

# 6.830 Lab 5: B+ Tree Index

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**Assigned: Wednesday, April 21, 2021**

**Due: Tuesday, May 4, 2021 11:59 PM EDT**

## 0. Introduction

In this lab you will implement a B+ tree index for efficient lookups and range scans. We supply you with all of the low-level code you will need to implement the tree structure. You will implement searching, splitting pages, redistributing tuples between pages, and merging pages.

You may find it helpful to review sections 10.3--10.7 in the textbook, which provide detailed information about the structure of B+ trees as well as pseudocode for searches, inserts and deletes.

As described by the textbook and discussed in class, the internal nodes in B+ trees contain multiple entries, each consisting of a key value and a left and a right child pointer. Adjacent keys share a child pointer, so internal nodes containing  $m$  keys have  $m+1$  child pointers. Leaf nodes can either contain data entries or pointers to data entries in other database files. For simplicity, we will implement a B+tree in which the leaf pages actually contain the data entries. Adjacent leaf pages are linked together with right and left sibling pointers, so range scans only require one initial search through the root and internal nodes to find the first leaf page. Subsequent leaf pages are found by following right (or left) sibling pointers.

## 1. Getting started

You should begin with the code you submitted for Lab 4 (if you did not submit code for Lab 4, or your solution didn't work properly, contact us to discuss options). Additionally, we are providing extra source and test files for this lab that are not in the original code distribution you received.

You will need to add these new files to your release and set up your lab4 branch. The easiest way to do this is to change to your project directory (probably called simple-db-hw), set up the branch, and pull from the master GitHub repository:

```
$ cd simple-db-hw $ git pull upstream master
```

## 2. Search

Take a look at `index/` and `BTreeFile.java`. This is the core file for the implementation of the B+Tree and where you will write all your code for this lab. Unlike the HeapFile, the BTreeFile consists of four different kinds of pages. As you would

expect, there are two different kinds of pages for the nodes of the tree: internal pages and leaf pages. Internal pages are implemented in `BTreeInternalPage.java`, and leaf pages are implemented in `BTreeLeafPage.java`. For convenience, we have created an abstract class in `BTreePage.java` which contains code that is common to both leaf and internal pages. In addition, header pages are implemented in `BTreeHeaderPage.java` and keep track of which pages in the file are in use. Lastly, there is one page at the beginning of every `BTreeFile` which points to the root page of the tree and the first header page. This singleton page is implemented in `BTreeRootPtrPage.java`. Familiarize yourself with the interfaces of these classes, especially `BTreePage`, `BTreeInternalPage` and `BTreeLeafPage`. You will need to use these classes in your implementation of the B+Tree.

Your first job is to implement the `findLeafPage()` function in `BTreeFile.java`. This function is used to find the appropriate leaf page given a particular key value, and is used for both searches and inserts. For example, suppose we have a B+Tree with two leaf pages (See Figure 1). The root node is an internal page with one entry containing one key (6, in this case) and two child pointers. Given a value of 1, this function should return the first leaf page. Likewise, given a value of 8, this function should return the second page. The less obvious case is if we are given a key value of 6. There may be duplicate keys, so there could be 6's on both leaf pages. In this case, the function should return the first (left) leaf page.

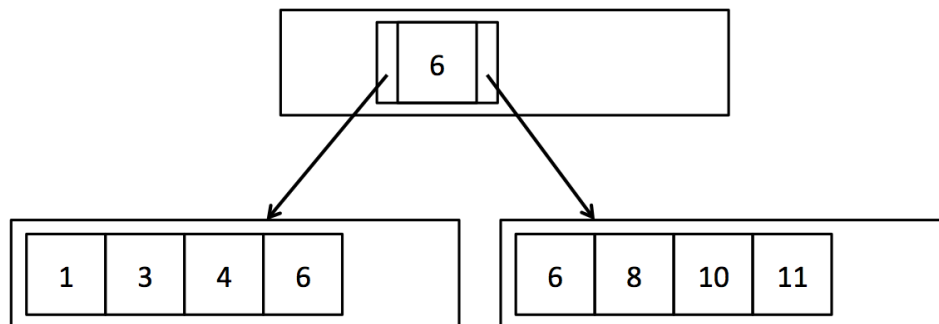


Figure 1: A  
simple B+ Tree with duplicate keys

Your `findLeafPage()` function should recursively search through internal nodes until it reaches the leaf page corresponding to the provided key value. In order to find the appropriate child page at each step, you should iterate through the entries in the internal page and compare the entry value to the provided key value. `BTreeInternalPage.iterator()` provides access to the entries in the internal page using the interface defined in `BTreeEntry.java`. This iterator allows you to iterate through the key values in the internal page and access the left and right child page ids for each key. The base case of your recursion happens when the passed-in `BTreePageId` has `pgcateg()` equal to `BTreePageId.LEAF`, indicating that it is a leaf page. In this case, you should just fetch the page from the buffer pool and return it. You do not need to confirm that it actually contains the provided key value f.

Your `findLeafPage()` code must also handle the case when the provided key value `f` is null. If the provided value is null, recurse on the left-most child every time in order to find the left-most leaf page. Finding the left-most leaf page is useful for scanning the entire file. Once the correct leaf page is found, you should return it. As mentioned above, you can check the type of page using the `pgcateg()` function in `BTreePageId.java`. You can assume that only leaf and internal pages will be passed to this function.

Instead of directly calling `BufferPool.getPage()` to get each internal page and leaf page, we recommend calling the wrapper function we have provided, `BTreeFile.getPage()`. It works exactly like `BufferPool.getPage()`, but takes an extra argument to track the list of dirty pages. This function will be important for the next two exercises in which you will actually update the data and therefore need to keep track of dirty pages.

Every internal (non-leaf) page your `findLeafPage()` implementation visits should be fetched with `READ_ONLY` permission, except the returned leaf page, which should be fetched with the permission provided as an argument to the function. These permission levels will not matter for this lab, but they will be important for the code to function correctly in future labs.

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### Exercise 1: `BTreeFile.findLeafPage()`

Implement `BTreeFile.findLeafPage()`.

After completing this exercise, you should be able to pass all the unit tests in `BTreeFileReadTest.java` and the system tests in `BTreeScanTest.java`.

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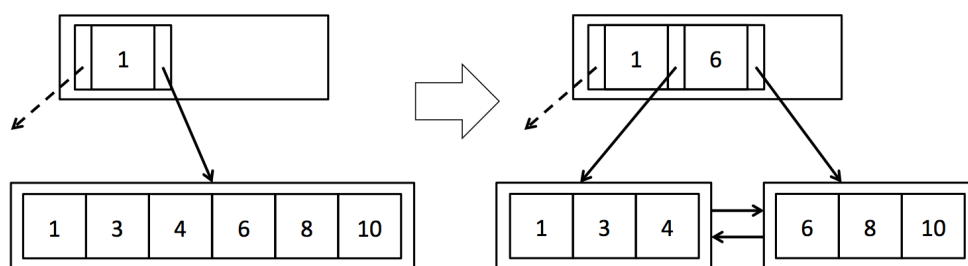
## 3. Insert

In order to keep the tuples of the B+Tree in sorted order and maintain the integrity of the tree, we must insert tuples into the leaf page with the enclosing key range. As was mentioned above, `findLeafPage()` can be used to find the correct leaf page into which we should insert the tuple. However, each page has a limited number of slots and we need to be able to insert tuples even if the corresponding leaf page is full.

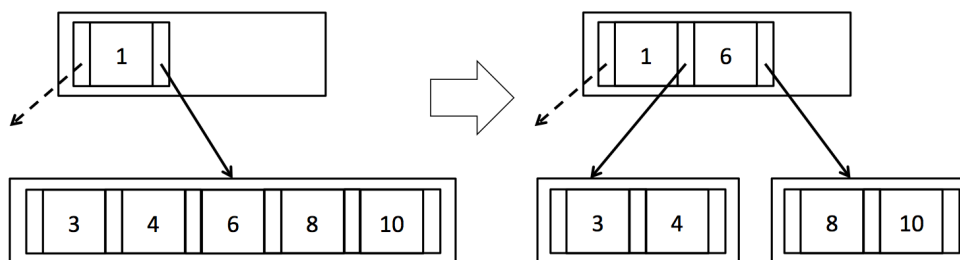
As described in the textbook, attempting to insert a tuple into a full leaf page should cause that page to split so that the tuples are evenly distributed between the two new pages. Each time a leaf page splits, a new entry corresponding to the first tuple in the second page will need to be added to the parent node. Occasionally, the internal node may also be full and unable to accept new entries. In that case, the parent should split and add a new entry to its parent. This may cause recursive splits and ultimately the creation of a new root node.

In this exercise you will implement `splitLeafPage()` and `splitInternalPage()` in `BTreeFile.java`. If the page being split is the root page, you will need to create a new internal node to become the new root page, and update the `BTreeRootPtrPage`. Otherwise, you will need to fetch the parent page with `READ_WRITE` permissions, recursively split it if necessary, and add a new entry.

You will find the function `getParentWithEmptySlots()` extremely useful for handling these different cases. In `splitLeafPage()` you should "copy" the key up to the parent page, while in `splitInternalPage()` you should "push" the key up to the parent page. See Figure 2 and review section 10.5 in the text book if this is confusing. Remember to update the parent pointers of the new pages as needed (for simplicity, we do not show parent pointers in the figures). When an internal node is split, you will need to update the parent pointers of all the children that were moved. You may find the function `updateParentPointers()` useful for this task. Additionally, remember to update the sibling pointers of any leaf pages that were split. Finally, return the page into which the new tuple or entry should be inserted, as indicated by the provided key field. (Hint: You do not need to worry about the fact that the provided key may actually fall in the exact center of the tuples/entries to be split. You should ignore the key during the split, and only use it to determine which of the two pages to return.)



Splitting a Leaf Page



Splitting an Internal Page

Figure 2: Splitting pages

Whenever you create a new page, either because of splitting a page or creating a new root page, call `getEmptyPage()` to get the new page. This function is an abstraction which will allow us to reuse pages that have been deleted due to merging (covered in the next section).

We expect that you will interact with leaf and internal pages using `BTreeLeafPage.iterator()` and `BTreeInternalPage.iterator()` to iterate through the tuples/entries in each page. For convenience, we have also provided reverse iterators for both types of pages: `BTreeLeafPage.reverseIterator()` and `BTreeInternalPage.reverseIterator()`. These reverse iterators will be especially useful for moving a subset of tuples/entries from a page to its right sibling.

As mentioned above, the internal page iterators use the interface defined in `BTreeEntry.java`, which has one key and two child pointers. It also has a `recordId`, which identifies the location of the key and child pointers on the underlying page. We think working with one entry at a time is a natural way to interact with internal pages, but it is important to keep in mind that the underlying page does not actually store a list of entries, but stores ordered lists of  $m$  keys and  $m+1$  child pointers. Since the `BTreeEntry` is just an interface and not an object actually stored on the page, updating the fields of `BTreeEntry` will not modify the underlying page. In order to change the data on the page, you need to call `BTreeInternalPage.updateEntry()`. Furthermore, deleting an entry actually deletes only a key and a single child pointer, so we provide the functions `BTreeInternalPage.deleteKeyAndLeftChild()` and `BTreeInternalPage.deleteKeyAndRightChild()` to make this explicit. The entry's `recordId` is used to find the key and child pointer to be deleted. Inserting an entry also only inserts a key and single child pointer (unless it's the first entry), so `BTreeInternalPage.insertEntry()` checks that one of the child pointers in the provided entry overlaps an existing child pointer on the page, and that inserting the entry at that location will keep the keys in sorted order.

In both `splitLeafPage()` and `splitInternalPage()`, you will need to update the set of `dirtyPages` with any newly created pages as well as any pages modified due to new pointers or new data. This is where `BTreeFile.getPage()` will come in handy. Each time you fetch a page, `BTreeFile.getPage()` will check to see if the page is already stored in the local cache (`dirtyPages`), and if it can't find the requested page there, it fetches it from the buffer pool. `BTreeFile.getPage()` also adds pages to the `dirtyPages` cache if they are fetched with read-write permission, since presumably they will soon be dirtied. One advantage of this approach is that it prevents loss of updates if the same pages are accessed multiple times during a single tuple insertion or deletion.

Note that in a major departure from `HeapFile.insertTuple()`, `BTreeFile.insertTuple()` could return a large set of dirty pages, especially if any internal pages are split. As you may remember from previous labs, the set of dirty pages is returned to prevent the buffer pool from evicting dirty pages before they have been flushed.

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**Warning:** as the B+Tree is a complex data structure, it is helpful to understand the properties necessary of every legal B+Tree before modifying it. Here is an informal list:

1. If a parent node points to a child node, the child nodes must point back to those same parents.
2. If a leaf node points to a right sibling, then the right sibling points back to that leaf node as a left sibling.
3. The first and last leaves must point to null left and right siblings respectively.
4. Record Id's must match the page they are actually in.
5. A `key` in a node with non-leaf children must be larger than any key in the left child, and smaller than any key in the right child.
6. A `key` in a node with leaf children must be larger or equal than any key in the left child, and smaller or equal than any key in the right child.
7. A node has either all non-leaf children, or all leaf children.

8. A non-root node cannot be less than half full.

We have implemented a mechanized check for all these properties in the file `BTreeChecker.java`. This method is also used to test your B+Tree implementation in the `systemtest/BTreeFileDeleteTest.java`. Feel free to add calls to this function to help debug your implementation, like we did in `BTreeFileDeleteTest.java`.

**N.B.**

1. The checker method should always pass after initialization of the tree and before starting and after completing a full call to key insertion or deletion, but not necessarily within internal methods.
2. A tree may be well formed (and therefore pass `checkRep()`) but still incorrect. For example, the empty tree will always pass `checkRep()`, but may not always be correct (if you just inserted a tuple, the tree should not be empty). \*\*\*

## Exercise 2: Splitting Pages

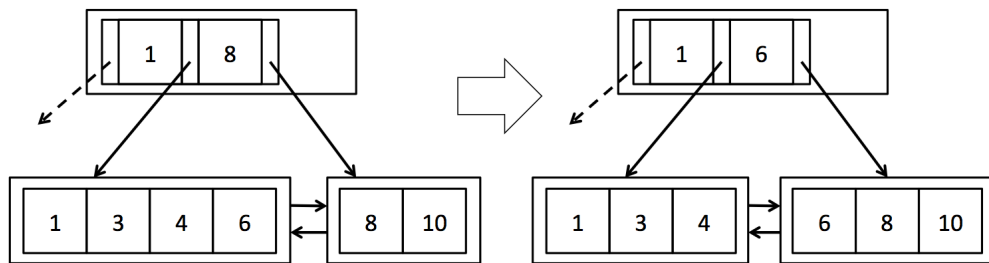
Implement `BTreeFile.splitLeafPage()` and `BTreeFile.splitInternalPage()`.

After completing this exercise, you should be able to pass the unit tests in `BTreeFileInsertTest.java`. You should also be able to pass the system tests in `systemtest/BTreeFileInsertTest.java`. Some of the system test cases may take a few seconds to complete. These files will test that your code inserts tuples and splits pages correctly, and also handles duplicate tuples.

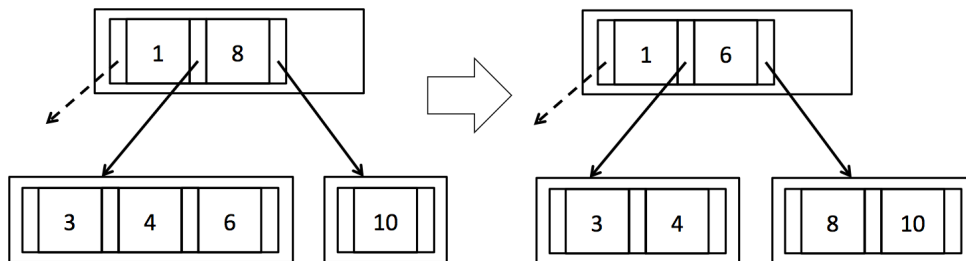
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## 4. Delete

In order to keep the tree balanced and not waste unnecessary space, deletions in a B+Tree may cause pages to redistribute tuples (Figure 3) or, eventually, to merge (see Figure 4). You may find it useful to review section 10.6 in the textbook.

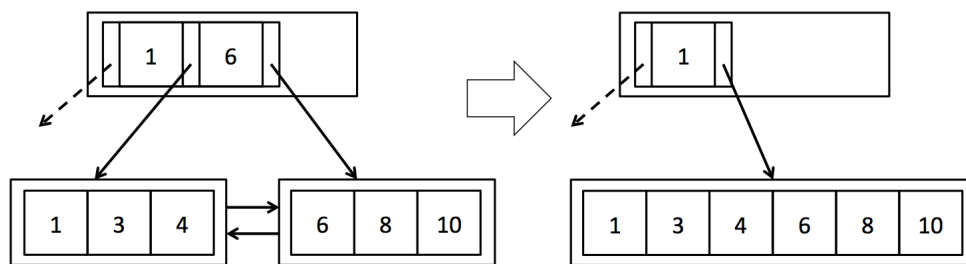


Redistributing Leaf Pages

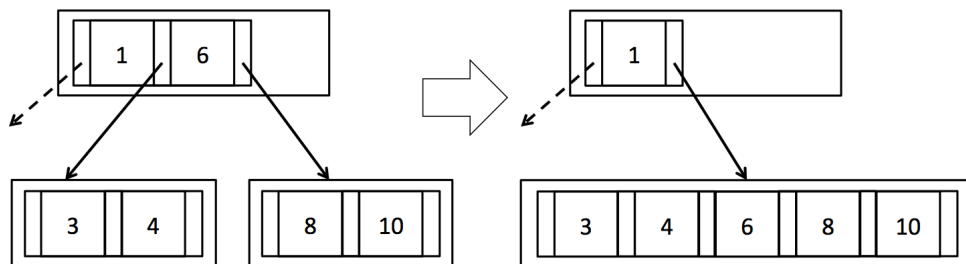


Redistributing Internal Pages

Figure 3: Redistributing pages



Merging Leaf Pages



Merging Internal Pages

Figure 4: Merging pages

As described in the textbook, attempting to delete a tuple from a leaf page that is less than half full should cause that page to either steal tuples from one of its siblings or merge with one of its siblings. If one of the page's siblings has tuples to spare, the tuples should be evenly distributed between the two pages, and the parent's entry should be updated accordingly (see Figure 3). However, if the sibling is also at minimum occupancy, then the two pages should merge and the entry deleted from the parent (Figure 4). In turn, deleting an entry from the parent may cause the parent to become less than half full. In this case, the parent should steal entries from its siblings or merge with a sibling. This may cause recursive merges or even deletion of the root node if the last entry is deleted from the root node.

In this exercise you will implement `stealFromLeafPage()`, `stealFromLeftInternalPage()`, `stealFromRightInternalPage()`, `mergeLeafPages()` and `mergeInternalPages()` in `BTreeFile.java`. In the first three functions you will implement code to evenly redistribute tuples/entries if the siblings have tuples/entries to spare. Remember to update the corresponding key field in the parent (look carefully at how this is done in Figure 3 - keys are effectively "rotated" through the parent). In `stealFromLeftInternalPage()/stealFromRightInternalPage()`, you will also need to update the parent pointers of the children that were moved. You should be able to reuse the function `updateParentPointers()` for this purpose.

In `mergeLeafPages()` and `mergeInternalPages()` you will implement code to merge pages, effectively performing the inverse of `splitLeafPage()` and `splitInternalPage()`. You will find the function `deleteParentEntry()` extremely useful for handling all the different recursive cases. Be sure to call `setEmptyPage()` on deleted pages to make them available for reuse. As with the previous exercises, we recommend using `BTreeFile.getPage()` to encapsulate the process of fetching pages and keeping the list of dirty pages up to date.

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### Exercise 3: Redistributing pages

Implement `BTreeFile.stealFromLeafPage()`, `BTreeFile.stealFromLeftInternalPage()`, `BTreeFile.stealFromRightInternalPage()`.

After completing this exercise, you should be able to pass some of the unit tests in `BTreeFileDeleteTest.java` (such as `testStealFromLeftLeafPage` and `testStealFromRightLeafPage`). The system tests may take several seconds to complete since they create a large B+ tree in order to fully test the system.

### Exercise 4: Merging pages

Implement `BTreeFile.mergeLeafPages()` and `BTreeFile.mergeInternalPages()`.

Now you should be able to pass all unit tests in `BTreeFileDeleteTest.java` and the system tests in `systemtest/BTreeFileDeleteTest.java`.

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## 5. Transactions

You may remember that B+ trees can prevent phantom tuples from showing up between two consecutive range scans by using next-key locking. Since SimpleDB uses page-level, strict two-phase locking, protection against phantoms effectively comes for free if the B+ tree is implemented correctly. Thus, at this point you should also be able to pass `BTreeNextKeyLockingTest`.

Additionally, you should be able to pass the tests in `test/simpledb/BTreeDeadlockTest.java` if you have implemented locking correctly inside of your B+ tree code.



If everything is implemented correctly, you should also be able to pass the BTreeTest system test. We expect many people to find BTreeTest difficult, so it's not required, but we'll give extra credit to anyone who can run it successfully. Please note that this test may take up to a minute to complete.

## 6. Extra Credit

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### Bonus Exercise 5: (10% extra credit)

Create and implement a class called BTreeReverseScan which scans the BTreeFile in reverse, given an optional IndexPredicate.

You can use BTreeScan as a starting point, but you will probably need to implement a reverse iterator in BTreeFile. You will also likely need to implement a separate version of BTreeFile.findLeafPage(). We have provided reverse iterators on BTreeLeafPage and BTreeInternalPage which you may find useful. You should also write code to test that your implementation works correctly. BTreeScanTest.java is a good place to look for ideas.

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## 7. Logistics

You must submit your code (see below) as well as a short (1 page, maximum) writeup describing your approach. This writeup should:

- Describe any design decisions you made, including anything that was difficult or unexpected.
- Discuss and justify any changes you made outside of BTreeFile.java.
- How long did this lab take you? Do you have any suggestions for ways to improve it?
- Optional: If you did the extra credit exercise, explain your implementation and show us that you thoroughly tested it.

### 7.1. Collaboration

This lab should be manageable for a single person, but if you prefer to work with a partner, this is also OK. Larger groups are not allowed. Please indicate clearly who you worked with, if anyone, on your writeup.

### 7.2. Submitting your assignment

We will be using gradescope to autograde all programming assignments. You should have all been invited to the class instance; if not, please let us know and we can help you set up. You may submit your code multiple times before the deadline; we will use the latest version as determined by gradescope. Place the write-up in a file called lab3-writeup.txt with your submission. You also need to explicitly add any other files you create, such as new \*.java files.

The easiest way to submit to gradescope is with `.zip` files containing your code. On Linux/MacOS, you can do so by running the following command:

```
$ zip -r submission.zip src/ lab5-writeup.txt
```

### 7.3. Submitting a bug

SimpleDB is a relatively complex piece of code. It is very possible you are going to find bugs, inconsistencies, and bad, outdated, or incorrect documentation, etc.

We ask you, therefore, to do this lab with an adventurous mindset. Don't get mad if something is not clear, or even wrong; rather, try to figure it out yourself or send us a friendly email.

Please submit (friendly!) bug reports to [6.830-staff@mit.edu](mailto:6.830-staff@mit.edu). When you do, please try to include:

- A description of the bug.
- A `.java` file we can drop in the `test/simpledb` directory, compile, and run.
- A `.txt` file with the data that reproduces the bug. We should be able to convert it to a `.dat` file using `HeapFileEncoder`.

You can also post on the class page on Piazza if you feel you have run into a bug.

### 7.4 Grading

75% of your grade will be based on whether or not your code passes the system test suite we will run over it. These tests will be a superset of the tests we have provided. Before handing in your code, you should make sure it produces no errors (passes all of the tests) from both `ant test` and `ant systemtest`.

**Important:** before testing, gradescope will replace your `build.xml`, `HeapFileEncoder.java` and the entire contents of the `test` directory with our version of these files. This means you cannot change the format of `.dat` files! You should also be careful changing our APIs. You should test that your code compiles the unmodified tests.

You should get immediate feedback and error outputs for failed tests (if any) from gradescope after submission. The score given will be your grade for the autograded portion of the assignment. An additional 25% of your grade will be based on the quality of your writeup and our subjective evaluation of your code. This part will also be published on gradescope after we finish grading your assignment.

We had a lot of fun designing this assignment, and we hope you enjoy hacking on it!