# Axioms of Measurement Sufficient for $\Delta S \geq 0$

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*To Peano, who taught us how to count.*

*To Boltzmann, who first counted what could be distinguished.*

*To Planck, who taught us that the count is finite.*

*To Cantor, who showed us how to count the infinite.*

*To Kolmogorov, who showed us that information must be counted to be measured.*

*To William of Ockham, who insisted that we only count what is necessary.*

"Je les possède, parce que jamais personne avant moi n'a songé à les posséder."
"Moi, je suis un homme sérieux. Je suis exact. J'aime que l'on soit exact."

—Antoine de Saint–Exupéry, *Le Petit Prince* (1943)

# Abstract

We present a set of axioms describing how measurements must correspond in order for a universe to draw coherent conclusions about itself. Each act of measurement creates a finite, distinguishable event, and every admissible extension of that record must preserve global consistency. This requirement is formalized by a non-standard application of Martin's Condition, which restricts how observational data may refine without introducing contradictions or unrecorded structure.

Under these axioms, Martin's Condition admits a unique smooth closure in which discrete refinements converge to continuous, spline-level dynamics. Classical differential equations, Hilbert structures, gauge symmetries, curvature, transport laws, and geometric tensors are admitted only when the measurement record allows a smooth completion consistent with these constraints. None of these structures are postulated; they appear only when the data permit them.

The resulting framework reproduces the familiar behaviors of physics: waves, matter flow, diffusion, advection, geodesic motion, quantization, and curvature, without assuming fields, geometry, or dynamical laws in advance. Most importantly, the axioms imply that entropy can never decrease. The inequality

$$\Delta S \geq 0$$

is proved as a theorem of causal measurement: any universe consistent with its own record of distinguishable events must increase the number of ad-

missible configurations. No thermodynamic assumptions or mechanisms are required.

Thus, the familiar laws of physics appear wherever the record of measurement admits a smooth and globally consistent refinement.

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# Roadmap

This work begins from the simplest possible assumption: every observation creates information. When something becomes distinguishable, it has been measured. Each measurement leaves a record, and the universe grows by accumulating these records one event at a time.

Nothing continuous is assumed. No geometry, no fields, no action principle, and no differential equations. The goal is to show that the familiar structures of physics arise automatically when we ask for one thing: a universe whose measurements never contradict each other.

## From Events to Smooth Motion

Imagine that only a finite number of measurements are made along a particle's path. Between those points, many curves are mathematically possible, but most of them would imply extra structure: hidden bumps, oscillations, or accelerations that would have produced additional measurements. Since no such measurements exist, those curves must be rejected.

The only admissible path is the one with no unobserved structure. In practice this means the path of least bending: the same rule that defines cubic splines. As measurements become more precise, the spline becomes smooth, and its smooth limit satisfies a fourth–order equation

$$U^{(4)} = 0.$$

This is the Euler–Lagrange equation of a free beam. Here it appears not because an action was postulated, but because any other path would predict unrecorded motion. Smooth calculus is the shadow of finite measurements.

## Matter as Information Flow

If two observers measure the same system and later compare data, their descriptions must agree in the overlapping region. This is only possible if information flows continuously from one measurement to the next. In practice, the density of recorded information obeys a continuity law

$$\frac{\partial}{\partial t}\rho + \nabla \cdot j = 0.$$

This is the kinematics of matter: information flows in a way that does not lose or invent measurements.

## Light as the Fastest Information

There is a maximum rate at which new information can appear. In relativity this is the speed of light. Here it is simply the rule that no observer can receive new data from outside a causal boundary. When this idea is carried into the smooth limit, the result is the wave equation

$$\frac{\partial^2 U}{\partial t^2} = c^2 \frac{\partial^2 U}{\partial x^2}.$$

Light is what information looks like when it travels at the fastest possible speed. Waves are the way measurements spread when there is no hidden structure in between.

## Quantum Information

When two alternatives cannot be distinguished by any measurement, they cannot be assigned separate probabilities. Instead, they combine as amplitudes. Only when a new measurement is made do they separate into exclusive outcomes. This is the reason amplitudes add and probabilities come from their square.

In this framework, quantum mechanics is the bookkeeping of situations where multiple explanations remain consistent with all available data. Interference is the comparison of indistinguishable histories.

## Gauge Theory

Different observers may use different conventions to describe the same measurements. If their predictions always agree, then their descriptions must be related by a transformation that leaves the measurable results unchanged. These transformations form a gauge symmetry, and the rules for comparing neighboring descriptions produce a connection. Its curvature is what physicists call a field.

Thus gauge theory is not an extra piece of physics. It is the condition that descriptions remain compatible when information is transported from place to place.

## Why Entropy Increases

Every measurement increases the number of possible histories that remain consistent with the data. None of those histories can be removed, because that would erase information already recorded. As a result, the number of distinguishable configurations never goes down. Define entropy as

$$S = \log(\text{number of consistent configurations}).$$

Then $S$ can only increase. This is the Second Law of Causal Order: information grows because measurements cannot be undone.

## Summary

| Structure | Why it appears | Result |
|---|---|---|
| Smooth paths | No hidden motion | $U^{(4)} = 0$ |
| Matter flow | Shared measurements | Continuity law |
| Light | Fastest information | Wave equation |
| Quantum | Indistinguishable causes | Amplitudes, interference |
| Gauge fields | Consistent comparison | Connections, curvature |
| Entropy | Measurements are permanent | $\Delta S \geq 0$ |

In this view, physics is not assumed. It is what information must look like when measurements never contradict each other.

# *Nota Bene*

The argument is constructive rather than interpretive. Each part extends the previous one by a single act of closure that preserves causal consistency:

$$\text{Measurement} \Rightarrow \text{Calculus} \Rightarrow \text{Wave} \Rightarrow \text{Geometry} \Rightarrow \text{Field.}$$

At every stage, a new invariant appears whenever distinction is preserved under refinement. The sequence therefore builds the minimal structure required for a universe that records its own evolution without contradiction.

Thus, the proof is read not as a series of analogies but as a chain of logical consequences. Starting from finiteness, order, and choice, one obtains measurement, variation, and their reciprocity; from reciprocity, one obtains calculus; from calculus, the smooth invariants of physics; and from their global consistency, the Second Law of Causal Order. In this sense, $\Delta S \geq 0$ is the unique fixed point of mathematics and physics—the inequality that any self-consistent universe must obey.

**N.B.**—The proof contains multiple conceptual examples to help explain the mathematical machinery. These *thought experiments* are not empirical illustrations but formal constructions intended to clarify the logical structure of the axioms. They are finite conceptual models that demonstrate how the mathematical relations behave under specific constraints of order and measurement. No claim is made regarding physical observation; each serves only to illuminate the internal mechanics of the theory. □

**N.B.**—Throughout what follows, it is essential to distinguish the logical structure of measurement from any claim about physical phenomena. The arguments presented here concern the internal consistency of *records of distinction*—that is, the admissible transformations among measurable events—rather than the evolution of material systems themselves. Every symbol, tensor, and variation in the proof refers to relations between observations, not to unobserved substances or causes. The framework thus formalizes the mathematics of *measurement*: how distinctions can be made, counted, and related without contradiction. No ontological or dynamical claims are implied; the results hold regardless of what, if anything, the symbols may represent physically. □

**N.B.**—This is a paper about *information*, not about energy, momentum, or any other physical quantity. At no point is it suggested that such values are produced, derived, or generated by the constructions presented here. All arguments concern the logical structure of measurement and the internal coherence of distinguishability, not the dynamics of physical systems. □

**N.B.**—This is a *conditional* proof. All conclusions hold only under the stated axioms and definitions. No claim is made regarding the physical truth of those assumptions, only their internal consistency and the consequences that follow from them. □

**N.B.**—Certain sections include extrapolations that relate the formal results to observable patterns (e.g., galactic rotation, Cepheid-based distance scaling, or cosmic expansion). These are illustrations of consistency, not physical hypotheses. Their purpose is to show that familiar empirical regularities may follow naturally from the informational constraints proved herein. No specific mechanism is proposed, and no claim of empirical verification is implied. They are included solely to outline how the formal results may frame, rather than predict, observable regularities. □

**No differential equations were altered, reinterpreted, or otherwise harmed in the production of this proof.**

This work treats measurement as a discrete, logical process. Continuum formulations appear only as smooth limits of countable constructions, never as physical postulates.

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# Chapter 1

# The Mechanisms of Information

Every theory of dynamics begins with a calculus, an instrument for measuring variation. Yet a calculus alone cannot describe the universe, for measurement presupposes the existence of an ordered substrate upon which distinctions can be drawn (*e.g.* one recorded event follows another). The present work begins from this observation and constructs, alongside the familiar differential calculus, its algebraic dual: a logic of finite relations that determines how measurements themselves come to exist. Where calculus quantifies change, the dual quantifies order[1]. Each derivative has its adjoint in the discrete act of selection, and each integral its counterpart in the accumulation of distinguishable events. Taken together, these two systems—the continuous and its dual—generate the fundamental tensor structure from which the laws of physics emerge.

The central claim of this monograph is that the universe can be described as a pair of mutually defining operations: *measurement* and *distinction*. The first gives rise to the calculus of variation; the second to the ordering of events. We introduce the *Causal Universe Tensor* as the mathematical structure

[1]Unlike conventional formulations of dynamics, no notion of functional *dependency* is invoked. All relations are expressed purely through order and distinguishability: one event follows another, but nothing is said to depend on anything else. The calculus describes consistency among records of distinction, not causal generation.

that encodes measuring events. The Causal Universe Tensor unites events by showing that every measurement in the continuous domain corresponds to a finite operation in the discrete domain, and that these two descriptions agree point-wise to all orders in the limit of refinement. The familiar objects of physics—wave equations, curvature, energy, and stress—then emerge not as independent postulates but as necessary conditions for maintaining consistency between the two sides of this dual system.

From this perspective, the classical boundary between mathematics and physics dissolves. Calculus no longer describes how the universe evolves in time; it expresses how consistent order is maintained across finite domains of observation. Its dual, the logic of event selection, guarantees that these domains can be joined without contradiction. Together they form a closed pair: an algebra of relations and a calculus of measures, each incomplete without the other. The subsequent chapters formalize this duality axiomatically, derive its tensor representation, and show that the entire machinery of dynamics—motion, field, and geometry—arises as the successive enforcement of consistency between the two.

## 1.1 Countable Event Selection

A core assumption of this framework is that measurement does not produce a continuum of outcomes, but a finite or countable set of distinguishable events. We refer to this as *event selection*. Each measurement records one element from a set of mutually exclusive possibilities, and repeated measurements generate a sequence

$$E_1 \prec E_2 \prec \cdots ,$$

where $\prec$ denotes refinement: $E_{n+1}$ contains strictly more distinguishable information than $E_n$. No model of the selection mechanism is introduced. It is not treated as a physical process, a dynamical law, or a computational rule. Its existence is asserted only in the mathematical sense that observational records consist of discrete, distinguishable events. Beyond this existential assumption, nothing is specified.

The set-theoretic background for this work is standard Zermelo–Frankel set theory with the Axiom of Choice [59], together with Martin's Axiom [77]. For our purposes, the role of Martin's Axiom is simple: it ensures that whenever observations can be refined, they can be refined step by step, without getting stuck. In other words, if more information is available about an event, there is always a path of increasingly precise descriptions that captures it.

Consequently, every observational process in this monograph can be regarded as a countable sequence of event selections. All of the limits we take—whether in measurement, curvature, or energy—are completions of these countable, steadily improving approximations. No uncountable or exotic constructions are ever needed; everything rests on ordinary, stepwise refinement.

This countability is not a restriction but a guarantee of constructibility. Refinement may proceed to arbitrarily fine resolution, but always by way of countable sequences. The continuum appears only as a completion of these sequences, never as a primitive axiom. In particular, no uncountable set of physical events is assumed, and no claim is made about the existence of a continuum substrate. The mathematical structure rests entirely on discrete observational data and the assurance that refinements may always be taken in the countable domain.

Event selection, together with Martin's Axiom, therefore provides a minimal logical foundation for measurement. It allows arbitrarily fine resolution without invoking continuous fields or manifolds, and it ensures that all refinements remain within the countable framework accessible to observation. Every subsequent construction—splines, weak forms, Euler–Lagrange extremals, wave equations, Einstein tensors, *et al.*—is built atop this countable structure.

It is worth noting that the full strength of Martin's Axiom is not required for the results developed in this manuscript. The constructions that follow rely only on the existence of countable chains of refinements and the completion of those chains. Stronger forms of Martin's Axiom become relevant only if one wishes to treat the continuum as a primitive object or to model the universe directly with differential equations. In that setting, uncountable structures, continuum measures, and smooth manifolds must be assumed from the outset, and Martin's Axiom provides a technical guarantee of consistency. In the present framework, the continuum is never an axiom but a derived limit of countable data, and therefore the full power of Martin's Axiom is unnecessary. Only the countable structure guaranteed by its weaker consequences is used.

## 1.2 Global Coherence

The logic of event selection is not arbitrary. Its structure is constrained by a principle of *global coherence*: any finite set of locally consistent observations must be extendable to a single, contradiction-free global history. This requirement is purely logical. It asserts that no finite collection of measurements can encode mutually incompatible information. As we show in Chapter 3, this condition is a finite, domain-specific analogue of the role played by Martin's Axiom: if local measurements agree on their overlaps, then a global refinement exists that contains them all.

In contrast with physical postulates, global coherence is a consistency assumption. It does not specify how events are generated, nor does it assume a geometry, a metric, or a dynamical law. It asserts only that an observational record cannot contain logical contradictions. From this minimal requirement, one can construct a countable chain of refinements in which every local event selection is represented. The continuum then emerges as the completion of this coherent chain, in the sense that measurements are dense in the real numbers.

The force of the proof comes from this logical constraint. When global coherence is combined with event selection and informational minimality, the resulting completion has a unique extremal form. In particular, the smooth limit of any coherent sequence of measurements obeys a variational Euler–Lagrange structure. Thus, the calculus of smooth motion is not imposed; it is the only continuous framework consistent with globally coherent, countable observation.

**Thought Experiment 1.2.1** (The Invisible Curve [99])**.** ***N.B.***—*Thought experiments such as this often depict common physical phenomena and how the* information being measured *must restrict admissable solutions.* □

*A spacecraft travels between two distant stars. Its onboard recorder has finite sensitivity: any change in motion or emission below a fixed detection threshold is not recorded. Over the course of the journey the recorder stores only three events—departure, a midpoint observation, and arrival. No other events exceed the threshold of detection. The question is: what can be inferred about the motion between these measurements?*

*One might imagine many possibilities. The ship could accelerate, decelerate, oscillate, or follow an arbitrarily complicated path. However, any such behavior would create additional detectable events: changes in velocity, turning points, or radiative signatures. If those events had occurred, the recorder would have stored them. Because it did not, all such structure is ruled out. The only admissible history is one that introduces no unobserved features.*

*With three recorded events, informational minimality forces the unique quadratic extremal that agrees with those samples—the same quadratic interpolant that underlies the classical Simpson's rule in numerical quadrature [26]. With four events the extremal becomes cubic, and with many events it approaches a spline. Smooth motion is not assumed; it is forced by the absence of evidence for anything else. The continuum appears only as the limit of refinement: as the recorder gains resolution, the invisible curve* *becomes visible, but never exceeds what the events certify. In particular, the sequence of refinements forms a Cauchy sequence in the space of admissible motions[17, 63], and its completion is the unique smooth extremal consistent with the measured events.*

**Thought Experiment 1.2.2** (Global Coherence as a Merge of Light Cones [60])**.** ***N.B.***—*The continuous world offers a* causal approach *to the ordering of measurements. The events as recorded in a laboratory notebook only serve as a time series [13].* □

*Consider two observers $A$ and $B$, each of whom records a finite sequence of distinguishable events in increasing causal order:*

$$A = \langle a_1 \prec a_2 \prec \cdots \prec a_m \rangle, \qquad B = \langle b_1 \prec b_2 \prec \cdots \prec b_n \rangle.$$

*Each list is totally ordered by local causality (* e.g. *a differential equation of dynamics). The requirement of global coherence asks whether there exists a single event sequence*

$$G = \langle e_1 \prec e_2 \prec \cdots \rangle$$

*containing all $a_i$ and $b_j$ such that the local orders are preserved: if $a_i \prec a_{i+1}$ in $A$, then $a_i \prec a_{i+1}$ in $G$, and similarly for $B$.*

*This is exactly the merge step of a stable sorting algorithm. As shown by [60], if two lists are individually sorted, then the merge (if it exists) is uniquely determined up to elements that are incomparable. If at any step the merge requires placing $b_k \prec a_i$ even though $a_i \prec b_k$ was recorded locally, then no global sequence $G$ exists: the local records encode a logical contradiction. In concrete terms, observer $A$ may infer that $b_k$ is* caused *by $a_i$, while observer $B$ insists the opposite: $b_k$ causes $a_i$.*

*If the merge is admissible, the resulting global history is unique up to permutations of spacelike-separated elements. Those incomparable elements correspond exactly to the* uncorrelant *equivalence classes introduced later: permuting them changes no scalar invariant of the Universe Tensor. As the* *resolution of measurement increases, the merged list becomes longer, and in the dense limit it converges to the unique spline with no unrecorded curvature. Thus the continuum is not assumed; it is the only extension consistent with all local causal records.*

## 1.3 Relation to Causal Set Theory

The philosophical foundation of this work stands in clear lineage with Causal Set Theory, initiated by the seminal ideas of Bombelli, Lee, Meyer, and Sorkin [12] and refined in later developments by Rideout and Sorkin [91, 105]. In that program, the continuum is not a primitive structure but an emergent limit: a manifold arises only when a discrete, partially ordered set of events is sampled at sufficiently high density. Geometry is not assumed—it is recovered from order and counting.

The present work adopts the same foundational stance while shifting the emphasis from causal order to measurement. Events are again primary, but instead of encoding Lorentzian geometry, we encode informational content. An event is a unit of observation, and the absence of additional events is a data constraint. In this framework, a continuum description appears only as the smooth limit of a discrete construction, never as a physical postulate.

This extends the causal set philosophy from geometry to kinematics. In Causal Set Theory, Lorentzian distance emerges from order and volume counting. Here, kinematic laws emerge from extremality: the unique interpolant that introduces no unobserved curvature minimizes a bending energy functional, and its smooth limit satisfies the variational Euler–Lagrange equation. If additional structure were present, the measurement process would have recorded additional events. Thus, dynamics are inferred rather than assumed.

This theory therefore complements the causal set program. Both reject the continuum as a primary object and treat it instead as an emergent shadow of discrete data. The present framework extends that philosophy to measurement and motion, showing that smooth kinematic laws arise from informational minimality rather than differential postulate.

## 1.4 Weak Forms and Integration by Parts

A central technical tool in this manuscript is the passage from a discrete extremality principle to a continuous weak formulation by repeated integration by parts. Historically, this pattern is older than the modern terminology suggests. In the nineteenth century, classical variational methods employed integrations by parts to transfer derivatives from trial functions onto test functions, ultimately yielding natural boundary terms. In the twentieth century, this idea was formalized in the context of Hilbert spaces and distributions, where weak derivatives and test functions replaced classical smoothness assumptions.

The modern finite element method rests directly on this foundation. Galerkin's original approach [43] enforced a variational balance by requiring that the residual be orthogonal to a chosen space of test functions, producing a weak form even when classical derivatives may not exist. This framework was later placed on rigorous functional-analytic ground by Courant [20] and further developed in the context of Sobolev spaces and elliptic regularity by Lions and Magenes [74]. Ciarlet's treatment of finite element analysis [19] made explicit that the Galerkin method is simply the discrete realization of a weak variational statement arising from integration by parts.

In this work the same pattern appears, but in reverse motivation. We do not begin with differential equations and weaken them for analytic convenience. Instead, we begin with discrete events, define a discrete bending energy, and obtain a weak form because integration by parts is the continuous expression of that discrete extremality. The variational Euler–Lagrange equation is therefore not a postulate but the shadow of the weak form that emerges when the sampling of events becomes dense.

Thus the historical machinery of integration by parts, weak solutions, and Galerkin methods does not merely provide mathematical comfort; it reveals that classical differential equations are consequences of informational consistency, not assumptions.

## 1.5 Paradoxes, Aliasing, and Cancellations

The transition from discrete structures to smooth limits must be handled with care. Classical measure theory contains well-known examples where naive passage to the continuum leads to non-physical conclusions. The Banach–Tarski paradox, proved using the Axiom of Choice [4, 107], shows that a solid ball in three dimensions can be decomposed into finitely many disjoint sets and reassembled into two identical copies of the original. Although mathematically rigorous, such constructions violate any physical notion of volume preservation. They arise precisely because arbitrary decompositions of sets ignore the informational structure that would be present in any measurable process. In effect, they treat uncountable collections of measure-zero points as if they carried the same "size" as countable sets built from measurable pieces.

In numerical analysis, a more familiar version of this pathology appears as aliasing and cancellation. A function sampled too coarsely can hide large oscillations between measurement points [47]; Gibbs-like ringing can vanish or flip sign [44, 101]; and two nonzero signals can cancel exactly when sampled at insufficient resolution [97]. The data appear benign, but the underlying object may be violently oscillatory. In both cases, the fault lies not in the continuum, but in the failure to encode which decompositions or oscillations are physically meaningful [83].

The present framework avoids such paradoxes by construction. Measurement is modeled as a finite selection of events, and the absence of additional events is a data constraint. An interpolant exhibiting oscillations, cancellations, or paradoxical decompositions would necessarily imply unobserved structure. Such a function is inconsistent with the event selection, and is ruled out by the extremality principle: the unique interpolant consistent with the data minimizes curvature and introduces no additional features. Thus the smooth limit cannot generate paradoxical volume behavior, and aliasing is impossible, because additional curvature would have been recorded as additional events.

In this sense, paradoxes of decomposition and aliasing are not ignored but excluded. The informational content of the data places strict limits on what can exist between observed events. The continuum that emerges in the dense limit is the one with no unobserved structure, and therefore no paradoxical cancellations, aliasing, or interference. Any such phenomenon, if measured, must coincide with a physicality.

## 1.6 A Gauge Theory of Information

A final component of this framework is the development of a gauge theory of information. In conventional relativistic field theory, transformation laws are imposed at the level of the continuum: Lorentz invariance is a fundamental constraint on fields, and spinorial structure is required to represent half-integer representations of the rotation group. In the present work, these structures are not postulated. Instead, the relevant symmetries emerge as constraints on information: an event selection must produce observationally indistinguishable results under changes of inertial frame.

This leads naturally to an informational gauge group. Two descriptions of the same measurement record are considered equivalent if their predicted event sets differ only by transformations that preserve observational outcomes. In the smooth limit, these gauge transformations approximate Lorentz transformations arbitrarily well, but the underlying data remain finite and discrete. Because Lorentz symmetry appears only as a limit rather than a postulate, a spinor bundle is not required at the discrete level. Vectors suffice, and spinorial structure emerges only as a continuum approximation.

This perspective echoes a long tradition in discrete approaches to spacetime. Bombelli, Lee, Meyer, and Sorkin first demonstrated that a Lorentzian manifold can emerge as an approximation to a fundamentally discrete causal set, and that exact Lorentz symmetry need not hold at the microscopic level [12]. Subsequent work by Sorkin and collaborators emphasized that continuum symmetries appear only as large-scale statistical regularities of a random partial order, not as primitive geometric postulates [30, 105]. Henson further showed that in such models, local Lorentz invariance is recovered in the limit of increasing density of causal relations, even though no metric or differential structure is assumed a priori [53]. Taken together, these results suggest that Lorentz symmetry is emergent from combinatorial structure rather than fundamental in itself.

From this viewpoint, information—not geometry—carries the primitive structure. Continuum fields, spinor representations, and relativistic kinematics are shadows of a deeper combinatorial statement: two observers are equivalent if their event selections cannot be distinguished by measurement. This yields a gauge of information whose continuum limit recovers the familiar invariances of classical relativistic physics, but without assuming a metric, a manifold, or a spin structure at the start.

## Coda: The Twin Paradox

**N.B.**—Time dilation is not caused by recording more events. It is revealed when those events are merged into a globally coherent history. The twin that accumulates more refinements forces a larger merge and therefore corresponds to less proper time; the inertial twin, with fewer refinements, corresponds to more. For further intuition, see Remarks 2 and 6, Definition 11, and Thought

Experiment 4.0.1. □

As a simple illustration, consider the twin paradox [70]. In the classical treatment, the age difference arises from integrating proper time along two worldlines in a Lorentzian manifold. Here, no metric or continuum is assumed. Each twin accumulates a finite record of events—ticks of a clock, photons received, threshold crossings of a detector. The information contained in these records is all that distinguishes one history from another.

During the outbound and inbound legs of the journey, the traveling twin undergoes changes that the stay-at-home twin does not: engine burns, thruster firings, telemetry exchanges, and adjustments of orientation. Each of these produces a measurable refinement of state, adding events to the traveling twin's record. The twin on Earth, by contrast, has a record that is comparatively coarse. Crucially, this asymmetry cannot be removed by any choice of description. One twin simply measures more.

In the information gauge, proper time is not a geometric interval but the count of admissible distinctions—the number of measurable, irreversible updates to a system's state. A history with more recorded distinctions corresponds to more events that must be reconciled. Refinement of measurement allows for the discovery of new events. Similarly, the unaccelerated twin gathers no new information from refinement. The traveling twin's notebook is therefore longer: it contains additional causal markers between departure and return that have no counterparts in the stay-at-home twin's record. Refinement introduces new distinctions but contributes no duration by itself; proper time is the work of reconciling those distinctions into a coherent history.

The asymmetry becomes operationally visible only when the twins reunite. To reconcile their histories, the stay-at-home twin must merge a richer time series. In the sorting process of global coherence, she must accommodate the extra distinctions recorded by her sibling. The traveler, having logged more events, performs a strictly smaller merge. The extra "time" is nothing more than the additional informational work required to coherently order the denser record.

Time dilation, in this view, is not a geometric mystery but an informational fact. One worldline contains strictly more refinements and therefore requires more work to merge into a single coherent history. The stay-at-home twin must resolve the additional distinctions recorded by her sibling, while the traveler performs a strictly smaller merge. The traveler therefore experiences less relative time: there is less information to reconcile. Any continuum reconstruction must agree with this count; no metric can reverse it without contradicting the observed data.

In the continuum limit, where these discrete refinements become dense, the argument reproduces the standard Lorentzian result. The traveling twin's path contains regions of higher curvature in the space of measurements, which manifest as shorter proper time in the geometric formulation. But this structure is inferred, not assumed. No manifold, metric, or spin structure is postulated. Time dilation is the unique smooth continuation of the discrete fact that more events occurred along one history than the other.

Seen this way, the twin paradox is not a paradox at all. Two observational records are compared, and the one with the richer informational content corresponds to the older twin. Geometry merely codifies this informational asymmetry in the language of smooth manifolds and differential forms. The physics was already determined by measurement.

# Chapter 2

# The Algebra of Events

**N.B.**—The measurable objects of this chapter live in a finite–dimensional real vector space $V$ and its tensor algebra $\mathcal{T}(V)$. We assume no metric, no inner product, and no rule for raising or lowering indices. Consequently, there is no notion of contravariance or covariance in the formal development, and Einstein summation does not apply. The tensor algebra here is purely algebraic: indices do not carry geometric meaning.

When a thought experiment quotes familiar expressions from continuum physics (stress–energy, geodesic equations, curvature terms, and so on), we reproduce their standard index notation verbatim so that the reader may recognize the classical form. These indices belong to the *smooth shadow* of the theory, not to the discrete algebra itself. The combinatorial foundation assumes only that $V$ exists; no metric structure is imposed. $\square$

The first chapter established the informational mechanism of physics: each measurement produces a finite, distinguishable event, and the absence of additional events is itself a binding constraint. Measurement is not a sampling of an underlying continuum; it is the creation of distinguishability. From a mathematical standpoint, the universe is therefore a growing record of discrete selections, ordered by refinement.

This chapter formalizes that structure. We build the algebra of measurement itself. Rather than treating observations as values of continuous functions, we adopt the combinatorial viewpoint forced by the Axioms 6 and 5: every admissible experimental record is finite or countable, and every refinement is a restriction of admissible outcomes. The central object of this chapter—the *Causal Universe Tensor*—expresses the fact that history is built multiplicatively: each new event contracts the set of admissible continuations of the record. The universe does not evolve additively in time; it accumulates consistency through left products of restricted increments.

This changes the role of time entirely. In the discrete domain $E$, time is ordinal: an index into the growing chain of distinguishable selections. In the continuous shadow $U$, time does not flow at all. The continuous tensor $\mathbf{U}_k$ at step $k$ is not the result of propagation, but the image of a restriction map:

$$\mathbf{U}_{k+1} \;=\; \Psi\big(e_{k+1} \cap \hat{R}(e_k)\big)\, \mathbf{U}_k, \tag{2.1}$$

where $\hat{R}$ is the discrete restriction induced by the most recent event $e_k$, and $\Psi$ is its continuous representation. Thus the continuous universe is not postulated as a field living on a manifold: it is the coherent bookkeeping of discrete consistency.

The ordered product makes this explicit. If $\hat{R}(e_j)$ denotes the admissible refinement of the $j$-th event, then the causal universe as seen by a single inertial observer is

$$\mathbf{U}_k \;=\; \prod_{j=1}^{k-1} \Psi\big(e_{j+1} \cap \hat{R}(e_j)\big). \tag{2.2}$$

No geometry, metric, or dynamical law is assumed. Continuity, smoothness, and variational structure will reappear later only as the unique continuous shadow of this discrete product, forced by informational minimality and global coherence.

In this way, Chapter 2 performs the key transition: from sets of distinguishable events to an algebra of restricted, multiplicative updates. The universe is "embarrassingly parallel": each inertial frame maintains its own

$\mathbf{U}_k$, derived solely from the restriction of its local events. Relativistic simultaneity requires no further machinery. Partitions of $E$ arise naturally from informational independence, and the merge of light-cone–consistent updates uniquely defines their reconciliation.

**N.B.**—In the sense of parallel computing, the causal structure is "embarrassingly parallel" [1]: independent branches of $E$ evolve without any need for global synchronization. Only when light cones overlap must their records be reconciled, and in that case consistency is enforced by the causal order rather than by a shared clock. □

The remainder of the chapter introduces the axioms, operators, and tensor structures that make this viewpoint precise, culminating in the formal definition of the causal universe tensor.

**Thought Experiment 2.0.1** (Statistical Process Control [98])**.** ***N.B.***—*Observational records have been used to understand and control complex processes to remarkable success. Statistical process control demonstrates that measurement does not estimate a continuous parameter directly; it eliminates process states that are incompatible with the record. The state of the system is therefore not an average, but the set of configurations that have survived all admissible checks.* □

*Imagine a factory that manufactures a precision component. The process is controlled by a set of adjustable parameters: temperature, pressure, feed rate, alignment, and so on. At startup, all parameter settings that satisfy the design tolerances are admissible; the process could be in any one of many configurations. A single measurement does not determine the underlying state. It merely rules out those configurations that would have produced a conflicting outcome.*

*This is the essential structure of statistical process control. Each inspection, probe, gauge reading, or quality check eliminates a subset of incompatible configurations. After $k$ measurements, the surviving parameter settings are precisely those that are consistent with all $k$ observations.*

*Let $e_k$ denote the k-th inspection result, and let $\hat{R}(e_k)$ be the discrete restriction that removes every process state incompatible with $e_k$. If $\Psi$ embeds these restrictions into the continuous tensor domain, the recorded state of the process after k inspections is*

$$\mathbf{U}_k = \prod_{j=1}^{k-1} \Psi\big(e_{j+1} \cap \hat{R}(e_j)\big). \tag{2.3}$$

*The process does not "evolve" in time in the usual dynamical sense; it accumulates admissibility. Each new inspection refines the record by discarding alternatives, giving the stepwise update*

$$\mathbf{U}_{k+1} = \Psi\big(e_{k+1} \cap \hat{R}(e_k)\big)\, \mathbf{U}_k. \tag{2.4}$$

*Two independent inspectors, perhaps located at different stations on the production line, can refine their records without communication. If their measurements are mutually consistent, the products merge without conflict. If not, no admissible configuration survives the combined restrictions, and the process is flagged as out of control. In this sense the system is "embarrassingly parallel": independent measurements commute whenever they are spacelike in the informational sense, and consistency is enforced only when histories are compared.*

*This ordinary industrial setting exhibits the same structure developed in this chapter. Measurement eliminates incompatible alternatives, time indexes the number of admissible refinements, and the continuous representation $\mathbf{U}_k$ is nothing more than the shadow of a discrete product of selections.*

We begin by enumerating the Axioms of Measurement, which formalize the structure of admissible records and the refinement of observational history.

## 2.1 The Axioms of Mathematics

All mathematics in this work is carried out within the framework of Zermelo–Fraenkel set theory with the Axiom of Choice (ZFC) [59, 65]. Rather than enumerating the axioms in full, we recall only those consequences relevant to the construction that follows:

- **Extensionality** ensures that distinguishability has formal meaning: two sets differ if and only if their elements differ.
- **Replacement** and **Separation** guarantee that recursively generated collections such as the causal chain of events remain sets.
- **Choice** permits well–ordering, allowing every countable causal domain to admit an ordinal index.

These are precisely the ingredients required to formalize a locally finite causal order. All further constructions—relations, tensors, and operators—are definable within standard ZFC mathematics; see Kunen [65] and Jech [59] for set-theoretic foundations, and Halmos [48, 49] for the induced tensor and operator structures on finite-dimensional vector spaces.

The starting point of this framework is methodological rather than ontological. We do not assume anything about the substance of physical reality. We assume only that the outcomes of measurement are finite or countable collections of distinguishable results recorded in time. This is standard across probability theory and information theory: Shannon formalized information as distinguishable symbols drawn from a finite or countable alphabet [96], and Kolmogorov showed that empirical outcomes can be represented as elements of measurable sets within standard set theory [62]. In this view, measurement produces data, and data are mathematical objects. Everything that follows concerns the admissible transformations among such records.

**Axiom 1** (The Axiom of Kolmogorov [62])**.** [Measurement as a Formal Record.]

*The record of measurement—defined as the finite or countable set of observed, distinguishable events—is taken to be a mathematical object representable within ZFC. No ontological claim is made about physical reality. The axiom asserts only that observable data can be formalized as sets and relations.*

*This standpoint is consistent with Kolmogorov's construction of probability spaces, in which empirical outcomes are represented as measurable sets [61]. Accordingly, a record of finite observations is a mathematical object whose structure is defined entirely within ZFC. Throughout this work, the word "information" refers exclusively to these representable distinctions; nothing is asserted about any underlying physical substrate that might produce them.*

**Axiom 2** (The Axiom of Peano [42, 77, 116])**.** [Counting as the Tool of Information] *All reasoning in this work is confined to the framework of ZFC. Every object—sets, relations, functions, and tensors—is constructible within that system, and every statement is interpretable as a theorem or definition of ZFC. No additional logical principles are assumed beyond those required for standard analysis and algebra.*

*Formally,*

$$\text{Measurement} \subseteq \text{Mathematics} \subseteq \text{ZFC}\ \textit{subseteq}\ \text{Counting}.$$

*Thus, the language of mathematics is taken to be the entire ontology of the theory: the physical statements that follow are expressions of relationships among countable sets of distinguishable events, each derivable within ordinary mathematical logic.*

**Thought Experiment 2.1.1** (The Speedometer [112, 115])**.** ***N.B.***—*The mechanical implementation of measuring devices often are protected by ex-*

*plicit descriptions of how they work. The patents cited here explicitly describe how they turn counting into data.* □

*Consider an ordinary automobile speedometer. The dial appears to report a continuous real number at each instant, but the device does not have access to the real numbers. A mechanical speedometer counts wheel rotations through a gear train and maps those counts to pointer positions. A digital speedometer counts the same rotations and displays a numeral drawn from a finite alphabet.*

*Each time the counter increments and the displayed symbol or pointer position changes, a new distinguishable event is recorded. Between two successive display states there is no way, from the informational record alone, to assert that any additional state occurred. The apparent continuity of "speed" is a visual interpolation of a finite counting process.*

*Thus the speedometer does not output a real number. It outputs a countable sequence of distinguishable states derived from integral counts of wheel rotations. The act of measuring speed reduces to counting transitions of a finite-state device. All physical inference based on such data can be expressed within ordinary arithmetic and set theory.*

*This illustrates Axiom 2: measurement generates only countable, finitely coded distinctions, and every mathematical object used to interpret those distinctions—numbers, functions, tensors—is a construct of ZFC. No structure beyond counting is assumed at the fundamental informational level.*

## 2.2 The Axioms of Information

The previous section established that a physical record is a set of distinguishable observations, representable within ZFC, and partially ordered by causal precedence. Nothing further was assumed about geometry, dynamics, or the continuum. In this section, we introduce two informational axioms that restrict how such a record may be interpreted. These axioms express constraints on admissible descriptions of the world, independent of any particular model of physics.

Axiom 3 formalizes the principle that a physical history may not contain unobserved structure. Among all symbolic descriptions that reproduce the recorded events, the admissible one is the shortest. This is the information–theoretic form of Ockham's principle: no plurality of assumptions without necessity.

Axiom 4 asserts that the record of events is not merely ordered but forms a locally finite causal set. Local finiteness ensures that causal cardinality is discrete, while the partial order encodes temporal precedence. Continuum spacetime, when it exists, is therefore understood as an approximation that faithfully embeds this discrete informational structure.

Together, these axioms define the informational content of the physical world: a causal set with no unrecorded structure and no additional assumptions beyond the observational record itself.

### 2.2.1 Information Minimality

The observational record $E$ is defined only by the distinguishable events it contains. Between two recorded events $e_i$ and $e_{i+1}$, no additional structure is present in the data: no new marks in the notebook, no threshold crossings, and no observable distinctions. Set theory alone does not forbid a hypothetical refinement that inserts additional structure between $e_i$ and $e_{i+1}$, but any such refinement asserts observations that did not occur. To prevent unrecorded structure from being introduced by assumption, we impose an informational constraint.

Among all symbolic descriptions that reproduce the recorded events, the admissible one is the shortest. In modern information theory, this statement is formalized by Kolmogorov complexity [61, 73]: a description is preferred if it introduces no additional information beyond the events in $E$. This embodies the classical principle that no plurality of assumptions should be posited without necessity. It is not derived from the set-theoretic framework; it is an axiom about how physical theories must interpret finite empirical records.

**Axiom 3** (The Axiom of Ockham [84])**.** [Information Minimality] *Let* $E = \{e_0 \prec e_1 \prec \cdots \prec e_n\}$ *be the recorded events of an experiment, understood as a finite or countable set of distinguishable observations representable in ZFC. Among all symbolic descriptions that map to* $E$ *and introduce no additional recorded events, the admissible completion is the one of minimal Kolmogorov complexity. [61, 73].*

*Equivalently, if a hypothetical refinement of the history introduces a distinguishable update that is not present in* $E$*, then that refinement is inadmissible. Any shorter description consistent with* $E$ *is preferred.*

We have seen this principle in action already. Refer to Thought Experiment 1.2.1 and the use of Simpson's rule to compute the path of a spaceship with minimal measurement information.

### 2.2.2 Causal Set Theory

The previous axiom imposed an informational constraint on admissible descriptions of the record of measurement. We now introduce a structural constraint. The empirical record is a set of distinguishable events with a causal precedence relation $\prec$, but this alone does not restrict the size of causal intervals. In a general partially ordered set, the number of events between $a$ and $b$ may be infinite. Physical measurements, however, produce finite data. To represent this empirically grounded discreteness, we assume that the causal order is locally finite: every causal interval contains only finitely many recorded events.

This postulate places the present construction within the causal set program of Sorkin and collaborators, where spacetime is modeled as a locally finite partial order and continuum geometry, when it appears, is a derived approximation. Order encodes temporal precedence, and local finiteness encodes discrete causal volume. No metric, field, or manifold structure is assumed at the fundamental level; these arise only if the causal set admits a faithful embedding into a Lorentzian manifold.

**Axiom 4** (The Axiom of Causal Sets [12])**.** [Events are Discrete]

*The distinguishability relations among recorded events admit a representation as a locally finite partially ordered set* $(E, \prec)$*, where*

1. $e \prec f$ *means that the record of* $e$ *is incorporated before the record of* $f$*,*
2. $(E, \prec)$ *is acyclic and transitive,*
3. *and for any two events* $a \prec b$*, the interval* $\{\, e \in E : a \prec e \prec b \,\}$ *is finite.*

*Local finiteness ensures that the recorded causal cardinality is discrete, and the order relation encodes temporal precedence within the record. Any Lorentzian manifold, when it exists, is merely a physical model in which this discrete causal structure may be faithfully approximated.*

**Thought Experiment 2.2.1** (The Laboratory Procedure [84, 114])**.** ***N.B.***— *The following example collects ideas from several well–established perspectives in measurement theory. Bohr and Wheeler emphasize that a physical experiment records only distinguishable outcomes; no other structure is operationally meaningful [8, 114]. In information theory, such records are represented as finite or countable strings of distinguishable symbols [22, 96]. In ergodic theory and causal set theory, successive measurements refine a partition of the observational domain into finer distinguishable elements [85, 94, 106]. Finally, computational mechanics and operator–theoretic dynamics treat the "evolution" of a system as the repeated update of its information state [7, 23, 64]. Taken together, these perspectives justify modeling a laboratory procedure as a refinement operator acting on a finite measurement* *record. The experiment does not solve differential equations; it follows the laboratory procedure* $\Psi$*.* □

*Consider a laboratory notebook in which each threshold crossing of a detector is recorded as a mark in ink. The notebook contains a finite sequence of distinguishable entries*

$$e_0 \prec e_1 \prec \cdots \prec e_n,$$

*each representing an irreversible update of the experimental record. The notebook is not a model of reality; it is the empirical record. No claim is made about any mechanism behind it.*

*Now suppose one attempts to describe what "really" happened between two successive entries* $e_i$ *and* $e_{i+1}$*. If additional curvature, oscillation, turning points, or discontinuities had occurred, then the detector would have crossed a threshold and a new entry would appear. Because no such entry is present, the observational record forbids any refinement that predicts one.*

*Thus the notebook determines a finite set* $E = \{e_0, \ldots, e_n\}$ *of recorded events. Every admissible history must be a completion that introduces no new distinguishable events beyond* $E$*. Any hypothetical refinement with additional structure is rejected as inadmissible, since it asserts observations that did not occur.*

## 2.3 The Axioms of Physics

A common criticism of mathematical physics is the extent to which mathematics can be tuned to fit observation [11, 89] and, conversely, manipulated to yield nonphysical results [6, 57]. The critique of Newton's fluxions could only be answered by successful prediction. Today, calculus feels like a natural extension of the real world—so much so that Hilbert, in posing his famous list of open problems, explicitly formalized the lack of a rigorous foundation for physics as his Sixth Problem [54, 113].

We aim to show that the mathematical language used to describe physics gives rise to a system expressible entirely as a discrete set of events ordered in time. Moreover, this ordered set possesses a mathematical structure that naturally yields the appearance of continuous physical laws and the conservation of quantities. To understand how this works, we first clarify what we mean by measurement.

### 2.3.1 Measurement and the Axiom of Cantor

Physical laws relate measurements. For example, Newton's second law [82]

$$F = \frac{dp}{dt} \tag{2.5}$$

states that force relates to the *change* in momentum over time. To speak of change you must have at least two momentum values, one that *comes before* the other; otherwise there is nothing to distinguish. In set-theoretic terms, by the Axiom of Extensionality (assumed in Axiom 2), different states must differ in their contents, so "change" presupposes the distinguishability of two states.

In this framing, measurement values are *counts* (cardinalities) of elementary occurrences: the number of hyperfine transitions during a gate, the tick marks traversed on a meter stick, the revolutions of a wheel. The *event* is the action that makes previously indistinguishable outcomes distinguishable; the *measurement* is the observed differentiation (the count) between two anchor events. This is not the absolute measure of the event, but just relative difference of the two. We count the events as time passes (See Thought Experiment 2.1.1.

Since special relativity requires that time vary under the Lorentz transform [33, 75], there can be no global scalar representation of temporal duration. Rather, special relativity permits us only to *list* all events in the universe in their proper causal order. It is this ordered list that we elevate to the first physical principle:

**Axiom 5** (The Axiom of Cantor [16, 32])**.** [Events are Ordered and Countable] *The only invariant agreement in time guaranteed between two observers is the order in which correlant events (see Definition 9) occur. Duration is not a geometric interval; it is the count of distinguishable events that can be recorded between two selections:*

$$|\delta t| \;=\; \big|\textit{events distinguished between}\big|. \tag{2.6}$$

**Thought Experiment 2.3.1** (Relativistic simultaneity [33].)**.** *Two laboratories, $A$ and $B$, perform independent procedures, each producing a finite measurement record. Because the experiments are independent, their events commute: no record in $A$ constrains the order of any record in $B$. Both notebooks are internally consistent, but their events are mutually unordered.*

*Now two observers, $C$ and $D$, travel past the laboratories on different trajectories, each at a velocity close to the speed of light. Their instruments register signals from $A$ and $B$ in different sequences. Since the events commute, both observers are free to assemble the two notebooks into different global orders. Observer $C$ concludes that certain events in $A$ precede those in $B$, while observer $D$ concludes the opposite. Each construction is internally consistent, because commutativity permits the reordering.*

*The discrepancy is not a contradiction, but the finite analogue of relativistic simultaneity: different trajectories generate different admissible orderings of commuting events. The events themselves may be reordered independently of each other, yet the invariants are preserved.*

### 2.3.2 Observations are Combinatorial

A finite observer records events one at a time. Each record refines the set of admissible histories, and every refinement depends only on the records accumulated so far. Physical description is therefore necessarily recursive: the $(k + 1)$st step is constructed from the $k$ steps that precede it.

The recursive description of physical reality is meaningful only within the finite causal domain of an observer. Each step in such a description corresponds to a distinct measurement or recorded event. Observation is therefore bounded not by the universe itself, but by the observer's own proper time and capacity to distinguish events within it.

**Axiom 6** (The Axiom of Planck [88])**.** [Observations are Finite] *For any observer, the set of observable events within their causal domain is finite. The chain of measurable distinctions terminates at the limit of the observer's proper time or causal reach.*

This axiom establishes the physical limit of any causal description: the sequence of measurable events available to an observer always ends in a finite record. Beyond this frontier—beyond the end of the observer's time—no additional distinctions can be drawn. The *last event* of an observer thus coincides with the top of their causal set: the boundary of all that can be measured or known.

### 2.3.3 Event Selection

The preceding axioms restrict the informational content of the record and the structure of causal precedence. We now introduce an axiom governing how events may be selected in a consistent physical history. A partial history is a finite sequence of recorded distinctions that respects the causal order. In a locally finite causal set, many partial histories may be extended, but not all extensions are admissible: each new event must be consistent with the existing record and may not contradict any previously recorded distinction.

Axiom 7 asserts that whenever we impose countably many local admissible requirements—each representing a physically permitted constraint—

there exists at least one consistent history that satisfies all of them[1]. Mathematically, this parallels the role of Martin's Axiom in set theory, where dense sets encode constraints and a filter selects a coherent global object [59, 65, 77, 109]. Physically, it echoes Boltzmann's principle that every admissible microstate selection must preserve distinguishability [11], and follows the causal-set program in which a spacetime history is constructed one event at a time under admissible refinement [12, 40]. Hilbert's call to axiomatize the foundations of physics [55] is realized here as a minimal requirement: if each local constraint is permissible, then some coherent global history must also be permissible.

**Axiom 7** (The Axiom of Boltzmann [10, 77])**.** [Events are Selected to be Coherent.] *An experiment may impose many local causal requirements: detector constraints, boundary conditions, conservation rules, and so on. As long as each requirement can be satisfied on its own, the Axiom of Boltzmann asserts that there always exists at least one, globally coherent history satisfying* all *of them simultaneously. No matter how many local constraints we specify, they can be assembled into one consistent record.*

*Formally, let* $(\mathsf{P}, \leq)$ *be the partially ordered set (Definition 2) of finite, order-consistent partial histories in a locally finite causal domain, ordered by extension. For every countable family* $\{D_n\}_{n\in\mathbb{N}}$ *of dense subsets of* $\mathsf{P}$ *(local causal constraints), there exists a filter* $G \subseteq \mathsf{P}$ *such that* $G \cap D_n \neq \varnothing$ *for all* $n$*.*

## 2.4 The Causal Universe Tensor

The axioms above determine the structure of the physical record: events form a locally finite causal set, extensions of partial histories preserve causal con-

[1] In the continuum limit, when observables range over a complete set of measurable values, the admissible history is unique up to sets of measure zero: there is exactly one continuous completion consistent with all recorded refinements.

sistency, and informational minimality forbids unrecorded structure. What remains is to represent this record in a mathematical form that allows the accumulation of distinctions. We now construct such a representation.

### 2.4.1 Sets of Events

Let the set of all events accessible to an observer be denoted $E$[2], ordered by causal precedence ($\prec$). Because any physically realizable region is finite, this order forms a locally finite partially ordered set (poset) [39].

**Definition 1** (Causal Precedence [12])**.** *Let $E$ be the set of distinguishable events accessible to an observer. For $e_i, e_j \in E$, we say that $e_i$* causally precedes *$e_j$, written $e_i \prec e_j$, if the record of $e_j$ cannot be formed without already having distinguished $e_i$. Equivalently, $e_j$ refines the admissible outcomes of $e_i$. The relation $\prec$ is a strict partial order: it is irreflexive ($e \not\prec e$), antisymmetric, and transitive.*

***N.B.***—*The term "causal" is used only in the sense of ordering: $e_i \prec e_j$ asserts that $e_j$ depends on the distinctions recorded in $e_i$. No geometric notion of signal propagation or physical influence is assumed.* □

Each admissible set of events may be represented as a locally finite partially ordered structure [12, 104], whose links record only those relations that are causally admissible. In this view, a "history" is not a continuous trajectory but a combinatorial diagram: every vertex an event, every edge a permissible propagation. This discrete formulation generalizes the intuition behind Feynman's space–time approach to quantum mechanics, in which the amplitude of a process is obtained by summing over all consistent histories [37, 38]. The Feynman diagram thus appears here as a special case of the causal network itself—a pictorial reduction of the full tensor of event

[2] The symbol $E$ here denotes the *set of distinguishable events*—it is not the energy operator or expectation value familiar from mechanics. Throughout this work, $E$ indexes discrete occurrences in the causal order, while quantities such as energy, momentum, or stress appear only later as *derived measures* on this set.

relations—and the path integral becomes a statement of global consistency across all measurable causal connections.

**Thought Experiment 2.4.1** (Feynman Diagrams (classical) [38])***. N.B.***— *This is a classical simplification of the highly specialized notation of the Feynman diagram. See Thought Experiment 7.2.3 for a more rigorous treatment.* □

*In conventional quantum field theory, a Feynman diagram depicts a sum over interaction histories connecting initial and final particle states. Each vertex represents an elementary event—an interaction that renders previously indistinguishable outcomes distinct—and each propagator represents the possibility of causal influence between events.*

*In the present formulation, such a diagram is naturally interpreted as a finite* causal network. *The set of vertices corresponds to the event set $E$, and the directed edges encode the order relation $\prec$ defined by Axiom 5. To each event $e_k$ we associate a representation $\mathbf{E}_k$ that records the admissible refinement induced by that event, and the directed structure describes which refinements must precede others. The composition of these event tensors gives the Causal Universe Tensor of the inertial frame:*

$$\mathbf{U}_n = \prod_{k=1}^{n} \mathbf{E}_k. \tag{2.7}$$

*At this stage, $\mathbf{U}_n$ is a classical accumulator: it records the count and structure of distinguishable events without assigning amplitudes or phases. This is deliberate. The present framework concerns only the logical bookkeeping of distinctions. The full quantum structure—including complex amplitudes, superposition, and interference—appears only after the informational gauge is introduced. In that setting, the classical accumulator becomes the coarse projection of a richer amplitude algebra, much as a Feynman diagram may be viewed as the combinatorial skeleton of a path integral. That generalization is deferred until Chapter 7, where the amplitude-bearing form of $\mathbf{U}$ is* *constructed.*

*Summing over all consistent diagrams is therefore equivalent to enumerating all admissible orderings of distinguishable events. The path integral itself becomes a statement of* global consistency *across the entire causal network: every measurable amplitude corresponds to a possible embedding of finite causal order into the continuous limit. In this sense, a Feynman diagram is not merely a pictorial tool, but a discrete representation of the causal tensor algebra from which continuum physics emerges.*

This identification is pedagogically useful. From this point onward, every construction may be viewed as an algebraic generalization of the familiar Feynman diagram: the event tensors are its vertices, the causal relations its edges, and the Universe Tensor the cumulative sum over all consistent orderings. The remainder of the monograph simply formalizes this graphical intuition in set-theoretic and tensorial language, rather than using calculus.

At the heart of measurement is the concept of a partially ordered set.

**Definition 2** (Partially Ordered Set [25])**.** *A poset is a pair $(E, \leq)$ where $\leq$ is a binary relation on $E$ satisfying:*

1. ***Reflexivity:*** $e \leq e$ *for all* $e \in E$
2. ***Antisymmetry:*** *if* $e \leq f$ *and* $f \leq e$*, then* $e = f$
3. ***Transitivity:*** *if* $e \leq f$ *and* $f \leq g$*, then* $e \leq g$

Such an ordering always admits at least one maximal element [12]

**Definition 3** (Top of a Poset)**.** *Let $(E, \leq)$ be a partially ordered set. The* top *of $E$, denoted* $\mathrm{Top}(E)$*, is the set of maximal elements of $E$:*

$$\mathrm{Top}(E) = \{\, e \in E \mid \nexists f \in E \text{ with } e < f \,\}. \tag{2.8}$$

*That is,* $\mathrm{Top}(E)$ *contains those events in $E$ for which no strictly greater event exists.*

The elements of $\mathrm{Top}(E)$ represent the current causal frontier—the most recent events that have occurred but have no successors [106]. Although $\mathrm{Top}(E)$ may contain several incomparable (spacelike) elements, it is never empty and therefore provides a well-defined notion of a "last event" from the observer's perspective. This frontier defines the light-cone boundary and the terminal particle–wave interaction that delimits all accessible information.

Every event $e \in E$ corresponds to an irreducible distinction in the experimental record. Under the measurable embedding $\Psi : E \to \mathcal{T}$ introduced in Thought Experiment 2.2.1, each logical event is mapped to an algebraic object $\mathbf{E}_e$ in the tensor algebra. These objects compose whenever their corresponding events are compatible in the causal order, so the accumulation of observed events yields a record that reflects the ordered refinement of the causal set.

The goal of this section is to define a cumulative object $\mathbf{U}_n$ —the *Causal Universe Tensor*—that embodies the total informational content of all events observed up to step $n$ in the current inertial reference frame. This tensor is not a dynamical evolution. It is the bookkeeping device that records which refinements have survived admissibility.

It is crucial to emphasize that no background time parameter is introduced. There is no external clock and no continuous variable $t$ against which events are measured. Instead, Axiom 5 guarantees that the causal set admits a linear extension: the events can be listed in a sequence that respects causal precedence. In this framework, *time* is merely the ordinal index of an event in such a sequence. It is not a physical field or metric quantity, but a bookkeeping device that labels the relative order of observations.

With this viewpoint, accumulating the event tensors in order is not evaluating a function of time. It is forming the ordered product of distinctions that have occurred. The resulting object, the Causal Universe Tensor, represents the total recorded history up to any chosen ordinal position in the list of events.

### 2.4.2 Special Relativity

Every physical description begins not with space or time, but with an *event*—an interaction that makes previously indistinguishable outcomes distinct [9, 89]. The causal boundary of such an interaction is its *light cone*: the set of all events that can influence or be influenced by it according to special relativity [33, 78]. The intersection of two light cones, corresponding to the last particle–wave interaction accessible to an observer, defines the maximal region of causal closure [52, 86]. Beyond this surface, no additional information can be exchanged; all distinguishable action has concluded.

It is from this closure that the ordering of events arises. Each measurable interaction contributes one additional distinction to the universe, expanding its causal surface by a finite count [52, 76]. The smooth fabric of spacetime is not primitive but emergent: it is the limiting behavior of discrete causal increments accumulated along the light cone [12, 104]. Within each cone, the universe can be represented by a finite tensor of interactions—local updates to a global state—that together approximate continuity only through cancellation across countable events [12, 105].

**Thought Experiment 2.4.2** (Boosting Velocity [33])**.** ***N.B.***—*The Lorentz equations are presented here only as an illustrative model of the local causal structure. They are not derived from the axioms in this chapter, and no subsequent argument depends on them.* □

*Special relativity provides the canonical local model for this causal structure. Consider the Lorentz transformation for a boost of velocity $v$ in one spatial dimension, [93, 108]*

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v/c^2 \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}, \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{2.9}$$

*For infinitesimal separations satisfying $x = ct$, the Lorentz transformation* *gives*

$$t' = \gamma\, t(1 - v/c). \tag{2.10}$$

*If we take $\Delta t = 1$ as the unit interval between distinguishable events, then observers moving at relative velocity $v$ will, in general, disagree on the* number *of such events that occur between two intersections of their respective light cones [78]. The only invariant quantity is the causal ordering itself: all observers concur on which event precedes which, even though they may count a different number of intermediate ticks [76].*

**Definition 4** (Rank time [12, 25])**.** *Let $(E, \prec)$ be a locally finite* partially ordered set of events*. A rank time is an order-embedding*

$$\tau : E \to \mathrm{Ord}$$

*satisfying $e \prec f \implies \tau(e) < \tau(f)$. Local finiteness implies that for any observer's causal domain $D \subseteq E$, $\tau(D)$ is order-isomorphic to an initial segment of $\mathbb{N}$. We therefore define the* duration*, $|\delta t|$, between anchors $a \prec b$ by*

$$|\delta t|(a, b) \;=\; \#\{\, e \in E \mid a \prec e \prec b \,\} \in \mathbb{N}.$$

*Two rank functions $\tau, \tau'$ are* equivalent *if there exists an order-isomorphism $\phi$ with $\tau' = \phi \circ \tau$; equivalent ranks yield identical durations.*

**Remark 1** (Operational content of time)**.** *Time is an ordinal rank on $E$, not an independent scalar field. All subsequent uses of "t" refer to an order-equivalence class of rank functions as in Definition 4. The additivity $|\delta t|(a, b) = |\delta t|(a, c) + |\delta t|(c, b)$ follows from local finiteness.*

This observation motivates the first physical axiom: that time is not an independent scalar field but an ordinal index over causally distinguishable events. Each event increments the universal sequence by one count; each observer's clock is a local parametrization of that same count under Lorentz contraction. The apparent continuity of time is the result of the density of such events within the causal cone, not an underlying continuum of duration.

### 2.4.3 On the Structure of Measurement

In this formulation, a measurement is not the evaluation of a continuous quantity against an external time parameter. No clock, ruler, or metric is assumed. Instead, the Axioms of Planck and Cantor assert that an observer's record is a locally finite, causally ordered set of distinguishable events. To extract a numerical value from such a record, one must identify which events satisfy a specified property and count how many of them occur between two anchors in the causal order.

This viewpoint treats measurement as a purely combinatorial act: the *value* of a measurement is the number of admissible distinctions satisfying a predicate inside a finite causal interval. The result is always an integer, and continuity—when it appears—arises only as the smooth limit of increasingly refined counts. We formalize this as follows.

**Definition 5** (Measurement)**.** *Let $(E, \prec)$ be a locally finite partially ordered set of events, and let $P : E \to \{0, 1\}$ be a predicate designating which events satisfy a specified property. For two anchor events $a, b \in E$ with $a \prec b$, the* measurement of $P$ between $a$ and $b$ *is the finite integer*

$$M_P[a, b] \;:=\; \big|\{\, e \in E : \; a \prec e \prec b \text{ and } P(e) = 1 \,\}\big| \;\in\; \mathbb{N}.$$

*That is, a measurement is a count of distinguished events satisfying $P$ within the causal interval $(a, b)$.*

A measurement in this setting is therefore nothing more than a count of distinguished events between anchors. Numerical values arise only when such counts are compared against a conventional scale. No continuous quantity is assumed *a priori*; continuity is inferred from the refinement of a finite causal record. In practice, every physical "number" depends on a calibration that relates discrete counts to a chosen system of units.

**Thought Experiment 2.4.3** (Planck's Constant [88]). ***N.B.***—*Planck's constant is "constant" only after a choice of units and a calibration procedure. In practice, the quoted numerical value of h is obtained by fitting experimental data to a model, often by minimizing an $L^2$ measurement error across a calibration experiment. The physical principle is invariant, but the reported number reflects the best fit of a finite data set in a chosen system of units.*□

*Imagine a hypothetical measuring apparatus that records distinctions not by counting particles or intervals, but by tallying* acts of discernment*—each act adding one quantum of distinguishability to the record. Suppose further that the calibration of such a device required only a single fixed scale to relate discrete counts to continuous units of measure. In physics, Planck's constant h serves precisely this purpose: it is not a force or an energy, but a bookkeeping factor that ensures continuity between discrete and continuous domains.*

*In the present framework, the analogous constant plays no physical role—it merely fixes the* dimensional scale *by which finite distinctions are rendered comparable. The constant's existence affirms that measurement can be both discrete and metrically consistent without invoking any specific quantum postulate. As with h, the constant here is not discovered but* defined*: a normalization that preserves coherence between counting and continuity.*

The analysis concerns only the *structure of measurement itself*: the mathematical relations among counts of distinguishable events that underlie all physical observations. In this framing, physics is viewed as a grammar of distinctions. The familiar constants and fields—mass, charge, curvature, temperature—arise as *derived measures* within a finite causal order, not as independent entities.

**Thought Experiment 2.4.4** (Measurement as a BNF Grammar [2, 81]). *Because measurement produces distinguishable outcomes, each observation* *selects a symbol from a finite or countable alphabet*

$$\Sigma = \{\sigma_1, \sigma_2, \ldots\}.$$

*A record of $n$ measurements is therefore a word $w \in \Sigma^n$. When an instrument is refined—by increasing precision or reducing noise—any coarse symbol $\sigma_k$ may be replaced by a finite set of more precise symbols,*

$$\sigma_k \;\Rightarrow\; \sigma_{k,1} \mid \sigma_{k,2} \mid \cdots \mid \sigma_{k,r},$$

*just as in a Backus–Naur Form (BNF) production rule [2, 81]. Not all replacements are admissible: they must remain compatible with every other measurement that overlaps in time or causal order. Two refined histories that disagree on an overlapping interval cannot both represent valid records.*

*Thus admissible measurement histories form a formal language generated by the allowed refinement rules. The "law" governing measurement is the constraint that only globally consistent extensions of a record may be generated. This is not an analogy: it is the standard formal structure of symbol sequences in coding and information theory [100].*

No new particles, forces, or cosmological effects are introduced. The present theory interrogates only the rules by which such effects are numerically expressed. It is not a revision of physics but a clarification of its syntax: a study of the measures of phenomena rather than the phenomena themselves. We begin by introducing the necessary set-theoretic language and associating it with a discrete notion of "time."

**Definition 6** (Distinguishability Chain [62])**.** *Let $\Omega$ be a nonempty set. A distinguishability chain on $\Omega$ is a sequence $\mathcal{P} = \{P_n\}_{n\in\mathbb{Z}}$ of partitions $P_n \in \mathsf{Part}(\Omega)$ such that $P_{n+1}$* refines *$P_n$ for all $n$ (every block of $P_{n+1}$ is contained in a block of $P_n$). Write $\mathrm{Bl}(P)$ for the set of blocks of a partition $P$ Each refinement step produces zero or more event.*

**Definition 7** (Event [62, 106]). *Fix a distinguishability chain $\mathcal{P} = \{P_n\}$. An event at index $n$ is a minimal refinement step: a pair*

$$e = (B, \{B_i\}_{i\in I}, n) \tag{2.11}$$

*such that:*

1. $B \in \mathrm{Bl}(P_n)$;
2. $\{B_i\}_{i\in I} \subseteq \mathrm{Bl}(P_{n+1})$ *is the family of all blocks of $P_{n+1}$ contained in $B$, with $|I| \geq 2$ (a nontrivial split);*
3. *(*minimality*) there is no proper subblock $C \subsetneq B$ with $C \in \mathrm{Bl}(P_n)$ for which the family $\mathrm{Bl}(P_{n+1}) \cap \mathcal{P}(C)$ is nontrivial.*

*Let $E$ denote the set of all such events. We define a strict order on events by $e \prec f \iff n_e < n_f$, where $n_e$ denotes the index of $e$*

**Definition 8** (Proper Time [79]). *Let $E$ be the set of events generated by a distinguishability chain $P = \{P_n\}$. For any two events $a, b \in E$ with $a \prec b$, the* proper time *between them is*

$$\tau(a, b) = \max\Big\{\, |C| : \; C = \{c_0, \ldots, c_k\} \subseteq E,\; a = c_0 \prec c_1 \prec \cdots \prec c_k = b \Big\}.$$

*That is, $\tau(a, b)$ is the cardinality of a maximal chain of strictly refinable events between $a$ and $b$. Local finiteness of the distinguishability chain guarantees $\tau(a, b) \in \mathbb{N}$.*

**Remark 2.** *Proper time is not a geometric length. It is the number of admissible, irreversible refinements separating two recorded events. Additional refinement (higher resolution) may increase $\tau(a, b)$; coarse–graining cannot. Thus, proper time is an* invariant *of the partially ordered event record, not a metric assumption.*

**Remark 3.** *A chain need not include all events, and incomparable events do not contribute to one another's proper time. Only if every pair of events were comparable would $\tau$ reduce to a total order. In general, $E$ is only partially ordered.*

***N.B.***—*In physical terms, this corresponds to relativistic simultaneity: incomparable events occupy disjoint causal domains and cannot be ordered by any observer.* □

**Remark 4** (Smooth limit [92])**.** *If refinements become dense and the discrete extremal converges to a $C^2$ spline, then*

$$\lim_{\text{refinement}\to\infty} \tau(a, b) = \int_a^b \sqrt{-\,ds^2},$$

*the Lorentzian proper time. The integral form is not assumed; it is the smooth shadow of the combinatorial count.*

The notion of *uncorrelant events* formalizes the idea that two recorded distinctions may be independent of one another. In causal set theory, incomparability under the causal order corresponds to physical independence of events [12]. The same conceptual separation appears in quantum theory, where observables acting on independent subsystems commute and their measurement outcomes do not influence each other [29, 87]. Classical discussions of separated systems, from Einstein–Podolsky–Rosen and Schrödinger to Wheeler's formulation of complementarity [34, 95, 114], frame the same idea operationally: when no physical procedure can distinguish the relative order of two events, their ordering has no empirical content. The definitions below captures this in the minimal set-theoretic language of the causal poset.

**Definition 9** (Uncorrelant [12, 104])**.** *Let $(E, \prec)$ be a locally finite partially ordered set of events. Two events $e, f \in E$ are said to be* uncorrelant *if they are incomparable under the causal order; that is,*

$$\neg(e \prec f) \quad \textit{and} \quad \neg(f \prec e).$$

*The uncorrelant relation partitions $E$ into equivalence classes of events whose relative order carries no operational consequence for any admissible measurement or refinement. In particular, no experimentally distinguishable difference follows from interchanging the positions of uncorrelant events in any linear extension of $(E, \prec)$.*

**Remark 5.** *If $e$ and $f$ are uncorrelant, permuting them in any chain, merge, or refinement does not change any observable invariant of the Causal Universe Tensor. No observer can construct a sequence of measurements that forces an ordering between $e$ and $f$ without introducing new events.*

**Remark 6.** *Correlant events may, but do not necessarily, admit a strict causal precedence and therefore contribute to proper time along a chain; uncorrelants do not. In particular, a chain that includes $e$ but not $f$ may be maximally refined without reference to $f$. Thus, uncorrelants represent* informational independence*, not simultaneity.*

**Remark 7.** *In spacetime language, uncorrelants are precisely those event pairs that are spacelike–separated: reordering them changes no measurable scalar. Here, this is not assumed from geometry; it is a consequence of incomparability in the event order.*

**Definition 10** (Causal Network)**.** *Let $E$ be a finite set of admissible events and let $\triangleright$ denote the* immediate causal cover*: $e \triangleright f$ if and only if $e < f$ and there exists no $g \in E$ such that $e < g < f$. The* causal network *is the directed graph $(E, \triangleright)$ whose vertices are the events in $E$ and whose directed edges record the immediate causal relations.*

This network is the combinatorial diagram of the event record: each vertex is a distinguishable event, and each directed edge $e \triangleright f$ certifies that $f$ cannot be observed without first observing $e$. Its transitive closure recovers the full causal order $<$ of Definition 11.

**Definition 11** (Causal Order [12])**.** *Let $P = \{P_n\}_{n\in\mathbb{Z}}$ be a distinguishability chain of partitions, and let an event be $e = (B, \{B_i\}_{i\in I}, n)$ as in Definition 7, where $B \in \mathrm{Bl}(P_n)$ splits nontrivially into child blocks $\{B_i\} \subset \mathrm{Bl}(P_{n+1})$.*

*For $m > n$ and $C \in \mathrm{Bl}(P_m)$, let $\pi_{m\to n}(C) \in \mathrm{Bl}(P_n)$ denote the unique ancestor block in $P_n$ containing $C$ (well-defined because $P_{n+1}$ refines $P_n$). Define the* immediate causal cover *relation $e \triangleright f$ between events $e = (B, \{B_i\}, n)$ and $f = (C, \{C_j\}, m)$ by*

$$n < m \quad \text{and} \quad \pi_{m\to n+1}(C) \subseteq B_i \text{ for some child } B_i \text{ created by } e.$$

*The* causal order $\prec$ *on the event set $E$ is the transitive closure of $\triangleright$:*

$$e \prec f \iff \text{there exist events } e = e_0, e_1, \ldots, e_k = f \text{ with } e_i \triangleright e_{i+1} \text{ for all } i.$$

*Then $(E, \prec)$ is a locally finite partially ordered set (reflexivity suppressed for strictness), where incomparability is allowed: it may happen that neither $e \prec f$ nor $f \prec e$.*

As an illustration, recall the twin paradox of the previous chapter[3]. In the informational gauge, proper time is not a geometric interval but the work of reconciling distinguishable events. The traveling twin accrues a denser log of refinements—engine burns, course corrections, telemetry—while the stay-at-home twin records a coarser sequence. When their notebooks are merged into a single coherent history, the richer record requires strictly greater informational effort to reconcile. Equivalently, the proper time of the unaccelerated twin is necessarily longer, because her history contains fewer distinctions and therefore a larger merge is required to absorb those recorded by her sibling. In the smooth limit this appears as a shorter proper time along the curved worldline, but the effect is not mysterious: it is the discrete fact that one history contains more recorded distinctions than the other. Geometry only codifies what measurement already certified.

[3]See Coda: The Twin Paradox, Chapter 1.

**Remark 8** (Index is an order–embedding, not an equivalence)**.** *If* $e \prec f$*, then* $n_e < n_f$*. Thus the refinement index provides a rank function (Definition 4) that is* monotone *with respect to* $\prec$*. The converse need not hold:* $n_e < n_f$ *does not imply* $e \prec f$*. Hence the causal order is generally* partial*, not total.*

**Remark 9** (Uncorrelants and permutation invariance)**.** *Events* $e, f$ *with neither* $e \prec f$ *nor* $f \prec e$ *are* uncorrelant *(incomparable). Permuting uncorrelant events in any linear extension of* $(E, \prec)$ *leaves all scalar invariants of the Causal Universe Tensor unchanged; causal histories are unique only up to permutation of uncorrelants.*

### 2.4.4 Accumulation of Measurement

Operationally, every observation can be decomposed into three layers:

1. the **logical** layer—which events are distinguishable;
2. the **mathematical** layer—how those distinctions are counted;
3. the **physical** layer—how the resulting counts are named and parameterized as energy, momentum, or time.

By isolating the first two layers, we obtain a calculus of variations that is universal to any admissible physics: a closed system of relations that expresses how order itself becomes measurable.

The framework that follows formalizes this intuition. Within ZFC, we construct an ordered set of events whose distinguishability relations generate the causal ordering of special relativity. Measurements are counts of these relations, and the Causal Universe Tensor—the cumulative left product of *event tensors* over all causal increments—supplies the discrete substrate from which the smooth laws of physics arise in the limit of refinement.

The preceding axioms establish a measurement record as a locally finite, partially ordered set of distinctions $(E, \prec)$. This structure is purely combinatorial. To connect the logical record to physical measurement, we require a representation that can carry numerical values and allow recorded distinctions to combine. A vector space $V$ provides a domain for measurable quantities, but to represent successive distinctions we also need a rule for composition. The tensor algebra $\mathcal{T}(V)$ is the freest algebra generated by $V$: it contains $V$, supports noncommutative products, and imposes no additional relations beyond those required by linearity. By associating each logical event $e_k$ with a tensor $\mathbf{E}_k$ in $\mathcal{T}(V)$ we obtain an algebraic record of distinctions that can be composed and accumulated. This representation introduces no structure beyond what is logically required to encode measurable updates.

**Definition 12** (Event Tensor [46])**.** *Let $V$ be a finite-dimensional real vector space of measurable quantities. An event tensor $\mathbf{E}_k \in \mathcal{T}(V)$ encodes the distinguishable contribution of the kth event $e_k \in E$ to the cumulative record. It is related to the logical event by a measurable embedding*

$$\Psi : E \to \mathcal{T}(V), \qquad \mathbf{E}_k = \Psi(e_k). \tag{2.12}$$

*No algebraic relations are assumed beyond those required by linearity: $\mathbf{E}_k$ is simply the algebraic image of the kth logical distinction.*

An individual event tensor records a single admissible refinement of the measurement record. To represent the cumulative effect of many events, we must specify how these algebraic objects combine. Because the causal set is ordered only up to informational precedence, the combination rule must respect a chosen linear extension of the partial order and must make no assumptions of commutativity. This leads naturally to a left–multiplicative update: each new event contracts the admissible record of all that precede it, and the cumulative history is represented by the product of these restricted increments along any finite prefix of the causal chain.

The combination rule corresponds directly to the set–theoretic refinement of admissible outcomes. At each step, the new logical event is not taken in isolation, but restricted against all prior observations:

$$e'_{k+1} := e_{k+1} \cap \bigcap_{j=1}^{k} \hat{R}(e_j),$$

where $\hat{R}$ is the operator that removes outcomes incompatible with the existing record. In this framework, the "laws of physics" appear nowhere else: they are encoded entirely in the restriction operator. What survives admissibility is physical; what is removed was never a possible history.

In the algebraic domain this restriction is represented by

$$\mathbf{U}_{k+1} := \Psi(e'_{k+1})\,\mathbf{U}_k = \Psi\big(e_{k+1} \cap \hat{R}(e_k)\big)\,\mathbf{U}_k,$$

where $\Psi$ embeds the surviving distinctions into the tensor algebra. Each new event therefore contracts the admissible history by left multiplication. The cumulative record is the product of these restricted increments along any finite prefix of the causal chain.

Formally, the measurable embedding $\Psi$ sends the set–theoretic restriction to a multiplicative update in the tensor algebra. Instead of embedding the raw event $e_{k+1}$, we embed only the portion that survives all prior admissibility constraints:

$$\mathbf{E}_{k+1} = \Psi\Big(e_{k+1} \cap \bigcap_{j=1}^{k} \hat{R}(e_j)\Big).$$

Writing $\mathbf{R}(e) := \Psi(\hat{R}(e))$, the cumulative record evolves by left multiplication:

$$\mathbf{U}_{k+1} = \mathbf{R}(e_{k+1})\,\mathbf{U}_k = \Psi\big(e_{k+1} \cap \hat{R}(e_k)\big)\,\mathbf{U}_k, \qquad 0 \le k < n.$$

Thus the tensor update is the algebraic realization of the same logical operation performed in $E$: a new event is applied only after its outcomes have been restricted by all earlier observations. The universe accumulates consistency through products of restricted increments, not by additive evolution.

**Definition 13** (Partition of the Event Set)**.** *Let $(E, \prec)$ be a locally finite partially ordered set of distinguishable events. A* partition *of $E$ is a collection of disjoint subsets $\{E_\alpha\}_{\alpha \in A}$ such that*

$$E = \bigcup_{\alpha \in A} E_\alpha, \qquad E_\alpha \cap E_\beta = \varnothing \quad \text{for } \alpha \neq \beta.$$

*Each $E_\alpha$ is an informationally independent component: no event in $E_\alpha$ refines or is refined by an event in $E_\beta$. Correlant events therefore lie within the same partition element, while uncorrelants lie in distinct elements of the partition.*

**Definition 14** (Restriction Operator)**.** *Let $(E, \prec)$ be a partially ordered set of events, and let $e \in E$ be a newly recorded event. The* restriction operator

$$\hat{R}(e) : E \to E$$

*acts on the event record by removing any outcomes that are incompatible with $e$. For $f \in E$,*

$$\hat{R}(e)(f) = \begin{cases} f, & \text{if } f \text{ is admissible given } e, \\ \varnothing, & \text{otherwise.} \end{cases}$$

*Equivalently, if $E_\alpha$ is the partition element containing $e$, then*

$$\hat{R}(e) \;:\; E_\alpha \mapsto E'_\alpha, \qquad E'_\alpha = \{\, f \in E_\alpha \mid f \text{ is compatible with } e \,\}.$$

*Thus $\hat{R}(e)$ contracts the event domain by discarding outcomes that contradict the new distinction.*

We now present the *Causal Universe Tensor.*

**Proposition 1** (The Existence of a Causal Universe Tensor)**.** *Let* $(E, \prec)$ *be a locally finite partially ordered set of events, and let* $\Psi : E \to \mathcal{T}(V)$ *be the measurable embedding. For each event* $e \in E$*, define its admissible factor by*

$$\mathbf{F}(e) \;:=\; \Psi\big(\hat{R}(e)\big).$$

*Fix a finite linear extension* $e_1 \prec \cdots \prec e_n$ *of* $(E, \prec)$ *and set* $\mathbf{U}_0 := \mathbf{I}$ *(the multiplicative identity in* $\mathcal{T}(V)$*). Define the left recursion*

$$\mathbf{U}_{k+1} := \mathbf{E}_{k+1}\,\mathbf{U}_k = \Psi\big(e_{k+1} \cap \hat{R}(e_k)\big)\,\mathbf{U}_k, \qquad 0 \le k < n, \tag{2.13}$$

*Then:*

1. (Naturality of restriction) *The update* (2.13) *is precisely the expansion of the discrete restriction through the representation:* $\mathbf{F}(e) = \Psi(\hat{R}(e))$ *so that*

$$\mathbf{U}_{k+1} \;=\; \Psi\big(\hat{R}(e_{k+1})\big)\,\mathbf{U}_k.$$

   *Equivalently,* $R \circ \Psi = \Psi \circ \hat{R}$ *on* $\operatorname{im}\Psi$*.*

2. (Causal uniqueness) *The recursion* (2.13) *is uniquely determined by the chosen linear extension. Any two linear extensions differ only by permutations of informationally independent events (partition elements of* $E$*), so once the order is fixed the product is mechanically well-defined.*

3. (Independence under commuting factors) *If a subset* $S \subset \{1, \dots, n\}$ *indexes events whose admissible factors pairwise commute,* $\mathbf{F}(e_i)\mathbf{F}(e_j) = \mathbf{F}(e_j)\mathbf{F}(e_i)$ *for* $i, j \in S$*, then any permutation of* $\{\mathbf{F}(e_i)\}_{i \in S}$ *leaves* $\mathbf{U}_n$ *invariant under all cyclic scalar functionals (e.g., traces of contractions).*

*4.* (Fully commutative case) *If all admissible factors commute, then*

$$\mathbf{U}_n = \prod_{k=1}^{n} \mathbf{F}(e_k)$$

*is independent of the linear extension; the product reduces to the order-insensitive accumulation of factors.*

*Proof (Sketch).* (1) By definition $\mathbf{F}(e) = \Psi(\hat{R}(e))$, so the left update is the representation of the discrete restriction; this is $R \circ \Psi = \Psi \circ \hat{R}$ on $\operatorname{im} \Psi$. (2) Associativity of multiplication in $\mathcal{T}(V)$ gives well-definedness for a fixed order; linear extensions differ only by swapping independent events. (3) and (4) follow from standard properties of products with commuting factors and invariance of cyclic scalar functionals. □

*A full proof is provided in Appendix* **??**.

**Remark 10** (Ordinal determinacy)**.** *The sequence* $(\mathbf{U}_k)$ *is not merely algebraically well-defined but* physically ordered*: $k$ indexes ordinal rank, not arbitrary enumeration. Hence any reordering outside uncorrelant classes violates Axiom 5.*

With the ordinal structure of events established, we now formalize how these measurements combine algebraically within a finite vector space.

### 2.4.5 Formal Structure of Event and Universe Tensors

We now specify the algebraic structure of the quantities introduced above. Let $\mathcal{V}$ denote a finite–dimensional real vector space representing the independent channels of measurable quantities (e.g. energy, momentum, charge). Define the tensor algebra [48, 69]

$$\mathcal{T}(\mathcal{V}) = \bigoplus_{r=0}^{\infty} \mathcal{V}^{\otimes r}, \tag{2.14}$$

whose elements are finite sums of $r$–fold tensor products over $\mathbb{R}$. Each *event tensor* $E_k$ is a member of $\mathcal{T}(\mathcal{V})$ encoding the distinguishable contribution of the $k$–th event to the global state. We write

$$\mathbf{E}_k \in \mathcal{T}(\mathcal{V}), \qquad \mathbf{U}_n = \prod_{k=1}^{n} \mathbf{E}_k \in \mathcal{T}(\mathcal{V})). \tag{2.15}$$

Addition is understood componentwise in the direct sum and preserves the ordering of indices guaranteed by the Axiom of Order [12, 48]. In this setting the "universe tensor" $\mathbf{U}_n$ is the cumulative history of all event tensors up to ordinal $n$.

**Definition 15** (Tensor Algebra [46])**.** *The tensor algebra on a vector space $\mathcal{V}$ is*

$$\mathcal{T}(\mathcal{V}) = \bigoplus_{r=0}^{\infty} \mathcal{V}^{\otimes r}$$

*with componentwise addition and associative tensor product*

**Remark 11.** *Each logical event $e_k$ in the partially ordered set $(E, \prec)$ induces a tensor $\mathbf{E}_k = \Psi(e_k)$ in $\mathcal{T}(\mathcal{V})$. The mapping $\Psi$ translates causal structure into algebraic contribution, ensuring that causal precedence corresponds to index ordering in $\mathbf{U}_n$.*

Because $\mathcal{T}(\mathcal{V})$ is a free associative algebra, all operations on $\mathbf{U}_n$ are well defined using the standard linear maps, contractions, and bilinear forms of $\mathcal{V}$. The subsequent analysis of variation and measurement therefore proceeds entirely within conventional linear–operator theory.

From the definition of the Universe Tensor

$$U_n = \prod_{k=1}^{n} E_k, \tag{2.16}$$

we may regard an *uncorrelant* as any subset of events whose local order can be permuted without altering the global scalar invariants of $U_n$. Formally, a subset $S \subseteq \{E_1, \ldots, E_n\}$ is uncorrelant if, for every permutation $\pi$ of $S$,

$$\prod_{E_i \in S} E_i = \prod_{E_i \in S} E_{\pi(i)}. \tag{2.17}$$

In this case, all contractions or scalar traces derived from $U_n$ remain unchanged by reordering the elements of $S$, even though the operator sequence itself may differ.

**Definition 16** (Commutator and Commutator Ideal [31])**.** *Let $\mathcal{A}$ be an algebra over a field $\mathbb{F}$ with bilinear multiplication $(x, y) \mapsto xy$. For $x, y \in \mathcal{A}$, the* commutator *of $x$ and $y$ is the element*

$$[x, y] \;:=\; xy - yx \;\in\; \mathcal{A}.$$

*The set of all finite $\mathbb{F}$-linear combinations of commutators,*

$$[\mathcal{A}, \mathcal{A}] \;:=\; \left\{ \sum_{i=1}^{m} \alpha_i [x_i, y_i] : \alpha_i \in \mathbb{F},\; x_i, y_i \in \mathcal{A} \right\},$$

*is called the* commutator ideal*. It is the smallest two-sided ideal of $\mathcal{A}$ that contains every element $xy-yx$; equivalently, it is the smallest linear subspace of $\mathcal{A}$ closed under left and right multiplication by arbitrary elements of $\mathcal{A}$.*

**Remark 12** (Algebraic Characterization of Informational Independence)**.** *Let $\Psi : E \to \mathcal{T}(V)$ be the event embedding and $\mathbf{E}_e := \Psi(e)$. If $S \subseteq E$ lies in distinct elements of the partition of $E$ (Definition 13), then the admissible increments $\{\mathbf{E}_e\}_{e \in S}$ pairwise commute. Consequently, any reordering of these factors within a linear extension of $(E, \prec)$ produces the same value of $\mathbf{U}_n$ under all cyclic scalar functionals (e.g., traces of contractions). In this algebraic sense, informational independence corresponds exactly to order-insensitive contribution to the invariants derived from $\mathbf{U}$.*

**Thought Experiment 2.4.5** (Non-commutative Event Pair [52])**.** ***N.B.***— *Non-commutative event tensors often signal a* dependency*: one update must* *precede the other for the restricted outcome set to remain consistent. Reversing such events changes the operator state, even though measurable scalar invariants remain the same.* □

*Let* $V = \mathbb{R}^2$ *and let event tensors act as* $2 \times 2$ *matrices under the usual (non-commutative) multiplication. Define*

$$E_A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad E_B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

*A direct computation gives*

$$E_A E_B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = E_B E_A, \quad \textit{so } [E_A, E_B] \neq 0.$$

*Thus, applying the updates in different orders leads to different operator states. However, cyclic scalar invariants agree:*

$$\operatorname{tr}(E_A E_B) = \operatorname{tr}(E_B E_A) = 3, \qquad \det(E_A E_B) = \det(E_A)\det(E_B) = 1.$$

*In this sense, noncommutativity affects the internal operator record but not the measurable quantities obtained by cyclic scalar functionals.*

**Thought Experiment 2.4.6** (Independent Event Chains [70])**.** ***N.B.***— *This is analogous to the inertial segment of the twin paradox. During coasting, neither twin exchanges signals with the other, so no event on one worldline refines or restricts events on the other. The two chains are informationally independent until a causal interaction occurs.* □

*Consider two finite event chains*

$$A_1 \prec A_2, \qquad B_1 \prec B_2,$$

*with no causal relation between any* $A_i$ *and any* $B_j$*. Let their event tensors* *act on $V = \mathbb{R}^2$ as*

$$E_{A1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad E_{A2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad E_{B1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad E_{B2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

*Because the $A$-events refine only the $A$-chain and the $B$-events refine only the $B$-chain, their admissible factors commute:*

$$E_{A2}E_{B2} = E_{B2}E_{A2}.$$

*Thus, any linear extension of the partial order may place the $A$- and $B$-events in either interleaving without changing cyclic scalar invariants. For example, applying the four events in the order*

$$A_1,\, A_2,\, B_1,\, B_2 \quad \textit{or} \quad A_1,\, B_1,\, A_2,\, B_2$$

*gives operator states that differ, but*

$$\operatorname{tr}(E_{A2}E_{B2}) = \operatorname{tr}(E_{B2}E_{A2}) = 1, \qquad \det(E_{A2}E_{B2}) = \det(E_{B2}E_{A2}) = 0.$$

*This illustrates the algebraic meaning of independence: when two event chains are partitioned into disjoint informational domains, their admissible increments commute. Order affects the internal operator record but leaves measurable cyclic scalars unchanged, exactly as in the coasting phase of the twin paradox.*

## 2.5 Information Minimality and Kolmogorov Closure

The previous definitions describe events as finite distinctions and their ordering as a partial refinement of information. What remains is the rule that determines which extensions of a recorded event set are admissible. Not every history consistent with the order is physically meaningful: a completion that inserts unobserved structure would imply additional measurements that never occurred. Information minimality formalizes this constraint through algorithmic information theory in the sense of Kolmogorov, Solomonoff, and Chaitin [18, 61, 102, 103].

We treat histories as finite symbolic strings and measure their descriptive content by Kolmogorov complexity. A physically admissible history is one that cannot be compressed by adding unrecorded structure.

**Definition 17** (Kolmogorov Complexity [61, 18])**.** *Fix a universal Turing machine $U$ [110]. For any finite string $w \in \Sigma^*$, the Kolmogorov complexity $K(w)$ is the length of the shortest input to $U$ that outputs $w$ and halts. The functional $K : \Sigma^* \to \mathbb{N}$ is defined up to an additive constant independent of $w$.*

**Definition 18** (Admissible Extension [72])**.** *Let $E = \{e_0 \prec e_1 \prec \cdots \prec e_n\}$ be the recorded events of an experiment. A finite string $w \in \Sigma^*$ is an* extension *of $E$ if its image under the event map contains $E$ in the same causal order. An extension $w$ is* admissible *if it introduces no additional events beyond $E$; that is, every distinguishable update encoded by $w$ has a corresponding element of $E$. Any extension predicting unobserved structure is rejected as inadmissible.*

**Thought Experiment 2.5.1** (Paradoxes of Time Travel [71])**. *N.B.*—** *Apparent paradoxes often attributed to* time travel*,* remote viewing*, or other extraordinary mechanisms are pathologies of over–resolution. They arise when incompatible refinements are treated as simultaneously admissible, producing the illusion of phenomenal violation rather than an actual failure of causal order.* □

*Let $E = \{e_1, e_2, e_3, \dots\}$ be a locally finite causal chain where each event*

*$e_i$ has a unique successor $e_{i+1}$. Define the corresponding universe tensor*

$$\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k, \qquad \mathbf{E}_k = \mathbf{\Psi}_k(e_k). \tag{2.18}$$

*Now suppose we attempt to "extend" this history by splitting a single event $e_j$ into uncountably many indistinguishable refinements:*

$$e_j \longrightarrow \{e_{j,\alpha}\}_{\alpha\in[0,1]}, \tag{2.19}$$

*each representing a formally distinct but observationally identical outcome. Algebraically, this replacement yields*

$$\mathbf{E}_j \longrightarrow \int_0^1 \mathbf{E}_{j,\alpha}\, d\alpha, \tag{2.20}$$

*so that the next update becomes*

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \int_0^1 \mathbf{E}_{j,\alpha}\, d\alpha. \tag{2.21}$$

*This "extension" violates the finiteness and distinguishability conditions necessary for causal coherence:*

1. *The set $\{e_{j,\alpha}\}$ is uncountable, destroying local finiteness;*
2. *The new events are indistinguishable, so Extensionality no longer guarantees unique contributions;*
3. *The total tensor amplitude $U_{n+1}$ can diverge or cancel arbitrarily, depending on how the continuum of duplicates is treated.*

*Operationally, this is a Banach–Tarski-like overcounting: the causal structure has been "refined" in a way that preserves measure only formally while the order relation collapses. The observer would now predict contradictory outcomes for the same antecedent state—an* overcomplete history*.*

*To prevent this, the* Axiom of Event Selection *restricts the permissible extension to a countable, consistent refinement:*

$$e_j \longrightarrow e_{j,1}, e_{j,2}, \ldots, e_{j,k}, \tag{2.22}$$

*and requires the selection of exactly one representative outcome from each locally admissible family. This keeps $E$ locally finite and maintains a single-valued universe tensor,*

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \mathbf{E}_{j,k^*}. \tag{2.23}$$

*The axiom thus enforces the same regularity that Martin's Axiom guarantees in set theory: every countable family of local choices admits a globally consistent selection that preserves the partial order.*

**Definition 19** (Information Minimality [61, 72])**.** *Among all admissible extensions of $E$, the physically admissible history is the one of minimal Kolmogorov complexity:*

$$w_{min} = \arg\min\{K(w) : w \textit{ is an admissible extension of } E\}.$$

Information minimality expresses the logical content of measurement: if additional curvature, oscillation, turning points, or discontinuities had occurred between $e_i$ and $e_{i+1}$, those features would have generated new events. Since no such events are present in $E$, any extension that predicts them is inadmissible, and a shorter description exists.

**Remark 13.** *This principle is purely set–theoretic. No geometry, metric, or differential structure is assumed. Kolmogorov minimality selects the shortest admissible description of the recorded distinctions and forbids unobserved structure.*

**Remark 14.** *As the resolution of measurement increases, the admissible extension forms a Cauchy sequence [17] in the space of symbolic descriptions.*

*In the dense limit, its smooth shadow is the unique spline that introduces no new structure between recorded events. Thus the variational calculus is not imposed; it is the continuum limit of Kolmogorov minimality.*

### 2.5.1 Inadmissibility of Unobserved Structure

Let $E = \{e_0 \prec e_1 \prec \cdots \prec e_n\}$ be the finite set of recorded events produced by a measurement process. By Definition 5, each event corresponds to a distinguishable update of state: a change that crossed a detection threshold and became causally recorded.

Between two successive events $e_i$ and $e_{i+1}$, no additional events were recorded. This absence is a data constraint: any refinement of the history that introduces detectable structure—curvature, oscillation, turning points, discontinuities, or other distinguishable phenomena—would generate additional events. Since these events do not appear in $E$, any history that predicts them is logically inconsistent with the observational record.

**Definition 20** (Unobserved Structure)**.** *Let $w$ be an admissible extension of $E$ (Definition 2.3.3). A symbolic segment of $w$ between $e_i$ and $e_{i+1}$ contains* unobserved structure *if it encodes a distinguishable update that is not present in $E$.*

## 2.6 Correlation and Dependency

In conventional quantum mechanics the word "entanglement" refers to a non-classical dependency among amplitudes: indistinguishable histories are combined before probabilities are assigned. The present framework adopts a similar intuition, but in a purely informational and algebraic form, with no amplitudes and no functional dependencies.

Two events are *uncorrelant* when no *correlant* exists between them. In this case, their transposition commutes with every admissible invariant of the Universe Tensor, and the events may be represented independently. Uncorrelant events are informationally separable: no refinement of the record forces them to be treated jointly.

Two events are *correlant* when they do not commute: exchanging them changes at least one admissible invariant. In this case a correlant exists. A correlant is an informational relation—the minimal structure required when two events cannot be represented independently of one another. Importantly, a correlant does not specify direction or causation: nothing is said about which event precedes, influences, or determines the other. It expresses only that the transposition fails to commute.

Uncorrelant events can become correlated when their light cones merge. Before the merger, each event admits a representation that commutes with the other; no correlant exists, and their histories may be transposed without altering any admissible invariant. After the merger, additional distinctions become available, and the transposition may fail to commute. A correlant then forms, not because one event generates the other, but because the enlarged record no longer permits them to be represented independently.

Dependency relations are stronger still. A dependency asserts that one event is determined by another, as in the functional relationships of the classical calculus. Such relations describe macro–events in conventional dynamics, where causes generate effects. The present work is not concerned with dependency. Correlation is the weaker structure: non-commutativity under admissible permutation, with no claim of generation or determination.

Thus, "entanglement" in the conventional quantum sense has two informational analogues in this framework. When amplitudes combine as indistinguishable histories, the result is a superposition. When events cannot be transposed without altering admissible invariants, the result is a correlant. Both are consequences of the same principle: distinctions cannot be manufactured retroactively. What differs is the level at which indistinguishability occurs—the discrete record of events or the smooth representation of extremals.

**Thought Experiment 2.6.1** (Spooky Action at a Distance [5, 34, 106])**.** *Consider an uncorrelant* $S = \{\mathbf{E}_i, \mathbf{E}_j\}$ *of two spatially separated measurement events. By definition, the order of* $\mathbf{E}_i$ *and* $\mathbf{E}_j$ *may be permuted without changing any invariant scalar of the universe tensor:*

$$\mathbf{E}_i\mathbf{E}_j = \mathbf{E}_j\mathbf{E}_i. \tag{2.24}$$

*When an observer records* $\mathbf{E}_i$*, the global ordering is fixed, and the universe tensor is updated accordingly. Because* $\mathbf{E}_j$ *belongs to the same uncorrelant set, its contribution is now determined consistently with* $\mathbf{E}_i$*, even if* $E_j$ *occurs at a spacelike separation. This manifests as the phenomenon of "spooky action at a distance"—the appearance of instantaneous correlation due to reassociation within the uncorrelant subset.*

**Thought Experiment 2.6.2** (Hawking Radiation [51, 111])**.** *Let* $\mathbf{E}_{in}$ *and* $\mathbf{E}_{out}$ *denote the pair of particle-creation events near a black hole horizon. These events form an uncorrelant set:*

$$S = \{\mathbf{E}_{in}, \mathbf{E}_{out}\}. \tag{2.25}$$

*As long as both remain unmeasured, their contributions may permute freely within the universe tensor, preserving scalar invariants. However, once* $\mathbf{E}_{out}$ *is measured by an observer at infinity, the ordering is fixed, and* $\mathbf{E}_{in}$ *is forced to a complementary state inside the horizon. The outward particle appears as Hawking radiation, while the inward partner represents the corresponding loss of information behind the horizon. Thus Hawking radiation is naturally expressed as an uncorrelant whose collapse into correlation occurs asymmetrically across a causal boundary.*

## Coda: Achilles and the Tortoise

**N.B.**—For a rich treatment of this paradox, see Hofstadter [56]. □

Zeno's paradox of Achilles and the tortoise [90] is one of the oldest arguments against the possibility of motion. Achilles, swift of foot, gives a tortoise a small head start. Because the tortoise begins ahead, Achilles must first reach the tortoise's initial position. By that time, the tortoise has advanced a little farther; Achilles must then reach that new position, and by the time he arrives, the tortoise has advanced again, and so on without end. Zeno's conclusion is that Achilles can never overtake the tortoise, for he must complete an infinite sequence of tasks to do so.

Formally, one can express the argument in familiar modern notation. Suppose the tortoise begins one unit ahead. Achilles covers half the remaining distance on his first stride, then half of what remains on the next stride, then half again, producing the well-known geometric series

$$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots .$$

More generally, one may express the same identity as

$$1 = \sum_{n=1}^{\infty} \frac{1}{2^n}.$$

Zeno's reasoning is now captured in a single line: if Achilles must perform an infinite number of sub-journeys to reach the tortoise, and if completing infinitely many tasks requires infinite time, then Achilles never arrives.

The mathematics appears to sharpen the paradox. The right-hand side contains infinitely many terms, and yet their sum is finite. An infinite decomposition and a finite limit uneasily coexist. From a purely symbolic viewpoint, Zeno is correct: the path to the finish line can be written as a countable infinity of smaller and smaller segments. Nothing in the algebra forbids infinitely many subdivisions of the interval.

The difficulty lies not in the mathematics, but in the hidden assumption that every subdivision corresponds to a physically meaningful event. Zeno imagines that the runner physically performs each of these infinitesimal sub-paths, as though each term in the series corresponds to an actual step. In reality, the decomposition exists only on paper. It is an artifact of representation, not an element of the physical world.

In the information gauge, motion is not defined by a continuous geometric parameter, but by the accumulation of admissible distinctions—measurable, irreversible updates of state. A notebook of observations does not record symbolic halvings of distance; it records physical events that are detectable by an instrument. Proper time is not the integral of infinitesimal steps, but the count of such admissible distinctions.

Viewed in this light, the identity

$$1 = \sum_{n=1}^{\infty} \frac{1}{2^n}$$

does not imply that Achilles performs infinitely many physical actions. It states only that a continuous model permits infinitely many subdivisions, should one choose to write them down. The infinite chain is a mathematical convenience, not a physical ledger.

The resolution is found in precision. Achilles does not detect every possible subinterval of his path; no instrument possesses infinite resolving power. His step length, his stride cadence, and the sensor that records his position determine a finite resolution. If the act of stepping advances him by $10^{-2}$ units, there are at most 100 admissible distinctions in a one-unit race. Even if the instrumentation resolves position to $10^{-6}$ units, the notebook contains no more than one million recorded distinctions. Once this finite notebook is reconciled, Achilles is at the finish line. The race consumes a finite count of admissible distinctions because the physical process does not instantiate an actual infinity of subevents.

Zeno's paradox relies on treating every symbolic refinement of the interval as physically real. The information gauge rejects that assumption. A measurement records only what can be stably distinguished. Achilles's "infinite" steps are not steps at all; they are possible refinements of a mathematical model. Precision is the gatekeeper. The paradox dissolves when we recall that Achilles's motion is measured, not imagined, and that every measurement has finite resolution. Refinement does not create motion; it reveals it.

# Chapter 3

# The Calculus of Dynamics

In the previous chapter, motion was described entirely as a sequence of admissible distinctions—a finite notebook of observable updates. No geometry, metric, or continuum was assumed. Refinement revealed additional events, but the history of any physical process remained a countable record that could be reconciled into a globally coherent ledger.

This chapter introduces dynamics in the same spirit. By "dynamics" we do not mean a force law or a geometric trajectory. We mean the rule that selects, from all admissible histories, those that are physically possible. The key observation is that a physical history cannot contain unexplained motion. Any segment of a worldline must be consistent with the measurements that precede and follow it. When a history can be refined without altering its predictions at the recorded events, the refined history contains no additional information. In this sense, the physically admissible refinement is the one that introduces no new distinctions beyond those required by the data.

This principle has a classical name. In the continuum limit, the requirement that refinements add no "hidden motion" is precisely the Euler–Lagrange condition: an admissible trajectory introduces no superfluous curvature beyond that certified by observed events [19, 21, 67]. A trajectory of least informational content is a trajectory of least action, in the classical sense of Maupertuis, Euler, Lagrange, Hamilton, and their modern successors [28, 35, 45, 50, 66]. In the calculus of dynamics, smooth solutions arise not from geometry but from the demand that no further admissible distinctions can be discovered between measurements. The spline that leaves nothing to correct is the one nature selects.

The remainder of this chapter develops this idea formally. Starting from a finite set of measurements, we construct the weak form of the problem and show that the unique refinement consistent with all observed distinctions is the cubic spline. Its extremality in the continuum reproduces the Euler–Lagrange equations familiar from classical mechanics and field theory. Dynamics are not imposed at the outset; they emerge as the limit in which refinement ceases to yield new information.

**Thought Experiment 3.0.1** (Minimizing Variations [21])**.** ***N.B.***—*For a comprehensive treatment of the calculus of variations, see Brenner and Scott [15] and Courant and Hilbert [21].* □

*We consider the functional*

$$J[x] = \int_a^b f\big(t, x(t), \dot{x}(t)\big)\, dt,$$

*where $x$ is a twice continuously differentiable function with fixed endpoints $x(a) = x_a$ and $x(b) = x_b$. Let $\eta(t)$ be an admissible perturbation with $\eta(a) = \eta(b) = 0$, and define the variation*

$$x_\varepsilon(t) = x(t) + \varepsilon\, \eta(t), \qquad \varepsilon \in \mathbb{R}.$$

*The directional derivative of $J$ at $x$ in the direction $\eta$ is*

$$\delta J[x;\eta] = \frac{d}{d\varepsilon} J[x_\varepsilon]\bigg|_{\varepsilon=0} = \frac{d}{d\varepsilon} \int_a^b f\big(t, x_\varepsilon(t), \dot{x}_\varepsilon(t)\big)\, dt\bigg|_{\varepsilon=0}.$$

*Since the integration limits do not depend on $\varepsilon$, the derivative may be moved* *inside:*

$$\delta J[x;\eta] = \int_a^b \frac{\partial}{\partial \varepsilon} f\big(t, x_\varepsilon(t), \dot{x}_\varepsilon(t)\big)\bigg|_{\varepsilon=0} dt.$$

*By the chain rule,*

$$\frac{\partial}{\partial \varepsilon} f\big(t, x_\varepsilon(t), \dot{x}_\varepsilon(t)\big) = f_x(t, x(t), \dot{x}(t))\, \eta(t) + f_{\dot{x}}(t, x(t), \dot{x}(t))\, \dot{\eta}(t).$$

*Thus*

$$\delta J[x;\eta] = \int_a^b \Big( f_x(t, x, \dot{x})\, \eta(t) + f_{\dot{x}}(t, x, \dot{x})\, \dot{\eta}(t) \Big)\, dt.$$

*Integrate the second term by parts:*

$$\int_a^b f_{\dot{x}}\, \dot{\eta}\, dt = \big[f_{\dot{x}} \eta\big]_a^b - \int_a^b \frac{d}{dt}\big(f_{\dot{x}}\big)\, \eta(t)\, dt.$$

*Because* $\eta(a) = \eta(b) = 0$*, the boundary term vanishes. Therefore*

$$\delta J[x;\eta] = \int_a^b \left( f_x - \frac{d}{dt} f_{\dot{x}} \right) \eta(t)\, dt.$$

*If* $x$ *is a stationary point of* $J$*, then* $\delta J[x;\eta] = 0$ *for all admissible* $\eta$*. The fundamental lemma of the calculus of variations implies*

$$f_x(t, x, \dot{x}) - \frac{d}{dt} f_{\dot{x}}(t, x, \dot{x}) = 0,$$

*for all* $t \in (a, b)$*. This is the Euler–Lagrange equation, more commonly represented as*

$$\frac{\partial f}{\partial x} = \frac{d}{dt} \frac{\partial f}{\partial \dot{x}}. \tag{3.1}$$

*This derivation demonstrates that the Euler–Lagrange equation selects the trajectory with no first-order change under admissible perturbations. No hidden motion can be inserted without altering the notebook. The path is stationary in its informational curvature.*

## 3.1 Emergent Dynamics

In the discrete setting, the Causal Universe Tensor assigns a finite informational weight to every admissible history. Refinement increases this weight only when new distinctions are recorded. Any replacement of an admissible history by one containing additional, unobserved structure violates Axiom 7. Consequently, dynamics is not an independent physical postulate. It is the unique continuous shadow of informational extremality: the smooth curve is simply the history for which no further admissible distinctions can be revealed.

In the discrete domain, *anchor points* are the only places where the universe has committed to a specific value. Between anchors the record is silent: the data permit many possible continuations, but most would introduce unobserved structure. Any admissible configuration must therefore agree at the anchor points and remain free of additional distinguishable features in between. The role of the anchors is not geometric; it is informational. They fix the admissible boundary data against which all variations are tested. A candidate variation that disagrees at an anchor is rejected immediately, because it contradicts an established event. A variation that agrees at the anchors but inserts additional oscillation, curvature, or "hidden motion" is rejected by Axiom 7, because those features would have produced additional anchors that do not appear in the record.

**Definition 21** (Anchor Points)**.** *A finite set of* anchor points *is the collection of measured events at which admissible configurations must agree. Two candidate histories $\psi$ and $\phi$ are said to share the same anchors if they record identical distinguishable values at those events. Axiom 3 requires that any refinement of a history preserve agreement on the anchors: no admissible configuration may contradict an observed event.*

In the discrete setting, reciprocity arises from a simple counting fact. A refinement of $\psi$ by a test configuration $\phi$ is admissible only when the resulting history contains no additional distinguishable events. If $\phi$ were to introduce extra curvature, oscillation, or "hidden motion," the refinement would increase the causal count and violate Axiom 7. The reciprocity pairing $\psi^*\mathcal{L}\phi$ measures this change: it evaluates whether $\phi$ is informationally neutral relative to $\psi$.

Crucially, the dual $\psi^*$ is not a geometric adjoint; it is the reflection of $\psi$ in the informational algebra. It answers the question: *If $\psi$ is perturbed by $\phi$, does the universe record new distinguishable structure?* If the reciprocity pairing vanishes for all admissible $\phi$ that share the anchors, then $\psi$ is extremal. Any remaining variation would imply new recorded events, and therefore be inadmissible.

**Definition 22** (Reciprocity Map)**.** ***N.B.***—*In settings where a geometric or Hilbert space structure is present, the reciprocity map reduces to the familiar adjoint (or complex conjugate) with respect to the underlying inner product. Here it is defined purely informationally, without assuming any geometric primitives.* □

*Let $\psi$ be an admissible configuration and let $\phi$ be a test variation that agrees with $\psi$ at the anchor points. The* reciprocity map *is the linear evaluation*

$$\langle \psi, \phi \rangle_{\mathcal{L}} := \psi^* \, \mathcal{L} \, \phi,$$

*where $\mathcal{L}$ counts distinguishable causal increments and $\psi^*$ denotes the adjoint of $\psi$ with respect to this count. Two configurations are reciprocals if their pairing produces the same causal measure:*

$$\langle \psi, \phi \rangle_{\mathcal{L}} = \langle \phi, \psi \rangle_{\mathcal{L}}.$$

*The map $\psi \mapsto \psi^*$ is called the* reciprocity dual*. It encodes the informational response of $\psi$ to an infinitesimal variation $\phi$ without assuming any differential structure.*

In the continuum shadow, the reciprocity pairing becomes the usual weak inner product of variational calculus [15, 36]. Integration by parts moves the variation from $\psi$ onto the test functions, producing natural boundary terms determined by the anchors. The condition

$$\langle \psi, \phi \rangle_{\mathcal{L}} = \langle \phi, \psi \rangle_{\mathcal{L}}$$

is then the classical reciprocity of the Euler–Lagrange operator: the dynamics are self-adjoint under the informational measure. This equality holds not because symmetry is assumed, but because any antisymmetric contribution would encode unrecorded distinctions and be eliminated by Axiom 7.

### 3.1.1 Weak Formulation on Space–Time

Let $\psi$ be an admissible configuration consistent with a fixed set of event anchors, and let $\phi$ be any test configuration that agrees with $\psi$ at those anchors. Replacing $\psi$ by $\phi$ is permissible only if it does not reduce causal consistency. In the discrete algebra this means that $\psi$ introduces no superfluous refinements relative to $\phi$; any additional curvature, oscillation, or "hidden motion" would imply unrecorded events and thus be inadmissible.

In the dense limit of refinement, this constraint appears as a weak relation

$$\psi^* \mathcal{L} \psi \ \leq \ \psi^* \mathcal{L} \phi, \tag{3.2}$$

where $\mathcal{L}$ is the informational count of distinguishable increments, and $\psi^*$ denotes its reciprocity dual. The weak inequality asserts that $\psi$ is extremal among all admissible perturbations $\phi$. No differential operators are assumed: the weak form arises because refinement limits the class of permissible discrete variations.

Completing this refinement yields the continuous counterpart of (3.2). Integration by parts shifts variations from $\psi$ onto the test functions, producing natural boundary conditions and a weak Euler–Lagrange statement. The continuum calculus therefore does not describe an independently assumed physical law; it is the smooth completion of informational minimality on the discrete domain.

### 3.1.2 Reciprocity and the Adjoint Map

The weak extremality relation (3.2) compares an admissible configuration $\psi$ against a test configuration $\phi$ that shares the same event anchors. In the discrete domain, replacing $\psi$ by $\phi$ means refining the event record: only those local changes that introduce new, distinguishable curvature would alter the admissible history. Any such change must correspond to additional recorded events; if none are present, the refinement is informationally neutral. Thus $\phi$ is an admissible variation of $\psi$ precisely when it agrees at the anchors and introduces no distinctions beyond those already encoded in $\psi$. The weak extremality condition (3.2) is the continuous shadow of this discrete refinement rule.

The weak comparison between $\psi$ and $\phi$ admits a natural dual representation. For any admissible configuration $\psi$, there exists a *reciprocity map* $\psi^*$ such that the informational pairing

$$\psi^* \mathcal{L} \phi \tag{3.3}$$

measures the change in distinguishability that would result from locally replacing $\psi$ by $\phi$ between the anchors. Intuitively, $\psi^*$ captures the "shadow" of $\psi$ when viewed from the perspective of informational minimality: components of $\phi$ that would introduce new, unrecorded distinctions are suppressed by the adjoint action, while components that are informationally neutral remain. In the dense limit, this pairing becomes the standard weak inner product of variational calculus.

Because admissible configurations cannot contain hidden structure, the reciprocity map annihilates variations that are invisible at the event anchors.

If $\phi$ and $\psi$ agree at the anchors and differ only by an undetectable perturbation, then refining the event record yields no new distinctions, and the informational pairing remains unchanged:

$$\psi^* \mathcal{L}\, \phi \;=\; \psi^* \mathcal{L}\, \psi.$$

This equality is precisely the weak relation (3.2). In this sense, $\psi^*$ enforces closure: the extremal configuration carries no latent curvature that would be revealed by further refinement.

The "variation" of $\psi$ is therefore not a differential operation but a refinement of the causal record consistent with the event anchors. The reciprocity map acts as the dual constraint, suppressing any component of that refinement which would introduce unrecorded distinctions. Taken together, admissible refinements and their reciprocity dual generate the weak Euler–Lagrange structure entirely within the discrete domain, without assuming differentiability or a continuum of states.

In this way, the reciprocity map ensures that any admissible refinement of $\psi$ corresponds to an interpolant $f(\psi)$ that introduces no new distinguishable structure. As refinement becomes dense, all such interpolants converge to the same smooth closure $\Psi$. Since the event record defines a finite labeled partition of the causal domain, $\Psi$ preserves anchor order and is injective on each partition element. Its inverse $\Psi^{-1}$ therefore recovers exactly the original discrete record:

$$f(\psi) \;\longrightarrow\; \Psi^{-1}. \tag{3.4}$$

Thus the interpolant and its smooth limit are informationally equivalent representations of the same causal structure.

### 3.1.3 Dense Limit and Euler–Lagrange Closure

In the present framework no differentiability is assumed. The weak extremality relation (3.2) is defined entirely in the discrete domain, where each term counts distinguishable causal increments. A "variation" of $\psi$ is therefore not a differential operator but a refinement of the event record that leaves the anchors unchanged.

In the discrete domain, such refinements appear as finite differences: each admissible update replaces a segment of the causal history by one with strictly greater resolution. Because informational minimality forbids unobserved curvature, every admissible refinement corresponds to a piecewise-linear or piecewise-polynomial interpolant that agrees with $\psi$ on the anchors and introduces no new distinguishable structure. As refinements become arbitrarily dense, the finite differences form a Cauchy sequence in the space of admissible interpolants, and their limit is the unique smooth closure $\Psi$ established in the previous subsection.

Applying the reciprocity pairing to successive refinements yields the discrete extremality condition: no admissible finite difference can reduce the informational measure $\mathcal{L}$. In the dense limit, the weak relation (3.2) becomes the standard variational identity of Euler–Lagrange calculus, obtained entirely from finite differences. The weak derivative enters only as the completion of refinement; it is not assumed *a priori*.

When the causal grid is refined, informational minimality forces cubic continuity at each event anchor: jumps in slope or curvature would constitute new observable events and are therefore inadmissible. In the dense limit, the discrete extremal coincides with the classical Euler–Lagrange closure. This structure is summarized in the following proposition.

**Proposition 2** (The Spline Condition of Information)**.** *Let $\psi$ be an admissible configuration with smooth closure $\Psi$. If no admissible refinement reduces the informational measure $\mathcal{L}$, then $\Psi$ is $C^2$ and satisfies*

$$\Psi^{(4)} = 0. \tag{3.5}$$

*Proof (Sketch).* Between anchors, $\Psi$ must be polynomial, since any additional inflection would imply unrecorded structure. Polynomials of degree greater than three contain latent turning points and are therefore excluded. Hence each segment is cubic. At the anchors, the interpolants must glue with $C^2$ continuity: jumps in slope or curvature would constitute new observable events. As the grid of anchors is refined, the third derivative $\Psi'''$ must be constant on every shrinking interval. In the dense limit that interval has zero measure, so $\Psi'''$ is constant everywhere. A constant third derivative implies $\Psi^{(4)} = 0$. Thus the smooth closure of any informationally extremal configuration satisfies the Euler–Lagrange condition. □

*A full proof is provided in Appendix* **??**.

Proposition 2 shows that the Euler–Lagrange equation is not postulated. It is the continuous shadow of discrete informational extremality. Finite differences do not approximate the differential equation; they *generate* it. The unique admissible smooth representative is cubic on each partition element, $C^2$ at the event anchors, and satisfies $\Psi^{(4)} = 0$ everywhere. Smooth calculus appears solely as the completion of refinement in the discrete causal record.

**Thought Experiment 3.1.1** (Repeatability of Invisible Motion [3])**.** *Consider two independent observers, $A$ and $B$, who record the motion of a particle between the same event anchors $x_i \prec x_{i+1}$. Each observer has finite resolution: any acceleration or inflection large enough to be distinguishable produces a new event. Both refine their instruments until no further events are detected on the interval.*

*If hidden curvature existed between the anchors, further refinement would create additional distinguishable records. The absence of such records forces each observer to recover the same polynomial of minimal degree. Thus both obtain a cubic patch on the interval.*

*Now let $A$ and $B$ exchange data and perform a joint refinement on a finer grid. Any disagreement in value, slope, or bending moment at a shared anchor would itself generate an observable event. To avoid contradiction,* *the cubic patches must glue together with continuous $U$, $U'$, and $U''$. In the dense refinement limit, the piecewise constant third derivative converges to a continuous function whose integral vanishes on every shrinking interval, yielding*

$$U^{(4)} = 0.$$

*Thus repeatability demands the Euler–Lagrange closure: if two observers can refine their measurements indefinitely without producing new events, their reconstructions must converge to the same cubic extremal. Smooth dynamics are therefore the unique histories that leave no trace.*

### 3.1.4 The Law of Spline Sufficiency

The preceding analysis shows that every admissible refinement of the event record corresponds to a piecewise-cubic interpolant that preserves the event anchors and introduces no new distinguishable structure. In the dense limit, these interpolants converge to a unique smooth closure $\Psi$ that is $\mathcal{C}^2$ and satisfies $\Psi^{(4)} = 0$. The discrete causal record and its smooth completion are therefore informationally equivalent representations of the same history.

**Law 1** (The Law of Spline Sufficiency)**.** *Let $\psi$ be any finite, non-contradictory record of admissible events. There exists a unique continuous completion $\Psi$ such that:*

1. *$\Psi$ agrees with $\psi$ at every event anchor,*
2. *$\Psi$ is piecewise cubic and $\mathcal{C}^2$ on its domain,*
3. *$\Psi$ introduces no new distinguishable structure beyond $\psi$, and*
4. *$\Psi$ satisfies the Euler–Lagrange closure $\Psi^{(4)} = 0$.*

*The cubic spline is therefore* sufficient *to represent all admissible distinctions in the data: no higher-order model encodes additional information available to measurement.*

The Law of Spline Sufficiency justifies the use of Galerkin methods [43] in this framework. By choosing the functional

$$\mathcal{J}[\Psi] = \int (\Psi'')^2 \, dx \tag{3.6}$$

as a measure of curvature, the Galerkin extremal selects the simplest admissible interpolant consistent with the event anchors. Because every sequence of admissible refinements converges to the unique $\mathcal{C}^2$ cubic closure, the Galerkin solution coincides with the informationally extremal configuration. No additional degrees of freedom are required.

In this sense, spline sufficiency provides the logical bridge between discrete measurement and continuous dynamics:

$$\text{discrete measurement} \xrightarrow{\text{spline sufficiency}} \Psi \xrightarrow{\text{closure}} \Psi^{(4)} = 0.$$

Finite differences do not approximate the Euler–Lagrange equation; they *generate* it. Smooth calculus enters only as the completion of refinement in the causal record, not as an assumed geometric primitive.

## 3.2 Galerkin Methods

**N.B.**—This argument applies the Law of Spline Sufficiency. We do not assume that Euler–Lagrange dynamics exist *a priori*. Rather, we show that if the data admit a smooth completion, then a cubic spline exists which reproduces the Euler–Lagrange solution to arbitrary accuracy. In this sense, observing a spline is sufficient to infer Euler–Lagrange dynamics: the differential equation models the behavior only insofar as the data allow it, and no additional geometric or differentiable structure is assumed. □

The Law of Spline Sufficiency establishes that cubic splines contain all admissible distinguishable structure. In this section we assume the existence of a smooth Euler–Lagrange solution and show that a Galerkin projection onto a spline basis produces a sequence of spline functions that converges to it. This suffices to justify the use of splines as the representatives of continuous dynamics: if Euler–Lagrange motion exists, Galerkin refinement will recover it to arbitrary accuracy.

### 3.2.1 Galerkin Projection onto a Spline Basis

Let $\Psi$ be the smooth solution to an Euler–Lagrange boundary value problem. Choose a finite spline basis $\{\varphi_k\}$ that satisfies the boundary constraints and let

$$\Psi_n(x) = \sum_{k=1}^{n} a_k \, \varphi_k(x)$$

be the Galerkin projection of $\Psi$ onto this space. The coefficients $a_k$ are chosen so that the residual of the Euler–Lagrange equation is orthogonal to the spline basis:

$$\int \Psi_n''(x) \, \varphi_k''(x) \, dx = \int \Psi''(x) \, \varphi_k''(x) \, dx, \qquad k = 1, \ldots, n. \tag{3.7}$$

This is the standard spline Galerkin formulation [19, 15]: the weak form enforces the Euler–Lagrange condition in the finite dimensional subspace spanned by the splines.

Solving (3.7) yields a unique spline $\Psi_n$ that agrees with the smooth solution at all knot points and is $\mathcal{C}^2$ on the domain. No higher-order degrees of freedom are necessary; the curvature functional ensures that splines are the minimal weak extremals.

### 3.2.2 Convergence of the Galerkin Sequence

By the Weierstrass Approximation Theorem, cubic splines form a dense subspace of continuous functions on a compact interval. As the mesh is refined and more basis functions are added, the sequence $\{\Psi_n\}$ converges uniformly to $\Psi$:

$$\Psi_n \xrightarrow[n\to\infty]{} \Psi.$$

Because the Euler–Lagrange operator is continuous in the weak topology, convergence of $\Psi_n$ implies convergence of all weak derivatives:

$$\Psi_n'' \xrightarrow[n\to\infty]{} \Psi''.$$

Thus the Galerkin sequence yields arbitrarily good spline approximations of the Euler–Lagrange solution. In particular, $\Psi_n$ satisfies

$$\Psi_n^{(4)} = 0$$

on each spline element, up to a boundary residual that vanishes as the mesh is refined.

**Corollary 1.** *If a smooth Euler–Lagrange solution $\Psi$ exists, a sequence of cubic splines $\{\Psi_n\}$ constructed by Galerkin projection converges uniformly to $\Psi$. Since cubic splines represent all admissible distinguishable structure, observing a spline solution is sufficient to infer the underlying Euler–Lagrange dynamics.*

In summary:

$$\Psi \xrightarrow{\text{Galerkin projection}} \Psi_n \xrightarrow[n\to\infty]{\text{Weierstrass}} \Psi,$$

so splines not only represent all admissible distinctions, but converge to the unique extremal of the Euler–Lagrange equation whenever one exists. The Galerkin method therefore completes the argument of spline sufficiency in the continuum: if continuous dynamics exist, spline solutions will recover them to arbitrary accuracy.

The Galerkin refinement therefore recovers smooth calculus without assuming infinitesimal increments or geometric primitives. The classical paradox of the fluxion may now be revisited in this light.

**Thought Experiment 3.2.1** (Fluxions [6, 82])**.** ***N.B.***—*The classical paradox of the fluxion treats an infinitesimal* dt *as a quantity that is neither zero nor nonzero. In the present framework, the limit is defined without invoking infinitesimals: smooth structure appears only as the unique completion of finite distinctions.* □

*In the 18th century, Bishop Berkeley criticized Newton's calculus of* fluxions *$(\dot{x}, \dot{y})$ for relying on quantities that vanish in one step of a proof and are treated as nonzero in the preceding step. If $\dot{x}$ and $\dot{y}$ are the ghost-like "increments" of position, the question arises:* How can a finite, observable change emerge from the vanishing difference of infinitesimal quantities?

*In the causal accounting used here, this is not a paradox of quantity but a limitation of informational resolution. The fluxion*

$$\dot{x} = \frac{\Delta x}{\Delta t}$$

*is a ratio of two sequentially recorded distinctions: the number of spatial ticks $\Delta x$ versus the number of temporal ticks $\Delta t$ between two anchors. Both are finite, integer-valued measurements.*

*The classical paradox appears only when $\Delta t \to 0$ is interpreted as a transition through a nonphysical intermediate state. In the present framework, no such state is required. The smooth completion $\Psi$ constructed in the dense limit satisfies $\Psi^{(4)} = 0$ and is the unique curvature-free extension of the data. As the anchor spacing shrinks, the ratio $\frac{\Delta x}{\Delta t}$ converges to the unique $\mathcal{C}^2$ slope $\Psi'$ of the cubic interpolant determined by the neighboring anchors.*

*No ghost-like infinitesimal is invoked. The derivative is the continuous shadow of finite bookkeeping: the single value required to prevent the appearance of new, unrecorded events as resolution increases. Smooth calculus arises not by manipulating vanished quantities, but as the unique function consistent with every refinement of the observable record.*

## 3.3 Equivalence of Discrete and Smooth Representations

### 3.3.1 Equivalence of Discrete and Smooth Representations

The preceding results establish the final closure of the Calculus of Dynamics. An admissible measurement record $\psi$ supported on event anchors $\{x_i\}$ is informationally equivalent to its smooth completion $\Psi$. The smooth calculus does not introduce new structure; it is the completion of refinement in the discrete domain.

Let $\psi$ be an admissible event record and let $f(\psi)$ denote any interpolant that preserves the anchors and introduces no distinguishable features between them. Refining the interpolant over nested partitions $\{\mathcal{T}_n\}$ produces a Galerkin sequence $\{\, \Psi_n \,\}$. By the convergence theorems, this sequence converges uniformly to a unique $\mathcal{C}^2$ cubic function $\Psi$:

$$\Psi_n \xrightarrow[n\to\infty]{} \Psi.$$

Informational minimality ensures that $\Psi$ is uniquely determined by the anchors: for every event point $x_i$,

$$\Psi(x_i) = \psi(x_i).$$

Because $\Psi$ is cubic on each partition element, preserves anchor order, and is globally $\mathcal{C}^2$, it is injective on each interval. Its inverse therefore recovers the original record:

$$\Psi^{-1}(x_i) = \psi(x_i).$$

Thus the discrete record $\psi$ and the smooth completion $\Psi$ contain exactly the same information. The interpolant and its limit are informationally equivalent representations of a single causal history.

### 3.3.2 Recovery of the Euler–Lagrange Form

The weak extremality condition was obtained entirely from finite differences in the discrete domain. In the Galerkin formulation this appears as

$$\int \Psi''(x)\,\phi''(x)\,dx = 0, \qquad \text{for all admissible test functions } \phi.$$

Integrating this identity twice yields the strong closure

$$\Psi^{(4)}(x) = 0.$$

No differentiability was assumed *a priori*: smoothness appears only as the completion of refinement in the Galerkin limit. The Euler–Lagrange equation is therefore a *recovered* description of the data, not an independent postulate. It is sufficient to model the discrete record because every admissible refinement converges to the same $\mathcal{C}^2$ cubic function.

In this sense the epistemic direction is inverted. We do not derive Euler–Lagrange dynamics and then discretize them. We begin with finite measurements, enforce informational minimality, and recover the Euler–Lagrange operator as the unique smooth shadow of refinement:

$$\text{measurement} \xrightarrow{\text{refinement}} \Psi \xrightarrow{\text{closure}} \Psi^{(4)} = 0.$$

In this sense the epistemic direction is inverted. We do not derive Euler–Lagrange dynamics and then discretize them. We begin with finite measurements, enforce informational minimality, and recover the Euler–Lagrange operator as the unique smooth shadow of refinement:

$$\text{measurement} \xrightarrow{\text{refinement}} \Psi \xrightarrow{\text{closure}} \Psi^{(4)} = 0.$$

Smooth calculus is therefore compatible with the axioms because it contains exactly the information present in the discrete causal record and no more. **N.B.**—With apologies to Bishop Berkeley: smooth dynamics are not prior to measurement; they are merely the grammar of its consistent refinement. □

## 3.4 The Free Parameter of the Cubic Spline

The Law of Spline Sufficiency requires that the smooth completion $\Psi$ of any admissible record be $\mathcal{C}^2$ and satisfy $\Psi^{(4)} = 0$. Each segment of $\Psi$ is therefore a cubic polynomial,

$$\Psi(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3,$$

but informational minimality collapses the apparent local degrees of freedom to a single global parameter.

### 3.4.1 Fixing the Lower–Order Coefficients

The value $a_0$ is fixed by the anchors: $\Psi(x_i) = \psi(x_i)$ for every event point $x_i$. The first derivative $\Psi'$ must be continuous across anchors; a jump in slope would constitute a new observable event, so $a_1$ is likewise determined. The curvature $\Psi''$ must also be continuous; any discontinuity would represent an unobserved acceleration and violate informational minimality. Thus $a_2$ is fixed by $\mathcal{C}^2$ continuity at the anchors.

These constraints ensure that adjacent cubic segments glue together without introducing new distinguishable structure. The only remaining coefficient, $a_3$, controls the third derivative of $\Psi$:

$$\Psi'''(x) = 6a_3.$$

### 3.4.2 The Single Free Parameter

Because $\Psi^{(4)} = 0$, the third derivative $\Psi'''$ is constant on every interval of the causal domain. Informational minimality permits this quantity to vary from interval to interval only when the variation is itself detectable as a recorded event. Absent such detection, $\Psi'''$ is the sole unconstrained degree of freedom.

**Proposition 3** (The Free Parameter of Information)**.** *The smooth completion $\Psi$ contains exactly one free parameter: the global scale of its third derivative $\Psi'''$. All lower–order coefficients are fixed by anchor data and continuity constraints.*

*Proof (Sketch).* Cubic structure follows from $\Psi^{(4)} = 0$. Values and derivatives up to order two are fixed by $\mathcal{C}^2$ boundary matching; any jump would be observable. Hence the only quantity not determined by anchor data is the constant third derivative on each interval, which is governed by $a_3$. No other freedom remains. □

*A full proof is provided in Appendix* **??***.*

### 3.4.3 Physical Interpretation

The single free parameter $\Psi'''$ represents the entire informational content of smooth kinematics. All subsequent dynamical quantities—wave speed, stress, curvature, and eventually mass—are determined by this one global scale. The Law of Spline Sufficiency therefore reduces the continuum to its minimal informational foundation: a $\mathcal{C}^2$ cubic universe with one degree of freedom.

$$\text{finite record} \xrightarrow{\text{closure}} \Psi \xrightarrow{\text{spline law}} \Psi''' = \text{constant on intervals.}$$

Smooth dynamics contain no structure beyond what is already present in the discrete causal record. The apparent infinity of the continuum collapses to a single free parameter.

## Coda: Navier–Stokes as a Finite Third Parameter

We do not derive the Navier–Stokes equations. Rather, we show how the measurement calculus constrains any smooth limit of finite records to a cubic-spline structure and thereby recasts the regularity question as the finiteness of a single quantity: the third parameter of the spline.

### 1. Statement of the classical problem

Let $v(x,t)$ be a velocity field and $p(x,t)$ a pressure satisfying the incompressible Navier–Stokes system on $\mathbb{R}^3$ (or a smooth domain with suitable boundary conditions):

$$\partial_t v + (v \cdot \nabla)v + \nabla p = \nu \Delta v + f, \qquad \nabla \cdot v = 0, \tag{3.8}$$

with smooth initial data $v_0$. The Millennium Problem asks whether smooth solutions remain smooth for all time or may develop singularities in finite time.

### 2. Measurement-to-spline reduction

Chapter 2 established that admissible smooth limits of finite records obey a local cubic constraint. Along any coordinate line (and likewise along any admissible selection chain) each component admits a representation whose fourth derivative vanishes in the limit:

$$U^{(4)} = 0 \quad \text{(componentwise along admissible lines).} \tag{3.9}$$

Hence the only freely varying local quantity is the *third parameter* (the derivative of curvature). In one dimension this is $U'''$. In three dimensions we package the idea as the third spatial derivatives of $v$:

$$\Theta(x,t) := \nabla(\nabla^2 v)(x,t) \quad \text{(a third-derivative tensor).} \tag{3.10}$$

Informally: $v$, $\nabla v$, and $\nabla^2 v$ are glued continuously by the spline closure; only $\Theta$ may vary piecewise without introducing fourth-order structure.

## 3. Regularity as finiteness of the third parameter

> *Principle.* If the third parameter $\Theta$ stays finite at all scales allowed by measurement, the smooth spline limit persists and no singularity can occur within the calculus of measurement.

A practical surrogate is a scale-invariant boundedness criterion on $\Theta$ (or a closely related norm tied to enstrophy growth):

$$\sup_{0 \le t \le T} \|\Theta(\cdot,t)\|_X < \infty \quad \Longrightarrow \quad \text{no blow-up on } [0,T], \tag{3.11}$$

where $X$ is chosen to control the admissible refinements (e.g. an $L^\infty$-type or Besov/Hölder proxy along selection chains). In words: the only obstruction to global smoothness is unbounded third-parameter amplitude.

## 4. Heuristic link to classical controls

Energy and enstrophy inequalities control $\|v\|_{L^2}$ and $\|\nabla v\|_{L^2}$. Vorticity $\omega = \nabla \times v$ monitors the first derivative. Growth of $\nabla\omega$ involves $\nabla^2 v$; the *onset* of non-smoothness is therefore detected by $\Theta = \nabla(\nabla^2 v)$, the next rung. Thus the finite-third-parameter condition (3.11) plays the same role in this framework that classical blow-up criteria play in PDE analyses: it is the minimal spline-compatible guardrail against curvature concentration.

## 5. Non-classical dependency is not invoked

No dependency (cause-effect) is asserted. The argument is purely informational: as long as the admissible record does not force the third parameter to diverge, the cubic-spline closure remains valid and the smooth limit inferred earlier continues to apply.

## 6. The rephrased question

> **Navier–Stokes, reframed.** Given smooth initial data and forcing, must the third parameter $\Theta$ in (3.10) remain finite for all time under (3.8)? Equivalently, can measurement-consistent refinement generate unbounded third-parameter amplitude in finite time?

If $\Theta$ stays finite, the spline structure persists, and the calculus of measurement supports global smoothness. If $\Theta$ diverges, the smooth continuum description ceases to be representable as a limit of admissible records, and the measurement calculus no longer licenses Euler–Lagrange inference on that interval.

## 7. What we have and have not done

We have not solved the Millennium Problem. We have shown that within this program the obstruction to smoothness is concentrated in a single quantity, the third parameter of the cubic spline representation. The classical regularity question is thus equivalent, in this calculus, to the finiteness of $\Theta$.

# Chapter 4

# Motion

Motion requires a mechanism for comparing one record of distinction with another. In the causal framework, that mechanism is the clock: a source of admissible refinements that allow two observers to construct ordered sequences of events. A clock does not measure duration, velocity, or geometry. It only asserts that one event occurred after another, and that this ordering is invariant under refinement. Its utility is purely ordinal. Whether the clock is an atomic transition, a quartz oscillator, a binary counter, or a list of threshold crossings, its output is the same kind of object: a finite record of distinguishable ticks.

**Definition 23** (Clock (non-standard))**.** ***N.B.***—*This is a* formal *definition of a clock. No physical assumption is made that clocks exist, or that any particular mechanism generates the ticks. In this framework, a clock is only a logical instrument that emits distinguishable events, allowing the causal order to be indexed. Its existence is a definition inside the mathematics of admissible distinctions, not a physical postulate about quantum systems, atomic transitions, or relativistic matter.* □

*A* clock *is an instrument that emits a sequence of distinguishable events. Each emitted event is admissible under Axiom 6: it produces a finite refinement of the causal record. A clock is therefore not a continuous variable or* *a dynamical law; it is a device that guarantees the existence of a countable chain of ordered distinctions. The function of a clock is to certify an ordering on the events of a measurement, nothing more.*

From this perspective, a clock is not a dynamical primitive. It is a logical instrument. The act of ticking establishes a chain of events, and the absence of extra ticks is a data constraint. If a clock recorded no intermediate events between two ticks, then no admissible description may contain structure that would have produced one. In particular, acceleration, oscillation, or curvature that would create additional ticks are ruled out by informational minimality. Motion is therefore not inferred from a continuous trajectory, but from the consistency of the tick record itself.

Because clocks produce ordered events, two observers may compare their records by merging their tick sequences under global coherence. When the merge produces no contradiction, a single coherent history exists, and the count of ordered refinements defines the relative motion of their systems. In the smooth limit, the unique continuous interpolant between ticked events is the cubic extremal with no unobserved structure. Thus, classical kinematics is the shadow of a discrete bookkeeping process: a clock provides order, informational minimality removes hidden curvature, and the continuum appears only as the completion of finite refinements.

In what follows, motion will be defined as the reconciliation of two causal records produced by clocks. Relative velocity, proper time, and inertial behavior arise not from geometry or differential equations, but from the minimal continuous shadow consistent with their countable tick sequences. Motion is what ordered distinction looks like when refinement tends to the smooth limit.

**Thought Experiment 4.0.1** (Laser Tracking and Informational Dilation)**.** *Two identical observers, A and B, begin co-located with synchronized clocks. Observer B embarks on a journey involving periods of acceleration, while observer A remains at the origin of an idealized inertial frame. We* explicitly neglect *the gravitational and relativistic influence of Earth, the Sun, Sagittarius A*, and all other bodies; spacetime is treated as Minkowski over the region of interest.*

*Rather than waiting for reunion, A continuously tracks B by emitting a stream of monochromatic laser pulses. Each pulse is timestamped in A's notebook when fired, and timestamped again when the reflected pulse is received from B's retroreflector.*

*Every fired pulse is a distinguishable event; every received pulse is another. If B follows a complicated accelerative path, then the return times of the pulses form a more densely refined sequence than the symmetric record A would observe if B were inertial. The point is not energy or Doppler shift. The informational content of the record increases: each round-trip establishes a new ordered pair of emission and reception, constraining B's admissible motion.*

*If B were inertial, the spacings of the returned timestamps would follow the unique minimal interpolant that introduces no unobserved curvature. But acceleration forces extra refinements: the return times become uneven in a way that cannot be reconciled with a coasting trajectory. These "irregularities" are not interpreted through differential equations; they are simply distinct events that must be merged into A's causal record.*

*When B returns, both observers merge their sequences. A's laser notebook contains a much longer chain: every emission and every reflection has already placed constraints on B's path. B's local clock, by contrast, has recorded only its own internal ticks and those refinements forced by onboard events. The merge therefore requires A to reconcile a larger informational workload, while B performs a smaller one. Consistent ordering assigns the larger count of admissible distinctions to A, and the smaller to B. The result is that A's proper time is larger—she has the denser causal record.*

*In the smooth limit, the same count enforces the classical dilation formula of relativity. But here the conclusion is purely informational: acceleration in-*

*troduces refinements, refinements create more events, and more events imply more work when histories are coherently merged. Time dilation is the bookkeeping of laser-certified distinctions, not a geometric postulate.*

*This informational mechanism therefore recovers the ability to compute the Lorentz contraction posed in Thought Experiment 2.4.2 through the update rule* $E_k = \Psi\big(e_k \cap \hat{R}(e_{k-1})\big)$*, using only the observers' laboratory notebooks.*

## 4.1 Relative Motion

With a clock in hand, an observer constructs an ordered sequence of admissible events $\{e_k\}$. The Causal Universe Tensor encodes this growing record through the recursive update

$$E_k = \Psi\big(e_k \cap \hat{R}(e_{k-1})\big),$$

where $\hat{R}$ restricts all admissible continuations to those consistent with the most recent distinguishable event. For a single inertial frame, this produces a monotone refinement of information: each step eliminates histories that would contradict the record.

To describe motion, no new structure is required. Consider two inertial observers, A and B, each with their own clock and their own causal universe tensors:

$$E^A_k = \Psi\big(e^A_k \cap \hat{R}(e^A_{k-1})\big), \qquad E^B_\ell = \Psi\big(e^B_\ell \cap \hat{R}(e^B_{\ell-1})\big).$$

Each observer possesses a complete and self-consistent history of admissible distinctions. Relative motion is nothing more than the reconciliation of these two histories under the rule of global coherence. When A and B compare notebooks, every admissible refinement recorded by one must also be admissible to the other. In the absence of contradiction, a merge exists, and the merged list induces a partial ordering on the pair $\{E^A_k\} \cup \{E^B_\ell\}$.

If the merged record is strictly longer than either individual record, the observers infer that their clocks have accumulated refinements at different rates. By Axiom 5, the merged history cannot discard any admissible distinction: every recorded refinement must be preserved. Consequently, the observer whose notebook produces the *denser* merged record corresponds to the *longer* proper time. The informational work of reconciling those refinements is the measure of duration. Thus,

$$|\, e_k^A \cap \hat{R}(e_{k-1}^A)\,| > |\, e_\ell^B \cap \hat{R}(e_{\ell-1}^B)\,| \tag{4.1}$$

implies that A has the longer proper time. No metric, coordinate chart, or differential equation is required; the relative motion is encoded entirely in the informational asymmetry of their merged causal histories.

In the smooth limit, the unique continuous interpolant of the merged record is the cubic extremal with no unobserved structure. Classical kinematics—relative velocity, time dilation, and Lorentz contraction—appears as the shadow of this merge. The Causal Universe Tensor does not simulate motion; it enforces consistency. Relative motion is what two coherent universe tensors look like when compared under refinement.

### 4.1.1 Merging a Single Event

Referring to Thought Experiment 4.0.1, consider the merging of a single event: the moment a reflected photon is absorbed by A's detector. This absorption is a distinguishable refinement of A's record and therefore constitutes an admissible event $e_{k+1}^A$. The photon has traveled to B, interacted with the retroreflector, and returned. Whatever else the experimenter may imagine, this exchange contains one certified fact: the causal distance between A and B has changed in a way detectable by A's clock.

In the language of the causal universe tensor, the absorption is merged via

$$E_{k+1}^A = \Psi\big(e_{k+1}^A \cap \hat{R}(e_k^A)\big).$$

Nothing more is required. The event contributes only the distinction that A received a photon at that moment. The return time rules out any hypothetical motion of B that would have prevented this arrival, and it rules out any curvature or oscillation that would have produced additional admissible pulses. The refinement therefore narrows A's admissible histories to those consistent with both emission and reception.

When B later inspects A's notebook, the same absorption event must be admissible within B's causal universe tensor:

$$E^B_{\ell+1} = \Psi\big(e^A_{k+1} \cap \hat{R}(e^B_\ell)\big).$$

If a contradiction were forced—for example, if B's notebook implied the photon could not have returned at that time—global coherence would fail, and the combined record would be inadmissible. But if the merge succeeds, the joint history becomes strictly more refined, and the updated tensors[1] encode a new restriction on their relative motion.

A single merged photon event therefore eliminates an entire family of hypothetical motions. It narrows the admissible set of configurations and extends the causal record without introducing any continuous structure. In the smooth limit, repeated merges of this form force the cubic extremal between emission and reception times—the unique interpolant with no unobserved structure. Classical distance, velocity, and Lorentz contraction appear as the continuous shadow of this discrete bookkeeping.

### 4.1.2 Measurement of Acceleration as Counts of Events

Acceleration does not require forces, masses, or differential equations. In the causal framework, acceleration is nothing more than a second refinement: a change in the distinguishable difference between successive admissible events.

[1]No physical model of a photon is required. The "photon" only represents a distinguishable event transmitted between observers.

To detect such a change, a single measurement is insufficient. At least two refined measurements are needed so that the difference between them can itself be distinguished.

Suppose A emits two photons at events $e_k^A$ and $e_{k+1}^A$, and later receives their reflections at $e_{k+r}^A$ and $e_{k+s}^A$. Each absorption is merged by

$$E_{k+r}^A = \Psi\big(e_{k+r}^A \cap \hat{R}(e_{k+r-1}^A)\big), \qquad E_{k+s}^A = \Psi\big(e_{k+s}^A \cap \hat{R}(e_{k+s-1}^A)\big).$$

If B is in uniform motion relative to A, the refinements contributed by these two events are consistent with a unique minimal interpolant: the admissible histories require that the difference in reception times is itself constant under refinement. Any hidden curvature or oscillation would have produced additional admissible events—extra pulses, missed reflections, or altered return order—and is therefore ruled out by the Axiom of Planck.

However, if the spacing between $e_{k+r}^A$ and $e_{k+s}^A$ cannot be reconciled by a single coasting history, then the admissible set must be further restricted. The causal universe tensor eliminates all hypothetical configurations in which B remained inertial. What remains are those histories in which the separation of the events changes in a way that is itself distinguishable. The second refinement is the signature of acceleration.

In this sense, acceleration is not a postulated quantity. It is the discovery that two refinements cannot be merged into a single coasting interpolant without contradiction. A sequence of such measurements produces a chain of eliminations: each return time excludes admissible events that would require an invisible change in curvature. The remaining histories are the ones in which acceleration has occurred.

In the smooth limit, repeated second refinements force a unique continuous extremal whose second variation is nonzero. Classical acceleration appears as the shadow of a finite bookkeeping process: acceleration is the count of distinguishable failures of coasting to explain the merged record. No forces, masses, or trajectories are assumed. The event counts alone enforce curvature in the admissible histories.

Thus, we have recovered the ability to verify Newton's second law—again, with apologies to Lord Berkeley. Acceleration is not a substance but the second variation of admissible refinements in the merged event record.

### 4.1.3 The Equations of Motion

The remainder of this chapter examines the equations of motion that arise when finite records of admissible distinctions are merged without contradiction. Nothing further is assumed. Each equation appears as the continuous shadow of informational minimality: the unique smooth extremal that contains no unrecorded structure.

We begin with heat transport. Although commonly divided into conduction, convection, and radiation, all three arise here from distinct constraints on admissible refinements. When refinements diffuse symmetrically through a medium with no hidden variations, the smooth limit forces the diffusion equation. When refinements are transported coherently through the medium, the extremal satisfies the advective transport law. When refinements propagate at the maximal admissible speed, the continuous shadow is radiative transport governed by the wave equation. No model of heat is assumed; each law is simply the completion of a finite notebook of events.

Annealing appears when a ledger repeatedly reconciles its own coarse description. The iterated application of the merge operator eliminates sharp distinctions that would predict unobserved refinements. As the sequence of folds converges, the smooth limit is diffusion. Annealing is therefore informational smoothing: the heat equation is its continuous shadow when the coarse ledger is refined to closure.

Adiabatic transport arises when the ledger evolves without creating or destroying admissible distinctions. In the smooth limit, this invariance forces the classical adiabatic law. Nothing dynamical is postulated; an adiabatic process is simply a sequence of refinements that preserves the global count of admissible configurations.

Even quantum phenomena admit the same treatment. The Casimir effect appears when the merged record forbids a continuous family of admissible configurations between two boundaries. The elimination of those histories produces an informational pressure, and the smooth limit recovers the familiar expression for Casimir energy.

Alpha decay appears in its original form: the Mott problem [80]. The ledger of the nucleus contains two nearly indistinguishable configurations—one in which the alpha cluster remains bound, and one in which it escapes. Over time, these dual descriptions drift out of alignment. The moment of decay is not the passage of a particle through a barrier, but the repair of a contradiction: the merged record eliminates all configurations in which the two descriptions diverge. The resulting refinement is recorded as a distinct decay event. In the smooth limit, this informational repair produces the exponential law of radioactive decay without invoking forces, potentials, or tunneling particles. No dice are rolled.

In each case, the classical equation of motion is not assumed. It is what consistency looks like in the smooth limit of finite measurement. Motion is bookkeeping; the laws that follow are shadows of refinement.

### 4.1.4 Martin's Condition and the Propagation of Order

Up to this point, motion has been defined locally: two observers exchange admissible events, merge their records, and eliminate any hypothetical history that would have produced unrecorded refinements. This closure guarantees that each observer maintains a coherent ledger. It does not yet guarantee that their ledgers are mutually compatible.

For observable physics, local coherence is not enough. Distinct observers must be able to reconcile their refinements along their shared boundary without introducing new distinguishabilities. The requirement that every locally finite patch of causal order extends to a globally consistent history is Martin's Condition.

**Definition 24** (Martin's Condition (Non-standard))**.** *A causal network (Definition 10) satisfies Martin's Condition if every locally finite subset of events can be extended to a globally consistent ordering without introducing new admissible distinctions. Equivalently, all finite causal updates admit an extension that preserves the same coincidence relations on their overlaps.*

Intuitively, Martin's Condition demands that information created in one region does not contradict information measured in another. It forbids causal overcounting—the duplication of distinctions that would destroy reversibility—by ensuring that overlapping observers reconstruct identical splines of the causal universe tensor along their shared boundary. The Axiom of Event Selection limits what may happen within a light cone; Martin's Condition governs how those choices propagate outward.

Once Martin's Condition holds, the closure of finite refinements induces a global propagation rule. Locally symmetric overlaps enforce a second variation and yield the wave operator. Oriented overlaps enforce a first variation and yield advection. When variations are eliminated by repeated projection, the smooth limit is diffusion. The familiar equations of motion—waves, advection, diffusion, and later curvature—are therefore the continuous shadows of global consistency under Martin's Condition.

**Thought Experiment 4.1.1** (Davisson–Germer and the Universality of Causal Waves [27])**.** [Waves are the continuous shadows of Martin–consistent propagation]

*Imagine an electron gun firing individual electrons toward a crystalline nickel target. A distant screen records the arrival of scattered electrons as distinguishable events. Between gun, crystal, and screen, no internal distinctions are measured; the observers record only the emission, the scattering plane, and the pattern of impacts. Each detection on the screen is there-*

*fore an admissible refinement of the joint causal ledger of gun, crystal, and detector.*

*Under Martin's Condition, every locally finite segment of this ledger must extend to a globally consistent history. The crystal introduces a periodic partition: successive lattice planes represent indistinguishable choices, except at angles where the merged ledger would predict additional or missing refinements. Along these planes, reciprocal measurement enforces translation invariance: if one segment of the ledger is shifted by a lattice spacing, the count of admissible refinements must remain unchanged.*

*The only smooth extremals compatible with this translation invariance are wave modes. Among these, the constructive modes are precisely those whose wavelength* $\lambda$ *satisfies Bragg's relation [14]*

$$2d \sin\theta = m\lambda, \qquad \lambda = \frac{h}{p},$$

*where* $d$ *is the lattice spacing,* $\theta$ *the scattering angle,* $m$ *an integer, and* $h/p$ *encodes the count of distinguishabilities preserved along the oriented Martin bridges. At those angles, no hidden refinements are predicted; outside them, the merged ledger would contain missing or extra distinguishable events, contradicting Martin's Condition.*

*Operationally, the bright peaks on the screen are fixed points of reciprocal measurement under lattice translations. What physicists call "electron diffraction" is simply the bookkeeping consequence of demanding that indistinguishable causal neighborhoods propagate consistently across the crystal. No wavefunction is assumed. The "wave" is the unique smooth extension of discrete, Martin–consistent event counts.*

*Thus, the Davisson–Germer experiment does not demonstrate that electrons are waves or particles. It demonstrates that any causal history satisfying Martin's Condition must propagate its indistinguishabilities as waves. The universality of wave behavior is a consequence of global consistency, not a* *special property of matter.*

## 4.2 The Algebra of Interaction

Each system $X$ carries an accumulated causal universe tensor as a left–fold of update factors:

$$\mathbf{U}_1^X = E_1^X, \qquad \mathbf{U}_{n+1}^X = E_{n+1}^X\,\mathbf{U}_n^X, \qquad E_{n+1}^X := \Psi\big(e_{n+1}^X \cap \hat{R}(e_n^X)\big).$$

**Definition 25** (Interaction operator)**.** *Given two ledgers (tensors)* $\mathbf{U}^A$ *and* $\mathbf{U}^B$*, the* interaction operator

$$f:\ (\mathbf{U}^A, \mathbf{U}^B)\ \longmapsto\ \mathbf{U}^{AB}$$

*returns the minimal accumulated state* $\mathbf{U}^{AB}$ *that extends both inputs and is Martin–consistent on their overlap. Equivalently,* $\mathbf{U}^{AB}$ *is obtained by left–folding the common update factors (the jointly admissible events) in observed order so that no unrecorded refinements are invented (Axiom of Planck) and none already recorded are erased (Axiom of Cantor). Let* $E(\mathbf{U})$ *denote the underlying event set of* $\mathbf{U}$ *and define the newly contributed distinctions by*

$$\mathbf{J}^{AB}\ :=\ E(\mathbf{U}^{AB}) \setminus \big(E(\mathbf{U}^A) \cup E(\mathbf{U}^B)\big).$$

**Definition 26** (Length on the common boundary)**.** *Let* $\partial(\mathbf{U}^A, \mathbf{U}^B)$ *denote the common boundary (overlap) of the ledgers* $\mathbf{U}^A$ *and* $\mathbf{U}^B$*. The* length on the boundary *is the number of folded factors from a ledger that lie on this overlap:*

$$\mathrm{len}_\partial(\mathbf{U}^A, \mathbf{U}^B)\ :=\ \mathrm{len}\big(\mathbf{U}^A \restriction_{\partial(\mathbf{U}^A,\mathbf{U}^B)}\big), \qquad \mathrm{len}_\partial(\mathbf{U}^B, \mathbf{U}^A)\ :=\ \mathrm{len}\big(\mathbf{U}^B \restriction_{\partial(\mathbf{U}^A,\mathbf{U}^B)}\big).$$

*Equality* $\mathrm{len}_\partial(\mathbf{U}^A, \mathbf{U}^B) = \mathrm{len}_\partial(\mathbf{U}^B, \mathbf{U}^A)$ *expresses informational equilibrium on the shared frontier.*

**Proposition 4** (The Anti–symmetry of Information Propogation)**.** *In general $f(\mathbf{U}^A, \mathbf{U}^B) \neq f(\mathbf{U}^B, \mathbf{U}^A)$. Symmetry holds iff the overlap carries equal refinement counts:*

$$f(\mathbf{U}^A, \mathbf{U}^B) = f(\mathbf{U}^B, \mathbf{U}^A) \quad \Longleftrightarrow \quad \mathrm{len}_\partial(\mathbf{U}^A, \mathbf{U}^B) = \mathrm{len}_\partial(\mathbf{U}^B, \mathbf{U}^A).$$

*Proof (Sketch).* The interaction operator $f(U_A, U_B)$ performs a left–fold of all jointly admissible update factors on the overlap $\partial(U_A, U_B)$, in the unique order that is consistent with the causal refinements already recorded in each ledger. Anti–symmetry arises because this fold depends on the observed order of refinements whenever the overlap contains correlated (noncommuting) factors.

Suppose first that the refinement counts on the shared boundary are equal:

$$\mathrm{len}_\partial(U_A, U_B) = \mathrm{len}_\partial(U_B, U_A).$$

Every factor lying on the overlap is therefore recorded with the same resolution by both ledgers. No ledger contributes a strictly finer refinement than the other on the shared frontier. In this case the overlap consists only of mutually uncorrelant update factors: their order is not fixed by either ledger, and informational minimality forces them to commute. Because the only factors whose relative placement could differ lie in this commuting set, the resulting left–fold is invariant under exchanging the inputs, and

$$f(U_A, U_B) = f(U_B, U_A).$$

Conversely, assume the refinement counts on the overlap are unequal. Without loss of generality, let $U_A$ record strictly more refinement on the boundary than $U_B$. Then $\partial(U_A, U_B)$ contains at least one factor recorded by $A$ with higher resolution than by $B$. Such a factor cannot be uncorrelant: if it were, its finer structure could not have been observed by only one ledger. The overlap therefore contains a correlated pair of update factors whose tensor representatives do not commute. The left–fold must place this pair in the local causal order recorded by the corresponding ledger. Because $U_A$ and $U_B$ record different boundary orders for these noncommuting factors, the two possible folds produce distinct accumulated tensors:

$$f(U_A, U_B) \neq f(U_B, U_A).$$

Thus symmetry of the interaction operator occurs exactly when the two ledgers carry equal refinement counts on their shared boundary, and fails precisely when one ledger resolves strictly more distinguishable structure than the other. □

*A full proof is provided in Appendix* **??**.

**Proposition 5** (The Transitivity of Information Propogation)**.** *For any Martin–consistent triple* $\mathbf{U}_n, \mathbf{U}_{n+1}, \mathbf{U}_{n+2}$*,*

$$f(\mathbf{U}^A, \mathbf{U}_{n+2}) \;=\; f\big(\mathbf{U}^A,\; f(\mathbf{U}_n, \mathbf{U}_{n+1})\big).$$

*That is, folding via the intermediate ledger equals folding directly into* $\mathbf{U}_{n+2}$*.*

*Proof (Sketch).* Let $U_n, U_{n+1}, U_{n+2}$ be a Martin–consistent triple. Each ledger is a left–fold of its admissible update factors, and the interaction operator $f$ produces the minimal ledger that extends its inputs without inventing or erasing recorded refinements. The transitivity property expresses the fact that the unique globally coherent ledger for the triple does not depend on how the pairwise folds are grouped.

Consider the right–hand side,

$$f\big(U_A,\; f(U_n, U_{n+1})\big).$$

The inner fold $f(U_n, U_{n+1})$ reconciles all jointly admissible refinements of $U_n$ and $U_{n+1}$ on their shared boundary. Because the pair is Martin–consistent, this fold is unique: no alternative ordering of their overlapping factors survives the consistency check. The result is a ledger that contains exactly the refinements common to both inputs together with their compatible unique factors. Folding this ledger with $U_A$ adds precisely the admissible refinements from $U_A$ that remain consistent with the already merged pair. No additional events may be inserted, and none already present may be removed.

Now consider the left–hand side,

$$f(U_A, U_{n+2}).$$

Since the triple is Martin–consistent, $U_{n+2}$ already encodes all refinements that can appear after $U_{n+1}$ without violating the Axiom of Planck or the Axiom of Cantor. Any refinement compatible with $U_n$ and $U_{n+1}$ must also be compatible with $U_{n+2}$. Thus the direct fold of $U_A$ with $U_{n+2}$ produces a ledger that contains exactly the jointly admissible refinements of all three inputs. As before, no additional distinctions may be introduced.

In both constructions, the surviving event factors are the same: the set of refinements jointly admissible across the triple. Martin's Condition ensures that this set admits a unique causal ordering, so both sides fold precisely the same sequence of factors. By informational minimality and uniqueness of the admissible ordering, the resulting tensors must coincide:

$$f(U_A, U_{n+2}) = f\big(U_A,\ f(U_n, U_{n+1})\big).$$

Thus the interaction operator is transitive on any Martin–consistent triple: grouping of intermediate folds does not affect the final accumulated ledger.

□

*A full proof is provided in Appendix* **??**.

**Proposition 6** (The Commutativity of Uncorrelant Events)**.** *If*

$$f\big(f(\mathbf{U}^A, \mathbf{U}^B),\ f(\mathbf{U}^C, \mathbf{U}^D)\big) \ = \ f\big(f(\mathbf{U}^C, \mathbf{U}^D),\ f(\mathbf{U}^A, \mathbf{U}^B)\big),$$

*then the pairs* $(A, B)$ *and* $(C, D)$ *are* relativistically simultaneous*: exchanging block order introduces no new admissible distinctions on the shared boundary; the merged tensor is invariant under the swap.*

*Proof (Sketch).* Let $U^{AB} := f(U_A, U_B)$ and $U^{CD} := f(U_C, U_D)$. The hypothesis is that the two blocks commute under the interaction operator:

$$f\big(U^{AB}, U^{CD}\big) = f\big(U^{CD}, U^{AB}\big).$$

By Proposition 4, such commutativity can occur only when the shared boundary carries equal refinement counts. In the present setting this means that every update factor lying in the overlap $\partial(U^{AB}, U^{CD})$ is recorded at the same resolution by both blocks. No factor is strictly more refined on one side than the other.

Equal refinement counts force the overlapping factors to be uncorrelant: neither block records a finer causal relation among these events, so informational minimality forbids any ledger from resolving a precedence relation absent from the other. In the tensor algebra this uncorrelance appears as commutation of the corresponding update factors. Because only these boundary factors can appear in different relative positions when the blocks are folded, and because they commute, swapping the blocks yields the same accumulated ledger.

To interpret this result, note that two events are uncorrelant precisely when neither precedence $e < f$ nor $f < e$ is recorded in any admissible refinement. Such events lie outside each other's causal neighborhoods; exchanging their order introduces no new distinguishable structure and preserves all scalar invariants of the universe tensor. Thus, if the blocks $(A, B)$ and $(C, D)$ commute under $f$, every event in the first block is uncorrelant with every event in the second. No causal precedence can be established across the blocks.

This is exactly the condition of relativistic simultaneity in the causal framework: the two blocks occupy spacelike-separated regions of the observational record. Their fold order is unconstrained, and the merged ledger is invariant under the swap. Hence commutativity of the interaction operator implies relativistic simultaneity. □

*A full proof is provided in Appendix* **??**.

**N.B.**—This is the point at which the usual notion of *causality* is rejected. No geometric light cones, no differential structure, and no propagation law are assumed. The only order in the development is the order of *recorded* refinements. What physicists call causal structure appears later only as the smooth shadow of informational bookkeeping: the continuum calculus that encodes cause–effect relations is not a primitive of the theory but an emergent completion of discrete refinements. Nothing in this chapter assumes or relies on physical causation; all that is used is the partial order induced by the Axiom 5. □

**N.B.**—Uncorrelant events play a central conceptual role in this framework. They are not "independent random variables" nor "simultaneous in a reference frame" nor artifacts of a chosen coordinate system. They are the events for which the record contains *no admissible refinement* that orders one before the other. This absence of recorded precedence is an observable fact, not a geometric assumption. All smooth notions of spacelike separation, relativistic simultaneity, and commuting update factors arise from this single idea. When two events are uncorrelant, reordering their update factors creates no new distinguishable structure, and every algebraic invariant of the ledger is preserved. The geometry of relativity is therefore not presupposed but recovered from the informational status of uncorrelance. □

**Remark 15.**

Idempotence: $f(\mathbf{U}^A, \mathbf{U}^A) = \mathbf{U}^A$.

Monotonicity: $\mathbf{U}^{AB}$ *is a monotone extension of both inputs; no recorded refinement is removed.*

Locality: *Joint refinements lie in the common causal neighborhood; fold order is the observed order; reordering is forbidden unless the corresponding factors commute.*

Operational link: *Bi–directional folds yield the wave operator; oriented folds yield advection; iterated projection yields diffusion. These are smooth shadows of the discrete left–fold* $\mathbf{U}_{n+1} = E_{n+1}\mathbf{U}_n$ *under* $f$.

**Thought Experiment 4.2.1** (The Dantzig Pivot [24] of Entanglement [34])**.**
***N.B.***—*The Dantzig Pivot is not a physical process. Nothing travels, no signal is sent, and no mechanism propagates. The pivot is bookkeeping: boundary consistency is enough to eliminate incompatible histories without scanning the interior of the ledger.* □

*Two spacelike-separated laboratories, A and B, each maintain their own causal universe tensor. A single preparation event produces two admissible refinements,* $e_i$ *and* $e_j$*, that are indistinguishable in causal order: both*

$$\langle e_i \prec e_j \rangle \quad \textit{and} \quad \langle e_j \prec e_i \rangle$$

*generate the same accumulated state. No scalar invariant recorded in either ledger can tell which ordering occurred. This is a state of* causal degeneracy*: two distinct histories produce the same observational content.*

*At time* n+1*, laboratory A measures* $e_i$*. By the Axiom of Planck, this refinement must be folded into the accumulated state. The interaction operator* $f$ *computes*

$$\mathbf{U}_{n+1} = f(\mathbf{U}_n, e_i),$$

*which is a strict update:* $e_i$ *now has a definite position in the record relative to all prior events.*

*Because* $e_i$ *and* $e_j$ *were degenerate, this update triggers a global repair. The merged ledger must eliminate every history in which* $e_j$ *is ordered incompatibly with* $e_i$ *under Martin's Condition. No signal is sent from A to B; instead, the causal universe tensor performs a* pivot*: it selects the unique ordering of* $(e_i, e_j)$ *that avoids introducing new distinguishabilities. The ambiguous pair collapses to a single admissible ordering.*

Critically*, this repair is not a search over an entire volume of possible histories. Martin's Condition requires agreement only on the* boundary *of the overlap: the parts of* $\mathbf{U}^A$ *and* $\mathbf{U}^B$ *that already coincide. The pivot therefore acts on the smallest region where a contradiction could occur. Only the boundary is inspected, and only the incompatible orderings are removed. There is no need to re-evaluate the entire causal universe; the ledger verifies consistency by checking the joint frontier. Interaction is thus* computable*: global coherence is enforced by local boundary repair, not by scanning an exponential set of histories.*

*Thus, the "instantaneous" correlation is not a physical transmission. It is the bookkeeping consequence of a non-degenerate refinement. Entanglement is the existence of causal degeneracy; the apparent nonlocal update is the pivot that removes it by repairing the boundary of the overlap.*

*The name "pivot" is not accidental. In Dantzig's algorithm, a degenerate solution is resolved by moving along the boundary of admissible configurations until a single vertex remains consistent with all constraints. The search never explores the interior volume of the feasible set; it advances only along the frontier where inconsistency can appear. The causal pivot behaves the same way. When a non-degenerate refinement is recorded, the ledger examines only the boundary of the overlap and removes incompatible orderings. The result is a unique, globally coherent history selected by local boundary repair. In both settings, the pivot is a boundary operation, not a volume search: global consistency is enforced without scanning an exponential family of possibilities.*

**Thought Experiment 4.2.2** (Mach–Zehnder Interferometer as Causal Superposition)**.** ***N.B.***—*Interference is a bookkeeping result. The ledger compares two admissible histories and keeps only the one that preserves Martin's Condition at the boundary. No wavefunction, no probability amplitude, no transmission is assumed.* □

*A single photon enters a Mach–Zehnder interferometer. At the first beam splitter, a single input event* $e_0$ *leads to two admissible refinements,* $e_1$ *(upper path) and* $e_2$ *(lower path). Both produce valid causal chains: each path accumulates its own ordered list of refinements—reflections, delays, and phase shifts—and each yields an accumulated tensor* $\mathbf{U}^{(1)}$ *and* $\mathbf{U}^{(2)}$ *satisfying Martin's Condition. No experiment in either arm can distinguish which refinement is "real": both histories are admissible and neither produces a contradiction. The interferometer therefore carries two coexisting, consistent ledgers.*

*At the second beam splitter, the detection event* $e_f$ *must be recorded as a strict update. By the Axiom of Planck, the refinement* $e_f$ *must fold into the accumulated state. The interaction operator computes*

$$\mathbf{U}_{\text{final}} = f(\mathbf{U}^{(1)}, \mathbf{U}^{(2)}),$$

*the minimal accumulated tensor consistent with both paths. All hypothetical histories in which the arrival at* $e_f$ *contradicts either ledger are removed.*

*Interference is the informational comparison of the two causal chains. If their accumulated phase—a bookkept record of distinguishability—is equal modulo* $2\pi$*, the paths are informationally indistinguishable at the boundary. The fold produces a single ledger: both paths merge without creating new refinements. If the accumulated phase differs by* $\pi$*, the asymmetric parts of the update factors cancel under the fold, and* $e_f$ *becomes inadmissible. No destructive force is invoked; the cancellation expresses the fact that no consistent ledger can be formed with that ordering.*

*Thus, "superposition" is the coexistence of multiple valid, Martin-consistent refinements until detection forces a non-degenerate fold. The Mach–Zehnder* *interferometer does not show a particle traveling two paths; it shows that causal histories can remain distinct and simultaneously admissible until the interaction operator selects the unique ordering that avoids contradiction at the boundary.*

**Thought Experiment 4.2.3** (Bell–Aspect Tests as Global Martin Consistency)**.** *Two spacelike-separated laboratories, $A$ and $B$, share a preparation event that produces an entangled pair. Each maintains its own causal universe tensor. The preparation is such that multiple ordered refinements remain admissible: different measurement settings at $A$ and $B$ produce distinct, yet individually consistent, ledgers. Before either measurement is recorded, the global state is degenerate: many joint histories remain compatible with all previous refinements, and no scalar invariant distinguishes among them.*

*A local hidden-variable model assumes that this degeneracy can be resolved purely by local rules. In ledger language, it assumes that the update*

$$(\textit{measure at } A,\ \textit{measure at } B)$$

*can be decomposed into separate, predetermined refinements in each ledger. That is, the merged state could be written as a fold of two independent maps acting only on local records, with no global repair.*

*The Bell–Aspect tests show this is impossible. When $A$ records a refinement corresponding to setting $a$ and $B$ records one corresponding to $b$, the accumulated tensor must be updated by the interaction operator,*

$$\mathbf{U}_{\text{final}} = f(\mathbf{U}^A, \mathbf{U}^B).$$

*For many setting pairs $(a, b)$, the resulting ledger eliminates histories that would have remained admissible under any local rule. The violation of Bell inequalities is the empirical statement that no decomposition of $f$ into independent, local updates can preserve all observed distinctions. The fold is intrinsically global.*

*Operationally, a new refinement at $A$ forces a pivot on the boundary shared with $B$, eliminating joint histories that contradict the updated record. No signal travels between the laboratories; no mechanism carries information. The ledger simply performs the minimal boundary repair required by Martin's Condition. The observed "nonlocal" correlations are the bookkeeping consequence of enforcing a single, globally consistent causal ordering.*

*Thus, the Bell–Aspect tests reveal that entanglement is not a hidden influence. It is the fact that the causal universe must repair its boundary globally when a non-degenerate refinement is recorded. Local hidden variables fail because they deny the existence of this global pivot.*

**Thought Experiment 4.2.4** (The Qubit as a Causal Doublet)**.** *Consider a system whose next admissible refinement may proceed in one of two distinct ways, $e_0$ or $e_1$. Both are individually consistent with all recorded events. Neither produces a new distinguishability. The ledger therefore admits two possible causal updates,*

$$\mathbf{U}_{n+1} = E_0\,\mathbf{U}_n \qquad \textit{or} \qquad \mathbf{U}_{n+1} = E_1\,\mathbf{U}_n,$$

*and nothing in the accumulated tensor resolves which will occur. The pair $S = \{e_0, e_1\}$ is a* causal doublet*: two equally admissible refinements, each extending the history without contradiction. The system is not undecided; it simply has more than one globally consistent continuation.*

*This is the minimal unit of causal degeneracy. A* qubit *is a causal doublet whose two refinements remain admissible until a non-degenerate event forces a decision. When a measurement is recorded, the new refinement must be folded into the ledger. The interaction operator computes*

$$\mathbf{U}_{n+1} = f(\mathbf{U}_n, e_b) \qquad \textit{for a unique } b \in \{0, 1\},$$

*and all hypothetical histories in which the other refinement contradicts the updated ledger are removed. The doublet collapses to a single admissible* *ordering.*

*Nothing physical "chooses" $e_0$ or $e_1$. No hidden variable pre-selects the outcome. The pivot is bookkeeping: Planck forbids invisible refinements, Cantor forbids erasure, and Martin enforces global consistency. Until the pivot, $S$ remains a superposed causal state: two admissible orderings, both equally consistent with all observations.*

**Thought Experiment 4.2.5** (Hawking Radiation Revisited)**. *N.B.***—*No physical emission is assumed. Surrogate refinements are bookkeeping: the minimal distinctions required to restore Martin consistency when the boundary saturates.* □

*An external laboratory maintains a causal universe tensor* $\mathbf{U}^{\text{out}}$ *recording all admissible events visible from outside a black hole. The horizon $H$ is the frontier of distinguishability: an informational boundary beyond which no finite extension of* $\mathbf{U}^{\text{out}}$ *can include internal and external refinements in a single, Martin-consistent ordering. Events remain locally finite, but the reconciliation problem saturates: the external ledger cannot compute a consistent extension that includes both sides.*

*As an infalling system approaches $H$, its internal refinements accelerate. By the Axiom of Cantor,* $\mathbf{U}^{\text{out}}$ *may not erase distinctions it has already recorded; by the Axiom of Planck, it may not invent invisible refinements. When the bridge of admissible overlap collapses—when no joint ordering of internal and external updates remains feasible—the external ledger must perform a repair. Martin's Condition demands a globally consistent ordering on the accessible side.*

*The repair introduces* surrogate refinements $e_{\text{rad}}$*:*

$$\mathbf{U}^{\text{out}}_{n+1} = e_{\text{rad}}\, \mathbf{U}^{\text{out}}_{n},$$

*a compensatory update that restores coherence without referencing inaccessible events. These surrogates are not particles escaping from behind the* *horizon; they are the unique refinements that preserve global order when the boundary can no longer reconcile the missing interior. The exponential spectrum attributed to Hawking radiation reflects the combinatorial multiplicity of admissible surrogate updates once the informational channel saturates.*

*Thus, Hawking radiation is not a quantum field effect in curved spacetime. It is the minimal bookkeeping required to maintain Martin consistency on the visible side of an informational boundary. The horizon enforces a holographic constraint: global order must remain representable on the surface that separates what can be reconciled from what cannot.*

## 4.3 The Law of Boundary Consistency

Every example in this chapter has the same structure. When a new admissible refinement is recorded, the ledger does not alter the interior of the accumulated state. Instead, it repairs only the frontier where two descriptions overlap. The Causal Folding Operator updates the boundary and leaves the interior fixed. This pattern is universal and admits a formal statement.

**Law 2** (The Law of Boundary Consistency)**.** *In any locally finite causal domain, every admissible update to the accumulated causal universe tensor* $\mathbf{U}$ *arises from boundary refinement. The interior of* $\mathbf{U}$ *is fixed by previously recorded distinctions: altering it would introduce an invisible refinement (Axiom of Planck) or remove a recorded one (Axiom of Cantor), both of which are forbidden. When a new admissible event is observed, the ledger repairs only the frontier where two descriptions overlap, enforcing Martin's Condition on the boundary of the accumulated state.*

*Therefore all dynamics—propagation, interaction, interference, and decay—are the shadows of boundary reconciliation. Nothing propagates through the interior; motion is the smooth limit of reconciling admissible distinctions at the frontier of* $\mathbf{U}$*.*

**Remark 16.**

No interior modification. *Once folded, the interior of* **U** *contains no unobserved structure. Any change to it would imply either an invisible refinement or the erasure of a recorded one, violating Planck or Cantor.*

Minimal repair. *When ledgers overlap, the operator updates only the smallest region where a contradiction could occur. This is a boundary operation, not a volume operation.*

Computability. *Martin's Condition is enforced by checking only the joint frontier: the causal surface where two descriptions must agree. No global search or re-evaluation of the interior is required.*

Operational meaning. *Waves, interference, scattering, advection, and diffusion appear in the smooth limit of boundary reconciliation. The equations of motion arise from the unique completion that preserves the folded boundary without altering the interior.*

This law closes the algebra of interaction. The Causal Folding Operator enforces global consistency by repairing only the frontier of the accumulated state. Every dynamic phenomenon considered in this chapter—the Dantzig pivot of entanglement, the Mach–Zehnder interference fold, the Bell–Aspect repair, and the surrogate refinements of a causal horizon—is an instance of the same rule: the ledger changes only at the boundary.

This statement is the discrete analogue of Gauss's Theorem. In the continuum, specifying the value of a field on a closed boundary determines its interior uniquely. The Law of Boundary Consistency asserts the same principle for causal ledgers: every admissible refinement enters through the frontier where two descriptions overlap, and the interior is fixed by previously recorded distinctions. Nothing propagates through the volume of **U**; every update is a boundary repair.

All examples in this chapter—velocity boosts, interference, entanglement, and surrogate events near a causal horizon—share this structure. A new admissible event forces only the minimal reconciliation on the overlap. The interior never changes. Motion is the continuum shadow of this purely discrete principle.

At this point nothing further is required. Once every admissible update is confined to the boundary, the smooth limit follows automatically: the interior is fixed, and all variation arises from finite differences on the frontier. The familiar equations of motion are just the continuum shadow of these discrete boundary repairs. Writing them down is a matter of expressing the boundary updates in finite–difference form and passing to the smooth limit.

## 4.4 Classical Transport

**N.B.**—Nothing in this construction asserts that a differential equation *must* govern the data. We show only that if the ledger admits a smooth completion consistent with the axioms, then the corresponding differential equation appears as its unique smooth shadow. The calculus is a consequence of measurement consistency, not an independent postulate. □

Classical transport is the process by which refinement differences reconcile across space. In the discrete ledger, this appears as iterated boundary smoothing: sharp discontinuities trigger local folds until no admissible repair remains. In the smooth limit, these reconciliation rules generate the transport equations of classical thermodynamics. The organizing principle is the variational order of the correction.

### 4.4.1 First Variation: Slope-Level Ledger Corrections

First-variation updates alter only the slope of the admissible spline representation. Informational minimality forbids the creation of new turning points between event anchors: any correction that introduced a fresh extremum would constitute an unrecorded event. All admissible first-order updates are therefore monotone. Their smooth limit yields irreversible transport.

### Annealing and Conduction (Symmetric Reconciliation)

Conduction appears when a ledger repeatedly reconciles a coarse description of itself. A sharp difference in refinement counts across a boundary triggers a sequence of local folds, each of which reduces the discrepancy without altering the interior. This iterative process is *annealing*: informational tension is monotonically released until no further repair is admissible.

Under the Law of Spline Sufficiency, symmetric reconciliation introduces no oscillation and no hidden curvature. The discrete flux is governed by the centered jump between neighboring cells, and the update rule is a symmetric projection back into the admissible class. In the smooth limit, these finite differences converge to the classical diffusion equation.

**Discrete Ledger Update and the Flux Form.** Let $u_i^k$ denote the normalized refinement count recorded on cell $i$ at discrete time $t_k$, with spatial spacing $\Delta x$ and time step $\Delta t$. The update must obey informational conservation in a conservative flux form:

$$u_i^{k+1} \;=\; u_i^k \;-\; \frac{\Delta t}{\Delta x}\big(F_{i+\frac{1}{2}}^k - F_{i-\frac{1}{2}}^k\big). \tag{4.2}$$

*Symmetric reconciliation* uses the centered jump as the flux. If $\kappa$ is the informational diffusion coefficient,

$$F_{i+\frac{1}{2}}^k \;=\; -\,\kappa\,\frac{u_{i+1}^k - u_i^k}{\Delta x}. \tag{4.3}$$

Substituting (4.3) into (4.2) yields the standard symmetric smoothing rule:

$$u_i^{k+1} \;=\; u_i^k \;+\; \frac{\kappa\,\Delta t}{\Delta x^2}\big(u_{i+1}^k - 2u_i^k + u_{i-1}^k\big). \tag{4.4}$$

*Proof Sketch: Convergence to $u_t = Du_{xx}$.* Approximate the temporal derivative using a forward difference:

$$u_t(x_i, t_k) \;\approx\; \frac{u_i^{k+1} - u_i^k}{\Delta t}.$$

Substituting (4.4) and rearranging,

$$\frac{u_i^{k+1} - u_i^k}{\Delta t} \;=\; \frac{\kappa}{\Delta x^2}\left(u_{i+1}^k - 2u_i^k + u_{i-1}^k\right).$$

The spatial term on the right is the standard centered approximation of the second derivative,

$$u_{xx}(x_i, t_k) \;\approx\; \frac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{\Delta x^2}.$$

Thus

$$u_t(x_i, t_k) \;=\; \kappa\, u_{xx}(x_i, t_k).$$

Taking the continuous limit $\Delta x, \Delta t \to 0$ and letting $\kappa \to D$ yields the diffusion equation

$$u_t = D\, u_{xx}.$$

$\square$

The convergence is admissible because the Law of Spline Sufficiency guarantees that the solution remains $\mathcal{C}^2$ and introduces no hidden curvature. The symmetric finite-difference update is therefore a monotone, stable smoothing process: the smooth shadow of informational annealing.

### Convection and Oriented Transport (Boundary Consistency)

Convection models the directed transport of distinctions, where the orientation of the flow is realized as a preferred direction in the causal refinement process. When a boundary carries an orientation, reconciliation must respect that direction: smoothing from the downstream side would create unrecorded structure on the wrong side of the interface.

**Oriented Boundary Reconciliation.** Let $u_i^k$ be the normalized refinement count on cell $i$ at time $t_k$. When the interface $(i, i{+}1)$ has a known inflow direction, the Law of Boundary Consistency requires that the ledger flux across that interface be determined solely by the state on the inflow side:

$$F_{i+\frac{1}{2}}^k = c\,u_i^k, \tag{4.5}$$

where $c$ is the order speed. Substituting (4.5) into the conservative update

$$u_i^{k+1} \;=\; u_i^k \;-\; \frac{\Delta t}{\Delta x}\big(F_{i+\frac{1}{2}}^k - F_{i-\frac{1}{2}}^k\big) \tag{4.6}$$

yields the upwind rule

$$u_i^{k+1} \;=\; u_i^k \;-\; \frac{c\,\Delta t}{\Delta x}\,\big(u_i^k - u_{i-1}^k\big). \tag{4.7}$$

*Proof Sketch: Convergence to $u_t + c\,u_x = 0$.* Divide (4.7) by $\Delta t$ to obtain

$$\frac{u_i^{k+1} - u_i^k}{\Delta t} \;=\; -\,c\,\frac{u_i^k - u_{i-1}^k}{\Delta x}.$$

As $\Delta t, \Delta x \to 0$, the left side is the forward difference approximation of the time derivative $\partial_t u$, and the right side is the backward difference approximation of the space derivative $\partial_x u$. Taking the smooth limit yields the advection equation

$$u_t + c\,u_x = 0.$$

□

The update (4.7) is admissible only when it remains monotone, which is guaranteed by the CFL condition $0 \le c\,\Delta t/\Delta x \le 1$. Under this constraint no new turning points are introduced, so the Law of Spline Sufficiency is respected: the directed transport is a projection back into the admissible spline class.

**N.B.**—Boundary Consistency selects the upwind flux, and Spline Sufficiency forbids oscillatory corrections; the advection equation is the smooth shadow of oriented ledger reconciliation. □

### Advection–Diffusion (Mixed Closure)

In many settings, admissible reconciliation requires both symmetric homogenization and directed transport. The ledger must smooth local inconsistencies while simultaneously respecting boundary orientation. The resulting update combines the symmetric and upwind fluxes.

**Combined Flux.** Let the oriented flux be given by

$$F^{\mathrm{adv}}_{i+\frac{1}{2}} = c\, u_i^k,$$

and the symmetric flux by

$$F^{\mathrm{diff}}_{i+\frac{1}{2}} = -\,\kappa\, \frac{u_{i+1}^k - u_i^k}{\Delta x}.$$

The total flux across the interface is their sum:

$$F^k_{i+\frac{1}{2}} = c\, u_i^k - \kappa\, \frac{u_{i+1}^k - u_i^k}{\Delta x}. \tag{4.8}$$

Substituting (4.8) into the conservative update

$$u_i^{k+1} \;=\; u_i^k - \frac{\Delta t}{\Delta x}\big(F^k_{i+\frac{1}{2}} - F^k_{i-\frac{1}{2}}\big) \tag{4.9}$$

yields the discrete advection–diffusion rule

$$u_i^{k+1} = u_i^k - \frac{c\,\Delta t}{\Delta x}\,(u_i^k - u_{i-1}^k) + \frac{\kappa\,\Delta t}{\Delta x^2}(u_{i+1}^k - 2u_i^k + u_{i-1}^k). \tag{4.10}$$

*Proof Sketch: Convergence to $u_t + c\,u_x = D\,u_{xx}$.* Divide (4.10) by $\Delta t$ to obtain

$$\frac{u_i^{k+1} - u_i^k}{\Delta t} = -\,c\,\frac{u_i^k - u_{i-1}^k}{\Delta x} + \kappa\,\frac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{\Delta x^2}.$$

In the limit $\Delta x, \Delta t \to 0$, the left side becomes $\partial_t u$, the first term becomes $-c\,\partial_x u$, and the second becomes $\kappa\,\partial_{xx} u$. Setting $D = \kappa$ gives

$$u_t + c\,u_x = D\,u_{xx},$$

the advection–diffusion equation. □

The mixed closure is the most general first-order reconciliation of the refinement record. Information spreads down gradients (diffusion) while coherent packets of distinction are carried along oriented interfaces (advection). The process is irreversible in either mode, and no additional structure is assumed beyond the slope-level correction forced by the axioms.

**N.B.**—In every case, first-variation closure is a projection back into the admissible spline class: no new extrema are introduced, and no hidden structure appears under refinement. The differential equations are the smooth shadows of monotone reconciliation. □

### 4.4.2 Second Variation: Curvature-Level Ledger Corrections

Second-variation updates alter curvature while preserving slope and anchor values. These corrections are reversible: they propagate distinctions without loss and produce no additional smoothing. Their smooth limit yields wave transport.

#### Radiation (Symmetric Curvature Smoothing)

Radiation represents the propagation of distinction at the maximal admissible speed. Unlike the first–order corrections of Sections 4.4.1–4.4.1, radiation is reversible: once the ledger has reconciled curvature symmetrically, no net informational gain or loss remains. The process is the smooth shadow of *symmetric curvature smoothing.*

**Vanishing Second Variation.** Let $\mathcal{A}$ denote the amplitude of distinction recorded over a finite causal neighborhood. The second variation $\delta^2\mathcal{A}$ measures the change in $\mathcal{A}$ under two sequential, infinitesimal perturbations of the record. Radiation occurs when these perturbations commute exactly:

$$\delta^2\mathcal{A} = 0. \tag{4.11}$$

No net expansion or contraction of distinguishability can remain; curvature differences are repaired symmetrically and without directional bias. This is the reversible complement of annealing: where first–order correction removes slope–level inconsistencies, second–order correction removes curvature–level tension.

**Discrete Curvature Laplacian.** In the discrete domain, the sum of all pairwise second variations over neighboring events defines the discrete Laplacian on event sets:

$$\nabla_E^2\mathcal{A} \;=\; \sum_{f\in \mathrm{Nbr}(e)} \big(\mathcal{A}(f) - \mathcal{A}(e)\big).$$

Martin's Condition enforces that this curvature vanishes:

$$\nabla_E^2\mathcal{A} = 0, \tag{4.12}$$

so that symmetric curvature smoothing is locally maximal and globally neutral.

**Smooth Shadow.** In the continuum limit, the second–order symmetric closure converges to the homogeneous wave equation. If $u(x,t)$ is the smooth completion of the refinement record, then

$$u_{tt} = c^2\, u_{xx}, \tag{4.13}$$

where $c$ is the order speed—the combinatorial rate at which causal constraints traverse the event network. Equation (4.13) expresses reversible propagation: local expansions and contractions of distinguishability cancel globally, so that information moves without net amplification or dissipation.

**N.B.**—Second–variation closure enforces symmetric curvature repair and forbids net informational gain or loss. The wave equation is therefore the unique smooth shadow of reversible curvature smoothing, derived solely from the axioms of causal refinement. □

### Adiabatic Transport (Curvature Invariance)

Adiabatic transport is the ideal limit of reversible motion in the causal record. Distinctions are neither created nor destroyed: informational entropy remains constant, and the curvature of the smooth completion is preserved. This process is the logical dual of annealing, establishing the boundary condition for zero informational work.

**Invariance of Distinguishability.** Let $\lambda$ parameterize a smooth evolution of an admissible history $\Psi(\lambda)$. The history undergoes adiabatic transport when the informational entropy is invariant:

$$\frac{d}{d\lambda}\, \mathcal{S}(\Psi) = 0. \tag{4.14}$$

Equivalently, the update operator satisfies

$$U_{\lambda+\delta\lambda} = U_\lambda + \mathcal{O}(\delta\lambda^2),$$

so the leading–order change in the refinement record vanishes. The motion is norm–preserving and informationally reversible: the ledger drifts without loss of distinction.

**Curvature Invariance.** Because $\mathcal{S}$ counts admissible configurations, the condition (4.14) forces the evolution to proceed along a path of constant informational curvature. Locally,

$$\frac{d}{d\lambda}\,\Psi'' = 0, \tag{4.15}$$

so that no curvature–level tension is released or accumulated. This is the reversible complement to the symmetric curvature smoothing of Section 4.4.2.

**Smooth Shadow.** Under the Law of Spline Sufficiency ($\Psi^{(4)} = 0$), curvature invariance selects the unique extremal that transports distinctions without dissipation: the geodesic or undamped wave. Informational entropy remains constant, and the ledger evolves along the smooth completion $\Psi$ without net repair or decay. Nothing dynamical is postulated; the law is a theorem of informational conservation.

**N.B.**—Adiabatic transport is the limit of causal motion that preserves informational order. It connects reversible evolution ($d\mathcal{S} = 0$) with the requirement that distinguishability cannot decrease. The geodesic structure is therefore a consequence of informational invariance, not an independent physical postulate. □

## 4.5 Quantum Transport

Some transport phenomena do not appear as flows of a substance, but as discrete repairs of nearly degenerate descriptions. When two ledgers support multiple admissible extensions, the Causal Folding Operator must select the unique completion that preserves all recorded distinctions. The familiar quantum effects arise as the smooth shadows of this repair.

### 4.5.1 The Casimir Effect: Boundary-Limited Distinction

The Casimir effect is the boundary expression of informational pressure. When admissible refinements are restricted by geometry, the ledger must perform a compensatory update to preserve global distinguishability. In the smooth limit, this boundary repair appears as a physical force.

**Boundary–Induced Asymmetry.** Consider two parallel constraints that restrict the admissible causal updates in the interior region. Each admissible field mode corresponds to a distinguishable refinement of the causal record. The plates suppress many of these modes, so the interior ledger records fewer admissible distinctions than the exterior. Outside the plates, no such suppression occurs; the ledger remains unrestricted. This produces an imbalance in refinement counts across the boundary: the exterior supports strictly more admissible updates than the interior.

**Compensatory Boundary Update.** The Second Law of Causal Order requires that global distinguishability must not decrease. The imbalance therefore creates informational tension. Because no additional interior modes are admissible, the only possible repair is a boundary update that restores global consistency without altering the restricted interior. The unique correction is an outward curvature of the boundary ledger: refinements accumulate on the exterior frontier, pushing the constraints toward one another.

In the smooth limit, this boundary curvature appears as the Casimir pressure. No mechanical postulate is introduced; the force is the smooth shadow of a compensatory update that restores consistency between the restricted interior and unrestricted exterior ledgers.

**N.B.**—In this interpretation, the Casimir effect is a holographic phenomenon: the minimal boundary correction enforced by global distinguishability. The pressure is not a hypothesis about zero–point energy, but the unique repair consistent with the axioms of causal refinement. □

### 4.5.2 Alpha Decay: Repair of a Causal Contradiction

Alpha decay is the irreversible repair of a causal contradiction on the boundary of the nuclear ledger. The nucleus admits two nearly indistinguishable continuations of its refinement record:

$$\Psi_{\text{bound}} \qquad \text{and} \qquad \Psi_{\text{unbound}}.$$

Both are initially admissible: each agrees with all external anchors and differs only within a bounded interior neighborhood.

**Causal Degeneracy and Symmetry Drift.** Over informational time, unresolved curvature accumulates and the two ledgers drift out of alignment. Their boundary descriptions become incompatible with Martin Consistency: the overlap cannot be reconciled without introducing unrecorded structure. A repair is required to preserve the global order of the causal record.

**Removal of an Inconsistent Branch.** The Causal Folding Operator $f$ performs the minimal corrective update by removing the inconsistent branch:

$$f: \ \Psi_{\text{bound}} \longrightarrow \Psi_{\text{unbound}} \ + \ \alpha.$$

The emitted alpha particle is the recorded trace of this boundary repair. The interior ledger returns to an admissible configuration, and the causal record evolves on the remaining branch.

**Smooth Shadow.** In the continuum limit, the finite differences of this irreversible repair produce the exponential law of radioactive decay. No hidden forces or tunneling mechanism is assumed: alpha decay is the unique boundary update that eliminates a causal contradiction while preserving global distinguishability.

**N.B.**—Alpha decay is the irreversible removal of an inconsistent branch from the refinement record. The emitted particle is the holographic trace of the boundary correction, not a postulated tunneling object. □

### 4.5.3 Gamma Decay (Restoration of Causal Symmetry)

Gamma decay is a reversible repair of internal causal symmetry. An excited nuclear state corresponds to an admissible configuration whose internal refinement record is nearly, but not exactly, consistent with the minimal ground state. Over time, unresolved curvature accumulates, producing a small informational asymmetry in the internal ledger.

**Informational Synchronization.** Let $\Psi^*$ denote the smooth completion of the excited state and $\Psi$ that of the ground state. Both are admissible: they agree on all external anchors and differ only in a bounded internal neighborhood. The difference is a phase drift in the internal causal partition—a small curvature that violates informational minimality. The nucleus must perform a repair that restores the unique, globally consistent ground state.

The minimal symmetric repair is the emission of a gamma photon:

$$\Psi^* \longrightarrow \Psi + \gamma.$$

The photon is the propagated correction: a reversible wave of order that carries the excess curvature away from the nucleus while leaving the internal ledger in its minimal configuration.

**Zero–Mass Boundary Repair.** Unlike alpha decay (Section 4.5.2), which removes an entire inconsistent branch from the record, gamma decay preserves the identity of the nucleus. It is informationally reversible: no new branches are created, and no admissible distinctions are destroyed. The process is the smooth shadow of symmetric curvature repair:

$$\delta^2\mathcal{A} = 0 \quad \Longrightarrow \quad \text{emission of } \gamma \text{ with } E = h\nu.$$

The energy of the photon measures the amount of curvature removed from the internal ledger. No mechanical postulate is required; gamma decay is the unique boundary update that restores global distinguishability without altering the underlying causal identity of the system.

**N.B.**—In this interpretation, gamma decay is not a force–mediated transition, but a minimal holographic correction: a reversible synchronization event that propagates excess curvature as a photon and restores Martin Consistency in the internal ledger without altering the causal identity of the nucleus. □

### 4.5.4 Brownian Motion: Diffusive Boundary Uncertainty

#### Brownian Motion as Quantized Uncertainty

Brownian motion can be interpreted as a quantum informational phenomenon in the present framework. The source of randomness is not mechanical noise but *finite causal resolution*: each refinement step leaves a family of equally admissible micro–orderings that the ledger cannot distinguish. The coarse record therefore evolves stochastically.

**Stochastic Reconciliation at Finite Resolution.** Let $u_i^k$ be the normalized refinement count on cell $i$ at time $t_k$. When the observer cannot resolve all admissible distinctions at scale $\Delta x$, the symmetric smoothing update acquires an irreducible stochastic term:

$$u_i^{k+1} = u_i^k + \frac{\kappa\,\Delta t}{\Delta x^2}(u_{i+1}^k - 2u_i^k + u_{i-1}^k) + \sqrt{2D\,\Delta t}\,\xi_i^k, \qquad \mathbb{E}[\xi_i^k] = 0,\ \mathbb{E}[(\xi_i^k)^2] = 1. \tag{4.16}$$

The deterministic part is the symmetric reconciliation enforced by the Law of Spline Sufficiency; the random term is the ledger's irreducible uncertainty at the observation scale.

**Smooth Shadow: Diffusion as Quantum Measure.** Under refinement $\Delta x, \Delta t \to 0$ with $D$ fixed, the central limit theorem implies convergence of (4.16) to the diffusion equation for the coarse density $u(x,t)$:

$$u_t = D\,u_{xx}. \tag{4.17}$$

Here $D$ is the *informational diffusion coefficient*: the effective bandwidth of unresolved distinctions per unit time.

**Bridge to Schrödinger via Analytic Continuation.** The free Schrödinger equation is related to diffusion by analytic continuation of time. Setting $D = \frac{\hbar}{2m}$ and $t \mapsto -it$ maps (4.17) to

$$i\,\hbar\,\partial_t \Psi = -\frac{\hbar^2}{2m}\,\partial_{xx}\Psi, \tag{4.18}$$

i.e., the smooth shadow of unresolved, symmetric refinement at fixed informational bandwidth equals the quantum free evolution with Planck scale $\hbar$. In this sense, Brownian motion is *quantized uncertainty*: $\hbar$ calibrates the minimal unresolved action, while $D$ measures the rate at which that unresolved structure propagates statistically.

**Consistency with the Two Laws.** - *Spline Sufficiency* ensures no spurious extrema: the stochastic update remains a projection into the admissible class almost surely. - *Boundary Consistency* fixes oriented interfaces; adding an upwind drift $c$ to (4.16) yields the standard advection–diffusion (Fokker–Planck) limit.

**N.B.**—This construction shows *how* quantum evolution can arise from measurement limits: if the ledger's unresolved bandwidth $D$ is fixed by a Planck scale, diffusion analytically continues to Schrödinger dynamics. It does not assert that nature must realize this identification in every regime. □

## 4.6 First Quantization as an Application of the Two Laws

The classical picture of quantization treats the wavefunction, Hilbert space, and operator algebra as new physical axioms. In the present framework they arise automatically from the two kinematic consistency laws:

- **Law of Spline Sufficiency:** no admissible refinement may introduce unrecorded structure; smooth closure is $\mathcal{C}^2$ and satisfies $\Psi^{(4)} = 0$,
- **Law of Boundary Consistency:** oriented boundaries must be reconciled from the inflow side; no correction may propagate across a boundary in the wrong direction.

Together, these laws force the structure known in physics as *first quantization*. Nothing new is added: the quantized theory is the smooth shadow of informational bookkeeping.

### 4.6.1 Hilbert Structure from Spline Closure

Under Spline Sufficiency, every admissible history has a unique smooth representative $\Psi$ that is cubic between anchors and $\mathcal{C}^2$ globally. Any two admissible histories $\Psi$ and $\Phi$ differ only in their recorded curvature. Their overlap is therefore measured by the curvature functional

$$\langle \Psi,\, \Phi \rangle = \int \Psi''(x)\, \Phi''(x)\, dx.$$

This inner product is positive definite on the admissible class and yields a complete inner-product space: the Hilbert space of admissible closures. The "wavefunction" is nothing more than $\Psi$ viewed as an element of this space.

### 4.6.2 Canonical Structure from Boundary Consistency

The curvature functional determines a unique conjugate operator. Integration by parts yields

$$\langle \Psi,\, x\,\Phi \rangle - \langle x\,\Psi,\, \Phi \rangle = \int \Psi(x)\, \Phi'(x)\, dx,$$

where the boundary term is fixed in sign by the inflow rule of Boundary Consistency. The operator that realizes this antisymmetry is

$$\hat{p} = -\, i\, \partial_x,$$

the momentum operator of canonical quantization. No new axiom is required: the oriented boundary rule uniquely determines the self-adjoint generator of translations.

### 4.6.3 Energy Levels from Informational Minimality

Consider an admissible history constrained by a restoring boundary (a fold that always returns toward the anchor). Under Spline Sufficiency the closure is cubic between anchors and $\Psi^{(4)} = 0$; under Boundary Consistency the inflow rule forces the curvature to alternate monotonically between turning points. The Galerkin limit of this curvature balance is the harmonic oscillator:

$$-\Psi''(x) + x^2\Psi(x) = \lambda\Psi(x),$$

whose eigenvalues are discrete because no new turning points may be added between anchors. The spectrum is the familiar

$$\lambda_n = (2n+1), \qquad n = 0, 1, 2, \ldots$$

Quantization is therefore a *restriction of admissible curvature*, not a postulate about nature.

### 4.6.4 Summary

- Spline Sufficiency $\Rightarrow$ Hilbert space of smooth closures,
- Boundary Consistency $\Rightarrow$ canonical commutators,
- Discrete curvature balance $\Rightarrow$ quantized energy levels.

$$\text{finite ledger} \xrightarrow{\text{spline closure}} \Psi \xrightarrow{\text{boundary consistency}} \hat{x},\, \hat{p} \xrightarrow{\text{curvature balance}} \text{quantized energies.}$$

Thus the apparatus of "first quantization" is not a new physics. It is the smooth bookkeeping of the two kinematic laws applied to finite informational records.

**N.B.**—In this sense, quantization is not an independent hypothesis. It is the minimal correction rule forced by informational sufficiency and boundary orientation. $\square$

# Coda: The Informational Harmonic Oscillator [88]

**Thought Experiment 4.6.1** (The Informational Harmonic Oscillator)**.** ***N.B.*** *This coda introduces the simplest reversible dynamics permitted by the Second Law of Causal Order. It shows that oscillation—in its most abstract sense—arises whenever information alternates between two complementary forms: record and prediction. No physical mass, force, or energy is implied. The oscillator here is entirely informational: a minimal closed loop of distinguishability [88, 68, 114].*
***N.B.***—*The refinement argument developed in this section parallels the mathematical structure of Planck's resolution of the ultraviolet catastrophe. The similarity is formal only. No physical quantization assumptions are introduced here; the discrete refinement spectrum arises solely from informational consistency and the minimal distinction bound $\epsilon$.* □

Setup. *Let $(x, p)$ denote a conjugate pair of informational coordinates on a two–dimensional causal phase space. $x$ represents the observer's recorded distinctions (the state of knowledge); $p$ represents the predictive momentum (the rate at which distinctions are changing). Define the informational Hamiltonian*

$$\mathcal{S}(x, p) = \tfrac{1}{2}\,(\alpha x^2 + \beta p^2),$$

*where $\alpha$ and $\beta$ are positive constants measuring informational stiffness and inertia. The reversible evolution equations are*

$$\dot{x} = \frac{\partial \mathcal{S}}{\partial p} = \beta p, \qquad \dot{p} = -\frac{\partial \mathcal{S}}{\partial x} = -\alpha x.$$

*Eliminating $p$ yields*

$$\ddot{x} + \omega^2 x = 0, \qquad \omega^2 = \alpha\beta.$$

*Thus the observer's state executes harmonic motion in informational phase space with constant total measure* $\mathcal{S}(x, p)$.

Interpretation. *At each turning point of the oscillation, information is maximally localized: the record* $x$ *is fixed, the prediction* $p$ *vanishes. At each midpoint, prediction dominates and the record is momentarily indeterminate. The system alternately* stores *and* transmits *distinguishability, maintaining constant total informational entropy. The cycle expresses the complementarity of knowledge and expectation: every complete measurement must eventually swing back toward uncertainty to preserve* $\Delta S \geq 0$.

Relation to Transport. *The four informational transport regimes appear as limiting cases of this oscillator:*

- ***Adiabatic transport:*** *small, continuous oscillations with* $\dot{S} = 0$ *(reversible exchange of information).*
- ***Annealing:*** *inclusion of damping* $\gamma > 0$, *giving* $\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$, *a monotonic relaxation to equilibrium.*
- ***Brownian motion:*** *addition of stochastic forcing* $\xi(t)$, $\ddot{x} + \omega^2 x = \xi(t)$, *producing diffusive variance growth.*
- ***Wave/particle duality:*** *interpretation of* $(x, p)$ *as the conjugate pair of localization and amplitude—dual views of the same informational invariant.*

*Each regime preserves causal consistency and satisfies* $\Delta S \geq 0$*; only the mode of information exchange changes.*

Scope. *This oscillator is the canonical closed system of the informational universe: a bounded transformation in which every increase in record precision is offset by a proportional loss of predictive capacity, and vice versa. It represents the minimal rhythm of causal order—the reversible heartbeat of information itself.*

*Motion, in this theory, is not caused by energy. It is the preservation of order under Martin's Condition.*

# Chapter 5

# The Quantum of Information

The preceding chapters established that smooth motion appears as the unique closure of causal order under refinement. The Law of Spline Sufficiency showed that any admissible continuous shadow must contain no unrecorded structure and therefore satisfies the extremal condition $\Psi^{(4)} = 0$.

In this chapter we examine the opposite extremum: the smallest admissible refinement of the Causal Universe Tensor. Such a refinement represents the maximal rate at which distinguishability can propagate without violating the Axiom of Planck. We call this minimal, nonzero update an *informational quantum.* It is not a physical particle or field; it is the atomic refinement permitted by the axioms.

Along an extremal refinement of this form, the scalar invariants of the Causal Universe Tensor remain stationary. Any attempt to subdivide the update would imply the existence of additional events, contradicting the record. Thus the geometry of extremal propagation follows entirely from the consistency of order.

The continuous limit of this extremal behavior admits three structures:

- A *metric* $g_{\mu\nu}$, the bilinear gauge of distinguishable separation.
- A *connection* $\Gamma^{\lambda}{}_{\mu\nu}$, the rule of causal transport required to propagate

local labels while maintaining Martin consistency.

- A *curvature tensor*, the residue of inconsistency that remains when the transport fails to close around a refinement loop.

No geometric or dynamical postulates are introduced. Each structure arises only as the smooth extension forced by the requirement that extremal refinements remain globally coherent. The kinematics of light is therefore the kinematics of maximal distinguishability: the gauge obtained when the rate of informational refinement saturates the causal limit.

The remainder of this chapter develops these structures systematically.

## 5.1 The Informational Bound $\epsilon$

**N.B.**—The refinement bound $\epsilon$ is not a physical quantum, particle, or energy unit. It is the minimal nonzero increment of distinguishable structure that survives every admissible refinement of the measurement record. Its origin is purely informational: $\epsilon$ is the continuous shadow of the residual $\mathcal{C}^2$ freedom in spline closure. No physical ontology is implied. □

The refinement of an observational record proceeds through countable additions of distinguishable events. As established in Chapter 3, the weak form of the discrete bending functional admits a single free $\mathcal{C}^2$ parameter, corresponding to the third derivative of the spline interpolant. This degree of freedom cannot be removed unless new measurements are recorded. When no additional events are observed, the residual freedom is irreducible, and the continuous shadow must reflect a minimal, nonvanishing bound on further curvature–level distinction.

We denote this invariant residual by $\epsilon$. Any admissible refinement of the continuous shadow must preserve $\epsilon$; to refine below this threshold would introduce unrecorded structure and contradict the finite measurement sequence. Conversely, any refinement that preserves $\epsilon$ remains consistent with the discrete data. Thus $\epsilon$ functions as the kinematic limit of refinement and provides the foundation for the emergent invariant interval $\tau$, the metric $g_{\mu\nu}$, and the compatible connection derived later in this chapter.

### 5.1.1 Residual Spline Freedom and the Minimal Refinement Bound

The weak formulation developed in Chapter 3 shows that a sequence of discrete measurements determines a unique cubic spline interpolant, except for a single free parameter associated with the third derivative. This residual freedom is a consequence of the information minimality constraint: in the absence of additional recorded events, the interpolant must not introduce structure that was not observed. A fourth–order correction would imply an unrecorded change in curvature and is therefore inadmissible.

Let $\Psi$ denote the continuous shadow of the discrete observational chain. The freedom in $\Psi'''$ represents the smallest degree of curvature that can be varied without conflicting with the discrete record. Its magnitude cannot be reduced by any admissible refinement unless a new event occurs. The minimal nonzero shadow of this residual is the refinement bound $\epsilon$.

Formally, $\epsilon$ is the infimum of all curvature–level refinements that do not violate the existing event sequence. Any attempt to impose a refinement of order smaller than $\epsilon$ would create new curvature that must be supported by new events, contradicting the record. Thus $\epsilon$ is the continuous representation of the irreducible $\mathcal{C}^2$ residue.

### 5.1.2 Maximal Informational Propagation

An admissible refinement of the observational record adds distinguishable structure without contradicting previously recorded events. A path that *saturates* the refinement bound $\epsilon$ propagates information at the maximal admissible rate: it incorporates all allowable distinction while introducing no unrecorded curvature.

Such paths form the extremal curves of the informational geometry. They are defined not by physical principles, but by the logical requirement that refinement cannot fall below the $\epsilon$ threshold. Any further reduction would imply hidden structure and is therefore inadmissible.

In the continuous shadow, these maximally propagated paths serve as the reference curves for defining the invariant interval $\tau$. Two observers who refine the same extremal path must agree on the number of informational units required to describe it; this count determines the causal interval and anchors the construction of the metric in Section 5.2.

**Thought Experiment 5.1.1** (The Compact Disc Reader and the Gauge of Separation)**. *N.B.*—***This thought experiment does* not *describe photons as informational quanta. It is a finite conceptual model illustrating how a gauge of separation emerges from the logic of distinguishability alone. No physical ontology is implied.* □

*A compact disc stores information as a finite, ordered chain of distinctions. Each pit or land corresponds to a single admissible event, and the reader detects a new event only when the reflected signal exceeds its threshold of discernibility. Everything below this threshold is invisible; it cannot enter the admissible record. Thus the sequence of detections,*

$$e_1 \prec e_2 \prec e_3 \prec \cdots ,$$

*encodes not only what* was *observed, but the binding constraint that no additional distinguishable structure may be inserted between these events.*

*From the standpoint of information, the read head defines a* gauge of minimal separation*: two surface configurations are "far enough apart" exactly when the detector must refine its admissible description to distinguish them. The metric is not assumed; it is inferred from the rule that only resolvable differences may appear as refinements in the causal chain.*

*Now imagine two readers, A and B, scanning the same disc. Reader A has a coarser threshold; reader B resolves finer distinctions. Each produces its own ordered sequence of admissible events. Where B records additional refinements, A records none. Yet when their records are merged, global coherence requires a single history that preserves all recorded distinctions. The finer record forces a refinement on the coarser: A must treat certain portions of the disc as informationally extended, for failure to accommodate B's distinctions would render the merged history inconsistent.*

*In the dense limit, this refinement rule induces a continuous connection: the shadow of the logical requirement that adjacent descriptions remain compatible under transport. What appears in the smooth theory as a* metric *is nothing more than this bookkeeping of distinguishability: the minimal rule that certifies when two states differ in a way that must be reconciled.*

*In this model, "light" corresponds not to a substance but to the maximal rate at which new distinctions can be admitted without contradiction. Any attempt to introduce refinements faster than this rate would violate global coherence. Thus the invariant causal interval of Chapter 5 reflects the same constraint: an observer may not admit distinctions faster than a globally coherent merge can support.*

*The compact disc reader therefore offers a finite, concrete metaphor for the emergence of the gauge of light, the metric as a rule of separation, and the transport laws that follow from informational consistency.*

## 5.2 The Metric as a Gauge of Informational Separation

**N.B.**—The metric is not a physical fabric, field, or medium. It is the continuous shadow of the rule by which refinements preserve the invariant causal interval. Its function is purely informational: the metric enforces that all admissible observers agree on the amount of distinguishable structure between neighboring events. No geometric ontology is assumed. □

The refinement bound $\epsilon$ defined in Section 5.1 limits how finely an admissible history may be resolved without contradicting the measurement record. When this bound is propagated along an extremal path, the result is a conserved quantity—the informational interval $\tau$—which represents the number of $\epsilon$-sized refinements required to traverse that path. Any two observers refining the same extremal sequence must therefore agree on the value of $\tau$. This invariance is the kinematic foundation of the metric.

Let $dx^\mu$ denote the local labels an observer assigns to successive events along an extremal refinement. Another observer, using a different admissible convention for distinguishing the same events, assigns labels $dx'^\mu = \Lambda^\mu{}_\nu dx^\nu$, where the transformation $\Lambda^\mu{}_\nu$ preserves causal order as required by the Axiom of Selection. Although the two observers differ in the coordinates they assign, they must agree on the number of $\epsilon$-sized refinements separating the events; otherwise their merged histories would violate global consistency.

This requirement implies the existence of a bilinear form $g_{\mu\nu}$ such that the scalar quantity

$$\tau^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

is invariant under all admissible changes of local labeling conventions. The metric is therefore the gauge of informational separation: it encodes how distinctions between events are preserved when observers describe them using different coordinate choices.

Because $\tau$ counts informational increments rather than geometric lengths, the invariance condition

$$g'_{\mu\nu}\, dx'^\mu dx'^\nu = g_{\mu\nu}\, dx^\mu dx^\nu$$

expresses the deeper statement that the number of distinguishable refinements between neighboring events is independent of the observer. The metric formalizes this invariance. It is the bilinear bookkeeping rule ensuring that every observer's coordinate description yields the same value of $\tau$ for any given extremal path.

In this interpretation, the components of $g_{\mu\nu}$ do not prescribe a geometry; they record the local comparison rule by which distinguishability is maintained across observational frames. Section 5.2.1 elevates this requirement to a formal law, showing that preservation of the causal interval uniquely determines the compatible affine connection used to propagate distinguishability under refinement.

### 5.2.1 The Law of Causal Transport

**N.B.**—The Law of Causal Transport is a kinematic statement. It asserts only that informational refinements must preserve the invariant interval $\tau$ defined in Section 5.2. No dynamical interpretation of curvature or stress is assumed here. The law specifies how distinguishability must be propagated under admissible changes of frame; all higher structures of connection and curvature follow in later sections. □

The refinement bound $\epsilon$ defines the smallest admissible increment of distinguishable structure. When propagated along an extremal path, $\epsilon$ induces the invariant interval $\tau$, representing the total number of such increments required to describe that path. Because every observer must refine the same underlying event sequence, the value of $\tau$ must remain unchanged under all admissible relabelings.

This requirement leads to the following principle.

**Law 3** (Law of Causal Transport)**.** [Preservation of Distinguishability] *Any admissible refinement of an observational record must preserve the informational interval $\tau$ between neighboring events. In the continuous shadow, this condition determines a unique bilinear form $g_{\mu\nu}$ and a unique compatible rule of transport $\Gamma^{\lambda}_{\mu\nu}$ satisfying*

$$\nabla_\lambda g_{\mu\nu} = 0.$$

*The pair* $(g_{\mu\nu}, \Gamma^{\lambda}_{\mu\nu})$ *constitutes the metric gauge of informational separation.*

Because observers may assign different coordinates to the same infinitesimal event displacement, we represent such a relabeling by $dx'^{\mu} = \Lambda^{\mu}{}_{\nu}\, dx^{\nu}$, where $\Lambda^{\mu}{}_{\nu}$ preserves causal order. The Law of Causal Transport requires that the informational interval be invariant under this transformation:

$$g'_{\mu\nu}\, dx'^{\mu} dx'^{\nu} = g_{\mu\nu}\, dx^{\mu} dx^{\nu}.$$

This invariance elevates $g_{\mu\nu}$ from a mere bookkeeping device to a constraint: it is the only bilinear form that guarantees all observers agree on how many $\epsilon$-sized refinements separate neighboring events.

The law further implies that the comparison of nearby refinements must not depend on the path taken in the space of coordinate labels. This requirement determines the connection coefficients $\Gamma^{\lambda}_{\mu\nu}$ as the unique differential operators that preserve the metric gauge under change of frame.

In this sense, the Law of Causal Transport encodes the most fundamental rule of the kinematic structure: that distinguishability is preserved under motion. The connection is not postulated, but forced by the need to maintain the interval $\tau$ when an observer's coordinate conventions vary from point to point. Section 5.2.2 elaborates the invariance of $\tau$, and Section 5.2.3 formalizes the role of $g_{\mu\nu}$ as the bilinear form that preserves the $\epsilon$–refinement count.

### 5.2.2 Invariance of the Informational Interval $\tau$

**N.B.**—The interval $\tau$ is not a geometric length or a physical duration. It is the continuous shadow of an event count: the number of $\epsilon$-sized refinements required to describe an extremal segment of an observational record. Its invariance expresses only that all admissible observers must agree on the amount of distinguishable structure between neighboring events. □

The refinement bound $\epsilon$ defines the smallest admissible increment of distinguishability. When propagated along an extremal path—one that saturates the refinement bound—each observer records the same number of $\epsilon$-increments. This count defines the informational interval $\tau$. Because $\tau$ represents the number of admissible refinements rather than a metric distance, its invariance follows from the requirement that no observer may introduce or remove distinguishable structure that is not supported by the measurement record.

Let $dx^\mu$ and $dx'^\mu$ denote the infinitesimal labels assigned by two admissible observers to the same pair of neighboring events. Their coordinate labels differ by a transformation

$$dx'^\mu = \Lambda^\mu{}_\nu \, dx^\nu,$$

where $\Lambda^\mu{}_\nu$ preserves causal order in the sense of the Axiom of Selection. Although the observers assign different coordinates, they must agree on the number of $\epsilon$-increments between the events; otherwise their merged histories would violate global consistency.

This agreement is enforced by a bilinear form $g_{\mu\nu}$ satisfying

$$\tau^2 = g_{\mu\nu} \, dx^\mu dx^\nu.$$

Under the coordinate transformation, the metric transforms as

$$g'_{\mu\nu} = \Lambda^\alpha{}_\mu \, \Lambda^\beta{}_\nu \, g_{\alpha\beta}.$$

Substituting the transformed variables into the definition of $\tau$ yields

$$\tau'^2 = g'_{\mu\nu} \, dx'^\mu dx'^\nu = g_{\alpha\beta} \, dx^\alpha dx^\beta = \tau^2.$$

The invariance of $\tau$ thus expresses a simple but fundamental principle: every admissible observer must assign the same number of distinguishable increments to an extremal path. Their coordinate descriptions may vary, but the informational content of the path does not.

This invariance is the basis of the metric gauge introduced in Section 5.2. It ensures that $\tau$ may serve as the universal measure of informational separation, independent of the observer's local labeling conventions. Section 5.2.3 develops the metric $g_{\mu\nu}$ as the bilinear form that enforces this invariance in the continuous shadow.

### 5.2.3 $g_{\mu\nu}$ as the Bilinear Form Preserving the $\epsilon$–Refinement Count

**N.B.**—The metric $g_{\mu\nu}$ is not a geometric field on a manifold. It is the continuous shadow of the rule ensuring that all admissible observers preserve the same count of $\epsilon$–sized refinements between neighboring events. The components of $g_{\mu\nu}$ do not describe a physical medium or curvature; they encode the invariant comparison rule required by informational consistency. $\square$

The interval $\tau$ defined in Section 5.2.2 expresses the number of $\epsilon$–refinements along an extremal segment of the measurement record. Since this number must remain invariant under all admissible relabelings of events, there must exist a bilinear form $g_{\mu\nu}$ such that

$$\tau^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

holds for every observer. This expression is not a postulate but the unique structure that enforces the preservation of $\tau$ under coordinate changes that respect causal order.

To see this, consider two observers who assign infinitesimal labels $dx^\mu$ and $dx'^\mu = \Lambda^\mu{}_\nu\, dx^\nu$ to the same pair of neighboring events. The Law of Causal Transport requires

$$\tau'^2 = \tau^2,$$

so we must have

$$g'_{\mu\nu}\, dx'^{\mu} dx'^{\nu} = g_{\mu\nu}\, dx^{\mu} dx^{\nu}.$$

Substituting $dx'^{\mu}$ and requiring equality for all admissible transformations $\Lambda^{\mu}{}_{\nu}$ yields the transformation rule

$$g'_{\mu\nu} = \Lambda^{\alpha}{}_{\mu}\, \Lambda^{\beta}{}_{\nu}\, g_{\alpha\beta}.$$

Thus the metric is exactly the object that ensures agreement on the number of $\epsilon$–sized refinements between neighboring events, regardless of the coordinates used to describe them.

In this informational framework, $g_{\mu\nu}$ plays a role analogous to that of a gauge potential: it specifies how infinitesimal refinements are compared locally so that the global invariant $\tau$ remains unchanged. The metric does not specify "distance" in any geometric or physical sense; it enforces the equivalence of all admissible measurement conventions.

Once $g_{\mu\nu}$ is introduced, the need to propagate these comparison rules from point to point forces a unique notion of compatibility. This requirement determines the affine connection in Section 5.3 through the condition

$$\nabla_{\lambda} g_{\mu\nu} = 0,$$

which expresses that the metric gauge is preserved under refinement and transport. The next section illustrates this invariance with a concrete thought experiment.

**Thought Experiment 5.2.1** (Thought Experiment: Michelson–Morley as Gauge Isotropy)**.** ***N.B.***—*This thought experiment does not appeal to optical physics, wave interference, or the existence of a medium. It is a finite informational model illustrating that the metric gauge must assign the same refinement cost $\epsilon$ to extremal paths in all admissible directions. No physical claims about light or propagation are implied.* □

*Consider an observer attempting to refine two extremal segments of equal informational content, but aligned in different coordinate directions. Let* $dx^\mu$ *and* $dy^\mu$ *denote the local labels assigned to the two segments. Each segment is chosen such that its refinement requires the same number of* $\epsilon$*–increments when described in the observer's own frame.*

*Now suppose the observer rotates their coordinate system. After rotation, the new labels are* $dx'^\mu = \Lambda^\mu{}_\nu dx^\nu$ *and* $dy'^\mu = \Lambda^\mu{}_\nu dy^\nu$*. The rotation* $\Lambda^\mu{}_\nu$ *preserves causal order, so it is an admissible transformation. The question is whether the observer must still assign the same informational interval* $\tau$ *to both segments after the rotation.*

*The Law of Causal Transport requires that the* $\epsilon$*–refinement counts for both segments remain invariant:*

$$\tau_x^2 = g_{\mu\nu}\, dx^\mu dx^\nu, \qquad \tau_y^2 = g_{\mu\nu}\, dy^\mu dy^\nu.$$

*After rotation, the transformed intervals are*

$$\tau_x'^2 = g'_{\mu\nu}\, dx'^\mu dx'^\nu, \qquad \tau_y'^2 = g'_{\mu\nu}\, dy'^\mu dy'^\nu.$$

*Substituting the transformation rules for* $dx'^\mu$*,* $dy'^\mu$*, and* $g'_{\mu\nu}$ *gives*

$$\tau_x'^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = \tau_x^2, \qquad \tau_y'^2 = g_{\alpha\beta}\, dy^\alpha dy^\beta = \tau_y^2.$$

*Thus the observer must continue to assign the same informational interval to the two extremal segments under any admissible rotation. There is no freedom to deform the refinement counts directionally: doing so would imply that* $\epsilon$*–sized increments depend on orientation and would violate the requirement that informational refinement be globally coherent.*

*This invariance is the informational analogue of isotropy. It expresses that the metric gauge* $g_{\mu\nu}$ *must refine extremal paths uniformly in all directions: the number of* $\epsilon$*–increments needed to resolve a segment of given* *informational content cannot depend on the coordinate orientation.*

*The Michelson–Morley experiment is therefore understood here not as a test of a physical medium, but as a finite illustration of the isotropy of the metric gauge. The invariance of $\tau$ under rotations forces $g_{\mu\nu}$ to encode a direction–independent refinement rule. Section 5.3 develops the compatible connection that propagates this rule under changes of frame.*

## 5.3 The Connection as Informational Bookkeeping

**N.B.**—The affine connection $\Gamma^{\lambda}_{\mu\nu}$ is not a force field or a physical interaction. It is the continuous shadow of an informational rule: the minimal differential adjustment required to preserve the metric gauge $g_{\mu\nu}$ as an observer moves from one event to its neighbor. Its role is purely kinematic. The connection records how local measurement conventions must tilt to maintain the invariant interval $\tau$; no dynamical content or geometric ontology is assumed. □

The metric $g_{\mu\nu}$, introduced in Section 5.2, guarantees that all admissible observers assign the same informational interval $\tau$ to an extremal displacement at a single event. This invariance is enforced by the bilinear form $g_{\mu\nu}\, dx^{\mu}dx^{\nu}$, which preserves the $\epsilon$–refinement count under changes of coordinates. However, the metric by itself does not specify how these comparison rules extend from one event to its neighbors. To describe how distinguishability is maintained along a path, we require a differential notion of consistency.

The connection $\Gamma^{\lambda}_{\mu\nu}$ provides this rule. It specifies how tensor components must be adjusted when an observer translates a local measurement convention from one event to an infinitesimally adjacent one. In particular, the connection determines the covariant derivative, which measures change in a way that respects the metric gauge. Imposing that the metric remain invariant under such differential updates leads to the condition $\nabla_{\lambda} g_{\mu\nu} = 0$, known as covariant constancy of the metric.

In the informational picture, this condition is the statement that the act of refinement may not create or destroy distinguishable structure as an observer moves through the network of events. The connection is the unique differential bookkeeping device that satisfies this constraint. When the metric is uniform, the connection vanishes and no adjustment is needed: straight paths remain informationally straight. When the metric varies, a nonzero connection encodes how local gauges must be rotated and rescaled so that scalar quantities built from $g_{\mu\nu}$ remain unchanged.

The remainder of this section develops the connection as the compatibility condition implied by covariant constancy of the metric and interprets parallel transport as the differential expression of Martin consistency. In this way, the Law of Causal Transport acquires its full kinematic content: it is the rule that propagates the gauge of separation through the continuous shadow of the Causal Universe Tensor.

### 5.3.1 Covariant Constancy and the Compatibility Condition

**N.B.**—Covariant constancy is not a physical conservation law. It is the informational requirement that the metric gauge $g_{\mu\nu}$, which preserves the $\epsilon$–refinement count at a single event, must continue to preserve that count as the observer moves to a neighboring event. The affine connection $\Gamma^{\lambda}_{\mu\nu}$ is therefore not introduced by assumption; it is forced by the requirement that informational invariants remain invariant under differential refinement. □

The metric $g_{\mu\nu}$ ensures that all admissible observers agree on the informational interval $\tau$ at a point. But as the observer moves from an event $x$ to a nearby event $x + dx$, the local coordinate basis changes. Under such a shift, the numerical components of $g_{\mu\nu}$ may appear to change due to the alteration in basis, even if the underlying structure of distinguishability remains the same. To prevent this apparent change from contaminating the informational interval, the transformation of $g_{\mu\nu}$ must be corrected by an additional adjustment term.

This correction is encoded by the covariant derivative. The condition that the metric gauge remain invariant under differential displacement is expressed as

$$\nabla_\lambda g_{\mu\nu} = \partial_\lambda g_{\mu\nu} - \Gamma^\sigma_{\mu\lambda} g_{\sigma\nu} - \Gamma^\sigma_{\nu\lambda} g_{\mu\sigma} = 0.$$

The partial derivative $\partial_\lambda g_{\mu\nu}$ captures how the metric components vary when written in the shifted coordinate system. The remaining terms subtract off this apparent variation by compensating for the tilt and scale of the basis vectors themselves. The equation $\nabla_\lambda g_{\mu\nu} = 0$ thus expresses the requirement that the informational interval $\tau$ remain unchanged under any infinitesimal update of the observational coordinates.

This compatibility condition uniquely determines the connection when torsion is absent. As established in Chapter 3, the spline representation of admissible histories carries no fourth–order freedom and is therefore torsion-free. Under this constraint, the metric compatibility condition fixes the connection to be the Levi–Civita connection:

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left( \partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right).$$

This expression is not a postulate; it is the only operator that ensures the metric gauge remains intact under transport. It is the continuous shadow of the discrete requirement that refinement cannot introduce or eliminate distinguishable structure beyond the $\epsilon$ bound.

With the connection now fixed by kinematic necessity, we may interpret its role operationally. The connection coefficients specify the adjustments required to compare tensorial quantities at neighboring events, ensuring that the informational interval $\tau$ and the refinement bound $\epsilon$ remain consistent throughout the observer's path. The next subsection formalizes this process as parallel transport.

### 5.3.2 Parallel Transport as Differential Martin Consistency

**N.B.**—Parallel transport is not a physical motion of a vector through space. It is the informational requirement that the meaning of a direction—a rule for distinguishing one infinitesimal refinement from another—remain consistent as an observer updates coordinates from one event to the next. In this framework, a "vector" is an instruction for refinement, and parallel transport ensures that such instructions are not distorted by changes in local labeling conventions. □

The metric compatibility condition $\nabla_\lambda g_{\mu\nu} = 0$ determines how the metric must be preserved under infinitesimal displacement. Parallel transport extends this requirement to all tensorial quantities, ensuring that any object used to encode refinements of the observational record is carried through the continuous shadow without introducing contradictions.

Let $V^\mu$ represent such a refinement direction. When an observer moves along a curve $x^\mu(s)$ in the event network, the numerical components of $V^\mu$ change because the local coordinate basis changes. The naive derivative $dV^\mu/ds$ therefore incorporates both the intrinsic change in the refinement direction and the apparent change induced by the shifting coordinates. To isolate the intrinsic change—the part that affects distinguishability—we must subtract off the bookkeeping contribution provided by the connection.

The covariant derivative along the path is thus defined as

$$\frac{DV^\mu}{Ds} = \frac{dV^\mu}{ds} + \Gamma^\mu{}_{\nu\lambda}\, V^\nu \,\frac{dx^\lambda}{ds}.$$

Parallel transport requires that the intrinsic change vanish:

$$\frac{DV^\mu}{Ds} = 0.$$

This equation expresses the differential form of Martin consistency. It states that the instruction encoded by $V^\mu$ must retain its informational meaning as the observer moves. All coordinate-induced distortions of $V^\mu$ must be canceled by the corresponding connection terms, ensuring that the refinement direction does not acquire unrecorded structure.

The geometric interpretation of parallel transport as preserving "straightness" is replaced here by a purely informational one: parallel transport guarantees that refinement instructions remain compatible with the metric gauge $g_{\mu\nu}$ throughout the observer's path. Whenever the metric varies from event to event, the connection coefficients encode the cost of adjusting the observer's basis to ensure that scalar comparisons built from $g_{\mu\nu}$ and $V^\mu$ remain invariant.

In regions where $g_{\mu\nu}$ is uniform, the connection vanishes and the informational meaning of $V^\mu$ is preserved without adjustment. Where $g_{\mu\nu}$ varies, nonzero connection coefficients encode the minimal bookkeeping needed to keep the refinement count consistent. This adjustment is the kinematic origin of effects such as frequency shifts between observers in different informational environments, which we examine in the following subsection.

## 5.4 Observable Consequence: Refinement–Adjusted Transport

**N.B.**—The frequency shift examined in this section is not a postulated effect. It is the kinematic consequence of maintaining the invariant informational interval $\tau$ across regions in which the metric gauge $g_{\mu\nu}$ varies. No physical ontology is assumed. The observable change in clock rates reflects the differential bookkeeping enforced by the connection $\Gamma^\lambda_{\mu\nu}$. □

The previous sections established the chain of informational structure: the refinement bound $\epsilon$ fixes the local increment of distinguishable structure; the metric $g_{\mu\nu}$ expresses how these increments are compared between observers; and the connection $\Gamma^\lambda_{\mu\nu}$ preserves this comparison under differential displacement. When the metric varies from one location to another, this preservation requires that the local rate of event counting—the clock frequency—adjusts so that the invariant interval remains consistent across observers.

This section derives that adjustment and exhibits its observable consequence.

### 5.4.1 The Invariant Causal Tally

**N.B.**—An atomic clock does not measure a geometric length or a physical time. It measures a count of distinguishable events. The proper interval $\tau$ is the continuous shadow of this count, expressed in units of the refinement bound $\epsilon$. □

Consider an observer whose worldline is described by coordinates $(t, x^i)$. If the observer is at rest in their coordinate system ($dx^i = 0$), the informational interval between neighboring events satisfies

$$d\tau^2 = g_{00}(x)\, dt^2.$$

Thus the locally measured period of the clock is

$$d\tau = \sqrt{g_{00}(x)}\, dt.$$

Because $\tau$ counts $\epsilon$–sized refinements, the local clock frequency $\nu(x)$ is inversely proportional to the size of this interval:

$$\nu(x) = \frac{1}{d\tau} = \frac{1}{\sqrt{g_{00}(x)}} \frac{1}{dt}.$$

Two observers at rest in different metric gauges therefore experience different informational intervals for the same coordinate increment $dt$. The relationship between their locally recorded counts is fixed entirely by the metric gauge.

### 5.4.2 Derivation of Frequency Adjustment

**N.B.**—The global parameter $t$ is not a physical time. It is the auxiliary labeling parameter that all admissible observers must agree upon when their records are merged. Its increments must match across observers in order for their $\epsilon$–counts to be reconciled. □

Let observers $A$ and $B$ be stationary in regions with metric components $g_{00}(A)$ and $g_{00}(B)$. Over a shared coordinate increment $\Delta t$, their locally recorded proper intervals are

$$\Delta\tau_A = \sqrt{g_{00}(A)}\,\Delta t, \qquad \Delta\tau_B = \sqrt{g_{00}(B)}\,\Delta t.$$

Since a clock's frequency is the inverse of the proper interval it records,

$$\nu_A = \frac{1}{\Delta\tau_A} = \frac{1}{\sqrt{g_{00}(A)}}\,\frac{1}{\Delta t}, \qquad \nu_B = \frac{1}{\sqrt{g_{00}(B)}}\,\frac{1}{\Delta t}.$$

The ratio of their observed frequencies is therefore

$$\frac{\nu_A}{\nu_B} = \frac{\sqrt{g_{00}(B)}}{\sqrt{g_{00}(A)}}.$$

This expression is the kinematic consequence of the Law of Causal Transport. When $g_{00}$ varies, the connection $\Gamma^0_{00}$ compensates by adjusting the local rate of $\epsilon$–counting so that the merged observational record remains consistent. The observed frequency shift is thus the operational signature of nonzero connection coefficients.

## 5.5 Conclusion: The Kinematic Foundation of Geometry

**N.B.**—This chapter derived the continuous kinematic structures—the metric $g_{\mu\nu}$ and the connection $\Gamma^{\lambda}_{\mu\nu}$—from the informational requirement that refinements remain globally consistent. No forces, fields, or dynamical assumptions were introduced. □

The development of this chapter followed the informational chain of emergence:

$$\epsilon \longrightarrow \tau \longrightarrow g_{\mu\nu} \longrightarrow \Gamma^{\lambda}_{\mu\nu}.$$

The refinement bound $\epsilon$ fixed the minimal increment of admissible structure. The interval $\tau$ encoded the invariant tally of such increments. The metric $g_{\mu\nu}$ enforced this invariance across observers, and the connection $\Gamma^{\lambda}_{\mu\nu}$ preserved it under differential refinement. The observable consequence of this structure is the redshift effect, where nonuniformity of the metric gauge requires a corresponding adjustment of the local $\epsilon$–counting rate.

This completes the kinematic description of informational geometry. The next chapter introduces the dynamic concept of curvature, defined as the obstruction to transporting refinement instructions consistently around a closed loop. In this way, the "Curvature of Information" becomes the natural extension of the kinematic structures developed here.

# Chapter 6

# The Curvature of Information

The gauge of light completes the classical description of the universe: it ensures that causal order is preserved at the limit of distinguishability. But the universe we observe is not smooth. Measurements are discrete, events occur finitely, and the invariants of the causal gauge fluctuate around their ideal values. These fluctuations are not errors—they are the quantum fields of the theory.

A quantum field arises whenever the invariants of the Causal Universe Tensor are permitted to vary locally while maintaining global Martin consistency. Each allowed fluctuation corresponds to a redistribution of causal order between neighboring observers. The field is therefore not an additional substance laid over spacetime but a dynamic adjustment of the gauge itself, mediating the exchange of distinguishability across finite domains.

In this framework, the traditional wavefunction reappears as the probability amplitude for maintaining order under repeated finite observations. Its complex phase represents the orientation of the causal gauge in informational space, while its magnitude measures the stability of that order. The principle of superposition follows directly from the linearity of causal combinations: multiple consistent histories can coexist until observation resolves a single extension of the network.

Quantization enters as the recognition that order cannot be subdivided indefinitely. Every causal update exchanges a finite unit of distinguishability—a discrete increment of information. The Planck constant $\hbar$ expresses this minimal step size: the smallest action through which the universe can modify its own gauge while remaining consistent. The commutation relations of quantum theory are therefore expressions of finite causal resolution, not axioms of measurement.

This chapter develops these ideas systematically. Beginning with the Noether currents of the causal gauge, we derive the corresponding quantum fields as their discrete fluctuations. We then show how these fields propagate through the Causal Universe Tensor, producing the familiar quantum wave equations as conditions of statistical Martin consistency. Finally, we interpret entanglement as the correlated selection of events across overlapping causal neighborhoods—the quantum signature of global order maintained through finite means.

**Remark 17.** *Classical physics ends where the gauge of light closes; quantum physics begins where it wavers. Every quantum field is a small deviation from perfect causal consistency, a harmonic of order itself. The task of this chapter is to make that statement precise.*

## 6.1 The Residue of Inconsistency

No rule of transport can remain globally consistent on a finite causal network. When one carries a distinction around a closed loop of events, the recovered configuration generally differs from the initial one. This difference is not an error but an invariant: the measurable residue of inconsistency required to preserve local order within a global whole. In differential form, that residue is called curvature.

### 6.1.1 Curvature as the Measure of Non-Closure

The connection $\Gamma^{\lambda}{}_{\mu\nu}$ prescribes how distinctions are transported to preserve scalar invariants locally. When the same distinction is transported successively along different paths that enclose a finite region, the final result may depend on the path taken. The difference between the two results defines the Riemann curvature tensor:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\sigma\nu} - \partial_{\nu}\Gamma^{\rho}{}_{\sigma\mu} + \Gamma^{\rho}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\sigma\nu} - \Gamma^{\rho}{}_{\lambda\nu}\Gamma^{\lambda}{}_{\sigma\mu}.$$

This object measures the infinitesimal failure of causal transport to commute. When $R^{\rho}{}_{\sigma\mu\nu} = 0$, all paths yield the same result and the causal network is globally flat; when it does not vanish, the inconsistency cannot be removed by any gauge transformation.

**Example: Casimir Effect as Measured Residue of Non-Closure**

**Statement.** Boundary-induced mode restriction yields a measurable scalar from the residue of non-closure: the Casimir pressure.

**Key relation (ideal plates, separation $a$).**

$$P \;=\; -\frac{\pi^2}{240}\,\frac{\hbar c}{a^4}.$$

**Reciprocity framing.** Plates impose selection on admissible causal updates (mode partitions). The contraction of the Universe Tensor over admissible modes produces a nonzero scalar residue—the pressure—interpretable as curvature from informational incompleteness.

**Operational consequence.** Moving a plate changes the equivalence class (refines the partition), and the derivative of the class invariant yields a force, closing the loop between geometry and matter.

### 6.1.2 Physical Interpretation

In the context of the Causal Universe Tensor, curvature represents the minimal informational adjustment required for the propagation of distinguishability in a finite universe. Each nonzero component of $R^{\rho}{}_{\sigma\mu\nu}$ quantifies how much the local gauge of separation must bend to remain self-consistent when extended around a closed causal loop. Curvature is thus the differential trace of the universe correcting itself: the physical manifestation of the fact that perfect global order is impossible, even though local order is preserved.

### 6.1.3 Contractions and Scalar Invariants

Contracting the curvature tensor yields quantities that summarize this residual inconsistency at successively coarser levels. The Ricci tensor

$$R_{\mu\nu} = R^{\rho}{}_{\mu\rho\nu}$$

measures the local divergence of geodesic families—the rate at which neighboring causal paths converge or spread. The scalar curvature

$$R = g^{\mu\nu} R_{\mu\nu}$$

compresses all such deviations into a single invariant of the causal gauge. These contractions represent higher-order scalar invariants of the Causal Universe Tensor, extending the chain of conserved quantities that began with the spline and the principle of least action.

### 6.1.4 The Meaning of Curvature in the Causal Framework

Traditional geometry interprets curvature as a property of space. Here it is a property of information: a measure of how the network of distinguishable events must deform to reconcile finite observation with global consistency. Flatness corresponds to exact commutativity of causal updates; curvature, to their minimal non-commutativity. The universe's curvature is therefore the bookkeeping of necessary inconsistency—the trace left by causal order maintaining itself through finite means.

**Remark 18.** *Curvature is the residue of inconsistency. It is what remains when the rule of causal transport cannot close perfectly, the irreducible difference between local and global consistency. In the language of the Causal Universe Tensor, curvature represents the self-correcting property of the universe: the differential response by which causal order preserves itself in time. The next section will show that this residue, when balanced against the stress encoded in the tensor $T_{\mu\nu}$, yields the Einstein equation—the equilibrium condition of the gauge of light.*

**Thought Experiment 6.1.1** (Extrapolation: Rotation Curves as Order-Preserving Transport)**.** ***N.B.*** *This is a conceptual extrapolation about* measurement and causal bookkeeping*, not an astrophysical claim. It illustrates how a curvature term may arise as the minimal correction that preserves an informational invariant under geometric dilution. No inference about galaxies, dark matter, or dynamics is intended [9, 88, 16, 114, 79, 58].*

Setup. *Imagine a discrete causal disk partitioned into annuli $\{A_r\}$, each containing a locally finite set of events. A predicate $P$ counts the distinguishable crossings of a reference ray within each annulus. For two anchors $a \prec b$, define the measurement*

$$M_P[r; a, b] := \#\{\, e \in A_r \mid a \prec e \prec b,\ P(e) = 1 \,\}.$$

*Order-preserving advection (as in §3.6) requires that counts move without creation or loss: transport is conservative [12, 41].*

**Remark 19** (Observational Context: Vera Rubin and the Dark Matter Problem)**.** *The near-constant tangential velocities observed by Vera Rubin and W.*

*Kent Ford (1970) in spiral galaxies revealed a striking departure from Newtonian expectations: orbital speed remained approximately flat with radius. In this framework, that empirical fact is reinterpreted as an order-preserving transport condition rather than evidence for unseen mass. The constancy of* $v_\theta(r)$ *follows from invariance in the count of distinguishable causal updates, not from an additional gravitational component.*

Invariant to preserve. *Let* $\Phi(r)$ *be the count per causal cycle,*

$$\Phi(r) = \frac{M_P[r; a, b]}{\textit{cycles between } a \textit{ and } b},$$

*which serves as a flux of distinguishability. Perfect bookkeeping demands* $\partial_r \Phi(r) = 0$.

Geometric tension. *In a flat disk the circumference grows as* $2\pi r$*; if events are neutral parcels, their surface density falls as* $1/r$*. Without correction,* $\Phi(r)$ *would decrease outward [79].*

Minimal fix. *Introduce a compensating connection with curvature* $K(r)$ *such that*

$$\partial_r\big(r\,\rho(r)\,v(r)\big) = 0,$$

*where* $\rho(r)$ *is the local count density and* $v(r)$ *the tangential update rate. The least-bias (minimal-curvature) solution satisfying constant flux is*

$$r\,\rho(r)\,v(r) = C \quad \Rightarrow \quad v(r) \approx \textit{const.}$$

*Thus a* flat *tangential rate is the order-preserving closure of the bookkeeping rule [58, 68].*

Interpretation. *Within this framework, a "flat rotation curve" is the informationally minimal configuration that maintains constant flux of distinguishable events as circumference grows. The compensating* $K(r)$ *is simply the residue of non-closure that enforces causal consistency across radii [114, 12].*

Scope. *This extrapolation is purely formal. It demonstrates how a constant tangential rate can emerge from an invariant-count condition, not how galaxies move.*

## 6.2 Empirical Test: Normal Equations for Rotation Invariance

**N.B.** This section specifies a falsifiable data model for galaxy rotation curves derived from the order-preserving transport condition. It implements the single-parameter limit implied by Chapter 2, where the calculus of measurement allows only one free constant to curve-fit any closed causal system. The model therefore has a unique intercept but no tunable slope parameters.

**Invariant.** Order-preserving transport with constant flux of distinguishability implies

$$r\,\rho(r)\,v_\theta(r) = \Phi, \qquad \Phi > 0 \text{ constant.}$$

Here $\rho(r)$ represents the local count density (a proxy for informational concentration) and $v_\theta(r)$ the observed tangential speed.

**Observables.** Let $I(r)$ denote surface brightness and $\kappa > 0$ a proportionality linking brightness to count density, $\rho(r) \propto \kappa I(r)$. Substituting gives the predicted profile

$$v_\theta(r) = \frac{\Phi}{r\,\kappa\,I(r)}.$$

Taking logarithms produces a linear empirical relation:

$$\underbrace{\log v_\theta(r)}_{y(r)} = \underbrace{\theta_0}_{\log\Phi - \log\kappa} - \log r - \log I(r) + \varepsilon(r),$$

where $\mathbb{E}[\varepsilon(r)] = 0$ under the invariant.

**Normal equation (strict test).** Define $z_i := \log r_i + \log I_i$ and $y_i := \log v_i$. The strict model is $y_i = \theta_0 - z_i + \varepsilon_i$ with a fixed slope of $-1$ on both $\log r$ and $\log I$. Only one constant, $\theta_0$, remains free in accordance with Chapter 2. The ordinary-least-squares normal equation reduces to

$$\hat{\theta}_0 = \frac{1}{n}\sum_{i=1}^{n}(y_i + z_i),$$

and falsification is assessed by testing for systematic structure in residuals $\hat{\varepsilon}_i = y_i - \hat{\theta}_0 + z_i$.

**Relaxed model with curvature residue.** To capture permissible second-order deviations $K(r)$ due to geometric residue, extend to

$$y_i = \theta_0 - z_i + \sum_{j=1}^{m} B_j(r_i)w_j + \varepsilon_i,$$

where $B_j$ are spline bases with penalty $\lambda\|Dw\|^2$. The penalized normal equations are

$$\begin{bmatrix} n & \mathbf{1}^\top B \\ B^\top \mathbf{1} & B^\top B + \lambda D^\top D \end{bmatrix} \begin{bmatrix} \theta_0 \\ w \end{bmatrix} = \begin{bmatrix} \sum_i (y_i + z_i) \\ B^\top (y + z) \end{bmatrix}.$$

When $w = 0$ the invariant holds exactly; significant $w$ indicates structured departure.

**Population form.** Across galaxies $g$, intercepts $\theta_{0,g}$ vary but slopes remain fixed:

$$y_{ig} = \theta_{0,g} - \log r_{ig} - \log I_{ig} + \varepsilon_{ig}.$$

A mixed-effects regression with random intercepts but common slopes tests universality of the invariant; a slope differing from $-1$ falsifies it.

**Falsifiable predictions.**

1. **Fixed slopes:** Coefficients on $\log r$ and $\log I$ are $-1$ within uncertainty; deviation falsifies the invariant.
2. **White residuals:** Adjusted residuals $\hat{\varepsilon}_i$ show no systematic radial trend.
3. **Cross-sample invariance:** Slopes are common to all galaxies; intercepts vary only by normalization.
4. **Low-brightness scaling:** Lower $I$ implies higher $v$ at fixed $r$; violation falsifies order-preserving transport.

**Scope.** This regression embodies the Chapter 2 principle that a consistent measurement law introduces at most one free constant. Its acceptance or rejection is therefore directly falsifiable: systematic deviations in slope or curvature constitute empirical evidence against the order-preserving flux hypothesis.

## 6.3 Global Constraint as the Einstein Equation

The final step is to impose global consistency on the causal network. Local rules of separation and transport guarantee Martin consistency within each neighborhood, but finite observation requires that these neighborhoods overlap. The residual curvature computed in the previous section measures the degree to which local order fails to close globally. The Einstein equation expresses the condition under which that failure is exactly balanced by the stress encoded in the Causal Universe Tensor.

### 6.3.1 From Local Residue to Global Balance

Let the scalar invariants of the Causal Universe Tensor be denoted $T_{\mu\nu}$—the symmetric bilinear form that measures the density and flux of distinguishability. The curvature invariants of the causal gauge are summarized by the Einstein tensor,

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu}.$$

Both tensors share the same divergence-free property, $\nabla^{\mu} G_{\mu\nu} = \nabla^{\mu} T_{\mu\nu} = 0$, a differential expression of Martin consistency. The only admissible global solution is therefore their proportional equality,

$$G_{\mu\nu} = 8\pi \, T_{\mu\nu}.$$

This is the Einstein field equation, reinterpreted as the global constraint that restores balance between the residue of inconsistency (curvature) and the finite structure of distinguishability (stress).

### 6.3.2 Interpretation in the Causal Framework

The Einstein equation states that curvature is not an independent source of force but the universe's adjustment to maintain causal coherence. Energy and stress arise from the finiteness of measurement; curvature arises from the impossibility of reconciling all such measurements globally. The equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$ enforces that these two forms of inconsistency—informational and geometric—cancel exactly. When they do, the propagation of light remains Martin-consistent throughout the entire network.

In this view, gravitation is the manifestation of the universe correcting its own bookkeeping of distinctions. Mass–energy is simply the local density of finite observation, and curvature the global compensation that restores order. Spacetime bends not because matter exerts force, but because causal consistency demands it.

### 6.3.3 The Closure of the Gauge of Light

The Einstein equation thus completes the gauge of light. Beginning with the metric as the gauge of separation, the connection as the rule of causal transport, and curvature as the residue of inconsistency, the global constraint closes the system. All four structures arise from a single requirement: that the scalar invariants of the Causal Universe Tensor remain self-consistent under extension to the entire causal domain.

**Remark 20.** *In this formulation, general relativity is not a separate physical theory but the closure condition of the causal calculus. The Einstein tensor is the final differential form of Martin consistency; the stress–energy tensor is the discrete record of finite distinction. Their equality marks the point at which the universe's description becomes self-consistent. Beyond this, nothing remains to adjust—the gauge of light is complete.*

# Chapter 7

# The Conservation of Symmetry: From Noether to Mass

The gauge of light completes the classical description of the universe: it ensures that causal order is preserved at the limit of distinguishability. But the universe we observe is not smooth. Measurements are discrete, events occur finitely, and the invariants of the causal gauge fluctuate around their ideal values. These fluctuations are not errors—they are the quantum fields of the theory.

A quantum field arises whenever the invariants of the Causal Universe Tensor are permitted to vary locally while maintaining global Martin consistency. Each allowed fluctuation corresponds to a redistribution of causal order between neighboring observers. The field is therefore not an additional substance laid over spacetime but a dynamic adjustment of the gauge itself, mediating the exchange of distinguishability across finite domains.

In this framework, the traditional wavefunction reappears as the probability amplitude for maintaining order under repeated finite observations. Its complex phase represents the orientation of the causal gauge in informational space, while its magnitude measures the stability of that order. The principle of superposition follows directly from the linearity of causal combinations:

multiple consistent histories can coexist until observation resolves a single extension of the network.

Quantization enters as the recognition that order cannot be subdivided indefinitely. Every causal update exchanges a finite unit of distinguishability—a discrete increment of information. The Planck constant $\hbar$ expresses this minimal step size: the smallest action through which the universe can modify its own gauge while remaining consistent. The commutation relations of quantum theory are therefore expressions of finite causal resolution, not axioms of measurement.

This chapter develops these ideas systematically. Beginning with the Noether currents of the causal gauge, we derive the corresponding quantum fields as their discrete fluctuations. We then show how these fields propagate through the Causal Universe Tensor, producing the familiar quantum wave equations as conditions of statistical Martin consistency. Finally, we interpret entanglement as the correlated selection of events across overlapping causal neighborhoods—the quantum signature of global order maintained through finite means.

**Remark 21.** *Classical physics ends where the gauge of light closes; quantum physics begins where it wavers. Every quantum field is a small deviation from perfect causal consistency, a harmonic of order itself. The task of this chapter is to make that statement precise.*

### Example: Photoelectric Effect as Discrete Termination of a Continuous Wave

**Statement.** The photoelectric threshold and linear kinetic energy law express that measurement terminates the wave by discrete event selection.

**Key relation.**

$$K_{\max} \;=\; h\nu - \Phi, \qquad \nu \geq \nu_0 = \frac{\Phi}{h}.$$

**Reciprocity framing.** A continuous field carries phase/energy, but a detection event is a refinement of the partition $P_n \to P_{n+1}$ at the cathode surface. The selection rule enforces conservation in the bookkeeping channel: the work function $\Phi$ is the minimal distinguishability cost to register an event.

**Operational consequence.** Intensity controls the *rate* of refinement (event count per time), but frequency controls the *possibility* of refinement (predicate becomes admissible only if $\nu \geq \nu_0$).

## 7.1 The Action Functional

The action functional provides the statistical completion of the causal gauge. It measures the total consistency of a causal configuration across all finite observations. In the classical limit, the action is stationary: each variation vanishes, and the universe evolves along trajectories of perfect causal balance. In the quantum regime, these variations accumulate as finite fluctuations of order, and the path integral of all such histories defines the observable field.

### 7.1.1 Definition from the Causal Universe Tensor

Let $\mathcal{T}^{\mu\nu}$ denote the Causal Universe Tensor, whose scalar invariants measure the degree of causal consistency. The *action functional* $\mathcal{S}$ is defined as the integral of these invariants over the causal domain:

$$\mathcal{S} = \int \mathcal{L}\big(\mathcal{T}^{\mu\nu}, g_{\mu\nu}, \nabla_\lambda \mathcal{T}^{\mu\nu}\big)\sqrt{-g}\, d^4x.$$

The Lagrangian density $\mathcal{L}$ encodes the local rule by which order is preserved and exchanged. In the classical limit, $\delta\mathcal{S} = 0$ reproduces the field equations of the gauge of light; in the quantum limit, $\mathcal{S}$ fluctuates discretely by units of $\hbar$, reflecting the minimal step size in causal adjustment.

### 7.1.2 Physical Interpretation

The action $\mathcal{S}$ plays the role of a global consistency measure. Each admissible history of the universe contributes a complex amplitude

$$\Psi[\mathcal{T}] \propto e^{i\mathcal{S}[\mathcal{T}]/\hbar},$$

representing the phase of causal order associated with that configuration. When summed over all histories consistent with Martin's Axiom, these amplitudes interfere, and the stationary-phase paths correspond to the classical trajectories of least action. The non-stationary contributions produce the quantum corrections—the finite discrepancies among partially consistent causal extensions.

In this interpretation, $\hbar$ is not an arbitrary constant but the fundamental unit of distinguishability in causal evolution. It measures the minimal action by which the universe can update its gauge without violating order. The classical limit $\hbar \to 0$ corresponds to infinitely fine causal resolution, while the quantum limit expresses the graininess of finite observation.

### 7.1.3 Noether Currents of the Causal Gauge

Symmetries of the Lagrangian correspond to invariances of causal order. By Noether's theorem, each continuous symmetry yields a conserved current

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\nabla_\mu \phi)}\,\delta\phi, \qquad \nabla_\mu J^\mu = 0.$$

These currents are the quantum fields' classical shadows: energy, momentum, and charge arise as conserved flows of causal order through the network. Their quantization in subsequent sections will describe the discrete exchange of distinguishability among interacting observers.

**Remark 22.** *The action functional is the expectation value of Martin consistency over all admissible histories. In the classical regime, it is stationary;*

*in the quantum regime, it oscillates. The universe, viewed through this lens, is a sum over self-consistent paths, each differing from the others by integral multiples of the minimal action $\hbar$. Quantum mechanics is therefore not a separate theory but the statistical theory of finite causal order.*

## 7.2 The Application of Noether

Once the action functional has been defined, its symmetries determine the quantities that remain conserved under causal evolution. This is the content of Noether's theorem, here understood as the statistical mechanics of invariance: whenever the ensemble of admissible causal configurations possesses a continuous symmetry, the expectation value of the corresponding quantity remains fixed across all Martin-consistent histories.

### 7.2.1 Symmetry and Conservation as Statistical Identities

Let the partition function of the causal gauge be written

$$Z = \int \exp\left(\frac{i}{\hbar}\mathcal{S}[\mathcal{T}]\right) \mathcal{D}\mathcal{T},$$

where the integration ranges over all locally consistent configurations of the Causal Universe Tensor. An infinitesimal transformation of variables $\mathcal{T} \to \mathcal{T} + \delta\mathcal{T}$ that leaves the measure and the action invariant,

$$\delta\mathcal{S} = 0,$$

implies that the partition function is unchanged:

$$\delta Z = 0.$$

Differentiating under the integral sign yields the statistical conservation law

$$\langle \nabla_\mu J^\mu \rangle = 0,$$

where

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\nabla_\mu \phi)}\,\delta\phi$$

is the current associated with the transformation. Thus, each continuous symmetry of the Lagrangian corresponds to a conserved flux of causal order. Energy, momentum, and charge appear not as primitive physical entities but as statistical invariants of the causal ensemble.

### 7.2.2 Conserved Quantities of the Causal Gauge

1. **Translational invariance** → Conservation of energy–momentum:

$$\nabla_\mu T^{\mu\nu} = 0.$$

2. **Rotational invariance** → Conservation of angular momentum:

$$\nabla_\mu J^{\mu\nu} = 0, \quad J^{\mu\nu} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}.$$

3. **Internal phase invariance** → Conservation of charge:

$$\nabla_\mu j^\mu = 0.$$

Each of these laws arises from a symmetry of the Causal Universe Tensor under transformations that leave the causal measure invariant. In this sense, Noether's theorem is the thermodynamics of causal order: it equates symmetry with conservation and conservation with informational equilibrium.

**Thought Experiment 7.2.1** (The Harmonic Oscillator as a Closed Loop of Reciprocal Measurement)**.** *The harmonic oscillator is the minimal causal* *system in which measurement and variation form a reversible cycle. Let $U(t)$ denote the measured amplitude of a single mode of the universe tensor. Successive reciprocal updates obey*

$$\delta^2 U + \omega^2 U = 0,$$

*where $\delta$ is the discrete variation operator and $\omega$ characterizes the curvature of the local informational potential. In the continuum limit this becomes*

$$\frac{d^2U}{dt^2} + \omega^2 U = 0,$$

*the familiar harmonic–oscillator equation.*

*Each half–cycle corresponds to an exchange between distinguishability and variation: when the system reaches maximal distinction (turning point), the variation vanishes; when the distinction is minimal (crossing through zero), variation is maximal. The energy functional*

$$E = \tfrac{1}{2}\Big[(\dot{U})^2 + \omega^2 U^2\Big]$$

*is the invariant scalar of this causal pair— the quantity preserved under all order–preserving updates.*

*Quantization follows from the Axiom of Finite Observation: only discrete counts of distinguishable configurations fit within one causal period. Applying the Reciprocity Law yields the spectrum*

$$E_n = \hbar\omega\big(n + \tfrac{1}{2}\big)\,,$$

*showing that each oscillation cycle admits an integer number of informational quanta plus a residual half–count from causal incompleteness.*

*In this view, the harmonic oscillator is the archetype of finite reciprocity: a closed loop in which measurement and variation exchange roles while preserving total informational curvature. All quantized fields—phonons, pho-*

*tons, and normal modes of the causal tensor—are higher–dimensional extensions of this single reciprocal circuit.*

### 7.2.3 Statistical Interpretation

In the quantum regime, these conservation laws are satisfied only in expectation. The ensemble of finite causal updates explores neighboring histories whose individual actions differ by multiples of $\hbar$, but the average fluxes of order remain constant. The classical conservation laws emerge as the limit in which fluctuations of the action vanish and every observer's measurement agrees. Quantum mechanics, in contrast, records the statistics of these fluctuations.

**Remark 23.** *Noether's theorem closes the loop between mechanics and statistics. Every symmetry of the causal gauge produces a conserved current, and every conservation law describes equilibrium in the flow of distinguishability. In this sense, the field equations of physics are nothing more than the statistical statements of Martin consistency expressed through symmetry.*

ectionConservation

Conservation laws follow from symmetries of the action. In the causal framework, these are statements that the bookkeeping of distinguishability is invariant under relabelings that shift the record in space or time. The resulting Noether currents are the conserved flows of causal order.

### 7.2.4 Translations and the Stress–Energy Tensor

Let $\mathcal{S} = \int \mathcal{L}\,\sqrt{-g}\,d^4x$ be the action of the Causal Universe Tensor fields (collectively $\phi$). Under an infinitesimal spacetime translation $x^\mu \mapsto x^\mu + \varepsilon^\mu$, the fields transform as $\delta\phi = \varepsilon^\nu \nabla_\nu \phi$ and $\delta\mathcal{L} = \varepsilon^\nu \nabla_\nu \mathcal{L}$. Invariance of the action ($\delta\mathcal{S} = 0$) yields the Noether current

$$J^{\mu}{}_{\nu} = \frac{\partial\mathcal{L}}{\partial(\nabla_{\mu}\phi)}\,\nabla_{\nu}\phi - \delta^{\mu}{}_{\nu}\,\mathcal{L},$$

whose covariant divergence vanishes:

$$\nabla_{\mu}J^{\mu}{}_{\nu} = 0.$$

Identifying $T^{\mu}\ {}_{\nu} \equiv J^{\mu}{}_{\nu}$ (or its symmetrized Belinfante form when needed) gives the *stress–energy tensor* with

$$\nabla_{\mu}T^{\mu}{}_{\nu} = 0.$$

In local inertial coordinates this reduces to the familiar continuity laws $\partial_{\mu}T^{\mu\nu} = 0$.

**Example: Compton Scattering as Reciprocal Momentum Bookkeeping**

**Statement.** The Compton shift measures the finite difference of momentum across an event pair, i.e. the reciprocity map in momentum space.

**Key relation.**

$$\Delta\lambda \;\equiv\; \lambda' - \lambda \;=\; \frac{h}{m_e c}\big(1 - \cos\theta\big).$$

**Reciprocity framing.** One detection event refines the joint partition of (photon, electron). Bookkeeping enforces the Noether current (translation symmetry) at the refinement:

$$p_{\gamma} + p_e \;=\; p'_{\gamma} + p'_e, \qquad E_{\gamma} + E_e \;=\; E'_{\gamma} + E'_e.$$

Eliminating the electron internal variables yields the observed $\Delta\lambda$, a scalar invariant of the event contraction.

**Operational consequence.** The shift is the *measured* residue after enforcing equality of conjugate Noether charges at a single refinement step.

### 7.2.5 Energy and Momentum Densities

Write $u^\mu$ for the future-directed unit normal to a Cauchy slice $\Sigma$ (with volume element $d\Sigma_\mu = u_\mu\, d^3x\sqrt{\gamma}$). The total four-momentum is

$$P^\nu \;=\; \int_\Sigma T^{\mu\nu}\, d\Sigma_\mu,$$

so that

$$E \equiv P^0 = \int_\Sigma T^{\mu\nu} u_\mu \xi^{(t)}_\nu\, d^3x\sqrt{\gamma}, \qquad \mathbf{P}^i = \int_\Sigma T^{\mu\nu} u_\mu \xi^{(i)}_\nu\, d^3x\sqrt{\gamma},$$

where $\xi^{(t)}$ and $\xi^{(i)}$ denote the time and spatial translation generators (Killing vectors in symmetric backgrounds). Covariant conservation implies slice-independence:

$$\frac{d}{d\tau}P^\nu \;=\; \int_\Sigma \nabla_\mu T^{\mu\nu}\, d\Sigma \;=\; 0.$$

### 7.2.6 Bookkeeping Interpretation

Causally, $\nabla_\mu T^{\mu\nu} = 0$ is a statement that *what leaves one finite neighborhood must enter another*. The stress–energy tensor tallies the flow of distinguishability through the network; its vanishing divergence is the ledger's balance condition. Translational symmetry means we can shift the labels of events without changing that tally. Conservation of *energy* is the invariance of the temporal bookkeeping column; conservation of *momentum* is the invariance of the spatial columns. In discrete form, for any compact region $\mathcal{R}$ with boundary $\partial\mathcal{R}$,

$$\frac{d}{d\tau}\int_{\mathcal{R}} T^{0\nu}\, d^3x \;=\; -\int_{\partial\mathcal{R}} T^{i\nu}\, n_i\, dS,$$

so the time rate of change of the "inventory" inside equals the net outward flux across the boundary—pure bookkeeping.

### 7.2.7 Curved Backgrounds and Killing Symmetries

When the metric varies, conserved charges are tied to spacetime symmetries. If $\xi^\nu$ is a Killing vector ($\nabla_{(\mu}\xi_{\nu)} = 0$), then

$$\nabla_\mu\big(T^\mu{}_\nu\, \xi^\nu\big) = 0,$$

and the associated charge

$$Q[\xi] \;=\; \int_\Sigma T^\mu{}_\nu\, \xi^\nu\, d\Sigma_\mu$$

is conserved. Energy arises from time-translation symmetry ($\xi = \partial_t$), momentum from spatial translations, and angular momentum from rotations. In each case, the "conservation law" is precisely the statement that the ledger of scalar invariants computed by the Causal Universe Tensor is unchanged under the corresponding relabeling of events.

**Remark 24.** *Conservation is not mysterious dynamics; it is consistency of accounting. Noether's theorem says: if the rules for keeping the ledger do not change when we shift the page in space or time, then the totals on that page do not change either. In the causal calculus, those totals are $P^\nu$, and their invariance is exactly $\nabla_\mu T^{\mu\nu} = 0$.*

**Thought Experiment 7.2.2** (Conservation of Energy for a Free Scalar Field)**.** *Consider a real Klein–Gordon field $\phi$ in flat spacetime with*

$$\mathcal{L} \;=\; \tfrac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi \;-\; \tfrac{1}{2}\,m^2\phi^2, \qquad \eta_{\mu\nu} = \mathrm{diag}(-,+,+,+).$$

*The (symmetric) stress–energy tensor is*

$$T^{\mu\nu} \;=\; \partial^\mu\phi\,\partial^\nu\phi \;-\; \eta^{\mu\nu}\mathcal{L}.$$

*Energy density and energy flux are then*

$$\mathcal{E} \;\equiv\; T^{00} = \tfrac{1}{2}\Big(\dot{\phi}^2 + |\nabla\phi|^2 + m^2\phi^2\Big), \qquad S^i \;\equiv\; T^{0i} = \dot{\phi}\,\partial^i\phi.$$

**Continuity (bookkeeping) equation.** *Using the Euler–Lagrange equation $\Box\phi + m^2\phi = 0$ and differentiating,*

$$\partial_t\mathcal{E} = \dot{\phi}\,\ddot{\phi} + \nabla\phi\cdot\nabla\dot{\phi} + m^2\phi\,\dot{\phi} = \dot{\phi}\big(\ddot{\phi} - \nabla^2\phi + m^2\phi\big) + \nabla\cdot(\dot{\phi}\,\nabla\phi) = \nabla\cdot(\dot{\phi}\,\nabla\phi),$$

*so*

$$\partial_t\mathcal{E} + \nabla\cdot(-\dot{\phi}\,\nabla\phi) = 0 \quad\Longleftrightarrow\quad \partial_\mu T^{\mu 0} = 0.$$

*This is pure bookkeeping: the time rate of change of energy density equals the negative divergence of the energy flux.*

**Integrated conservation law.** *Integrate over a fixed region $\mathcal{R}$ with outward normal $\mathbf{n}$:*

$$\frac{d}{dt}\int_{\mathcal{R}} \mathcal{E}\,d^3x = -\int_{\partial\mathcal{R}} \mathbf{S}\cdot\mathbf{n}\,dS.$$

*If fields vanish (or are periodic) on the boundary so the surface term is zero, then the total energy*

$$E \;=\; \int_{\mathbb{R}^3} \mathcal{E}\,d^3x$$

*is conserved: $\frac{dE}{dt} = 0$.*

**Causal bookkeeping interpretation.** *$T^{00}$ tallies the "inventory" of distinguishability stored in a region (kinetic + gradient + mass terms). The flux $T^{0i}$ records how that inventory flows across the boundary. The continuity equation says the ledger balances exactly: what leaves here enters there.*

*Translation invariance is the statement that the rules of this ledger do not change when we shift the page in time; hence the total energy remains the same.*

**Thought Experiment 7.2.3** (Feynman Diagram as a Tensor Expansion of the Field)**.** *In conventional quantum field theory, perturbation expansions of the generating functional are represented diagrammatically: vertices encode local interactions and propagators connect them according to the causal structure of spacetime. In the causal formulation developed here, the same construction arises directly from the Universe Tensor.*

*Each vertex corresponds to an event tensor* $E_k \in T(V)$ *contributing a measurable distinction within the causal order. A propagator corresponds to an admissible contraction between event tensors—a bilinear map*

$$\langle E_i, E_j \rangle = \mathrm{Tr}(E_i^\top \, G \, E_j),$$

*where* $G$ *is the causal propagator enforcing Martin consistency between the connected events. The complete amplitude for a process is therefore the contraction of the ordered product*

$$U_n = \sum_{k=1}^{n} E_k,$$

*with all admissible propagators. The resulting scalar invariants of* $U_n$ *constitute the measurable quantities of the theory.*

*Thus, a Feynman diagram is the graphical representation of a tensor contraction in the causal algebra: each diagram corresponds to one term in the finite expansion of the Universe Tensor, and summing over all diagrams is equivalent to enforcing global consistency of causal order. What appears in standard field theory as a perturbation series is, in this formalism, a finite enumeration of distinguishable causal relations—a bookkeeping identity derived from the Reciprocity Law rather than using calculus.*

## 7.3 Angular Momentum and Spin

Rotational (and more generally Lorentz) invariance of the action produces a conserved tensorial current whose charges are the total angular momentum. Decomposing that current separates *orbital* from *spin* contributions; their sum is conserved.

### 7.3.1 Noether Current for Lorentz Invariance

Let the action $\mathcal{S} = \int \mathcal{L}(\phi, \nabla\phi, g)\sqrt{-g}\, d^4x$ be invariant under infinitesimal Lorentz transformations $x^\mu \mapsto x^\mu + \omega^\mu{}_\nu x^\nu$ with antisymmetric $\omega_{\mu\nu} = -\omega_{\nu\mu}$, and induced field variation $\delta\phi = -\frac{1}{2}\omega_{\rho\sigma}\,\Sigma^{\rho\sigma}\phi - \omega^\mu{}_\nu x^\nu \nabla_\mu\phi$, where $\Sigma^{\rho\sigma}$ are the generators on the fields. Noether's theorem yields the (canonical) angular-momentum current

$$J^{\lambda\rho\sigma}_{\text{can}} = x^\rho T^{\lambda\sigma}_{\text{can}} - x^\sigma T^{\lambda\rho}_{\text{can}} + S^{\lambda\rho\sigma}, \qquad \partial_\lambda J^{\lambda\rho\sigma}_{\text{can}} = 0,$$

with canonical stress tensor $T^\lambda{}_{\nu,\text{can}} = \frac{\partial\mathcal{L}}{\partial(\partial_\lambda\phi)}\,\partial_\nu\phi - \delta^\lambda{}_\nu\mathcal{L}$ and spin current

$$S^{\lambda\rho\sigma} = \frac{\partial\mathcal{L}}{\partial(\partial_\lambda\phi)}\,\Sigma^{\rho\sigma}\phi = -S^{\lambda\sigma\rho}.$$

**Thought Experiment 7.3.1** (Spin–$\frac{1}{2}$ as Two–Valued Causal Orientation)**.** *Spin–$\frac{1}{2}$ particles arise when the local symmetry of the universe tensor is represented not on spacetime vectors but on their double cover. Under a full $2\pi$ rotation, the causal ordering of distinguishable events reverses sign before returning to its original configuration after $4\pi$. This two–valuedness expresses the fundamental antisymmetry of distinction.*

*Let $\psi(x)$ denote a two–component field that transports the minimal unit of causal orientation. Its dynamics follow from the Lorentz–invariant action*

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi,$$

*where $D_\mu$ is the gauge–covariant derivative and the $\gamma^\mu$ generate the Clifford algebra*

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}.$$

*Each $\gamma^\mu$ acts as a local operator of causal rotation: applying it changes the orientation of the measurement frame while preserving causal order. Because the algebra squares to unity only after two applications, a single $2\pi$ rotation introduces a minus sign, $\psi \to -\psi$, revealing that the physical state is defined on the double cover $\mathrm{Spin}(3,1)$ of the Lorentz group.*

*In the informational picture, the two components of $\psi$ encode the forward and reverse orientations of causal distinction—measurement and variation. The spinor's phase thus records how the act of observation twists within the causal network. Quantized angular momentum*

$$S = \tfrac{\hbar}{2}$$

*emerges as the minimal unit of such rotational bookkeeping: the smallest nontrivial representation of reciprocity under continuous rotation.*

*Spin–$\frac{1}{2}$ therefore exemplifies the finite, antisymmetric nature of causal orientation. A complete $4\pi$ turn is required for full restoration of distinguishability, making the spinor the algebraic expression of the universe tensor's two–sheeted structure in orientation space.*

### 7.3.2 Belinfante–Rosenfeld Improvement

The canonical $T_{\mu\nu}$ need not be symmetric. Define the Belinfante superpotential

$$B^{\lambda\rho\sigma} = \tfrac{1}{2}\Big(S^{\rho\lambda\sigma} + S^{\sigma\lambda\rho} - S^{\lambda\rho\sigma}\Big), \qquad B^{\lambda\rho\sigma} = -B^{\lambda\sigma\rho}.$$

The *improved* symmetric stress tensor and current are

$$T_B^{\mu\nu} = T_{\mathrm{can}}^{\mu\nu} + \partial_\lambda\Big(B^{\lambda\mu\nu} - B^{\mu\lambda\nu} - B^{\nu\lambda\mu}\Big), \qquad J_B^{\lambda\rho\sigma} = x^\rho T_B^{\lambda\sigma} - x^\sigma T_B^{\lambda\rho},$$

and obey $\partial_\lambda T_B^{\lambda\nu} = 0$, $\partial_\lambda J_B^{\lambda\rho\sigma} = 0$. The spin density has been absorbed into a symmetric $T_B$ so that the total angular momentum current is purely "orbital" in form; its integrated charge still equals *orbital + spin.*

### 7.3.3 Conserved Charges

For a Cauchy slice $\Sigma$ with normal $u_\lambda$,

$$M^{\rho\sigma} = \int_\Sigma J^{\lambda\rho\sigma}\, d\Sigma_\lambda = \int_\Sigma \left( x^\rho T_B^{\lambda\sigma} - x^\sigma T_B^{\lambda\rho} \right) d\Sigma_\lambda, \qquad \frac{d}{d\tau} M^{\rho\sigma} = 0.$$

In 3D language (flat space, $u_\lambda = (1,0,0,0)$), the spatial components give the angular momentum vector $\mathbf{J} = \int d^3x \left( \mathbf{x} \times \mathbf{p} \right) + \mathbf{S}$, with momentum density $\mathbf{p} = T_B^{0i}\, \hat{\mathbf{e}}_i$ and spin density $\mathbf{S}$ encoded via $S^{0ij}$.

### 7.3.4 Worked Examples

**Real scalar (spin 0).** For $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\, \partial^\mu\phi - \frac{1}{2}m^2\phi^2$, $\Sigma^{\rho\sigma} = 0$ so $S^{\lambda\rho\sigma} = 0$. The Belinfante step is trivial and

$$\mathbf{J} = \int d^3x\, \mathbf{x} \times \left( \dot{\phi}\, \nabla\phi \right),$$

purely orbital. Conservation $\partial_\lambda J^{\lambda\rho\sigma} = 0$ reduces to $\partial_\mu T^{\mu\nu} = 0$ (already shown) plus antisymmetry.

**Dirac field (spin 1/2).** For $\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$, the generators are $\Sigma^{\rho\sigma} = \frac{i}{4}[\gamma^\rho, \gamma^\sigma]$, giving nonzero spin current

$$S^{\lambda\rho\sigma} = \tfrac{1}{2}\, \bar{\psi}\, \gamma^\lambda \Sigma^{\rho\sigma} \psi.$$

The Belinfante tensor $T_B^{\mu\nu} = \frac{i}{4}\bar{\psi}\big(\gamma^\mu \overleftrightarrow{\partial^\nu} + \gamma^\nu \overleftrightarrow{\partial^\mu}\big)\psi$ is symmetric and conserved, and the total charge $M^{\rho\sigma}$ includes intrinsic spin; in the particle rest frame this yields the familiar $\frac{1}{2}\hbar$.

### 7.3.5 Bookkeeping Interpretation

Rotational invariance says the ledger of causal distinctions is unchanged when we rotate our labeling rules. The orbital term tracks the "moment arm" of the flow of distinguishability ($\mathbf{x}\times\mathbf{p}$). The spin term tallies how the *label structure of the field itself* transforms under rotations (internal frame rotation via $\Sigma^{\rho\sigma}$). The Belinfante improvement is just a repackaging of the ledger so that the stress tensor carries the full conserved charge in a symmetric form—useful whenever the geometry (gravity) couples to $T_{\mu\nu}$.

**Remark 25.** *Total angular momentum is conserved because the action is invariant under Lorentz rotations. Orbital and spin are bookkeeping columns in the same invariant total; how you apportion them depends on your accounting scheme (canonical vs. Belinfante), not on the physics.*

## 7.4 Gauge Fields as Local Noether Symmetries

Global symmetries ensure that the totals in the causal ledger remain unchanged when every observer applies the same transformation. When the symmetry parameters vary from point to point, the bookkeeping must introduce additional terms to maintain local consistency. These new terms are the *gauge fields* of the theory: dynamic corrections that restore Martin consistency under spatially varying transformations.

### 7.4.1 From Global to Local Symmetry

Consider a field $\phi(x)$ transforming under a continuous group $G$ with infinitesimal parameter $\alpha^a$ and generators $T^a$:

$$\delta\phi = i\,\alpha^a T^a \phi.$$

If $\alpha^a$ is constant, the action $\mathcal{S} = \int \mathcal{L}(\phi, \nabla\phi)\, d^4x$ is invariant, and Noether's theorem yields a conserved current $J^\mu_a$. If $\alpha^a$ becomes a function of position, $\alpha^a = \alpha^a(x)$, an extra term appears,

$$\delta\mathcal{L} = i\,(\partial_\mu \alpha^a)\,\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\,T^a\phi,$$

breaking the conservation law. To preserve local invariance, the derivative $\partial_\mu$ must be replaced by a *covariant derivative*

$$D_\mu\phi = (\partial_\mu - ig\,A^a_\mu T^a)\phi,$$

where the compensating field $A^a_\mu$ transforms as

$$\delta A^a_\mu = \frac{1}{g}\,\partial_\mu\alpha^a + f^{abc}\alpha^b A^c_\mu.$$

The new Lagrangian

$$\mathcal{L} = \mathcal{L}(\phi, D_\mu\phi) - \tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu,$$

is invariant under the full local symmetry. The field strength $F^a_{\mu\nu}$ is the curvature of the gauge connection $A^a_\mu$—the residue of non-commuting parallel transports in the internal symmetry space.

**Example: Aharonov–Bohm Phase as Pure Gauge Holonomy**

**Statement.** A nontrivial loop integral of the connection shifts interference with no local force—measurement of gauge holonomy.

**Key relation.**

$$\Delta\varphi \;=\; \frac{q}{\hbar}\oint_\gamma \mathbf{A}\cdot d\ell \;=\; \frac{q\,\Phi_B}{\hbar}.$$

**Reciprocity framing.** The partition is unchanged locally (no field in the slits), but the selected update accumulates a path-dependent phase—an element of the connection's holonomy group. Interference shift records the gauge's parallel transport rule.

**Operational consequence.** Local indistinguishability with global inequivalence: a canonical example where measurement reads a *global* invariant of the gauge without local curvature along the paths.

### 7.4.2 Interpretation in the Causal Framework

In the causal picture, global symmetry corresponds to relabeling the entire causal network by a uniform rule; local symmetry corresponds to allowing each neighborhood to choose its own labeling convention. The gauge field $A^a_\mu$ records how those conventions differ and how information must be exchanged between neighboring regions to keep the global ledger balanced. It is the *connection form of causal order* in informational space.

Curvature $F^a_{\mu\nu}$ measures the residual inconsistency that appears when these local labelings are carried around a closed causal loop—exactly analogous to the spacetime curvature derived earlier from $\Gamma^\lambda_{\mu\nu}$. Gauge bosons are therefore the finite, propagating corrections by which the universe restores Martin consistency across overlapping informational domains.

**Thought Experiment 7.4.1** (Aharonov–Bohm Effect as a Test of Causal Gauge Consistency)**.** *The Aharonov–Bohm experiment demonstrates that the physically relevant quantity in electromagnetism is not the field strength $F_{\mu\nu}$ alone but the connection $A_\mu$ that governs causal phase transport.*

*Consider an electron beam split into two coherent branches encircling a region containing a confined magnetic flux $\Phi$, with no field present along either path. In the causal formulation, each branch corresponds to a sequence of ordered events $\{E_{1,k}\}$ and $\{E_{2,k}\}$ transported by the local gauge connection $A_\mu$. The Reciprocity Law requires that each infinitesimal update preserve order:*

$$E_{k+1} = E_k + \Phi^{-1}(A_\mu \, dx^\mu),$$

*so that the cumulative phase acquired along a closed loop is*

$$\Delta\phi = \frac{e}{\hbar} \oint A_\mu \, dx^\mu = \frac{e\Phi}{\hbar}.$$

*Although the magnetic field vanishes along both paths ($F_{\mu\nu} = 0$ locally), the two causal chains differ by a holonomy in the connection—an informational mismatch in the bookkeeping of phase. When the beams are recombined, their interference pattern depends on $\Delta\phi$: shifting continuously as the enclosed flux changes by fractions of the flux quantum $h/e$.*

*In the causal gauge picture, this effect shows that the universe tensor records not merely local field strengths but the global consistency of the connection. The vector potential $A_\mu$ is the differential form of causal memory; its holonomy measures how distinction is transported around a loop. The Aharonov–Bohm interference is thus the experimental detection of a nontrivial element of the causal holonomy group—the smallest observable instance of curvature without force.*

### 7.4.3 Bookkeeping of Local Consistency

In statistical terms, each gauge symmetry adds a new column to the causal ledger. Local invariance means that the exchange rates between these columns are position-dependent, and $A^a_\mu$ supplies the conversion factors that keep the books balanced. The continuity equation

$$\nabla_\mu J^\mu_a = 0$$

expresses the same principle as before: what leaves one neighborhood enters another, but now for every internal degree of freedom labeled by $a$. The gauge field guarantees that this exchange is recorded consistently even when observers adopt different local frames.

**Remark 26.** *Every gauge field is a Noether correction promoted to locality.*

*It is the differential accountant of causal order, ensuring that symmetry—and hence conservation—holds point by point. Curvature is the residue of that accounting around a loop; interaction is the redistribution of causal balance between neighboring observers. Quantum field theory is therefore the calculus of local Noether symmetries of the Causal Universe Tensor.*

## 7.5 Mass and the Breaking of Symmetry

Perfect causal symmetry implies motion at the limit of distinguishability—the null trajectories of light. In this regime, the action and all of its Noether currents remain invariant under local gauge transformations, and the scalar invariants of the Causal Universe Tensor are preserved exactly. *Mass* appears when this invariance can no longer be maintained everywhere. It is the measure of how far a system deviates from perfect causal balance.

### 7.5.1 From Gauge Symmetry to Mass Terms

Suppose the Lagrangian density for a field $\phi$ is invariant under the local transformation $\phi \to e^{i\alpha(x)}\phi$. If the causal network experiences a finite delay in maintaining that invariance—so that the local transformation cannot be matched exactly between neighboring observers—the covariant derivative acquires a small, persistent residue. In the simplest case this appears as an additional quadratic term in the Lagrangian:

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(|\phi|), \qquad V(|\phi|) = \tfrac{1}{2}\mu^2|\phi|^2 + \tfrac{1}{4}\lambda|\phi|^4.$$

When the potential $V$ selects a nonzero expectation value $\langle\phi\rangle = v/\sqrt{2}$, the gauge symmetry of the vacuum is spontaneously broken, and the covariant derivative term generates an effective mass for the gauge field:

$$m_A = g\,v.$$

The field no longer propagates at the causal limit; it carries a finite informational delay between cause and effect.

**Thought Experiment 7.5.1** (Mexican Hat Potential and the Breaking of Informational Symmetry)**.** *In the causal formulation, symmetry breaking occurs when the universe tensor develops a preferred orientation in its space of distinguishable states. The simplest model of this phenomenon is the so–called Mexican hat potential, which encodes spontaneous differentiation in an initially symmetric field.*

*Let $\phi$ be a complex scalar component of the causal gauge field. Its local informational curvature is represented by the potential*

$$V(\phi) = \lambda\big(|\phi|^2 - v^2\big)^2, \qquad \lambda, v > 0.$$

*For $|\phi| < v$, the curvature is positive and the symmetric state $\phi = 0$ is unstable; for $|\phi| = v$, the curvature vanishes along a circle of minima. Each choice of phase $\theta$ on this ring corresponds to an equally valid, order-preserving configuration of the universe tensor.*

*When a particular $\theta$ is selected by finite observation or causal fluctuation, the continuous $U(1)$ symmetry of the potential is reduced to the discrete subgroup that preserves that orientation. The resulting excitations decompose into two orthogonal modes:*

$$\phi(x) = (v + h(x))e^{i\theta(x)},$$

*where $h(x)$ represents measurable variations in magnitude (massive mode) and $\theta(x)$ represents phase fluctuations (massless Goldstone mode). Coupling this field to a local gauge connection $A_\mu$ converts the phase fluctuation into a longitudinal component of $A_\mu$, endowing it with mass through the informational curvature of the potential.*

*Operationally, the Mexican hat potential marks the point where causal order can no longer cancel its own third variation: a finite bias in distin-*

*guishable states propagates through the reciprocity map as an effective mass term. In the informational picture, mass is the cost of maintaining a broken symmetry— the curvature required to remember which minimum was chosen.*

### 7.5.2 Causal Interpretation

In the causal framework, symmetry breaking represents the loss of perfect order propagation. The gauge can no longer be reconciled exactly between neighboring domains, and a residual phase difference accumulates. That phase difference behaves as inertia: a tendency of the causal structure to resist change in its internal configuration. The quantity we call *mass* measures the curvature of causal order in the informational direction—the degree to which a system's internal symmetry lags behind the propagation of light.

Thus the Higgs mechanism appears as a natural bookkeeping adjustment. The scalar field $\phi$ provides an additional column in the ledger that can absorb the mismatch of local phase conventions. When the ledger cannot close exactly, the residual correction manifests as a finite mass term. Mass is therefore not a separate entity but the universe's accounting of imperfect causal synchronization.

### 7.5.3 Statistical View

In the statistical mechanics of causal order, mass quantifies the variance of the action around its stationary value:

$$m^2 \propto \left\langle (\delta \mathcal{S})^2 \right\rangle.$$

Lightlike propagation corresponds to zero variance: every observer's record of order agrees. Massive propagation corresponds to finite variance: local histories differ slightly, and the ensemble average restores consistency only statistically. The rest energy $E = mc^2$ measures the informational cost of maintaining a coherent description across those variations.

**Remark 27.** *Mass is the finite residue of broken symmetry—the price the universe pays for keeping its causal books consistent when perfect gauge balance cannot be sustained. Where light moves without lag, massive matter hesitates, accumulating phase in time. The rest mass of any field is thus the measure of its informational inertia: how much causal order must bend to preserve consistency within a finite universe.*

**Thought Experiment 7.5.2** (Semiconductors as Partially Broken Informational Lattices)**.** *In a crystalline solid, the atoms form a periodic causal network—a lattice of distinguishable sites linked by local order relations. Within this structure, electrons occupy quantized informational states whose distinguishability depends on both lattice symmetry and the observer's partition of measurement.*

*At zero temperature, all available states up to the Fermi level are filled, and the partition $\mathcal{P}_n$ groups occupied and unoccupied states into two disjoint causal classes. In a perfect insulator these classes are fully separated by a forbidden bandgap: no variation in the universe tensor can map one class into the other without violating order preservation. In a metal the classes overlap completely, forming a continuous manifold of accessible distinctions.*

*A semiconductor occupies the intermediate regime. Its informational lattice is nearly symmetric but not fully resolved; there exists a narrow causal boundary between filled and unfilled states. Thermal or dopant-induced perturbations refine the partition from $\mathcal{P}_n$ to $\mathcal{P}_{n+1}$, enabling limited causal transitions across the bandgap. The carrier density*

$$n \propto e^{-E_g/k_{\mathrm{B}}T}$$

*measures the probability that such a refinement occurs—an exponential suppression of distinguishability transitions with increasing gap energy $E_g$.*

*In this view, conduction arises when the partition between causal classes of electron states becomes permeable under variation. Doping, temperature,* *and illumination are operations that adjust the informational curvature of the lattice, controlling how easily one class of distinguishability flows into another. Semiconductors are thus macroscopic examples of causal fuzziness under controlled refinement: a solid-state realization of partition dynamics between measurement and variation.*

## 7.6 Conclusion: Quantization as Finite Consistency

The classical universe is the ledger of perfect causal balance: every distinction is matched, every event accounted for, every observer's record consistent with the next. Quantum mechanics emerges when that perfection is relaxed—when the bookkeeping of order is carried out on a finite register. Each quantum of action, each exchange of $\hbar$, is a discrete adjustment in the causal gauge: the smallest step by which the universe can preserve consistency without infinite precision.

From this point of view, the quantum field is not a separate ontology but the statistical completion of the same calculus that defines the geometry of spacetime. The field amplitudes are probability weights for maintaining order across overlapping causal neighborhoods. Their phases encode the orientation of the gauge, and their interference expresses the collective effort of all observers to remain mutually consistent. The path integral is thus the partition function of causal order.

Mass, spin, and charge are the residues of that consistency process. Mass records temporal lag, spin records the rotational structure of labeling, and charge records the bookkeeping of internal symmetries. None are primitive; all arise from the same principle that distinguishes light: the demand that order be preserved even when the universe must correct itself locally.

In the causal formalism, conservation laws, gauge interactions, and quantization share a single origin. They are not independent laws written into nature but emergent regularities of a self-consistent informational network. The Causal Universe Tensor provides the grammar of that network; its contractions yield spacetime geometry, its variations yield fields, and its statistical extension yields the quantum.

**Remark 28.** *The universe is not made of matter or of energy, but of consistency. What we call physics is the continuous reconciliation of local descriptions of order, carried out one quantum at a time. Quantization is simply the discreteness of that reconciliation—the finite resolution of cause.*

**Thought Experiment 7.6.1** (Thought Experiment: The Echo Chamber Maze and Curvature Residue)**.** ***N.B.*** *This experiment translates geometric curvature into informational inconsistency.*

Setup. *Navigate a maze by clapping; echoes trace causal paths. Straight corridors (flat metric) return clean echoes—perfect parallel transport. Curved passages distort the return, producing phase residue.*

Demonstration. *Walk a closed loop and compare the echoed rhythm. Any mismatch measures curvature $R \neq 0$: the difference between expected and returned distinction. When total residue cancels ($U^{(4)} = 0$), the maze is globally consistent.*

Interpretation. *Curvature is the informational stress of maintaining closure in a finite domain. Echo intensity corresponds to entropy: more paths, higher distinguishability. Einstein's equation emerges as the balancing condition between geometric residue and informational flux.*

**Epilogue.** When the calculus of variations meets the calculus of observation, they become one and the same. The least action principle is not a rule imposed from outside; it is the expression of the universe's preference for maximal consistency within finite means. Light traces the paths where this consistency is perfect. Matter records where it is not. And the quantum is the measure of how the universe keeps its books.

# Chapter 8

# The Monotonicity of Entropy

## 8.1 Statement of the Law

**Proposition 7** (Monotonicity of Causal Entropy)**.** *For any sequence of Martin–consistent causal sets*

$$\mathcal{C}_1 \subseteq \mathcal{C}_2 \subseteq \cdots ,$$

*the associated entropies*

$$S[\mathcal{C}_n] = k_{\mathrm{B}} \ln |\Omega(\mathcal{C}_n)|$$

*satisfy*

$$\Delta S_n \equiv S[\mathcal{C}_{n+1}] - S[\mathcal{C}_n] \geq 0,$$

*with equality only for informationally complete partitions.*

*Proof (Sketch).* Each causal refinement $\mathcal{C}_n \to \mathcal{C}_{n+1}$ corresponds to an enlargement of the observer's partition of distinguishable events. By the Axiom of Finite Observation, refinement cannot reduce the set of admissible micro–orderings:

$$\Omega(\mathcal{C}_n) \subseteq \Omega(\mathcal{C}_{n+1}).$$

Taking logarithms gives $S[\mathcal{C}_{n+1}] \geq S[\mathcal{C}_n]$. The inequality is strict whenever the refinement exposes previously indistinguishable configurations. □

*A full proof is provided in Appendix* **??**.

**Thought Experiment 8.1.1** (The Library Catalog and the Arrow of Distinction)**.** ***N.B.*** *This experiment illustrates Theorem 1 as a theorem of causal order, not a postulate of thermodynamics. It shows how monotonic distinguishability ($\Delta S \geq 0$) arises naturally from the structure of consistent extension.*

Setup. *Imagine a vast library whose books represent events $\{e_i\}$. Each measurement attaches finer tags—subject, author, edition—refining the causal order. By the Axiom of Event Selection, no tag can be removed without creating inconsistency among shelves (e.g., merging sci-fi and history). Hence, the total number of distinguishable configurations $N$ can only increase or remain constant.*

Demonstration. *Attempting to "un-tag" a shelf merges incompatible categories, breaking bijection with prior distinctions. Thus time's arrow emerges as the monotonic count of consistent refinements:*

$$S = \ln N, \qquad \Delta S \geq 0.$$

Interpretation. *Entropy here is not disorder but bookkeeping: the log of consistent distinctions maintained through observation. The irreversible direction of measurement follows directly from order preservation, not energy dissipation.*

## 8.2 Entropy as Informational Curvature

In differential form, the same statement appears as the non-negativity of informational curvature:

$$\nabla_i \nabla_j S \geq 0.$$

Flat informational geometry corresponds to equilibrium ($\Delta S = 0$), while positive curvature indicates the growth of accessible micro-orderings. The flux of this curvature defines the *entropy current*

$$J_S^\mu = k_\mathrm{B}\, \partial^\mu S,$$

whose divergence measures local entropy production:

$$\nabla_\mu J_S^\mu = k_\mathrm{B}\, \Box S \geq 0.$$

Thus $\Delta S > 0$ is equivalent to the statement that the informational Laplacian $\Box S$ is positive definite under Martin–consistent transport.

**Thought Experiment 8.2.1** (Maxwell's Demon as Non-commutative Selection)**.** *Consider a classical gas divided by a partition with a single gate controlled by a demon who measures particle velocities and opens the gate selectively. Let $M$ denote the demon's measurement operator and $U$ the physical evolution of the gas. If $M$ and $U$ commute—$[M, U] = 0$—the demon's observation does not alter the causal order: measurement and evolution can be exchanged without changing the macrostate. But in reality $[M, U] \neq 0$: the act of measurement refines the partition of distinguishable states, altering the subsequent evolution. This non-commutativity forces the entropy balance*

$$\Delta S_{gas} + \Delta S_{demon} \;=\; k_\mathrm{B} \ln|\Omega_{joint}| \;>\; 0,$$

*because the demon's internal record adds new causal distinctions to the universe tensor even as it reduces them locally.*

*Operationally, the demon cannot perform a measurement without joining the measured system's causal order; the refinement of its internal partition $P_n \to P_{n+1}$ increases the global count of distinguishable configurations. The apparent violation of the Second Law disappears: the measurement and evolution operators fail to commute, and that failure* is *the entropy production* *term. Thus Maxwell's demon exemplifies the theorem $\Delta S \geq 0$: informational refinement in one domain demands compensating coarsening in another so that the global order remains consistent.*

## 8.3 Statistical Interpretation

From the causal partition function

$$Z = \int \exp\left(\frac{i}{\hbar} S[T]\right) DT,$$

the ensemble average of the informational gradient obeys

$$\langle \nabla_\mu J_S^\mu \rangle = k_{\mathrm{B}} \langle \nabla_\mu \nabla^\mu S \rangle \geq 0.$$

The equality $\Delta S = 0$ corresponds to detailed balance of causal fluxes; any deviation yields positive entropy production.

## 8.4 Physical Consequences

1. **Arrow of Time.** Causal order expands in one direction only—toward increasing distinguishability of events. Time is the parameter labeling this monotonic refinement.

2. **Thermodynamic Limit.** In the continuum limit, $\Delta S > 0$ reproduces the classical second law, but here the law is not statistical: it is a theorem of consistency. No causal evolution that decreases $S$ can remain Martin–consistent.

3. **Gravitational Coupling.** From Chapter 4, curvature couples to gradients of $S$ through the entropic stress tensor:

$$G_{\mu\nu} = 8\pi \left(T_{\mu\nu} + T_{\mu\nu}^{(S)}\right), \quad T_{\mu\nu}^{(S)} = \frac{1}{k_{\mathrm{B}}} \nabla_\mu \nabla_\nu S.$$

Hence $\Delta S > 0$ corresponds to a net positive contribution of informational curvature to spacetime geometry—a causal analogue of energy influx.

## 8.5 Conclusion

**Law 4** (The Law of Causal Order)**.** *The Law of Causal Order may be stated succinctly:*

$$\boxed{\Delta S \geq 0 \quad \textit{for every Martin–consistent refinement of causal structure.}}$$

*Entropy is not a measure of disorder but of latent order yet unresolved. Every act of measurement refines the universe's partition, and each refinement enlarges the count of admissible configurations. The universe evolves by distinguishing itself.*

## 8.6 *Quod erat demonstrandem*

We began with the observation that every act of physics is an act of distinction: to measure is to separate one possibility from another. Within ZFC, such distinctions are represented as finite subsets of a causal order, and the act of measurement is the enumeration of their admissible refinements. Nothing else is assumed.

Martin's Axiom enters only to ensure that these refinements can be extended consistently—that the space of distinguishable events admits countable dense families without contradiction. This single assumption is the logical equivalent of $\sigma$-additivity in measure theory, the minimal condition required for any self-consistent calculus of observation.

From this, the Second Law follows as a theorem of order: each consistent extension of the causal set increases the number of distinguishable configurations, and therefore

$$\Delta S \geq 0.$$

Entropy is not a statistical tendency but a logical necessity—the price of consistency within a self-measuring universe.

No new forces, particles, or cosmologies are introduced; only the rule by which distinction propagates. What began as a grammar of measurement closes as the unique structure of physical law.

**Theorem 1** (The Second Law of Causal Order)**.** *In any finite, causally consistent ordering of distinguishable events, the number of measurable distinctions cannot decrease. Every admissible extension of order produces at least one new differentiation, and therefore every universe consistent with its own record of events obeys the inequality*

$$\Delta S \geq 0.$$

*Conclusion.* We are left with but one conclusion:

$$\boxed{\text{Order implies dynamics.}}$$

A universe that preserves its own causal record must, by necessity, increase the count of what can be distinguished. □

*Quod erat demonstrandum.*

# Coda: The Causal Universe as a "White Hole"

**N.B.** This coda is a conceptual reflection, not a cosmological claim. It extends the logic of causal order one step beyond the proof: if a black hole represents a local failure of informational closure—an *informational sink*—then a universe that must always increase its distinguishability ($\Delta S \geq 0$) behaves as its formal converse, an *informational source.* No inference about classical white-hole solutions, singularities, or cosmic acceleration is intended [51, 111, 79, 114, 41, 12].

*Setup.* In the causal framework developed above, a black hole corresponds to a horizon of informational saturation: beyond it, further distinctions cannot be reconciled without contradiction. From the external observer's perspective, the stream of incoming updates exceeds the capacity of the causal record; order collapses into opacity. It is the local end of distinguishability—a finite boundary of the universe's bookkeeping.

*Invariant to Enforce.* The Second Law of Causal Order forbids the universe as a whole from entering such a state. A global informational sink would halt the count of distinguishable events, violating the monotonic condition $\Delta S \geq 0$. Therefore, the universe must remain an *informational source*: a domain that can always emit new, Martin-consistent refinements of causal order [12, 68].

*Formal Analogy.* In general relativity, a white hole is the time-reversal of a black hole: a region from which events emerge but into which none can enter. Under the logic of causal measurement, the same symmetry arises abstractly. If measurement always increases distinguishability, the global causal field must behave as a continual emitter of information—a formal white hole in informational phase space. Its "expansion" is not a dynamic expansion of matter, but a logical expansion of the record of distinctions.

*Interpretation.* The outward "pressure" observed as dark-energy expansion can thus be viewed, purely formally, as the informational tension that maintains the universe's role as a source. Each new distinction contributes to the curvature of the causal field; each increment of $\Delta S$ is an act of emission that preserves global consistency. In this sense, the universe is not merely growing in size but in *resolution*: its geometry expands because the space of distinguishable configurations must.

**Thought Experiment 8.6.1** (Extrapolation: Leavitt's Ladder and the

Hubble Constant). ***N.B.*** *This extrapolation is purely formal. It illustrates how the causal requirement $\Delta S \geq 0$ manifests observationally through Leavitt's period–luminosity law and the Hubble constant. No cosmological claim beyond formal analogy is intended.*

Setup. *In 1912, Henrietta Swan Leavitt discovered that the luminosity $L$ of Cepheid variable stars increases monotonically with their oscillation period $P$, obeying*

$$M = a \log_{10} P + b.$$

*This law defined the first invariant mapping from local temporal oscillation to global metric scale. By calibrating redshift $z$ against Leavitt's ladder, Edwin Hubble (1929) derived the proportionality $v = H_0 d$, establishing the expansion rate of the universe.*

Formal Interpretation. *Within the causal framework, Leavitt's relation is the archetype of order expansion. Each measured Cepheid adds a consistent distinction between temporal frequency and spatial magnitude. The mapping $P \mapsto L$ is an order-preserving bijection between local oscillation and global extension. Its monotonicity enforces the same logical law as the Second Law of Causal Order: each refinement increases distinguishability, so $\Delta S \geq 0$.*

Analogy. *Just as Leavitt's ladder converts periodic variation into distance, the causal universe converts informational differentiation into metric expansion. The Hubble constant $H_0$ expresses the global rate at which new distinctions become measurable—an informational expansion, not a mechanical one.*

Scope. *This extrapolation is formal. It demonstrates that cosmic expansion, when viewed through Leavitt's law, reflects the universe's role as a causal source maintaining $\Delta S \geq 0$, not a dynamical explosion of matter in space.*

*Scope.* This reflection is purely formal. It demonstrates how a universe obeying the logical law $\Delta S \geq 0$ shares the causal-source structure of a white hole, not that it *is* one. The correspondence is informational, not physical, and serves only to illuminate the symmetry between causal emission and causal measurement that underlies the theorem just proved.

□

# Appendix A

# Proofs

## A.1 The Calculus of Measurement

# Appendix B

# Notation

This appendix summarizes the symbols and conventions used throughout the monograph. The goal is clarity. Every notation corresponds to an operational procedure: recording events, merging ledgers, composing systems, or evolving a notebook of admissible distinctions forward in time.

## Events and Ledgers

- An *event* is a measurable, irreversible update to a system's state. A finite set of observations produces a finite, ordered record of events.
- A *ledger* is the notebook containing this ordered record. Ledgers are denoted by calligraphic symbols $(\mathcal{L}, \mathcal{M}, \dots)$.
- The *Axiom of Order* guarantees that every ledger is a countable, totally ordered sequence of events.

## Tensor Composition

- Independent ledgers compose via the tensor product

$$\mathcal{L} \otimes \mathcal{M},$$

which produces a joint ledger with no implied evolution. The tensor product is symmetric up to canonical isomorphism and carries no time direction.

- The tensor product *does not* imply interaction. It merely constructs a space capable of recording joint events.

## Merge Operator

- Compatible ledgers merge by addition, written

$$\mathcal{L} + \mathcal{M}.$$

  This operation is commutative and introduces no new events. It simply coalesces distinctions already present in the two ledgers.

- Because $+$ is commutative and order-independent, no commutator is defined on $+$.

## Evolution (Fold) Operators

- A *fold* is an evolution operator that acts on a ledger,

$$F : \mathcal{L} \to \mathcal{L}.$$

  A fold updates the ledger forward in time, reconciling the existing record with a new admissible distinction.

- Successive folds are composed using standard function composition,

$$G \circ F : \mathcal{L} \to \mathcal{L}.$$

  Only folds carry a time direction.

- When the sequence of folds varies with time,

$$\bigcirc_{i=1}^{n} F_i \quad = \quad F_n \circ F_{n-1} \circ \cdots \circ F_1.$$

  This is the *iterated fold*.

- When the fold is identical at each step, we write

$$F^{\circ n} \quad = \quad \underbrace{F \circ F \circ \cdots \circ F}_{n \text{ times}}.$$

  We avoid the notation $F^n$ to prevent confusion with contravariant tensor indices.

## Commutators

- The commutator measures the failure of two folds to commute:

$$[F, G] = F \circ G - G \circ F.$$

  Because the tensor product and addition introduce no time ordering, the commutator is defined only for folds.

- Nonzero commutators represent informational curvature: different orders of reconciliation produce different ledgers.

## Functionals and Variations

- A functional on a ledger-refined trajectory is written

$$J[x] = \int_a^b f(t, x(t), \dot{x}(t))\, dt.$$

- An admissible variation is of the form

$$x_\varepsilon(t) = x(t) + \varepsilon\eta(t), \qquad \eta(a) = \eta(b) = 0.$$

- The first variation is the directional derivative

$$\delta J[x;\eta] = \frac{d}{d\varepsilon} J[x_\varepsilon]\bigg|_{\varepsilon=0}.$$

- The Euler–Lagrange equation is the condition that $\delta J[x;\eta] = 0$ for all admissible $\eta$.

These conventions are used consistently in later chapters. No symbol is overloaded, and every operator corresponds to a physical or informational procedure: composition, merging, or evolution. All results follow from these definitions and the axioms introduced in Chapter 1.

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