4.1 Równanie transportu ciepła

$$-k(x)\frac{d^2u(x)}{dx^2} = 0$$
$$u(2) = 3$$
$$\frac{du(0)}{dx} + u(0) = 20$$
$$k(x) = \begin{cases} 1 & \text{dla } x \in [0,1] \\ 2 & \text{dla } x \in (1,2] \end{cases}$$

Gdzie u to poszukiwana funkcja

$$[0,2] \ni x \to u(x) \in \mathbb{R}$$

$$-k(x)u''(x) = 0$$

$$-\int_0^2 k(x)u''(x)v(x)dx = 0$$

$$\int_0^2 k(x)u'(x)v'(x)dx - 2u'(2)v(2) + u'(0)v(0) = 0$$

$u = w + \tilde{u}$ $\quad w(2) = 0$ $\quad \tilde{u}(2) = 3$

$\tilde{u}(x) = 3$

$$u(x) = w(x) + 3$$
$$u'(x) = w'(x)$$
$$u''(x) = w''(x)$$

$$\int_0^2 k(x)w'(x)v'(x)dx \underbrace{- 2w'(2)v(2)}_{=0} + \left[20 - (w(0)+3)\right]v(0) = 0$$

$$\underbrace{\int_0^2 k(x)w'(x)v'(x)dx - w(0)v(0)}_{B(w,v)} = \underbrace{-17v(0)}_{L(v)}$$