Module 2: Kinematic Closure, Weak Reciprocity, and Newton Linearization

Validating G2: Minimal Curvature Condition $\mathbf{U}^{(4)} = 0$

G2S — Module Contract Fulfillment (V.tex Compliant)

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1 Input Node: Reciprocity and Variational Setup

The unique analytic closure for the admissible history $\mathbf{U}$ minimizes the *Informational Curvature Action* $\mathcal{A}[\mathbf{U}]$ over the Hilbert space $H^2([x_0, x_n])$:

$$\mathcal{A}[\mathbf{U}] \;=\; \frac{1}{2}\int_{[x_0,x_n]} \big(\mathbf{U}''\big)^2 \, dx.$$

The minimization is subject to fixed interpolation nodes and the *natural boundary conditions* $\mathbf{U}''(x_0) = \mathbf{U}''(x_n) = 0$.

Definition 1 (Bilinear Form of Reciprocity)**.** *The symmetric bilinear form* $\mathsf{B} : H^2([x_0, x_n]) \times H^2([x_0, x_n]) \to \mathbb{R}$ *corresponding to the weak form of the variational principle is*

$$\mathsf{B}(\mathbf{U}, \varphi) \;=\; \int_{[x_0,x_n]} \big(\mathbf{U}''\big)\, \varphi'' \, dx.$$

2 Theorem: Kinematic Closure ($\mathbf{U}^{(4)} = 0$)

Theorem 1 (Strong Kinematic Closure ($\mathbf{U^{(4)} = 0}$))**.** *The unique minimal-curvature solution* $\mathbf{U}^{\star}$ *compatible with Event Selection is the stationary point of* $\mathcal{A}[\mathbf{U}]$*, which yields the Euler–Lagrange strong form* $\mathbf{U}^{(4)} = 0$*.*

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G2 Proof Obligation Fulfillment*.*

G2 Proof Obligation. *(Provide stationarity, reciprocity, and refinement consistency.)*

Reciprocity and coercivity. The bilinear form $\mathsf{B}(\mathbf{U}, \varphi)$ is symmetric. Coercivity holds by a Poincaré–type inequality on $H^2([x_0, x_n])$. **Weak form (stationarity).** The first variation vanishes:

$$\text{Find } \mathbf{U} \in H^2([x_0, x_n]) \text{ such that } \mathsf{B}(\mathbf{U}, \varphi) = 0 \; \forall \, \varphi \in \mathcal{V}.$$

Strong form (Euler–Lagrange). Integrating by parts twice, boundary terms vanish by $\mathbf{U}''(x_0) = \mathbf{U}''(x_n) = 0$ and the test space definition, yielding

$$0 \;=\; \mathsf{B}(\mathbf{U}, \varphi) \;=\; \int_{[} x_0, x_n] \big(\mathbf{U}^{(4)}\big) \, \varphi \, dx \quad \Longrightarrow \quad \mathbf{U}^{(4)} = 0.$$

Refinement consistency (spline). The stationary solution $\mathbf{U}^{\star}$ is the cubic spline interpolant. Convergence of the discrete scheme confirms the continuum limit.

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Remark 1 (**Kernel elimination**)**.** *The homogeneous strong form* $\mathbf{U}^{(4)} = 0$ *admits the four-dimensional kernel* $\operatorname{span}\{1, x, x^2, x^3\}$*. Fixed interpolation* *nodes together with* $\mathbf{U}''(x_0) = \mathbf{U}''(x_n) = 0$ *remove the kernel, ensuring the* unique cubic spline solution $\mathbf{U}^\star$.

3 G2: Operational Interpretation and Noether Bridge

Physical meaning of $\mathbf{U}^{(4)} = 0$. The closure condition $\mathbf{U}^{(4)} = 0$ selects the minimal-information path consistent with global reciprocity.

[Davisson–Germer and kinematic consistency] Electron diffraction demonstrates that a discrete event chain (particle) must obey the continuous propagation law induced by the reciprocity principle behind $\mathbf{U}^{(4)} = 0$. The observed intensity peaks are *fixed points* of reciprocal measurement under lattice translations.

Residual and Newton step (index-free). The kinematic closure is interpreted as a numerically tractable root-finding problem. Define the residual R and (Fréchet) Jacobian J:

$$\mathsf{R}(\mathbf{U}) \;:=\; \mathbf{U}^{(4)}, \qquad \mathsf{J}(\mathbf{U}) \;:=\; \frac{\partial \mathsf{R}}{\partial \mathbf{U}}.$$

Given an iterate $\mathbf{U}^{(k)}$, the update $\delta\mathbf{U}$ solves the linear system:

$$\mathsf{J}(\mathbf{U}^{(k)})\,\delta\mathbf{U} \;=\; -\,\mathsf{R}(\mathbf{U}^{(k)}).$$

Noether bridge (API-level, index-free). Stationarity $\delta\mathcal{A} = 0$ is the hypothesis for conservation. From a local density L and symmetry Ξ,

$$\mathbf{T} \;\leftarrow\; \mathsf{N}[L, \mathbf{U}; \Xi]\,, \qquad \nabla\cdot\mathbf{T} \;=\; 0.$$

Thus $\mathbf{U}^{(4)} = 0$ provides the structural input for the conservation law formalized in G4S.