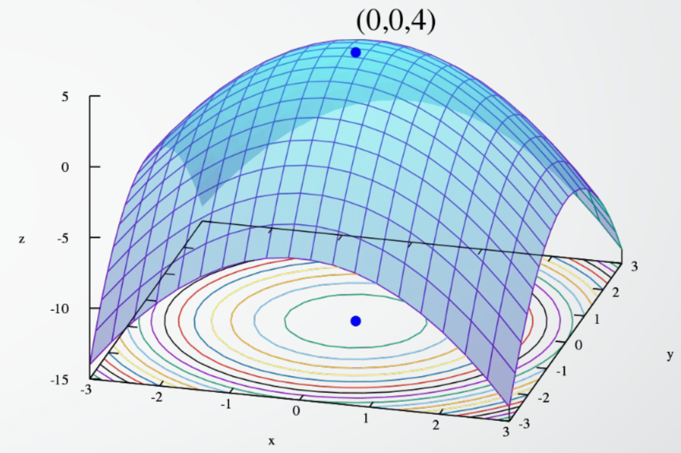
Lecture 2 – Basics of Optimization

Module 1 – Welcome and Introduction

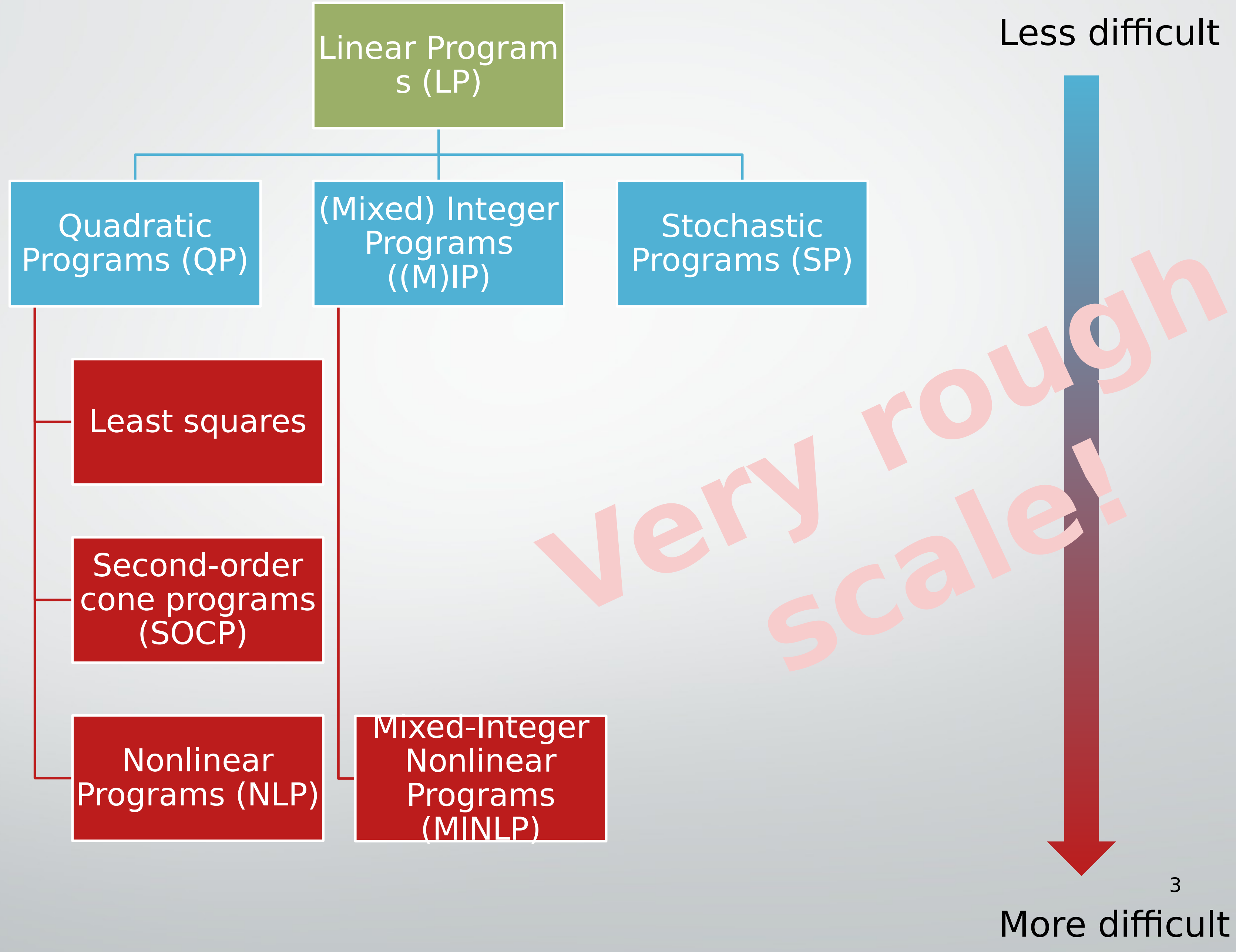
CS/ISyE/ECE 524

Okay, what is this "optimization" business?

From Wikipedia: "In mathematics, computer science and operations research, mathematical **optimization** or mathematical **programming** is the selection of a best element (with regard to some criterion) from some set of available alternatives."



- Very broad – can be more easily understood by looking at different *classes* of optimization
- In this course, we'll investigate each class a bit
  - Almost all these classes of optimization have entire courses dedicated to them
  - We'll only scratch the surface!


Linear Program
s (LP)
Quadratic Programs (QP)
(Mixed) Integer Programs ((M)IP)
Stochastic Programs (SP)
Least squares
Second-order cone programs (SOCP)
Nonlinear Programs (NLP)
Mixed-Integer Nonlinear Programs (MINLP)
Less difficult
More difficult
Very rough scale!

Categories of optimization models

Either	Or
Linear	Nonlinear
Convex	Nonconvex
Discrete	Continuous
Deterministic	Stochastic

These categorizations can have a *significant* impact on the **computational tractability** of a problem instance

How do we solve more complex optimization problems?

Modeling languages

- How you interface with **solvers**
- CVX, GAMS, AMPL, JuMP, ...

Solvers

- Implementation of **algorithms**
- CPLEX, Gurobi, Ipopt,Mosek, ...

Algorithms

- How we solve specific instances of **models**
- Gradient descent, simplex, …

Models

- Mathematical representation of problem
- Categorized based on objective, variables, constraints (LP, QP, SOCP,MIP, SP, ...)
- We’ll spend most of the semester here!

A 3-ingredient recipe for optimization problems

Decisions that we (the decision maker) need to make (**decision variables**)

A single* goal written as a real-valued function (**objective function**)

Rules we need to follow (**constraints**)

Every optimization problem can be broken down into these three ingredients!

*There's a type of optimization called "multi-objective optimization," which we'll very briefly discuss in this course

Top Brass Trophy Company makes large championship trophies for youth athletic leagues. At the moment, they are planning production for fall sports: football and soccer. Each football trophy has a wood base, an engraved plaque, a large brass football on top, and returns $12 in profit. Soccer trophies are similar (a brass ball, a wood base, and an engraved plaque) except that a brass soccer ball is on top, and the unit profit is only $9. Since the football has an asymmetric shape, its base requires 4 board feet of wood; the soccer base requires only 2 board feet. At the moment there are 1000 brass footballs in stock, 1500 soccer balls, 1750 plaques, and 4800 board feet of wood. What trophies should be produced from these supplies to maximize total profit assuming that all that

Top Brass Model

Need those three ingredients:

1. Decision variables:
   - # football trophies: x_f
   - # soccer trophies: x_s

2. Goal (objective):

   max $\$12 x_f + \$9 x_s$

3. Constraints (in words):
   - Can't use more b f. ________ than we have
   - Can't use more b s. ________ than we have
   - Can't use more plaques ________________ than we have
   - Can't use more wood ________________ than we have

Is that it?

- Can't make negative trophies

Let’s turn those constraints into math (remember all those high school math word problems?)

Constraints (in math):

- Can't use more brass footballs than we have

$x_f \le 1000$

- Can't use more brass soccer balls than we have

$x_s \le 1500$

- Can't use more plaques than we have

$x_f + x_s \le 1750$

- Can't use more wood than we have

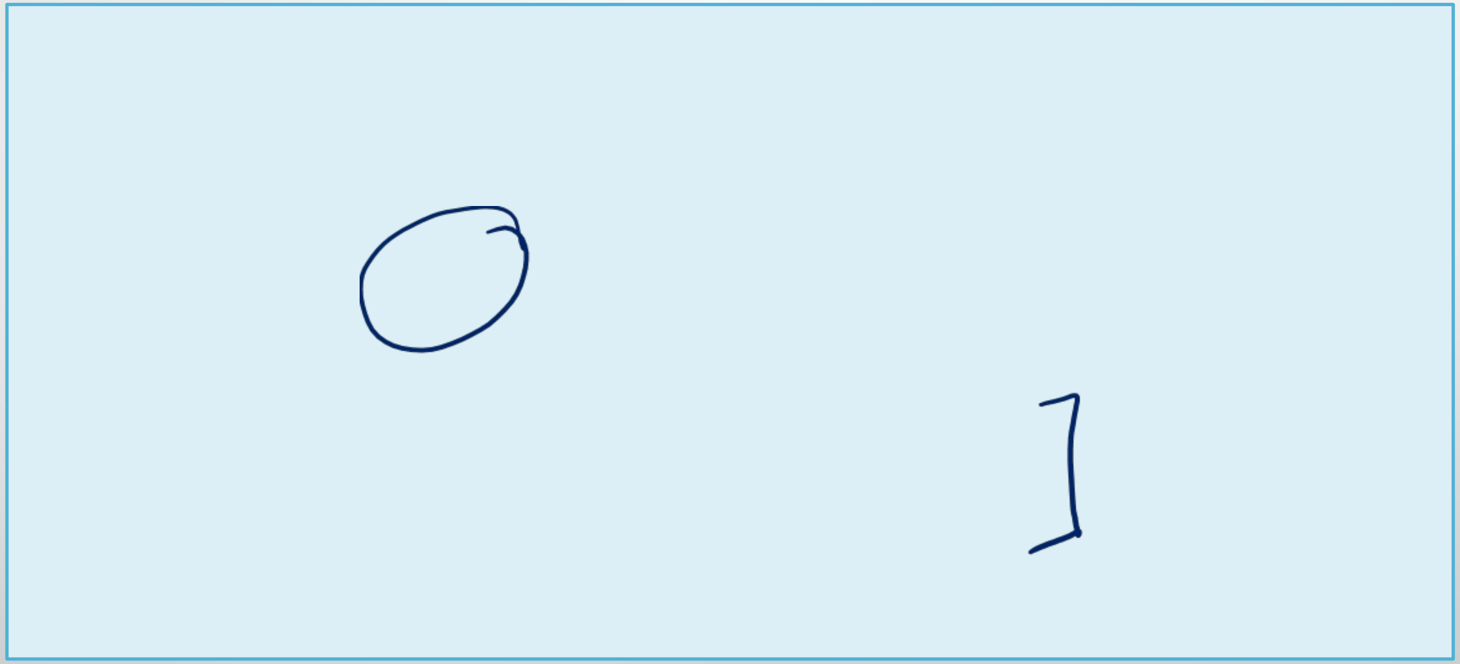
$4x_f + 2x_s \le 4800$

- Can’t make negative trophies

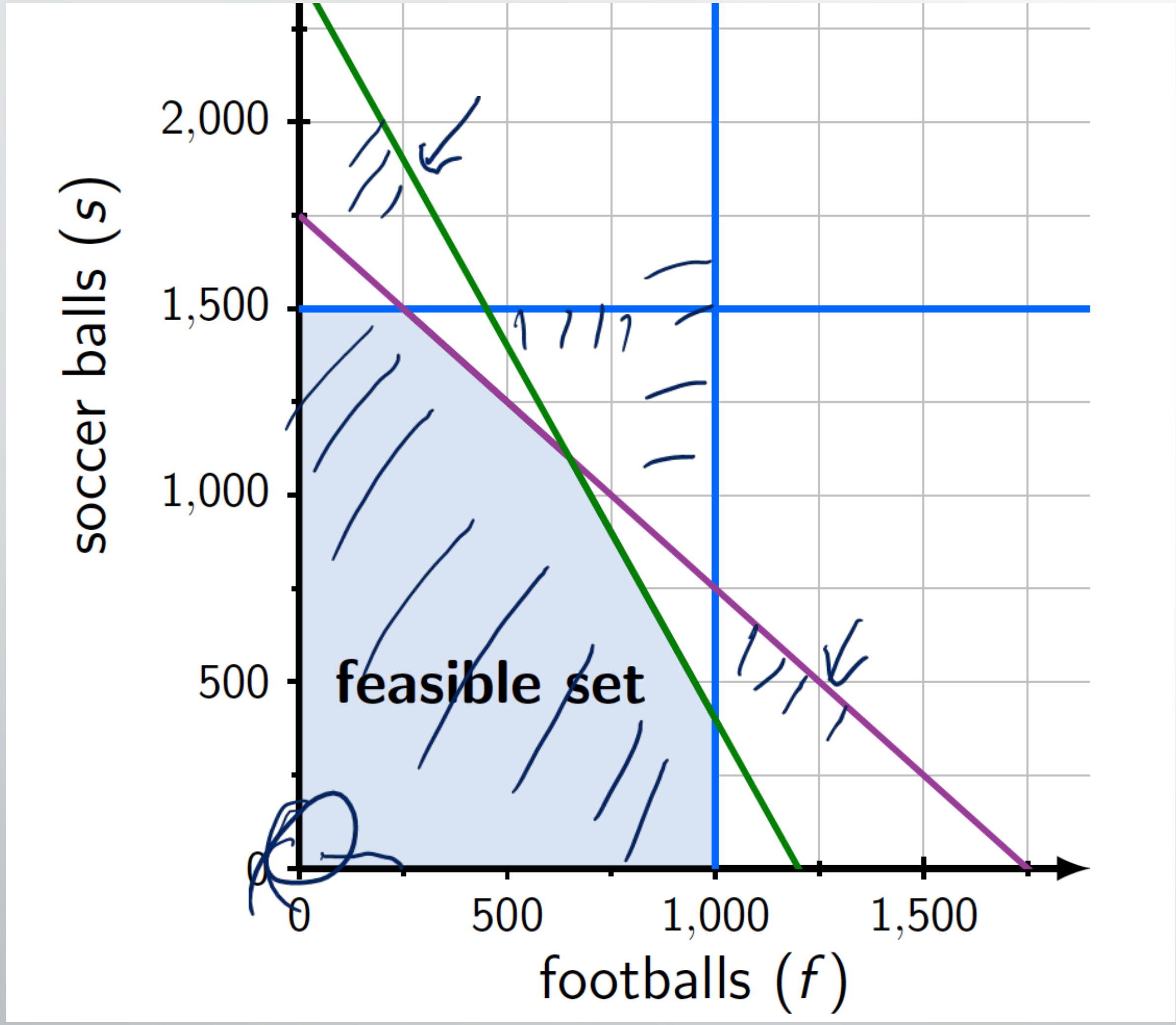
$x_f \geq 0$

$x_s \geq 0$

Full Math Model of Top Brass



Geometry of Top Brass



$$\max_{f,s}\ 12f + 9s$$

$$\text{s.t.}\quad 4f + 2s \le 4800$$

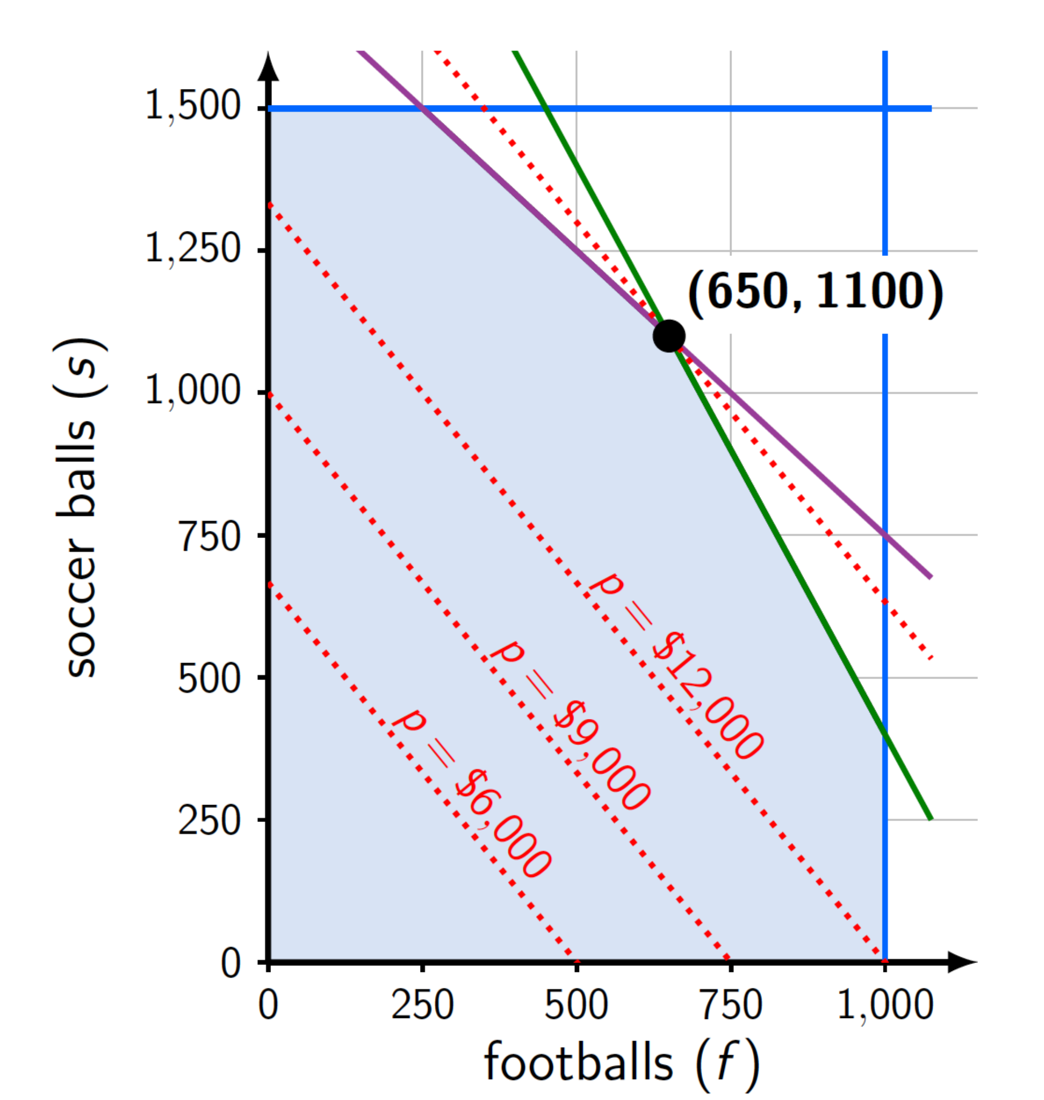
$$f + s \le 1750$$

$$0 \le f \le 1000$$

$$0 \le s \le 1500$$

Any point (f,s) in the feasible set is a **feasible** decision

Geometry of Top Brass



$$\max_{f,s} \; 12f + 9s$$
$$\text{s.t.} \quad 4f + 2s \leq 4800$$
$$f + s \leq 1750$$
$$0 \leq f \leq 1000$$
$$0 \leq s \leq 1500$$

Question: Which feasible point (f,s) gives the *maximum* profit (p = 12f + 9s)?

Welcome & Introduction Module Learning Outcomes

Now, you should….

- Know what to expect in this class
  - Logistics
  - Tentative(!) schedule of topics
- Understand the basic components of optimization (languages, solvers, algorithms, models)
- Have seen an example of an optimization problem
  - Be able to solve the problem with the graphical method