Lecture 1 – Review of Matrices & Vectors

Module 3 – Linear Algebra
CS/ISyE/ECE 524

Linear Algebra Module Learning Outcomes

By the end of this module, you should be able to ....

- Define key concepts, such as:
  - Matrix
  - Transpose
  - Inner/outer product
  - Linear/affine functions
  - Affine combinations
  - Convex combinations
  - Hyperplane
  - Subspace
  - Dimension
  - Halfspace
  - Polyhedron (polytope)
- Perform some simple linear algebra operations, such as:
  - Matrix multiplication
  - Write lists of linear/affine functions as matrices
- Start thinking about constraints as intersections of halfspaces

What is a matrix?

A **matrix** is an array of numbers with *m* rows and *n* columns. We denote the space where a matrix "lives" by:

- The most general form of a matrix is as follows:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

- We can multiply matrices together if the **inner dimension** of both matrices are equal:

$$\underset{(m\times p)}{C} \; ¿ \; \underset{(m\times n)}{A} \times \underset{(n\times p)}{B}$$

- The individual elements of the new matrix C are calculated with the formula:

(Not that kind of matrix)

$$C = A \times B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 4 & 3 \\ 8 & 9 \end{bmatrix} = ?$$

$$c_{11} = \sum_{k=1}^{2} a_{1k} b_{k1} = ¿ \; 1 \cdot 4 + 2 \cdot 8 = 4 + 16 = 20$$

$$C = ¿ \begin{bmatrix} 20 & \overset{21}{\overset{\shortparallel}{1 \cdot 3 + 2 \cdot 9}} \\ 44 & 45 \\ 68 & 69 \end{bmatrix}$$

Matrix multiplication example

Basic Definitions: **Transpose**

The transpose operator has some important properties:

- The transpose of the transpose is the original matrix:

- The transpose of two matrices multiplied together is the transpose of the second matrix multiplied by the transpose of the first matrix:

The **transpose** operator swaps rows and columns of a matrix:

and

EXAMPLE:

Basic Definitions: **Vector**

- We typically write vectors as follows:

where

This is just an matrix

- The transpose of a vector is a row matrix (or row vector):

A **vector** is a matrix with only one column ("column matrix").

- The transpose of a row vector (row matrix) is a column vector (column matrix) or just a "vector."

Basic Definitions: **Inner and Outer Products**

The **inner product** operator is a way to multiply two vectors and and produce a scalar:

or

- Another way of writing an inner product operation:

The **outer product** operator is a way to multiply two vectors and and produce a matrix:

- Another way of writing an outer product operation:

Note: These product operations are just special cases of…?

Basic Definitions: **Linear and Affine Functions**

A function is **linear** in if there exist constants such that:

A function is **affine** in if there exist constants such that:

Are these functions of affine, linear, or neither?

Linear

Neither

Affine

Matrix Notation for Lists of Functions

- If we have a list of several linear (or affine) functions of , we can "collect" them in matrices to write them in a more compact format:
- More simply, we can write:
- Now we can define terms related to *vector-valued functions:*

A vector-valued function is **linear** in if there exists a **constant matrix** such that

A vector-valued function is **affine** in if there exists a **constant matrix** and a vector such that