V.tex — Minimal Verification Contract

Keeps P_clean correct without redefining it (V2025-10-13-min)

V (compatibility layer)

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Purpose

This minimal layer:

- defines shared macros only if missing,
- pins two semantics used by `P_clean`: entropy monotonicity and biharmonic closure,
- and warns if required structural anchors are missing.

1 G1: Monotone Entropy (Contract)

Definition 1 (Observable Classes)**.** *For a causal structure $\mathcal{C}$ and observable π, let $\Omega(\mathcal{C})$ be the induced equivalence classes. Define entropy $\mathrm{S}[\pi](\mathcal{C}) = \ln|\Omega(\mathcal{C})|$ (units absorbed).*

Definition 2 (Compatible Refinement)**.** *A refinement $\phi : \mathcal{C} \to \mathcal{C}'$ is compatible if it is order-embedding and non-merging at the level of classes.*

Proposition 1 (Injective Class Map). *Compatibility induces* $\iota : \Omega(\mathcal{C}) \to \Omega(\mathcal{C}')$, $\iota([x]) = [\phi(x)]$, *which is injective.*

Theorem 1 (Second Law (Minimal Form)). *For any compatible refinement,* $\Delta\mathrm{S} = \mathrm{S}[\pi'](\mathcal{C}') - \mathrm{S}[\pi](\mathcal{C}) \geq 0$.

2 G2: Kinematic Closure (Contract)

Definition 3 (Action and Reciprocity). *For* $\mathbf{U} \in \mathcal{H}$ *and* $\mathbf{V} \in \mathcal{V}$,

$$\mathcal{A}[\mathbf{U}] = \tfrac{1}{2}\int (\mathrm{D}^2\mathbf{U})^2\, dx, \qquad \mathsf{B}(\mathbf{U},\mathbf{V}) = \int (\mathrm{D}^2\mathbf{U})(\mathrm{D}^2\mathbf{V})\, dx.$$

Stationarity of $\mathcal{A}$ *gives the weak form* $\mathsf{B}(\mathbf{U},\mathbf{V}) = 0$ *for all* $\mathbf{V} \in \mathcal{V}$.

Theorem 2 (Biharmonic Strong Form). *Under appropriate BCs, stationarity implies the strong form* $\mathrm{D}^4\mathbf{U} = 0$.

Spline Consistency (notational hook). If $\mathbf{U}_h$ is a discrete solution and S_h a reconstruction, statements of the form $\|\mathsf{S}_h\mathbf{U}_h - \mathbf{U}^\star\| \to 0$ as $h \to 0$ are admissible and well-typed here.