V.tex: The Verification Tensor

A Jacobian-Free Newton–Krylov (JFNK) Specification for Manuscript Verification

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Abstract

This document formalizes a verifier $\mathcal{V}$ for a research manuscript produced by $\mathcal{P}$ as a nonlinear system $F(x) = \mathbf{0}$. We decompose F into orthogonal residual channels (logic, structure, notation, citation), minimize a weighted norm, and solve the induced Newton step with a Jacobian-Free Newton–Krylov (JFNK) method using finite-difference Jacobian actions, block preconditioning, eigenmode deflation, and a trust-region line search. Gold checks (unit tests) on $\Delta S \geq 0$, $U^{(4)} = 0$, the spline$\Rightarrow$Euler–Lagrange chain, reciprocity, and MA$\leftrightarrow$continuity provide hard constraints for acceptance.

1 State, Residual, and Objective

Let the manuscript state be $x \in \mathbb{R}^n$, encoding a finite set of units (claims, lemmas, definitions, equations, figures, citations) plus a sparse dependency graph. The verifier maps x to residuals

$$F(x) \;=\; \mathcal{F}(x) \;=\; \begin{bmatrix} r_{\text{logic}}(x) \\ r_{\text{structure}}(x) \\ r_{\text{notation}}(x) \\ r_{\text{citation}}(x) \end{bmatrix} \;\in\; \mathbb{R}^{m_\ell + m_s + m_n + m_c}. \tag{1}$$

We minimize a composite norm

$$\|F(x)\| \;=\; 10\,\|r_{\text{logic}}(x)\|_2 + 4\,\|r_{\text{structure}}(x)\|_2 + 3\,\|r_{\text{notation}}(x)\|_2 + 2\,\|r_{\text{citation}}(x)\|_2, \tag{2}$$

with $10 \gg 4 \geq 3 \geq 2$ so logical soundness dominates.

1.1 Channel Models (engineering forms)

Logic channel. Let $B \in \mathbb{R}^{E \times V}$ be the incidence matrix of the premise$\to$claim DAG and $s_i \in [0,1]$ a satisfaction score for premise i (1 = stated & proved, 0 = missing). For each edge $(i \to j)$ with threshold $\tau \in (0,1]$, define

$$r_{\text{logic}}^{(i\to j)}(x) = [\tau - s_i]_+ \,. \tag{3}$$

Stack all edges to obtain r_{logic}.

Structure channel. Let $A_{\text{struct}}x = b$ encode linear placement/templating constraints (e.g. "each theorem has exactly one statement and one proof block"). Then

$$r_{\text{structure}}(x) = A_{\text{struct}}x - b. \tag{4}$$

Notation channel. Let T be the symbol table; for symbol σ, P_σ projects x onto the components using σ and t_σ is the canonical anchor (definition+scope). With positive-definite weights W_σ,

$$r_{\text{notation}}(x) \; = \; \sum_{\sigma \in T} W_\sigma \big(P_\sigma x - t_\sigma\big). \tag{5}$$

Citation channel. For obligations $Cx + \xi \geq d$ with slacks $\xi \geq 0$ (one inequality per claim that requires a cite), define

$$r_{\text{citation}}(x) = \xi, \qquad \xi = [d - Cx]_+ \text{ (elementwise).} \tag{6}$$

2 Jacobian-Free Newton–Krylov

We seek a Newton step δx solving $J(x)\delta x = -F(x)$. We do not form J explicitly; we approximate its action via directional differencing.

2.1 Finite-difference Jacobian actions

For any direction v,

$$J(x)v \; \approx \; \frac{F\big(x + \varepsilon v\big) - F(x)}{\varepsilon}, \qquad \varepsilon \; = \; \varepsilon_0 \, \frac{1 + \|x\|_2}{1 + \|v\|_2}, \tag{7}$$

with $\varepsilon_0 \in [10^{-6}, 10^{-4}]$ (text-space is noisy; start at 10^{-4}).

2.2 Krylov subspace and GMRES

Given $r_0 = F(x)$, build

$$\mathcal{K}_m(J, r_0) = \operatorname{span}\{r_0, Jr_0, J^2 r_0, \ldots, J^{m-1} r_0\} \tag{8}$$

using modified Gram–Schmidt (optionally blocked by channel). Solve the linearized system with (F)GMRES, right-preconditioned (next section).

Algorithm 1 JFNK with Trust Region and Deflation (single outer iteration)

Require: state x, tolerance τ, max Krylov dim m, restart m_r, initial step cap $\alpha_{\max}$

$r \leftarrow F(x)$; $\quad r \leftarrow \textsc{Deflate}(r)$

Solve $J(x)\delta x = -r$ with (F)GMRES using actions (**??**) and preconditioner M^{-1}

Choose $\alpha \in (0, \alpha_{\max}]$ by ratio test $\rho = \dfrac{\|r\| - \|F(x + \alpha\delta x)\|}{\text{model decrease}}$

if $\rho < \rho_{\min}$ **then**

  $\alpha \leftarrow \alpha/2$ and retry (up to a few cuts)

end if

$x^+ \leftarrow x + \alpha\,\delta x$; $\quad$ **return** x^+

3 Preconditioning and Deflation

3.1 Block preconditioner

Use a block-diagonal right preconditioner

$$M^{-1} \approx \begin{bmatrix} M_\ell^{-1} & & & \\ & M_s^{-1} & & \\ & & M_n^{-1} & \\ & & & M_c^{-1} \end{bmatrix}, \tag{9}$$

where:

- M_ℓ^{-1}: *premise fill* (auto-insert missing premises if present elsewhere), *cycle break* (promote/demote nodes to restore DAG);
- M_s^{-1}: template normalization (Theorem→Proof→Corollary), duplicate squash (minhash/cosine);
- M_n^{-1}: symbol unification (single source-of-truth macros), conflict rename;
- M_c^{-1}: obvious BibTeX fixes; otherwise mark `needs-cite`.

3.2 Eigenmode deflation

Maintain a set $\{q_i\}$ of solved residual shapes (low-eigenvalue modes). Before Krylov builds, project them out:

$$r \;\leftarrow\; r - \sum_i \frac{\langle r,\, q_i\rangle}{\langle q_i,\, q_i\rangle}\, q_i, \qquad v \;\leftarrow\; v - \sum_i \frac{\langle v,\, q_i\rangle}{\langle q_i,\, q_i\rangle}\, q_i. \tag{10}$$

4 Trust Region and Edit Budget

Accept $x^+ = x + \alpha\,\delta x$ with $\alpha \in (0, 1]$ chosen by the ratio test in Algorithm 1. Impose a per-iteration edit budget (*e.g.*, $\leq 2\%$ tokens changed; at most one theorem moved) to avoid overshoot.

5 Gold Checks (Hard Constraints)

These must pass for acceptance:

G1. Formal statement and scope of $\Delta S \geq 0$ with explicit premises (discrete causal counting) and no hidden thermodynamic assumptions.

G2. Coherence condition $U^{(4)} = 0$ appears exactly where used; $U^{(4)}$ is defined once without symbol drift.

G3. Spline $\Rightarrow$ Euler–Lagrange chain occurs once, cleanly:

$$J[y] = \int \big(\lambda \|y''(t)\|^2 + \phi(y(t))\big)\, dt \;\Rightarrow\; \frac{d}{dt}\big(\partial_{y'}\mathcal{L}\big) - \partial_y \mathcal{L} = 0.$$

G4. Reciprocity functional is stated with correct variational form and boundary terms; no sign inconsistencies.

G5. MA $\leftrightarrow$ continuity boundary is clearly delimited; set-theoretic heuristics are annotated as meta-assumptions, not proof steps.

6 Stopping Criteria and Acceptance

Stop when all hold:

(i) $\|F(x)\| \leq \tau$ for (**??**);

(ii) All **G1**–**G5** pass;

(iii) No high-severity logic items remain (logic channel empty);

(iv) Last accepted step has $\alpha = 1$ or Krylov stagnates with negligible residual.

7 Audit Trail (Deterministic Handoff to $\mathcal{P}$)

Each iteration emits:

- channel-wise residual summary and norms;
- ranked findings with suggested local edits (preconditioner actions flagged);
- updated deflation set $\{q_i\}$;
- accepted step size α and observed decrease.

Remark 1 (Minimal integration steps). *To wire this into your loop: (1) build the four residual channels from the current manuscript graph and symbol/citation tables; (2) expose $F(\cdot)$ and the action oracle $v \mapsto J(x)v$ via (**??**); (3) enable the preconditioner blocks as cheap, local transforms; (4) enforce the edit budget and gold checks at accept time.*