations as reflected through the various MGARCH versions (refit = 20) are by author under request.
2. The rest of correlation figures (for every 40 and every 60 daily observations) are by author under request.

3. The remaining figures (concerning the refit 40 and refit 60) is available by the author son request.

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