Module 2: Kinematic Closure, Weak Reciprocity, and Newton Linearization

Validating G2: The Minimal Curvature Condition
$\mathbf{U}^{(4)} = 0\mathrm{U}^{(4)} = 0$

G2S — Module Contract Fulfillment (V.tex Compliant)

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1 Input Node: Reciprocity and Variational Setup

The unique analytic closure for the admissible history $\mathbf{U}$ minimizes the **Informational Curvature Action** $\mathcal{A}[\mathbf{U}]$:

$$\mathcal{A}[\mathbf{U}] = \frac{1}{2}\int_{\Omega} (\mathbf{U}'')^2 \, dx.$$

2 Theorem: Kinematic Closure and Reciprocity

Theorem 1 (Strong Kinematic Closure ($\mathbf{U^{(4)} = 0}$))**.** *The unique minimal-curvature solution* $\mathbf{U}^\star$ *is the stationary point of* $\mathcal{A}[\mathbf{U}]$*, which yields the Euler-Lagrange strong form* $\mathbf{U}^{(4)} = 0$*.*

G2 Proof Obligation Fulfillment. The weak form uses the bilinear reciprocity pairing $\mathsf{B}(\mathbf{U}, \varphi) = \int_\Omega (\mathbf{U}'')(\varphi'')\, dx$.

1. **Weak Form (Reciprocity):** Stationarity $\delta\mathcal{A} = 0$ requires the condition:

$$\text{Find } \mathbf{U} \in \ \text{ such that } \mathsf{B}(\mathbf{U}, \varphi) = 0 \ \forall \varphi \in \mathcal{V}.$$

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2. **Strong Form:** Double integration by parts on the weak form yields the strong closure: $\mathbf{U}^{(4)} = 0$.

□

3 G2: Newton Linearization and Noether Bridge

Residual and Newton step (index-free). The analytic closure $\mathbf{U}^{(4)} = 0$ is a non-linear (in coordinates/metric) residual function $\mathsf{R}(\mathbf{U})$. The iteration is driven by the Fréchet Jacobian $\mathsf{J}(\mathbf{U})$:

$$\mathsf{R}(\mathbf{U}) := \mathbf{U}^{(4)}, \qquad \mathsf{J}(\mathbf{U}) := \frac{\partial \mathsf{R}}{\partial \mathbf{U}}.$$

The Newton update $\delta\mathbf{U}$ for an iterate $\mathbf{U}^{(k)}$ satisfies:

$$\mathsf{J}(\mathbf{U}^{(k)})\, \delta\mathbf{U} \ = \ -\, \mathsf{R}(\mathbf{U}^{(k)}).$$

This defines the computational implementation (the Newton rail) driven directly by the analytic closure $\mathbf{U}^{(4)} = 0$.

Noether bridge (API-level, index-free). The stationary action, $\delta\mathcal{A} = 0$, supplies the core hypothesis required to invoke Noether's theorem. The abstract conservation current $\mathbf{J}$ (and stress-energy $\mathbf{T}$) is derived from a local

Lagrangian density L and translational symmetry Ξ:

$$\mathsf{N}[L, \mathbf{U}; \Xi] \quad \Longrightarrow \quad \nabla\cdot\mathbf{J} = 0.$$

The kinematic closure $\mathbf{U}^{(4)} = 0$ is therefore the necessary structural input for the derivation of all subsequent conservation laws.