

Module 4: The Noether Bridge

Validating G4: Derivation of Conserved Currents $\nabla \cdot \mathbf{T} = 0$

G4S — Proof of Conservation (V.tex Compliant)

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1 Input Node: Stationarity from Kinematic Closure (G2)

The conservation law relies on the *action principle* being stationary. The kinematic closure $\mathbf{U}^{(4)} = 0$ guarantees that the fundamental field \mathbf{U} is a solution to the Euler–Lagrange equations in the sense needed to invoke stationarity $\delta S = 0$.

Definition 1 (Action Functional \mathcal{S}). *The action functional is the integral of the local Lagrangian density \mathcal{L} that encapsulates informational curvature:*

$$\mathcal{S}[\mathbf{U}] = \int \mathcal{L}(\mathbf{U}, \nabla \mathbf{U}) d\tau,$$

where the measure $d\tau$ is left abstract (index-free).

2 Theorem: Conservation Law (Index-Free Noether)

Theorem 1 (Noether Conservation). *For every continuous symmetry Ξ that leaves the action invariant ($\delta\mathcal{S} = 0$), there exists an associated current \mathbf{J} whose divergence vanishes:*

$$\nabla \cdot \mathbf{J} = 0.$$

G4 Proof Obligation Fulfillment (Translational Case). 1. **Hypothesis (stationarity).** The closure $\mathbf{U}^{(4)} = 0$ positions \mathbf{U} at a stationary point: $\delta\mathcal{S} = 0$.

2. **Symmetry selection.** We select the translational symmetry Ξ_{trans} .

3. **Noether bridge (API).** Invoke the abstract construction

$$\mathbf{T} \leftarrow \mathbf{N}[\mathcal{L}, \mathbf{U}; \Xi_{\text{trans}}].$$

4. **Conservation identity.** Symmetry implies the resulting current is conserved, establishing the global bookkeeping constraint:

$$\nabla \cdot \mathbf{T} = 0.$$

Thus the Noether bridge carries the geometric constraint $\mathbf{U}^{(4)} = 0$ into the required structural conservation statement. \square
