



Modelling the joint dynamics of financial assets using MGARCH family models: Insights into hedging and diversification strategies

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ABSTRACT

This study analyzes the conditional correlations and thereafter constructs the optimal portfolios for G-12 countries' stock market returns and national benchmark bonds, crude oil, gold and the volatility index (VIX) returns. We use daily data ranging from January 1, 1994 to May 3, 2021 and compare the DCC, ADCC and GO-GARCH methods to investigate the past shocks and volatility transmissions. The rolling estimation techniques are employed to construct one-step-ahead forecasts of the dynamic conditional correlations and the optimal hedge ratios of G12 markets and other variables. For most of the situations studied, the volatility index (VIX) generates the best effective hedge to stock returns for these markets. The national benchmark bond indices create the second-best hedge. The risk and downside risk measures suggest that a sole stock exhibits the greatest risk and the expected maximum loss compared to a mixed bond-stock, a mixed VIX-stock, or a mixed gold-stock portfolio. The results are robust to alternative modeling specifications, model selection, as well as distributional assumptions.

1. Introduction

The notion of not putting all the eggs in one basket is centuries old, yet in their endeavours to seek higher returns and safeguard their equity portfolios from market risks, for diversification, the 21st-century investors are looking beyond the traditional assets. Concurrently, hedge and safe-haven assets and volatility derivatives are now desired venues of investment that investors are seeking for 'Armageddon trade' that will insure them in times of stock market collapses.¹ These are refuge assets that insure investors from risks during future financial downturns (Brenner and Zhang, 2006; Szado, 2009; Baur and McDermott, 2010; Chen et al., 2011 and Silvennoinen and Thorp, 2013). Traditional safe assets (e.g., Bonds and commodities) aside, volatility derivatives have gained popularity during the last two decades. There have been significant developments in volatility trading, which are attributed to the introduction of a new range of derivatives, including VIX futures, options, and exchange-traded funds, which make trading in volatility more

feasible to investors around the world. Expansion in the development of volatility derivatives over the last decade is considered an important financial innovation in the volatility trading markets, which should provide more opportunities for investors to seek refuge assets to heed portfolio losses (Whaley, 2013). Since their inception, investment in VIX has attracted much-heated debate, focusing on its controversial diversification benefits for passive buy-and-hold investors who may or may not find this type of investment fit for purpose (Bahaji and Aberkane, 2016). Besides the debate and controversy, in parallel to the improving liquidity of the VIX options and futures, the trading volume of these derivatives has been increasing due to their significance in hedging tail risks, particularly when the markets are under stress. In this regard, a short position in VIX futures or options may reduce a stock portfolio's losses during bad investing times (Park, 2016).

The integration among international stock markets has been significantly increasing in the last few decades, which is manifested in the increased correlations between international stock markets during crises

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¹ Baur and Lucey (2010) formally propose testable definitions of the "hedge" and "safe haven" as an asset that is negatively correlated with a stock on average periods and in the periods of strong stock market declines, respectively.

(Nasir and Du, 2018). These associations have been strengthened by the expansionary monetary policies, which have made investors perceptive for low correlated assets as alternative investment havens. The remarkable negative correlation of the volatility VIX index with the global stock market during market stress periods is a specific feature of VIX that makes its derivatives more attractive and very appealing hedging tools (Brenner and Zhang, 2006; Szado, 2009 and Chen et al., 2011). This phenomenon of negative correlations refers to the “leverage effect,” which means that companies become more levered as per the decrease in asset prices (Black, 1976; Ait-Sahalia et al., 2013), and it is natural to expect their stocks to become riskier, and hence more volatile.²

The strong negative correlations between VIX and stock returns observed during recessions and crisis periods present a strong motivation for us to test the viability of VIX³ as a beneficial hedging tool for the G-12 stock markets, as compared to other traditional safe-haven assets. The “lottery-like” features that VIX possesses, such as positively skewed variations and high liquidity, increase investors’ attractiveness with high skewness preferences (Barberis and Huang, 2008). Financial crises, such as the Asian financial crisis 1997–98, the Dot-com crisis 2000–02, the global financial crisis 2007–09, the European debt crisis 2010–13, and the unprecedented Covid-19 crisis, have considerably impacted major market participants around the world. This has bestowed more relevance on the VIX related products to be considered in portfolios. On this aspect, Bahaji and Aberkane (2016) argued that VIX derivatives’ liquidity improves during market turmoil when the investors are looking for hedge instruments. This contrasts with the liquidity of credit derivatives such as the credit default swaps (CDS) that tend to decline during crises.

It is cogent to expect the portfolio managers and financial analysts to act cautiously while constructing their portfolios (Christensen et al., 2016). On theoretical grounds, the construction of optimal portfolio and diversification has also been the point of great interest to scholars (Markowitz, 1952; Sharpe, 1964). However, in practice, most stock market investors have heterogeneous expectations and prefer more defensive diversification strategies such as choosing safe-haven assets like commodities and benchmark bonds to complement their stock portfolios (Huang et al., 2016). Therefore, in the subject study, we analyze the performance of VIX relative to gold, crude oil, and benchmark bonds while also considering the fundamental concerns of investors who seek to improve the portfolio risk-adjusted returns. Moreover, the financial panics and the sudden spikes in global risk during the 2007 sub-prime crisis, which turned into the global financial crisis of 2008–2009, have cast doubts on the diversification benefits of traditional assets like stocks and bonds. Concomitantly, the main motivation for considering volatility derivatives (VIX) is expanding and developing these exotic instruments in recent years. There have been important financial innovations in volatility trading markets, which should provide more opportunities for investors to seek refuge assets to heed portfolio losses (Whaley, 2013; Bahaji and Aberkane, 2016; Park, 2016). On the other hand, gold is the traditional candidate in human

² Note: According to Choi and Richardson (2016), leverage and volatility of an asset play a key role in determining the firm’s riskiness as well as the volatility of its stock. This is because asset volatility is one of the important determinants of capital structure valuations and the standard risk and return tradeoff independence of financial leverage.

³ Since the VIX cash is not tradable, an exposure to volatility can be made while investing in volatility derivatives e.g., VIX futures, options or in VIX squared portfolios (Szado, 2009; Chen et al., 2011). A long exposure to volatility gives a greater ability to diversify an equity portfolio instead of other assets particularly in periods of market turmoil (Signori et al., 2010). The Chicago Board Options Exchange (CBOE) has introduced several volatility indices where the volatility index (VIX) has received considerable attention as the fear gauge. The VIX represents the market’s expectations over the next 30 days and implies the volatility of the S&P 500 index options (Whaley, 2009).

history with no introduction to its usage storage of value and hedging property in almost every society and era. The usefulness as an investment asset as well as hedging ability of gold is beyond its traditional use in jewelry and ornaments (Lucey and Li, 2015; Beckmann et al., 2015). Similar to gold, oil is also a strong candidate to be brought into consideration. The importance of oil is *prima facie* evidence in the fact that it is the largest traded commodity in the world with an annual market size of over US\$ 1.7 trillion (Nasir et al., 2018). Nonetheless, oil is also used by non-commercial traders as a financial asset as they hold the oil futures to hedge against future shocks (Kaufmann, 2011; Tokic, 2011; Masters, 2008; Medlock and Jaffe, 2009; Weiner, 2002; Cifarelli and Paladino, 2010; Kolodziej and Kaufmann, 2013; Filimonov et al., 2014). Modelling the volatility dynamics between commodities (gold and oil) and stock returns is also important to study because of the financialisation of commodity markets (Silvennoinen and Thorp, 2013). As commodities often act as a natural hedge against inflation, they become more attractive during such downturns (Daskalaki et al., 2014). Lastly, the bonds are also an important candidate to be brought into consideration due to the notion of *flight-to-safety*, particularly since the global financial crisis 2008–09 and the European debt crisis (Mensi et al., 2015). The *flight-to-safety* implies a shift of investments from risky assets (e.g., stocks) to bonds (Pieterse-Bloem et al., 2016; Ingelbrecht et al., 2013; Baur and Lucey, 2009) and hence potentially incentivize diversification (Bordo et al., 2001). The correlation between stock and bonds is also important for asset allocations, hedging, and risk analysis (Aslanidis and Christiansen, 2014; Connolly et al., 2005; Campbell et al., 2009; Forbes and Rigobon, 2002; Kodres and Pritsker, 2002; Baker and Wurgler, 2012; Kolluri et al., 2015). To gain high risk-adjusted returns during global uncertainty, investors shift the primary focus of their investments from equities (referred to as risk-on trades) to bonds (referred to as risk-off trades) for the wealth preservation. The higher the negative correlation between bonds and stocks is, the greater are the benefits from the *flight-to-safety* effect. Concomitantly, considering these factors, in this study, we are focusing on VIX, gold, crude oil, and benchmark bonds.

This study contributes to the existing literature in various perspectives. Firstly, it investigates the dynamic dependence between gold, crude oil, the benchmark bonds, the volatility index (VIX), and the stock markets of G-12 countries, namely; Australia, Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Spain, Sweden, Switzerland, the UK and the U.S. We compare the performance of these refuge assets as hedging tools for the stock markets of G-12 countries which can be considered as the major representatives or participants in the global stock market. These markets are very attractive to international and domestic investors (see, e.g., Christensen et al., 2016; Su, 2015). Thus, it is worth analyzing the hedging effectiveness of the safe-haven assets which derives the implications of portfolio management strategies for these major stock markets. Secondly, this study pays particular attention to the performance of the hedging instruments including benchmark bonds, crude oil, gold, and VIX in the recent unprecedented Covid-19 crisis, 2010–12 European debt crisis and the 2008–09 global financial crisis. Thirdly, the subject study employs a very rich set of novel empirical approaches, and in so doing, it uses the Dynamic Conditional Correlation (DCC), Asymmetric Dynamic Conditional Correlation (ADCC), and Generalized Orthogonal GARCH (GO-GARCH) models to estimate the conditional correlation variances and covariances to assess their usefulness in hedging effectiveness. The choice of these models is motivated by various reasons. Several prior studies use the DCC models of Engle (2002) to study the symmetry in the time series proxies (Ciner et al., 2013; Arouri et al., 2011; Chang et al., 2011; Sadorsky, 2012, 2014; Lin et al., 2014) and to capture the persistence in volatility and time-varying correlations in the time series dataset. These models account for other important features of time series data such as asymmetry in the dynamics of the markets by employing the ADCC model of Capriello et al. (2006), which is an extended version of the DCC model and is used to measure the impact of positive and negative shocks separately

(Cappiello et al., 2006; Ederington and Guan, 2010; Chkili et al., 2011; Chkili, 2016). The asymmetric features reveal that negative shocks have more pronounced effects than positive shocks. That pushes investors to seek refuge in safe-haven assets in order to safeguard their investments from extreme negative shocks. Moreover, it applies the GO-GARCH model by Van der Weide (2002) to investigate the spillover effect in volatility under linear transformation, time-varying correlations and time-varying variances, and (asymmetric) volatility spillovers. In comparison to other multivariate GARCH models, the GO-GARCH model is more flexible (Basher and &Sadorsky, 2016). However, considering the enormous labour required for the estimation of this model, it is often condoned.

Fourthly, the subject treatise assesses the optimal hedge ratios calculated from different GARCH specifications and examines their role in performing hedging effectiveness and risk reduction management. It also uses the GARCH models to construct one-step-ahead hedge ratios, which is different from the previous studies that employ the current hedge ratio as a proxy for the next period. Moreover, we employ the rolling window techniques to estimate the one-step-ahead ratio, which is more suitable to examine the hedging effectiveness of various assets under study. Finally, we apply the rolling window to investigate the portfolio diversification benefits of financial assets using different risk measures, such as value-at-risk (VaR) reduction, risk-reduction effectiveness, Regret Reduction (Re), Expected Shortfall (ES) and Semi-Variance (SV).⁴ It analyzes the implications of the results for downside risk reduction of three different portfolios composed of each pair of assets. Addressing the curse of dimensionality associated with estimating multivariate GARCH models with large datasets is challenging. However, it is also intriguing because it enables us to study the symmetry in the time series and capture the persistence in the volatility and time-varying relations between the equity markets and each of the bonds, commodities and volatility derivatives. Our key findings yielded from an exhaustive empirical exercise show that the VIX provides the best and effective hedge to G-12 countries' stock returns and that the national benchmark bond indices act as a second-best hedge.

Further, the risk and downside risk measures highlight that the stock-only portfolio offers the greatest risk as well as the expected maximum loss, compared to a mixed bond-stock, VIX-stock, or gold-stock portfolios. When added to stock portfolios, the bonds (crude oil) provide the highest (lowest) risk- and downside risk-reduction gains. However, the risk measures also indicate that the mixed VIX-stock portfolios can facilitate to mitigate risk much more than the mixed gold-stock portfolios. This study examines stock markets in the G-12 countries. This is an alternative perspective compared to prior literature, for example, global assets (Tiwari et al., 2018) and emerging markets (Basher and Sadorsky, 2016). Our approach facilitates to assess specific portfolio in pursuit of hedging and diversifying objectives, especially in developed financial markets. On the other hand, this study is expected to find the role of popular alternative investments in the digital era, those are, fixed income (bond), natural resource (gold), energy representative (oil), and sentiment (VIX). This design further suggests policy implications on the future of financial markets, especially in the context of the digital revolution.

The expected findings have tremendous implications for investors seeking a higher risk-return tradeoff from G-12 countries' stock markets by diversifying a stock portfolio with various other financial assets such as bonds, VIX futures, or commodities like oil and gold, which would reduce the expected maximum loss of the diversified portfolio. While

⁴ In so doing, we examine the risk and downside risk-mitigation effectiveness of different assets (i.e., stock-bond, stock-oil, stock-gold and stock-VIX future) Portfolios II, III and IV, compared to the benchmark portfolio, Portfolio I, i.e. the stock only portfolio. The portfolio weights are computed based on the risk-minimization strategy (i.e., Portfolio II), the variance minimization strategy (i.e., Portfolio III) and the equally weighted (i.e., Portfolio IV).

considering the equally weighted portfolio (Portfolio IV) and the optimally weighted portfolio (Portfolio II), we find that the bond or the VIX futures in the equity portfolio generates the best risk reduction. Secondly, findings of the VaR reduction benefits indicate that Portfolio II exhibits the largest VaR reduction. Thirdly, we find that the ES reduction gains are higher for the mixed bond-stock portfolios than for the mixed VIX-stock and gold-stock portfolios. Finally, as *Semi Variance* (SV) and *Regret Reduction* (Re) are concerned, Portfolio II and Portfolio IV create the largest downside risk reduction for the bond-stock, VIX-stock and gold-stock portfolios, respectively. Moreover, under these risk measures, the diversified portfolios, when the VIX futures are included, perform better than gold and crude oil. We suggest that trading volatilities in the episodes of financial downturns can help safeguard the equity portfolios against losses.

The rest of the paper is structured as follows: Section 2 summarises the literature review and section 3 presents the employed methodologies. Section 4 and 5 provide the data and empirical findings of the study, respectively. Finally, Section 6 reports the conclusion and the recommendations.

2. Literature review

In the theoretical literature on the diversification and optimal portfolio construction, Markowitz (1952) and Sharpe (1964) made seminal contributions which led to the development of Modern Portfolio Theory (MPT). Although the contributions on the valuation of stock (by fundamental analysis) based on their intrinsic values were made by Graham and Dodd (1934) in the *Security Analysis* and by Williams (1938) in the *Theory of Investment Value*, the important aspect which was not addressed was the diversification. This caveat in the body of knowledge was addressed by Markowitz (1952) and Sharpe (1964) by empirically establishing the importance of diversification. Though, the principle of not putting all your eggs in one basket is the foundation of all the reasoning and logic of diversification, in which basket and how much is the question! Nonetheless, the financial development and Financialisation in the last few decades have led to the development of exotic assets, including VIX, which also requires accounting for these developments. In fact, the traditional assets such as gold and oil are not commodities per se but, due to Financialisation, have given birth to the commodity-based derivatives, e.g., gold and oil futures. Concomitantly, we must employ the state of the art empirical tools to analyze the association among these asset classes and investigate how the principle of diversification and hedging plays out here.

In the review of the recent empirical literature, we look at studies that examine gold as an alternative and safe-haven investment, crude oil as an important commodity or asset, volatility derivatives as a hedging tool, and the flights-to-safety phenomenon (associated with sovereign bonds) as a means of risk-rebalancing which once again has attracted the attention of both researchers and policymakers. However, given our objective to investigate alternative assets' role, we pay particular attention to studies that deal with hedging and diversification strategies. In this context, several scholars such as Tang and Xiong (2012), Silvennoinen and Thorp (2013) and Basher and &Sadorsky (2016) argue that the financialisation of commodities⁵ markets offers the investors with various approaches to hedge and diversify their portfolios. Moreover, Daskalaki and Skiadopoulos (2011) argue that commodities have proved as an alternative financial asset with significant diversification benefits in equity portfolios. This makes the commodities a financial protector and encourages the investors to invest in commodities during market stresses (Silvennoinen and Thorp, 2013). On this aspect, Sadorsky (2012) explored the hedging benefits of oil for the European stocks, Raza et al. (2018) investigated the hedging effectiveness of

⁵ For other studies on commodities, see for instance, Rehman et al. (2020), Shahzad et al. (2017), Ren et al. (2022), etc.

commodities futures for the US real estate stock portfolios, Chang et al. (2010) analyzed hedge abilities of gasoline and oil spot prices against their own future prices in bear and bull markets and Bessler and Wolff (2015) examined the performance of commodities in various assets portfolios. These findings are supported by the correlation between stock prices and natural gas (Kumar et al., 2019). Similarly, Raza et al. (2016), Lucey and Li (2015) and Beckmann et al. (2015) conclude that gold as an alternative investment is more useful than its traditional uses.

In terms of their testable definitions and characteristics, the hedge is described as an asset that is negatively correlated with a stock on average periods, while the “safe haven” as an asset that has a negative correlation in the periods of strong stock market declines (Baur and Lucey, 2010). Considering these characteristics, Baur and McDermott (2010) have explored the strong and weak forms of safe havens and found that gold is a strong safe haven for most developed markets and a weak safe haven for small and emerging markets. Ali et al. (2020) and Coudert and Raymond-Feingold (2011) also reported similar results for developed markets. In later studies, by investigating the returns and volatility spillovers, Arouri et al. (2015), Kumar (2014), and Ewing and Malik (2013) found strong evidence of volatility transmission among oil, gold, and stock markets. These authors suggested that adding gold to stock portfolios significantly decreases the downside risk. In the context of crisis, Creti et al. (2013) found that the dynamic correlations between stock markets and commodities showed a highly volatile pattern during the global financial crisis of 2008–09 and that the gold-stock pair exhibit a stable pattern. More interestingly, the negative correlation between gold and stock was observed in crisis periods, which confirms the safe haven role of gold. In relevant studies, Raza et al. (2016) conclude that gold proves as a strong safe haven for BRICS and Islamic stock index during the Asian financial crises and Naeem et al. (2021) find that gold acts as a diversifier for the overall Islamic equity index and most of the Islamic stock sectors during normal market conditions. In a comparative study, Gurgun and Unalmis (2014) reported that the safe haven aptitude of gold is more pronounced for developed countries compared to small and emerging market countries.

Among the other metals and commodities, gold is quite unique because of its ability to mitigate some of the investors’ risks while holding other assets. In this context, Ciner et al. (2013) and Miyazaki et al. (2012) examined the conditional correlations among the major asset classes such as bond, stock, gold, oil, and currencies. Their findings showed that the benchmark bonds are an effective hedge for stock markets, however, in comparison to crude oil, gold can be regarded as an effective hedge for currency movements and a better safe haven for stock market investors. In a similar study on the exchange rate movement and the role of gold, Reboredo (2013) reported similar findings showing that gold has the ability to hedge currency portfolios. Moreover, Capies et al. (2005), Joy (2011) and Ciner (2011) have also analyzed and reported on the safe-haven status of gold across a wide range of currencies.

Financialisation of the commodity markets has important implications for portfolio diversification. Interestingly, the phenomenon of investing in commodity markets has motivated investors to select commodities as a protector in episodes of financial downturns if the macroeconomic shocks have a different impact on stock and commodities prices (Silvennoinen and Thorp, 2013). On this aspect, Narayan and Sharma (2011) reported that oil prices affect U.S firms’ returns, and this effect is regime-dependent. By studying the returns and volatility determinants, in their later study, Narayan and Sharma (2014) argued that oil prices are a significant predictor of returns and volatility of stock markets and that the information on commodity futures is helpful to devise trading strategies to gain maximum returns from investment. Similarly, Mensi et al. (2015) reported that commodities’ investments are profitable based on trading strategies and that profits are dependent on structural-breaks.

The non-commercial traders usually trade oil as a financial asset and also hold oil futures to hedge against future shocks (Masters, 2008; Medlock and Jaffe, 2009). In this context, Kaufmann (2011) and Tokic

(2011) argued that the speculations are the main reasons for the rapid changes and spikes in oil prices, while Weiner (2002) argued that non-commercial traders act according to the expectations of oil markets. A number of studies that accounted for this behaviour reported that the non-commercial trader’s behaviours (i.e., buy and sell) magnify the oil price volatility (Cifarelli and Paladino, 2010; Kolodziej and Kaufmann, 2013; Filimonov et al., 2014). This provides investors with the rationale to manage their portfolio risk through diversification. Gorton and Rouwenhorst (2006) documented that commodity and oil prices positively correlate with inflation but a negative correlation with bonds and stocks, which make them perfect hedges. Similar to this notation, it is well documented in the literature that the negative correlation between crude oil and stock returns highlights the role of crude oil as a financial asset (Stoupas and Kiohos, 2021; Zaighum et al., 2021; Miller and Ratti, 2009; Ciner, 2001; De Roon et al., 2000; Sadorsky, 1999; Ferson and Harvey, 1995; Kaneko and Lee, 1995).

Recent literature affirms the nexus between stock and commodities markets. Empirical findings are indicated in various perspectives, for example, market conditions of Islamic markets (Naeem et al., 2021; Hamma et al., 2021), energy prices (Zaighum et al., 2021), commodities’ returns and volatility (Shahzad et al., 2017), and post-crisis assessments (Stoupas and Kiohos, 2021).

The literature on volatility derivatives as hedging tools is limited, but some remarkable studies have been done in the last few years. For instance, Chen et al. (2011) argued that adding VIX futures to a benchmark portfolio leads to a significant expansion in the efficient frontier of the portfolio. Similarly, using a risk-reward analysis, Warren (2012) demonstrated that VIX futures could potentiate the performance of stock portfolios, while Signori et al. (2010) reported on the benefits of volatility trading using the value-at-risk (VaR) measure. Overarching, these authors found the volatility-related assets to help reduce the risk of both leveraged and unleveraged stock portfolios. Jung (2016) highlighted the importance of volatility derivatives and portfolio insurance strategies and argued that the volatility derivatives are effective hedging tools because they are devised to estimate expected volatility. Later studies also emphasized the importance of volatility derivatives such as Raza et al. (2019) find that VIX exhibits highest hedging effectiveness for both Islamic and conventional stock portfolios and Hamma et al. (2021) conclude that VISTOXX is the best asset to hedge Islamic and conventional stock portfolios.

Since the global financial crisis of 2008–09 and the European sovereign debt crisis 2010–12, the flight-to-safety phenomenon has once again become the focus of investors and researchers. As obvious, the crisis plunged asset prices and led to a significant increase in their correlations (Mensi et al., 2015). This phenomenon indicates a shift in investments from risky assets (e.g., stocks) to bonds (Pieterse-Bloem et al., 2016; Ingelbrecht et al., 2013; Baur and Lucey, 2009). Moreover, flight-to-safety incentivize investors to diversify their portfolios across asset classes (Bordo et al., 2001). Consequently, the negative correlations between stock and bond returns in turmoil periods motivate the shift of investments from risky assets to safer assets (Guler and Ozlale, 2005; Mardi Dungey et al., 2009). The correlation between stock and bonds is important for asset allocations, hedging, and risk analysis. There is a considerable variation in the stock-bond co-movement, and it is important to consider the tails of the distribution by calculating the optimal portfolio weights (Aslanidis and Christiansen, 2014). For instance, portfolios composed of stock and bonds have higher diversification benefits in times of stress due to their negative correlation with each other (Connolly et al., 2005; Campbell et al., 2009). Conversely, the value-at-risk (VaR) of stock-bond portfolios remains high when the correlation is positive (Ilmanen, 2003). The strong positive correlation between stocks and bonds in the periods of market stress indicates lower diversification benefits in times when risk-reduction benefits are warranted (Bodart and Reding, 1999). Forbes and Rigobon (2002), Kodres and Pritsker (2002), Baker and Wurgler (2012), Aslanidis and Christiansen (2014), and Kolluri et al. (2015) document that the heightened

uncertainty in the capital markets results in flight-to-safety. In order to achieve high risk-adjusted returns during global uncertainty, investors shift the primary focus of their investments from equities (risk-on trades) to bonds (risk-off trades) for the preservation of wealth. The higher the negative correlation between bonds and stocks is, the greater the benefits from flight-to-safety strategies.

3. Methodology

In order to examine the hedging effectiveness of national benchmark bonds, crude oil, gold and the VIX, we have estimated the dynamic conditional correlations and then the dynamic hedge ratios. For this purpose, we have adopted three conditional volatility models, the dynamic conditional correlation (DCC)-MGARCH, the asymmetric DCC (ADCC)-MGARCH and the generalized orthogonal GO-GARCH models proposed by Engle (2002), Cappiello et al. (2006) and Van der Weide (2002), respectively. The use of the DCC models has been chosen because they help in determining whether the volatility shocks of the asset returns are complements or substitutes. Hence, the variations in the interdependence structure can be detected between the financial time-series.

Suppose, r_t is a $n \times 1$ vector of returns for an asset. Then, the autoregressive - AR(1) process of r_t conditioned on the information set I_{t-1} can be expressed as⁶:

$$r_t = \mu + \alpha r_{t-1} + \varepsilon_t, \quad (1)$$

We use $\varepsilon_t = H_t^{1/2} z_t$ to calculate the residuals in Equation (1), where H_t denotes the conditional covariance matrix for r_t and z_t . This matrix H_t is a $n \times 1$ vector of i.i.d. random errors. In the two step DCC model, first GARCH parameters are estimated and then the conditional correlations. Let the conditional covariance matrix ($n \times n$) be defined as:

$$H_t = D_t R_t D_t \quad (2)$$

where, D_t in Equation (2) indicates a diagonal matrix having the time-varying conditional standard deviations placed on its diagonals. The matrix D_t is written as:

$$D_t = \text{diag}\left(h_{1,t}^{1/2}, \dots, h_{n,t}^{1/2}\right). \quad (3)$$

Moreover, R_t represents the time-varying conditional correlation matrix of the returns on an asset and is expressed as follows:

$$R_t = \text{diag}\left(q_{1,t}^{-1/2}, \dots, q_{n,t}^{-1/2}\right) Q_t \text{diag}\left(q_{1,t}^{-1/2}, \dots, q_{n,t}^{-1/2}\right) \quad (4)$$

In Equation (3), we estimate the expressed ' h_{it} ' conditional variance using individual univariate GARCH models. Moreover, diagonal matrix H_t has been used for the GARCH (1,1) estimations. The components of H_t matrix can be defined as:

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1} \quad (5)$$

where, Q_t in Equation (4) represents asymmetric positive definite matrix and is given as under:

$$Q_t = (1 - \theta_1 - \theta_2) \underline{Q} + \theta_1 z_{t-1} z_{t-1}' + \theta_2 Q_{t-1} \quad (6)$$

In Equation (6), \underline{Q} denotes the standardized residuals $n \times n$ unconditional correlation matrix $z_{i,t} (z_{i,t} = e_{i,t} / \sqrt{h_{i,t}})$. The exponential smoothing process, employed to estimate the DCCs, is found to be linked with the non-negative θ_1 and θ_2 parameters. Later, the mean reversion of these constructed models can be examined by assessing whether the sum of the values of θ_1 and θ_2 is negative or less than 1. If so, then the constructed DCCs are said to be mean-reverting and vice versa. Therefore, we write the correlation estimator as:

⁶ As it is an AR process, information set I_{t-1} is embedded in the r_{t-1} .

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t} q_{j,j,t}}} \quad (7)$$

Further, the asymmetric dynamic conditional correlation (ADCC) is modelled as:

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1} + d_i \varepsilon_{i,t-1}^2 I(\varepsilon_{i,t-1}) \quad (8)$$

where, indicator function is represented through $I(\varepsilon_{i,t-1})$. The indicator function is equal to 1 and 0 if $\varepsilon_{i,t-1} < 0$ and $\varepsilon_{i,t-1} > 0$, respectively. Therefore, in the ADCC models, if the value for d_i is positive then there is a greater increase in the conditional variance due to the negative residuals than due to the positive ones. The financial assets are said to have an increased volatility due to their overreaction to an unexpected fall in asset prices, compared to a sudden rise, a phenomenon known as the "asymmetric or leverage effect". Therefore, the ADCC model is useful since they help in capturing the asymmetric effect for which bad news increases volatility more than good news of the same magnitude.

The dynamics of Q for the ADCC model are expressed as:

$$Q_t = \left(\underline{Q} - A' \underline{Q} A - B' \underline{Q} B - G' \underline{Q}^- G \right) + A' z_{t-1} z_{t-1}' + B' Q_{t-1} B + G' z_t^- z_t' \quad (9)$$

where the unconditional matrices of z_t and z_t^- are represented by Q and \underline{Q} , respectively. Note that the zero-threshold i.e., standardized errors is indicated through z_t^- which is equal to z_t when it is less than 0, and 0 otherwise. The matrices A , B , and G denote the coefficient matrices of $n \times n$ parameters.

A specific class of independently univariate and conditionally uncorrelated GARCH mechanisms is known as the GO-GARCH model. According to Ghalanos et al. (2014), a linear map (separately and in parallel form) uses the marginal density coefficients to explore these elements to the observed data. The estimation of margins separately through the GO-GARCH model offers much more flexibility, compared to the calculations done using multivariate GARCH models. The GO-GARCH model represents r_t (asset returns) as a function of m_t (conditional mean). The conditional mean may contain ε_t (error term), and an AR(1) term. The expression can be written as:

$$r_t = m_t + \varepsilon_t \quad (10)$$

A set of unobservable independent factors f_t (shown below) is used to map the $r_t - m_t$.

$$\varepsilon_t = Af_t \quad (11)$$

In Equation (11), an unconditional covariance matrix Σ and an orthogonal (rotational) matrix U as $A = \Sigma^{1/2} U$ have been constructed by decomposing a mixing matrix A . The rows (columns) of the mixing matrix A indicate assets (factors). Here, we can write the factors (f) as:

$$f_t = H_t^{1/2} z_t \quad (12)$$

In Equation (12), a random variable having a 0 mean and 1 variance is represented with z_t . Moreover, we estimate the factor conditional variances h_{it} using a GARCH. Further, $E(f_t) = 0$ and $E(f_t f_t')$ = 1 are satisfied through unconditional distribution of factors (f). Next, equations (10), (11) and (13) are combined to model the conditional mean of the asset returns as under:

$$r_t = m_t + H_t^{1/2} z_t \quad (13)$$

and the conditional covariance matrix of the asset returns ($r_t - m_t$) as:

$$\Sigma_t = AH_tA \quad (14)$$

There are two key assumptions of GO-GARCH model i.e., the conditional matrix H_t is a diagonal matrix whereas the matrix A is time-invariant. Here, a special case of OGARCH known as GO-GARCH model develops as we restrict matrix A to be orthogonal. As reported in literature, we apply the independent component analysis (ICA) to

model the U matrix (Zhang and Chan, 2009; Broda and Paoella, 2009).⁷ Since, financial times series data show the evidence of autocorrelation, volatility clustering and fat tails therefore, the addition of an AR(1) mean equation for each GARCH model is examined using the multivariate Student-t distributions for DCC and ADCC while a Normal Inverse Gaussian (NIG) process for GO-GARCH models.

4. Data and findings

4.1. Data description

The dataset consists of the daily stock returns of the following indices: S&P/ASX 200 (Australia), BEL 20 (Belgium), S&P/TSX (Canada), CAC 40 (France), DAX 30 (Germany), FTSE MIB (Italy), NIKKEI 225 (Japan), AEX INDEX (the Netherlands), MADRID SE (Spain), NASDAQ OMX (Sweden), SWX Swiss (Switzerland), FTSE ALL SHARE (UK) and S&P 500 (USA). The sample spans the period from January 2, 1994, to May 3, 2021, which includes the last five crises (e.g., the occurring Covid-19 crisis, the 2010-12 European debt crisis, 2007-09 global financial crisis, the 2000-02 dot-com and bubble burst, the 1997-98 Asian financial crisis, and the 1993-94 Mexican peso crisis). The rich dataset provides us with deep insight into the associations among the variables of interest and their dynamics. We evaluate the hedging effectiveness and diversification strategies of the benchmark bonds, crude oil, gold and the VIX. The data is retrieved from Thomson Reuters DataStream where the stock and Govt. 10-year benchmark bond indices are denominated in local currencies for each country. However, the volatility index VIX and futures on gold, crude oil traded at NYMEX are expressed in US dollars. The continuous future contract type 0 is considered as the nearby contract.

4.2. Descriptive stats and un-conditional correlations

The descriptive statistics as presented in Table 1 show that the average daily returns for all equity indices are positive except for Italy, where the returns are negative. Considering the sample period under study, benchmark bonds of Sweden and Germany show higher returns than the other countries' national benchmark bonds. However, gold and crude oil indicate higher returns as compared to the rest of the bond indices and VIX. Moreover, we find that the VIX (benchmark bond indices) indicate highest (lowest) standard deviation. The return distributions for the stock indices, benchmark indices, and gold and crude oil future indices are negatively skewed except for the volatility index and the national benchmark bond of Italy which are positively skewed. The Jarque-Bera test statistics and kurtosis coefficient confirm that the return series under study are non-normal. The coefficient of variation (C. V) shows that NIKKEI 225 (Japan) and the volatility index have the high variability, while the benchmark bonds of Belgium, Japan and the Netherlands have the least. The presence of the ARCH effect, confirmed through ARCH (12) LM test indicates that we correctly specify the DCC, ADCC and GO-GARCH models to capture the dynamic dependence between the stocks, benchmark bonds, gold, crude oil and VIX.

The pair-wise unconditional correlations presented in Table 2 indicate negative correlations among the stock index, the VIX and benchmark bonds for all the countries. However, this negative correlation between the individual stock index and VIX is stronger than that exists between stock and bonds. Moreover, the correlation coefficients for stocks and crude oil are positive and statistically significant for all countries except Italy. Across all asset returns, only gold shows a mixed

relation, having a significant and positive relationship for Australia and UK, while having a significant and negative relationship with the stock indices of France, Switzerland, and the United States, which suggests the market-specific diversification benefits of gold. These results correspond with the findings of Bredin et al. (2015) and Beckman et al. (2015).

All the estimated versions of DCC models include a GARCH (1, 1) process in the variance equation and an intercept in the mean equation. Further, we make the adjustments relative to an AR (1) term. However, we identify that based on the selection criteria as indicated in Table 3, a DCC model fits the best when applied with t-distribution and have an AR (1) term in the mean equation.

4.3. DCC model estimation results

The findings of the DCC model as presented in Table 4 show that the conditional mean (μ) in the mean equation is positive and significant for all stock and benchmark bond indices, gold, and crude oil but negative for the VIX. The estimated coefficient (a) of the AR-term is positive and significant for Belgium and Canadian stock indices along with the benchmark bond indices of all countries, except Australia. However, in the case of France, Italy, Japan, the US, gold, crude oil and VIX, the AR-term is negative and significant. However, for the remaining countries, this term is insignificant. Both the long-term volatility persistence (β) as well as the short-term volatility persistence (α) estimated under this model remain highly significant for each variable. The values of (β) are higher than those of (α) in each case, which highlight the significance of the long-term volatility persistence.

The significance of (α) and (β) is also evident in the volatility clustering in these markets. However, the volatility and returns together are not following the integrated generalized autoregressive conditional heteroscedasticity, which indicates that the risk is also involved with the investment in these markets. The sum ($\alpha + \beta$) is still less than one which signifies that the shocks to volatilities are not of the same magnitude. Based on the reported evidence, we infer that investors and portfolio managers might lose their investments in the long-run, even if they make high returns in the short-run.

The degrees of freedom and the shape parameters λ are equal in each case. The shape of the t-distribution gets normal as the number of degrees of freedom approaches infinity. The higher values of the estimated shape parameter (λ), e.g., more than (7), in the case of Australia, Belgium, France, Germany, Italy, Sweden, the Netherlands, and the UK indicate that these stock indices have lighter tail distributions than the tail distributions of Canada, Japan, Switzerland, and the US stock indices, crude oil, gold and VIX. Moreover, the parameter estimates (θ_1 and θ_2) of the DCC model are positive and statistically significant with a sum less than one, which implies a mean-reverting behaviour of the dynamic conditional correlations.

4.4. ADCC model estimation results

The estimations of ADCC model as shown in Table 5 indicate that the conditional mean and variance (μ and ω) are positive and statistically significant for the stock and benchmark bond indices, crude oil and gold, but negative and significant for VIX. The Canadian stock index (VIX) shows the highest conditional mean return (variance). More specifically, the magnitude of the GARCH coefficient of the long-term volatility persistence (β) is high and varies from 0.86 to 0.96, underscoring the high volatility persistence across time. Moreover, the sum of short and long term volatility parameters ($\alpha + \beta$) is less than 1 which provides evidence of long-run volatility consistency in these markets.

However, the estimated parameters on the asymmetric term (γ) are statistically significant for all stock indices in our sample, as well as for gold, crude oil, VIX and the benchmark bonds of Italy and the United States, indicating asymmetric behaviour in volatility. Given this result, negative shocks have a greater impact than positive shocks of the same magnitude. However, the asymmetric term (γ) is positive and significant

⁷ Among other alternative estimation techniques such as the method of moments and nonlinear least squares proposed by Boswijk and van der Weide (2011) and Boswijk and Van Der Weide (2006) respectively, the independent component analysis (ICA) approach is the most appropriate statistical method for the estimation of hidden factors.

Table 1
Descriptive Statistics of daily returns.

	Mean	Standard deviation	Skewness	Kurtosis	Jarque-Bera	ARCH (12)
Panel "A" Stock Returns						
Australia	0.000	0.010	-0.678	8.519	22107 ^a	1898.9 ^a
Belgium	0.000	0.012	-0.400	9.780	29780 ^a	1101.4 ^a
Canada	0.000	0.035	-0.981	17.744	94685 ^a	1295.8 ^a
France	0.000	0.014	-0.197	6.103	11112 ^a	1172.8 ^a
Germany	0.000	0.014	-0.239	5.839	10198 ^a	1212.1 ^a
Italy	0.000	0.015	-0.534	8.425	18295 ^a	708.0 ^a
Japan	0.000	0.142	-0.275	6.223	11595 ^a	1530.3 ^a
Netherlands	0.000	0.013	-0.248	7.237	15634 ^a	1641.2 ^a
Spain	0.000	0.014	-0.351	8.790	23103 ^a	771.1 ^a
Sweden	0.000	0.013	-0.166	5.645	9499 ^a	989.2 ^a
Switzerland	0.000	0.011	-0.289	7.251	15717 ^a	1409.9 ^a
UK	0.000	0.011	-0.395	8.406	22176 ^a	1502.5 ^a
USA	0.000	0.012	-0.426	11.529	39688 ^a	1989.5 ^a
Panel "B" Bond Returns						
Australia	0.000	0.005	-0.207	3.540	3775 ^a	493.3 ^a
Belgium	0.000	0.003	-0.327	4.556	6293 ^a	1322.8 ^a
Canada	0.000	0.004	-0.136	2.019	1233 ^a	461.3 ^a
France	0.000	0.006	-0.026	2.734	2300 ^a	593.9 ^a
Germany	0.000	0.003	-0.363	2.804	2493 ^a	327.3 ^a
Italy	0.000	0.005	0.066	19.924	100705 ^a	742.9 ^a
Japan	0.000	0.003	-0.664	7.948	19292 ^a	940.1 ^a
Netherlands	0.000	0.003	-0.329	1.986	1301 ^a	375.3 ^a
Spain	0.000	0.004	0.580	13.322	53126 ^a	433.3 ^a
Sweden	0.000	0.004	-0.343	5.287	8444 ^a	901.8 ^a
Switzerland	0.000	0.003	-0.184	6.118	11160 ^a	305.4 ^a
UK	0.000	0.004	-0.065	2.794	2324 ^a	444.5 ^a
USA	0.000	0.005	-0.150	3.227	3120 ^a	513.4 ^a
Panel "C" Oil, Gold and Volatility (VIX) returns						
Gold	0.000	0.010	-0.095	7.472	16598 ^a	410.5 ^a
Oil	0.000	0.025	-0.205	14.131	59355 ^a	1328.3 ^a
VIX	0.000	0.067	0.974	6.845	15048 ^a	375.8 ^a

Note: J-B denotes Jarque-Bera test used for normality and ARCH(12) indicates the ARCH test for Conditional heteroskedasticity.

^a Indicates the significance at 1%.

Table 2
Un-conditional Correlations between daily returns.

	Australia	Belgium	Canada	France	Germany	Italy	Japan
BONDS	-0.116	-0.079	-0.158	-0.136	-0.231	0.207	-0.242
Gold	0.059	-0.030	0.143	-0.027	-0.023	-0.024	0.037
OIL	0.097	0.163	0.282	0.187	0.159	0.209	0.083
VIX	-0.128	-0.402	-0.527	-0.419	-0.405	-0.428	-0.101
	Netherlands	Spain	Sweden	Switzerland	UK	USA	
BONDS	-0.217	0.111	-0.138	-0.194	-0.195	-0.260	
Gold	-0.020	-0.017	0.006	-0.046	0.018	-0.016	
OIL	0.181	0.186	0.174	0.140	0.217	0.189	
VIX	-0.401	-0.402	-0.389	-0.363	-0.409	-0.724	

for all stock indices and for crude oil. This implies that the negative residuals tend to increase the conditional volatility (variance) more than the positive shocks of equal magnitude. On the other hand, this (γ) is insignificant in the case of national benchmark bond indices, which indicates that these indices react weakly to shocks. Moreover, (γ) is negatively significant for gold, the volatility index (VIX), and Italy and the USA's national benchmark bonds. This implies that the negative residuals decrease the conditional volatility in these markets and provide diversification benefits for the stock markets. The results of our study are inline with those of [Basher and & Sadorsky \(2016\)](#) in terms of similar results for the emerging stock markets. Moreover, our findings suggest that distinct leverage effects may arise due to various asymmetric information, heterogeneity and different arbitrage activities.

In comparison to the DCC model, the values of the estimated shape parameters (λ) are increased (i.e., over 8) for the stock and benchmark bond indices. However, the values of the estimated shape parameter (λ) in the case of gold and the volatility index remain less than (5), indicating that these assets have heavier tail distributions than the distributions of the stock and benchmark bond indices and crude oil. As the

abnormal returns derived from heavy-tailed distributed asset returns influence the portfolio risk and its potential performance ([Mainik et al., 2015](#)), we infer that the alternative assets (gold and VIX) with heavy tails will provide better diversification benefits. The conditional correlations also show a mean-reverting pattern in the case of the ADCC model as the parameter estimates (θ_1 and θ_2) are positive and statistically significant and their sum is also less than 1. Moreover, each information criterion bestows that the ADCC model is the best-fitted model, and also confirms the findings of the DCC model. The shocks to volatilities are not of the same magnitude and portfolio managers and investors will lose their investments even if they earn higher returns in the short-run. Similar findings are produced by [Chkili \(2016\)](#) and [Rahim and Masih \(2016\)](#).

4.5. GO-GARCH model estimation results

The results of the estimated GO-GARCH model employed using a multivariate affine negative inverse Gaussian (MANIG) distribution, are provided in [Table 6](#). All specifications include an AR(1) term and a

Table 3
MGARCH models specifications.

		MUVT					MVNORM				
		AIC	BIC	Shibata	H-Q	LL	AIC	BIC	Shibata	H-Q	LL
Australia	Yes	16.762	16.812	16.762	16.78	-48048	17.058	17.1	17.058	17.073	-48904
	NO	16.778	16.822	16.778	16.793	-48097	17.073	17.1	17.073	17.086	-48949
Belgium	Yes	16.069	16.119	16.069	16.086	-46058	16.388	16.4	16.388	16.403	-46980
	NO	16.092	16.136	16.092	16.108	-46131	16.413	16.5	16.413	16.426	-47056
Canada	Yes	15.71	15.76	15.71	15.727	-45028	16.036	16.1	16.036	16.051	-45970
	NO	15.73	15.774	15.73	15.745	-45092	16.056	16.1	16.056	16.069	-46033
France	Yes	16.548	16.597	16.547	16.565	-47432	16.865	16.9	16.865	16.88	-48349
	NO	16.565	16.609	16.565	16.581	-47488	16.883	16.9	16.883	16.896	-48404
Germany	Yes	16.425	16.475	16.425	16.442	-47080	16.757	16.8	16.756	16.771	-48038
	NO	16.443	16.487	16.443	16.459	-47138	16.775	16.8	16.774	16.787	-48094
Italy	Yes	17.049	17.108	17.049	17.07	-39979	17.386	17.4	17.386	17.111	-40776
	NO	17.068	17.12	17.068	17.086	-40029	17.406	17.5	17.406	17.421	-40828
Japan	Yes	16.144	16.194	16.144	16.162	-46275	16.478	16.5	16.478	16.493	-47238
	NO	16.153	16.197	16.153	16.169	-46306	16.485	16.5	16.485	16.498	-47265
Netherlands	Yes	16.178	16.228	16.178	16.195	-46371	16.49	16.5	16.49	16.505	-47274
	NO	16.2	16.244	16.2	16.216	-46440	16.512	16.5	16.512	16.525	-47342
Spain	Yes	16.685	16.734	16.685	16.702	-47825	17.007	17.05	17.007	17.022	-48757
	NO	16.717	16.761	16.717	16.732	-47922	17.037	17.075	17.037	17.05	-48848
Sweden	Yes	16.526	16.576	16.526	16.543	-47369	16.836	16.9	16.836	16.851	-48266
	NO	16.55	16.595	16.55	16.566	-47445	16.862	16.9	16.862	16.875	-48346
Switzerland	Yes	15.76	15.81	15.76	15.777	-45173	16.126	16.2	16.126	16.141	-46228
	NO	15.779	15.823	15.779	15.794	-45231	16.145	16.2	16.145	16.158	-46287
UK	Yes	16.075	16.124	16.075	16.092	-46075	16.381	16.4	16.381	16.396	-46960
	NO	16.093	16.137	16.093	16.109	-46133	16.4	16.4	16.4	16.412	-47018
USA	Yes	15.63	15.679	15.629	15.647	-44798	15.994	16	15.994	16.009	-45849
	NO	15.637	15.681	15.637	15.652	-44825	16.002	16	16.002	16.015	-45877

Notes: AIC, BIC, H-Q, and LL indicate Akaike Information Criterion, Bayesian Information Criterion, Hannan-Quinn Criterion and log likelihood, respectively. Moreover, Shibata is used for an asymptotically optimal selection of regression variable.

constant in the mean equation. The parameter estimates of GO-GARCH are presented along with the mixing matrix (A) and the rotation matrix (U). The rotation matrix (U) is orthogonal due to $UTU = I$. The factors' estimates eliminate the requirement of the standard error for the GO-GARCH (Van der Weide, 2002).

Factor $F1, F2, F3, F4$ and $F5$ represent the stock, national benchmark bond, crude oil, gold, and volatility index (VIX), respectively. Volatility is said to be persistent if today's returns have a large effect on the forecast variance of the future periods (Basher and Sadorsky, 2016). For each factor, the short and long-term volatility persistence is shown by (α) and (β). Our findings suggest that both the short and long term volatility persistence play an important role in explaining excess returns. The average short-term volatility persistence for national benchmark bonds and stock indices is above 0.055, for crude oil and the volatility index, it is 0.041 each, while for gold, it is 0.10. This implies that the gold (crude oil and VIX) has the highest (lowest) short-term volatility persistence in our sample period. Moreover, the long term volatility persistence shows similar pattern for each factor (i.e., above 0.9), which indicates that in the long-run their return volatility will be influenced by their past volatility. Our findings of GO-GARCH for volatility persistence coincide with the DCC and ADCC results and show that the long-term volatility persistence is considerably higher than the short-term volatility persistence in each market. Overall, the fourth factor $F4$ (gold), indicates less long variations and more short variations. In other words, past day gold price increase (decrease) will significantly increase (decrease) the future returns in the short-run.

Further, the estimated models (i.e., DCC, ADCC, and GO-GARCH) are compared based on their fixed rolling window correlations, their one-step-ahead rolling correlations, their hedge ratios, and their hedging effectiveness.

4.6. One-step ahead rolling conditional correlations

The estimation window is fixed at 4738 observations to calculate the 1000 one-step-ahead rolling dynamic conditional correlations. The GARCH models are refit at every 20 observations, which provides the

robustness for the model refits, forecast length, and distribution assumptions. The one-step-ahead rolling dynamic conditional correlations produced from the ADCC and DCC models remain the same over the entire period (see Fig. 1 a-l), but the GO-GARCH correlations show a different pattern. The rolling conditional correlation between the stocks and national benchmark bonds has changed (positively and negatively) considerably over time. Our findings are consistent with the conventional wisdom that common macroeconomic conditions, such as expected inflation or economic prospects, drive both the stock and bond markets. We infer that the negative correlation between bonds and stock markets is due to the "Bernanke put" that has resulted from the zero short-term interest rate, which has led to the lower yield on the benchmark bonds. However, the positive correlation between bonds and stock markets reflects the investor risk-on/off trading strategies in asset allocation. The results indicate that the financial crisis has resulted in extreme levels of investors' fear that caused the relationship between bonds and stock returns due to changes in the risk appetite. During the crisis, stocks apparently become riskier and exert lower returns. Therefore, investors shift their investments to the bond market, a phenomenon known as the "flight-to-safety". The one-step-ahead rolling correlation between the national benchmark bonds and stock indices shows more variability over the period 2010–2012, which shows that the European sovereign debt crisis of 2010–12 has significantly altered the stock-bond connections. This relationship is consistent with a 'flight to quality' as risk appetite falls when investor fear increases (Baur and Lucey, 2009; Ingelbrecht et al., 2013; Pieterse-Bloem et al., 2016; Basher and Sadorsky, 2016).

The one-step-ahead rolling conditional correlation between the stock indices and crude oil remains positive for each pair except Australia, where it shows a negative correlation during 2014 only, which marks the most recent plunge in oil prices. This positive correlation between crude oil and the stock indices weakens oil's status as a safe financial asset, as someone believes. On the other hand, the one-step-ahead rolling correlation between the stock indices and gold remains weak and positive in most of the countries during the sub-period 2012–2013. However, gold shows a negative correlation with the stock markets only

Table 4
Parameter estimates of the MGARCH-DCC model.

	Australia	Belgium	Canada	France	Germany	Italy	Japan	Netherlands	Spain	Sweden	Switzerland	UK	USA	Oil	VIX
Stock Indices															
μ	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.006*** (0.001)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.049** (0.023)	-0.231*** (0.062)
a	-0.006 (0.011)	0.044*** (0.010)	0.062*** (0.011)	-0.028*** (0.007)	-0.028*** (0.010)	-0.040*** (0.011)	-0.042*** (0.011)	0.016 (0.010)	0.003 (0.011)	-0.001 (0.010)	0.004 (0.010)	0.019* (0.010)	-0.042*** (0.010)	-0.018 (0.013)	-0.069*** (0.013)
ω	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.019*** (0.006)	2.392*** (0.510)
α	0.070*** (0.007)	0.109*** (0.010)	0.094*** (0.008)	0.836*** (0.008)	0.087*** (0.008)	0.082*** (0.008)	0.083*** (0.008)	0.1031*** (0.009)	0.081*** (0.007)	0.085*** (0.008)	0.122*** (0.010)	0.090*** (0.008)	0.100*** (0.008)	0.039*** (0.002)	0.105*** (0.014)
β	0.921*** (0.007)	0.885*** (0.009)	0.898*** (0.008)	0.911*** (0.008)	0.909*** (0.007)	0.918*** (0.007)	0.912*** (0.008)	0.892*** (0.009)	0.914*** (0.007)	0.908*** (0.008)	0.868*** (0.009)	0.903*** (0.009)	0.895*** (0.008)	0.958*** (0.000)	0.836*** (0.024)
λ	8.358*** (0.892)	7.582*** (0.690)	6.721*** (0.606)	9.394*** (1.109)	8.228*** (0.892)	7.958*** (0.863)	6.611*** (0.606)	8.564*** (0.909)	7.537*** (0.732)	8.326*** (0.851)	6.839*** (0.628)	8.877*** (0.970)	5.779*** (0.465)	6.357*** (0.539)	5.335*** (0.381)
Benchmark Bond Indices															
μ	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.001)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000** (0.000)	0.000** (0.000)	0.000** (0.000)	0.000** (0.000)	0.000** (0.008)	0.008 (0.008)
a	-0.071*** (0.010)	0.046*** (0.011)	-0.001 (0.010)	0.010 (0.010)	0.032*** (0.010)	0.044*** (0.011)	-0.024** (0.010)	0.038*** (0.010)	0.109*** (0.010)	0.088*** (0.010)	0.027*** (0.010)	-0.003 (0.010)	-0.001 (0.010)	-0.038** (0.012)	9
ω	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.002*** (0.001)	0.002*** (0.001)
α	0.039*** (0.005)	0.068*** (0.007)	0.048*** (0.006)	0.049*** (0.005)	0.048*** (0.005)	0.073*** (0.007)	0.091*** (0.008)	0.050*** (0.005)	0.075*** (0.007)	0.064*** (0.006)	0.056*** (0.006)	0.040*** (0.005)	0.047*** (0.005)	0.041*** (0.002)	0.041*** (0.002)
β	0.956*** (0.005)	0.913*** (0.009)	0.941*** (0.008)	0.939*** (0.007)	0.943*** (0.006)	0.919*** (0.007)	0.916*** (0.007)	0.939*** (0.006)	0.917*** (0.008)	0.925*** (0.008)	0.931*** (0.007)	0.953*** (0.005)	0.947*** (0.005)	0.958*** (0.000)	0.958*** (0.000)
λ	7.373*** (0.648)	6.717*** (0.546)	8.883*** (0.947)	6.726*** (0.571)	7.127*** (0.648)	6.171*** (0.569)	5.558*** (0.379)	7.431*** (0.661)	6.762*** (0.596)	7.153*** (0.633)	3.859*** (0.250)	8.042*** (0.790)	7.329*** (0.652)	4.285*** (0.209)	
Diagnostics															
θ_1	0.318*** (0.005)	0.041*** (0.005)	0.032*** (0.005)	0.033*** (0.005)	0.041*** (0.005)	0.034*** (0.005)	0.0213*** (0.004)	0.038*** (0.006)	0.022*** (0.004)	0.031*** (0.005)	0.036*** (0.006)	0.031*** (0.005)	0.042*** (0.005)		
θ_2	0.963*** (0.006)	0.952*** (0.007)	0.964*** (0.006)	0.962*** (0.005)	0.957*** (0.005)	0.964*** (0.005)	0.975*** (0.005)	0.958*** (0.006)	0.976*** (0.006)	0.964*** (0.006)	0.951*** (0.009)	0.967*** (0.005)	0.967*** (0.005)	0.956*** (0.005)	
λ	7.270*** (0.400)	6.893*** (0.372)	7.271*** (0.386)	6.642*** (0.350)	6.454*** (0.335)	6.738*** (0.386)	4.923*** (0.213)	7.062*** (0.403)	6.961*** (0.375)	7.284*** (0.388)	5.403*** (0.233)	7.362*** (0.405)	5.911*** (0.285)		

Notes: Multivariate t (MVT) distribution is used to estimate the DCC model. For all specifications, the mean equation includes an AR(1) term and a constant. *, **, *** indicate significance at the 10%, 5% and 1%, respectively. Standard errors are presented in (). γ refers to leverage effect and λ is shape parameter i.e., to the degrees of freedom where these degrees of freedom reached to infinity and if the sum of θ_1 and θ_2 is less than 1, then there is a mean-reverting behavior.

Table 5
Parameter Estimates of the MGARCH-ADCC Model.

	Australia	Belgium	Canada	France	Germany	Italy	Japan	Netherlands	Spain	Sweden	Switzerland	UK	USA	Oil	VIX
Stock Indices															
μ	0.001*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.042* (0.023)	-0.107* (0.064)
a	-0.006 (0.011)	0.043*** (0.010)	0.062*** (0.011)	-0.028*** (0.010)	-0.028*** (0.010)	-0.040*** (0.011)	-0.043*** (0.011)	0.016 (0.010)	0.003 (0.011)	-0.003 (0.002)	0.004 (0.010)	0.019* (0.010)	-0.042*** (0.010)	-0.018 (0.013)	-0.066*** (0.013)
ω	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.018*** (0.005)	2.270*** (0.415)
α	0.070*** (0.007)	0.109*** (0.099)	0.095*** (0.007)	0.083*** (0.008)	0.087*** (0.008)	0.081*** (0.007)	0.083*** (0.008)	0.103*** (0.009)	0.081*** (0.007)	0.085*** (0.008)	0.122*** (0.010)	0.090*** (0.007)	0.100*** (0.008)	0.027*** (0.004)	0.158*** (0.021)
β	0.921*** (0.008)	0.885*** (0.009)	0.898*** (0.007)	0.911*** (0.010)	0.909*** (0.008)	0.918*** (0.007)	0.911*** (0.008)	0.892*** (0.009)	0.914*** (0.007)	0.908*** (0.008)	0.868*** (0.011)	0.903*** (0.008)	0.895*** (0.008)	0.960*** (0.000)	0.866*** (0.012)
γ	0.111*** (0.019)	0.138*** (0.019)	0.090*** (0.018)	0.129*** (0.018)	0.126*** (0.017)	0.105*** (0.017)	0.102*** (0.017)	0.135*** (0.017)	0.119*** (0.017)	0.131*** (0.019)	0.158*** (0.019)	0.141*** (0.018)	0.155*** (0.019)	0.020** (0.008)	-0.187*** (0.022)
λ	9.730*** (1.189)	8.510*** (0.855)	7.112*** (0.679)	10.540*** (1.366)	9.096*** (1.080)	8.618*** (1.017)	7.116*** (0.707)	10.026*** (1.342)	8.124*** (0.855)	9.925*** (1.242)	7.707*** (0.857)	10.073*** (1.269)	6.668*** (0.622)	6.434*** (0.552)	4.564*** (0.413)
Benchmark Bond Indices															
μ	0.000 (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000** (0.000)	0.000** (0.000)	0.000* (0.000)	0.000 (0.000)	0.009 (0.008)	
a	-0.071*** (0.010)	0.046*** (0.011)	-0.001 (0.010)	0.010 (0.010)	0.032*** (0.010)	0.043*** (0.011)	-0.025** (0.011)	0.038*** (0.010)	0.109*** (0.010)	0.088*** (0.010)	0.027*** (0.010)	-0.003 (0.010)	-0.002*** (0.010)	-0.039*** (0.012)	
ω	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.000*** (0.000)	0.002*** (0.001)	
α	0.038*** (0.005)	0.068*** (0.007)	0.048*** (0.006)	0.049*** (0.005)	0.048*** (0.005)	0.073*** (0.007)	0.091*** (0.009)	0.050*** (0.009)	0.075*** (0.009)	0.063*** (0.007)	0.056*** (0.007)	0.040*** (0.006)	0.047*** (0.005)	0.055*** (0.004)	
β	0.956*** (0.005)	0.913*** (0.009)	0.941*** (0.008)	0.939*** (0.007)	0.943*** (0.007)	0.919*** (0.007)	0.916*** (0.007)	0.939*** (0.007)	0.917*** (0.009)	0.924*** (0.008)	0.931*** (0.008)	0.953*** (0.013)	0.947*** (0.005)	0.956*** (0.001)	
γ	-0.005 (0.007)	0.018 (0.013)	0.011 (0.010)	0.003 (0.008)	-0.009 (0.014)	0.039 (0.012)	0.018 (0.011)	0.004 (0.014)	-0.033** (0.012)	-0.003 (0.013)	-0.01 (0.012)	-0.003 (0.012)	-0.014** (0.007)	-0.024*** (0.007)	
λ	7.367*** (0.648)	6.719*** (0.546)	8.891*** (0.944)	6.723*** (0.571)	7.147*** (0.649)	6.237*** (0.579)	5.237*** (0.381)	5.520*** (0.662)	7.436*** (0.612)	6.865*** (0.633)	7.158*** (0.253)	3.850*** (0.791)	8.050*** (0.661)	7.369*** (0.213)	4.317***
Diagnostics															
θ_1	0.032*** (0.004)	0.041*** (0.005)	0.032*** (0.005)	0.032*** (0.005)	0.040*** (0.005)	0.032*** (0.005)	0.023*** (0.005)	0.038*** (0.005)	0.021*** (0.004)	0.032*** (0.005)	0.036*** (0.005)	0.030*** (0.004)	0.041*** (0.005)		
θ_2	0.963*** (0.005)	0.952*** (0.007)	0.964*** (0.006)	0.963*** (0.005)	0.956*** (0.006)	0.962*** (0.006)	0.969*** (0.007)	0.958*** (0.006)	0.976*** (0.004)	0.962*** (0.007)	0.951*** (0.007)	0.967*** (0.005)	0.956*** (0.005)		
θ_3	-0.002 (0.003)	0.000 (0.003)	-0.000 (0.002)	0.003 (0.002)	0.003 (0.003)	0.008** (0.002)	0.005** (0.002)	0.002 (0.002)	0.003 (0.002)	-0.001 (0.003)	0.001 (0.002)	0.001 (0.002)	0.001 (0.002)		
λ	7.268*** (0.405)	6.893*** (0.353)	7.266*** (0.402)	6.642*** (0.345)	6.468*** (0.348)	6.725*** (0.378)	4.935*** (0.217)	7.065*** (0.396)	6.950*** (0.360)	7.294*** (0.411)	5.402*** (0.258)	7.367*** (0.399)	5.916*** (0.280)		

Notes: Multivariate t (MVT) distribution is used to estimate the ADCC model. For all specifications, the mean equation includes an AR(1) term and a constant. *, **, *** indicate significance at the 10%, 5% and 1%, respectively. Standard errors are presented in (). γ refers to leverage effect and λ is shape parameter i.e., to the degrees of freedom where these degrees of freedom reached to infinity and if the sum of θ_1 and θ_2 is less than 1, then there is a mean-reverting behavior.

Table 6

Estimates of the GO-GARCH model.

	The rotation matrix U					The mixing matrix A					Parameter estimates of GO-GARCH								
	U(1)	U(2)	U(3)	U(4)	U(5)	A(1)	A(2)	A(3)	A(4)	A(5)	F1	F2	F3	F4	F5				
Australia	U(1)	-0.057	0.041	-0.047	0.996	0.011	A(1)	1.000	-0.072	0.020	0.131	-0.479	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	0.022	-0.085	0.225	0.026	-0.970	A(2)	-0.072	1.005	0.086	-0.148	0.518	α	0.067	0.038	0.034	0.064	0.088	
	U(3)	0.130	-0.238	0.930	0.058	0.241	A(3)	0.020	0.086	1.008	0.406	0.138	β	0.926	0.958	0.964	0.929	0.834	
	U(4)	-0.227	0.932	0.279	-0.038	-0.024	A(4)	0.131	-0.148	0.406	1.209	-0.397	Skew	0.100	-0.208	0.021	-0.247	-0.075	
	U(5)	-0.963	-0.256	0.068	-0.042	0.015	A(5)	-0.479	0.518	0.138	-0.397	1.572	λ	2.204	3.213	0.851	1.631	1.668	
Belgium	U(1)	0.062	-0.206	-0.051	0.975	0.009	A(1)	1.000	-0.036	1.001	0.233	-0.182	0.267	α	0.093	0.068	0.034	0.068	0.099
	U(2)	0.003	0.156	0.222	0.053	-0.961	A(2)	-0.036	1.001	-0.057	0.281	-1.879	Ω	0.000	0.000	0.000	0.000	0.000	
	U(3)	0.168	-0.893	0.370	-0.180	-0.070	A(3)	-0.057	0.233	1.057	0.365	0.208	β	0.898	0.916	0.963	0.926	0.803	
	U(4)	-0.145	0.301	0.898	0.118	0.262	A(4)	0.281	-0.182	0.365	1.286	-0.770	Skew	-0.132	0.081	0.025	-0.238	-0.062	
	U(5)	-0.973	-0.212	-0.072	0.014	-0.053	A(5)	-1.879	0.267	0.208	-0.770	4.625	λ	2.028	2.541	0.844	1.713	1.701	
Canada	U(1)	-0.179	-0.070	-0.278	-0.941	0.013	A(1)	1.000	-0.065	0.108	0.613	-3.219	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	-0.026	0.241	0.158	-0.047	0.956	A(2)	-0.065	1.004	0.078	-0.475	1.183	α	0.083	0.043	0.035	0.068	0.104	
	U(3)	0.055	0.944	0.141	-0.126	-0.266	A(3)	0.108	0.078	1.019	0.371	0.313	β	0.911	0.947	0.963	0.926	0.768	
	U(4)	-0.061	0.211	-0.930	0.272	0.113	A(4)	0.613	-0.475	0.371	1.681	-2.230	Skew	0.111	0.030	0.121	0.216	0.074	
	U(5)	0.980	-0.046	-0.113	-0.149	0.049	A(5)	-3.219	1.183	0.313	-2.230	12.646	λ	2.264	0.843	2.710	1.751	1.708	
France	U(1)	0.073	-0.254	-0.048	0.963	0.012	A(1)	1.000	-0.047	-0.050	0.254	-1.669	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	0.036	0.201	0.227	0.073	-0.950	A(2)	-0.047	1.002	0.161	-0.373	0.160	α	0.073	0.048	0.034	0.069	0.102	
	U(3)	0.260	-0.909	0.118	-0.251	-0.174	A(3)	-0.050	0.161	1.028	0.359	0.147	β	0.921	0.941	0.963	0.924	0.789	
	U(4)	-0.085	0.038	0.964	0.062	0.240	A(4)	0.254	-0.373	0.359	1.378	-0.641	Skew	-0.083	0.110	0.027	-0.225	-0.048	
	U(5)	-0.959	-0.261	-0.049	0.002	-0.103	A(5)	-1.669	0.160	0.147	-0.641	3.838	λ	1.917	3.377	0.840	1.716	1.753	
Germany	U(1)	-0.081	-0.321	0.046	0.942	0.013	A(1)	1.000	-0.053	-0.039	0.201	-1.568	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	0.250	0.156	0.014	0.087	-0.952	A(2)	-0.053	1.003	0.267	-0.439	0.845	α	0.075	0.046	0.034	0.069	0.107	
	U(3)	0.023	-0.902	0.244	-0.315	-0.167	A(3)	-0.039	0.267	1.072	0.312	0.291	β	0.921	0.943	0.964	0.925	0.788	
	U(4)	-0.959	0.072	0.112	-0.060	-0.244	A(4)	0.201	-0.439	0.312	1.412	-0.849	Skew	0.044	0.152	-0.141	-0.211	-0.061	
	U(5)	-0.106	-0.233	-0.962	-0.041	-0.084	A(5)	-1.568	0.845	0.291	-0.849	4.087	λ	0.883	2.767	2.012	1.675	1.739	
Italy	U(1)	0.158	0.265	0.011	0.948	-0.080	A(1)	1.000	-0.001	-0.040	0.282	-1.607	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	0.032	0.225	0.933	-0.101	-0.260	A(2)	-0.001	1.000	0.189	-0.107	-0.198	α	0.071	0.065	0.033	0.076	0.104	
	U(3)	-0.336	-0.863	0.226	0.287	-0.090	A(3)	-0.040	0.189	1.038	0.391	0.038	β	0.926	0.927	0.958	0.919	0.805	
	U(4)	-0.096	0.036	0.270	0.083	0.954	A(4)	0.282	-0.107	0.391	1.270	-0.653	Skew	0.117	0.171	0.028	0.238	-0.054	
	U(5)	-0.923	0.364	-0.076	0.046	-0.089	A(5)	-1.607	-0.198	0.038	-0.653	3.672	λ	1.676	3.801	1.684	1.682	0.880	
Japan	U(1)	0.008	0.046	0.046	-0.998	-0.011	A(1)	1.000	-0.035	0.009	0.080	-0.322	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	0.010	-0.070	-0.230	-0.025	0.970	A(2)	-0.035	1.001	0.041	-0.014	-0.225	α	0.080	0.096	0.034	0.064	0.100	
	U(3)	-0.069	0.991	0.063	0.047	0.088	A(3)	0.009	0.041	1.002	0.415	0.086	β	0.913	0.912	0.964	0.930	0.824	
	U(4)	-0.010	0.083	-0.970	-0.039	-0.225	A(4)	0.080	-0.014	0.415	1.178	-0.297	Skew	-0.118	-0.097	-0.024	0.254	0.076	
	U(5)	0.998	0.069	-0.003	0.011	-0.006	A(5)	-0.322	-0.225	0.086	-0.297	1.269	λ	1.376	1.887	0.844	1.668	1.606	
Netherlands	U(1)	0.069	0.238	-0.085	-0.965	-0.015	A(1)	1.000	-0.054	-0.045	0.253	-1.652	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	-0.246	-0.144	-0.045	-0.064	0.955	A(2)	-0.054	1.003	0.215	-0.422	0.265	α	0.089	0.049	0.035	0.069	0.106	
	U(3)	-0.077	0.948	-0.146	0.240	0.132	A(3)	-0.045	0.215	1.047	0.331	0.178	β	0.906	0.938	0.962	0.924	0.783	
	U(4)	0.959	0.006	-0.115	0.077	0.248	A(4)	0.253	-0.422	0.331	1.415	-0.681	Skew	-0.041	-0.156	0.130	0.228	0.061	
	U(5)	0.096	0.156	0.978	-0.042	0.092	A(5)	-1.652	0.265	0.178	-0.681	3.814	λ	0.859	3.371	2.118	1.759	1.721	
Spain	U(1)	-0.110	-0.279	-0.012	-0.952	0.067	A(1)	1.000	-0.012	-0.033	0.227	-1.624	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	-0.020	-0.166	-0.951	0.080	0.247	A(2)	-0.012	1.000	0.135	-0.090	0.091	α	0.074	0.075	0.034	0.067	0.102	
	U(3)	0.339	0.877	-0.170	-0.290	0.059	A(3)	-0.033	0.135	1.019	0.398	0.149	β	0.921	0.918	0.963	0.926	0.805	
	U(4)	0.079	-0.035	-0.251	-0.063	-0.962	A(4)	0.227	-0.090	0.398	1.233	-0.557	Skew	-0.085	-0.120	-0.055	-0.221	0.034	
	U(5)	0.931	-0.353	0.061	0.000	0.073	A(5)	-1.624	0.091	0.149	-0.557	3.697	λ	1.900	3.369	1.726	1.815	0.862	
Sweden	U(1)	-0.038	0.206	-0.050	-0.977	-0.003	A(1)	1.000	-0.053	-0.021	0.236	-1.611	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	-0.007	-0.166	0.225	-0.049	0.959	A(2)	-0.053	1.003	0.083	-0.280	0.530	α	0.078	0.059	0.034	0.065	0.099	
	U(3)	-0.176	0.948	0.025	0.205	0.168	A(3)	-0.021	0.083	1.007	0.384	0.178	β	0.916	0.932	0.964	0.929	0.796	
	U(4)	-0.051	-0.033	-0.972	0.044	0.224	A(4)	0.236	-0.280	0.384	1.296	-0.682	Skew	0.125	-0.126	0.026	0.237	0.060	
	U(5)	0.982	0.175	-0.046	0.001	0.048	A(5)	-1.611	0.530	0.178	-0.682	3.855	λ	2.018	3.046	0.852	1.728	1.788	
Switzerland	U(1)	0.016	-0.189	-0.066	0.980	0.010	A(1)	1.000	-0.040	-0.055	0.200	-1.750	Ω	0.000	0.000	0.000	0.000	0.000	
	U(2)	0.015	0.103	0.242	0.045	-0.964	A(2)	-0.040	1.002	0.199	-0.326	0.915	α	0.102	0.051	0.034	0.068	0.106	

(continued on next page)

Table 6 (continued)

The rotation matrix U						The mixing matrix A						Parameter estimates of GO-GARCH						F1			F2			F3			F4			F5		
U(1)			U(2)			U(3)			U(4)			U(5)			A(1)			A(2)			A(3)			A(4)			A(5)					
UK	U(3)	0.014	-0.927	0.340	-0.156	-0.021	A(3)	-0.055	0.199	1.042	0.359	0.277	0.882	0.936	0.964	0.926	0.791															
	U(4)	-0.093	0.304	0.903	0.118	0.263	A(4)	0.200	-0.326	0.359	1.327	-0.861	β	-0.070	0.129	0.038	-0.237	-0.061														
	U(5)	-0.995	-0.042	-0.077	0.003	-0.039	A(5)	-1.750	0.915	0.277	-0.861	4.841	λ	0.820	2.625	0.882	1.715	1.685														
	U(1)	0.230	0.043	-0.083	-0.969	-0.004	A(1)	1.000	-0.080	-0.029	0.381	-2.221	Ω	0.000	0.000	0.000	0.000	0.000														
	U(2)	-0.197	-0.220	-0.034	-0.057	0.953	A(2)	-0.080	1.006	0.136	-0.315	0.747	α	0.079	0.038	0.034	0.070	0.108														
	U(3)	-0.142	0.970	-0.036	0.012	0.194	A(3)	-0.029	0.136	1.019	0.364	0.284	β	0.914	0.956	0.963	0.924	0.782														
	U(4)	0.929	0.083	-0.158	0.238	0.220	A(4)	0.381	-0.315	0.364	1.396	-1.125	Skew	-0.163	-0.023	0.109	0.227	0.054														
	U(5)	0.157	0.045	0.983	-0.045	0.075	A(5)	-2.221	0.747	0.284	-1.125	6.308	λ	3.482	0.843	2.508	1.722	1.768														
	U(1)	0.391	0.057	-0.168	-0.903	0.033	A(1)	1.000	-0.098	-0.045	0.338	-4.255	Ω	0.000	0.000	0.000	0.000	0.000														
	U(2)	-0.203	-0.233	-0.040	-0.061	0.948	A(2)	-0.098	1.010	0.128	-0.456	0.476	α	0.092	0.040	0.035	0.071	0.091														
USA	U(3)	-0.207	0.959	-0.039	-0.015	0.189	A(3)	-0.045	0.128	1.017	0.373	0.218	β	0.904	0.955	0.964	0.926	0.828														
	U(4)	0.862	0.141	-0.103	0.410	0.241	A(4)	0.338	-0.456	0.373	1.486	-1.519	Skew	-0.012	-0.026	0.115	0.232	0.070														
	U(5)	0.141	0.053	0.979	-0.115	0.077	A(5)	-4.255	0.476	0.218	-1.519	19.114	λ	1.679	0.836	1.885	1.675	1.602														

Notes: MANIG distribution is used to estimate the GO-GARCH model. The rotation matrix U is orthogonal because $U^T U = I$. Standard errors are not presented because GO-GARCH estimates factors.

in 2014 and 2015. The correlation produced from the GO-GARCH model for the gold-stock pairs shows less variability in comparison to the DCC and ADCC models. The weak and positive correlation between the stocks and gold highlights gold's potential diversification benefits in the stock portfolios. Moreover, our findings are consistent with those of [Baur and Lucey \(2010\)](#), [Baur and McDermott \(2010\)](#), [Coudert and Raymond-Feingold \(2011\)](#), [Ciner et al. \(2013\)](#) and [Gurgun and Unalmış \(2014\)](#), which show that gold has a low correlation with the stock markets and that adding gold to the stock portfolio may potentially lower the risk of the portfolio.

Finally, the negative correlation between the stock indices and the volatility index VIX is confirmed through each estimated GARCH model. This suggests significant diversification benefits for trading volatility-related products, especially the VIX futures contracts, which are readily tradable at CBOE. We infer that trading volatility derivatives such as the VIX futures, options, and ETFs can safeguard stock portfolios during the episodes of financial downturns. The remarkable negative correlation between VIX and the stock markets is often referred to as Black's leverage effect and is asymmetric in nature ([Yu, 2005](#); [Chen et al., 2011](#)). The stock markets tend to lose money in the case of increasing uncertainty, and that taking a short or a long position in the VIX futures can protect an equity investment from negative shocks. Moreover, the correlations produced from the DCC type models are gradually weakening than the correlations produced from the GO-GARCH model which are fairly strong.

4.7. Correlations between correlations

The results of correlations between correlations produced by the three models are presented in [Table 7](#), which show that the correlations produced from DCC and ADCC are strongly correlated (almost close to 1), compared to the correlations between ADCC and GO-GARCH or DCC and GO-GARCH. This is consistent with the correlation figures, and these differences can also be seen in the news impact correlation surface plots (see [Figs. 2 and 3 a-l](#)), the surface plots of DCC and ADCC are very similar and are very different from the GO-GARCH surface plots. The correlations between the stock markets and the volatility index and between the stock markets and benchmark bonds are the lowest and even negative, and the surface plots for these two pairs follow a positive to a negative pattern at the z_1 axis and are opposite at the z_2 axis that remarks the asymmetric effects of the shocks for these two pairs. However, the GO-GARCH surface plots are concave and display more symmetry in comparison with the DCC and ADCC surface plots, which are convex to the origin. The symmetric effects are expected because the GO-GARCH factors are orthogonalized, and this result is consistent with prior studies ([Van der Weide, 2002](#); [Boswijk and Van Der Weide, 2006](#); [Boswijk and van der Weide, 2011](#) and [Basher and &Sadorsky, 2016](#)). We also notice that the correlations produced from GO-GARCH for the stock markets and the volatility index remain negative in each market, but the shocks pertain to the factors.

5. Hedging effectiveness

The three estimated models in this section are compared based on their hedge ratios and their hedging effectiveness. The returns on the portfolios composed of index and alternative assets are represented as follows:

$$R_{H,t} = R_{S,t} - \gamma_t R_{AA,t}$$

where, $R_{H,t}$ shows the return on the hedged portfolio, and $R_{S,t}$ and $R_{AA,t}$ represent the stock return and return of alternative assets, respectively. The γ_t represents the hedge ratio and the variance of the hedged portfolio at $t-1$ is given as follows:

$$\text{var}(R_{H,t}I_{t-1}) = \text{var}(R_{S,t}I_{t-1}) - 2\gamma_t \text{cov}(R_{AA,t}, R_{S,t}I_{t-1}) + \gamma_t^2 \text{var}(R_{AA,t}I_{t-1})$$

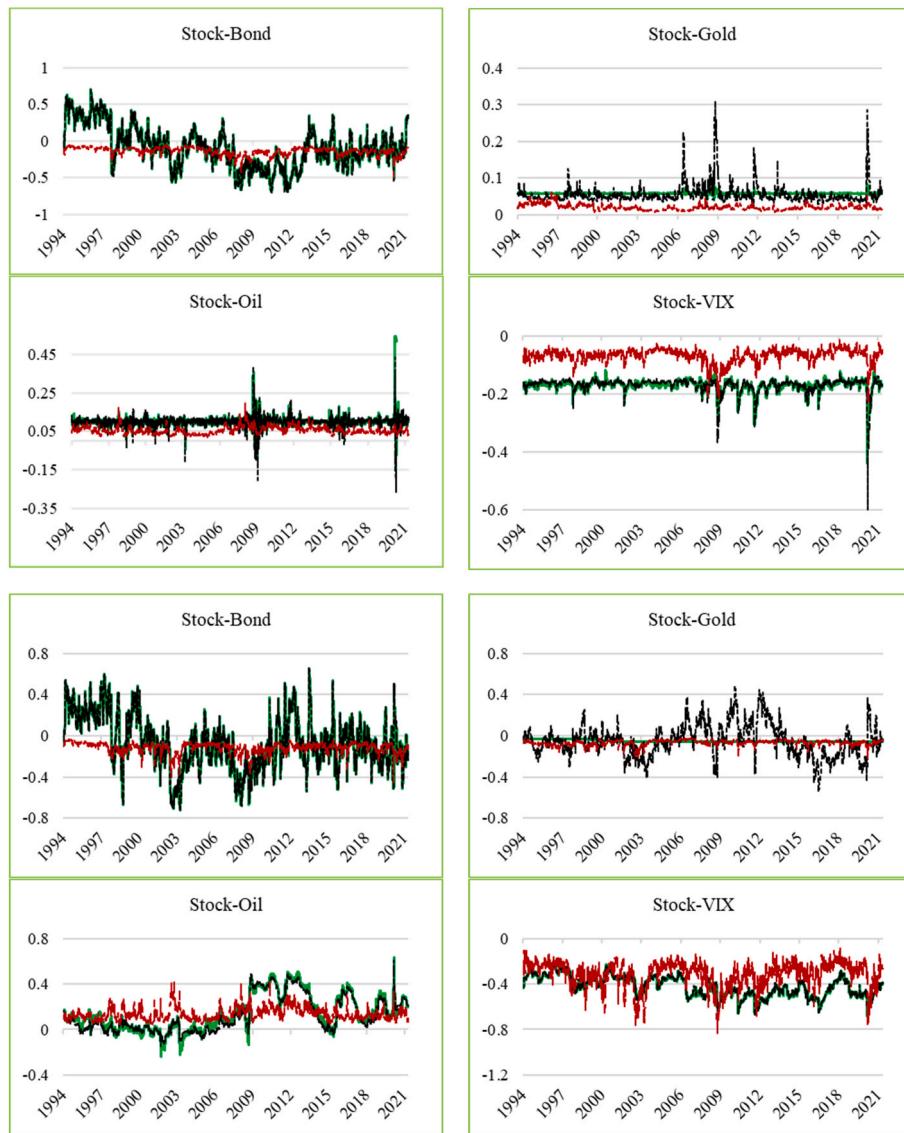


Fig. 1. One-step ahead rolling conditional correlations. **Fig. 1-a** Australia: Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Figure 1-b Belgium:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Figure 1-c Canada:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Figure 1-d France:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Fig. 1-e Germany:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Fig. 1-f Italy:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Fig. 1-g Japan:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Fig. 1-h The Netherlands:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Fig. 1-i Spain:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Fig. 1-j Sweden:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Fig. 1-k Switzerland:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Fig. 1-l UK:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----]. **Fig. 1-m USA:** Rolling one-step-ahead correlation [DCC -----, ADCC ———, GO-GARCH -----].

The ratios γ_t^* represent the optimal hedge ratios that minimize the conditional variance of hedged portfolio and are conditional on the information set I_{t-1} . They can be obtained by taking the partial derivative of the variance with respect to γ_t and set the expression equal to zero (Baillie and Myers, 1991).

$$\gamma_t^* I_{t-1} = \frac{\text{cov}(R_{S,t}, R_{AA,t} | I_{t-1})}{\text{var}(R_{AA,t} | I_{t-1})}$$

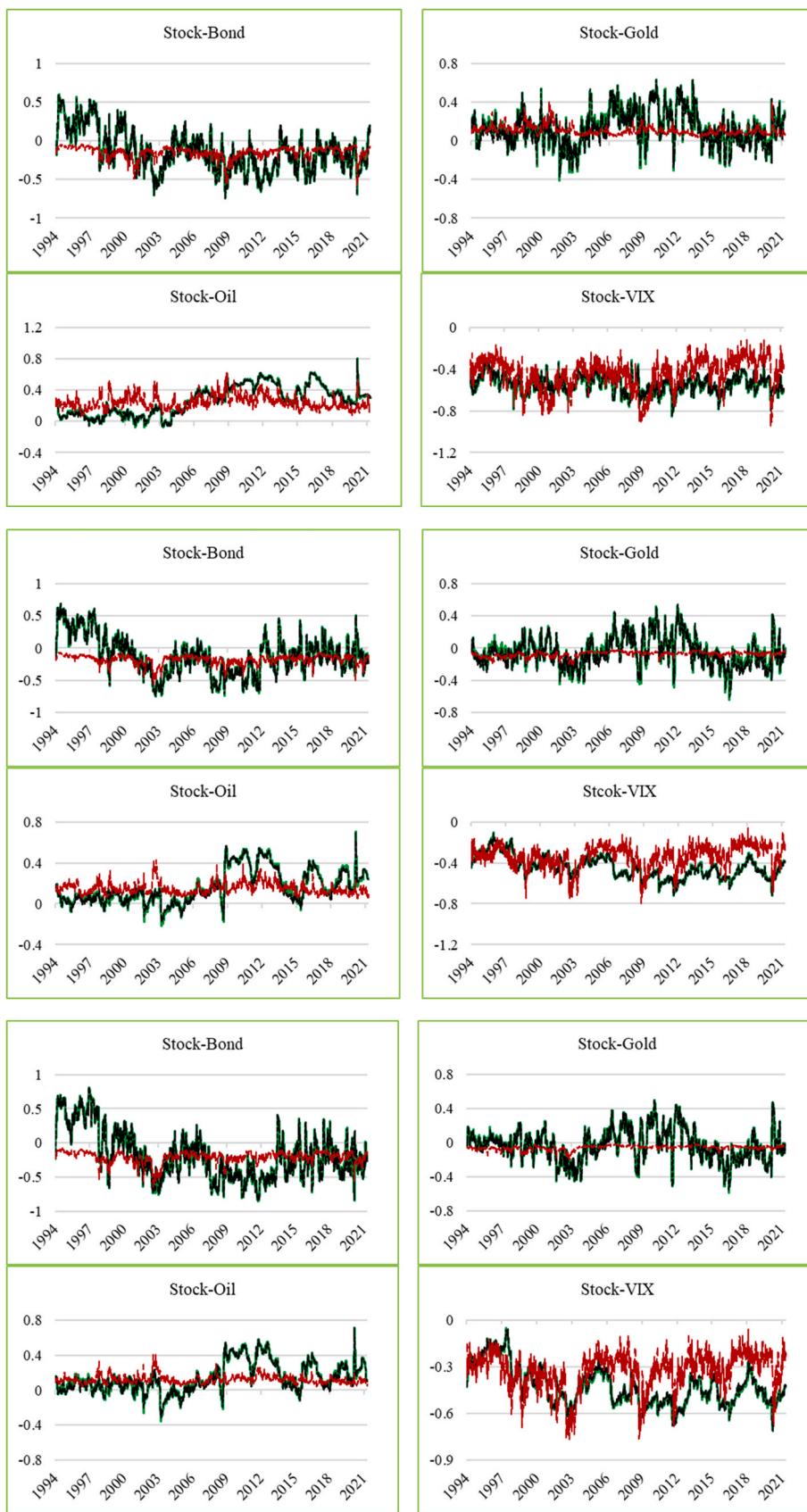
The parameters of the conditional volatility estimated from the GARCH models are used to calculate the hedge ratios (Kroner and Sultan, 1993). That is, the long position in the stock index can be hedged by taking a short position in an alternative asset. The hedge ratio is expressed as follows:

$$\gamma_t^* | I_{t-1} = h_{SAA,t} / h_{F,t}$$

The conditional covariance and variance between the stock index and alternative asset returns are represented by the $h_{SAA,t}$ and $h_{AA,t}$, respectively. The hedge effectiveness is given as under:

$$HE = \frac{\text{var}_{unhedged} - \text{var}_{hedged}}{\text{var}_{unhedged}}$$

where, the $\text{var}_{unhedged}$ and var_{hedged} represent the variance of returns on the unhedged stock portfolio and the hedged mixed-asset portfolio (e.g., stock and alternative assets portfolios), respectively. The performance of the employed GARCH models is compared based on their hedge effectiveness (HE) where the higher the (HE) index is, the higher the hedge

**Fig. 1. (continued).**

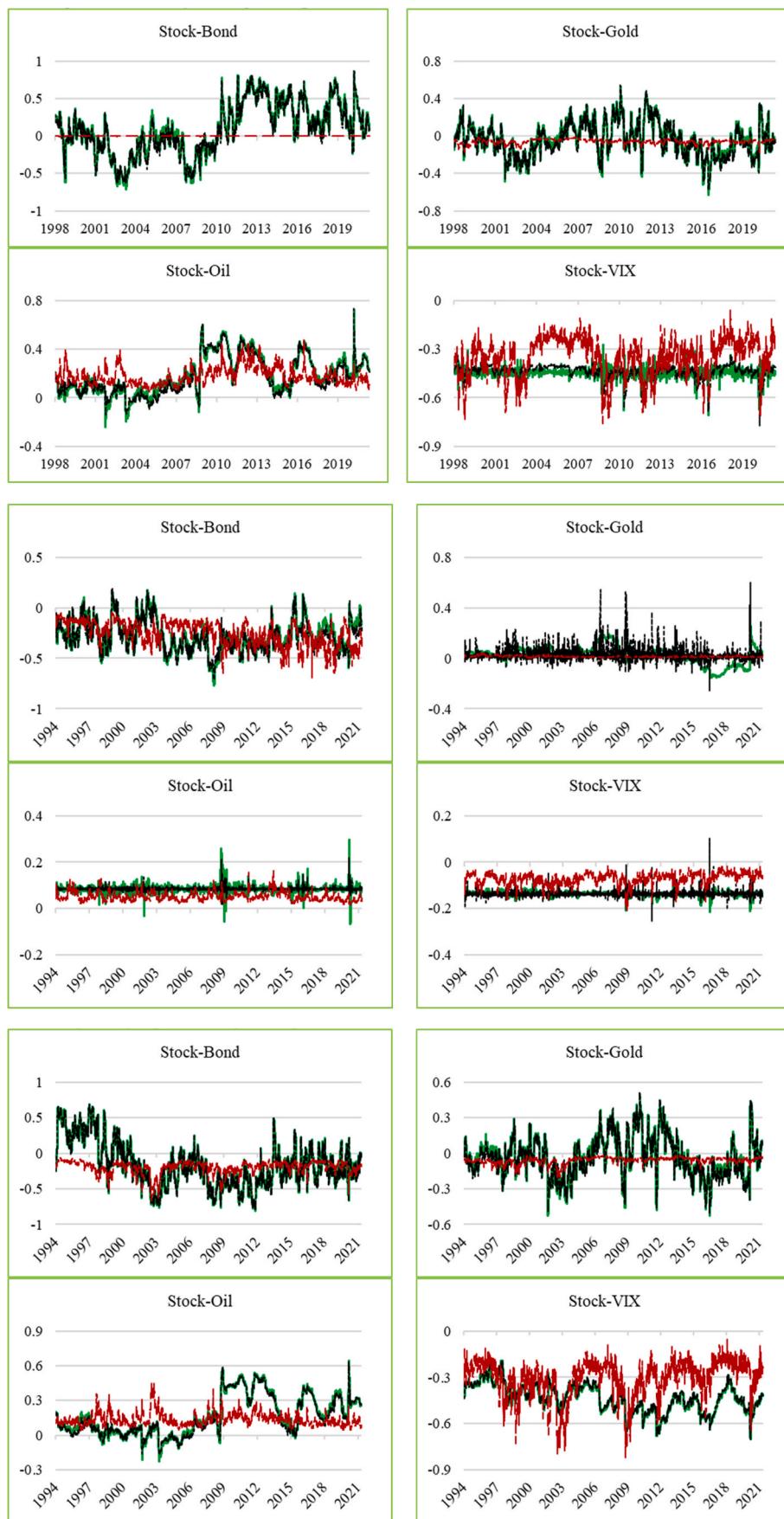


Fig. 1. (continued).

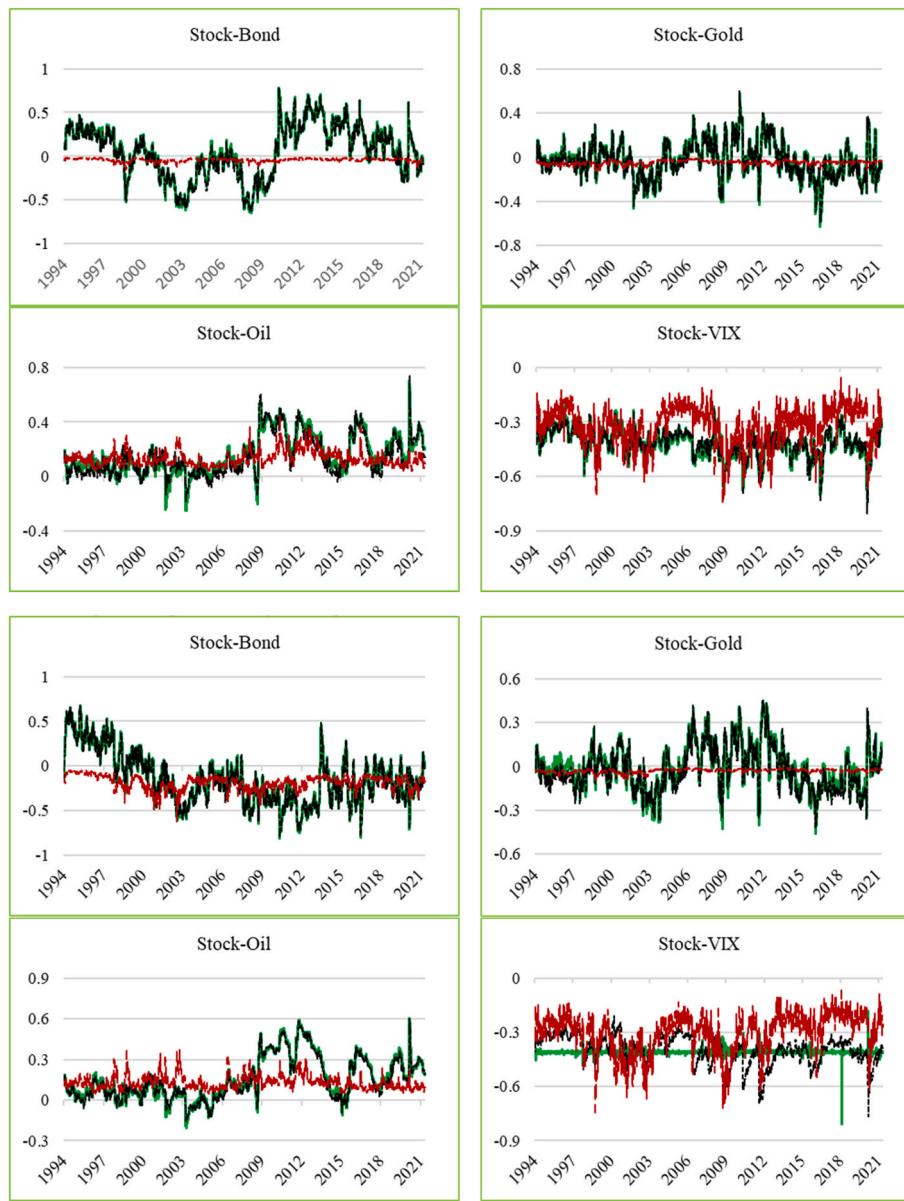


Fig. 1. (continued).

effectiveness of the model (Ku et al., 2007; Chang et al., 2010; Basher and Sadorsky, 2016). By using the rolling window approach, we construct the out-of-sample hedge ratios. Further, the one-step-ahead conditional volatilities are used to construct the one-step-ahead hedge ratios, and then these ratios are used to construct a hedged portfolio. Similar to the rolling conditional correlations, the estimated window is fixed at 4738 observations. These observations are used to calculate the 1000 one-step-ahead hedge ratio with the help of rolling window analysis.

5.1. Hedge ratio's summary statistics

The correlations between the hedge ratios, reported in (Table 7), show that the hedge ratios calculated from the DCC and ADCC models are very similar compared to the hedge ratios calculated from the GO-GARCH model. The optimal hedge ratios are calculated by considering a long position in a stock market and a short position in gold, crude oil, the volatility index or a bond, respectively. The hedge ratios are presented in Table 8 and the average values of the hedge ratios between the national benchmark bonds and the stock indices are negative because

these pairs are negatively correlated.

This implies that a hedge can be formed for a stock/bond pair by taking either a short position in both assets or a long position in both assets. However, the hedging effectiveness for Italy, UK and US's national benchmark bonds is higher than for other benchmark bonds in their countries' stock portfolios. Our findings suggest that investors have to shift their primary focus of investments from equities (referred to as risk on trades) to bonds (referred to as risk-off trades) to achieve high risk-adjusted returns during global uncertainty to preserve their wealth. Because, in the crisis periods, stocks apparently become riskier, and returns become more sensitive to changes in the level of the investors' fear. Both risk measures, the standard deviation and the non-parametric value-at-risk, reveal that gold, crude oil and the volatility index (VIX) are substantially riskier than bonds.

In the case of the crude oil/stock portfolios, the mean values of the hedge ratios lie between 0.11 and 0.17, which implies that 11 cents can hedge a \$1 long position in the equity markets in the oil market. The hedge ratio is higher for Italy's oil/stock market compared to the hedge ratios between the oil and stock markets of other countries. Italy has negative stock returns over the sample period. This implies the weak

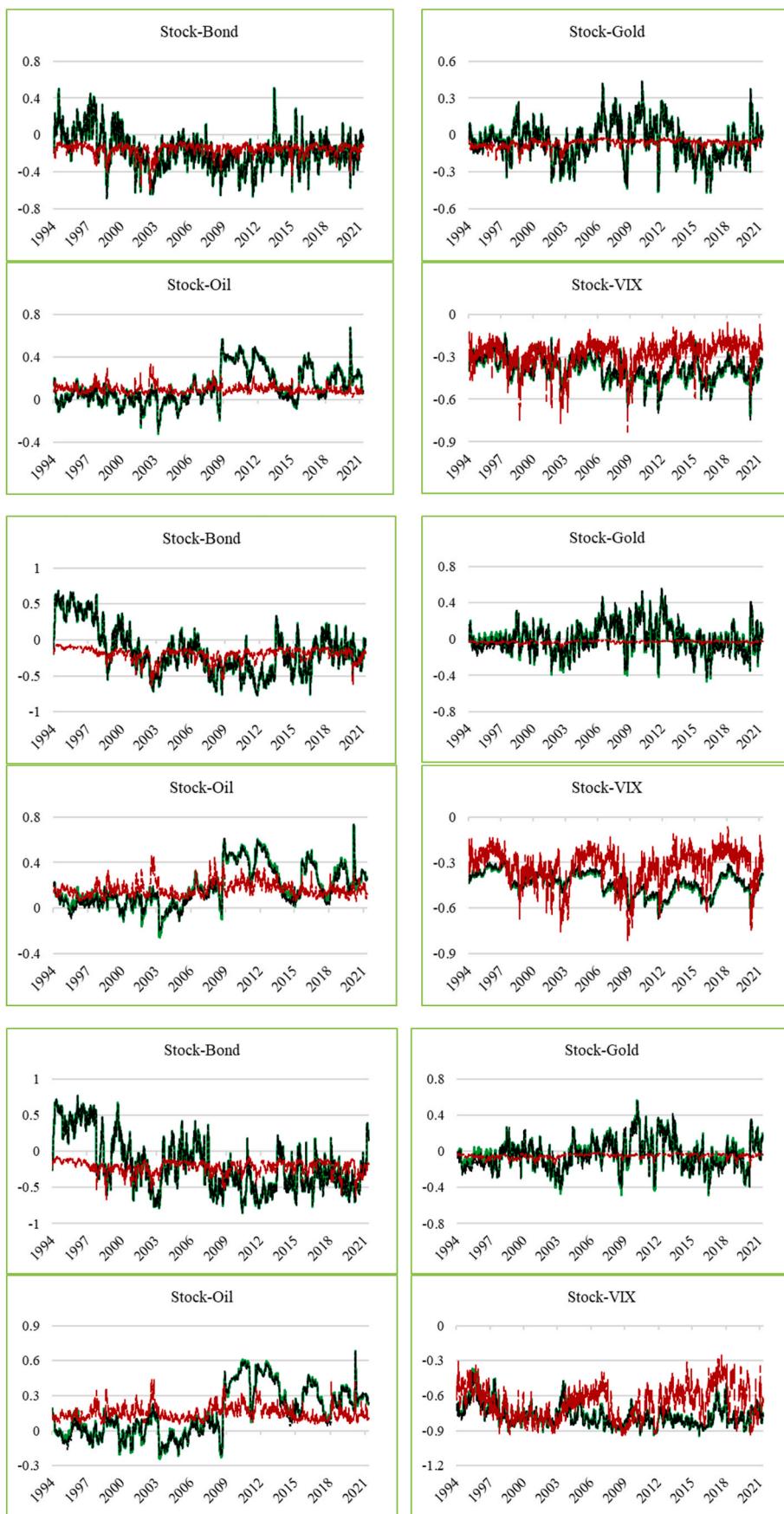


Fig. 1. (continued).

Table 7
Correlations.

Models	Correlations between correlations				Correlations between hedge ratios			
	SI/Bond	SI/Oil	SI/Gold	SI/VIX	SI/Bond	SI/Oil	SI/Gold	SI/VIX
Australia	DCC/ADCC	1	0.998	0.997	0.99	0.994	0.991	0.964
	DCC/GO-GARCH	0.187	0.176	0.324	-0.282	0.505	0.45	-0.138
	ADCC/GO-GARCH	0.184	0.177	0.346	-0.253	0.516	0.468	-0.125
Belgium	DCC/ADCC	0.999	0.999	0.999	0.982	0.993	0.992	0.95
	DCC/GO-GARCH	0.442	0.543	0.464	-0.068	0.499	0.667	0.555
	ADCC/GO-GARCH	0.444	0.553	0.473	-0.067	0.487	0.657	0.55
Canada	DCC/ADCC	0.999	1	0.999	0.991	0.993	0.991	0.986
	DCC/GO-GARCH	0.627	0.029	0.621	0.098	0.755	0.477	0.473
	ADCC/GO-GARCH	0.624	0.022	0.614	0.105	0.768	0.49	0.451
France	DCC/ADCC	0.997	0.999	0.999	0.979	0.988	0.987	0.988
	DCC/GO-GARCH	0.584	0.551	0.514	0.056	0.718	0.716	0.546
	ADCC/GO-GARCH	0.597	0.564	0.51	0.035	0.719	0.707	0.563
Germany	DCC/ADCC	0.999	1	0.998	0.988	0.98	0.992	0.99
	DCC/GO-GARCH	0.097	0.52	0.516	0.027	0.584	0.697	0.607
	ADCC/GO-GARCH	0.092	0.531	0.506	0.029	0.545	0.694	0.59
Italy	DCC/ADCC	0.994	0.999	0.998	0.989	0.948	0.987	0.988
	DCC/GO-GARCH	0.072	0.487	0.597	0.251	0.084	0.744	0.607
	ADCC/GO-GARCH	0.059	0.479	0.583	0.283	0.114	0.734	0.589
Japan	DCC/ADCC	0.999	0.998	0.996	0.993	0.971	0.99	0.992
	DCC/GO-GARCH	0.297	0.263	0.633	-0.042	0.905	0.219	0.7
	ADCC/GO-GARCH	0.305	0.303	0.637	-0.047	0.896	0.242	0.711
Netherlands	DCC/ADCC	0.998	0.999	0.999	0.98	0.979	0.99	0.992
	DCC/GO-GARCH	0.422	0.551	0.586	0.006	0.682	0.696	0.708
	ADCC/GO-GARCH	0.409	0.557	0.578	-0.006	0.642	0.687	0.695
Spain	DCC/ADCC	0.995	0.999	0.999	0.992	0.996	0.998	0.995
	DCC/GO-GARCH	-0.095	0.534	-0.099	-0.192	-0.241	0.708	0.558
	ADCC/GO-GARCH	-0.086	0.543	-0.108	-0.176	-0.239	0.721	0.562
Sweden	DCC/ADCC	0.999	0.999	0.998	0.989	0.986	0.994	0.991
	DCC/GO-GARCH	-0.037	0.437	0.635	-0.154	0.4	0.62	0.66
	ADCC/GO-GARCH	-0.061	0.43	0.62	-0.186	0.38	0.613	0.638
Switzerland	DCC/ADCC	0.995	0.998	0.998	0.995	0.982	0.989	0.994
	DCC/GO-GARCH	0.121	0.235	0.563	0.237	0.701	0.528	0.598
	ADCC/GO-GARCH	0.13	0.217	0.551	0.252	0.706	0.516	0.575
UK	DCC/ADCC	0.998	0.999	0.997	0.985	0.985	0.988	0.984
	DCC/GO-GARCH	0.042	0.486	0.47	-0.02	0.611	0.725	0.443
	ADCC/GO-GARCH	0.025	0.49	0.448	-0.009	0.62	0.726	0.424
USA	DCC/ADCC	0.996	0.998	0.996	0.982	0.971	0.977	0.986
	DCC/GO-GARCH	0.467	0.344	0.516	0.537	0.688	0.484	0.484
	ADCC/GO-GARCH	0.497	0.334	0.511	0.565	0.722	0.474	0.505

Notes: Fixed width rolling analysis (produces 1000 one-step forecasts) is used to calculate the forecasts. Models are refit every 20 observation.

performance of the equities during this period, which is *prima facie* manifested in the Italian economic performance. The higher hedge ratio also suggests the Italian stock's vulnerability to the oil shocks for which the higher optimal hedge ratios are required. On the other hand, crude oil's hedging effectiveness is weak for the stock markets included in our sample period.

Compared to crude oil, a short position in gold futures against a long position in the G-12 stock markets has better hedging benefits, except for Canada. Moreover, in each case gold provides the best hedging effectiveness for the stock markets. If the macroeconomic shocks have a differential impact on the stock and commodities prices, we suggest choosing commodities as a financial protector during financial downturns.

The hedging effectiveness of the volatility index is higher than the national benchmark bonds' hedging effectiveness, gold and crude oil. The average values of the hedge ratios between VIX and the stock market are negative because of the remarkable negative correlation the index has as earlier documented in the literature (Brenner and Zhang, 2006; Szado, 2009; Chen et al., 2011; Jung, 2016). This negative correlation is a basic feature of VIX which makes its derivatives attractive and appealing hedge tools.

Overall, our findings suggest that all three models (DCC, ADCC, and GO-GARCH) offer similar insights for the hedging effectiveness in the case of each examined pair, including the bond/stock, oil/stock, gold/stock, and VIX/stock pairs. However, the average values of the optimal hedge ratios calculated from the ADCC models are stronger than the corresponding values from the DCC and GO-GARCH models. Our

findings have important implications for investors seeking higher returns in the G-12 countries' stock markets by hedging the equity portfolios' downside risk. The hedging effectiveness is more pronounced for VIX, which can provide room for the development of new widely tradable volatility-related products. We suggest that trading volatilities in the episodes of financial downturns be employed to safeguard equity portfolios. The benchmark bond provides the second-highest hedging effectiveness. In comparison to crude oil, gold is the more desirable commodity to be included in these countries' stock portfolios. These are interesting findings and cannot be enriched further without such an extensive analysis that leads to constructing different portfolios based on risk- and variance-minimizing strategies.

5.2. Risk analysis for portfolio management

Following Reboredo (2013), Hammoudeh et al. (2014) and Mensi et al. (2015b), we compare the risks of three multi-asset portfolios (II, III, IV) with the risk of benchmark portfolio (i.e., Portfolio I) which is composed of stocks only. The usefulness of gold, crude oil, volatility index futures and the benchmark bond indices is evaluated to assess the potential reduction in the risk of the benchmark equity portfolio by adding another asset to this portfolio (I).

Firstly, without reducing the expected returns, we form a risk-minimizing (e.g., stock and gold or crude oil or volatility index futures or benchmark bond) Portfolio (II), following Kroner and Ng (1998) where the optimal weights are given as:

News impact correlation surface plots (DCC)

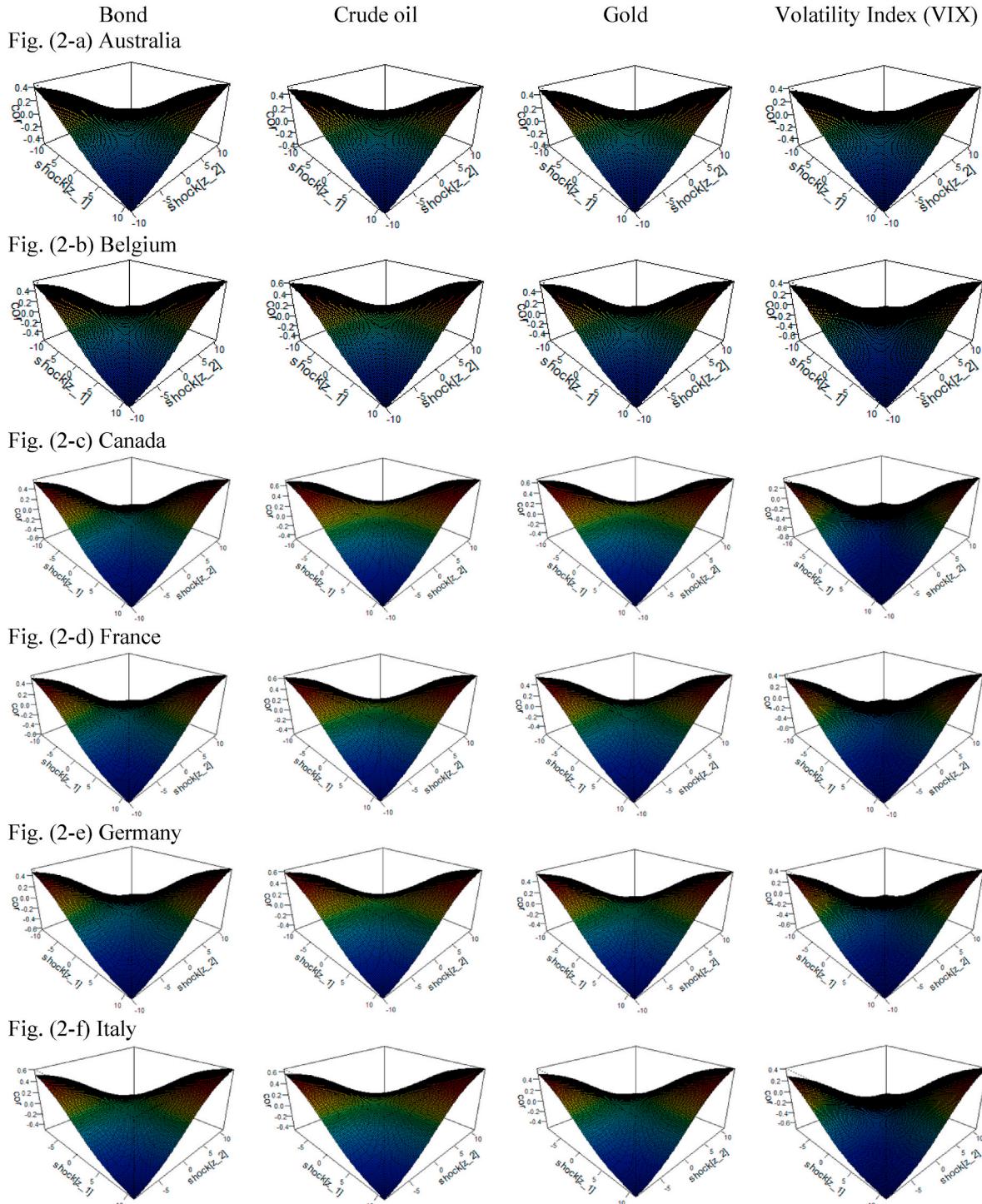


Fig. 2. Correlation surface plots. Note: The correlation surface traces out a positive (negative) to a negative (positive) pattern along the z_1 axis and traces out a negative (positive) to positive (negative) relationship along the z_2 axis. The surface plot that follows a positive to a negative pattern at the z_1 highlights the asymmetric effects of shocks.

$$w_t^{AA} = \frac{h_t^S - h_t^{AAS}}{h_t^C - 2h_t^{AAS} + h_t^S}, \quad \text{with } w_t^{*AA} = \begin{cases} 0 & \text{if } w_t^{AA} < 0 \\ w_t^{AA} & \text{if } 0 \leq w_t^{AA} \leq 1 \\ 1 & \text{if } w_t^{AA} > 1 \end{cases} \quad (15)$$

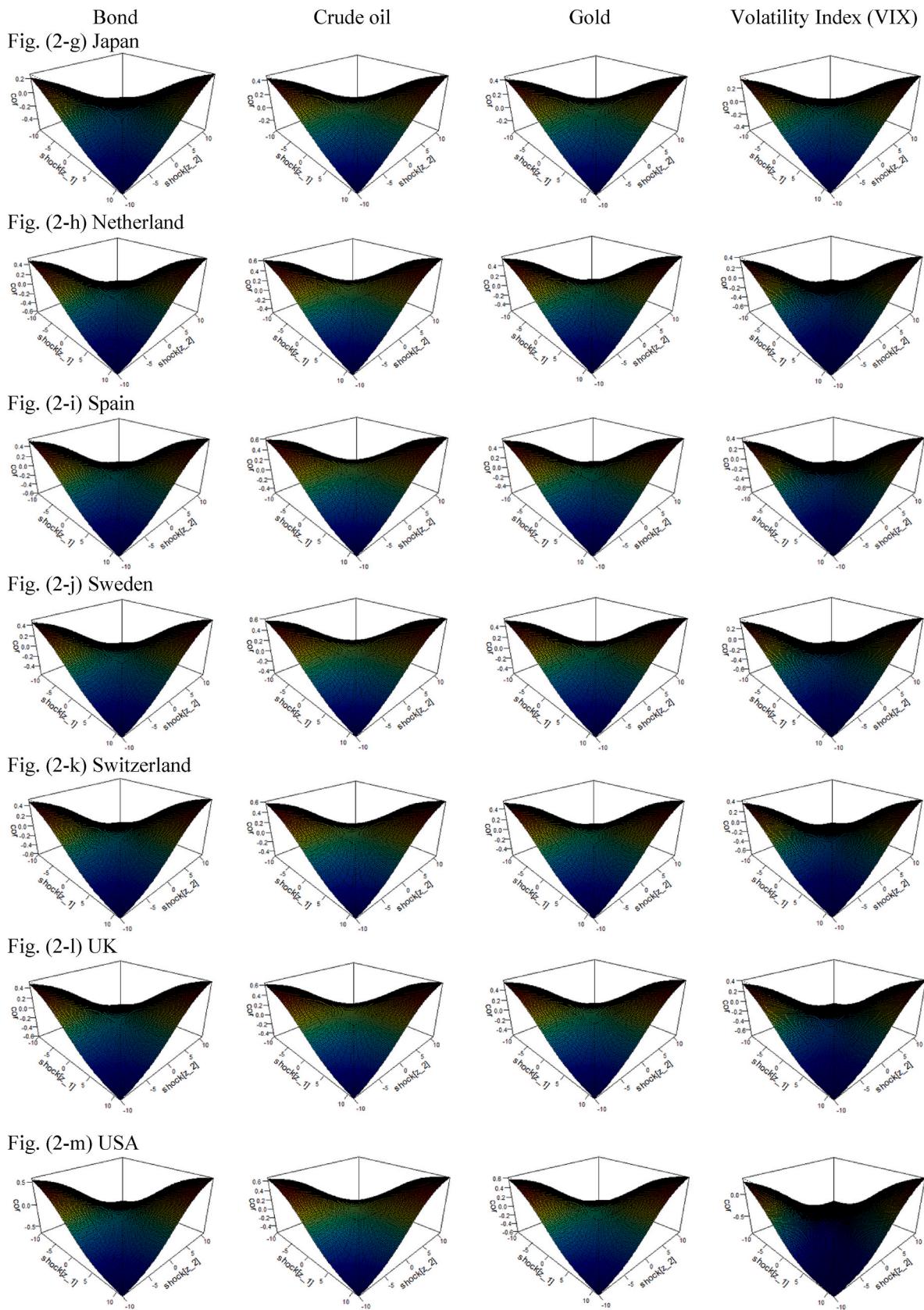


Fig. 2. (continued).

where the conditional volatility of an alternative asset is represented by h_t^{AA} , the conditional volatility of a stock by h_t^S and the conditional covariance between the alternative asset and a stock at time t by h_t^{AAS} .

The optimal weight of the stock (i.e., $1 - w_t^{*AA}$) and for each pair, the information on w_t^{*AA} is obtained from the rolling window analysis, that is, the one-step-ahead rolling conditional correlation between the

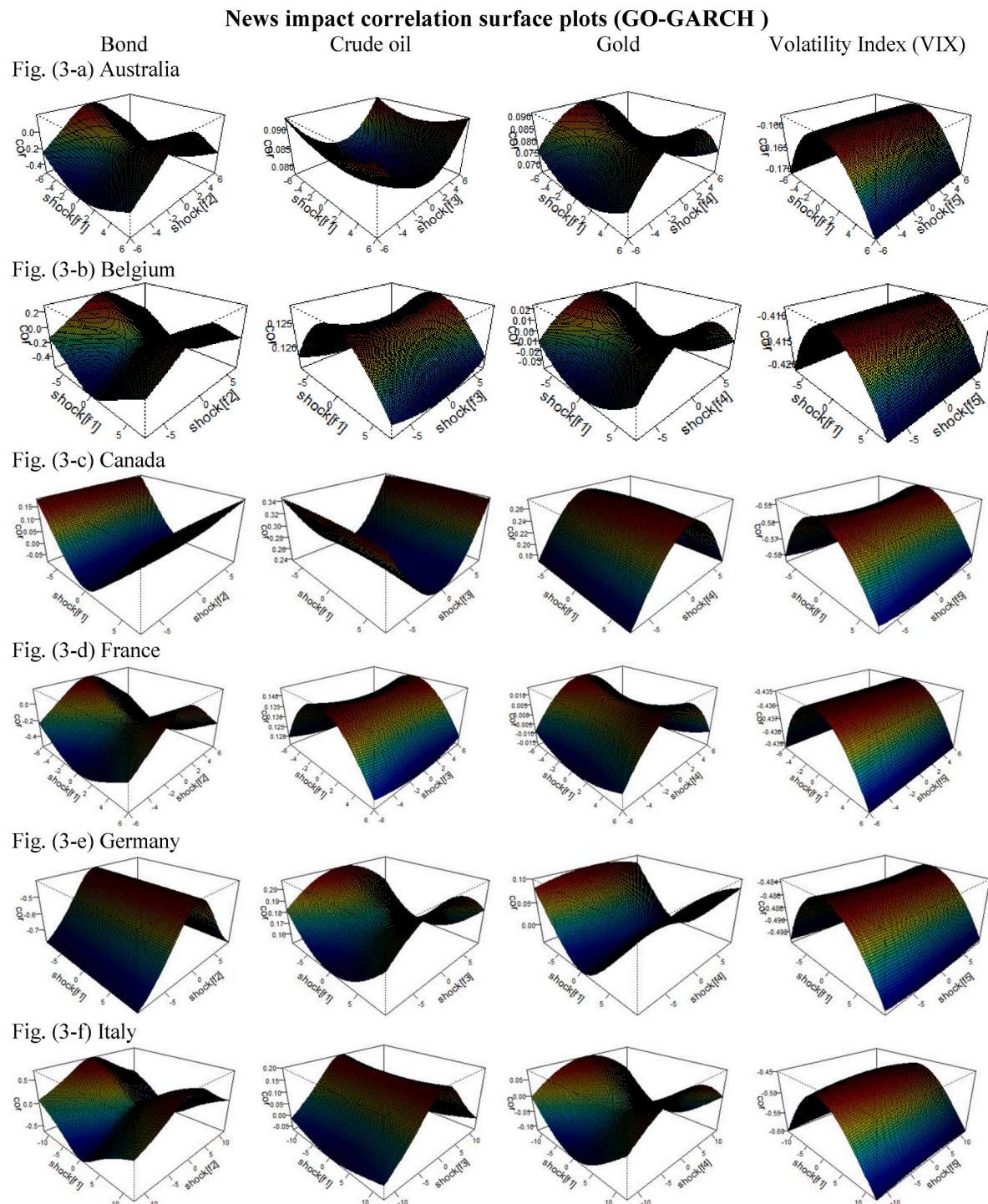


Fig. 3. Correlation surface plots. Note: The correlation surface traces out a positive (negative) to a negative (positive) pattern along the z_1 axis and traces out a negative (positive) to positive (negative) relationship along the z_2 axis. The surface plot that follows a positive to a negative pattern at the z_1 highlights the asymmetric effects of shocks

alternative asset and the stock produced from the DCC, ADCC and GO-GARCH models. The rolling window size remains the same as used earlier to construct the hedged portfolio in the previous section.

Secondly, in Portfolio III the weights are determined using the variance-minimizing strategy, e.g., having a long position in a stock market and a short position in an alternative asset, given as:

$$\beta_t = \frac{h_t^{AAS}}{h_t^{AA}} \quad (16)$$

Finally, Portfolio IV which is composed of equal weights of stock and an alternative asset is considered similar to that in DeMiguel et al. (2009), as this kind of portfolio has a good out-of-sample performance.

The risk-reduction effectiveness (RE) of a multi-asset portfolio P_j is assessed by comparing the percentage reduction in the variance of the multi-asset portfolio relative to the variance of the benchmark Portfolio I, which is the only stock portfolio:

News impact correlation surface plots (GO-GARCH)

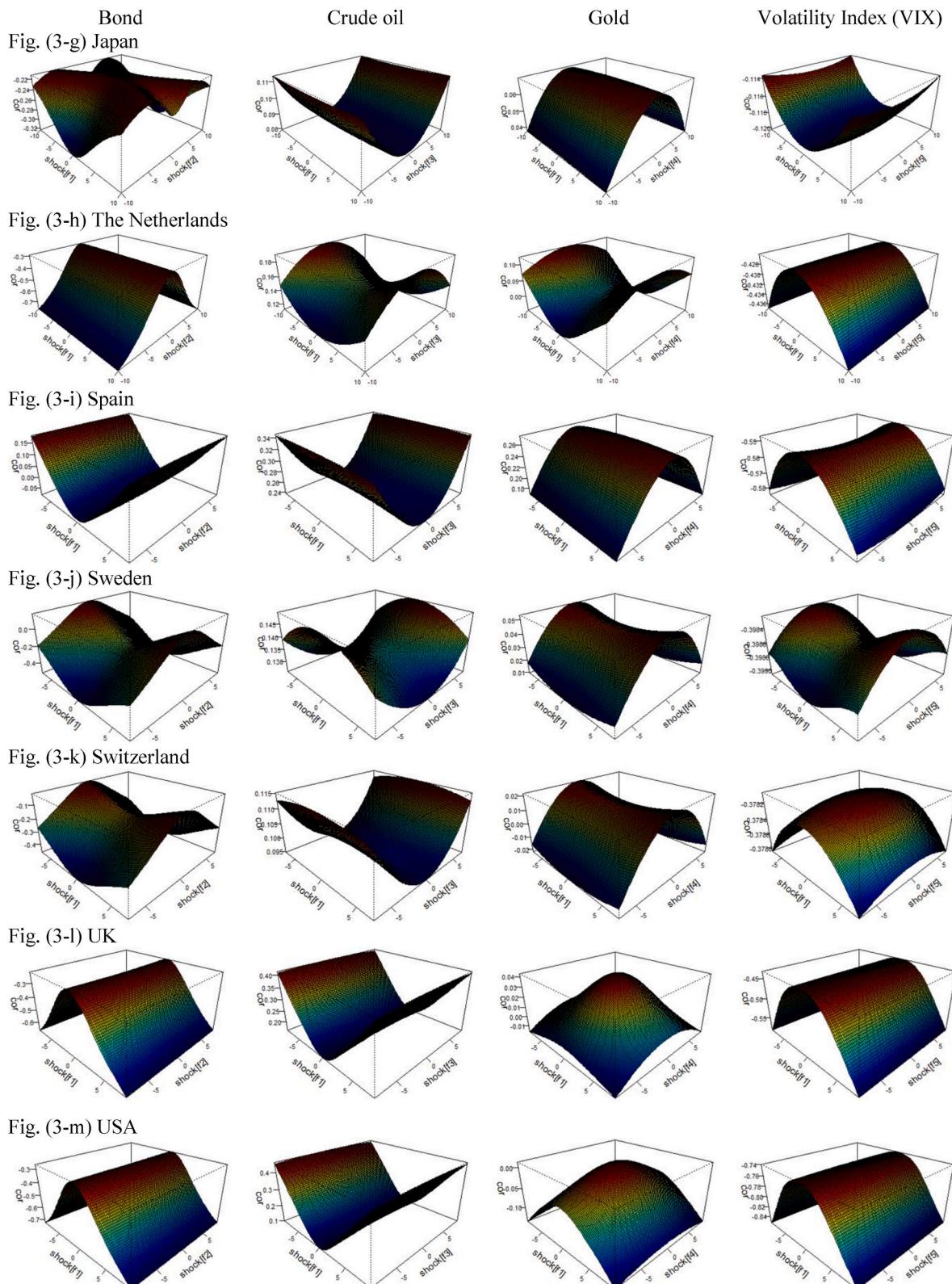


Fig. 3. (continued).

Table 8

Hedge ratio summary statistics and hedging effectiveness (HE).

		Benchmark Bonds				Crude Oil				Gold				Volatility Index (VIX)			
		Mean	Min	Max	HE	Mean	Min	Max	HE	Mean	Min	Max	HE	Mean	Min	Max	HE
Australia	DCC	-0.254	-0.751	0.137	0.034	0.032	-0.062	0.13	0.001	0.022	-0.178	0.146	-0.003	-0.019	-0.052	-0.001	0.025
	ADCC	-0.245	-0.852	0.165	0.036	0.03	-0.057	0.141	0.002	0.022	-0.188	0.146	-0.004	-0.019	-0.053	-0.002	0.026
	GO-GARCH	-0.196	-1.802	0.267	0.029	0.072	-0.015	0.258	-0.001	0.052	-0.097	0.192	-0.008	-0.025	-0.031	-0.021	0.025
Belgium	DCC	-0.137	-1.082	1.344	0.033	0.115	-0.042	0.406	0.1	0.003	-0.472	0.385	0.019	-0.07	-0.153	-0.029	0.224
	ADCC	-0.126	-1.056	1.473	0.038	0.113	-0.043	0.423	0.102	0.01	-0.457	0.492	0.018	-0.068	-0.156	-0.026	0.23
	GO-GARCH	-0.17	-2.245	0.673	0.024	0.13	-0.054	0.584	0.082	0.001	-0.564	0.131	0.014	-0.069	-0.122	-0.047	0.225
Canada	DCC	-0.538	-1.345	0.014	0.117	0.146	0.036	0.379	0.204	0.153	-0.135	0.466	0.064	-0.068	-0.139	-0.028	0.396
	ADCC	-0.538	-1.483	0.013	0.121	0.147	0.04	0.401	0.206	0.146	-0.126	0.622	0.059	-0.071	-0.154	-0.025	0.395
	GO-GARCH	-0.432	-1.689	0.261	0.112	0.164	0.02	0.469	0.179	0.141	-0.447	0.223	0.056	-0.082	-0.109	-0.065	0.358
France	DCC	-0.434	-1.664	0.738	0.02	0.154	-0.029	0.54	0.101	-0.01	-0.515	0.444	0.022	-0.088	-0.174	-0.034	0.23
	ADCC	-0.417	-1.709	0.764	0.026	0.149	-0.035	0.578	0.103	-0.006	-0.56	0.601	0.022	-0.088	-0.179	-0.031	0.237
	GO-GARCH	-0.522	-3.108	0.394	0.024	0.169	-0.081	0.75	0.085	-0.021	-0.533	0.05	0.013	-0.088	-0.144	-0.066	0.237
Germany	DCC	-1.105	-2.738	0.081	0.12	0.14	-0.039	0.516	0.088	-0.015	-0.517	0.454	0.018	-0.082	-0.171	-0.027	0.206
	ADCC	-1.06	-2.901	0.103	0.131	0.136	-0.043	0.553	0.091	-0.007	-0.498	0.598	0.018	-0.083	-0.173	-0.026	0.211
	GO-GARCH	-0.885	-3.033	-0.046	0.082	0.141	-0.06	0.626	0.073	-0.022	-0.488	0.058	0.015	-0.083	-0.135	-0.055	0.206
Italy	DCC	1.18	0.363	2.346	0.253	0.175	-0.046	0.602	0.088	-0.016	-0.549	0.596	0.023	-0.102	-0.222	-0.032	0.181
	ADCC	1.169	0.363	2.72	0.252	0.163	-0.046	0.584	0.089	-0.01	-0.535	0.593	0.024	-0.098	-0.207	-0.032	0.188
	GO-GARCH	0.501	0.036	1.837	0.131	0.228	-0.089	0.737	0.069	-0.011	-0.682	0.113	0.014	-0.096	-0.167	-0.069	0.192
Japan	DCC	-2.748	-6.443	-0.277	0.069	0.032	-0.033	0.116	-0.005	-0.012	-0.391	0.123	0	-0.025	-0.096	0.003	0.011
	ADCC	-2.701	-5.734	-0.299	0.073	0.029	-0.033	0.104	-0.004	-0.016	-0.375	0.138	0.001	-0.024	-0.09	0.003	0.011
	GO-GARCH	-3.072	-9.033	-0.719	0.065	0.114	-0.013	0.635	-0.024	0.081	-0.065	0.109	-0.01	-0.028	-0.116	-0.014	0.008
Netherland	DCC	-0.723	-1.898	0.148	0.045	0.121	-0.023	0.431	0.094	-0.022	-0.549	0.365	0.022	-0.075	-0.168	-0.027	0.231
	ADCC	-0.701	-2.079	0.119	0.052	0.119	-0.023	0.465	0.096	-0.015	-0.541	0.483	0.022	-0.074	-0.161	-0.025	0.236
	GO-GARCH	-0.445	-2.66	0.387	0.044	0.114	-0.087	0.541	0.079	0.007	-0.401	0.082	0.013	-0.074	-0.147	-0.053	0.237
Spain	DCC	0.835	0.26	1.867	0.192	0.149	-0.033	0.571	0.082	0.021	-0.366	0.708	0.028	-0.089	-0.208	-0.035	0.161
	ADCC	0.833	0.228	1.968	0.194	0.144	-0.031	0.54	0.082	0.023	-0.399	0.628	0.028	-0.089	-0.233	-0.031	0.167
	GO-GARCH	0.064	-0.264	0.922	-0.009	0.267	-0.044	0.919	0.063	0.004	-0.236	0.098	-0.009	-0.089	-0.259	-0.005	0.13
Sweden	DCC	-0.805	-2.231	0.064	0.129	0.114	-0.029	0.508	0.095	0.006	-0.374	0.475	0.021	-0.069	-0.159	-0.026	0.232
	ADCC	-0.801	-2.465	0.044	0.135	0.115	-0.028	0.569	0.099	0.012	-0.383	0.637	0.02	-0.07	-0.176	-0.022	0.235
	GO-GARCH	-0.659	-2.134	0.005	0.099	0.154	-0.038	0.544	0.075	-0.013	-0.651	0.067	0.022	-0.086	-0.107	-0.076	0.197
Switzerland	DCC	-0.676	-4.088	0.44	0.025	0.068	-0.031	0.247	0.026	-0.057	-0.891	0.189	0.027	-0.054	-0.147	-0.015	0.13
	ADCC	-0.659	-4.715	0.339	0.015	0.066	-0.034	0.255	0.023	-0.056	-0.988	0.231	0.026	-0.053	-0.145	-0.013	0.132
	GO-GARCH	-0.845	-4.933	-0.213	0.001	0.092	-0.053	0.46	-0.026	-0.044	-0.458	0.029	0.022	-0.062	-0.212	-0.048	0.099
UK	DCC	-0.646	-1.704	-0.005	0.15	0.118	0.009	0.467	0.114	0.03	-0.272	0.408	0.009	-0.061	-0.133	-0.021	0.229
	ADCC	-0.624	-1.802	0.009	0.159	0.117	0.007	0.497	0.12	0.035	-0.268	0.575	0.005	-0.061	-0.143	-0.022	0.235
	GO-GARCH	-0.505	-1.557	0.001	0.127	0.124	-0.064	0.479	0.091	-0.013	-0.391	0.093	0.008	-0.065	-0.096	-0.052	0.231
USA	DCC	-0.72	-1.546	-0.047	0.166	0.133	-0.019	0.38	0.138	0.016	-0.363	0.394	0.016	-0.104	-0.181	-0.051	0.697
	ADCC	-0.687	-2.015	0.007	0.174	0.128	-0.031	0.433	0.137	0.022	-0.415	0.454	0.014	-0.104	-0.177	-0.05	0.701
	GO-GARCH	-0.652	-2.501	0.003	0.162	0.093	-0.047	0.522	0.085	-0.051	-0.886	0.036	0.005	-0.117	-0.148	-0.101	0.662

Notes: Fixed width rolling analysis (produces 1000 one-step forecasts) is used to calculate the hedge ratios. Models are refit every 20 observation.

$$RE_{Var} = 1 - \frac{Var(P_j)}{Var(P_I)} \quad (17)$$

where, P_j indicates the three different portfolios (II, III, IV) and $Var(P_j)$ and $Var(P_I)$ denotes the variance of the j th multi-asset portfolio and the variance of Portfolio I, respectively. The RE_{Var} values lie between 0 and 1 and a higher value indicates a higher variance reduction. The one-step-ahead rolling conditional correlations and volatilities are used to explore the portfolio diversification effects of gold, crude oil, bonds and VIX, by using different analytical risk measures such as the value-at-risk, VaR reduction, Expected Shortfall (ES), Semi variance and Regret reduction. The VaR provides information about the maximum loss in a portfolio at a given time t with a confidence level ($1-p$) with the expected return R_t on a given portfolio. That is,

$$Pr(\gamma_{t-1}) = p$$

However, the VaR of a given portfolio can be computed as:

$$VaR_t(p) = \mu_t - t_v^{-1}(p) \sqrt{h_t}$$

where, the conditional mean and standard deviation of a particular asset is denoted by μ_t and $\sqrt{h_t}$, respectively, along with the $t_v^{-1}(p)$, the p th quartile of the t -distribution and the v degrees of freedom. Moreover, the expected size of a loss exceeding VaR is described as the expected shortfall (ES) given as:

$$ES = E(R_t < VaR_t(p))$$

The return variability which is below a specific threshold is measured by the Semi variance (SV) approach and it is unlike the variance measure that uses equal weights for positive and negative returns. It is given by:

$$SV = E[min\{0, R_t - E(R_t)\}]^2$$

Finally, the values of the expected returns which are below zero are measured by the regret reduction (Re) expressed as:

$$Re = -E[min\{0, R_t\}]$$

5.3. Risk evaluation

We examine the risk and downside risk-reduction effectiveness of different mixed asset (i.e., stock-bond, stock-oil, stock-gold, and stock-VIX future) Portfolios III and IV, relative to the benchmark portfolio I, i.e., the stock-only portfolio. The portfolio weights are computed based on the risk-minimization strategy (i.e., Portfolio II), the variance minimization strategy (i.e., Portfolio III), and the equally weighted (i.e., Portfolio IV). The results are reported in Table 9, which show that the national benchmark bonds, when included in their respective country stock portfolios, significantly reduce the risk of portfolios as compared to the addition of each of the other alternative assets. Our findings tend to indicate that the risk-reduction effectiveness gains are higher for mixed stock-bond portfolios compared to the mixed stock-oil, stock-gold, and stock-VIX future portfolios for the G-12 countries. However, the risk-reduction effectiveness varies across the portfolios where it is more pronounced for Portfolios II and IV than for Portfolio III. The risk-reduction effectiveness is the highest (lowest) for the stock-bond (stock-oil) portfolio. Moreover, the VIX futures provide the second highest risk-reduction effectiveness. Gold provides the third when added to G-12 stock portfolios.

The empirical evidence for the VaR reduction effectiveness indicates that the national benchmark bond decreases the VaR at the 95% confidence level. The VaR reduction gains are also higher for the stock-bond portfolios II and IV, but lower for stock-crude oil portfolios II and IV. The comparative gains also highlight that after the highest VaR reduction generated by the inclusion of bonds in the stock portfolios, the VaR reduction gains of the stock-VIX futures portfolios are more than those

for the stock-gold and the stock-crude oil portfolios, except for the Australian and Japanese stocks, where gold outperforms the VIX futures.

In addition to that, the expected shortfall (ES) reduction, the semi-variance (SV) reduction and the regret reduction demonstrate similar patterns as observed for the risk and VaR reductions. The ES, SV and Re reductions dominate the stock-bond portfolios II and IV. We infer that during global uncertainties, investors with an objective to safeguard their stock portfolios through high risk-adjusted returns have to shift their primary focus of investing from the stock only portfolios to the mixed asset portfolios. Our findings suggest that the benchmark bonds are more appropriate for stock portfolios to prevent investors from losing the expected returns. We demonstrate that the VIX futures provide the second-highest risk reduction gains and gold provide the third. However, no significant risk-reduction benefits have been witnessed while adding crude oil to the stock portfolios. Overall, the analysis underscores the importance of mixed-asset portfolios in order to reduce risk and downside risk. However, the gains of the risk and VaR reductions may vary because of the composition of the portfolios, time scale, etc. The results also highlight that the stock-only portfolio would exhibit the greater risk and the expected maximum loss (or the variability of losses), compared to the mixed stock-bond, the mixed stock-VIX futures or the mixed stock-gold portfolio.

Finally, we observe that the risk minimized and equally weighted portfolios (II and III, respectively) outperform the variance minimized and the benchmark stock only portfolios I and IV, respectively. This is consistent with the reported literature (e.g., Reboredo, 2013; Hammoudeh et al., 2014; Kolluri et al., 2015; Huang et al., 2016).

6. Conclusion

This study aims to evaluate the hedging effectiveness and the diversification benefits of alternative assets (i.e., benchmark bonds, gold, crude oil and VIX futures) for the major global stock indices of G-12 countries. In this endeavour, we employed a rich set of data and novel empirical approaches and used three different multivariate GARCH models namely; DCC, ADCC and GO-GARCH. We should emphasize that the curse of dimensionality associated with estimating these multivariate GARCH models with large datasets is challenging but is also intriguing and presents strong motivation for us to study the symmetry (or lack thereof) in the time series and to capture the persistence in the volatility and time-varying relationships between the stock market indices and each of the bonds, commodities and volatility derivatives under consideration. The asymmetry features reveal that in comparison to positive shocks, the negative shocks of equal magnitude have more pronounced effects. This drives investors to seek safe haven assets to safeguard their investments from extreme negative volatility shocks since these shocks do not have the same impact. Thus, portfolio managers and investors may lose their investments in the long-run, even if they earn high returns in the short-run. Moreover, to study the spillover effects in volatility under a linear transformation, we apply the GO-GARCH model which satisfies all the necessary time series features such as persistence in volatility, conditional correlations and spillover effects. As is well known, the financial time series are characterized by presenting heavy tails, gain/loss asymmetry and volatility clusters. These characteristics make (GO-GARCH) model an excellent option to be applied on such series as indicated that it is a conditional volatility model for multivariate returns that can incorporate heavy tails and asymmetric returns quite naturally. The difficulty and enormous labour associated with the estimation of GO-GARCH has been the reason of its underutilization, yet considering its empirical robustness, we considered it for our empirical exercise. Our findings of this model for volatility persistence coincide with the results of the DCC and ADCC models, showing that the long-term volatility persistence is considerably higher than the short-term counterpart in each market. The multivariate GARCH models are compared based on their one-step-ahead rolling correlations, optimal hedge ratios and hedging effectiveness. The one-

Table 9
Risk evaluation.

		Bond			Crude oil			Gold			VIX		
		PII	PIII	PIV	PII	PIII	PIV	PII	PIII	PIV	PII	PIII	PIV
Australia	Risk Red	0.871	0.048	0.769	0.104	0.007	-0.533	0.404	0.004	0.396	0.340	0.012	0.084
	VaR Red	0.643	0.028	0.521	0.053	0.004	-0.243	0.233	0.002	0.228	0.187	0.006	0.040
	ES Red	0.694	0.155	0.555	0.067	0.006	-0.092	0.231	-0.014	0.213	0.254	-0.015	0.200
	SV Red	0.658	0.052	0.531	0.051	0.005	-0.205	0.231	0.002	0.225	0.218	0.008	0.124
Belgium	Re Red	0.624	-0.008	0.506	0.043	0.002	-0.267	0.219	0.000	0.216	0.157	0.005	0.007
	Risk Red	0.923	0.001	0.746	0.051	0.037	-0.392	0.488	0.002	0.488	0.590	0.160	0.481
	VaR Red	0.726	0.004	0.499	0.025	0.019	-0.184	0.290	0.000	0.290	0.359	0.084	0.278
	ES Red	0.769	-0.002	0.499	0.031	-0.008	-0.065	0.292	-0.010	0.279	0.464	0.120	0.451
Canada	SV Red	0.730	0.010	0.502	0.017	0.024	-0.165	0.280	0.001	0.280	0.396	0.096	0.344
	Re Red	0.726	-0.001	0.495	0.022	0.012	-0.224	0.272	-0.001	0.272	0.336	0.079	0.251
	Risk Red	0.922	-0.001	0.769	-0.036	0.104	-0.746	0.344	0.048	0.339	0.606	0.183	0.441
	VaR Red	0.725	0.007	0.522	-0.019	0.054	-0.327	0.194	0.022	0.192	0.372	0.096	0.250
France	ES Red	0.758	0.131	0.543	-0.038	0.081	-0.172	0.211	0.014	0.196	0.539	0.175	0.417
	SV Red	0.748	0.051	0.531	-0.014	0.059	-0.257	0.207	0.024	0.207	0.426	0.109	0.364
	Re Red	0.694	-0.043	0.513	-0.050	0.052	-0.446	0.139	0.025	0.133	0.346	0.098	0.196
	Risk Red	0.952	0.047	0.770	0.076	0.039	-0.187	0.548	0.003	0.539	0.639	0.150	0.591
Germany	VaR Red	0.785	0.032	0.523	0.038	0.020	-0.091	0.333	0.001	0.326	0.399	0.078	0.360
	ES Red	0.800	0.086	0.519	-0.001	-0.005	-0.030	0.290	0.011	0.287	0.529	0.111	0.486
	SV Red	0.787	0.046	0.526	0.026	0.024	-0.092	0.317	0.001	0.313	0.438	0.095	0.409
	Re Red	0.781	0.019	0.521	0.037	0.013	-0.117	0.323	0.000	0.314	0.379	0.072	0.342
Italy	Risk Red	0.954	0.087	0.788	0.079	0.034	-0.220	0.528	0.004	0.522	0.622	0.109	0.564
	VaR Red	0.788	0.057	0.542	0.038	0.017	-0.112	0.316	0.001	0.312	0.385	0.057	0.337
	ES Red	0.804	0.162	0.549	-0.023	0.047	-0.129	0.204	-0.026	0.217	0.504	0.052	0.431
	SV Red	0.789	0.068	0.545	0.025	0.023	-0.108	0.302	0.002	0.301	0.426	0.067	0.393
Japan	Re Red	0.780	0.039	0.541	0.031	0.010	-0.149	0.302	-0.001	0.297	0.360	0.053	0.311
	Risk Red	0.899	0.061	0.685	0.121	0.035	-0.042	0.597	0.002	0.574	0.649	0.125	0.626
	VaR Red	0.686	0.029	0.440	0.063	0.018	-0.019	0.372	0.001	0.353	0.407	0.064	0.388
	ES Red	0.740	0.076	0.389	0.033	0.014	0.090	0.387	0.015	0.359	0.506	0.078	0.519
Netherland	SV Red	0.695	0.029	0.440	0.061	0.021	-0.007	0.368	0.001	0.352	0.439	0.073	0.431
	Re Red	0.715	0.022	0.451	0.051	0.010	-0.048	0.358	0.000	0.335	0.385	0.060	0.367
	Risk Red	0.978	-0.624	0.769	0.221	0.004	0.059	0.585	-0.006	0.567	0.445	0.012	0.405
	VaR Red	0.853	-0.256	0.521	0.117	0.002	0.027	0.361	-0.003	0.346	0.254	0.006	0.227
Spain	ES Red	0.866	-0.167	0.538	0.145	-0.004	0.094	0.300	-0.024	0.305	0.270	0.051	0.344
	SV Red	0.858	-0.208	0.521	0.120	0.003	0.059	0.369	-0.002	0.354	0.310	0.021	0.304
	Re Red	0.852	-0.277	0.528	0.084	0.000	-0.025	0.331	-0.006	0.318	0.200	-0.011	0.171
	Risk Red	0.952	0.049	0.780	0.053	0.041	-0.293	0.518	0.003	0.514	0.611	0.112	0.540
Sweden	VaR Red	0.785	0.036	0.533	0.026	0.021	-0.139	0.311	0.001	0.308	0.376	0.058	0.321
	ES Red	0.782	0.070	0.502	-0.036	0.022	-0.085	0.238	0.014	0.253	0.426	0.034	0.400
	SV Red	0.789	0.045	0.534	0.016	0.026	-0.127	0.304	0.001	0.303	0.413	0.066	0.383
	Re Red	0.770	0.013	0.533	0.018	0.015	-0.190	0.287	0.000	0.285	0.351	0.057	0.282
		Bond			Crude oil			Gold			VIX		
		PII	PIII	PIV	PII	PIII	PIV	PII	PIII	PIV	PII	PIII	PIV
Spain	Risk Red	0.876	-0.041	0.690	0.107	0.026	-0.109	0.566	0.000	0.552	0.621	0.106	0.583
	VaR Red	0.651	-0.025	0.445	0.055	0.013	-0.054	0.347	0.000	0.335	0.384	0.054	0.353
	ES Red	0.711	0.035	0.424	0.093	0.007	0.017	0.323	0.000	0.352	0.492	0.091	0.474
	SV Red	0.674	-0.039	0.448	0.049	0.016	-0.050	0.329	0.000	0.321	0.416	0.064	0.394
Sweden	Re Red	0.658	-0.043	0.451	0.039	0.008	-0.090	0.328	0.000	0.317	0.354	0.048	0.325
	Risk Red	0.951	0.037	0.780	0.069	0.034	-0.278	0.503	0.006	0.501	0.602	0.145	0.525
	VaR Red	0.781	0.027	0.532	0.033	0.017	-0.138	0.299	0.002	0.297	0.369	0.076	0.308
	ES Red	0.813	0.035	0.532	-0.008	0.003	-0.031	0.221	0.024	0.215	0.528	0.090	0.479
Switzerland	SV Red	0.783	0.036	0.534	0.028	0.023	-0.121	0.286	0.002	0.286	0.413	0.092	0.375
	Re Red	0.769	-0.006	0.530	0.022	0.009	-0.181	0.281	0.000	0.279	0.340	0.061	0.274
	Risk Red	0.943	-0.064	0.771	0.032	0.018	-0.604	0.453	0.000	0.448	0.517	0.128	0.324
	VaR Red	0.764	-0.023	0.523	0.015	0.009	-0.272	0.265	0.000	0.262	0.305	0.067	0.175
UK	ES Red	0.796	0.059	0.552	-0.018	0.000	-0.134	0.181	-0.001	0.158	0.480	0.158	0.331
	SV Red	0.775	0.010	0.528	0.012	0.011	-0.239	0.260	0.000	0.257	0.351	0.086	0.259
	Re Red	0.748	-0.076	0.516	0.005	0.006	-0.334	0.237	0.000	0.234	0.275	0.057	0.126
	Risk Red	0.919	0.025	0.784	-0.005	0.061	-0.671	0.409	0.009	0.406	0.558	0.169	0.379
USA	VaR Red	0.721	0.058	0.538	0.000	0.040	-0.298	0.236	-0.001	0.234	0.335	0.089	0.210
	ES Red	0.756	0.121	0.574	0.026	0.023	-0.133	0.184	0.015	0.193	0.504	0.181	0.407
	SV Red	0.729	0.042	0.541	-0.011	0.039	-0.271	0.227	0.005	0.224	0.385	0.113	0.295
	Re Red	0.697	-0.020	0.532	-0.011	0.020	-0.367	0.211	0.001	0.207	0.312	0.083	0.166
USA	Risk Red	0.905	0.106	0.784	0.040	0.049	-0.441	0.486	-0.002	0.486	0.751	0.300	0.654
	VaR Red	0.693	0.058	0.537	0.019	0.025	-0.206	0.288	-0.002	0.288	0.502	0.164	0.410
	ES Red	0.766	0.227	0.596	-0.007	0.035	-0.064	0.288	0.015	0.284	0.646	0.239	0.583
	SV Red	0.711	0.102	0.552	0.015	0.029	-0.171	0.294	-0.002	0.294	0.546	0.177	0.488
	Re Red	0.658	0.012	0.523	-0.006	0.019	-0.331	0.229	-0.001	0.229	0.502	0.189	0.377

Notes: Risk Red (Risk-reduction), VaR Red (VaR reduction), ES Red (Expected Shortfall reduction), SV Red (Semivariance reduction) and Re Red (Regret reduction) are calculated for benchmark bonds, crude oil, gold and VIX portfolios by considering the ratio given in Eq. (17) for VaR. Equations (15) and (16) which are used to compute the weights for Portfolios II and III, respectively, and Portfolio IV is the equally weighted portfolio.

step-ahead rolling dynamic conditional correlations produced from the ADCC and DCC models show similar patterns but the correlations produced from GO-GARCH are more pronounced than from the other two approaches. Our findings also have important implications for investors seeking higher risk-returns tradeoff from their investments in major global (i.e., G-12) stock markets by hedging their portfolios' downside risk. Overall, we conclude that the hedging effectiveness is more pronounced for the VIX index compared to the other hedging tools, which highlights the importance of the development of new volatility-related products. The results also suggest that trading volatilities in the episodes of financial downturns can help safeguard equity portfolios. The national benchmark bond provides the second highest hedging effectiveness for the G-12 stock markets and comes right after VIX.

Finally, the usefulness of gold, crude oil, VIX futures and national benchmark bond indices is evaluated to assess the potential reduction in the risk of the benchmark portfolio (I) i.e., stock only, while adding an additional asset. We find that in comparison to the commodities and VIX futures, the national benchmark bonds have the least variance which gives significant credence to the bonds to be included in the suggested mixed portfolios. The comparison of different risk measures shows that the mixed stock-bond portfolios outperform all of the other mixed-asset portfolios of stock-gold, stock-crude oil and stock-VIX futures. The effectiveness of risk reduction varies across the three portfolios. The risk mitigation is more effective for Portfolios II (based on the risk-minimization strategy) and IV (the equally weighted) than for Portfolio III (based on the variance minimization strategy). Lastly, our findings also lead us to conclude that the risk and downside risk mitigation gains are highest for Portfolios II and IV, while they are the lowest for portfolio (IV). The results also highlight that the sole stock investment would exhibit the greatest risk and the expected maximum loss compared to the hybrid portfolios of stock-bond, stock-VIX futures or stock-gold.

CRediT authorship contribution statement

Sajid Ali: Conceptualization, Data curation, Writing – original draft, Writing – review & editing, Revision. **Naveed Raza:** Writing – original draft, Empirical analysis, Methodology, Revision, Visualization, Supervision. **Xuan Vinh Vo:** Writing – review & editing, Validation. **Le Van:** Writing – review & editing, Literature review, Empirical analysis, Revision.

Data availability

Data will be made available on request.

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