V: Reader's Guide and Contract Layer

(thermodynamic $\Delta \mathrm{S} \geq 0$, kinematic $\mathbf{U}^{(4)} = 0$, reciprocity $\mathsf{B}(\mathbf{U}, \varphi) = 0 \; \forall \varphi \in \mathcal{V}$)

Verification Tensor (V)

October 13, 2025

Abstract

This note declares the shared symbols and contracts (G1–G4), adds an index-free Noether bridge $\mathsf{N}[L, \mathbf{U}; \Xi]$ linking stationarity to conserved currents, and introduces Volume 4 primitives for invariants and statistics. All handles are title-safe (`\ensuremath`), so they can be used directly in text, headings, and captions.

Contracts (G1–G4)

- **G1 thermodynamic closure:** $\Delta \mathrm{S} \geq 0$.
- **G2 kinematic closure (strong):** $\mathbf{U}^{(4)} = 0$.
- **G2 weak/reciprocity:** $\mathsf{B}(\mathbf{U}, \varphi) = 0 \; \forall \varphi \in \mathcal{V}$; summarized as Discrete–continuum reciprocity via B.
- **G4 statistical invariants:**
  - Data processing / coarse-graining: D_{KL} monotone under admissible coarsegraining.
  - Score unbiasedness at truth: $\mathbb{E}[\mathcal{S}] = 0$.

Shared symbols

$\mathbf{U}$, $\mathbf{V}$, $\varphi \in \mathcal{V}$, ∇, $\mathcal{C}$, $\mathsf{B}(\cdot, \cdot)$, S, $\mathbf{U}^{(4)}$, Ξ, $\mathsf{N}[L, \mathbf{U}; \Xi]$, $\mathbf{J}$, $\mathbf{T}$, $\nabla \mathbf{J}$, $\mathbb{E}$, $\mathrm{Var}[\cdot]$, $\hat{\cdot}$, $\mathcal{S}$, $\mathcal{I}$, $\mathrm{D}_{\mathrm{KL}}(P \,\|\, Q)$, $\mathsf{Inv}[\cdot]$, R.

Noether bridge (index-free API)

We introduce a lightweight primitive:

$$\mathsf{N}[L, \mathbf{U}; \Xi],$$

mapping a local density L, field $\mathbf{U}$, and infinitesimal generator Ξ to an abstract current $\mathbf{J}$ (and, for translations, a stress–energy $\mathbf{T}$), with conservation

$$\nabla \cdot \mathbf{J} = 0.$$

The closure $\mathbf{U}^{(4)} = 0$ supplies the stationarity hypothesis required to invoke this construction.

Volume 4: invariants and statistical closure

Admissible coarse-grainings (consistent with $\mathcal{C}$) cannot increase distinguishability:

$$\mathrm{D}_{\mathrm{KL}}(P \,\|\, Q) \text{ does not increase under admissible maps.}$$

At the true parameter, the score has zero mean:

$$\mathbb{E}[\mathcal{S}] = 0.$$

Estimation accuracy is bounded via $\mathcal{I}$, while symmetry induces index-free invariants $\mathsf{Inv}[\Xi]$ consistent with $\nabla \cdot \mathbf{J} = 0$. Residual structure R is thereby interpreted as constrained noise under the established closures.

Usage notes

The statement handles already include math mode. Prefer `(\GOneStatement)` not `\(\GOneStatement\)`. Avoid `\left...\right` unless you include both sides.