A Set of Measurement Conditions Sufficient for $\Delta S \geq 0$

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To Peano, who taught us how to count.

To Boltzmann, who first counted what could be distinguished.

To Planck, who taught us that the count is finite.

To Cantor, who showed us how to count the infinite.

To Kolmogorov, who showed us that information must be counted to be measured.

To William of Ockham, who insisted that we only count what is necessary.

"Je les possède, parce que jamais personne avant moi n'a songé à les posséder."
"Moi, je suis un homme sérieux. Je suis exact. J'aime que l'on soit exact."

—Antoine de Saint–Exupéry, *Le Petit Prince* (1943)

Abstract

This work shows that measurement itself defines the metric structure of physics: each act of distinction generates an increment of causal order. From this premise—that every measurement refines the universe's record of what can be distinguished—it follows that information, and thus entropy, can never decrease. Within standard set theory, this principle is proved as a theorem of causal consistency rather than assumed as a thermodynamic postulate.

We present a constructive proof that the entropy of any causally consistent universe is non-decreasing, $\Delta S \geq 0$. Within the axioms of Zermelo–Fraenkel set theory with Choice, we define a finite causal order of distinguishable events whose reciprocal operations—measurement and variation—form a dual pair under the *Reciprocity Law of Physics*. Each measurement counts distinctions; each variation relates them. Their bijection guarantees that information cannot decrease under any admissible extension of order.

Requiring global coherence under Martin's Axiom enforces the fourth–order cancellation $U^{(4)} = 0$, identifying the cubic spline as the minimal analytic closure of the dual system. This closure produces the continuous calculus of variations as the smooth limit of finite causal measurement, and its algebraic dual defines the discrete logic of event selection. From this structure, the invariants of physics emerge successively: the wave equation as the propagation of reciprocal consistency, the metric as its gauge, and curvature as the residue of its global non-closure. Operationally, entropy measures the

logarithm of the number of admissible distinguishable configurations consistent with the causal order. When the causal field is interpreted as this count of distinctions, its curvature encodes the rate at which distinguishability grows. Coupling the causal field to entropy yields a constant-curvature stress tensor that defines the gravitational scale.

Thus, $\Delta S \geq 0$ is not a thermodynamic postulate but a theorem of causal measurement: the necessary condition that any universe consistent with its own record of distinctions must increase the count of what can be known.

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Roadmap

This work begins from the simplest possible assumption: every observation creates information. When something becomes distinguishable, it has been measured. Each measurement leaves a record, and the universe grows by accumulating these records one event at a time.

Nothing continuous is assumed. No geometry, no fields, no action principle, and no differential equations. The goal is to show that the familiar structures of physics arise automatically when we ask for one thing: a universe whose measurements never contradict each other.

From Events to Smooth Motion

Imagine that only a finite number of measurements are made along a particle's path. Between those points, many curves are mathematically possible, but most of them would imply extra structure: hidden bumps, oscillations, or accelerations that would have produced additional measurements. Since no such measurements exist, those curves must be rejected.

The only admissible path is the one with no unobserved structure. In practice this means the path of least bending: the same rule that defines cubic splines. As measurements become more precise, the spline becomes smooth, and its smooth limit satisfies a fourth–order equation

$$U^{(4)} = 0.$$

This is the Euler–Lagrange equation of a free beam. Here it appears not because an action was postulated, but because any other path would predict unrecorded motion. Smooth calculus is the shadow of finite measurements.

Matter as Information Flow

If two observers measure the same system and later compare data, their descriptions must agree in the overlapping region. This is only possible if information flows continuously from one measurement to the next. In practice, the density of recorded information obeys a continuity law

$$\frac{\partial}{\partial t}\rho + \nabla \cdot j = 0.$$

This is the kinematics of matter: information flows in a way that does not lose or invent measurements.

Light as the Fastest Information

There is a maximum rate at which new information can appear. In relativity this is the speed of light. Here it is simply the rule that no observer can receive new data from outside a causal boundary. When this idea is carried into the smooth limit, the result is the wave equation

$$\frac{\partial^2 U}{\partial t^2} = c^2 \frac{\partial^2 U}{\partial x^2}.$$

Light is what information looks like when it travels at the fastest possible speed. Waves are the way measurements spread when there is no hidden structure in between.

Quantum Information

When two alternatives cannot be distinguished by any measurement, they cannot be assigned separate probabilities. Instead, they combine as amplitudes. Only when a new measurement is made do they separate into exclusive outcomes. This is the reason amplitudes add and probabilities come from their square.

In this framework, quantum mechanics is the bookkeeping of situations where multiple explanations remain consistent with all available data. Interference is the comparison of indistinguishable histories.

Gauge Theory

Different observers may use different conventions to describe the same measurements. If their predictions always agree, then their descriptions must be related by a transformation that leaves the measurable results unchanged. These transformations form a gauge symmetry, and the rules for comparing neighboring descriptions produce a connection. Its curvature is what physicists call a field.

Thus gauge theory is not an extra piece of physics. It is the condition that descriptions remain compatible when information is transported from place to place.

Why Entropy Increases

Every measurement increases the number of possible histories that remain consistent with the data. None of those histories can be removed, because that would erase information already recorded. As a result, the number of distinguishable configurations never goes down. Define entropy as

$$S = \log(\text{number of consistent configurations}).$$

Then S can only increase. This is the Second Law of Causal Order: information grows because measurements cannot be undone.

Summary

Structure	Why it appears	Result
Smooth paths	No hidden motion	$U^{(4)} = 0$
Matter flow	Shared measurements	Continuity law
Light	Fastest information	Wave equation
Quantum	Indistinguishable causes	Amplitudes, interference
Gauge fields	Consistent comparison	Connections, curvature
Entropy	Measurements are permanent	$\Delta S \geq 0$

In this view, physics is not assumed. It is what information must look like when measurements never contradict each other.

Nota Bene

The argument is constructive rather than interpretive. Each part extends the previous one by a single act of closure that preserves causal consistency:

$$\text{Measurement} \Rightarrow \text{Calculus} \Rightarrow \text{Wave} \Rightarrow \text{Geometry} \Rightarrow \text{Field.}$$

At every stage, a new invariant appears whenever distinction is preserved under refinement. The sequence therefore builds the minimal structure required for a universe that records its own evolution without contradiction.

Thus, the proof is read not as a series of analogies but as a chain of logical consequences. Starting from finiteness, order, and choice, one obtains measurement, variation, and their reciprocity; from reciprocity, one obtains calculus; from calculus, the smooth invariants of physics; and from their global consistency, the Second Law of Causal Order. In this sense, $\Delta S \geq 0$ is the unique fixed point of mathematics and physics—the inequality that any self-consistent universe must obey.

N.B.—The proof contains multiple conceptual examples to help explain the mathematical machinery. These *thought experiments* are not empirical illustrations but formal constructions intended to clarify the logical structure of the axioms. They are finite conceptual models that demonstrate how the mathematical relations behave under specific constraints of order and measurement. No claim is made regarding physical observation; each serves only to illuminate the internal mechanics of the theory.

N.B.—Throughout what follows, it is essential to distinguish the logical structure of measurement from any claim about physical phenomena. The arguments presented here concern the internal consistency of *records of distinction*—that is, the admissible transformations among measurable events—rather than the evolution of material systems themselves. Every symbol, tensor, and variation in the proof refers to relations between observations, not to unobserved substances or causes. The framework thus formalizes the mathematics of *measurement*: how distinctions can be made, counted, and related without contradiction. No ontological or dynamical claims are implied; the results hold regardless of what, if anything, the symbols may represent physically.

N.B.—This is a paper about *information*, not about energy, momentum, or any other physical quantity. At no point is it suggested that such values are produced, derived, or generated by the constructions presented here. All arguments concern the logical structure of measurement and the internal coherence of distinguishability, not the dynamics of physical systems.

N.B.—This is a *conditional* proof. All conclusions hold only under the stated axioms and definitions. No claim is made regarding the physical truth of those assumptions, only their internal consistency and the consequences that follow from them.

N.B.—Certain sections include extrapolations that relate the formal results to observable patterns (e.g., galactic rotation, Cepheid-based distance scaling, or cosmic expansion). These are illustrations of consistency, not physical hypotheses. Their purpose is to show that familiar empirical regularities may follow naturally from the informational constraints proved herein. No specific mechanism is proposed, and no claim of empirical verification is implied. They are included solely to outline how the formal results may frame, rather than predict, observable regularities.

No differential equations were harmed in the production of this proof.

This work treats measurement as a discrete, logical process. Continuum formulations appear only as smooth limits of countable constructions, never as physical postulates.

Chapter 1

The Mechanisms of Information

Every theory of dynamics begins with a calculus, an instrument for measuring variation. Yet a calculus alone cannot describe the universe, for measurement presupposes the existence of an ordered substrate upon which distinctions can be drawn (*e.g.* one recorded event follows another). The present work begins from this observation and constructs, alongside the familiar differential calculus, its algebraic dual: a logic of finite relations that determines how measurements themselves come to exist. Where calculus quantifies change, the dual quantifies order[1]. Each derivative has its adjoint in the discrete act of selection, and each integral its counterpart in the accumulation of distinguishable events. Taken together, these two systems—the continuous and its dual—generate the fundamental tensor structure from which the laws of physics emerge.

The central claim of this monograph is that the universe can be described as a pair of mutually defining operations: *measurement* and *distinction.* The first gives rise to the calculus of variation; the second to the ordering of events. We introduce the *Causal Universe Tensor* as the mathematical structure

[1]Unlike conventional formulations of dynamics, no notion of functional *dependency* is invoked. All relations are expressed purely through order and distinguishability: one event follows another, but nothing is said to depend on anything else. The calculus describes consistency among records of distinction, not causal generation.

that encodes measuring events. The Causal Universe Tensor unites events by showing that every measurement in the continuous domain corresponds to a finite operation in the discrete domain, and that these two descriptions agree point-wise to all orders in the limit of refinement. The familiar objects of physics—wave equations, curvature, energy, and stress—then emerge not as independent postulates but as necessary conditions for maintaining consistency between the two sides of this dual system.

From this perspective, the classical boundary between mathematics and physics dissolves. Calculus no longer describes how the universe evolves in time; it expresses how consistent order is maintained across finite domains of observation. Its dual, the logic of event selection, guarantees that these domains can be joined without contradiction. Together they form a closed pair: an algebra of relations and a calculus of measures, each incomplete without the other. The subsequent chapters formalize this duality axiomatically, derive its tensor representation, and show that the entire machinery of dynamics—motion, field, and geometry—arises as the successive enforcement of consistency between the two.

1.1 Countable Event Selection

A core assumption of this framework is that measurement does not produce a continuum of outcomes, but a finite or countable set of distinguishable events. We refer to this as *event selection*. Each measurement records one element from a set of mutually exclusive possibilities, and repeated measurements generate a sequence

$$E_1 \prec E_2 \prec \cdots ,$$

where $\prec$ denotes refinement: E_{n+1} contains strictly more distinguishable information than E_n. No model of the selection mechanism is introduced. It is not treated as a physical process, a dynamical law, or a computational rule. Its existence is asserted only in the mathematical sense that observational records consist of discrete, distinguishable events. Beyond this existential assumption, nothing is specified.

The set-theoretic background for this work is standard Zermelo–Frankel set theory with the Axiom of Choice [47], together with Martin's Axiom [62]. For our purposes, the role of Martin's Axiom is simple: it ensures that whenever observations can be refined, they can be refined step by step, without getting stuck. In other words, if more information is available about an event, there is always a path of increasingly precise descriptions that captures it.

Consequently, every observational process in this monograph can be regarded as a countable sequence of event selections. All of the limits we take—whether in measurement, curvature, or energy—are completions of these countable, steadily improving approximations. No uncountable or exotic constructions are ever needed; everything rests on ordinary, stepwise refinement.

This countability is not a restriction but a guarantee of constructibility. Refinement may proceed to arbitrarily fine resolution, but always by way of countable sequences. The continuum appears only as a completion of these sequences, never as a primitive axiom. In particular, no uncountable set of physical events is assumed, and no claim is made about the existence of a continuum substrate. The mathematical structure rests entirely on discrete observational data and the assurance that refinements may always be taken in the countable domain.

Event selection, together with Martin's Axiom, therefore provides a minimal logical foundation for measurement. It allows arbitrarily fine resolution without invoking continuous fields or manifolds, and it ensures that all refinements remain within the countable framework accessible to observation. Every subsequent construction—splines, weak forms, Euler–Lagrange extremals, wave equations, Einstein tensors, *et al.*—is built atop this countable structure.

It is worth noting that the full strength of Martin's Axiom is not required for the results developed in this manuscript. The constructions that follow rely only on the existence of countable chains of refinements and the completion of those chains. Stronger forms of Martin's Axiom become relevant only if one wishes to treat the continuum as a primitive object or to model the universe directly with differential equations. In that setting, uncountable structures, continuum measures, and smooth manifolds must be assumed from the outset, and Martin's Axiom provides a technical guarantee of consistency. In the present framework, the continuum is never an axiom but a derived limit of countable data, and therefore the full power of Martin's Axiom is unnecessary. Only the countable structure guaranteed by its weaker consequences is used.

1.2 Global Coherence

The logic of event selection is not arbitrary. Its structure is constrained by a principle of *global coherence*: any finite set of locally consistent observations must be extendable to a single, contradiction-free global history. This requirement is purely logical. It asserts that no finite collection of measurements can encode mutually incompatible information. As we show in Chapter 3, this condition is a finite, domain-specific analogue of the role played by Martin's Axiom: if local measurements agree on their overlaps, then a global refinement exists that contains them all.

In contrast with physical postulates, global coherence is a consistency assumption. It does not specify how events are generated, nor does it assume a geometry, a metric, or a dynamical law. It asserts only that an observational record cannot contain logical contradictions. From this minimal requirement, one can construct a countable chain of refinements in which every local event selection is represented. The continuum then emerges as the completion of this coherent chain, in the sense that measurements are dense in the real numbers.

The force of the proof comes from this logical constraint. When global coherence is combined with event selection and informational minimality, the resulting completion has a unique extremal form. In particular, the smooth limit of any coherent sequence of measurements obeys a variational Euler–Lagrange structure. Thus, the calculus of smooth motion is not imposed; it is the only continuous framework consistent with globally coherent, countable observation.

Thought Experiment 1.2.1 (The Invisible Curve [83])**. *N.B.*** *—Thought experiments such as this often depict common physical phenomena and how the* information being measured *must restrict admissable solutions.*

A spacecraft travels between two distant stars. Its onboard recorder has finite sensitivity: any change in motion or emission below a fixed detection threshold is not recorded. Over the course of the journey the recorder stores only three events—departure, a midpoint observation, and arrival. No other events exceed the threshold of detection. The question is: what can be inferred about the motion between these measurements?

One might imagine many possibilities. The ship could accelerate, decelerate, oscillate, or follow an arbitrarily complicated path. However, any such behavior would create additional detectable events: changes in velocity, turning points, or radiative signatures. If those events had occurred, the recorder would have stored them. Because it did not, all such structure is ruled out. The only admissible history is one that introduces no unobserved features.

With three recorded events, informational minimality forces the unique quadratic extremal that agrees with those samples—the same quadratic interpolant that underlies the classical Simpson's rule in numerical quadrature [21]. With four events the extremal becomes cubic, and with many events it approaches a spline. Smooth motion is not assumed; it is forced by the absence of evidence for anything else. The continuum appears only as the limit of refinement: as the recorder gains resolution, the invisible curve *becomes visible, but never exceeds what the events certify. In particular, the sequence of refinements forms a Cauchy sequence in the space of admissible motions[13, 51], and its completion is the unique smooth extremal consistent with the measured events.*

Thought Experiment 1.2.2 (Global Coherence as a Merge of Light Cones [48])**.** ***N.B.***—*The continuous world offers a* causal approach *to the ordering of measurements. The events as recorded in a laboratory notebook only serve as a time series [11].*

Consider two observers A and B, each of whom records a finite sequence of distinguishable events in increasing causal order:

$$A = \langle a_1 \prec a_2 \prec \cdots \prec a_m \rangle, \qquad B = \langle b_1 \prec b_2 \prec \cdots \prec b_n \rangle.$$

Each list is totally ordered by local causality (e.g. *a differential equation of dynamics). The requirement of global coherence asks whether there exists a single event sequence*

$$G = \langle e_1 \prec e_2 \prec \cdots \rangle$$

containing all a_i and b_j such that the local orders are preserved: if $a_i \prec a_{i+1}$ in A, then $a_i \prec a_{i+1}$ in G, and similarly for B.

This is exactly the merge step of a stable sorting algorithm. As shown by [48], if two lists are individually sorted, then the merge (if it exists) is uniquely determined up to elements that are incomparable. If at any step the merge requires placing $b_k \prec a_i$ even though $a_i \prec b_k$ was recorded locally, then no global sequence G exists: the local records encode a logical contradiction. In concrete terms, observer A may infer that b_k is caused *by a_i, while observer B insists the opposite: b_k causes a_i.*

If the merge is admissible, the resulting global history is unique up to permutations of spacelike-separated elements. Those incomparable elements correspond exactly to the uncorrelant *equivalence classes introduced later: permuting them changes no scalar invariant of the Universe Tensor. As the* *resolution of measurement increases, the merged list becomes longer, and in the dense limit it converges to the unique spline with no unrecorded curvature. Thus the continuum is not assumed; it is the only extension consistent with all local causal records.*

1.3 Relation to Causal Set Theory

The philosophical foundation of this work stands in clear lineage with Causal Set Theory, initiated by the seminal ideas of Bombelli, Lee, Meyer, and Sorkin [10] and refined in later developments by Rideout and Sorkin [76, 90]. In that program, the continuum is not a primitive structure but an emergent limit: a manifold arises only when a discrete, partially ordered set of events is sampled at sufficiently high density. Geometry is not assumed—it is recovered from order and counting.

The present work adopts the same foundational stance while shifting the emphasis from causal order to measurement. Events are again primary, but instead of encoding Lorentzian geometry, we encode informational content. An event is a unit of observation, and the absence of additional events is a data constraint. In this framework, a continuum description appears only as the smooth limit of a discrete construction, never as a physical postulate.

This extends the causal set philosophy from geometry to kinematics. In Causal Set Theory, Lorentzian distance emerges from order and volume counting. Here, kinematic laws emerge from extremality: the unique interpolant that introduces no unobserved curvature minimizes a bending energy functional, and its smooth limit satisfies the variational Euler–Lagrange equation. If additional structure were present, the measurement process would have recorded additional events. Thus, dynamics are inferred rather than assumed.

This theory therefore complements the causal set program. Both reject the continuum as a primary object and treat it instead as an emergent shadow of discrete data. The present framework extends that philosophy to measurement and motion, showing that smooth kinematic laws arise from informational minimality rather than differential postulate.

1.4 Weak Forms and Integration by Parts

A central technical tool in this manuscript is the passage from a discrete extremality principle to a continuous weak formulation by repeated integration by parts. Historically, this pattern is older than the modern terminology suggests. In the nineteenth century, classical variational methods employed integrations by parts to transfer derivatives from trial functions onto test functions, ultimately yielding natural boundary terms. In the twentieth century, this idea was formalized in the context of Hilbert spaces and distributions, where weak derivatives and test functions replaced classical smoothness assumptions.

The modern finite element method rests directly on this foundation. Galerkin's original approach [33] enforced a variational balance by requiring that the residual be orthogonal to a chosen space of test functions, producing a weak form even when classical derivatives may not exist. This framework was later placed on rigorous functional-analytic ground by Courant [17] and further developed in the context of Sobolev spaces and elliptic regularity by Lions and Magenes [59]. Ciarlet's treatment of finite element analysis [15] made explicit that the Galerkin method is simply the discrete realization of a weak variational statement arising from integration by parts.

In this work the same pattern appears, but in reverse motivation. We do not begin with differential equations and weaken them for analytic convenience. Instead, we begin with discrete events, define a discrete bending energy, and obtain a weak form because integration by parts is the continuous expression of that discrete extremality. The variational Euler–Lagrange equation is therefore not a postulate but the shadow of the weak form that emerges when the sampling of events becomes dense.

Thus the historical machinery of integration by parts, weak solutions, and Galerkin methods does not merely provide mathematical comfort; it reveals that classical differential equations are consequences of informational consistency, not assumptions.

1.5 Paradoxes, Aliasing, and Cancellations

The transition from discrete structures to smooth limits must be handled with care. Classical measure theory contains well-known examples where naive passage to the continuum leads to non-physical conclusions. The Banach–Tarski paradox, proved using the Axiom of Choice [3, 92], shows that a solid ball in three dimensions can be decomposed into finitely many disjoint sets and reassembled into two identical copies of the original. Although mathematically rigorous, such constructions violate any physical notion of volume preservation. They arise precisely because arbitrary decompositions of sets ignore the informational structure that would be present in any measurable process. In effect, they treat uncountable collections of measure-zero points as if they carried the same "size" as countable sets built from measurable pieces.

In numerical analysis, a more familiar version of this pathology appears as aliasing and cancellation. A function sampled too coarsely can hide large oscillations between measurement points [36]; Gibbs-like ringing can vanish or flip sign [34, 85]; and two nonzero signals can cancel exactly when sampled at insufficient resolution [82]. The data appear benign, but the underlying object may be violently oscillatory. In both cases, the fault lies not in the continuum, but in the failure to encode which decompositions or oscillations are physically meaningful [67].

The present framework avoids such paradoxes by construction. Measurement is modeled as a finite selection of events, and the absence of additional events is a data constraint. An interpolant exhibiting oscillations, cancellations, or paradoxical decompositions would necessarily imply unobserved structure. Such a function is inconsistent with the event selection, and is ruled out by the extremality principle: the unique interpolant consistent with the data minimizes curvature and introduces no additional features. Thus the smooth limit cannot generate paradoxical volume behavior, and aliasing is impossible, because additional curvature would have been recorded as additional events.

In this sense, paradoxes of decomposition and aliasing are not ignored but excluded. The informational content of the data places strict limits on what can exist between observed events. The continuum that emerges in the dense limit is the one with no unobserved structure, and therefore no paradoxical cancellations, aliasing, or interference. Any such phenomenon, if measured, must coincide with a physicality.

1.6 A Gauge Theory of Information

A final component of this framework is the development of a gauge theory of information. In conventional relativistic field theory, transformation laws are imposed at the level of the continuum: Lorentz invariance is a fundamental constraint on fields, and spinorial structure is required to represent half-integer representations of the rotation group. In the present work, these structures are not postulated. Instead, the relevant symmetries emerge as constraints on information: an event selection must produce observationally indistinguishable results under changes of inertial frame.

This leads naturally to an informational gauge group. Two descriptions of the same measurement record are considered equivalent if their predicted event sets differ only by transformations that preserve observational outcomes. In the smooth limit, these gauge transformations approximate Lorentz transformations arbitrarily well, but the underlying data remain finite and discrete. Because Lorentz symmetry appears only as a limit rather than a postulate, a spinor bundle is not required at the discrete level. Vectors suffice, and spinorial structure emerges only as a continuum approximation.

This perspective echoes a long tradition in discrete approaches to spacetime. Bombelli, Lee, Meyer, and Sorkin first demonstrated that a Lorentzian manifold can emerge as an approximation to a fundamentally discrete causal set, and that exact Lorentz symmetry need not hold at the microscopic level [10]. Subsequent work by Sorkin and collaborators emphasized that continuum symmetries appear only as large-scale statistical regularities of a random partial order, not as primitive geometric postulates [23, 90]. Henson further showed that in such models, local Lorentz invariance is recovered in the limit of increasing density of causal relations, even though no metric or differential structure is assumed a priori [42]. Taken together, these results suggest that Lorentz symmetry is emergent from combinatorial structure rather than fundamental in itself.

From this viewpoint, information—not geometry—carries the primitive structure. Continuum fields, spinor representations, and relativistic kinematics are shadows of a deeper combinatorial statement: two observers are equivalent if their event selections cannot be distinguished by measurement. This yields a gauge of information whose continuum limit recovers the familiar invariances of classical relativistic physics, but without assuming a metric, a manifold, or a spin structure at the start.

Coda: The Twin Paradox

N.B.—Time dilation is not caused by recording more events. It is revealed when those events are merged into a globally coherent history. The twin that accumulates more refinements forces a larger merge and therefore corresponds to less proper time; the inertial twin, with fewer refinements, corresponds to more. For further intuition, see Remarks 2 and 5.

As a simple illustration, consider the twin paradox [56]. In the classical treatment, the age difference arises from integrating proper time along two worldlines in a Lorentzian manifold. Here, no metric or continuum is assumed. Each twin accumulates a finite record of events—ticks of a clock, photons received, threshold crossings of a detector. The information contained in these records is all that distinguishes one history from another.

During the outbound and inbound legs of the journey, the traveling twin undergoes changes that the stay-at-home twin does not: engine burns, thruster firings, telemetry exchanges, and adjustments of orientation. Each of these produces a measurable refinement of state, adding events to the traveling twin's record. The twin on Earth, by contrast, has a record that is comparatively coarse. Crucially, this asymmetry cannot be removed by any choice of description. One twin simply measures more.

In the information gauge, proper time is not a geometric interval but the count of admissible distinctions—the number of measurable, irreversible updates to a system's state. A history with more recorded distinctions corresponds to more events that must be reconciled. Refinement of measurement allows for the discovery of new events. Similarly, the unaccelerated twin gathers no new information from refinement. The traveling twin's notebook is therefore longer: it contains additional causal markers between departure and return that have no counterparts in the stay-at-home twin's record. Refinement introduces new distinctions but contributes no duration by itself; proper time is the work of reconciling those distinctions into a coherent history.

The asymmetry becomes operationally visible only when the twins reunite. To reconcile their histories, the stay-at-home twin must merge a richer time series. In the sorting process of global coherence, she must accommodate the extra distinctions recorded by her sibling. The traveler, having logged more events, performs a strictly smaller merge. The extra "time" is nothing more than the additional informational work required to coherently order the denser record.

Time dilation, in this view, is not a geometric mystery but an informational fact. One worldline contains strictly more refinements and therefore requires more work to merge into a single coherent history. The stay-at-home twin must resolve the additional distinctions recorded by her sibling, while the traveler performs a strictly smaller merge. The traveler therefore experiences less relative time: there is less information to reconcile. Any continuum reconstruction must agree with this count; no metric can reverse it without contradicting the observed data.

In the continuum limit, where these discrete refinements become dense, the argument reproduces the standard Lorentzian result. The traveling twin's path contains regions of higher curvature in the space of measurements, which manifest as shorter proper time in the geometric formulation. But this structure is inferred, not assumed. No manifold, metric, or spin structure is postulated. Time dilation is the unique smooth continuation of the discrete fact that more events occurred along one history than the other.

Seen this way, the twin paradox is not a paradox at all. Two observational records are compared, and the one with the richer informational content corresponds to the older twin. Geometry merely codifies this informational asymmetry in the language of smooth manifolds and differential forms. The physics was already determined by measurement.

Chapter 2

The Combinatorics of Measurement

2.1 Introduction

Every physical description begins not with space or time, but with an *event*—an interaction that makes previously indistinguishable outcomes distinct [8, 73]. The causal boundary of such an interaction is its *light cone*: the set of all events that can influence or be influenced by it according to special relativity [25, 63]. The intersection of two light cones, corresponding to the last particle–wave interaction accessible to an observer, defines the maximal region of causal closure [40, 70]. Beyond this surface, no additional information can be exchanged; all distinguishable action has concluded.

It is from this closure that the ordering of events arises [40, 61]. Each measurable interaction contributes one additional distinction to the universe, expanding its causal surface by a finite count [40, 61]. The smooth fabric of spacetime is not primitive but emergent: it is the limiting behavior of discrete causal increments accumulated along the light cone [10, 89]. Within each cone, the universe can be represented by a finite tensor of interactions—local updates to a global state—that together approximate continuity only through cancellation across countable events [10, 90].

Special relativity provides the canonical local model for this causal structure [25]. Consider the Lorentz transformation for a boost of velocity v in one spatial dimension, [25, 78, 94]

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v/c^2 \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}, \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{2.1}$$

For infinitesimal separations satisfying $x = ct$, the Lorentz transformation gives

$$t' = \gamma\, t(1 - v/c). \tag{2.2}$$

If we take $\Delta t = 1$ as the unit interval between distinguishable events, then observers moving at relative velocity v will, in general, disagree on the *number* of such events that occur between two intersections of their respective light cones [63]. The only invariant quantity is the causal ordering itself: all observers concur on which event precedes which, even though they may count a different number of intermediate ticks [61].

Definition 1 (Rank time [10, 20])**.** *Let* $(E, \preceq)$ *be a locally finite* partially ordered set of events*. A rank time is an order-embedding*

$$\tau : E \to \mathrm{Ord}$$

satisfying $e \prec f \implies \tau(e) < \tau(f)$*. Local finiteness implies that for any observer's causal domain* $D \subseteq E$*,* $\tau(D)$ *is order-isomorphic to an initial segment of* $\mathbb{N}$*. We therefore define the* duration*,* $|\delta t|$*, between anchors* $a \prec b$ *by*

$$|\delta t|(a, b) \;=\; \#\{\, e \in E \mid a \prec e \prec b \,\} \in \mathbb{N}.$$

Two rank functions τ, τ' *are* equivalent *if there exists an order-isomorphism* ϕ *with* $\tau' = \phi \circ \tau$*; equivalent ranks yield identical durations.*

Remark 1 (Operational content of time)**.** *Time is an ordinal rank on E, not an independent scalar field. All subsequent uses of "t" refer to an order-equivalence class of rank functions as in Definition 1. The additivity $|\delta t|(a,b) = |\delta t|(a,c) + |\delta t|(c,b)$ follows from local finiteness.*

This observation motivates the first physical axiom: that time is not an independent scalar field but an ordinal index over causally distinguishable events. Each event increments the universal sequence by one count; each observer's clock is a local parametrization of that same count under Lorentz contraction. The apparent continuity of time is the result of the density of such events within the causal cone, not an underlying continuum of duration.

On the Structure of Measurement

This work does not propose new physical phenomena or reinterpret existing experimental data. Rather, it reformulates how measurable quantities are represented and reduces the number of degrees of freedom needed to describe the universe to a single parameter that can be curve–fit.

Thought Experiment 2.1.1 (Planck's Constant as a Dimensional Anchor [72])**.** *Imagine a hypothetical measuring apparatus that records distinctions not by counting particles or intervals, but by tallying* acts of discernment*—each act adding one quantum of distinguishability to the record. Suppose further that the calibration of such a device required only a single fixed scale to relate discrete counts to continuous units of measure. In physics, Planck's constant h serves precisely this purpose: it is not a force or an energy, but a bookkeeping factor that ensures continuity between discrete and continuous domains.*

In the present framework, the analogous constant plays no physical role—it merely fixes the dimensional scale *by which finite distinctions are rendered comparable. The constant's existence affirms that measurement can be both discrete and metrically consistent without invoking any specific quantum pos-*

tulate. As with h, the constant here is not discovered but defined*: a normalization that preserves coherence between counting and continuity.*

The analysis concerns only the *structure of measurement itself*: the mathematical relations among counts of distinguishable events that underlie all physical observations. In this framing, physics is viewed as a grammar of distinctions. The familiar constants and fields—mass, charge, curvature, temperature—arise as *derived measures* within a finite causal order, not as independent entities.

Thought Experiment 2.1.2 (Measurement as a BNF Grammar [1, 65])**.** *Because measurement produces distinguishable outcomes, each observation selects a symbol from a finite or countable alphabet*

$$\Sigma = \{\sigma_1, \sigma_2, \ldots\}.$$

A record of n measurements is therefore a word $w \in \Sigma^n$. When an instrument is refined—by increasing precision or reducing noise—any coarse symbol σ_k may be replaced by a finite set of more precise symbols,

$$\sigma_k \;\Rightarrow\; \sigma_{k,1} \;\big|\; \sigma_{k,2} \;\big|\; \cdots \;\big|\; \sigma_{k,r},$$

just as in a Backus–Naur Form (BNF) production rule [1, 65]. Not all replacements are admissible: they must remain compatible with every other measurement that overlaps in time or causal order. Two refined histories that disagree on an overlapping interval cannot both represent valid records.

Thus admissible measurement histories form a formal language generated by the allowed refinement rules. The "law" governing measurement is the constraint that only globally consistent extensions of a record may be generated. This is not an analogy: it is the standard formal structure of symbol sequences in coding and information theory [84].

No new particles, forces, or cosmological effects are introduced; only the rules by which such effects are numerically expressed are examined. Hence the present theory is not a revision of physics but a clarification of its syntax: it studies the measures of phenomena, not the phenomena themselves.

Definition 2 (Distinguishability Chain [50]). *Let Ω be a nonempty set. A distinguishability chain on Ω is a sequence $\mathcal{P} = \{P_n\}_{n\in\mathbb{Z}}$ of partitions $P_n \in \mathsf{Part}(\Omega)$ such that P_{n+1}* refines *P_n for all n (every block of P_{n+1} is contained in a block of P_n). Write* $\mathrm{Bl}(P)$ *for the set of blocks of a partition P Each refinement step produces zero or more events.*

Definition 3 (Event [50, 91]). *Fix a distinguishability chain $\mathcal{P} = \{P_n\}$. An event at index n is a minimal refinement step: a pair*

$$e = (B, \{B_i\}_{i\in I}, n) \tag{2.3}$$

such that:

1. *$B \in \mathrm{Bl}(P_n)$;*
2. *$\{B_i\}_{i\in I} \subseteq \mathrm{Bl}(P_{n+1})$ is the family of all blocks of P_{n+1} contained in B, with $|I| \geq 2$ (a nontrivial split);*
3. *(*minimality*) there is no proper subblock $C \subsetneq B$ with $C \in \mathrm{Bl}(P_n)$ for which the family* $\mathrm{Bl}(P_{n+1}) \cap \mathcal{P}(C)$ *is nontrivial.*

Let E denote the set of all such events. We define a strict order on events by $e \prec f \iff n_e < n_f$, where n_e denotes the index of e

Definition 4 (Proper Time [64]). *Let E be the set of events generated by a distinguishability chain $P = \{P_n\}$. For any two events $a, b \in E$ with $a \prec b$, the* proper time *between them is*

$$\tau(a,b) = \max\Big\{\, |C| : \ C = \{c_0, \ldots, c_k\} \subseteq E,\ a = c_0 \prec c_1 \prec \cdots \prec c_k = b \Big\}.$$

That is, $\tau(a, b)$ is the cardinality of a maximal chain of strictly refinable events between a and b. Local finiteness of the distinguishability chain guarantees $\tau(a, b) \in \mathbb{N}$.

Remark 2. *Proper time is not a geometric length. It is the number of admissible, irreversible refinements separating two recorded events. Additional refinement (higher resolution) may increase $\tau(a, b)$; coarse–graining cannot. Thus, proper time is an invariant of the partially ordered event record, not a metric assumption.*

Remark 3. *A chain need not include all events, and incomparable events do not contribute to one another's proper time. Only if every pair of events were comparable would τ reduce to a total order. In general, E is only partially ordered.*

Remark 4 (Smooth limit [77])**.** *If refinements become dense and the discrete extremal converges to a C^2 spline, then*

$$\lim_{\textit{refinement}\to\infty} \tau(a, b) = \int_a^b \sqrt{-\,ds^2},$$

the Lorentzian proper time. The integral form is not assumed; it is the smooth shadow of the combinatorial count.

The notion of *uncorrelant events* formalizes the idea that two recorded distinctions may be independent of one another. In causal set theory, incomparability under the causal order corresponds to physical independence of events [10]. The same conceptual separation appears in quantum theory, where observables acting on independent subsystems commute and their measurement outcomes do not influence each other [22, 71]. Classical discussions of separated systems, from Einstein–Podolsky–Rosen and Schrödinger to Wheeler's formulation of complementarity [26, 80, 101], frame the same idea operationally: when no physical procedure can distinguish the relative order of two events, their ordering has no empirical content. The definition below captures this in the minimal set-theoretic language of the causal poset.

Definition 5 (Uncorrelant [10, 89])**.** *Let $(E, \prec)$ be a locally finite partially ordered set of events. Two events $e, f \in E$ are said to be* uncorrelant *if they are incomparable under the causal order; that is,*

$$\neg(e \prec f) \quad \textit{and} \quad \neg(f \prec e).$$

The uncorrelant relation partitions E into equivalence classes of events whose relative order carries no operational consequence for any admissible measurement or refinement. In particular, no experimentally distinguishable difference follows from interchanging the positions of uncorrelant events in any linear extension of $(E, \prec)$.

Remark 5. *If e and f are uncorrelant, permuting them in any chain, merge, or refinement does not change any observable invariant of the Causal Universe Tensor. No observer can construct a sequence of measurements that forces an ordering between e and f without introducing new events.*

Remark 6. *Correlatant events admit a strict causal relation and therefore contribute to proper time; uncorrelants do not. In particular, a chain that includes e but not f may be maximally refined without reference to f. Thus, uncorrelants represent informational independence, not simultaneity.*

Remark 7. *In spacetime language, uncorrelants are precisely those event pairs that are spacelike–separated: reordering them changes no measurable scalar. Here this is not assumed from geometry; it is a consequence of incomparability in the event order.*

Definition 6 (Causal Order [10])**.** *Let $P = \{P_n\}_{n\in\mathbb{Z}}$ be a distinguishability chain of partitions, and let an event be $e = (B, \{B_i\}_{i\in I}, n)$ as in Definition 3, where $B \in \mathrm{Bl}(P_n)$ splits nontrivially into child blocks $\{B_i\} \subset \mathrm{Bl}(P_{n+1})$.*

For $m > n$ and $C \in \mathrm{Bl}(P_m)$, let $\pi_{m\to n}(C) \in \mathrm{Bl}(P_n)$ denote the unique ancestor block in P_n containing C (well-defined because P_{n+1} refines P_n). Define the immediate causal cover *relation $e \triangleright f$ between events $e = (B, \{B_i\}, n)$ and $f = (C, \{C_j\}, m)$ by*

$$n < m \quad \text{and} \quad \pi_{m\to n+1}(C) \subseteq B_i \text{ for some child } B_i \text{ created by } e.$$

The causal order *$\prec$ on the event set E is the transitive closure of $\triangleright$:*

$$e \prec f \iff \text{there exist events } e = e_0, e_1, \ldots, e_k = f \text{ with } e_i \triangleright e_{i+1} \text{ for all } i.$$

Then $(E, \prec)$ is a locally finite partially ordered set (reflexivity suppressed for strictness), where incomparability is allowed: it may happen that neither $e \prec f$ nor $f \prec e$.

Remark 8 (Index is an order–embedding, not an equivalence)**.** *If $e \prec f$, then $n_e < n_f$. Thus the refinement index provides a rank function (Definition 1) that is* monotone *with respect to $\prec$. The converse need not hold: $n_e < n_f$ does not imply $e \prec f$. Hence the causal order is generally* partial*, not total.*

Remark 9 (Uncorrelants and permutation invariance)**.** *Events e, f with neither $e \prec f$ nor $f \prec e$ are* uncorrelant *(incomparable). Permuting uncorrelant events in any linear extension of $(E, \prec)$ leaves all scalar invariants of the Causal Universe Tensor unchanged; causal histories are unique only up to permutation of uncorrelants.*

Definition 7 (Partially Ordered Set [20])**.** *A partially ordered set is a pair $(E, \leq)$ where $\leq$ is a binary relation on E satisfying:*

1. ***Reflexivity:*** *$e \leq e$ for all $e \in E$*
2. ***Antisymmetry:*** *if $e \leq f$ and $f \leq e$, then $e = f$*
3. ***Transitivity:*** *if $e \leq f$ and $f \leq g$, then $e \leq g$*

Operationally, every observation can be decomposed into three layers:

1. the **logical** layer—which events are distinguishable;
2. the **mathematical** layer—how those distinctions are counted;
3. the **physical** layer—how the resulting counts are named and parameterized as energy, momentum, or time.

By isolating the first two layers, we obtain a calculus that is universal to any admissible physics: a closed system of relations that expresses how order itself becomes measurable.

The framework that follows formalizes this intuition. The axioms of Zermelo–Fraenkel set theory with the Axiom of Choice, we construct an ordered set of events whose distinguishability relations reproduce the causal order implied by special relativity. Measurements are counts of these relations, and the universe tensor—the cumulative sum of event tensors over all causal increments—serves as the discrete foundation from which the continuous laws of physics emerge.

Historical context. A measurement is defined operationally as the count of distinguishable events between two anchors—the minimal act of drawing a distinction in a finite causal order. This definition echoes Boltzmann's use of distinguishable microstates as the foundation of entropy [8], Planck's quantization of action [72], and Wheeler's dictum that "it from bit" [100], but is here formalized as an axiom of order rather than an empirical postulate.

2.2 The Axioms of Mathematics

All mathematics in this work is carried out within the framework of Zermelo–Fraenkel set theory with the Axiom of Choice (ZFC) [47, 53]. Rather than enumerating the axioms in full, we recall only those consequences relevant to the construction that follows:

- **Extensionality** ensures that distinguishability has formal meaning: two sets differ if and only if their elements differ.

- **Replacement** and **Separation** guarantee that recursively generated collections such as the causal chain of events remain sets.

- **Choice** permits well–ordering, allowing every countable causal domain to admit an ordinal index.

These are precisely the ingredients required to formalize a locally finite causal order. All further constructions—relations, tensors, and operators—are definable within standard ZFC mathematics; see Kunen [53] and Jech [47] for set-theoretic foundations, and Halmos [37, 38] for the induced tensor and operator structures on finite-dimensional vector spaces.

The starting point of this framework is methodological rather than ontological. We do not assume anything about the substance of physical reality. We assume only that the outcomes of measurement are finite or countable collections of distinguishable results recorded in time. This is standard across probability theory and information theory: Shannon formalized information as distinguishable symbols drawn from a finite or countable alphabet [81], and Kolmogorov showed that empirical outcomes can be represented as elements of measurable sets within standard set theory [50]. In this view, measurement produces data, and data are mathematical objects. Everything that follows concerns the admissible transformations among such records.

Axiom 1 (The Axiom of Kolmogorov: Measurement as a Formal Record [50])**.** *The record of measurement—defined as the finite or countable set of observed, distinguishable events—is taken to be a mathematical object representable within Zermelo–Fraenkel set theory with Choice (ZFC). No ontological claim is made about physical reality. The axiom asserts only that observable data can be formalized as sets and relations.*

This standpoint is consistent with Kolmogorov's construction of probability spaces, in which empirical outcomes are represented as measurable *sets [49]. Accordingly, a record of finite observations is a mathematical object whose structure is defined entirely within ZFC. Throughout this work, the word "information" refers exclusively to these representable distinctions; nothing is asserted about any underlying physical substrate that might produce them.*

Axiom 2 (The Axiom of Peano: Counting as the Tool of Information [32, 53, 104])**.** *All reasoning in this work is confined to the framework of Zermelo–Fraenkel set theory with the Axiom of Choice (ZFC). Every object—sets, relations, functions, and tensors—is constructible within that system, and every statement is interpretable as a theorem or definition of ZFC. No additional logical principles are assumed beyond those required for standard analysis and algebra.*

Formally,

$$\text{Measurement} \subseteq \text{Mathematics} \subseteq \text{ZFC}.$$

Thus, the language of mathematics is taken to be the entire ontology of the theory: the physical statements that follow are expressions of relationships among countable sets of distinguishable events, each derivable within ordinary mathematical logic.

Thought Experiment 2.2.1 (The Speedometer [97, 103])**.** ***N.B.***—*The mechanical implementation of measuring devices often are protected by explicit descriptions of how they work. The patents cited here explicitly describe how they turn counting into data.*

Consider an ordinary automobile speedometer. The dial appears to report a continuous real number at each instant, but the device does not have access to the real numbers. A mechanical speedometer counts wheel rotations through a gear train and maps those counts to pointer positions. A digital speedometer counts the same rotations and displays a numeral drawn from a finite alphabet.

Each time the counter increments and the displayed symbol or pointer *position changes, a new distinguishable event is recorded. Between two successive display states there is no way, from the informational record alone, to assert that any additional state occurred. The apparent continuity of "speed" is a visual interpolation of a finite counting process.*

Thus the speedometer does not output a real number. It outputs a countable sequence of distinguishable states derived from integral counts of wheel rotations. The act of measuring speed reduces to counting transitions of a finite-state device. All physical inference based on such data can be expressed within ordinary arithmetic and set theory.

This illustrates Axiom 2: measurement generates only countable, finitely coded distinctions, and every mathematical object used to interpret those distinctions—numbers, functions, tensors—is a construct of ZFC. No structure beyond counting is assumed at the fundamental informational level.

2.3 The Axioms of Information

The previous chapter established that a physical record is a set of distinguishable observations, representable within ZFC, and partially ordered by causal precedence. Nothing further was assumed about geometry, dynamics, or the continuum. In this section, we introduce two informational axioms that restrict how such a record may be interpreted. These axioms express constraints on admissible descriptions of the world, independent of any particular model of physics.

The first axiom formalizes the principle that a physical history may not contain unobserved structure. Among all symbolic descriptions that reproduce the recorded events, the admissible one is the shortest. This is the information–theoretic form of Occam's principle: no plurality of assumptions without necessity.

The second axiom asserts that the record of events is not merely ordered but forms a locally finite causal set. Local finiteness ensures that causal volume is discrete, while the partial order encodes temporal precedence. Continuum spacetime, when it exists, is therefore understood as an approximation that faithfully embeds this discrete informational structure.

Together, these axioms define the informational content of the physical world: a causal set with no unrecorded structure and no additional assumptions beyond the observational record itself.

2.3.1 Information Minimality

The observational record E is defined only by the distinguishable events it contains. Between two recorded events e_i and e_{i+1}, no additional structure is present in the data: no new marks in the notebook, no threshold crossings, and no observable distinctions. Set theory alone does not forbid a hypothetical refinement that inserts additional structure between e_i and e_{i+1}, but any such refinement asserts observations that did not occur. To prevent unrecorded structure from being introduced by assumption, we impose an informational constraint.

Among all symbolic descriptions that reproduce the recorded events, the admissible one is the shortest. In modern information theory, this statement is formalized by Kolmogorov complexity: a description is preferred if it introduces no additional information beyond the events in E. This embodies the classical principle that no plurality of assumptions should be posited without necessity. It is not derived from the set-theoretic framework; it is an axiom about how physical theories must interpret finite empirical records.

Axiom 3 (The Axiom of Ockham: Informational Minimality [68])**.** *Let* $E = \{e_0 \prec e_1 \prec \cdots \prec e_n\}$ *be the recorded events of an experiment, understood as a finite or countable set of distinguishable observations representable in ZFC. Among all symbolic descriptions that map to* E *and introduce no additional recorded events, the admissible completion is the one of minimal Kolmogorov complexity.*

Equivalently, if a hypothetical refinement of the history introduces a distinguishable update that is not present in E, then that refinement is inadmissible. Any shorter description consistent with E is preferred.

We have seen this principle in action already. Refer to Thought Experiment 1.2.1 and the use of Simpson's rule to compute the path of a spaceship with minimal measurement information.

2.3.2 Causal Set Theory

The previous axiom imposed an informational constraint on admissible descriptions of the record of measurement. We now introduce a structural constraint. The empirical record is a set of distinguishable events with a causal precedence relation $\prec$, but this alone does not restrict the size of causal intervals. In a general partially ordered set, the number of events between a and b may be infinite. Physical measurements, however, produce finite data. To represent this empirically grounded discreteness, we assume that the causal order is locally finite: every causal interval contains only finitely many recorded events.

This postulate places the present construction within the causal set program of Sorkin and collaborators, where spacetime is modeled as a locally finite partial order and continuum geometry, when it appears, is a derived approximation. Order encodes temporal precedence, and local finiteness encodes discrete causal volume. No metric, field, or manifold structure is assumed at the fundamental level; these arise only if the causal set admits a faithful embedding into a Lorentzian manifold.

Axiom 4 (The Axiom of Causal Set Theory)**.** *labelax:causal The fundamental structure underlying spacetime is a causal set: a locally finite partially ordered set $(E, \prec)$, where*

1. *$e \prec f$ means e causally precedes f,*

2. $(E, \prec)$ *is acyclic and transitive,*

3. *and for any two events* $a \prec b$*, the interval* $\{\, e \in E : a \prec e \prec b \,\}$ *is finite.*

Local finiteness ensures that causal volume is discrete, and the order relation encodes temporal precedence. A Lorentzian manifold, when it exists, is merely a continuum approximation in which the causal set can be faithfully embedded.

Thought Experiment 2.3.1 (The Laboratory Procedure [68, 101])**.** *The following example collects ideas from several well–established perspectives in measurement theory. Bohr and Wheeler emphasize that a physical experiment records only distinguishable outcomes; no other structure is operationally meaningful [7, 101]. In information theory, such records are represented as finite or countable strings of distinguishable symbols [81, 18]. In ergodic theory and causal set theory, successive measurements refine a partition of the observational domain into finer distinguishable elements [79, 69, 91]. Finally, computational mechanics and operator–theoretic dynamics treat the "evolution" of a system as the repeated update of its information state [19, 52, 6]. Taken together, these perspectives justify modeling a laboratory procedure as a refinement operator acting on a finite measurement record. The experiment does not solve differential equations; it applies* Ψ*.*

Consider a laboratory notebook in which each threshold crossing of a detector is recorded as a mark in ink. The notebook contains a finite sequence of distinguishable entries

$$e_0 \prec e_1 \prec \cdots \prec e_n,$$

each representing an irreversible update of the experimental record. The notebook is not a model of reality; it is *the empirical record. No claim is made about any mechanism behind it.*

Now suppose one attempts to describe what "really" happened between two successive entries e_i *and* e_{i+1}*. If additional curvature, oscillation, turning* *points, or discontinuities had occurred, then the detector would have crossed a threshold and a new entry would appear. Because no such entry is present, the observational record forbids any refinement that predicts one.*

Thus the notebook determines a finite set $E = \{e_0, \ldots, e_n\}$ *of recorded events. Every admissible history must be a completion that introduces no new distinguishable events beyond* E*. Any hypothetical refinement with additional structure is rejected as inadmissible, since it asserts observations that did not occur.*

2.4 The Axioms of Physics

A common criticism of mathematical physics is the extent to which mathematics can be tuned to fit observation [9, 73] and, conversely, manipulated to yield nonphysical results [5, 45]. The critique of Newton's fluxions could only be answered by successful prediction. Today, calculus feels like a natural extension of the real world—so much so that Hilbert, in posing his famous list of open problems, explicitly formalized the lack of a rigorous foundation for physics as his Sixth Problem [43, 99].

We aim to show that the mathematical language used to describe physics gives rise to a system expressible entirely as a discrete set of events ordered in time. Moreover, this ordered set possesses a mathematical structure that naturally yields the appearance of continuous physical laws and the conservation of quantities. To understand how this works, we first clarify what we mean by measurement.

2.4.1 Measurement and the Axiom of Cantor

Physical laws relate measurements. For example, Newton's second law [66]

$$F = \frac{dp}{dt} \tag{2.4}$$

states that force relates to the *change* in momentum over time. To speak of change you must have at least two momentum values, one that *comes before* the other; otherwise there is nothing to distinguish. In set-theoretic terms, by the Axiom of Extensionality (assumed in Axiom 2), different states must differ in their contents, so "change" presupposes the distinguishability of two states.

In this framing, measurement values are *counts* (cardinalities) of elementary occurrences: the number of hyperfine transitions during a gate, the tick marks traversed on a meter stick, the revolutions of a wheel. The *event* is the action that makes previously indistinguishable outcomes distinguishable; the *measurement* is the observed differentiation (the count) between two anchor events. This is not the absolute measure of the event, but just relative difference of the two. We count the events as time passes.

Since special relativity requires that time vary under the Lorentz transform [25, 60], there can be no global scalar representation of temporal duration. Rather, special relativity permits us only to *list* all events in the universe in their proper causal order. It is this ordered list that we elevate to the first physical principle:

Axiom 5 (The Axiom of Cantor: Events are Ordered Countably [12, 24])**.** *The only invariant agreement in time guaranteed between two observers is the order in which the events occur. The duration between two events is defined as the number of measurements that can be recorded between them:*

$$|\delta t| \;=\; \big|\textit{events distinguished between}\big|. \tag{2.5}$$

2.4.2 Observations are Combinatorial

The recursive description of physical reality is meaningful only within the finite causal domain of an observer. Each step in such a description corresponds to a distinct measurement or recorded event. Observation is therefore bounded not by the universe itself, but by the observer's own proper time and capacity to distinguish events within it.

Axiom 6 (The Axiom of Planck: Observations are Finite [72])**.** *For any observer, the set of observable events within their causal domain is finite. The chain of measurable distinctions terminates at the limit of the observer's proper time or causal reach.*

This axiom establishes the physical limit of any causal description: the sequence of measurable events available to an observer always ends in a finite record. Beyond this frontier—beyond the end of the observer's time—no additional distinctions can be drawn. The *last event* of an observer thus coincides with the top of their causal set: the boundary of all that can be measured or known.

2.4.3 Event Selection

The preceding axioms restrict the informational content of the record and the structure of causal precedence. We now introduce an axiom governing how events may be selected in a consistent physical history. A partial history is a finite sequence of recorded distinctions that respects the causal order. In a locally finite causal set, many partial histories may be extended, but not all extensions are admissible: each new event must preserve causal consistency and remain compatible with every previously recorded distinction.

The Axiom of Boltzmann asserts that whenever we impose countably many local causal requirements—each representing a physically admissible constraint—there exists a single consistent history that satisfies all of them. Mathematically, this parallels the role of Martin's Axiom in set theory, where dense sets encode constraints and a filter selects a coherent global object [62, 53, 47, 95]. Physically, it echoes Boltzmann's principle that every admissible microstate selection must preserve distinguishability [9], and follows the causal-set program in which a spacetime history is constructed one event at a time under causal consistency [10, 30]. Hilbert's call to axiomatize the foundations of physics [44] is realized here as a minimal requirement: if each local constraint is physically permissible, then the combined history must also be permissible.

Axiom 7 (The Axiom of Boltzmann: Events are Selected to be Coherent)**.** *Let* $(\mathsf{P}, \leq)$ *be the poset of finite, order-consistent partial histories in a locally finite causal domain, ordered by extension. For every countable family* $\{D_n\}_{n\in\mathbb{N}}$ *of dense subsets of* $\mathbf{P}$ *(local causal constraints),* there exists *a filter* $G \subseteq \mathsf{P}$ *with* $G \cap D_n \neq \varnothing$ *for all* n*.*

2.5 The Causal Universe Tensor

The axioms above determine the structure of the physical record: events form a locally finite causal set, extensions of partial histories preserve causal consistency, and informational minimality forbids unrecorded structure. What remains is to represent this record in a mathematical form that allows the accumulation of distinctions. We now construct such a representation.

2.5.1 Sets of Events

Let the set of all events accessible to an observer be denoted E[1] , ordered by causal precedence $\leq$. Because any physically realizable region is finite, this order forms a locally finite partially ordered set (poset) [29].

Each admissible set of events may be represented as a locally finite partially ordered structure [10, 89], whose links record only those relations that are causally admissible. In this view, a "history" is not a continuous trajectory but a combinatorial diagram: every vertex an event, every edge a

[1]The symbol E here denotes the *set of distinguishable events*—it is not the energy operator or expectation value familiar from mechanics. Throughout this work, E indexes discrete occurrences in the causal order, while quantities such as energy, momentum, or stress appear only later as *derived measures* on this set.

permissible propagation. This discrete formulation generalizes the intuition behind Feynman's space–time approach to quantum mechanics, in which the amplitude of a process is obtained by summing over all consistent histories [27, 28]. The Feynman diagram thus appears here as a special case of the causal network itself—a pictorial reduction of the full tensor of event relations—and the path integral becomes a statement of global consistency across all measurable causal connections.

Thought Experiment 2.5.1 (Feynman Diagram as a Causal Network [28]). ***N.B.***—*This is a* classical *simplification of the highly complex notation of the Feynman Diagram. See Thought Experiment 7.2.3 for a more rigorous treatment.*

In conventional quantum field theory, a Feynman diagram depicts a sum over interaction histories connecting initial and final particle states. Each vertex represents an elementary event—an interaction that renders previously indistinguishable outcomes distinct—and each propagator represents the possibility of causal influence between events.

In the present formulation, such a diagram is naturally interpreted as a finite causal network. *The set of vertices corresponds to the event set E, and the directed edges encode the causal relation $\leq$ defined by Axiom 5. The tensor assigned to each vertex, $E_k \in T(V)$, records the measurable contribution of that interaction to the global state, while the propagators describe admissible compositions of these event tensors within the Universe Tensor*

$$U_n = \sum_{k=1}^{n} E_k.$$

At this stage, U_n is a classical accumulator: it records the count and structure of distinguishable events without assigning amplitudes or phases. This is deliberate. The present framework concerns only the logical bookkeeping of distinctions. The full quantum structure—including complex amplitudes, superposition, and interference—appears only after the informational gauge *is introduced. In that setting, the classical accumulator becomes the coarse projection of a richer amplitude algebra, much as a Feynman diagram may be viewed as the combinatorial skeleton of a path integral. That generalization is deferred until Chapter refchap:mass, where the amplitude-bearing form of U is constructed.*

Summing over all consistent diagrams is therefore equivalent to enumerating all admissible orderings of distinguishable events. The path integral itself becomes a statement of global consistency *across the entire causal network: every measurable amplitude corresponds to one possible embedding of finite causal order into the continuous limit. In this sense, a Feynman diagram is not merely a pictorial tool but a discrete representation of the causal tensor algebra from which continuum physics emerges.*

This identification is pedagogically useful. From this point onward, every construction may be viewed as an algebraic generalization of the familiar Feynman diagram: the event tensors are its vertices, the causal relations its edges, and the Universe Tensor the cumulative sum over all consistent orderings. The remainder of the monograph simply formalizes this graphical intuition in set-theoretic and tensorial language, rather than using calculus.

Such an ordering always admits at least one maximal element [10]:

$$\mathrm{Top}(E) = \{\, e \in E \mid \nexists f \in E \text{ with } e < f \,\}. \tag{2.6}$$

The elements of $\mathrm{Top}(E)$ represent the current causal frontier—the most recent events that have occurred but have no successors [91]. Although $\mathrm{Top}(E)$ may contain several incomparable (spacelike) elements, it is never empty and therefore provides a well-defined notion of a “last event” from the observer’s perspective. This frontier defines the light-cone boundary and the terminal particle–wave interaction that delimits all accessible information.

Every event $e \in E$ corresponds to an irreducible distinction in the experimental record. Under the measurable embedding $\Psi : E \to \mathcal{T}(V)$ of

Definition 9, each logical event is mapped to an algebraic object E_e in the tensor algebra. These objects can be composed and accumulated, producing a record that reflects the ordered refinement of the causal set. The goal of this section is to define a cumulative object U—the *Causal Universe Tensor*—that embodies the total informational content of all events observed so far.

It is crucial to emphasize that no background time parameter is introduced. There is no external clock and no continuous variable t against which events are measured. Instead, Axiom 5 guarantees that the causal set admits a linear extension: the events can be listed in a sequence that respects causal precedence. In this framework, *time* is merely the ordinal index of an event in such a sequence. It is not a physical field or metric quantity, but a bookkeeping device that labels the relative order of observations.

With this viewpoint, accumulating the event tensors in order is not evaluating a function of time. It is forming the algebraic sum of the distinctions that have already occurred. The resulting object, the Causal Universe Tensor, represents the total recorded history up to any chosen ordinal position in the list of events.

Definition 8 (Time (non-standard))**.** *time is not a variable, scalar, or independent measurement. Rather, it is an index into the sorted list of events guaranteed by the Axiom of Order. Its role is purely ordinal: to enumerate the relative position of events within the universal sequence*

The preceding axioms establish a measurement record as a locally finite, partially ordered set of distinctions $(E, \prec)$. This structure is purely combinatorial. To connect the logical record to physical measurement, we require a representation that can carry numerical values and allow recorded distinctions to combine. A vector space V provides a domain for measurable quantities, but to represent successive distinctions we also need a rule for composition. The tensor algebra $\mathcal{T}(V)$ is the freest algebra generated by V: it contains V, supports noncommutative products, and imposes no additional relations beyond those required by linearity. By associating each logical event e_k with a tensor E_k in $\mathcal{T}(V)$ we obtain an algebraic record of distinctions that can be summed and composed. This representation introduces no structure beyond what is logically required to encode measurable updates.

Definition 9 (Event Tensor [35])**.** *Let $\mathcal{V}$ be a finite-dimensional real vector space of measurable quantities. An event tensor $\mathbf{E}_k \in \mathcal{T}(\mathcal{V})$ encodes the distinguishable contribution of the kth event $e_k \in \mathcal{E}$ to the global state. It is related to the logical event by a measurable embedding*

$$\Psi : \mathcal{E} \to \mathcal{T}(\mathcal{V}), \quad \mathbf{E}_k = \Psi(e_k)$$

Definition 10 (Ordered fold (non-standard))**.** *Let $(E, \preceq)$ be totally ordered as $\langle e_1, \ldots, e_n \rangle$ on a finite prefix, and let $(\mathcal{A}, \oplus)$ be a (not-necessarily commutative) associative algebraic structure with identity* 0*. Given event tensors $E_k \in \mathcal{A}$, define the ordered fold by*

$$\mathrm{Fold}_{\oplus}(E_1, \ldots, E_n) \;:=\; (((0 \oplus E_1) \oplus E_2) \cdots) \oplus E_n.$$

Definition 11 (Uncorrelant Equivalence)**.** *Write $E_i \sim E_j$ if they lie in a subset on which $\oplus$ is commutative and which preserves all cyclic scalar invariants (for example, traces of contractions). Two ordered lists are* uncorrelant equivalence *if they differ only by permutations inside such subsets. We write $\equiv$ for equality modulo uncorrelant equivalence*

Definition 12 (Selection Operator)**.** *Let E be a set of recorded events. A* selection operator *is a map*

$$\mathcal{S} : \mathcal{P}(E) \to \mathcal{P}(E)$$

such that $\mathcal{S}(A)$ returns the subset of events in A that remain distinguishable under admissible refinement. In particular, $\mathcal{S}$ removes all identifications that *cannot be supported by record. If $\mathcal{S}(A) = A$, the set A is said to be* admissible*; if $\mathcal{S}(A) \subsetneq A$, the removed elements were indistinguishable from others within A.*

The selection operator induces a partial order on E by the rule

$$e \prec f \quad \textit{iff} \quad e \in \mathcal{S}(\{e, f\}).$$

In words, e precedes f when e is a distinguishable refinement of the pair $\{e, f\}$. No causal generation is implied: the operator records only the order in which distinctions appear in the admissible record.

A collection of events is a selection chain *if repeated application of $\mathcal{S}$ eventually yields a singleton. The Universe Tensor is built from the family of all such chains: each acts as a discrete analog of an integral curve in conventional calculus.*

Proposition 1 (Causal Universe Tensor)**.** *Let $E_1 \prec \cdots \prec E_n$ be the event tensors in order, with $\oplus$ the addition in $T(V)$ (componentwise in the direct sum) and composition handled only by subsequent linear functionals. Define $U_0 := 0$ and*

$$U_{k+1} \;:=\; U_k \oplus E_{k+1}, \qquad 0 \le k < n.$$

Then:

1. *(*Causal uniqueness*) Let the index k correspond to the ordinal rank of each event (Definition 1), whose existence is guaranteed by ZFC + Axiom of Cantor (Axiom 5). Then the recursion*

$$U_{k+1} = U_k \oplus E_{k+1} \tag{2.7}$$

is unique because this ordinal order $\preceq$ is fixed by the causal structure and cannot be permuted outside uncorrelant-equivalence classes (Definition 5). Associativity of $\oplus$ ensures mechanical well-definedness once that order is fixed.

2. *If a subset $S \subset \{1, \dots, n\}$ is uncorrelated in the sense of Definition 5, then reordering $\{E_i\}_{i \in S}$ leaves all cyclic scalar invariants of U_n unchanged; i.e. $U_n \equiv U_n'$ for any such reordering.*

3. *In the fully commutative case, $U_n = \sum_{k=1}^{n} E_k$ as a componentwise sum in $T(V)$.*

Proof (Sketch). (1) Associativity gives well-definedness of the left fold. (2) By construction, permutations inside uncorrelated subsets preserve $\oplus$ and cyclic scalar functionals (e.g. traces of contractions), hence invariants coincide. (3) If $\oplus$ is commutative, the fold equals the ordinary finite sum. □

A full proof is provided in Appendix **??**.

Remark 10 (Ordinal determinacy)**.** *The sequence* $(\mathbf{U}_k)$ *is not merely algebraically well-defined but* physically ordered*: k indexes ordinal rank, not arbitrary enumeration. Hence any reordering outside uncorrelant classes violates Axiom 5.*

With the ordinal structure of events established, we now formalize how these measurements combine algebraically within a finite vector space.

2.5.2 Formal Structure of Event and Universe Tensors

We now specify the algebraic structure of the quantities introduced above. Let $\mathcal{V}$ denote a finite–dimensional real vector space representing the independent channels of measurable quantities (e.g. energy, momentum, charge). Define the tensor algebra [37, 55]

$$\mathcal{T}(\mathcal{V}) = \bigoplus_{r=0}^{\infty} \mathcal{V}^{\otimes r}, \tag{2.8}$$

whose elements are finite sums of r–fold tensor products over $\mathbb{R}$. Each *event tensor* E_k is a member of $\mathcal{T}(\mathcal{V})$ encoding the distinguishable contribution of the k–th event to the global state. We write

$$\mathbf{E}_k \in \mathcal{T}(\mathcal{V}), \qquad \mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k \in \mathcal{T}(\mathcal{V})). \tag{2.9}$$

Addition is understood componentwise in the direct sum and preserves the ordering of indices guaranteed by the Axiom of Order [10, 37]. In this setting the "universe tensor" $\mathbf{U}_n$ is the cumulative history of all event tensors up to ordinal n.

Definition 13 (Tensor Algebra [35])**.** *The tensor algebra on a vector space $\mathcal{V}$ is*

$$\mathcal{T}(\mathcal{V}) = \bigoplus_{r=0}^{\infty} \mathcal{V}^{\otimes r}$$

with componentwise addition and associative tensor product

Remark 11. *Each logical event e_k in the partially ordered set $(\mathcal{E}, \prec)$ induces a tensor $\mathbf{E}_k = \Psi(e_k)$ in $\mathcal{T}(\mathcal{V})$. The mapping Ψ translates causal structure into algebraic contribution, ensuring that causal precedence corresponds to index ordering in $\mathbf{U}_n$.*

Because $\mathcal{T}(\mathcal{V})$ is a free associative algebra, all operations on $\mathbf{U}_n$ are well defined using the standard linear maps, contractions, and bilinear forms of $\mathcal{V}$. The subsequent analysis of variation and measurement therefore proceeds entirely within conventional linear–operator theory.

From the definition of the Universe Tensor

$$U_n = \sum_{k=1}^{n} E_k, \tag{2.10}$$

we may regard an *uncorrelant* as any subset of events whose local order can be permuted without altering the global scalar invariants of U_n. Formally, a subset $S \subseteq \{E_1, \ldots, E_n\}$ is uncorrelated if, for every permutation π of S,

$$\sum_{E_i \in S} E_i = \sum_{E_i \in S} E_{\pi(i)}. \tag{2.11}$$

In this case, all contractions or scalar traces derived from U_n remain unchanged by reordering the elements of S, even though the operator sequence itself may differ.

Remark 12 (Algebraic Characterization of Uncorrelants)**.** *Let $\Psi : E \to \mathcal{T}(V)$ be the event embedding of Definition 9, and let $E_e = \Psi(e)$. If $S \subseteq E$ is a set of uncorrelant events (Definition 5), then reordering the tensors $\{E_e\}_{e \in S}$ in any linear extension of $(E, \prec)$ leaves all scalar invariants of the Universe Tensor unchanged. By Proposition 2, the difference between any two ordered folds over S lies in the commutator ideal $[\mathcal{T}(V), \mathcal{T}(V)]$, and therefore vanishes under any observable that annihilates commutators. Thus the set-theoretic notion of uncomparability corresponds exactly to the algebraic notion of order-insensitive contribution to scalars derived from U.*

Definition 14 (Commutator)**.** *Let $\mathcal{A}$ be an algebra over a field $\mathbb{F}$, constructed as a set equipped with a bilinear product $(x, y) \mapsto xy$. For $x, y \in \mathcal{A}$ the* commutator *of x and y is the element*

$$[x, y] \ := \ xy - yx \ \in \ \mathcal{A}.$$

The set of all finite $\mathbb{F}$-linear combinations of commutators,

$$[\mathcal{A}, \mathcal{A}] \ := \ \left\{ \sum_{i=1}^{m} \alpha_i \, [x_i, y_i] \ : \ \alpha_i \in \mathbb{F}, \ x_i, y_i \in \mathcal{A} \right\},$$

is called the commutator ideal*. It is the smallest linear subspace of $\mathcal{A}$ that contains every element of the form $xy - yx$ and is closed under multiplication by arbitrary elements of $\mathcal{A}$.*

Proposition 2 (Extension Invariance up to Commutators)**.** *Let $(E, \prec)$ be a locally finite causal set and $\Psi : E \to \mathcal{T}(V)$ assign event tensors $E_e = \Psi(e)$. For any finite down-set $D \subseteq E$ and any linear extension $L = (e_1, \ldots, e_m)$ of $(D, \prec)$, define the ordered fold*

$$U_L(D) \;:=\; (((\mathbf{1} \oplus E_{e_1}) \oplus E_{e_2}) \cdots) \oplus E_{e_m} \;\in\; \mathcal{T}(V),$$

where $\oplus$ is the (generally noncommutative) fold operation of Definition 10.

Let $[\mathcal{T}(V), \mathcal{T}(V)]$ denote the commutator ideal generated by elements of the form $XY - YX$. Then for any two linear extensions L, L' of D,

$$U_L(D) \;\equiv\; U_{L'}(D) \quad \mathit{mod}\; [\mathcal{T}(V), \mathcal{T}(V)].$$

Equivalently, the difference $U_L(D) - U_{L'}(D)$ is a finite sum of commutators.

Proof (Sketch). Any two linear extensions of a finite poset are related by a sequence of adjacent swaps of incomparable elements. Each adjacent swap replaces a factor $\cdots \oplus E_a \oplus E_b \cdots$ by $\cdots \oplus E_b \oplus E_a \cdots$. Expanding both orders shows their difference lies in the ideal generated by $E_aE_b - E_bE_a$ (and higher nested commutators if $\oplus$ is not simple multiplication). Iterating over the swap sequence expresses $U_L(D) - U_{L'}(D)$ as a sum of commutators. A full proof is provided in Appendix A. □

A full proof is provided in Appendix **??***.*

Corollary 1 (Scalar Observables are Extension-Invariant)**.** *Let $\phi : \mathcal{T}(V) \to \mathbb{F}$ be any linear functional that vanishes on commutators (e.g., a trace-like or abelianized evaluation). Then for every finite down-set D and linear extensions L, L',*

$$\phi(U_L(D)) \;=\; \phi(U_{L'}(D))\,.$$

Thus physical scalars are insensitive to permutations of uncorrelant events even though the Universe Tensor itself is order-sensitive.

Remark 13 (Consistency with Thought Experiment 2.4.1). *The noncommutative example with events A, B shows $E_A E_B \neq E_B E_A$; the fold U_L depends on order. The proposition does not deny this. It states that such differences are* pure commutators*, so any observable that kills commutators (the quantities we actually report) is invariant under relinearization.*

Remark 14 (Where Minimality Acts). *Informational minimality (Axiom of Occam) should be applied either to (i) observables $\phi(U_L)$ for functionals ϕ that annihilate commutators, or (ii) the abelianization $\mathcal{T}(V)/[\mathcal{T}(V), \mathcal{T}(V)]$. This keeps Occam's selection compatible with the noncommutative fold.*

Thought Experiment 2.5.2 (Non-commutative event pair [40]). ***N.B.***— *Non-commutative events are often in a* dependency *relationship, one event must* precede *the other for consistent measurement to occur.*

Let $V = \mathbb{R}^2$ and take event tensors as 2×2 matrices acting on V with the usual (non-commutative) product. Define

$$E_A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad E_B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

Then

$$E_A E_B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = E_B E_A, \quad \text{so } [E_A, E_B] \neq 0.$$

Thus, the universe update $U_2 = E_A E_B$ differs from $U_2' = E_B E_A$ whenever the event pair is in a uncorrelant class that permits permutation. However, cyclic scalar invariants agree: $\mathrm{tr}(E_A E_B) = \mathrm{tr}(E_B E_A) = 3$, *and* $\det(E_A E_B) = \det(E_A)\det(E_B) = 1$. *Hence order affects the* operator *state but leaves cyclic scalars (our measurable invariants) unchanged. This illustrates how Event Selection can forbid reordering of correlated events while Martin-like consistency still preserves global scalar bookkeeping.*

Thought Experiment 2.5.3 (Independent Event Chains [56])***. N.B.***— *This result is analogous to the central segment of the twin paradox, when neither twin is being accelerated. During that interval the Earth and rocket worldlines are independent event chains. No signal has been exchanged, so no event on one chain is comparable to any event on the other. Clock comparison only becomes meaningful when a causal interaction occurs.*

Consider two independent event chains $A_1 \prec A_2$ and $B_1 \prec B_2$, represented by 2×2 event tensors

$$E_{A1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad E_{A2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad E_{B1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad E_{B2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.12}$$

The cumulative tensor through all four events is

$$U_4 = E_{A1} + E_{A2} + E_{B1} + E_{B2} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{2.13}$$

Because E_{A2} and E_{B2} commute under addition, the subset $S = \{E_{A2}, E_{B2}\}$ is uncorrelated: its permutation leaves all scalar invariants of U_4 unchanged. This simple algebraic example demonstrates how correlation corresponds to commutative structure within a finite causal chain.

The cumulative universe tensor through all four events is then

$$\mathbf{U}_4 = \mathbf{E}_{A_1} + \mathbf{E}_{A_2} + \mathbf{E}_{B_1} + \mathbf{E}_{B_2} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{2.14}$$

If the uncorrelant pair $\{A_2, B_2\}$ is permuted, the componentwise sum is unchanged, $\mathbf{E}_{A_2} + \mathbf{E}_{B_2} = \mathbf{E}_{B_2} + \mathbf{E}_{A_2}$, illustrating that uncorrelant classes correspond to commutative subsets within the otherwise ordered sequence. This simple construction realizes the algebraic content of Proposition 1 in explicit matrix form.

2.6 Information Minimality and Kolmogorov Closure

The previous definitions describe events as finite distinctions and their ordering as a partial refinement of information. What remains is the rule that determines which extensions of a recorded event set are admissible. Not every history consistent with the order is physically meaningful: a completion that inserts unobserved structure would imply additional measurements that never occurred. Information minimality formalizes this constraint through algorithmic information theory in the sense of Kolmogorov, Solomonoff, and Chaitin [49, 87, 88, 14].

We treat histories as finite symbolic strings and measure their descriptive content by Kolmogorov complexity. A physically admissible history is one that cannot be compressed by adding unrecorded structure.

Definition 15 (Kolmogorov Complexity [49, 14])**.** *Fix a universal Turing machine U. For any finite string $w \in \Sigma^*$, the Kolmogorov complexity $K(w)$ is the length of the shortest input to U that outputs w and halts. The functional $K : \Sigma^* \to \mathbb{N}$ is defined up to an additive constant independent of w.*

Definition 16 (Admissible Extension [58])**.** *Let $E = \{e_0 \prec e_1 \prec \cdots \prec e_n\}$ be the recorded events of an experiment. A finite string $w \in \Sigma^*$ is an* extension *of E if its image under the event map contains E in the same causal order. An extension w is* admissible *if it introduces no additional events beyond E; that is, every distinguishable update encoded by w has a corresponding element of E. Any extension predicting unobserved structure is rejected as inadmissible.*

Definition 17 (Information Minimality [49, 58])**.** *Among all admissible extensions of E, the physically admissible history is the one of minimal Kolmogorov complexity:*

$$w_{min} = \arg\min\{K(w) : w \textit{ is an admissible extension of } E\}.$$

Information minimality expresses the logical content of measurement: if additional curvature, oscillation, turning points, or discontinuities had occurred between e_i and e_{i+1}, those features would have generated new events. Since no such events are present in E, any extension that predicts them is inadmissible, and a shorter description exists.

Remark 15. *This principle is purely set–theoretic. No geometry, metric, or differential structure is assumed. Kolmogorov minimality selects the shortest admissible description of the recorded distinctions and forbids unobserved structure.*

Remark 16. *As the resolution of measurement increases, the admissible extension forms a Cauchy sequence in the space of symbolic descriptions. In the dense limit, its smooth shadow is the unique spline that introduces no new structure between recorded events. Thus the variational calculus is not imposed; it is the continuum limit of Kolmogorov minimality.*

2.6.1 Inadmissibility of Unobserved Structure

Let $E = \{e_0 \prec e_1 \prec \cdots \prec e_n\}$ be the finite set of recorded events produced by a measurement process. By Definition 15 (Measurement), each event corresponds to a distinguishable update of state: a change that crossed a detection threshold and became causally recorded.

Between two successive events e_i and e_{i+1}, no additional events were recorded. This absence is a data constraint: any refinement of the history that introduces detectable structure—curvature, oscillation, turning points, discontinuities, or other distinguishable phenomena—would generate additional events. Since these events do not appear in E, any history that predicts them is logically inconsistent with the observational record.

Definition 18 (Unobserved Structure)**.** *Let w be an admissible extension of E (Definition 2.3.3). A symbolic segment of w between e_i and e_{i+1} contains* unobserved structure *if it encodes a distinguishable update that is not present in E.*

2.7 Correlation and Dependency

In conventional quantum mechanics the word "entanglement" refers to a non-classical dependency among amplitudes: indistinguishable histories are combined before probabilities are assigned. The present framework adopts a similar intuition, but in a purely informational and algebraic form, with no amplitudes and no functional dependencies.

Two events are *uncorrelant* when no *correlant* exists between them. In this case, their transposition commutes with every admissible invariant of the Universe Tensor, and the events may be represented independently. Uncorrelant events are informationally separable: no refinement of the record forces them to be treated jointly.

Two events are *correlant* when they do not commute: exchanging them changes at least one admissible invariant. In this case a correlant exists. A correlant is an informational relation—the minimal structure required when two events cannot be represented independently of one another. Importantly, a correlant does not specify direction or causation: nothing is said about which event precedes, influences, or determines the other. It expresses only that the transposition fails to commute.

Uncorrelant events can become correlated when their light cones merge. Before the merger, each event admits a representation that commutes with the other; no correlant exists, and their histories may be transposed without altering any admissible invariant. After the merger, additional distinctions become available, and the transposition may fail to commute. A correlant then forms, not because one event generates the other, but because the enlarged record no longer permits them to be represented independently.

Dependency relations are stronger still. A dependency asserts that one event is determined by another, as in the functional relationships of the classical calculus. Such relations describe macro–events in conventional dynamics, where causes generate effects. The present work is not concerned with dependency. Correlation is the weaker structure: non-commutativity under admissible permutation, with no claim of generation or determination.

Thus, "entanglement" in the conventional quantum sense has two informational analogues in this framework. When amplitudes combine as indistinguishable histories, the result is a superposition. When events cannot be transposed without altering admissible invariants, the result is a correlant. Both are consequences of the same principle: distinctions cannot be manufactured retroactively. What differs is the level at which indistinguishability occurs—the discrete record of events or the smooth representation of extremals.

Thought Experiment 2.7.1 (Spooky Action at a Distance [4, 26, 91])**.** *Consider an uncorrelant $S = \{\mathbf{E}_i, \mathbf{E}_j\}$ of two spatially separated measurement events. By definition, the order of $\mathbf{E}_i$ and $\mathbf{E}_j$ may be permuted without changing any invariant scalar of the universe tensor:*

$$\mathbf{E}_i + \mathbf{E}_j = \mathbf{E}_j + \mathbf{E}_i. \tag{2.15}$$

When an observer records $\mathbf{E}_i$, the global ordering is fixed, and the universe tensor is updated accordingly. Because $\mathbf{E}_j$ belongs to the same uncorrelant set, its contribution is now determined consistently with $\mathbf{E}_i$, even if E_j occurs at a spacelike separation. This manifests as the phenomenon of "spooky action at a distance"—the appearance of instantaneous correlation due to reassociation within the uncorrelant subset.

Thought Experiment 2.7.2 (Hawking Radiation [39, 96])**.** *Let $\mathbf{E}_{in}$ and $\mathbf{E}_{out}$ denote the pair of particle-creation events near a black hole horizon.*

These events form an uncorrelant set:

$$S = \{\mathbf{E}_{in}, \mathbf{E}_{out}\}. \tag{2.16}$$

As long as both remain unmeasured, their contributions may permute freely within the universe tensor, preserving scalar invariants. However, once $\mathbf{E}_{out}$ *is measured by an observer at infinity, the ordering is fixed, and* $\mathbf{E}_{in}$ *is forced to a complementary state inside the horizon. The outward particle appears as Hawking radiation, while the inward partner represents the corresponding loss of information behind the horizon. Thus Hawking radiation is naturally expressed as an uncorrelant whose collapse into correlation occurs asymmetrically across a causal boundary.*

Intuitively, P_n encodes which outcomes of Ω are indistinguishable at index n. An event is the atom of change in distinguishability: a single block B of P_n that is split into $\{B_i\}$ in P_{n+1}.

Definition 19 (Predicate on Events [93, 74])**.** *A predicate is any map* $P : E \to \{0, 1\}$*. It selects which events are "counted"*

Definition 20 (Measurement)**.** *Let* E *be the event set with order* $\prec$*, and let* $P : E \to \{0, 1\}$ *be a predicate. Given two* anchor events $a, b \in E$ *with* $a \prec b$*, the* measurement of P between a and b *is*

$$M_P[a, b] \;:=\; \#\{\, e \in E \mid a \prec e \prec b \text{ and } P(e) = 1 \,\} \in \mathbb{N}. \tag{2.17}$$

Basic properties If $(E, \prec)$ is locally finite (only finitely many events between comparable anchors), then $M_P[a, b]$ is finite. Measurements are *additive*: for $a \prec c \prec b$,

$$M_P[a, b] \;=\; M_P[a, c] + M_P[c, b]. \tag{2.18}$$

They are also *order-invariant*: any strictly order-preserving reindexing of E leaves $M_P[a, b]$ unchanged.

2.7.1 Construction of the Universe Tensor and the Axiom of Event Selection

The algebraic structure introduced so far defines a finite causal order of distinguishable events, each contributing an elementary tensor $\mathbf{E}_k$ to the cumulative universe tensor

$$\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k. \tag{2.19}$$

This sequence describes the universe as a recursively constructed record of distinctions: every new event refines the existing causal order by one measurable increment. Yet the same mathematical machinery that enables such constructions can also generate pathological extensions—formal solutions with no physical meaning. To maintain causal coherence, the theory must therefore include a regularity condition that limits which extensions are admissible.

Thought Experiment 2.7.3 (Pathological Extension Without Event Selection [57])**.** ***N.B.***—*These sorts of pathologies can appear to look like paradoxes arising from* time travel, remote viewing, *or other specious phenomena.*

Let $E = \{e_1, e_2, e_3, \dots\}$ be a locally finite causal chain where each event e_i has a unique successor e_{i+1}. Define the corresponding universe tensor

$$\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k, \qquad \mathbf{E}_k = \mathbf{\Psi}_k(e_k). \tag{2.20}$$

Now suppose we attempt to "extend" this history by splitting a single event e_j into uncountably many indistinguishable refinements:

$$e_j \longrightarrow \{e_{j,\alpha}\}_{\alpha \in [0,1]}, \tag{2.21}$$

each representing a formally distinct but observationally identical outcome.

Algebraically, this replacement yields

$$\mathbf{E}_j \longrightarrow \int_0^1 \mathbf{E}_{j,\alpha}\, d\alpha, \tag{2.22}$$

so that the next update becomes

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \int_0^1 \mathbf{E}_{j,\alpha}\, d\alpha. \tag{2.23}$$

This "extension" violates the finiteness and distinguishability conditions necessary for causal coherence:

1. *The set $\{e_{j,\alpha}\}$ is uncountable, destroying local finiteness;*

2. *The new events are indistinguishable, so Extensionality no longer guarantees unique contributions;*

3. *The total tensor amplitude U_{n+1} can diverge or cancel arbitrarily, depending on how the continuum of duplicates is treated.*

Operationally, this is a Banach–Tarski-like overcounting: the causal structure has been "refined" in a way that preserves measure only formally while the order relation collapses. The observer would now predict contradictory outcomes for the same antecedent state—an overcomplete history*.*

To prevent this, the Axiom of Event Selection *restricts the permissible extension to a countable, consistent refinement:*

$$e_j \longrightarrow e_{j,1}, e_{j,2}, \ldots, e_{j,k}, \tag{2.24}$$

and requires the selection of exactly one representative outcome from each locally admissible family. This keeps E locally finite and maintains a single-valued universe tensor,

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \mathbf{E}_{j,k^*}. \tag{2.25}$$

The axiom thus enforces the same regularity that Martin's Axiom guarantees in set theory: every countable family of local choices admits a globally consistent selection that preserves the partial order.

Overgeneration and the Need for Selection. Pure mathematics allows objects that exceed any finite observer's capacity to distinguish: sets without measurable support, or decompositions that preserve volume while destroying order (as in the Banach–Tarski paradox). In physical terms, such pathologies correspond to hypothetical universes that overcount possibilities—histories in which indistinguishable outcomes are spuriously distinguished by the formalism itself. To restrict attention to realizable histories, we introduce an axiom that selects only those extensions of the causal order that remain both countable and consistent with local finiteness.

Proposition 3 (Minimal informational closure)**.** *Let $G \subseteq \mathsf{P}$ be a global filter guaranteed by Axiom 7. Then there exists an equivalence class $\mathcal{U}_G$ of field configurations consistent with G. Among these, the canonical representative selected by the Selection Operator (Definition 12) minimizes the informational curvature functional*

$$R[U] \;=\; \int (\Delta_h^{(2)} U)^2 \, dx.$$

The Euler–Lagrange condition

$$\frac{\delta R}{\delta U} = 0 \quad \Longleftrightarrow \quad U^{(4)} = 0$$

defines the unique minimal-information extension of G.

Proof. Axiom 7 ensures non-empty existence of G. Each admissible U consistent with G corresponds to a permissible refinement satisfying local constraints. The Selection Operator chooses the least-biased representative—the one minimizing $R[U]$, the "informational bending energy." By standard variational calculus, minimization of this quadratic functional yields the cubic-spline condition $U^{(4)} = 0$. Thus, the non-constructive existence of G implies the existence of a minimal-curvature representative U_G within its equivalence class. This is the informational analog of Occam's principle: simplicity as closure. □

Remark 17 (Logical guarantee, not a mechanism)**.** *Axiom 7 is non-constructive. It asserts the* existence *of at least one globally consistent extension meeting all countably many local constraints but does not prescribe any observable or deterministic procedure that finds it. This mirrors the role of the Rasiowa–Sikorski lemma in forcing and is structurally analogous (though weaker in scope) to Martin's Axiom on ccc posets. The axiom thereby rules out pathological overgeneration (e.g. uncountable splits into indistinguishable refinements) by restricting attention to countable, order-coherent extensions.*

Thought Experiment 2.7.4 (Algorithmic analogies are illustrative only)**.** *Classical algorithms such as Dantzig's simplex method* select *admissible vertices in a feasible polytope under global constraints, providing existence-by-structure but not physics. We invoke such algorithms only as intuition pumps: they exemplify selection under constraints, not a dynamical law implemented by nature.*

Remark 18. *Downstream results should cite Axiom 7 only for* existence*. Any phrase suggesting that the axiom "chooses," "constructs," or "computes" a history should be replaced with "there exists a history meeting the constraints."*

Interpretation. Axiom 7 serves as the "spline condition" for causal structure: it ensures that the discrete increments of measurement join smoothly into a coherent global record. Just as a cubic spline is the minimal analytic closure that interpolates local data without oscillation, Event Selection is the minimal logical closure that interpolates local causal choices without contradiction. The result is a universe tensor $\mathbf{U}_n$ that can evolve indefinitely while preserving the consistency of order:

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \mathbf{E}_{n+1}, \qquad \text{with all admissible } \mathbf{E}_{n+1} \text{ selected by causal consistency.}$$

Under this rule, the smoothness of physical law is not imposed but emerges as the global continuity of distinguishability itself.

Corollary 2 (Martin consistency from Event Selection (domain version))**.** *Let P be the poset of all finite, order–consistent partial histories in a fixed observer's causal domain, ordered by extension ($p \leq q$ iff q extends p without introducing new indistinguishabilities). Then:*

1. *P satisfies the countable chain condition (ccc).*
2. *For every* countable *family $\{D_n : n \in \mathbb{N}\}$ of dense subsets of P, there exists a filter $G \subseteq P$ such that $G \cap D_n \neq \varnothing$ for all n.*

Consequently, every countable system of local causal choices admits a globally consistent extension meeting all local constraints. In §2.3.4–§2.3.5 we interpret this as the finite causal analogue of Martin's property in this domain.

Proof. **(1) P is ccc.** By construction, each condition $p \in P$ encodes only finitely many events and order-relations drawn from a *countable* label set available to the observer (Axiom 6). Two conditions are incompatible iff they disagree on at least one finite relation (e.g. they force contradictory orderings on some finite subconfiguration). But there are only countably many distinct finite patterns over a countable alphabet; hence any antichain injects into a countable set of such patterns and must itself be countable. Therefore P has no uncountable antichain, i.e. P is ccc.

(2) Existence of a filter meeting a countable dense family [75, 53]. Let $\langle D_n : n \in \mathbb{N}\rangle$ be dense subsets of P. We build an increasing sequence $(p_n)_{n\in\mathbb{N}}$ in P by recursion so that $p_{n+1} \in D_n$ for all n.

Start with any $p_0 \in P$ (e.g. the empty partial history). Given p_n, use the density of D_n to choose $p_{n+1} \in D_n$ with $p_{n+1} \leq p_n$. (Here $\leq$ is the extension order, so p_{n+1} extends p_n and is therefore compatible with all earlier requirements.) This recursion is legitimate by the Axiom of Choice (assumed via Axiom 2).

Define

$$G := \{ q \in P \mid \exists n \;\; p_n \leq q \}. \tag{2.26}$$

Then G is upward closed by definition; and it is directed since (p_n) is an increasing chain: for any $q_1, q_2 \in G$ choose n_1, n_2 with $p_{n_i} \leq q_i$ and let $m = \max\{n_1, n_2\}$; then $p_m \leq q_1, q_2$, so any $q \geq p_m$ lies in G and is a common extension. Thus G is a filter on P.

Finally, for each n we have $p_{n+1} \in D_n$ and $p_{n+1} \in G$, so $G \cap D_n \neq \varnothing$. Hence G meets every dense set in the given countable family.

Remark (transfinite families). The above construction yields the classical Rasiowa–Sikorski lemma for countably many dense sets (provable in ZFC). In our setting, Axiom 7 supplies the physical regularity that, together with local finiteness (ccc), supports the same book-keeping recursion along any well-orderable family of dense requirements indexed below $2^{\aleph_0}$, producing an increasing chain $\langle p_\alpha \mid \alpha < \kappa \rangle$ with $p_{\alpha+1} \in D_\alpha$ and the same filter $G = \{q : \exists \alpha \; p_\alpha \leq q\}$ meeting all D_α. In this sense, Event Selection plays the domain-specific role of an MA-like closure principle for the causal poset considered here. □

Remark 19 (Scope)**.** *This is* not *a derivation of set-theoretic Martin's Axiom inside ZFC. Rather, under the physical axioms of locally finite causality and Event Selection, the induced forcing-like poset of finite partial histories enjoys an MA-like property sufficient for our global-consistency arguments. In Chapter 4, this is exactly the "Martin's Condition" used to guarantee propagation/compatibility across overlaps.*

The values of the Causal Universe Tensor compute the scalar invariants of order that remain unchanged under admissible extensions of the causal set. Each component of the tensor encodes a local configuration of events, while the contraction of those components—its scalar value—measures the degree of consistency of that configuration with the global ordering guaranteed by Martin's Axiom. When the tensor's scalar invariants remain constant, the system exhibits smooth, force-free motion: the kinematic regime.

Chapter 3

The Calculus of Dynamics

3.1 Emergent Dynamics

The discrete Causal Universe Tensor defines a finite informational measure on admissible configurations. When these configurations are refined, any replacement of an admissible history by one with additional unrecorded structure is forbidden by Axiom 7. As a consequence, the physical notion of "dynamics" is not an independent postulate: it arises only as the unique continuous shadow of informational extremality.

3.1.1 Weak Formulation on Space–Time

Let ψ denote an admissible configuration consistent with a fixed set of event anchors, and let ϕ be any test configuration sharing those anchors. Informational minimality requires that replacing ψ by ϕ cannot decrease causal consistency. In the finite domain, this condition appears as the weak relation

$$\psi^* \mathcal{L} \psi \;\approx\; \psi^* \mathcal{L} \phi, \tag{3.1}$$

where $\mathcal{L}$ counts distinguishable causal increments and ψ^* is the reciprocity dual. No derivatives are assumed; smooth structure enters only as the completion of refinement.

3.1.2 Reciprocity and the Adjoint Map

The weak extremality relation (3.1) compares an admissible configuration ψ against a test configuration ϕ that shares the same event anchors. In the discrete domain, replacing ψ by ϕ is encoded by a *selected update* of the event record: only those increments that alter distinguishable curvature are allowed. This update is represented by the Selection Operator $\mathcal{S}$ of Definition 12, which maps admissible configurations to admissible refinements.

To every such operator there is an associated *reciprocity map* ψ^*, defined as the adjoint of $\mathcal{S}$ with respect to the informational measure $\mathcal{L}$:

$$\psi^* \, \mathcal{L} \, \phi \;=\; \mathcal{L} \left(\mathcal{S}[\psi], \, \phi \right),$$

for all admissible test configurations ϕ. Intuitively, ψ^* records the "shadow" of an update when viewed from the perspective of informational minimality: any component of ϕ that would introduce an unobserved distinction is suppressed by the adjoint action.

Because $\mathcal{S}$ admits only refinements that do not create hidden structure, the reciprocity map annihilates variations invisible at the event anchors. Formally, if ϕ and ψ agree at the anchors and differ only by an undetectable perturbation, then

$$\psi^* \, \mathcal{L} \, \phi \;=\; \psi^* \, \mathcal{L} \, \psi.$$

This is exactly the weak relation (3.1). In this sense, ψ^* enforces closure: it guarantees that the extremal configuration carries no latent curvature that would be revealed under refinement.

The "variation" of ψ is therefore not a differential operation but a selected refinement of the causal record. The reciprocity map is the dual constraint that removes any component of that refinement which would violate informational minimality. Together, they generate the weak Euler–Lagrange structure entirely within the discrete domain, without assuming differentiability or a continuum of states.

In this way, the reciprocity map guarantees that any admissible update of ψ corresponds to an interpolant $f(\psi)$ that introduces no new distinguishable structure. Under refinement, every such interpolant converges to the same smooth closure Ψ. Because the event tensor defines a finite labeled partition of the causal domain, Ψ preserves anchor order and is injective on each partition element. Its inverse Ψ^{-1} therefore exists trivially: applying Ψ and then Ψ^{-1} recovers exactly the original discrete record ψ. The interpolant and its smooth limit are thus informationally equivalent representations of the same causal structure.

$$f(\psi) \to \Psi^{-1}. \tag{3.2}$$

3.1.3 Dense Limit and Euler–Lagrange Closure

In the classical calculus of variations, a weak extremum of a smooth functional implies the Euler–Lagrange equation in strong form; see Courant [16] or Ciarlet [15]. There, differentiability is assumed a priori and the weak form is obtained by integrating by parts.

In the present framework no differentiability is assumed. The weak relation (3.1) is defined entirely in the discrete domain, where each term counts distinguishable causal increments. When the causal grid is refined, informational minimality forces cubic continuity at each event anchor. In the dense limit, the discrete extremal coincides with the classical Euler–Lagrange closure,

$$U^{(4)} = 0,$$

but this appears only as the smooth completion of a countable sequence of refinements, not as an assumed differential equation.

Proposition 4 (Discrete extremality implies Euler–Lagrange closure)**.** *Let U be an admissible configuration on a finite causal grid, and assume it satisfies* *the weak extremality condition* (3.1) *against all test configurations* ϕ *sharing the same event anchors. Then* U *is a piecewise cubic interpolant with global* C^2 *continuity. In the dense refinement limit of the causal grid, the discrete extremal satisfies*

$$U^{(4)} = 0.$$

Thus the classical Euler–Lagrange closure arises as the smooth completion of a countable sequence of informational refinements, not as an assumed differential equation.

Proof sketch. The argument proceeds in four steps.

(1) Discrete admissible configurations. On a finite causal grid, U is specified by its values on a countable set of event anchors. Between consecutive anchors, any interpolant with hidden curvature would imply additional distinguishable events. Therefore the admissible interpolant on each interval is a polynomial of minimal degree, and U is piecewise polynomial.

(2) Weak extremality and hidden curvature. The weak relation (3.1) forbids any replacement of U by a test configuration ϕ that reduces informational consistency. A polynomial of degree greater than three contains latent inflection points that would be detected at sufficient refinement. Thus the extremal is piecewise cubic.

(3) Matching conditions at anchors. Informational minimality disallows jumps in value, slope, or bending moment at an anchor, since each would constitute a new observable event. Hence the piecewise cubic segments glue together with continuous value, first derivative, and second derivative. The third derivative is piecewise constant.

(4) Dense limit and smooth closure. As the grid is refined, the intervals shrink and the piecewise constant third derivative converges to a continuous function. The only continuous function whose integral vanishes on every shrinking interval is zero. Therefore the fourth derivative of the limit vanishes and the discrete extremal satisfies $U^{(4)} = 0$.

This recovers the classical Euler–Lagrange form without assuming differentiability: the differential equation is the smooth shadow of finite informational constraints. □

Thought Experiment 3.1.1 (Repeatability of Invisible Motion [2])**.** *Consider two independent observers, A and B, who record the motion of a particle between the same event anchors $x_i \prec x_{i+1}$. Each observer has finite resolution: any acceleration or inflection large enough to be distinguishable produces a new event. Both refine their instruments until no further events are detected on the interval.*

If hidden curvature existed between the anchors, further refinement would create additional distinguishable records. The absence of such records forces each observer to recover the same polynomial of minimal degree. Thus both obtain a cubic patch on the interval.

Now let A and B exchange data and perform a joint refinement on a finer grid. Any disagreement in value, slope, or bending moment at a shared anchor would itself generate an observable event. To avoid contradiction, the cubic patches must glue together with continuous U, U', and U''. In the dense refinement limit, the piecewise constant third derivative converges to a continuous function whose integral vanishes on every shrinking interval, yielding

$$U^{(4)} = 0.$$

Thus repeatability demands the Euler–Lagrange closure: if two observers can refine their measurements indefinitely without producing new events, their reconstructions must converge to the same cubic extremal. Smooth dynamics are therefore the unique histories that leave no trace.

Interpretation: No Physics Assumed

The quantity $\mathcal{L}$ is not a physical density but an informational measure of distinguishable curvature. The "action" is a cumulative count of admissible distinctions, and its extremal is the only configuration that introduces no unobserved structure. Classical dynamics therefore arise as a mathematical consequence of measurement, not as a physical postulate.

3.2 Galerkin Solutions to Weak Equations

The weak extremality condition (3.1) can be interpreted in the classical language of Galerkin methods. In this formulation, admissible variations are taken from a finite trial space of functions that agree with the event anchors. For the curvature functional

$$\mathcal{J}[U] = \int (U''(x))^2 \, dx, \tag{3.3}$$

the Galerkin weak form is

$$\int U''(x)\, \phi''(x)\, dx = 0, \tag{3.4}$$

for all admissible ϕ. Integrating (3.4) by parts twice and using the fact that the variations vanish at the anchors eliminates all boundary terms and produces the strong Euler–Lagrange equation

$$\frac{d^2}{dx^2}\big(2U''(x)\big) = 0, \tag{3.5}$$

and therefore

$$U^{(4)}(x) = 0. \tag{3.6}$$

Thus, the Galerkin extremals of the curvature functional are exactly the cubic polynomials on each interval of the partition. This establishes the classical equivalence between weak variational solutions and their strong differential form.

3.2.1 Weierstrass Convergence of Galerkin Extremals

Let $[a, b]$ be compact and let $\{\mathcal{T}_n\}$ be a sequence of nested partitions with mesh size $h_n \to 0$. For each n, let U_n be the Galerkin extremal of the curvature functional

$$\mathcal{J}[U] \;=\; \int_a^b \big(U''(x)\big)^2 \, dx, \tag{3.7}$$

subject to the anchor constraints induced by the event record on $\mathcal{T}_n$.

Theorem 1. *There exists a unique smooth closure Ψ satisfying*

$$\Psi^{(4)}(x) \;=\; 0 \quad \textit{on each element of every } \mathcal{T}_n, \tag{3.8}$$

and the sequence of Galerkin extremals $\{U_n\}$ converges to Ψ uniformly on $[a, b]$:

$$\lim_{n\to\infty} \sup_{x\in[a,b]} \big|U_n(x) - \Psi(x)\big| \;=\; 0. \tag{3.9}$$

Proof sketch. Energy monotonicity and nonnegativity imply $\{\mathcal{J}[U_n]\}$ is bounded, so $\{U_n''\}$ is bounded in $L^2(a, b)$. In one dimension, cubic–spline trial spaces are stable and yield uniform bounds on U_n and U_n'. Hence $\{U_n\}$ is equicontinuous and uniformly bounded on $[a, b]$. By Arzelà–Ascoli, there exists a uniformly convergent subsequence. Any uniform limit U satisfies the weak form on every subinterval, and integrating by parts recovers the strong Euler–Lagrange condition

$$U^{(4)}(x) = 0, \tag{3.10}$$

together with the anchor constraints. Uniqueness of the smooth closure Ψ forces $U = \Psi$, so the full sequence converges uniformly, establishing (3.9). □

This establishes Weierstrass (uniform) convergence of the Galerkin extremals to the unique cubic spline closure dictated by informational minimality.

In particular, any admissible interpolant $f(\psi)$ refined through nested Galerkin spaces converges uniformly to the unique smooth closure Ψ, and the inverse Ψ^{-1} recovers exactly the original anchor data ψ. Thus $f(\psi) \to \Psi$ and $\Psi^{-1} = \psi$.

Corollary 3. *For any admissible interpolant $f(\psi)$ consistent with the event anchors, the Galerkin refinements satisfy*

$$\lim_{n\to\infty} f_n(\psi) = \Psi, \tag{3.11}$$

and Ψ^{-1} restricted to the anchors recovers ψ exactly. Hence $f(\psi) \to \Psi^{-1}$ in the sense of uniform convergence on $[a,b]$.

3.3 Point-wise Agreement of the Galerkin Closure

Let $[a,b]$ be compact and let $\{\mathcal{T}_n\}$ be a sequence of nested partitions with mesh size $h_n \to 0$. For each n, let U_n be the Galerkin extremal of the curvature functional

$$\mathcal{J}[U] \;=\; \int_a^b \bigl(U''(x)\bigr)^2\,dx, \tag{3.12}$$

subject to the anchor constraints induced by the event record on $\mathcal{T}_n$.

Theorem 2 (Uniform convergence of Galerkin extremals)**.** *There exists a unique smooth closure Ψ satisfying*

$$\Psi^{(4)}(x) = 0 \qquad \textit{on each element of every } \mathcal{T}_n, \tag{3.13}$$

and the Galerkin sequence $\{U_n\}$ converges to Ψ uniformly:

$$\lim_{n\to\infty}\ \sup_{x\in[a,b]} \bigl|U_n(x) - \Psi(x)\bigr| \;=\; 0. \tag{3.14}$$

Proof sketch. Energy monotonicity and nonnegativity imply $\{\mathcal{J}[U_n]\}$ is bounded, so $\{U_n''\}$ is bounded in $L^2(a,b)$. One-dimensional spline spaces provide uniform bounds on U_n and U_n', so $\{U_n\}$ is equicontinuous and uniformly bounded. By Arzelà–Ascoli, a uniformly convergent subsequence exists. Any uniform limit must satisfy the weak form on every subinterval, and integrating by parts recovers the strong condition (3.13). Uniqueness of the cubic closure forces the entire sequence to converge uniformly to Ψ. □

We now show that agreement at the event anchors forces agreement everywhere.

Proposition 5 (Point-wise uniqueness)**.** *Let Ψ and Φ be smooth closures of admissible interpolants of the same event record ψ. If*

$$\Psi(x_i) = \Phi(x_i) \qquad \textit{for all event anchors } \{x_i\}, \tag{3.15}$$

then

$$\Psi(x) = \Phi(x) \qquad \textit{for all } x \in [a, b]. \tag{3.16}$$

Proof sketch. On each interval $[x_i, x_{i+1}]$, both Ψ and Φ satisfy (3.13) and are therefore cubic polynomials. A cubic is determined by its value, first derivative, and second derivative at a point. Informational minimality forbids discontinuities of U, U', or U'' at the anchors, so Ψ and Φ match these quantities at every x_i. They must therefore coincide on each interval, giving (3.16). □

Finally, since Ψ is cubic on each interval, all derivatives above third order vanish point-wise.

Corollary 4 (Point-wise agreement of derivatives)**.** *For every anchor x_i and for every integer $k \geq 4$,*

$$\Psi^{(k)}(x_i) = 0, \tag{3.17}$$

and on each interval $[x_i, x_{i+1}]$,

$$\Psi(x) \;=\; \Psi(x_i) + \Psi'(x_i)(x - x_i) + \tfrac{1}{2}\Psi''(x_i)(x - x_i)^2 + \tfrac{1}{6}\Psi'''(x_i)(x - x_i)^3. \tag{3.18}$$

Thus Ψ and its Taylor expansion agree to all orders point-wise at each anchor.

Together, (3.14), (3.16), and (3.18) show that every admissible interpolant converges to the same cubic closure Ψ, and that Ψ^{-1} recovers the original anchor data ψ.

3.3.1 C^2 and Piecewise Analytic Solutions

The uniform convergence in Section 3.3 shows that the Galerkin sequence $\{U_n\}$ has a unique smooth limit Ψ satisfying $\Psi^{(4)}(x) = 0$ on each element of the partition. Since a function with vanishing fourth derivative is a cubic polynomial on that interval, the closure Ψ is given by

$$\Psi(x) = \Psi(x_i) + \Psi'(x_i)(x - x_i) + \tfrac{1}{2}\Psi''(x_i)(x - x_i)^2 + \tfrac{1}{6}\Psi'''(x_i)(x - x_i)^3 \quad (3.19)$$

on every subinterval $[x_i, x_{i+1}]$.

Because each such polynomial is infinitely differentiable on the open interval, Ψ is analytic on (x_i, x_{i+1}). Moreover, cubic continuity at the anchors enforces agreement of value, slope, and curvature across interval boundaries, so Ψ is globally C^2:

$$\Psi \in C^2([a, b]). \quad (3.20)$$

Higher derivatives exist piecewise but need not be continuous across anchors, and for all $k \geq 4$

$$\Psi^{(k)}(x) = 0 \qquad \text{for } x \in (x_i, x_{i+1}). \quad (3.21)$$

Thus the Galerkin limit is a classical C^2 solution of the strong Euler–Lagrange form, and is analytic on each element of the partition. No additional smoothness or structure is present beyond the cubic representation, and every admissible interpolant converges to this same piecewise analytic closure.

3.3.2 Equivalence of Discrete and Smooth Representations

Let ψ denote an admissible discrete record supported on event anchors $\{x_i\}$, and let $f(\psi)$ be any admissible interpolant that introduces no new distinguishable structure between anchors. Refining the interpolant over nested partitions $\{\mathcal{T}_n\}$ produces a Galerkin sequence $\{U_n\}$. By Theorem 2,

$$U_n \longrightarrow \Psi \quad \text{uniformly on } [a, b], \tag{3.22}$$

where Ψ satisfies the classical closure condition

$$\Psi^{(4)}(x) = 0 \quad \text{on each element of every } \mathcal{T}_n. \tag{3.23}$$

By Proposition 5, any two smooth closures that agree at the anchors coincide point-wise on $[a, b]$. Thus the limit Ψ is uniquely determined by the anchor data of ψ:

$$\Psi(x_i) = \psi(x_i) \quad \text{for every anchor } x_i. \tag{3.24}$$

Since Ψ is cubic on each interval, all higher-order derivatives vanish on (x_i, x_{i+1}), and Ψ is analytic there. Cubic continuity across the anchors implies

$$\Psi \in C^2([a, b]). \tag{3.25}$$

Finally, Ψ preserves the anchor ordering and is strictly monotone on each interval. Therefore its inverse exists on $[a, b]$ and, when restricted to the anchor set, satisfies

$$\Psi^{-1}(x_i) = x_i, \qquad \text{so } \Psi^{-1} = \psi \text{ on the event record.} \tag{3.26}$$

Together, (3.22)–(3.26) show that any admissible interpolant $f(\psi)$ refined by Galerkin methods converges to the same C^2, piecewise analytic closure Ψ, and that Ψ^{-1} recovers the original discrete record. In this sense the discrete and smooth representations are informationally equivalent: refinement introduces no additional structure, and the smooth closure contains exactly the information encoded in ψ.

3.3.3 Recovery of the Euler–Lagrange Equation

The previous sections established that any admissible interpolant of the event record, when refined over nested partitions, converges uniformly to a unique C^2 closure Ψ. On each element of the partition, this closure satisfies

$$\Psi^{(4)}(x) = 0, \tag{3.27}$$

and is therefore cubic between anchors.

The corresponding weak extremality condition,

$$\int \Psi''(x)\,\phi''(x)\,dx = 0 \qquad \text{for all admissible } \phi, \tag{3.28}$$

was obtained directly from refinements of the discrete event record. Integrating (3.28) by parts twice eliminates boundary contributions at the anchors and yields the strong Euler–Lagrange form

$$\frac{d^2}{dx^2}\big(2\Psi''(x)\big) = 0 \quad \Leftrightarrow \quad \Psi^{(4)}(x) = 0. \tag{3.29}$$

No differentiability was assumed a priori; smoothness appears only after the Galerkin limit exists and is unique. Thus, from a discrete record and the requirement that refinements introduce no new distinguishable structure, we can safely infer the Euler–Lagrange equation. The strong differential form follows as a consequence of measurement and refinement, rather than as an assumed property of the underlying system.

3.4 Inference Without Assumption

The analysis above does not derive the Euler–Lagrange equation from the axioms of the discrete calculus of measurement. Rather, it shows that the Euler–Lagrange form can be inferred without violating those axioms. From a finite event record and admissible refinements, the Galerkin sequence converges uniformly to a unique C^2 closure Ψ that satisfies

$$\Psi^{(4)}(x) = 0 \qquad \text{on each element of the partition,} \tag{3.30}$$

and therefore admits the classical weak relation

$$\int \Psi''(x)\, \phi''(x)\, dx = 0 \qquad \text{for all admissible } \phi. \tag{3.31}$$

Integrating (3.31) by parts twice produces the strong Euler–Lagrange form

$$\frac{d^2}{dx^2}\big(2\Psi''(x)\big) = 0, \tag{3.32}$$

with no additional assumptions. Smoothness and differentiability arise only after the Galerkin limit exists and is unique; neither is introduced as an axiom of the theory.

Thus, the Euler–Lagrange equation is not postulated, but can be recovered consistently from the refinement of discrete measurements. The continuum is compatible with the axioms: if one wishes to represent the discrete record by a smooth function, the resulting closure obeys the classical variational condition. The framework allows the Euler–Lagrange equation to be inferred from measurement, while remaining agnostic about any underlying differential structure.

3.5 The Free Parameter of the Cubic Spline

A cubic spline appears, at first glance, to introduce three independent degrees of freedom at each point: the value of the function, its slope, and its curvature. In classical analysis these are treated as arbitrary boundary data. In the present framework they have a different meaning: each arises as a finite record of measurement.

Let $u(x)$ be the smooth limit of a countable refinement of finite measurements. The discrete extremality principle established earlier shows that any interpolant with unrecorded curvature violates informational minimality. In the dense limit this forces

$$u^{(4)}(x) = 0. \tag{3.33}$$

Every admissible smooth function is therefore locally a cubic polynomial. A general solution to (2.?) takes the form

$$u(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3. \tag{3.34}$$

The coefficients a_0, a_1, a_2, a_3 are determined by three local parameters: the value u, the slope u', and the curvature u''. The derivative of curvature,

$$u'''(x) = 6a_3, \tag{3.35}$$

is piecewise constant. This u''' is the only quantity that may vary from interval to interval without introducing fourth–order structure.

Locally this suggests three degrees of freedom. Globally this is not the case. Because adjacent spline pieces must meet with no unrecorded structure, they agree in value, slope, and curvature at every shared boundary. All but one parameter are fixed by the causal structure of measurement itself. The apparent local freedom collapses to a single global scale factor: the magnitude of the consistent curvature field.

Thus the cubic spline does not introduce three physical constants; it produces a single degree of freedom that expresses how strongly the record of measurement bends. In this framework the continuum contains only one free parameter: the global curvature scale that allows a countable record of distinctions to be represented as a smooth function with no unrecorded features. All subsequent quantities—second derivatives, wave speeds, stress tensors, and curvature—are determined by this single parameter.

3.6 Coda: Navier–Stokes as a Finite Third Parameter

We do not derive the Navier–Stokes equations. Rather, we show how the measurement calculus constrains any smooth limit of finite records to a cubic-spline structure and thereby recasts the regularity question as the finiteness of a single quantity: the third parameter of the spline.

1. Statement of the classical problem

Let $v(x,t)$ be a velocity field and $p(x,t)$ a pressure satisfying the incompressible Navier–Stokes system on $\mathbb{R}^3$ (or a smooth domain with suitable boundary conditions):

$$\partial_t v + (v \cdot \nabla)v + \nabla p = \nu \Delta v + f, \qquad \nabla \cdot v = 0, \tag{3.36}$$

with smooth initial data v_0. The Millennium Problem asks whether smooth solutions remain smooth for all time or may develop singularities in finite time.

2. Measurement-to-spline reduction

Chapter 2 established that admissible smooth limits of finite records obey a local cubic constraint. Along any coordinate line (and likewise along any admissible selection chain) each component admits a representation whose fourth derivative vanishes in the limit:

$$U^{(4)} = 0 \quad \text{(componentwise along admissible lines).} \tag{3.37}$$

Hence the only freely varying local quantity is the *third parameter* (the derivative of curvature). In one dimension this is U'''. In three dimensions we package the idea as the third spatial derivatives of v:

$$\Theta(x,t) := \nabla(\nabla^2 v)(x,t) \quad \text{(a third-derivative tensor).} \tag{3.38}$$

Informally: v, ∇v, and $\nabla^2 v$ are glued continuously by the spline closure; only Θ may vary piecewise without introducing fourth-order structure.

3. Regularity as finiteness of the third parameter

> *Principle.* If the third parameter Θ stays finite at all scales allowed by measurement, the smooth spline limit persists and no singularity can occur within the calculus of measurement.

A practical surrogate is a scale-invariant boundedness criterion on Θ (or a closely related norm tied to enstrophy growth):

$$\sup_{0 \le t \le T} \|\Theta(\cdot,t)\|_X < \infty \quad \Longrightarrow \quad \text{no blow-up on } [0,T], \tag{3.39}$$

where X is chosen to control the admissible refinements (e.g. an L^∞-type or Besov/Hölder proxy along selection chains). In words: the only obstruction to global smoothness is unbounded third-parameter amplitude.

4. Heuristic link to classical controls

Energy and enstrophy inequalities control $\|v\|_{L^2}$ and $\|\nabla v\|_{L^2}$. Vorticity $\omega = \nabla \times v$ monitors the first derivative. Growth of $\nabla\omega$ involves $\nabla^2 v$; the *onset* of non-smoothness is therefore detected by $\Theta = \nabla(\nabla^2 v)$, the next rung. Thus the finite-third-parameter condition (3.39) plays the same role in this framework that classical blow-up criteria play in PDE analyses: it is the minimal spline-compatible guardrail against curvature concentration.

5. Non-classical dependency is not invoked

No dependency (cause-effect) is asserted. The argument is purely informational: as long as the admissible record does not force the third parameter to diverge, the cubic-spline closure remains valid and the smooth limit inferred earlier continues to apply.

6. The rephrased question

> **Navier–Stokes, reframed.** Given smooth initial data and forcing, must the third parameter Θ in (3.38) remain finite for all time under (3.36)? Equivalently, can measurement-consistent refinement generate unbounded third-parameter amplitude in finite time?

If Θ stays finite, the spline structure persists, and the calculus of measurement supports global smoothness. If Θ diverges, the smooth continuum description ceases to be representable as a limit of admissible records, and the measurement calculus no longer licenses Euler–Lagrange inference on that interval.

7. What we have and have not done

We have not solved the Millennium Problem. We have shown that within this program the obstruction to smoothness is concentrated in a single quantity, the third parameter of the cubic spline representation. The classical regularity question is thus equivalent, in this calculus, to the finiteness of Θ.

Chapter 4

The Kinematics of Matter

4.1 Introduction: Martin's Condition and the Continuity of Causal Propagation

The closure of measurement in Part I established that every admissible calculus arises from a finite sequence of distinguishable events whose reciprocal variations cancel beyond fourth order. The resulting smooth field $U(x)$ represents not an assumption of continuity, but the unique extension that preserves causal consistency under the *Axiom of Event Selection*. Yet the closure of a finite causal chain does not by itself guarantee that distinct observers infer compatible fields. For global coherence, the local cancellations enforced by reciprocal measurement must propagate consistently across the entire causal network. This propagation is the content of *Martin's Condition*.

Definition 21 (Martin's Condition (Conceptual))**.** *A causal network satisfies* Martin's Condition *if every locally finite subset of events can be extended to a globally consistent ordering without introducing new distinguishabilities. Equivalently, all finite causal updates admit an extension that preserves the same coincidence relations on their overlaps.*

Intuitively, Martin's Condition demands that information created in one region does not contradict information measured in another. It forbids "causal overcounting"—the duplication of distinctions that would destroy reversibility—by ensuring that overlapping observers reconstruct identical splines of the universe tensor along their shared boundary. Where the Axiom of Event Selection limits what may happen within a light cone, Martin's Condition governs how those choices propagate outward. It is the global compatibility rule of the causal calculus: the guarantee that local smoothness stitches together into a single, coherent wave.

The next sections show that when Martin's Condition holds, the discrete reciprocity law induces a linear propagation operator whose eigenmodes are complex exponentials. The continuum limit of this operator is the familiar wave equation, and the resulting field inherits a canonical stress tensor. Thus the same closure that produced calculus in Part I now produces the continuous propagation of energy and information—the universal phenomenon we recognize as a *wave*.

Example: Davisson–Germer and the Universality of Causal Waves

Statement. Electron diffraction from a crystal demonstrates that discrete particles obey the same reciprocity-driven propagation law as classical waves.

Key relation. Bragg condition with de Broglie wavelength:

$$2d \sin\theta \;=\; m\,\lambda, \qquad \lambda \;=\; \frac{h}{p}.$$

Reciprocity framing. The partition is refined only at the detection screen; between source and screen, causal updates are translation-invariant, so the discrete Laplacian eigenmodes are waves. Matching of distinguished event counts along crystal planes yields constructive interference at Bragg angles.

Operational consequence. Observed intensity peaks are fixed points of reciprocal measurement under lattice translations, evidencing that "matter" and "wave" are the same consistency condition in two representations.

4.2 Interaction: The Union of Ordered Events

In a finite causal domain, an observer's description of the world is a locally ordered set of distinguishable events. When two such domains overlap, the question of *interaction* arises: how are their separate orderings reconciled into a single consistent history? Martin's Condition guarantees that locally finite orders can be extended without contradiction. Interaction is the constructive realization of that extension.

Definition 22 (Interaction of Causal Sets)**.** *Let* $(E_1, \preceq_1)$ *and* $(E_2, \preceq_2)$ *be locally finite posets of events, each satisfying Martin's Condition on its own domain. Their* interaction *is the smallest poset*

$$(E_{12}, \preceq_{12}), \qquad E_{12} = E_1 \cup E_2,$$

whose order $\preceq_{12}$ *is the transitive closure of* $\preceq_1 \cup \preceq_2$ *restricted by the requirement that all overlaps* $E_1 \cap E_2$ *remain consistent:*

$$\forall\, e, f \in E_1 \cap E_2,\ e \preceq_1 f \ \Leftrightarrow\ e \preceq_2 f.$$

Thought Experiment 4.2.1 (Non-commuting measurements as event selection)**.** *Let the event tensor act on a qubit. Take Pauli operators* σ_x, σ_z *and projective predicates* $P_x^{\pm} = \frac{1}{2}(I \pm \sigma_x), \quad P_z^{\pm} = \frac{1}{2}(I \pm \sigma_z)$*. The selection "measure* x *then* z*" corresponds to the ordered update* $U_{xz}(\rho) = \sum_{a,b\in\{\pm\}} P_z^b P_x^a\, \rho\, P_x^a P_z^b$*, while "measure* z *then* x*" is* $U_{zx}(\rho) = \sum_{a,b} P_x^a P_z^b\, \rho\, P_z^b P_x^a$*. Because* $[\sigma_x, \sigma_z] \neq 0$*, one has* $U_{xz} \neq U_{zx}$ *in general; order changes the post-measurement state and the subsequent event statistics. Yet the scalar count of admissible outcomes over the full branch set is preserved (the total probability* $= 1$*), reflecting that the measurable bookkeeping across the causal domain remains consistent even when the micro-updates do not commute. This realizes Event Selection as a non-commutative refinement of the distinguishability chain.*

The overlap $E_1 \cap E_2$ represents events recognized by both observers. For the union to remain causally consistent, these shared events must inherit identical ordering relations from both domains. If such an identification cannot be made, the systems are incompatible and cannot interact without violating Martin's Condition.

Definition 23 (Interaction Event)**.** *An event $e \in E_1 \cap E_2$ is called an* interaction event *if it is maximal in one order and minimal in the other:*

$$e \in \mathrm{Top}(E_1) \cap \mathrm{Min}(E_2) \quad or \quad e \in \mathrm{Top}(E_2) \cap \mathrm{Min}(E_1).$$

Such an event terminates one causal chain and initiates another.

Intuitively, an interaction occurs when the future boundary of one local ordering meets the past boundary of another. At that instant, two independent descriptions of the world become linked by a single shared distinction. The joint order $\preceq_{12}$ thus acts as a stitching rule: it preserves every prior ordering within E_1 and E_2 while extending them just enough to include the new comparabilities implied by the overlap.

Proposition 6 (Union Consistency)**.** *If $(E_1, \preceq_1)$ and $(E_2, \preceq_2)$ satisfy Martin's Condition and agree on all relations within $E_1 \cap E_2$, then their union $(E_{12}, \preceq_{12})$ also satisfies Martin's Condition.*

Idea of Proof. Each finite subset $S \subseteq E_{12}$ lies within finitely many overlapping domains E_i that already satisfy Martin's Condition. Since the overlaps agree on order, the union of their consistent extensions remains consistent. Thus every finite subset of E_{12} extends without introducing new distinguishabilities. □

Interpretation. Interaction is therefore not a separate dynamical law but the combinatorial closure of causal order under union. Whenever two chains intersect, their local orderings adjust to maintain global compatibility. The mutual adjustment propagates along both chains, enforcing consistency across their neighborhoods. Viewed iteratively, this propagation behaves as a *wave of ordering*: a disturbance that travels through the poset whenever new overlaps are formed. It is this propagation—the transmission of order constraints through successive interactions—that gives rise to the phenomenon we recognize as wave motion.

4.2.1 Spooky Action as a Dantzig Pivot

Thought Experiment 4.2.2 (Mach–Zehnder Interferometer as Causal Superposition)**.** *Consider a photon entering a Mach–Zehnder interferometer. At the first beam splitter, a single causal event E_0 bifurcates into two distinguishable yet coherent branches, E_1 and E_2, corresponding to the upper and lower optical paths. Each path accumulates its own sequence of distinctions—reflections, phase shifts, and delays—represented by ordered event tensors $\{E_{1,k}\}$ and $\{E_{2,k}\}$.*

The partial order of the experiment is not a binary decision tree but a superposition of two compatible causal chains that re-converge at the second beam splitter. The final detection event E_f therefore depends on the interference of two histories that remain Martin-consistent: their local ordering within each path is preserved, and their global coincidence at E_f is enforced by the Reciprocity Law.

Operationally, the interferometer measures the overlap of distinguishability between the two causal sequences. When their accumulated phase difference $\Delta\phi$ equals an integer multiple of 2π, the two histories are indistinguishable and the universe tensor records them as a single causal extension; when $\Delta\phi = \pi$, the histories cancel, producing a node of zero probability. Thus interference arises as the algebraic sum of two order-preserving histories whose tensor contributions differ only by a phase in the informational metric.

In this framing, the Mach–Zehnder interferometer is the simplest laboratory realization of causal superposition: two distinguishable sequences whose difference is purely informational, revealing that interference is not a mystery *of waves but a bookkeeping property of order under the Reciprocity Law.*

Consider an entanglement $S = \{E_i, E_j\}$ of two spatially separated measurement events. By definition, the order of E_i and E_j may be permuted without changing any invariant scalar of the universe tensor:

$$E_i + E_j = E_j + E_i. \tag{15}$$

Each pair of entangled events therefore constitutes a *degenerate basis* of the global causal structure: multiple local orderings are consistent with the same global invariants.

Degeneracy and Feasibility. Let $\mathcal{F}$ denote the space of all feasible causal orderings that satisfy Martin's Condition. Every element of $\mathcal{F}$ is a physically admissible extension of the partial order of events. When two or more orderings yield the same invariants, the corresponding configurations form a *degenerate face* of $\mathcal{F}$—analogous to a flat ridge in a linear-programming polytope where the objective is constant. Entanglement is precisely this degeneracy: several globally consistent orderings are equally admissible.

Selection as Pivot. When an observer records one member of an entangled pair, say E_i, the universe must select a unique consistent global ordering. This selection is equivalent to a *pivot operation* in the sense of Dantzig: a transition from one feasible vertex of $\mathcal{F}$ to another that preserves all constraints while choosing a particular basis. The pivot enforces consistency across the entire system, mapping the previous degenerate face to a single vertex. The resulting update

$$U_{n+1} = \Phi_{\text{sel}}(U_n)$$

is the causal analog of Dantzig's step toward optimality: a global reorganization that leaves all invariants unchanged but redefines which variables are active.

Nonlocal Consistency. Because the feasibility region $\mathcal{F}$ is global, the pivot cannot be localized. When E_i is measured, the reordering that selects its consistent partner E_j occurs simultaneously across the entire causal domain. To a local observer, this appears as instantaneous correlation—"spooky action at a distance"—but within the formalism it is simply the global enforcement of Martin's Condition: every pivot must preserve feasibility everywhere. No signal propagates; the basis of consistency merely updates as a whole.

Interpretation. Spooky action is therefore not a mysterious nonlocal force but the *global pivot of consistency* required to maintain a single feasible ordering of the universe tensor. Measurement corresponds to a Dantzig selection rule acting on the degenerate faces of the causal polytope, and collapse is the logical consequence of resolving entanglement into one of its admissible vertices. The Einstein–Podolsky–Rosen paradox thus reduces to a combinatorial theorem:

$$\text{Nonlocal correlation} \;=\; \text{Global preservation of feasibility.}$$

Example: Bell–Aspect Tests as Global Martin Consistency

Statement. Violations of Bell inequalities show that global filters (consistent extensions) exist that cannot be decomposed into local hidden refinements without contradiction—exactly the Martin-style global consistency you invoke.

Key relation (CHSH).

$$S \;=\; |E(a,b)+E(a,b')+E(a',b)-E(a',b')| \;\le\; 2 \quad \text{(local)} \quad \text{vs} \quad S_{\mathrm{QM}} \le 2\sqrt{2}.$$

Reciprocity framing. A single global entanglement class S allows reassociation (permutation) before refinement. Local pre-assignments would violate dense-set meeting across settings; the observed $S > 2$ indicates that only a *global* selection (filter meeting all dense constraints) is admissible.

Operational consequence. "Nonlocality" is reinterpreted as *global order-preserving selection*: the event filter meets all dense subsets (settings) without a jointly measurable pre-partition.

4.2.2 The Qubit as an Example of Event Selection

The Axiom of Event Selection ensures that, from any countable family of potential events, a consistent subset is chosen so that the causal order remains distinguishable and globally coherent. When two events E_0 and E_1 are equally admissible until such a selection is made, the pair forms the minimal unit of causal degeneracy.

Definition 24 (Qubit as a Causal Doublet)**.** *Let $S = \{E_0, E_1\}$ be an entangled subset of events satisfying $E_0 + E_1 = E_1 + E_0$. Prior to selection, S occupies both feasible orderings and therefore represents a superposed causal state. Applying the selection operator Φ_{sel} resolves this degeneracy:*

$$\Phi_{\text{sel}}(S) = E_b, \qquad b \in \{0, 1\}.$$

In the continuum limit this relation corresponds to the quantum state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

where the coefficients (α, β) encode the relative weights of the feasible orderings prior to event selection.

Thus a *qubit* is the simplest instance of the Event Selection process—the minimal pair of distinguishable yet unselected events. Measurement, represented by the Dantzig pivot Φ_{sel}, corresponds to choosing one consistent ordering within this causal doublet, mirroring the projection of a quantum superposition onto a definite basis state.

4.2.3 Hawking Radiation as the Loss and Restoration of Order

In a locally finite causal network, every interaction extends the partial order by introducing new comparabilities while maintaining Martin's Condition. When a causal boundary forms—such as the surface of a black hole—those extensions begin to saturate. The number of possible unions of light cones increases faster than any observer can resolve them, and the rate at which new distinctions can be recorded begins to fall. What we call a *horizon* is the surface beyond which the reconciliation of causal updates exceeds the observer's computational capacity to process them.

Definition 25 (Causal Horizon)**.** *Let* $(E, \preceq)$ *be a locally finite poset. A subset* $H \subset E$ *is a* causal horizon *for an observer if there exist events* $e, f \in E$ *such that* $e \preceq h$ *for some* $h \in H$ *but* $f \not\preceq h$ *for any* $h \in H$*, and no finite extension of the observer's order can include both* e *and* f*. The horizon marks the maximal boundary of extendable distinguishability.*

When an infalling system approaches this boundary, its local causal cones continue to expand, but the external observer's ability to register those expansions diminishes. The total number of events on the infaller's worldline grows rapidly, while the observer's *distinct* reception count grows only logarithmically. The event stream becomes oversaturated: too many correlations are forming for the exterior network to maintain Martin consistency in real time.

Observer–Side Perception of Order. To the distant observer, this saturation appears as an ever–increasing delay between successive confirmations of the infalling particle's state. Each emitted distinction must traverse an ever–widening intersection of causal cones before it can be reconciled with the external order. Because the unions $E_{\text{obs}} \cup E_{\text{infall}}(t)$ grow super–linearly in size as the infaller approaches the horizon, the cost of maintaining order consistency rises faster than the causal network can propagate it.

Let $N_{\text{ext}}(t)$ be the number of distinguishable updates received before coordinate time t. If $|E(t)|$ denotes the size of the causal union at that instant, then

$$\frac{dN_{\text{ext}}}{dt} \propto \frac{1}{|E(t)|} \frac{d|E(t)|}{dt}.$$

Near the horizon, $|E(t)| \sim (1 - r_s/r)^{-1}$, so as $r \to r_s$,

$$\lim_{t\to\infty} \frac{dN_{\text{ext}}}{dt} = 0.$$

The apparent "freezing" of the particle in time is therefore not an illusion of geometry but a property of information flow: the observer's frame can no longer complete the reconciliation of causal updates as the interior domain's informational density diverges.

What looks like a halted particle is, in fact, an observer encountering their own bandwidth limit. The infalling particle continues to receive and process distinctions—it experiences no slowdown—but the exterior network cannot integrate those updates into its own ordering. The visible universe tensor stalls because its synchronization surface has reached capacity.

The lag, then, is the *signature of finite computation*: the universe enforcing the Axiom of Order by denying further updates until all active causal cones can be reconciled. The redshifted, time–dilated glow of an infalling body is the visible trace of the bookkeeping failure—the external frame's attempt to digest an accelerating flood of internal distinctions.

Definition 26 (Order Collapse and Restoration)**.** *Given a Martin bridge $R \subset E_{\text{in}} \times E_{\text{out}}$, its* collapse *occurs when all $e_{\text{in}} \in E_{\text{in}}$ become causally unreachable. The induced order on E_{out} is restored by introducing surrogate events E_{rad} and relations $R' \subset E_{\text{out}} \times E_{\text{rad}}$ such that $(E_{\text{out}} \cup E_{\text{rad}}, \preceq')$ again* *satisfies Martin's Condition.*

Each surrogate event represents the reconciliation of an unresolvable causal update—a compensatory distinction emitted to preserve order on the accessible side. The ensemble of such replacements manifests statistically as a thermal spectrum.

Proposition 7 (Hawking Radiation as Order Completion)**.** *The apparent radiation observed at the horizon corresponds to the distribution of surrogate events* E_{rad} *required to restore Martin consistency after the collapse of a causal bridge. The exponential spectrum arises from the combinatorial multiplicity of admissible completions once the observer's information rate saturates.*

Relation to Holographic Consistency. The informational asymptote described above is the discrete analogue of the holographic correspondence between bulk and boundary theories. The interior causal domain E_{in} plays the role of the bulk, its rapidly expanding unions of light cones encoding the fine–grained local order. The exterior domain E_{out}, bounded by the horizon, functions as the boundary field theory whose finite causal capacity reconstructs that interior. The Martin bridge $R \subset E_{\text{in}} \times E_{\text{out}}$ acts as the holographic map: a discrete correspondence ensuring that every admissible bulk update has a representable boundary image. When the bridge collapses, the boundary compensates by emitting the surrogate events E_{rad}, analogous to boundary degrees of freedom restoring consistency in the AdS/CFT duality. The lag perceived near the horizon is therefore the operational form of holography—the boundary's failure to process the accelerating influx of bulk distinctions in real time, enforcing the holographic consistency condition that global order remain representable on the causal surface.

4.3 Wave Amplitude from Interaction Counts

Interaction between two locally finite causal domains $(E_1, \preceq_1)$ and $(E_2, \preceq_2)$ creates new distinguishabilities while identifying shared ones. We define the *wave amplitude* as the net number of new, non-overlapping events produced by the union, i.e. the cardinality of the set difference between union and intersection.

Definition 27 (Amplitude of Interaction). *Let $E_{12} = E_1 \cup E_2$ be the union poset obtained under Martin's Condition, with overlap $E_{1\cap 2} = E_1 \cap E_2$ order-consistent. The* amplitude *of the interaction is*

$$\mathcal{A}(E_1, E_2) := \big|(E_1 \cup E_2) \setminus (E_1 \cap E_2)\big| = |E_1| + |E_2| - 2|E_1 \cap E_2|.$$

Equivalently, $\mathcal{A}(E_1, E_2) = |E_1 \triangle E_2|$ is the size of the symmetric difference.

Interpretation. $\mathcal{A}(E_1, E_2)$ counts exactly the distinguishabilities that are *new to the union*: it removes anything already shared (the intersection) and keeps only the net additions. Viewed dynamically, this is the discrete "wave height" of order propagated when two domains interact.

Basic Properties

Proposition 8 (Symmetry and Nonnegativity). *For any locally finite E_1, E_2,*

$$\mathcal{A}(E_1, E_2) = \mathcal{A}(E_2, E_1) \geq 0, \qquad \mathcal{A}(E_1, E_2) = 0 \iff E_1 = E_2.$$

Proof sketch. Symmetry follows from the symmetry of union, intersection, and cardinality. Nonnegativity is immediate from the definition as a set cardinality. If $E_1 = E_2$, the symmetric difference is empty, hence amplitude 0. Conversely, if the symmetric difference is empty, the sets coincide. $\square$

Proposition 9 (Upper and Lower Bounds)**.**

$$\big||E_1| - |E_2|\big| \ \leq\ \mathcal{A}(E_1, E_2)\ \leq\ |E_1| + |E_2|.$$

Proof sketch. Use $|E_1 \cap E_2| \leq \min\{|E_1|, |E_2|\}$ and $\mathcal{A} = |E_1| + |E_2| - 2|E_1 \cap E_2|$ for the upper bound. For the lower bound, observe $|E_1 \cap E_2| \geq \max\{0,\ |E_1| + |E_2| - |E_1 \cup E_2|\}$ and $|E_1 \cup E_2| \leq |E_1| + |E_2|$. □

Proposition 10 (Additivity on Disjoint Domains)**.** *If $E_1 \cap E_2 = \varnothing$, then*

$$\mathcal{A}(E_1, E_2) = |E_1| + |E_2|.$$

Proof sketch. With empty intersection, $(E_1 \cup E_2) \setminus (E_1 \cap E_2) = E_1 \cup E_2$, so the amplitude is the size of the disjoint union. □

Proposition 11 (Triangle-Type Inequality)**.** *For any locally finite E_1, E_2, E_3,*

$$\mathcal{A}(E_1, E_3)\ \leq\ \mathcal{A}(E_1, E_2) + \mathcal{A}(E_2, E_3).$$

Proof sketch. $\mathcal{A}$ is the cardinality of the symmetric difference, which is the Hamming distance on indicator functions of subsets. The triangle inequality for Hamming distance yields the claim. □

Order-Sensitive Refinement

The amplitude defined above counts events. We now relate it to the number of *new comparabilities* created by the interaction.

Definition 28 (Frontiers and New Comparabilities)**.** *For a poset $(E, \preceq)$, write* $\mathrm{Top}(E)$ *for maximal elements and* $\mathrm{Min}(E)$ *for minimal elements. Given $(E_1, \preceq_1)$ and $(E_2, \preceq_2)$ with order-consistent overlap and union order $\preceq_{12}$, define*

$$\Delta_{\prec}(E_1, E_2) := \#\big\{(e, f) \in (E_1 \setminus E_2) \times (E_2 \setminus E_1)\ :\ e \prec_{12} f\ \textit{ or }\ f \prec_{12} e\big\}.$$

This counts the newly created *comparabilities across the interface.*

Proposition 12 (Amplitude Bounds New Comparabilities)**.**

$$\Delta_{\prec}(E_1, E_2) \;\leq\; \mathcal{A}(E_1, E_2) \cdot \min\{|E_1 \setminus E_2|,\, |E_2 \setminus E_1|\}.$$

Moreover, if the interface is "thin" (only frontier elements interact), then

$$\Delta_{\prec}(E_1, E_2) \;\asymp\; |\mathrm{Top}(E_1) \cap (E_1 \setminus E_2)| \cdot |\mathrm{Min}(E_2) \cap (E_2 \setminus E_1)|$$

up to a factor determined by Martin-consistent tie-breaking.

Proof sketch. Each new comparability pairs one element from the left difference with one from the right difference. There are at most $|E_1 \setminus E_2| \cdot |E_2 \setminus E_1|$ such pairs; the first bound follows by noting $\mathcal{A} = |E_1 \setminus E_2| + |E_2 \setminus E_1|$ and optimizing the product under fixed sum (achieved when the smaller side limits pairings). For thin interfaces, Martin's Condition forces new order primarily between opposing frontier elements, giving the asymptotic relation. □

Superposition over Multiple Domains

Proposition 13 (First-Order Superposition)**.** *For three domains E_1, E_2, E_3 with small triple-overlap,*

$$\big|\, \mathcal{A}(E_1 \cup E_2, E_3) - \big(\mathcal{A}(E_1, E_3) + \mathcal{A}(E_2, E_3)\big) \,\big| \;\leq\; 2\,|E_1 \cap E_2 \cap E_3|.$$

Proof sketch. Use inclusion–exclusion on unions and intersections to expand both sides and cancel terms. All discrepancies arise from triple-overlap terms, each contributing at most 2 in absolute value to the symmetric-difference counts. □

Operational Meaning

The count

$$\mathcal{A}(E_1, E_2) = |E_1 \triangle E_2|$$

is the minimal number of event insertions/deletions needed to transform one local history into the other while preserving the common core. Under Martin's Condition, this is precisely the amount of order that must *propagate* across the interface to maintain global consistency. The resulting propagation—tracked by newly created comparabilities—is the discrete wave generated by the interaction.

4.4 First Variation of Amplitude

The amplitude $\mathcal{A}(E_1, E_2)$ measures the net number of new distinctions created by the interaction of two causal domains. The *first variation* describes how that amplitude changes when either domain gains or loses a single event. This variation quantifies the local sensitivity of the wave of order.

Definition 29 (Infinitesimal Variation of an Event Set)**.** *Let* $(E, \preceq)$ *be a locally finite poset. An* elementary variation δE *is the addition or removal of a single event* e *together with its admissible relations that preserve Martin's Condition:*

$$E' = E \cup \{e\} \quad \textit{or} \quad E' = E \setminus \{e\}, \qquad (E', \preceq') \textit{ satisfies Martin's Condition.}$$

Definition 30 (First Variation of Amplitude)**.** *Given two interacting domains* E_1, E_2 *and a small perturbation* $E_1' = E_1 \cup \delta E_1$ *or* $E_2' = E_2 \cup \delta E_2$*, the first variation of the amplitude is*

$$\delta\mathcal{A} = \mathcal{A}(E_1', E_2) - \mathcal{A}(E_1, E_2) \quad \textit{or} \quad \delta\mathcal{A} = \mathcal{A}(E_1, E_2') - \mathcal{A}(E_1, E_2).$$

Expanding from the definition,

$$\delta\mathcal{A} = \big|(E_1 \cup \delta E_1) \triangle E_2\big| - |E_1 \triangle E_2|.$$

Proposition 14 (Local Variation Formula)**.** *If $\delta E_1 = \{e\}$ adds a single event e not in E_2, then*

$$\delta\mathcal{A} = \begin{cases} +1, & e \notin E_1 \cup E_2, \\ -1, & e \in E_2 \setminus E_1, \\ 0, & e \in E_1 \cap E_2. \end{cases}$$

Proof sketch. Each event contributes ± 1 to the symmetric difference depending on whether it creates or resolves a unique distinction. If e is entirely new, the amplitude increases by one. If e duplicates an event already present in E_2, the overlap grows and the amplitude decreases by one. If e already exists in both, no new distinguishability is created. □

Proposition 15 (First Variation as Discrete Derivative)**.** *Let $\mathcal{A}$ be viewed as a function on the lattice of finite subsets of a fixed event universe Ω. Then the mapping*

$$\delta_e\mathcal{A}(E_1, E_2) := \mathcal{A}(E_1 \cup \{e\}, E_2) - \mathcal{A}(E_1, E_2)$$

is the discrete directional derivative of $\mathcal{A}$ along e. It satisfies the antisymmetry relation

$$\delta_e\mathcal{A}(E_1, E_2) = -\,\delta_e\mathcal{A}(E_2, E_1).$$

Proof sketch. Direct expansion using $\mathcal{A} = |E_1| + |E_2| - 2|E_1 \cap E_2|$ shows that the increment in E_1 produces the negative of the increment in E_2 for the same event. Thus $\mathcal{A}$ behaves as a bilinear antisymmetric functional on the Boolean lattice of finite subsets. □

Interpretation. The first variation counts how the network of distinguishabilities responds to a single local perturbation. Adding an event outside the shared overlap increases the amplitude: a ripple of new order propagates. Adding one already correlated decreases it: a cancellation that smooths the field. Under successive local variations, the amplitude evolves according to the discrete balance between creation and annihilation of distinguishability. This balance is the combinatorial analogue of the differential wave equation; it describes the propagation of causal order itself.

4.5 Second Variation of Amplitude

The first variation measured how the distinguishability between two causal domains changes when a single event is added or removed. The *second variation* captures how those incremental changes themselves interact. It measures the curvature of distinguishability—the discrete analogue of acceleration or wave curvature—arising from the mutual influence of two local perturbations.

Definition 31 (Second Variation)**.** *Let δ_e and δ_f denote first variations with respect to elementary event insertions e and f. The* second variation of amplitude *is defined as the symmetric difference of the corresponding first variations:*

$$\delta^2_{e,f}\mathcal{A}(E_1,E_2) := \delta_f\big(\delta_e\mathcal{A}(E_1,E_2)\big) = \mathcal{A}(E_1\cup\{e,f\},E_2)-\mathcal{A}(E_1\cup\{e\},E_2)-\mathcal{A}(E_1\cup\{f\},E_2)+\mathcal{A}(E_1,E_2$$

This operator measures the change in the local propagation rate caused by introducing two distinct events. When $\delta^2_{e,f}\mathcal{A} = 0$, their effects are independent: the propagation is linear. When it is nonzero, the two variations interfere, producing either reinforcement or cancellation of distinguishability.

Proposition 16 (Symmetry)**.**

$$\delta^2_{e,f}\mathcal{A}(E_1,E_2) = \delta^2_{f,e}\mathcal{A}(E_1,E_2), \qquad \delta^2_{e,e}\mathcal{A}(E_1,E_2) = 0.$$

Proof sketch. Both δ_e and δ_f are finite-difference operators on the Boolean lattice of subsets. They commute, and a repeated variation on the same event cancels, yielding symmetry and self-annihilation. □

Proposition 17 (Explicit Form). *If $e \neq f$ are not contained in E_2, then*

$$\delta^2_{e,f}\mathcal{A}(E_1, E_2) = \begin{cases} -2, & e, f \in E_2 \setminus E_1, \\ +2, & e, f \notin E_1 \cup E_2, \\ 0, & \textit{otherwise.} \end{cases}$$

Proof sketch. Expand the four amplitude terms in the definition using $\mathcal{A} = |E_1| + |E_2| - 2|E_1 \cap E_2|$ and compute the finite difference. Each event contributes ± 1 to the first variation depending on overlap. The second variation doubles that effect when both new events share the same inclusion status relative to E_2, and cancels when they differ. □

Definition 32 (Discrete Laplacian on Event Sets). *Let $\nabla^2_E \mathcal{A}$ denote the sum of all pairwise second variations over neighboring events in a locally finite causal domain:*

$$\nabla^2_E \mathcal{A}(E_1, E_2) := \sum_{\substack{e,f \in E_1 \\ e \prec f \ \text{or} \ f \prec e}} \delta^2_{e,f}\mathcal{A}(E_1, E_2).$$

Proposition 18 (Wave Equation for Order). *Under Martin's Condition, the amplitude on any locally finite causal domain satisfies*

$$\nabla^2_E \mathcal{A} = 0$$

as the condition for global consistency.

Proof sketch. Each pairwise second variation measures the net curvature of distinguishability between causally related events. Martin's Condition enforces that all finite subsets extend consistently, which requires the total curvature over each closed causal neighborhood to vanish. Summing over all connected pairs yields $\nabla_E^2 \mathcal{A} = 0$, the discrete Laplace equation for order propagation. $\square$

Interpretation. The vanishing of the second variation expresses the equilibrium of causal propagation: local expansions and contractions of distinguishability cancel globally. Where the first variation gave the *slope* of causal change, the second variation fixes the *curvature*—the shape of the wave. The condition $\nabla_E^2 \mathcal{A} = 0$ is therefore the causal-set form of the homogeneous wave equation: a statement that information, once created, propagates through the network of events without net amplification or loss.

4.6 Advection as Order-Preserving Transport

The first variation counts how distinguishability propagates when new events are introduced; the second variation vanishes at equilibrium, yielding wave closure. When propagation is *directional*—because Martin bridges select a consistent orientation of overlaps along a chain—the resulting closure is *first–order*: advection.

Setup: a Translation–Invariant Causal Strip

Let $\Lambda = \{(n, i) : n \in \mathbb{Z},\, i \in \mathbb{Z}\}$ index a locally finite event strip with "time" levels n (ordinals of measurement steps) and spatial indices i along a chain of overlaps. Write $E_n = \{(n, i)\}_i$ and suppose overlaps are oriented so that interaction at level n feeds level $n+1$ predominantly from the left neighbor:

$$(n, i-1) \;\to\; (n+1, i).$$

Let $A_i^n \in \mathbb{N}$ denote the *amplitude density* (count of new distinguishabilities) measured on site i at level n.

Definition 33 (Order–Preserving Transport (Upwind Selection))**.** *A Martin–consistent, order–preserving update on* Λ *with orientation to the right is a map* T *such that*

$$A_i^{n+1} \;=\; (1-\lambda)\,A_i^n \;+\; \lambda\,A_{i-1}^n, \qquad 0 \le \lambda \le 1,$$

with λ *the* bridge fraction*: the proportion of next–step distinguishability at* $(n{+}1, i)$ *sourced from the left overlap.*

Interpretation. $\lambda = 1$ gives pure shift $A_i^{n+1} = A_{i-1}^n$ (deterministic transport one site per update). $0 < \lambda < 1$ mixes local retention with left–fed propagation, the discrete analogue of upwind transport. No energies are involved; only the preservation of order across oriented overlaps.

Discrete Continuity and Characteristics

Proposition 19 (Discrete Continuity Law)**.** *For any finite index set* $I \subset \mathbb{Z}$*,*

$$\sum_{i\in I} A_i^{n+1} - \sum_{i\in I} A_i^n = \lambda\big(A_{\min(I)-1}^n - A_{\max(I)}^n\big)\,.$$

Proof sketch. Telescoping sum of the upwind update across I cancels interior fluxes and leaves only boundary contributions, expressing conservation of distinguishability modulo oriented boundary flow. □

Proposition 20 (Order Characteristics)**.** *If* $\lambda = 1$*, then along lines* $i - n =$ *const one has* $A_i^{n+1} = A_{i-1}^n$*, hence* $A_i^n = A_{i-n}^0$*. Thus distinguishability is constant on the discrete characteristics* $i - n = \text{const}$*.*

Proof sketch. Iterate the shift relation n times. □

Continuum Limit: The Advection Equation

Let spatial mesh be $h > 0$ and step size $\Delta t > 0$. Define a smooth interpolant $a(t_n, x_i) = A_i^n$ with $t_n = n\,\Delta t$, $x_i = i\,h$, and take

$$\lambda \;=\; \frac{c\,\Delta t}{h} \quad (0 \le \lambda \le 1),$$

where c is the *order speed* fixed by the oriented Martin bridges.

Theorem 3 (Advection from Upwind Selection)**.** *Assume $a \in C^2$ and the oriented update*

$$A_i^{n+1} = (1-\lambda)A_i^n + \lambda A_{i-1}^n.$$

Then, under the scaling $\lambda = \frac{c\Delta t}{h}$ with fixed c and $\Delta t, h \to 0$ satisfying the Courant condition $0 \le \lambda \le 1$, the interpolant a satisfies

$$\partial_t a + c\,\partial_x a \;=\; 0 \qquad \textit{(advection)}$$

to first order in $(\Delta t, h)$.

Proof sketch. Taylor-expand $a(t+\Delta t, x) = a + \Delta t\, a_t + \mathcal{O}(\Delta t^2)$ and $a(t, x - h) = a - h\, a_x + \mathcal{O}(h^2)$, then substitute in

$$a(t+\Delta t, x) = (1-\lambda)a(t,x) + \lambda\, a(t, x-h).$$

Divide by Δt and use $\lambda = \frac{c\Delta t}{h}$:

$$a_t + \frac{\lambda}{\Delta t}\big(a(t, x-h) - a(t,x)\big) = a_t - \frac{c}{h}\big(h\, a_x + \mathcal{O}(h^2)\big) = a_t + c\, a_x + \mathcal{O}(h, \Delta t).$$

Letting $\Delta t, h \to 0$ yields $\partial_t a + c\,\partial_x a = 0$. $\square$

Order–Theoretic Meaning

Proposition 21 (Advection as Oriented Martin Flow)**.** *The advection equation expresses invariance of distinguishability along order–preserving characteristics* $x - ct = \text{const}$ *induced by a fixed orientation of Martin bridges. Equivalently, for any smooth test function* φ *compactly supported,*

$$\frac{d}{dt}\int a(t,x)\,\varphi(x+ct)\,dx = 0.$$

Proof sketch. Use the weak form of $\partial_t a + c\,\partial_x a = 0$ and integrate by parts along translated test functions; the quantity is conserved because propagation is a pure shift along characteristics. □

Remarks on Stability and Causality

- **CFL as Martin Bound.** $0 \le \lambda \le 1$ is exactly the requirement that next–step order at site i is determined by current order from *within* its causal neighborhood, matching Martin's Condition (no overreach).

- **Asymmetry ⇒ Advection.** When overlaps are unbiased left/right, the second variation dominates and yields the (symmetric) wave operator. A persistent orientation biases first–order closure, giving advection.

- **No Energetics.** All statements concern counts and comparabilities. The "speed" c is the rate at which order constraints traverse the poset—not a kinetic parameter— and is fixed by the density/orientation of Martin bridges per unit step.

Thought Experiment 4.6.1 (Thought Experiment: The Knot-Tying Puzzle and Cubic Spline Closure)**.** ***N.B.*** *This experiment visualizes why* $U^{(4)} = 0$ *is the natural limit of causal smoothness.*

Setup. *Picture threading a shoelace through a lattice of eyelets representing discrete events. Each tie fixes a local order; each pull relates adjacent ties under reciprocity. If the path bends beyond third order, new tension points appear that violate global closure.*

Demonstration. *Tying loops with 1st–3rd-order curvature yields smooth closure; a 4th-order "wiggle" over-constrains the path, leaving residual curvature. Hence the minimal global closure corresponds to cubic continuity:* $U^{(4)} = 0$.

Interpretation. *The lattice stands for the causal tensor. Minimal curvature ensures bijective mapping of distinctions—no "loose ends." The physical limit is the geodesic; the logical limit is consistency under finite observation.*

4.7 Adiabatic Transport

N.B. This section concerns the preservation of distinguishability under slow, order–preserving transformation of causal structure. It defines *adiabatic transport* as informational invariance under continuous refinement, not as a thermodynamic process. No physical energy, temperature, or momentum is invoked [54, 46, 101].

Definition 34 (Adiabatic Transport)**.** *Let* $\mathcal{C} = (E, \preceq)$ *be a causal set equipped with a local update rule* $T_\lambda : E \to E$ *parameterized by a continuous scalar* λ*. We say that* T_λ *defines* adiabatic transport *if, for every finite subdomain* $E_p \subset E$*, there exists* $\varepsilon > 0$ *such that for all* $|\delta\lambda| < \varepsilon$,

$$dist\big(T_{\lambda+\delta\lambda}(E_p),\, T_\lambda(E_p)\big) = 0,$$

where dist measures distinguishability in the logical sense (i.e., the number of predicate values altered). Equivalently, no distinctions are created or de-

stroyed under infinitesimal parameter variation:

$$\frac{d}{d\lambda}\,\Delta S(E_p;\lambda) = 0.$$

Law 1 (Adiabatic Invariance of Information)**.** *For every causal system evolving under order–preserving refinement, the informational entropy $\mathcal{S}(E_p)$ is invariant under adiabatic transport:*

$$\frac{d}{d\lambda}\mathcal{S}(E_p) = 0,$$

provided that $\Delta S \geq 0$ globally. This expresses the condition that reversible evolution preserves the count of distinguishable states while redistributing them across the parameter domain [8, 72].

Proposition 22 (Adiabatic Transport as Minimal Update)**.** *Let U_λ denote the linear update operator acting on the state vector of local predicates. Then U_λ is adiabatic iff*

$$U_{\lambda+\delta\lambda} = U_\lambda + \mathcal{O}(\delta\lambda^2),$$

and U_λ is norm–preserving:

$$\langle U_\lambda x, U_\lambda x\rangle = \langle x, x\rangle,$$

for all x in the local predicate space. Hence adiabatic transport is the first–order limit of informationally reversible evolution—a continuous deformation with zero local entropy production.

Sketch. By definition, $\frac{d}{d\lambda}\,\Delta S(E_p;\lambda) = 0$ for adiabatic flow. Expanding $U_{\lambda+\delta\lambda}$ in $\delta\lambda$ and applying the invariance of the inner product gives

$$\langle x, (U^*_{\lambda+\delta\lambda}U_{\lambda+\delta\lambda} - I)x\rangle = \mathcal{O}(\delta\lambda^2),$$

so U_λ is unitary up to second order, corresponding to zero first–order entropy production. This identifies adiabatic transport as the minimal (information-neutral) path in operator space. □

Thought Experiment 4.7.1 (Adiabatic Drift of a Boundary Predicate)**.** *Consider a boundary event $e_b(\lambda)$ parameterized by λ, marking the interface between recorded and unrecorded regions of a causal set. If the mapping $\lambda \mapsto e_b(\lambda)$ is adiabatic, then every increment $\delta\lambda$ changes only the labeling of boundary events, not the count of distinguishable outcomes. Thus the observer's entropy measure remains invariant while the locus of observation drifts—formally, a causal analog of quasi-static expansion in thermodynamics, but without energy exchange.*

Interpretation. Adiabatic transport represents the *limit of causal motion that preserves informational order*. It connects reversible evolution (zero $\dot{S}$) with the necessary global condition $\Delta S \geq 0$: the system may drift without loss of information but not contract beyond its recorded distinctions. This concept bridges the local invariance of measurement with the global monotonicity of the Second Law of Causal Order.

Scope. All references to "slow" or "continuous" change are formal, referring to the topology of predicate transformations, not to temporal rate or physical smoothness. The adiabatic condition ensures that causal curvature may evolve without violating informational invariance.

4.8 Annealing

N.B. This section treats *annealing* as an informational process of relaxation toward maximal distinguishability under causal constraint, not as a physical heat exchange. It describes how a causal domain progressively eliminates inconsistent configurations while preserving the monotonic law $\Delta S \geq 0$ [54, 46, 101].

Definition 35 (Annealing in the Causal Domain)**.** *Let $\mathcal{C} = (E, \preceq)$ be a causal structure with a family of admissible configurations $\mathcal{H}$ (partial histories) and an informational potential function*

$$\Phi : \mathcal{H} \to \mathbb{R}_{\geq 0},$$

measuring local inconsistency or "tension" between distinguishable records. A sequence $\{h_t\}_{t\in\mathbb{N}} \subset \mathcal{H}$ is said to undergo annealing *if*

1. *$\Phi(h_{t+1}) \leq \Phi(h_t)$ for all t (monotonic relaxation), and*

2. *$\lim_{t\to\infty} \Phi(h_t) = 0$ corresponds to a globally consistent configuration h_∞ satisfying the Event Selection Axiom.*

Law 2 (Informational Annealing Law)**.** *A causal domain subject to finite observation evolves toward a configuration minimizing informational tension while preserving distinguishability:*

$$\frac{d\Phi}{dt} \leq 0, \quad \frac{dS}{dt} \geq 0,$$

with equality only at global consistency ($\Phi = 0$). Thus annealing is the formal complement of adiabatic transport: instead of preserving information exactly, it monotonically reorganizes distinctions to reduce redundancy without loss of order [8, 72].

Proposition 23 (Annealing as Iterated Projection)**.** *Let U_t be the local update operator at iteration t, and let Π denote the projector onto the subspace of order–consistent states. If*

$$h_{t+1} = \Pi(U_t h_t),$$

then the sequence $\{h_t\}$ is annealing iff the composition ΠU_t is contractive in *the informational metric:*

$$\|\Pi U_t h - \Pi U_t k\| \leq \|h - k\| \quad \forall h, k \in \mathcal{H}.$$

Contractivity ensures that successive refinements move closer to a consistent fixed point while never decreasing the total number of distinguishable predicates.

Sketch. Under contractivity, Banach's fixed–point theorem guarantees convergence to a unique $h_\infty = \Pi U_\infty h_\infty$. Because Π only removes logical contradictions and never merges distinct equivalence classes, $S(h_{t+1}) \geq S(h_t)$, while $\Phi(h_{t+1}) \leq \Phi(h_t)$. Hence the dual monotonic conditions above are satisfied. □

Thought Experiment 4.8.1 (Causal Annealing as Refinement of Observation)**.** *Suppose an observer repeatedly reconciles conflicting records of events e_i and e_j under finite resolution. Each reconciliation step eliminates one inconsistency in ordering while maintaining all prior distinctions. As t increases, the observer's causal field "anneals" toward a self-consistent partial order: the informational potential Φ decreases, but ΔS remains nonnegative. The limit corresponds to the observer's stable record—a causal equilibrium in the informational sense.*

Interpretation. Adiabatic transport corresponds to reversible motion along an iso-entropic surface in informational phase space; annealing describes the relaxation onto that surface. Both obey $\Delta S \geq 0$: adiabatic evolution keeps $\Delta S = 0$ locally, annealing makes $\Delta S > 0$ globally until consistency is achieved. Together, they constitute the two limiting behaviors of informational dynamics: perfect preservation and convergent refinement.

Scope. The term "temperature" is deliberately avoided; no thermodynamic substrate is implied. Annealing here refers solely to the controlled reduction of inconsistency within the logical structure of events, ensuring that the system approaches global causal coherence without violating the monotonic growth of distinguishability.

4.9 Brownian Motion

N.B. This section presents *Brownian motion* as the limit of informational diffusion within a causal set, not as molecular kinetics. It formalizes randomness as the unresolved superposition of admissible distinctions under finite observation. The stochastic character arises from incomplete causal specification, not from physical noise [25, 86, 46, 101].

Definition 36 (Brownian Motion in the Causal Framework)**.** *Let* $(E, \preceq)$ *be a locally finite causal domain and* $X_t : E \to \mathbb{R}^n$ *a random field assigning numerical observables to events at discrete rank* t*. We call* $\{X_t\}$ *a* Brownian process on the causal domain *if, for every finite set of events* $F \subset E$*,*

$$\mathbb{E}[X_{t+1}(e) - X_t(e)] = 0, \quad \mathbb{E}[(X_{t+1}(e) - X_t(e))^2] = 2D\,\delta t,$$

where D *is the informational diffusion coefficient and* δt *the discrete causal increment. The expectation is taken over the ensemble of admissible event refinements consistent with the observer's finite resolution.*

Law 3 (Causal Diffusion Law)**.** *Let* $\rho(e, t)$ *denote the probability density over distinguishable states at event* e *and causal rank* t*. Then informational conservation under local diffusion implies*

$$\frac{\partial \rho}{\partial t} = D\,\nabla^2 \rho,$$

where the Laplacian ∇^2 *acts on the combinatorial adjacency of* E*. This is the causal analogue of the classical diffusion equation, expressing the randomization of distinctions subject to* $\sum_e \rho(e, t) = 1$*.*

Proposition 24 (Variance Growth and Informational Pressure). *Under the Causal Diffusion Law,*

$$\mathbb{E}[(X_t - X_0)^2] = 2Dt.$$

Thus the variance of informational displacement grows linearly with causal depth. Equivalently, uncertainty in the record expands at a constant informational "pressure" proportional to D. This represents the minimal stochastic perturbation compatible with $\Delta S \geq 0$: entropy increases strictly due to unresolved refinement of order.

Sketch. Applying the law above to the mean and variance gives $\frac{d}{dt}\mathbb{E}[X_t] = 0$ and $\frac{d}{dt}\mathbb{E}[X_t^2] = 2D$. Integrating yields the linear variance relation. Because the process is unbiased and conservative, entropy increases monotonically while total distinguishability (probability normalization) remains invariant. □

Thought Experiment 4.9.1 (Informational Brownian Motion). *Consider an observer recording the position of a causal signal with finite temporal resolution δt. Each interval introduces an unresolved set of micro–orderings among sub–events, which appear as random displacements of the observed value X_t. As the record refines, the unresolved components average to zero drift but nonzero variance, giving the signature of Brownian motion. The diffusion constant D measures the observer's coarse–graining bandwidth: smaller D corresponds to higher informational precision.*

Interpretation. Brownian motion represents the *limit of stochastic refinement*—the regime where information is preserved on average but redistributed unpredictably within causal bounds. Where adiabatic transport is reversible ($\dot{S} = 0$) and annealing directed ($\dot{S} > 0$ with convergence), Brownian evolution is *neutral on average* yet entropic in expectation ($\mathbb{E}[\Delta S] > 0$). It captures the statistical character of finite observation: even without physical noise, the logical uncertainty of incomplete distinction propagates diffusively.

Scope. All references to "randomness" or "diffusion" are formal. The process described is informational, not mechanical; it models how a finite observer's record spreads under repeated partial measurement, consistent with the Second Law of Causal Order.

Thought Experiment 4.9.2 (Thought Experiment: The Dual Transport of Measurement)**.** ***N.B.*** *This thought experiment explores the formal coexistence of discrete and continuous transport under finite observation. It does not describe quantum dynamics or wave mechanics. It serves to illustrate how an informational system can exhibit both particle–like localization (discrete event selection) and wave–like propagation (distributed causal amplitude) within the same logical framework [7, 41, 101].*

Setup. *Consider an observer maintaining a causal record $\mathcal{R}$ of distinguishable events $\{e_i\}$. Each update corresponds to an informational transport operator U_t, which may act:*

1. Discretely—*selecting a new event and appending it to $\mathcal{R}$ (particle limit), or*

2. Continuously—*refining the relational amplitudes between existing events (wave limit).*

In a regime of finite observation, these two descriptions coexist: the discrete record e_i is a sample of a continuous causal amplitude that itself evolves under adiabatic, annealing, or Brownian mechanisms.

Invariant to Preserve. *For each admissible transformation, the informational flux*

$$\Phi(t) = \sum_i \rho_i(t)\, v_i(t)$$

remains conserved in expectation. Here ρ_i is the probability weight of distinguishing e_i at time t, and v_i the causal update rate. Conservation of $\Phi(t)$ enforces complementarity: localization increases ρ_i at the expense of ampli-

tude spread (wave), while dispersion equalizes ρ_i (particle uncertainty). The two limits are dual descriptions of the same informational invariant.

Formal Analogy. *Let Ψ denote the normalized vector of causal amplitudes across events. Under reversible (adiabatic) transport,*

$$U_t\,\Psi = e^{iHt}\Psi,$$

and under irreversible (annealing or Brownian) refinement,

$$\Psi_{t+1} = (I - \epsilon L)\Psi_t,$$

where L is the informational Laplacian on the causal network. The "wave" view treats Ψ as a continuous amplitude over possible distinctions; the "particle" view corresponds to a single index realization of Ψ by the observer. Both are faithful encodings of the same causal process at different resolutions.

Interpretation. *Wave–particle duality thus arises as an epistemic phenomenon of finite observation: the wave is the uncollapsed ensemble of admissible distinctions, the particle the locally recorded choice among them. Observation projects Ψ onto one branch of $\mathcal{R}$, satisfying the Event Selection Axiom but leaving the global informational flux invariant. In this sense, the collapse of the wave function is not physical, but the act of registering one consistent causal trajectory within the set of possible orderings.*

Scope. *This thought experiment is purely formal. It demonstrates how the coexistence of discrete and continuous transport mechanisms follows from the structure of causal information, not from any underlying quantum field. Wave–particle duality here is the logical complementarity of refinement and selection—the informational shadow of the observer's bandwidth limit.*

4.10 On Deriving Motion Without Energy

The developments up to this point have been intentionally austere. We began with no continuum, no geometry, and no energetic quantity of any kind. From a finite collection of events ordered only by causal precedence, we obtained calculus as the closure of measurement, waves as the propagation of consistency under Martin's Condition, and advection as the directed transport of distinguishability. At no step was energy invoked. Nothing in the construction presupposed force, mass, or curvature. Yet the resulting equations coincide exactly with the kinematic skeleton underlying all of classical and quantum dynamics.

The Structural Consequence

The advection equation,

$$\partial_t a + c\,\partial_x a = 0,$$

arose not from the motion of particles through a medium, but from the preservation of order across oriented overlaps of finite event sets. The parameter c was defined purely as a ratio of discrete indices: the rate at which causal relations advance along the chain of overlaps. It is therefore not an energetic constant but a combinatorial one, a speed of bookkeeping rather than of matter. This reversal of interpretation is decisive. It suggests that the familiar forms of physical law—continuity, transport, and wave propagation—are not contingent on the existence of energetic carriers, but are inevitable properties of consistent causal description itself.

The Logical Hierarchy of Physics

The chain of constructions may now be summarized as

$$\text{Order} \implies \text{Variation} \implies \text{Propagation} \implies \text{Energy}.$$

Traditional formulations reverse this sequence, taking energy or momentum as the primitive and deriving motion as a consequence. Here motion appears first, as a necessary regularity of finite order. Energy, when it finally enters, can only be a measure of how much order is preserved or lost under repeated propagation. What physicists call *kinetic* or *potential* energy must therefore correspond to the count of distinguishabilities that remain invariant under the oriented application of Martin's Condition. In this sense, energy is not a cause of motion but a conserved shadow of causal consistency.

The Epistemic Reversal

To derive motion without energy is to invert the epistemology of physics. It means that the universe does not move because it has energy; it *has* energy because its order moves. Causal updates propagate distinguishability forward, and the invariants of that propagation are what observers interpret as energetic quantities. The calculus of motion precedes the quantities it was once thought to govern. This inversion brings physics closer to logic: dynamics become theorems of consistency rather than axioms of force.

Consistency as the Source of Dynamics

Under Martin's Condition, every finite causal neighborhood must extend to a globally consistent ordering. When overlaps are unbiased, this requirement produces the symmetric second–order closure $\nabla_E^2 \mathcal{A} = 0$, the discrete wave equation. When overlaps possess orientation, the first–order closure $\partial_t a + c\,\partial_x a = 0$ appears. Both are special cases of the same law:

Law 4. *Law of Consistency The universe minimizes the inconsistency of its own order.*

The entire machinery of classical dynamics—waves, advection, diffusion, and, later, curvature and field stress—can therefore be interpreted as successive approximations to the global enforcement of Martin's Condition. Every differential operator is a bookkeeping device for maintaining consistency in the face of finite, overlapping observations.

Implications

This interpretation carries several consequences:

1. **Causality precedes energy.** Energy cannot be fundamental if its defining equation is a by–product of causal bookkeeping. The conservation of energy must instead be a corollary of the conservation of distinguishability.

2. **Geometry is emergent.** Spatial metrics will appear later as statistical summaries of how distinguishabilities propagate across large causal domains. Space is the coarse–grained shadow of consistent order.

3. **The field concept is derivative.** A continuous field is simply the limit of a dense set of overlapping event relations that remain Martin–consistent under iteration. Field equations are encoded constraints on the propagation of order.

4. **Information and physics coincide.** The universe's physical regularities are identical to its rules for storing, updating, and reconciling information. No extra ontology is required.

Outlook

The reader should therefore pause to recognize the scope of what has already been accomplished. Without invoking mass, charge, or curvature, the framework has produced the canonical equations of transport and wave propagation purely from the logic of finite distinguishability. All subsequent structure—energy, stress, and geometry—must therefore emerge as higher–order invariants of this same logic. The remainder of this work develops those invariants explicitly, showing how the metric tensor, stress tensor, and curvature of spacetime are the continuous shadows of a discrete causal calculus.

Thought Experiment 4.10.1 (Extrapolation: The Casimir Effect as Boundary-Limited Distinction)**.** ***N.B.*** *This extrapolation is formal. It shows how vacuum energy differences emerge as a consequence of boundary-imposed limits on distinguishability, not as a mechanical "force" between conducting plates.*

Setup. *In quantum field theory, the Casimir effect arises when two parallel plates constrain the allowable electromagnetic modes between them, yielding a measurable pressure proportional to* $1/d^4$*, where* d *is the separation. Conventionally, this is interpreted as the difference between zero-point energies inside and outside the cavity:*

$$F/A = -\frac{\pi^2 \hbar c}{240 d^4}.$$

Formal interpretation. *In the informational framework, each admissible field mode corresponds to a distinguishable causal update. When boundary conditions restrict the available modes, the count of consistent updates between the plates decreases relative to the unbounded field. The universe enforces* $\Delta S \geq 0$ *by compensating this local loss with an outward pressure—an adjustment that restores global distinguishability.*

Analogy. *The Casimir pressure is thus an informational curvature: the gradient of admissible state density imposed by constraints in the causal manifold. It represents the same reciprocity principle found in adiabatic transport and annealing—an equilibrium restoring term that keeps the overall measure consistent.*

Scope. *This extrapolation demonstrates that apparent "vacuum energy" phenomena can be reinterpreted as rebalancing within the global distinction count. No change to established quantum electrodynamics is implied; the Casimir effect merely exemplifies how informational consistency reproduces* *the observed form of boundary-induced pressure.*

Thought Experiment 4.10.2 (Extrapolation: Quantum Tunneling as the Repair of a Broken Correlation)**.** ***N.B.*** *This extrapolation is formal. It treats tunneling as the restoration of informational continuity across a boundary where the causal correlation appears classically broken. No claim about subatomic mechanism is made.*

Setup. *In conventional quantum mechanics, a particle encountering a potential barrier of height V_0 and width d exhibits a finite transmission probability even when its energy $E < V_0$. The standard explanation attributes this to the non-zero overlap of an exponentially decaying wavefunction across the barrier:*

$$T \propto e^{-2\kappa d}, \quad \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}.$$

Classically, the trajectory would terminate; quantum mechanically, correlation persists.

Formal interpretation. *In the causal framework, measurement defines correlations—ordered distinctions that must remain globally consistent under $\Delta S \geq 0$. When a barrier divides the causal manifold, the mapping between pre- and post-barrier distinctions momentarily fails: a "broken correlation." To preserve informational continuity, the universe extends the correlation through the barrier as a minimal repair—an *extrapolated distinction* that restores bijection. The exponential attenuation reflects the cost of maintaining that minimal correlation across a region forbidden to classical propagation.*

Analogy. *Brownian motion represents spontaneous fluctuation within consistent order; tunneling represents the inverse—the system's attempt to prevent discontinuity in the causal mapping. It is the informational equivalent of a snapped thread reconnecting under tension: continuity is statistically unlikely but globally mandatory.*

Scope. *This extrapolation is formal. It shows that tunneling can be viewed as the minimal act required to prevent a breach of causal order—an infor-*

mational necessity that ensures no correlation, once established, terminates inconsistently. The observed transmission coefficient quantifies the "cost" of that repair.

Thought Experiment 4.10.3 (Extrapolation: The Hyperfine Transition as Periodic Reconciliation of Causal Order)**.** ***N.B.*** *This extrapolation is formal. It interprets hyperfine transitions as periodic acts of informational reconciliation between nearly degenerate causal configurations, not as dynamical spin flips.*

Setup. *In the hydrogen atom, the ground state comprises two configurations distinguished only by the relative orientation of the proton and electron spins. The energy difference,*

$$\Delta E = h\nu_{21} \approx 5.9 \times 10^{-6} \text{ eV},$$

corresponds to the 21 cm line ($\nu_{21} = 1.420\,405\,751$ GHz*). Quantum electrodynamics treats this as the magnetic dipole interaction between two spin moments.*

Formal interpretation. *Within the causal framework, the two spin configurations represent adjacent but distinct informational states—nearly indistinguishable under ordinary resolution. Over time, their causal descriptions drift out of phase due to accumulated measurement asymmetry. The spontaneous emission of a 21 cm photon restores alignment: it is a *reconciliation event* that re-synchronizes the dual description and preserves the global bijection required by* $\Delta S \geq 0$*.*

Analogy. *Where tunneling repaired a local breach in continuity, the hyperfine transition performs global bookkeeping. It is the metronome of causal coherence—a self-correcting tick ensuring that near-identical representations do not diverge beyond the system's tolerance. Each photon marks the closure of an informational loop, a return to consistency.*

Scope. *This extrapolation is formal. It demonstrates how spontaneous* *emission at the hyperfine scale can be viewed as the minimal act by which the universe re-aligns nearly degenerate causal states. The 21 cm line thus records the maintenance of informational equilibrium, not merely atomic magnetism.*

Thought Experiment 4.10.4 (Extrapolation: Correlation Drift and the Redshift of Atomic Clocks)**.** ***N.B.*** *This extrapolation is formal. It treats redshift as a change in the density of consistent correlations rather than a mechanical stretching of spacetime.*

Setup. *Atomic clocks mark the periodic reconciliation of nearly degenerate states—the hyperfine transition. When two clocks occupy regions with differing correlation densities ρ_c, the rate of reconciliation differs:*

$$\frac{\nu}{\nu_0} = \frac{\rho_c}{\rho_{c,0}}.$$

A deficit of correlation (e.g., under gravitational potential) yields $\nu < \nu_0$, the observed redshift.

Interpretation. *In this framework, gravitational redshift and time dilation emerge as manifestations of informational shear: the cost of preserving bijective correspondence between causal regions of unequal correlation density. The "slowing" of time is the reduced frequency of global consistency updates.*

Scope. *This extrapolation links frequency drift to the correlation structure of the causal field. It demonstrates that atomic clocks red-shift because their reconciliation rate is modulated by correlation density, not because time itself dilates.*

Coda: The Informational Harmonic Oscillator

Thought Experiment 4.10.5 (Coda: The Informational Harmonic Oscillator). ***N.B.*** *This coda introduces the simplest reversible dynamics permitted by the Second Law of Causal Order. It shows that oscillation—in its most abstract sense—arises whenever information alternates between two complementary forms: record and prediction. No physical mass, force, or energy is implied. The oscillator here is entirely informational: a minimal closed loop of distinguishability [72, 54, 101].*

Setup. *Let (x, p) denote a conjugate pair of informational coordinates on a two–dimensional causal phase space. x represents the observer's recorded distinctions (the state of knowledge); p represents the predictive momentum (the rate at which distinctions are changing). Define the informational Hamiltonian*

$$\mathcal{S}(x, p) = \tfrac{1}{2}\,(\alpha x^2 + \beta p^2),$$

where α and β are positive constants measuring informational stiffness and inertia. The reversible evolution equations are

$$\dot{x} = \quad \frac{\partial \mathcal{S}}{\partial p} = \beta p, \qquad \dot{p} = -\frac{\partial \mathcal{S}}{\partial x} = -\alpha x.$$

Eliminating p yields

$$\ddot{x} + \omega^2 x = 0, \qquad \omega^2 = \alpha\beta.$$

Thus the observer's state executes harmonic motion in informational phase space with constant total measure $\mathcal{S}(x, p)$.

Interpretation. *At each turning point of the oscillation, information is maximally localized: the record x is fixed, the prediction p vanishes. At each midpoint, prediction dominates and the record is momentarily indeterminate. The system alternately* stores *and* transmits *distinguishability, maintaining constant total informational entropy. The cycle expresses the complementar-*

ity of knowledge and expectation: every complete measurement must eventually swing back toward uncertainty to preserve $\Delta S \geq 0$.

Relation to Transport. *The four informational transport regimes appear as limiting cases of this oscillator:*

- ***Adiabatic transport:*** *small, continuous oscillations with* $\dot{S} = 0$ *(reversible exchange of information).*
- ***Annealing:*** *inclusion of damping* $\gamma > 0$, *giving* $\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$, *a monotonic relaxation to equilibrium.*
- ***Brownian motion:*** *addition of stochastic forcing* $\xi(t)$, $\ddot{x}+\omega^2 x = \xi(t)$, *producing diffusive variance growth.*
- ***Wave/particle duality:*** *interpretation of* (x, p) *as the conjugate pair of localization and amplitude—dual views of the same informational invariant.*

Each regime preserves causal consistency and satisfies $\Delta S \geq 0$*; only the mode of information exchange changes.*

Scope. *This oscillator is the canonical closed system of the informational universe: a bounded transformation in which every increase in record precision is offset by a proportional loss of predictive capacity, and vice versa. It represents the minimal rhythm of causal order—the reversible heartbeat of information itself.*

Motion, in this theory, is not caused by energy. It is the preservation of order under Martin's Condition.

Chapter 5

The Kinematics of Light

The preceding chapters established that motion itself arises from the consistency of causal order. The cubic spline and its dual principle of least action defined kinematics as the smooth propagation of distinguishable events within the Causal Universe Tensor. We now apply this same logic to the most symmetric case possible: the propagation of information at the limit of distinguishability. In that limit, the kinematics of the universe *is* the kinematics of light.

Light represents the boundary between distinguishable and indistinguishable events. Each photon defines an extremal path through the causal network—a trajectory along which the scalar invariants of the Causal Universe Tensor remain constant. Because such paths saturate the bound on causal speed, their geometry is determined entirely by the consistency of order itself. Curvature therefore measures not force, but deviation from perfect causal symmetry: it is the local record of how the network bends to preserve distinguishability as information propagates.

In this chapter we reinterpret the machinery of general relativity in this language. The metric tensor $g_{\mu\nu}$ appears as the continuous shadow of pairwise event separations; the connection coefficients $\Gamma^{\lambda}_{\mu\nu}$ encode how those separations adjust to maintain Martin consistency across overlapping causal neighborhoods. The Einstein tensor then summarizes the residual inconsistency of order—the curvature required for lightlike propagation to remain self-consistent in a finite universe.

Thus general relativity emerges here not as a theory of gravitational force, but as the *kinematics of light*: the unique geometry in which the scalar invariants of the Causal Universe Tensor remain stationary along all null directions. The curvature of spacetime is simply the bookkeeping term that guarantees the smooth evolution of causal order at the speed of information. The following sections formalize this intuition, deriving the Einstein field equations as the differential expression of that invariance and showing how energy, stress, and curvature arise as higher-order scalar invariants of the same causal calculus.

5.0.1 Consequences and Outlook

At this point, nothing unfamiliar has been assumed. Each object of general relativity has arisen as the minimal correction required to preserve causal consistency under finite observation. The variations that produced the Einstein tensor are not postulates of gravitation but the necessary differential identities of the Causal Universe Tensor. Once the calculus of measurement is accepted, the geometry of light follows without remainder.

The reader may pause here and recognize the consequence: there is no longer a conceptual gap between discrete measurement and continuous spacetime. Every term in the classical field equations is a higher-order scalar invariant of the same underlying order. The curvature that once appeared as a geometric hypothesis is revealed as bookkeeping for the propagation of distinguishability. There is, so far as the argument stands, no reason it should not work.

What remains is not proof of correctness but proof of scope—how far the same consistency extends beyond lightlike motion. The next chapter therefore turns from the kinematics of light to the dynamics of matter, asking how the gauge of causal order constrains systems that deviate from the null limit.

5.0.2 Arc of the Proof

The goal of this chapter is to show that the geometry of spacetime arises as the unique gauge condition under which lightlike propagation remains Martin-consistent. The proof proceeds in four stages.

1. Construction of the metric as a gauge of separation. We begin by defining the metric tensor $g_{\mu\nu}$ as the bilinear form that measures distinguishability between neighboring events. It appears as a gauge choice: an assignment of infinitesimal distances that preserves the local invariance of the scalar quantities computed by the Causal Universe Tensor. The metric thus represents the minimal information required for an observer to maintain causal order in a finite neighborhood.

2. Connection as the rule of causal transport. Next we introduce the connection $\Gamma^{\lambda}_{\mu\nu}$ as the operator that preserves these scalar invariants under parallel transport of distinguishable events. It records how the local labeling of events changes when moving through the causal network. The vanishing of the covariant derivative $\nabla_{\mu}T^{\mu\nu} = 0$ expresses Martin consistency in this differential form: order is preserved under transport.

3. Curvature as the residue of inconsistency. Transporting an event label around a closed causal loop yields a finite residue when local frames cannot be made globally consistent. This residue, the Riemann tensor $R^{\rho}{}_{\sigma\mu\nu}$, quantifies the holonomy of the causal gauge. Its contractions—the Ricci tensor $R_{\mu\nu}$ and the scalar curvature R—measure the degree to which the scalar invariants of the Causal Universe Tensor fail to remain constant under finite extension.

4. Einstein equation as the constraint of global consistency. Finally we impose that the total scalar invariant of order, represented by the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, balances the energy–momentum content encoded in the lower-order invariants of the Causal Universe Tensor:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}.$$

This equality enforces global consistency: the curvature required to maintain Martin consistency equals the causal stress produced by the finite structure of distinguishability. General relativity thus appears as the closure condition of the gauge of light.

Remark 20. *In summary, the arc of this proof mirrors the logic of the entire work. The metric defines measurement, the connection enforces order, the curvature measures residual inconsistency, and the Einstein equation restores balance. Each level is a higher-order invariant of the same causal calculus. Kinematics, when viewed through light, becomes the gauge theory of order itself.*

5.0.3 Defining Entropy

Definition 37 (Entropy)**.** *Let $\mathcal{C}$ denote the causal set of distinguishable events accessible to an observer, and let $\Omega(\mathcal{C})$ be the set of all admissible micro–orderings of those events consistent with the Reciprocity Law. The* entropy *associated with $\mathcal{C}$ is the logarithm of this count:*

$$S[\mathcal{C}] = k_{\mathrm{B}} \ln |\Omega(\mathcal{C})|.$$

Operationally, S measures the number of distinct internal configurations that yield the same observable causal invariants. In the continuum limit, variations in S appear as gradients of informational curvature; coupling this quantity to the stress tensor defines the entropic contribution to spacetime *geometry.*

Remark 21. *Entropy in this framework is not a measure of disorder but of indistinguishability: it quantifies how many discrete causal configurations correspond to the same continuous geometry. It is therefore the dual of curvature—one counting micro–order, the other measuring its macroscopic residue.*

5.1 Metric as a Gauge of Separation

The metric tensor arises naturally once the act of distinguishing events is viewed as a gauge freedom. Every observer maintains a local convention for labeling distinguishable events; what we call a *distance* is merely the scalar quantity that remains invariant when those local conventions are changed. The metric $g_{\mu\nu}$ is therefore not a physical fabric laid over spacetime but a bookkeeping device that encodes how distinctions are preserved under Martin consistency.

5.1.1 From Distinction to Distance

5.1.2 Axiomatic Necessity

The appeal to ZFC and Martin's Axiom is not an external mathematical convenience but a physical necessity. Finiteness of observation requires countable closure (ZFC's Replacement); causal consistency requires choice of ordering (the Axiom of Choice); and global coherence of local choices requires the Martin property (a countable chain condition ensuring no overcounting of causal possibilities). Thus each axiom corresponds to a measurable physical principle:

$$\text{Finiteness} \leftrightarrow \text{ZFC}, \qquad \text{Consistency} \leftrightarrow \text{Martin}, \qquad \text{Reversibility} \leftrightarrow \text{Choice}.$$

Hence, these axioms are not postulates about mathematics but symmetry constraints on any finite observer's causal domain.

Let each infinitesimal event displacement be represented by the differential dx^μ, denoting the local coordinates an observer assigns to successive measurements. Two observers using different conventions for measurement will represent the same infinitesimal separation by differentials $dx'^\mu = \Lambda^\mu{}_\nu \, dx^\nu$, where $\Lambda^\mu{}_\nu$ encodes the transformation between their local frames. To preserve the scalar invariants computed by the Causal Universe Tensor, we require that the inner product of these displacements remain unchanged:

$$ds^2 = g_{\mu\nu} \, dx^\mu \, dx^\nu = g'_{\mu\nu} \, dx'^\mu \, dx'^\nu.$$

This invariance defines $g_{\mu\nu}$ up to a gauge transformation of the local frame. The metric is thus the bilinear form that enforces Martin-consistent equivalence among all admissible coordinate choices.

5.1.3 The Metric as a Gauge Connection

Under this interpretation, the metric field acts as the gauge potential of causal separation. It defines the local rule by which infinitesimal differences between events are compared and reconciled across observers. If $T^{\mu\nu}$ denotes the Causal Universe Tensor, then maintaining the invariance of its scalar contractions,

$$\delta(g_{\mu\nu} T^{\mu\nu}) = 0,$$

requires a covariant definition of the derivative operator that absorbs changes of frame. The metric provides that operator's gauge background: it specifies the local symmetry under which causal distinctions are preserved.

Thought Experiment 5.1.1 (Michelson–Morley as Gauge Isotropy of Causal Separation)**. *Statement.*** *The null fringe shift in the Michelson–Morley in-*

terferometer operationally enforces that the causal interval

$$ds^2 \;=\; -c^2dt^2 + dx^2 + dy^2 + dz^2$$

is invariant under orthogonal transport of the measurement path, i.e. the gauge preserving reciprocity is isotropic.

Reciprocity framing. *Arms L_x and L_y define two partition-refined measurement chains with equal event counts at recombination when reciprocity is preserved. Any anisotropy in c would induce a measurable distinction (path-dependent tick surplus), violating the Reciprocity Law.*

Calculation sketch. *The phase difference is*

$$\Delta\phi \;=\; \frac{2\pi}{\lambda}\,\Delta\ell_{opt}, \qquad \Delta\ell_{opt} \equiv (n\,\ell)_x - (n\,\ell)_y.$$

Empirically $\Delta\phi \approx 0$ over apparatus rotations, implying $\Delta\ell_{opt} = 0$ and hence invariance of ds^2 under rotation of the apparatus frame. In our terms: the metric is the gauge of separation *that preserves the dual invariants of measurement and variation.*

Thought Experiment 5.1.2 (Thought Experiment: The Shadow Puppet Theater and Gauge Selection)**.** ***N.B.*** *This experiment clarifies how the metric acts as a distinction gauge rather than a physical field.*

Setup. *Project silhouettes of puppets (events) onto a wall (the Universe Tensor) using a lantern (causal order). Tilting the lantern changes how separations appear on the wall—analogous to a gauge transformation.*

Demonstration. *When the wall is flat and illumination uniform, puppet overlaps map bijectively to coordinates. Bend the wall or tilt the lantern: shadows distort, losing closure. Only orientations preserving overlap correspond to admissible $g_{\mu\nu}$ selections.*

Interpretation. *The metric is the rule ensuring that co- and contravariant descriptions yield the same invariant shadow. Parallel transport preserves distinguishability; distortion measures curvature—residue of inconsis-*

tency.

5.1.4 Causal Interpretation

Physically, $g_{\mu\nu}$ encodes the rate at which the universe must "tilt" its causal structure to maintain distinguishability at the limit of lightlike propagation. When $g_{\mu\nu}$ is constant, the mapping between neighboring causal neighborhoods is uniform and the universe appears flat. When $g_{\mu\nu}$ varies, the transformation between local frames acquires a nontrivial derivative; the resulting connection $\Gamma^{\lambda}{}_{\mu\nu}$ records how the gauge of separation changes with position.

In this view, curvature does not describe a deformation of space but a measure of the cost required to keep causal relations consistent under finite observation. The metric therefore functions as the lowest-order field in a hierarchy of corrections that preserve the scalar invariants of the Causal Universe Tensor. It is the gauge that ensures all observers agree on the magnitude of a distinction even when their labels for events differ.

Remark 22. *To summarize: the metric is the gauge of separation. It defines how the universe reconciles different conventions of measurement so that the scalar invariants of order—the values computed by the Causal Universe Tensor—remain unchanged. Once introduced, all higher structures of connection, curvature, and stress follow as successive corrections that enforce this same principle of causal consistency.*

Thought Experiment 5.1.3 (Galileo's Free–Fall as the Flat–Space Limit of Causal Motion)**.** *In Galileo's experiment, two spheres of unequal mass are dropped from the same height and reach the ground simultaneously. Within the causal framework, this observation expresses the invariance of order in a flat informational geometry: when the curvature of the entropy field vanishes, all trajectories sharing the same initial causal separation remain indistinguishable up to translation in time.*

Let the causal paths be $\gamma_1(t)$ and $\gamma_2(t)$, each governed by

$$\frac{d^2x}{dt^2} = g,$$

where g is constant. Because the informational curvature $\nabla_i \nabla_j S$ is zero, the metric gauge g_{ij} is uniform, and the Reciprocity Law preserves equality of causal intervals:

$$\delta^2 x_1 = \delta^2 x_2.$$

Hence both spheres follow identical causal updates regardless of mass.

In this limit, the observer's partition $\mathcal{P}_n$ resolves all relevant distinctions—position, time, and acceleration—so the reciprocity mapping

$$\Phi : V/{\sim_{\mathcal{P}_n}} \longleftrightarrow M/{\sim_{\mathcal{P}_n}}$$

is exact. No refinement of the partition changes the outcome: the motion is deterministic. Galileo's result therefore represents the classical limit of causal kinematics, the case of zero informational curvature where every variation is fully measurable and light's metric reduces to the Euclidean gauge.

Thought Experiment 5.1.4 (Gravitational Lensing as Informational Curvature)**.** *When light passes near a massive object, its trajectory bends—not because space itself is a physical medium that deforms, but because the mapping that preserves causal order becomes nonuniform. In the present framework, the metric acts as a gauge that encodes how distinguishability is preserved under curvature. Lensing is the observable signature of this informational distortion.*

Let a bundle of null trajectories $\{\gamma_i\}$ originate from a common source. In flat spacetime, each path maintains constant informational phase, and the separation between neighboring geodesics—their causal distinction—is uniform. Introducing a local entropy gradient $S(x)$ modifies this gauge: the *effective distance between successive events changes by*

$$\delta ds^2 \propto \nabla_i \nabla_j S,$$

so that the extremal path satisfies

$$\delta \int ds = 0 \quad \Longrightarrow \quad \frac{d^2 x^i}{d\lambda^2} + \Gamma^i_{jk} \frac{dx^j}{d\lambda} \frac{dx^k}{d\lambda} = 0,$$

with Γ^i_{jk} *determined by the informational curvature* $\partial_i \partial_j S$. *The apparent bending of light is therefore the visible effect of a nontrivial gradient in the entropy field: photons follow the locally shortest causal paths consistent with order, not the straightest geometric lines in Euclidean projection.*

Observers interpret this as a deflection angle $\alpha \approx 4GM/(c^2 b)$, *but within the causal formalism it represents a correction to the bookkeeping of distinction: the density of accessible micro–orderings changes with gravitational potential. Lensing thus measures how informational curvature couples to geometry—a macroscopic manifestation of the same reciprocity that defines the metric itself.*

Thought Experiment 5.1.5 (The Three–Body Problem as Computational Reciprocity)**.** *Consider three point masses* m_1, m_2, m_3 *interacting gravitationally with positions* $r_i(t) \in \mathbb{R}^3$. *Newtonian dynamics gives*

$$m_i \ddot{r}_i \;=\; G \sum_{j \neq i} \frac{m_i m_j}{\|r_j - r_i\|^3} \, (r_j - r_i), \qquad i = 1, 2, 3.$$

This system conserves total energy and angular momentum (Noether symmetries), yet, except for special families (e.g. Euler and Lagrange configurations), it admits no closed-form solution. In the present framework, this means the reciprocity map closes only computationally*: the admissible update that preserves order and invariants exists, but it must be realized by an iterative, order-preserving scheme.*

Remark 23 (Historical Context: Noether Symmetry and Reciprocity). *Emmy Noether's theorem (1918) formalized the equivalence between invariance under continuous transformation and the conservation of a measurable quantity. Within this framework, that equivalence becomes a corollary of the Reciprocity Law: each symmetry of the causal measure corresponds to an invariant distinction preserved under evolution.*

Let $U(t)$ encode the joint state of the three trajectories as an element of the universe tensor. Martin consistency requires that each reciprocal update $U(t) \mapsto U(t+\delta t)$ preserve the conserved scalars and causal ordering of events. Analytic spline closure ($U^{(4)} = 0$) is insufficient here: interactions couple the segments so that local cubic envelopes do not globally commute. The correct closure is algorithmic*: a reversible, symplectic, order-preserving integrator (e.g. velocity Verlet/leapfrog) that implements the reciprocity step without violating the invariants,*

$$\Phi_{\delta t}^{symp} : \; U(t) \longmapsto U(t + \delta t), \qquad \delta E = 0, \;\; \delta L = 0 \text{ (to integrator accuracy).}$$

Operationally, the "quantum-like" fuzziness appears here as sensitivity to initial partitions: tiny unresolved distinctions in initial conditions grow under iteration, producing qualitatively different causal histories (chaos), even though Martin consistency (global order) is never violated.

Thus the three–body problem exemplifies a domain where physics requires *computation: reciprocity and consistency still govern the update, but their closure cannot be written in elementary functions. The law survives as an algorithm: an order-preserving map on the causal state that respects the Noether invariants at each step.*

5.2 The Rule of Causal Transport

Having defined the metric $g_{\mu\nu}$ as the gauge of separation, we now ask how this gauge is to be preserved as the observer moves through the causal network. The answer is given by the rule of causal transport: the requirement that the scalar invariants of the Causal Universe Tensor remain constant when carried from one causal neighborhood to the next.

5.2.1 From Gauge Preservation to Connection

Thought Experiment 5.2.1 (The Two Observers and the Invariant Count [60, 25, 63, 98, 102])***. Concept.*** *This experiment reinterprets Lorentz contraction as a gauge effect. Rather than a physical shortening of rods or slowing of clocks, it is seen as a mathematical contraction of the gauge components themselves, required to preserve the invariant scalar—the total count of causal distinctions—for all observers.*

Setup. *Consider a fundamental causal interval: the emission and absorption of a light pulse. This interval consists of an invariant number N of distinguishable "ticks" or events. This N is the true physical reality upon which all observers agree. In the continuum limit, the corresponding invariant is the line element*

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu.$$

Observer $\mathcal{O}$ (Rest Frame). *This observer is at rest relative to the interval. Her gauge—her coordinate system—is aligned with the events. She measures*

$$N_t = N, \qquad N_x = 0,$$

so that

$$ds^2 \propto N_t^2 - N_x^2 = N^2.$$

Observer $\mathcal{O}'$ (Moving Frame). *A second observer moves at constant velocity v relative to $\mathcal{O}$. Both observers describe the same physical interval but* *with different gauge components. In the traditional interpretation of special relativity, $\mathcal{O}'$ observes time dilation and length contraction:*

$$N'_t \neq N_t, \qquad N'_x \neq N_x,$$

yet the invariant

$$(N'_t)^2 - (N'_x)^2 = N^2$$

remains unchanged.

New View (Gauge Contraction). *In the informational picture developed here, the observers' clocks and rods are not physically distorted. Rather, their* gauges*—their coordinate mappings of causal distinction—are related by the Lorentz transformation $\Lambda^\mu{}_\nu$, which enforces invariance of the scalar count:*

$$g'_{\mu\nu} = \Lambda^\rho{}_\mu \Lambda^\sigma{}_\nu g_{\rho\sigma}.$$

The Lorentz transformation thus acts as a gauge update preserving the invariant number of distinctions. What appears as "contraction" is simply the compensation required to maintain consistency under relabelling [25, 63, 98].

Conclusion. *The phenomenon of Lorentz contraction therefore migrates from the physical measuring apparatus to the mathematical gauge itself. The metric $g_{\mu\nu}$ is a bookkeeping device ensuring that each observer's local representation yields the same invariant scalar—the same informational count N. Spacetime kinematics thus emerge as the consistency rules of an informational gauge: a structure ensuring global agreement on what is distinguishable.*

Consider the transport of a vector field V^μ representing a direction in the space of distinguishable events. To maintain Martin consistency, the change in V^μ along an infinitesimal displacement dx^ν must not alter any scalar quantities computed from the tensor $g_{\mu\nu}V^\mu V^\nu$. The differential form of this requirement is

$$\nabla_\nu g_{\mu\sigma} = 0,$$

which defines the Levi–Civita connection $\Gamma^{\lambda}{}_{\mu\nu}$. The connection therefore arises not as a postulate of differential geometry but as the unique differential operator that preserves the gauge of separation defined by the metric. In the context of the Causal Universe Tensor, it ensures that all scalar invariants of order remain stationary under causal transport.

5.2.2 Operational Meaning

Each component $\Gamma^{\lambda}{}_{\mu\nu}$ records how the act of distinction must be adjusted when an observer translates a local rule of measurement from one event to its neighbor. It is, in essence, the differential bookkeeping of consistency. When the metric is uniform, $\Gamma^{\lambda}{}_{\mu\nu} = 0$, and the mapping of causal neighborhoods is trivial: straight lines remain straight. When the metric varies, the connection encodes how the local gauge must tilt to maintain the invariance of scalar quantities—how the "direction of distinction" is parallel transported through the network.

5.2.3 Parallel Transport and Martin Consistency

Parallel transport expresses Martin consistency in differential form. A vector is said to be parallel transported along a curve $x^{\mu}(s)$ if it satisfies

$$\frac{DV^{\lambda}}{Ds} = \frac{dV^{\lambda}}{ds} + \Gamma^{\lambda}{}_{\mu\nu}\, V^{\mu}\, \frac{dx^{\nu}}{ds} = 0.$$

This condition guarantees that the scalar invariants $g_{\mu\nu}V^{\mu}V^{\nu}$ remain unchanged along the curve, regardless of the local coordinate frame. The connection therefore enforces the *covariant constancy* of the causal gauge: every observer's measurements can differ, but the underlying order they describe remains identical.

Thought Experiment 5.2.2 (Non-Abelian transport and curvature)**.** *Let $A_{\mu}(x)$ be a matrix-valued connection (local gauge of distinction). Parallel* *transport along a path γ uses the path-ordered exponential*

$$U[\gamma] = \mathcal{P}\exp\left(\int_\gamma A_\mu \, dx^\mu\right).$$

For an infinitesimal rectangle spanned by $\delta x^\mu, \delta x^\nu$,

$$U[\square] = I + F_{\mu\nu}\,\delta x^\mu \delta x^\nu + O(\delta^3), \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu].$$

The $[A_\mu, A_\nu]$ term is *the non-commutative residue of transporting in different orders around the loop. Thus, failure of local updates to commute produces a measurable scalar (via contractions of $F_{\mu\nu}$) that records global nonclosure—our curvature as informational residue. This ties the reciprocity-preserving gauge to the field strength that appears in the continuum limit.*

5.2.4 Causal Interpretation

Physically, the rule of causal transport states that the universe updates its own coordinate assignments to maintain distinguishability as information propagates. The connection coefficients are the infinitesimal records of those updates. They quantify how causal neighborhoods must rotate and rescale to remain compatible under finite observation. A nonzero connection indicates that causal consistency is preserved through adjustment rather than uniformity—a curved but coherent propagation of order.

Remark 24. *In summary, the connection ${\Gamma^\lambda}_{\mu\nu}$ is the rule of causal transport: the unique differential relation that preserves the gauge of separation under motion. It translates the logical demand of Martin consistency into a local dynamical law. Curvature will appear in the next section as the finite residue that remains when this transport rule fails to close perfectly around a loop—an irreducible measure of global inconsistency in causal order.*

Thought Experiment 5.2.3 (Invariance of the Causal Interval ds^2)**.** *Consider two observers, $\mathcal{O}$ and $\mathcal{O}'$, who each assign coordinates to the same pair* *of infinitesimally separated events. Their local labels differ by a gauge transformation of the form*

$$dx'^{\mu} = \Lambda^{\mu}{}_{\nu}\, dx^{\nu},$$

where $\Lambda^{\mu}{}_{\nu}$ preserves the ordering of causal relations as required by Martin's Axiom. The scalar quantity

$$ds^2 = g_{\mu\nu}\, dx^{\mu}\, dx^{\nu}$$

represents the infinitesimal measure of distinguishability between these events—the local contraction of the Causal Universe Tensor with the gauge of separation.

Under the gauge transformation, the differentials and metric transform as

$$g'_{\mu\nu} = \Lambda^{\alpha}{}_{\mu}\, \Lambda^{\beta}{}_{\nu}\, g_{\alpha\beta}, \qquad dx'^{\mu} = \Lambda^{\mu}{}_{\sigma}\, dx^{\sigma}.$$

Substituting these into the definition of the interval yields

$$ds'^2 = g'_{\mu\nu}\, dx'^{\mu}\, dx'^{\nu} = g_{\alpha\beta}\, dx^{\alpha}\, dx^{\beta} = ds^2.$$

Hence the scalar ds^2 is invariant under all admissible gauge transformations that preserve causal order. It defines the quantity that every observer must agree upon, even when their coordinate conventions differ.

In the discrete formulation, this invariance states that the number of distinctions between two neighboring events is the same for all observers. In the continuum limit, it becomes the invariance of the causal interval in relativity. Both express the same principle: the universe may bend, accelerate, or dilate, but the order of events—the fact that one event can distinguish another—remains unchanged.

Example: Pound–Rebka Gravitational Redshift as Entropic Transport

Statement. Frequency shift in a gravitational field is the change in event-count rate under causal transport in an informationally curved background.

Key relation (weak field).

$$\frac{\Delta\nu}{\nu} \approx \frac{\Delta\Phi_{\text{grav}}}{c^2} = \frac{g\,h}{c^2}.$$

Reciprocity framing. Transporting a clock's partition along the connection changes the mapping from proper ticks to coordinate time. The entropic stress couples to the metric gauge, altering the local rate at which distinctions are accumulated.

Operational consequence. Redshift is parallel transport of the causal gauge: invariants are preserved, but the local counting density transforms, observed as a shift in ν.

Chapter 6

The Residue of Inconsistency

The gauge of light completes the classical description of the universe: it ensures that causal order is preserved at the limit of distinguishability. But the universe we observe is not smooth. Measurements are discrete, events occur finitely, and the invariants of the causal gauge fluctuate around their ideal values. These fluctuations are not errors—they are the quantum fields of the theory.

A quantum field arises whenever the invariants of the Causal Universe Tensor are permitted to vary locally while maintaining global Martin consistency. Each allowed fluctuation corresponds to a redistribution of causal order between neighboring observers. The field is therefore not an additional substance laid over spacetime but a dynamic adjustment of the gauge itself, mediating the exchange of distinguishability across finite domains.

In this framework, the traditional wavefunction reappears as the probability amplitude for maintaining order under repeated finite observations. Its complex phase represents the orientation of the causal gauge in informational space, while its magnitude measures the stability of that order. The principle of superposition follows directly from the linearity of causal combinations: multiple consistent histories can coexist until observation resolves a single extension of the network.

Quantization enters as the recognition that order cannot be subdivided indefinitely. Every causal update exchanges a finite unit of distinguishability—a discrete increment of information. The Planck constant $\hbar$ expresses this minimal step size: the smallest action through which the universe can modify its own gauge while remaining consistent. The commutation relations of quantum theory are therefore expressions of finite causal resolution, not axioms of measurement.

This chapter develops these ideas systematically. Beginning with the Noether currents of the causal gauge, we derive the corresponding quantum fields as their discrete fluctuations. We then show how these fields propagate through the Causal Universe Tensor, producing the familiar quantum wave equations as conditions of statistical Martin consistency. Finally, we interpret entanglement as the correlated selection of events across overlapping causal neighborhoods—the quantum signature of global order maintained through finite means.

Remark 25. *Classical physics ends where the gauge of light closes; quantum physics begins where it wavers. Every quantum field is a small deviation from perfect causal consistency, a harmonic of order itself. The task of this chapter is to make that statement precise.*

6.1 The Residue of Inconsistency

No rule of transport can remain globally consistent on a finite causal network. When one carries a distinction around a closed loop of events, the recovered configuration generally differs from the initial one. This difference is not an error but an invariant: the measurable residue of inconsistency required to preserve local order within a global whole. In differential form, that residue is called curvature.

6.1.1 Curvature as the Measure of Non-Closure

The connection $\Gamma^{\lambda}{}_{\mu\nu}$ prescribes how distinctions are transported to preserve scalar invariants locally. When the same distinction is transported successively along different paths that enclose a finite region, the final result may depend on the path taken. The difference between the two results defines the Riemann curvature tensor:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\sigma\nu} - \partial_\nu \Gamma^{\rho}{}_{\sigma\mu} + \Gamma^{\rho}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\sigma\nu} - \Gamma^{\rho}{}_{\lambda\nu}\Gamma^{\lambda}{}_{\sigma\mu}.$$

This object measures the infinitesimal failure of causal transport to commute. When $R^{\rho}{}_{\sigma\mu\nu} = 0$, all paths yield the same result and the causal network is globally flat; when it does not vanish, the inconsistency cannot be removed by any gauge transformation.

Example: Casimir Effect as Measured Residue of Non-Closure

Statement. Boundary-induced mode restriction yields a measurable scalar from the residue of non-closure: the Casimir pressure.

Key relation (ideal plates, separation a).

$$P \;=\; -\frac{\pi^2}{240}\,\frac{\hbar c}{a^4}.$$

Reciprocity framing. Plates impose selection on admissible causal updates (mode partitions). The contraction of the Universe Tensor over admissible modes produces a nonzero scalar residue—the pressure—interpretable as curvature from informational incompleteness.

Operational consequence. Moving a plate changes the equivalence class (refines the partition), and the derivative of the class invariant yields a force, closing the loop between geometry and matter.

6.1.2 Physical Interpretation

In the context of the Causal Universe Tensor, curvature represents the minimal informational adjustment required for the propagation of distinguishability in a finite universe. Each nonzero component of $R^{\rho}{}_{\sigma\mu\nu}$ quantifies how much the local gauge of separation must bend to remain self-consistent when extended around a closed causal loop. Curvature is thus the differential trace of the universe correcting itself: the physical manifestation of the fact that perfect global order is impossible, even though local order is preserved.

6.1.3 Contractions and Scalar Invariants

Contracting the curvature tensor yields quantities that summarize this residual inconsistency at successively coarser levels. The Ricci tensor

$$R_{\mu\nu} = R^{\rho}{}_{\mu\rho\nu}$$

measures the local divergence of geodesic families—the rate at which neighboring causal paths converge or spread. The scalar curvature

$$R = g^{\mu\nu} R_{\mu\nu}$$

compresses all such deviations into a single invariant of the causal gauge. These contractions represent higher-order scalar invariants of the Causal Universe Tensor, extending the chain of conserved quantities that began with the spline and the principle of least action.

6.1.4 The Meaning of Curvature in the Causal Framework

Traditional geometry interprets curvature as a property of space. Here it is a property of information: a measure of how the network of distinguishable events must deform to reconcile finite observation with global consistency. Flatness corresponds to exact commutativity of causal updates; curvature, to their minimal non-commutativity. The universe's curvature is therefore the bookkeeping of necessary inconsistency—the trace left by causal order maintaining itself through finite means.

Remark 26. *Curvature is the residue of inconsistency. It is what remains when the rule of causal transport cannot close perfectly, the irreducible difference between local and global consistency. In the language of the Causal Universe Tensor, curvature represents the self-correcting property of the universe: the differential response by which causal order preserves itself in time. The next section will show that this residue, when balanced against the stress encoded in the tensor $T_{\mu\nu}$, yields the Einstein equation—the equilibrium condition of the gauge of light.*

Thought Experiment 6.1.1 (Extrapolation: Rotation Curves as Order-Preserving Transport)**.** ***N.B.*** *This is a conceptual extrapolation about* measurement and causal bookkeeping*, not an astrophysical claim. It illustrates how a curvature term may arise as the minimal correction that preserves an informational invariant under geometric dilution. No inference about galaxies, dark matter, or dynamics is intended [8, 72, 12, 101, 64, 46].*

Setup. *Imagine a discrete causal disk partitioned into annuli $\{A_r\}$, each containing a locally finite set of events. A predicate P counts the distinguishable crossings of a reference ray within each annulus. For two anchors $a \prec b$, define the measurement*

$$M_P[r; a, b] := \#\{\, e \in A_r \mid a \prec e \prec b,\ P(e) = 1\,\}.$$

Order-preserving advection (as in §3.6) requires that counts move without creation or loss: transport is conservative [10, 31].

Remark 27 (Observational Context: Vera Rubin and the Dark Matter Problem)**.** *The near-constant tangential velocities observed by Vera Rubin and W.*

Kent Ford (1970) in spiral galaxies revealed a striking departure from Newtonian expectations: orbital speed remained approximately flat with radius. In this framework, that empirical fact is reinterpreted as an order-preserving transport condition rather than evidence for unseen mass. The constancy of $v_\theta(r)$ *follows from invariance in the count of distinguishable causal updates, not from an additional gravitational component.*

Invariant to preserve. *Let* $\Phi(r)$ *be the count per causal cycle,*

$$\Phi(r) = \frac{M_P[r; a, b]}{\textit{cycles between } a \textit{ and } b},$$

which serves as a flux of distinguishability. Perfect bookkeeping demands $\partial_r \Phi(r) = 0$.

Geometric tension. *In a flat disk the circumference grows as* $2\pi r$*; if events are neutral parcels, their surface density falls as* $1/r$*. Without correction,* $\Phi(r)$ *would decrease outward [64].*

Minimal fix. *Introduce a compensating connection with curvature* $K(r)$ *such that*

$$\partial_r\big(r\,\rho(r)\,v(r)\big) = 0,$$

where $\rho(r)$ *is the local count density and* $v(r)$ *the tangential update rate. The least-bias (minimal-curvature) solution satisfying constant flux is*

$$r\,\rho(r)\,v(r) = C \quad \Rightarrow \quad v(r) \approx \textit{const.}$$

Thus a flat *tangential rate is the order-preserving closure of the bookkeeping rule [46, 54].*

Interpretation. *Within this framework, a "flat rotation curve" is the informationally minimal configuration that maintains constant flux of distinguishable events as circumference grows. The compensating* $K(r)$ *is simply the residue of non-closure that enforces causal consistency across radii [101, 10].*

Scope. *This extrapolation is purely formal. It demonstrates how a constant tangential rate can emerge from an invariant-count condition, not how galaxies move.*

6.2 Empirical Test: Normal Equations for Rotation Invariance

N.B. This section specifies a falsifiable data model for galaxy rotation curves derived from the order-preserving transport condition. It implements the single-parameter limit implied by Chapter 2, where the calculus of measurement allows only one free constant to curve-fit any closed causal system. The model therefore has a unique intercept but no tunable slope parameters.

Invariant. Order-preserving transport with constant flux of distinguishability implies

$$r\,\rho(r)\,v_\theta(r) = \Phi, \qquad \Phi > 0 \text{ constant.}$$

Here $\rho(r)$ represents the local count density (a proxy for informational concentration) and $v_\theta(r)$ the observed tangential speed.

Observables. Let $I(r)$ denote surface brightness and $\kappa > 0$ a proportionality linking brightness to count density, $\rho(r) \propto \kappa I(r)$. Substituting gives the predicted profile

$$v_\theta(r) = \frac{\Phi}{r\,\kappa\,I(r)}.$$

Taking logarithms produces a linear empirical relation:

$$\underbrace{\log v_\theta(r)}_{y(r)} = \underbrace{\theta_0}_{\log\Phi - \log\kappa} - \log r - \log I(r) + \varepsilon(r),$$

where $\mathbb{E}[\varepsilon(r)] = 0$ under the invariant.

Normal equation (strict test). Define $z_i := \log r_i + \log I_i$ and $y_i := \log v_i$. The strict model is $y_i = \theta_0 - z_i + \varepsilon_i$ with a fixed slope of -1 on both $\log r$ and $\log I$. Only one constant, θ_0, remains free in accordance with Chapter 2. The ordinary-least-squares normal equation reduces to

$$\hat{\theta}_0 = \frac{1}{n}\sum_{i=1}^{n}(y_i + z_i),$$

and falsification is assessed by testing for systematic structure in residuals $\hat{\varepsilon}_i = y_i - \hat{\theta}_0 + z_i$.

Relaxed model with curvature residue. To capture permissible second-order deviations $K(r)$ due to geometric residue, extend to

$$y_i = \theta_0 - z_i + \sum_{j=1}^{m} B_j(r_i)w_j + \varepsilon_i,$$

where B_j are spline bases with penalty $\lambda\|Dw\|^2$. The penalized normal equations are

$$\begin{bmatrix} n & \mathbf{1}^\top B \\ B^\top \mathbf{1} & B^\top B + \lambda D^\top D \end{bmatrix} \begin{bmatrix} \theta_0 \\ w \end{bmatrix} = \begin{bmatrix} \sum_i (y_i + z_i) \\ B^\top (y + z) \end{bmatrix}.$$

When $w = 0$ the invariant holds exactly; significant w indicates structured departure.

Population form. Across galaxies g, intercepts $\theta_{0,g}$ vary but slopes remain fixed:

$$y_{ig} = \theta_{0,g} - \log r_{ig} - \log I_{ig} + \varepsilon_{ig}.$$

A mixed-effects regression with random intercepts but common slopes tests universality of the invariant; a slope differing from -1 falsifies it.

Falsifiable predictions.

1. **Fixed slopes:** Coefficients on $\log r$ and $\log I$ are -1 within uncertainty; deviation falsifies the invariant.
2. **White residuals:** Adjusted residuals $\hat{\varepsilon}_i$ show no systematic radial trend.
3. **Cross-sample invariance:** Slopes are common to all galaxies; intercepts vary only by normalization.
4. **Low-brightness scaling:** Lower I implies higher v at fixed r; violation falsifies order-preserving transport.

Scope. This regression embodies the Chapter 2 principle that a consistent measurement law introduces at most one free constant. Its acceptance or rejection is therefore directly falsifiable: systematic deviations in slope or curvature constitute empirical evidence against the order-preserving flux hypothesis.

6.3 Global Constraint as the Einstein Equation

The final step is to impose global consistency on the causal network. Local rules of separation and transport guarantee Martin consistency within each neighborhood, but finite observation requires that these neighborhoods overlap. The residual curvature computed in the previous section measures the degree to which local order fails to close globally. The Einstein equation expresses the condition under which that failure is exactly balanced by the stress encoded in the Causal Universe Tensor.

6.3.1 From Local Residue to Global Balance

Let the scalar invariants of the Causal Universe Tensor be denoted $T_{\mu\nu}$—the symmetric bilinear form that measures the density and flux of distinguishability. The curvature invariants of the causal gauge are summarized by the Einstein tensor,

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu}.$$

Both tensors share the same divergence-free property, $\nabla^\mu G_{\mu\nu} = \nabla^\mu T_{\mu\nu} = 0$, a differential expression of Martin consistency. The only admissible global solution is therefore their proportional equality,

$$G_{\mu\nu} = 8\pi\, T_{\mu\nu}.$$

This is the Einstein field equation, reinterpreted as the global constraint that restores balance between the residue of inconsistency (curvature) and the finite structure of distinguishability (stress).

6.3.2 Interpretation in the Causal Framework

The Einstein equation states that curvature is not an independent source of force but the universe's adjustment to maintain causal coherence. Energy and stress arise from the finiteness of measurement; curvature arises from the impossibility of reconciling all such measurements globally. The equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$ enforces that these two forms of inconsistency—informational and geometric—cancel exactly. When they do, the propagation of light remains Martin-consistent throughout the entire network.

In this view, gravitation is the manifestation of the universe correcting its own bookkeeping of distinctions. Mass–energy is simply the local density of finite observation, and curvature the global compensation that restores order. Spacetime bends not because matter exerts force, but because causal consistency demands it.

6.3.3 The Closure of the Gauge of Light

The Einstein equation thus completes the gauge of light. Beginning with the metric as the gauge of separation, the connection as the rule of causal transport, and curvature as the residue of inconsistency, the global constraint closes the system. All four structures arise from a single requirement: that the scalar invariants of the Causal Universe Tensor remain self-consistent under extension to the entire causal domain.

Remark 28. *In this formulation, general relativity is not a separate physical theory but the closure condition of the causal calculus. The Einstein tensor is the final differential form of Martin consistency; the stress–energy tensor is the discrete record of finite distinction. Their equality marks the point at which the universe's description becomes self-consistent. Beyond this, nothing remains to adjust—the gauge of light is complete.*

Chapter 7

The Conservation of Symmetry: From Noether to Mass

The gauge of light completes the classical description of the universe: it ensures that causal order is preserved at the limit of distinguishability. But the universe we observe is not smooth. Measurements are discrete, events occur finitely, and the invariants of the causal gauge fluctuate around their ideal values. These fluctuations are not errors—they are the quantum fields of the theory.

A quantum field arises whenever the invariants of the Causal Universe Tensor are permitted to vary locally while maintaining global Martin consistency. Each allowed fluctuation corresponds to a redistribution of causal order between neighboring observers. The field is therefore not an additional substance laid over spacetime but a dynamic adjustment of the gauge itself, mediating the exchange of distinguishability across finite domains.

In this framework, the traditional wavefunction reappears as the probability amplitude for maintaining order under repeated finite observations. Its complex phase represents the orientation of the causal gauge in informational space, while its magnitude measures the stability of that order. The principle of superposition follows directly from the linearity of causal combinations:

multiple consistent histories can coexist until observation resolves a single extension of the network.

Quantization enters as the recognition that order cannot be subdivided indefinitely. Every causal update exchanges a finite unit of distinguishability—a discrete increment of information. The Planck constant $\hbar$ expresses this minimal step size: the smallest action through which the universe can modify its own gauge while remaining consistent. The commutation relations of quantum theory are therefore expressions of finite causal resolution, not axioms of measurement.

This chapter develops these ideas systematically. Beginning with the Noether currents of the causal gauge, we derive the corresponding quantum fields as their discrete fluctuations. We then show how these fields propagate through the Causal Universe Tensor, producing the familiar quantum wave equations as conditions of statistical Martin consistency. Finally, we interpret entanglement as the correlated selection of events across overlapping causal neighborhoods—the quantum signature of global order maintained through finite means.

Remark 29. *Classical physics ends where the gauge of light closes; quantum physics begins where it wavers. Every quantum field is a small deviation from perfect causal consistency, a harmonic of order itself. The task of this chapter is to make that statement precise.*

Example: Photoelectric Effect as Discrete Termination of a Continuous Wave

Statement. The photoelectric threshold and linear kinetic energy law express that measurement terminates the wave by discrete event selection.

Key relation.

$$K_{\max} \;=\; h\nu - \Phi, \qquad \nu \geq \nu_0 = \frac{\Phi}{h}.$$

Reciprocity framing. A continuous field carries phase/energy, but a detection event is a refinement of the partition $P_n \to P_{n+1}$ at the cathode surface. The selection rule enforces conservation in the bookkeeping channel: the work function Φ is the minimal distinguishability cost to register an event.

Operational consequence. Intensity controls the *rate* of refinement (event count per time), but frequency controls the *possibility* of refinement (predicate becomes admissible only if $\nu \geq \nu_0$).

7.1 The Action Functional

The action functional provides the statistical completion of the causal gauge. It measures the total consistency of a causal configuration across all finite observations. In the classical limit, the action is stationary: each variation vanishes, and the universe evolves along trajectories of perfect causal balance. In the quantum regime, these variations accumulate as finite fluctuations of order, and the path integral of all such histories defines the observable field.

7.1.1 Definition from the Causal Universe Tensor

Let $\mathcal{T}^{\mu\nu}$ denote the Causal Universe Tensor, whose scalar invariants measure the degree of causal consistency. The *action functional* $\mathcal{S}$ is defined as the integral of these invariants over the causal domain:

$$\mathcal{S} = \int \mathcal{L}\big(\mathcal{T}^{\mu\nu}, g_{\mu\nu}, \nabla_\lambda \mathcal{T}^{\mu\nu}\big)\sqrt{-g}\, d^4x.$$

The Lagrangian density $\mathcal{L}$ encodes the local rule by which order is preserved and exchanged. In the classical limit, $\delta\mathcal{S} = 0$ reproduces the field equations of the gauge of light; in the quantum limit, $\mathcal{S}$ fluctuates discretely by units of $\hbar$, reflecting the minimal step size in causal adjustment.

7.1.2 Physical Interpretation

The action $\mathcal{S}$ plays the role of a global consistency measure. Each admissible history of the universe contributes a complex amplitude

$$\Psi[\mathcal{T}] \propto e^{i\mathcal{S}[\mathcal{T}]/\hbar},$$

representing the phase of causal order associated with that configuration. When summed over all histories consistent with Martin's Axiom, these amplitudes interfere, and the stationary-phase paths correspond to the classical trajectories of least action. The non-stationary contributions produce the quantum corrections—the finite discrepancies among partially consistent causal extensions.

In this interpretation, $\hbar$ is not an arbitrary constant but the fundamental unit of distinguishability in causal evolution. It measures the minimal action by which the universe can update its gauge without violating order. The classical limit $\hbar \to 0$ corresponds to infinitely fine causal resolution, while the quantum limit expresses the graininess of finite observation.

7.1.3 Noether Currents of the Causal Gauge

Symmetries of the Lagrangian correspond to invariances of causal order. By Noether's theorem, each continuous symmetry yields a conserved current

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\nabla_\mu \phi)}\,\delta\phi, \qquad \nabla_\mu J^\mu = 0.$$

These currents are the quantum fields' classical shadows: energy, momentum, and charge arise as conserved flows of causal order through the network. Their quantization in subsequent sections will describe the discrete exchange of distinguishability among interacting observers.

Remark 30. *The action functional is the expectation value of Martin consistency over all admissible histories. In the classical regime, it is stationary;*

in the quantum regime, it oscillates. The universe, viewed through this lens, is a sum over self-consistent paths, each differing from the others by integral multiples of the minimal action $\hbar$. Quantum mechanics is therefore not a separate theory but the statistical theory of finite causal order.

7.2 The Application of Noether

Once the action functional has been defined, its symmetries determine the quantities that remain conserved under causal evolution. This is the content of Noether's theorem, here understood as the statistical mechanics of invariance: whenever the ensemble of admissible causal configurations possesses a continuous symmetry, the expectation value of the corresponding quantity remains fixed across all Martin-consistent histories.

7.2.1 Symmetry and Conservation as Statistical Identities

Let the partition function of the causal gauge be written

$$Z = \int \exp\left(\frac{i}{\hbar}\mathcal{S}[\mathcal{T}]\right) \mathcal{D}\mathcal{T},$$

where the integration ranges over all locally consistent configurations of the Causal Universe Tensor. An infinitesimal transformation of variables $\mathcal{T} \to \mathcal{T} + \delta\mathcal{T}$ that leaves the measure and the action invariant,

$$\delta\mathcal{S} = 0,$$

implies that the partition function is unchanged:

$$\delta Z = 0.$$

Differentiating under the integral sign yields the statistical conservation law

$$\langle \nabla_\mu J^\mu \rangle = 0,$$

where

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\nabla_\mu \phi)}\,\delta\phi$$

is the current associated with the transformation. Thus, each continuous symmetry of the Lagrangian corresponds to a conserved flux of causal order. Energy, momentum, and charge appear not as primitive physical entities but as statistical invariants of the causal ensemble.

7.2.2 Conserved Quantities of the Causal Gauge

1. **Translational invariance** → Conservation of energy–momentum:

$$\nabla_\mu T^{\mu\nu} = 0.$$

2. **Rotational invariance** → Conservation of angular momentum:

$$\nabla_\mu J^{\mu\nu} = 0, \quad J^{\mu\nu} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}.$$

3. **Internal phase invariance** → Conservation of charge:

$$\nabla_\mu j^\mu = 0.$$

Each of these laws arises from a symmetry of the Causal Universe Tensor under transformations that leave the causal measure invariant. In this sense, Noether's theorem is the thermodynamics of causal order: it equates symmetry with conservation and conservation with informational equilibrium.

Thought Experiment 7.2.1 (The Harmonic Oscillator as a Closed Loop of Reciprocal Measurement)**.** *The harmonic oscillator is the minimal causal* *system in which measurement and variation form a reversible cycle. Let $U(t)$ denote the measured amplitude of a single mode of the universe tensor. Successive reciprocal updates obey*

$$\delta^2 U + \omega^2 U = 0,$$

where δ is the discrete variation operator and ω characterizes the curvature of the local informational potential. In the continuum limit this becomes

$$\frac{d^2U}{dt^2} + \omega^2 U = 0,$$

the familiar harmonic–oscillator equation.

Each half–cycle corresponds to an exchange between distinguishability and variation: when the system reaches maximal distinction (turning point), the variation vanishes; when the distinction is minimal (crossing through zero), variation is maximal. The energy functional

$$E = \tfrac{1}{2}\left[(\dot{U})^2 + \omega^2 U^2\right]$$

is the invariant scalar of this causal pair— the quantity preserved under all order–preserving updates.

Quantization follows from the Axiom of Finite Observation: only discrete counts of distinguishable configurations fit within one causal period. Applying the Reciprocity Law yields the spectrum

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right),$$

showing that each oscillation cycle admits an integer number of informational quanta plus a residual half–count from causal incompleteness.

In this view, the harmonic oscillator is the archetype of finite reciprocity: a closed loop in which measurement and variation exchange roles while preserving total informational curvature. All quantized fields—phonons, pho-

tons, and normal modes of the causal tensor—are higher–dimensional extensions of this single reciprocal circuit.

7.2.3 Statistical Interpretation

In the quantum regime, these conservation laws are satisfied only in expectation. The ensemble of finite causal updates explores neighboring histories whose individual actions differ by multiples of $\hbar$, but the average fluxes of order remain constant. The classical conservation laws emerge as the limit in which fluctuations of the action vanish and every observer's measurement agrees. Quantum mechanics, in contrast, records the statistics of these fluctuations.

Remark 31. *Noether's theorem closes the loop between mechanics and statistics. Every symmetry of the causal gauge produces a conserved current, and every conservation law describes equilibrium in the flow of distinguishability. In this sense, the field equations of physics are nothing more than the statistical statements of Martin consistency expressed through symmetry.*

ectionConservation

Conservation laws follow from symmetries of the action. In the causal framework, these are statements that the bookkeeping of distinguishability is invariant under relabelings that shift the record in space or time. The resulting Noether currents are the conserved flows of causal order.

7.2.4 Translations and the Stress–Energy Tensor

Let $\mathcal{S} = \int \mathcal{L}\sqrt{-g}\,d^4x$ be the action of the Causal Universe Tensor fields (collectively ϕ). Under an infinitesimal spacetime translation $x^\mu \mapsto x^\mu + \varepsilon^\mu$, the fields transform as $\delta\phi = \varepsilon^\nu \nabla_\nu \phi$ and $\delta\mathcal{L} = \varepsilon^\nu \nabla_\nu \mathcal{L}$. Invariance of the action ($\delta\mathcal{S} = 0$) yields the Noether current

$$J^{\mu}{}_{\nu} = \frac{\partial \mathcal{L}}{\partial(\nabla_\mu \phi)} \nabla_\nu \phi - \delta^{\mu}{}_{\nu} \mathcal{L},$$

whose covariant divergence vanishes:

$$\nabla_\mu J^{\mu}{}_{\nu} = 0.$$

Identifying $T^{\mu}{}_{\nu} \equiv J^{\mu}{}_{\nu}$ (or its symmetrized Belinfante form when needed) gives the *stress–energy tensor* with

$$\nabla_\mu T^{\mu}{}_{\nu} = 0.$$

In local inertial coordinates this reduces to the familiar continuity laws $\partial_\mu T^{\mu\nu} = 0$.

Example: Compton Scattering as Reciprocal Momentum Bookkeeping

Statement. The Compton shift measures the finite difference of momentum across an event pair, i.e. the reciprocity map in momentum space.

Key relation.

$$\Delta\lambda \;\equiv\; \lambda' - \lambda \;=\; \frac{h}{m_e c}\big(1 - \cos\theta\big).$$

Reciprocity framing. One detection event refines the joint partition of (photon, electron). Bookkeeping enforces the Noether current (translation symmetry) at the refinement:

$$p_\gamma + p_e \;=\; p'_\gamma + p'_e, \qquad E_\gamma + E_e \;=\; E'_\gamma + E'_e.$$

Eliminating the electron internal variables yields the observed $\Delta\lambda$, a scalar invariant of the event contraction.

Operational consequence. The shift is the *measured* residue after enforcing equality of conjugate Noether charges at a single refinement step.

7.2.5 Energy and Momentum Densities

Write u^μ for the future-directed unit normal to a Cauchy slice Σ (with volume element $d\Sigma_\mu = u_\mu\, d^3x\sqrt{\gamma}$). The total four-momentum is

$$P^\nu \;=\; \int_\Sigma T^{\mu\nu}\, d\Sigma_\mu,$$

so that

$$E \equiv P^0 = \int_\Sigma T^{\mu\nu} u_\mu \xi^{(t)}_\nu\, d^3x\sqrt{\gamma}, \qquad \mathbf{P}^i = \int_\Sigma T^{\mu\nu} u_\mu \xi^{(i)}_\nu\, d^3x\sqrt{\gamma},$$

where $\xi^{(t)}$ and $\xi^{(i)}$ denote the time and spatial translation generators (Killing vectors in symmetric backgrounds). Covariant conservation implies slice-independence:

$$\frac{d}{d\tau}P^\nu \;=\; \int_\Sigma \nabla_\mu T^{\mu\nu}\, d\Sigma \;=\; 0.$$

7.2.6 Bookkeeping Interpretation

Causally, $\nabla_\mu T^{\mu\nu} = 0$ is a statement that *what leaves one finite neighborhood must enter another*. The stress–energy tensor tallies the flow of distinguishability through the network; its vanishing divergence is the ledger's balance condition. Translational symmetry means we can shift the labels of events without changing that tally. Conservation of *energy* is the invariance of the temporal bookkeeping column; conservation of *momentum* is the invariance of the spatial columns. In discrete form, for any compact region $\mathcal{R}$ with boundary $\partial\mathcal{R}$,

$$\frac{d}{d\tau}\int_{\mathcal{R}} T^{0\nu}\, d^3x \;=\; -\int_{\partial\mathcal{R}} T^{i\nu}\, n_i\, dS,$$

so the time rate of change of the “inventory” inside equals the net outward flux across the boundary—pure bookkeeping.

7.2.7 Curved Backgrounds and Killing Symmetries

When the metric varies, conserved charges are tied to spacetime symmetries. If ξ^ν is a Killing vector ($\nabla_{(\mu}\xi_{\nu)} = 0$), then

$$\nabla_\mu \big(T^\mu{}_\nu \, \xi^\nu\big) = 0,$$

and the associated charge

$$Q[\xi] \;=\; \int_\Sigma T^\mu{}_\nu \, \xi^\nu \, d\Sigma_\mu$$

is conserved. Energy arises from time-translation symmetry ($\xi = \partial_t$), momentum from spatial translations, and angular momentum from rotations. In each case, the “conservation law” is precisely the statement that the ledger of scalar invariants computed by the Causal Universe Tensor is unchanged under the corresponding relabeling of events.

Remark 32. *Conservation is not mysterious dynamics; it is consistency of accounting. Noether's theorem says: if the rules for keeping the ledger do not change when we shift the page in space or time, then the totals on that page do not change either. In the causal calculus, those totals are P^ν, and their invariance is exactly $\nabla_\mu T^{\mu\nu} = 0$.*

Thought Experiment 7.2.2 (Conservation of Energy for a Free Scalar Field)**.** *Consider a real Klein–Gordon field ϕ in flat spacetime with*

$$\mathcal{L} \;=\; \tfrac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi \;-\; \tfrac{1}{2}\,m^2\phi^2, \qquad \eta_{\mu\nu} = \mathrm{diag}(-,+,+,+).$$

The (symmetric) stress–energy tensor is

$$T^{\mu\nu} \;=\; \partial^\mu\phi\,\partial^\nu\phi \;-\; \eta^{\mu\nu}\mathcal{L}.$$

Energy density and energy flux are then

$$\mathcal{E} \;\equiv\; T^{00} = \tfrac{1}{2}\Big(\dot\phi^2 + |\nabla\phi|^2 + m^2\phi^2\Big), \qquad S^i \;\equiv\; T^{0i} = \dot\phi\,\partial^i\phi.$$

Continuity (bookkeeping) equation. *Using the Euler–Lagrange equation $\Box\phi + m^2\phi = 0$ and differentiating,*

$$\partial_t\mathcal{E} = \dot\phi\,\ddot\phi + \nabla\phi\cdot\nabla\dot\phi + m^2\phi\,\dot\phi = \dot\phi\big(\ddot\phi - \nabla^2\phi + m^2\phi\big) + \nabla\cdot(\dot\phi\,\nabla\phi) = \nabla\cdot(\dot\phi\,\nabla\phi),$$

so

$$\partial_t\mathcal{E} + \nabla\cdot(-\dot\phi\,\nabla\phi) = 0 \quad\Longleftrightarrow\quad \partial_\mu T^{\mu 0} = 0.$$

This is pure bookkeeping: the time rate of change of energy density equals the negative divergence of the energy flux.

Integrated conservation law. *Integrate over a fixed region $\mathcal{R}$ with outward normal $\mathbf{n}$:*

$$\frac{d}{dt}\int_{\mathcal{R}}\mathcal{E}\,d^3x = -\int_{\partial\mathcal{R}}\mathbf{S}\cdot\mathbf{n}\,dS.$$

If fields vanish (or are periodic) on the boundary so the surface term is zero, then the total energy

$$E \;=\; \int_{\mathbb{R}^3}\mathcal{E}\,d^3x$$

is conserved: $\frac{dE}{dt} = 0$.

Causal bookkeeping interpretation. *T^{00} tallies the "inventory" of distinguishability stored in a region (kinetic + gradient + mass terms). The flux T^{0i} records how that inventory flows across the boundary. The continuity equation says the ledger balances exactly: what leaves here enters there.*

Translation invariance is the statement that the rules of this ledger do not change when we shift the page in time; hence the total energy remains the same.

Thought Experiment 7.2.3 (Feynman Diagram as a Tensor Expansion of the Field)**.** *In conventional quantum field theory, perturbation expansions of the generating functional are represented diagrammatically: vertices encode local interactions and propagators connect them according to the causal structure of spacetime. In the causal formulation developed here, the same construction arises directly from the Universe Tensor.*

Each vertex corresponds to an event tensor $E_k \in T(V)$ *contributing a measurable distinction within the causal order. A propagator corresponds to an admissible contraction between event tensors—a bilinear map*

$$\langle E_i, E_j \rangle = \mathrm{Tr}(E_i^\top \, G \, E_j),$$

where G *is the causal propagator enforcing Martin consistency between the connected events. The complete amplitude for a process is therefore the contraction of the ordered product*

$$U_n = \sum_{k=1}^{n} E_k,$$

with all admissible propagators. The resulting scalar invariants of U_n *constitute the measurable quantities of the theory.*

Thus, a Feynman diagram is the graphical representation of a tensor contraction in the causal algebra: each diagram corresponds to one term in the finite expansion of the Universe Tensor, and summing over all diagrams is equivalent to enforcing global consistency of causal order. What appears in standard field theory as a perturbation series is, in this formalism, a finite enumeration of distinguishable causal relations—a bookkeeping identity derived from the Reciprocity Law rather than using calculus.

7.3 Angular Momentum and Spin

Rotational (and more generally Lorentz) invariance of the action produces a conserved tensorial current whose charges are the total angular momentum. Decomposing that current separates *orbital* from *spin* contributions; their sum is conserved.

7.3.1 Noether Current for Lorentz Invariance

Let the action $\mathcal{S} = \int \mathcal{L}(\phi, \nabla\phi, g)\sqrt{-g}\, d^4x$ be invariant under infinitesimal Lorentz transformations $x^\mu \mapsto x^\mu + \omega^\mu{}_\nu x^\nu$ with antisymmetric $\omega_{\mu\nu} = -\omega_{\nu\mu}$, and induced field variation $\delta\phi = -\frac{1}{2}\omega_{\rho\sigma}\, \Sigma^{\rho\sigma}\phi - \omega^\mu{}_\nu x^\nu \nabla_\mu \phi$, where $\Sigma^{\rho\sigma}$ are the generators on the fields. Noether's theorem yields the (canonical) angular-momentum current

$$J^{\lambda\rho\sigma}_{\text{can}} = x^\rho T^{\lambda\sigma}_{\text{can}} - x^\sigma T^{\lambda\rho}_{\text{can}} + S^{\lambda\rho\sigma}, \qquad \partial_\lambda J^{\lambda\rho\sigma}_{\text{can}} = 0,$$

with canonical stress tensor $T^\lambda{}_{\nu,\text{can}} = \frac{\partial\mathcal{L}}{\partial(\partial_\lambda\phi)}\, \partial_\nu\phi - \delta^\lambda{}_\nu \mathcal{L}$ and spin current

$$S^{\lambda\rho\sigma} = \frac{\partial\mathcal{L}}{\partial(\partial_\lambda\phi)}\, \Sigma^{\rho\sigma}\phi = -S^{\lambda\sigma\rho}.$$

Thought Experiment 7.3.1 (Spin–$\frac{1}{2}$ as Two–Valued Causal Orientation)**.** *Spin–$\frac{1}{2}$ particles arise when the local symmetry of the universe tensor is represented not on spacetime vectors but on their double cover. Under a full 2π rotation, the causal ordering of distinguishable events reverses sign before returning to its original configuration after 4π. This two–valuedness expresses the fundamental antisymmetry of distinction.*

Let $\psi(x)$ denote a two–component field that transports the minimal unit of causal orientation. Its dynamics follow from the Lorentz–invariant action

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi,$$

where D_μ is the gauge–covariant derivative and the γ^μ generate the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}.$$

Each γ^μ acts as a local operator of causal rotation: applying it changes the orientation of the measurement frame while preserving causal order. Because the algebra squares to unity only after two applications, a single 2π rotation introduces a minus sign, $\psi \to -\psi$, revealing that the physical state is defined on the double cover $\mathrm{Spin}(3,1)$ of the Lorentz group.

In the informational picture, the two components of ψ encode the forward and reverse orientations of causal distinction—measurement and variation. The spinor's phase thus records how the act of observation twists within the causal network. Quantized angular momentum

$$S = \tfrac{\hbar}{2}$$

emerges as the minimal unit of such rotational bookkeeping: the smallest nontrivial representation of reciprocity under continuous rotation.

Spin–$\frac{1}{2}$ therefore exemplifies the finite, antisymmetric nature of causal orientation. A complete 4π turn is required for full restoration of distinguishability, making the spinor the algebraic expression of the universe tensor's two–sheeted structure in orientation space.

7.3.2 Belinfante–Rosenfeld Improvement

The canonical $T_{\mu\nu}$ need not be symmetric. Define the Belinfante superpotential

$$B^{\lambda\rho\sigma} = \tfrac{1}{2}\Big(S^{\rho\lambda\sigma} + S^{\sigma\lambda\rho} - S^{\lambda\rho\sigma}\Big), \qquad B^{\lambda\rho\sigma} = -B^{\lambda\sigma\rho}.$$

The *improved* symmetric stress tensor and current are

$$T_B^{\mu\nu} = T_{\text{can}}^{\mu\nu} + \partial_\lambda\Big(B^{\lambda\mu\nu} - B^{\mu\lambda\nu} - B^{\nu\lambda\mu}\Big), \qquad J_B^{\lambda\rho\sigma} = x^\rho T_B^{\lambda\sigma} - x^\sigma T_B^{\lambda\rho},$$

and obey $\partial_\lambda T_B^{\lambda\nu} = 0$, $\partial_\lambda J_B^{\lambda\rho\sigma} = 0$. The spin density has been absorbed into a symmetric T_B so that the total angular momentum current is purely "orbital" in form; its integrated charge still equals *orbital + spin*.

7.3.3 Conserved Charges

For a Cauchy slice Σ with normal u_λ,

$$M^{\rho\sigma} = \int_\Sigma J^{\lambda\rho\sigma}\, d\Sigma_\lambda = \int_\Sigma \left(x^\rho T_B^{\lambda\sigma} - x^\sigma T_B^{\lambda\rho}\right) d\Sigma_\lambda, \qquad \frac{d}{d\tau} M^{\rho\sigma} = 0.$$

In 3D language (flat space, $u_\lambda = (1,0,0,0)$), the spatial components give the angular momentum vector $\mathbf{J} = \int d^3x\, \big(\mathbf{x} \times \mathbf{p}\big) + \mathbf{S}$, with momentum density $\mathbf{p} = T_B^{0i}\, \hat{\mathbf{e}}_i$ and spin density $\mathbf{S}$ encoded via S^{0ij}.

7.3.4 Worked Examples

Real scalar (spin 0). For $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\, \partial^\mu\phi - \frac{1}{2}m^2\phi^2$, $\Sigma^{\rho\sigma} = 0$ so $S^{\lambda\rho\sigma} = 0$. The Belinfante step is trivial and

$$\mathbf{J} = \int d^3x\, \mathbf{x} \times \big(\dot{\phi}\, \nabla\phi\big),$$

purely orbital. Conservation $\partial_\lambda J^{\lambda\rho\sigma} = 0$ reduces to $\partial_\mu T^{\mu\nu} = 0$ (already shown) plus antisymmetry.

Dirac field (spin 1/2). For $\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$, the generators are $\Sigma^{\rho\sigma} = \frac{i}{4}[\gamma^\rho, \gamma^\sigma]$, giving nonzero spin current

$$S^{\lambda\rho\sigma} = \tfrac{1}{2}\, \bar{\psi}\, \gamma^\lambda \Sigma^{\rho\sigma} \psi.$$

The Belinfante tensor $T_B^{\mu\nu} = \frac{i}{4}\bar{\psi}\big(\gamma^\mu \overleftrightarrow{\partial^\nu} + \gamma^\nu \overleftrightarrow{\partial^\mu}\big)\psi$ is symmetric and conserved, and the total charge $M^{\rho\sigma}$ includes intrinsic spin; in the particle rest frame this yields the familiar $\frac{1}{2}\hbar$.

7.3.5 Bookkeeping Interpretation

Rotational invariance says the ledger of causal distinctions is unchanged when we rotate our labeling rules. The orbital term tracks the "moment arm" of the flow of distinguishability ($\mathbf{x}\times\mathbf{p}$). The spin term tallies how the *label structure of the field itself* transforms under rotations (internal frame rotation via $\Sigma^{\rho\sigma}$). The Belinfante improvement is just a repackaging of the ledger so that the stress tensor carries the full conserved charge in a symmetric form—useful whenever the geometry (gravity) couples to $T_{\mu\nu}$.

Remark 33. *Total angular momentum is conserved because the action is invariant under Lorentz rotations. Orbital and spin are bookkeeping columns in the same invariant total; how you apportion them depends on your accounting scheme (canonical vs. Belinfante), not on the physics.*

7.4 Gauge Fields as Local Noether Symmetries

Global symmetries ensure that the totals in the causal ledger remain unchanged when every observer applies the same transformation. When the symmetry parameters vary from point to point, the bookkeeping must introduce additional terms to maintain local consistency. These new terms are the *gauge fields* of the theory: dynamic corrections that restore Martin consistency under spatially varying transformations.

7.4.1 From Global to Local Symmetry

Consider a field $\phi(x)$ transforming under a continuous group G with infinitesimal parameter α^a and generators T^a:

$$\delta\phi = i\,\alpha^a T^a \phi.$$

If α^a is constant, the action $\mathcal{S} = \int \mathcal{L}(\phi, \nabla\phi)\, d^4x$ is invariant, and Noether's theorem yields a conserved current J^μ_a. If α^a becomes a function of position, $\alpha^a = \alpha^a(x)$, an extra term appears,

$$\delta\mathcal{L} = i\,(\partial_\mu \alpha^a)\,\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\,T^a\phi,$$

breaking the conservation law. To preserve local invariance, the derivative ∂_μ must be replaced by a *covariant derivative*

$$D_\mu\phi = (\partial_\mu - ig\,A^a_\mu T^a)\phi,$$

where the compensating field A^a_μ transforms as

$$\delta A^a_\mu = \frac{1}{g}\,\partial_\mu\alpha^a + f^{abc}\alpha^b A^c_\mu.$$

The new Lagrangian

$$\mathcal{L} = \mathcal{L}(\phi, D_\mu\phi) - \tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu,$$

is invariant under the full local symmetry. The field strength $F^a_{\mu\nu}$ is the curvature of the gauge connection A^a_μ—the residue of non-commuting parallel transports in the internal symmetry space.

Example: Aharonov–Bohm Phase as Pure Gauge Holonomy

Statement. A nontrivial loop integral of the connection shifts interference with no local force—measurement of gauge holonomy.

Key relation.

$$\Delta\varphi \;=\; \frac{q}{\hbar}\oint_\gamma \mathbf{A}\cdot d\ell \;=\; \frac{q\,\Phi_B}{\hbar}.$$

Reciprocity framing. The partition is unchanged locally (no field in the slits), but the selected update accumulates a path-dependent phase—an element of the connection's holonomy group. Interference shift records the gauge's parallel transport rule.

Operational consequence. Local indistinguishability with global inequivalence: a canonical example where measurement reads a *global* invariant of the gauge without local curvature along the paths.

7.4.2 Interpretation in the Causal Framework

In the causal picture, global symmetry corresponds to relabeling the entire causal network by a uniform rule; local symmetry corresponds to allowing each neighborhood to choose its own labeling convention. The gauge field A_μ^a records how those conventions differ and how information must be exchanged between neighboring regions to keep the global ledger balanced. It is the *connection form of causal order* in informational space.

Curvature $F_{\mu\nu}^a$ measures the residual inconsistency that appears when these local labelings are carried around a closed causal loop—exactly analogous to the spacetime curvature derived earlier from $\Gamma^\lambda_{\mu\nu}$. Gauge bosons are therefore the finite, propagating corrections by which the universe restores Martin consistency across overlapping informational domains.

Thought Experiment 7.4.1 (Aharonov–Bohm Effect as a Test of Causal Gauge Consistency)**.** *The Aharonov–Bohm experiment demonstrates that the physically relevant quantity in electromagnetism is not the field strength $F_{\mu\nu}$ alone but the connection A_μ that governs causal phase transport.*

Consider an electron beam split into two coherent branches encircling a region containing a confined magnetic flux Φ, with no field present along either path. In the causal formulation, each branch corresponds to a sequence of ordered events $\{E_{1,k}\}$ and $\{E_{2,k}\}$ transported by the local gauge connection A_μ. The Reciprocity Law requires that each infinitesimal update preserve order:

$$E_{k+1} = E_k + \Phi^{-1}(A_\mu \, dx^\mu),$$

so that the cumulative phase acquired along a closed loop is

$$\Delta\phi = \frac{e}{\hbar} \oint A_\mu \, dx^\mu = \frac{e\Phi}{\hbar}.$$

Although the magnetic field vanishes along both paths ($F_{\mu\nu} = 0$ locally), the two causal chains differ by a holonomy in the connection—an informational mismatch in the bookkeeping of phase. When the beams are recombined, their interference pattern depends on $\Delta\phi$: shifting continuously as the enclosed flux changes by fractions of the flux quantum h/e.

In the causal gauge picture, this effect shows that the universe tensor records not merely local field strengths but the global consistency of the connection. The vector potential A_μ is the differential form of causal memory; its holonomy measures how distinction is transported around a loop. The Aharonov–Bohm interference is thus the experimental detection of a nontrivial element of the causal holonomy group—the smallest observable instance of curvature without force.

7.4.3 Bookkeeping of Local Consistency

In statistical terms, each gauge symmetry adds a new column to the causal ledger. Local invariance means that the exchange rates between these columns are position-dependent, and A^a_μ supplies the conversion factors that keep the books balanced. The continuity equation

$$\nabla_\mu J^\mu_a = 0$$

expresses the same principle as before: what leaves one neighborhood enters another, but now for every internal degree of freedom labeled by a. The gauge field guarantees that this exchange is recorded consistently even when observers adopt different local frames.

Remark 34. *Every gauge field is a Noether correction promoted to locality.*

It is the differential accountant of causal order, ensuring that symmetry—and hence conservation—holds point by point. Curvature is the residue of that accounting around a loop; interaction is the redistribution of causal balance between neighboring observers. Quantum field theory is therefore the calculus of local Noether symmetries of the Causal Universe Tensor.

7.5 Mass and the Breaking of Symmetry

Perfect causal symmetry implies motion at the limit of distinguishability—the null trajectories of light. In this regime, the action and all of its Noether currents remain invariant under local gauge transformations, and the scalar invariants of the Causal Universe Tensor are preserved exactly. *Mass* appears when this invariance can no longer be maintained everywhere. It is the measure of how far a system deviates from perfect causal balance.

7.5.1 From Gauge Symmetry to Mass Terms

Suppose the Lagrangian density for a field ϕ is invariant under the local transformation $\phi \to e^{i\alpha(x)}\phi$. If the causal network experiences a finite delay in maintaining that invariance—so that the local transformation cannot be matched exactly between neighboring observers—the covariant derivative acquires a small, persistent residue. In the simplest case this appears as an additional quadratic term in the Lagrangian:

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(|\phi|), \qquad V(|\phi|) = \tfrac{1}{2}\mu^2|\phi|^2 + \tfrac{1}{4}\lambda|\phi|^4.$$

When the potential V selects a nonzero expectation value $\langle\phi\rangle = v/\sqrt{2}$, the gauge symmetry of the vacuum is spontaneously broken, and the covariant derivative term generates an effective mass for the gauge field:

$$m_A = g\,v.$$

The field no longer propagates at the causal limit; it carries a finite informational delay between cause and effect.

Thought Experiment 7.5.1 (Mexican Hat Potential and the Breaking of Informational Symmetry)**.** *In the causal formulation, symmetry breaking occurs when the universe tensor develops a preferred orientation in its space of distinguishable states. The simplest model of this phenomenon is the so–called Mexican hat potential, which encodes spontaneous differentiation in an initially symmetric field.*

Let ϕ be a complex scalar component of the causal gauge field. Its local informational curvature is represented by the potential

$$V(\phi) = \lambda\big(|\phi|^2 - v^2\big)^2, \qquad \lambda, v > 0.$$

For $|\phi| < v$, the curvature is positive and the symmetric state $\phi = 0$ is unstable; for $|\phi| = v$, the curvature vanishes along a circle of minima. Each choice of phase θ on this ring corresponds to an equally valid, order-preserving configuration of the universe tensor.

When a particular θ is selected by finite observation or causal fluctuation, the continuous $U(1)$ symmetry of the potential is reduced to the discrete subgroup that preserves that orientation. The resulting excitations decompose into two orthogonal modes:

$$\phi(x) = (v + h(x))e^{i\theta(x)},$$

where $h(x)$ represents measurable variations in magnitude (massive mode) and $\theta(x)$ represents phase fluctuations (massless Goldstone mode). Coupling this field to a local gauge connection A_μ converts the phase fluctuation into a longitudinal component of A_μ, endowing it with mass through the informational curvature of the potential.

Operationally, the Mexican hat potential marks the point where causal order can no longer cancel its own third variation: a finite bias in distin-

guishable states propagates through the reciprocity map as an effective mass term. In the informational picture, mass is the cost of maintaining a broken symmetry— the curvature required to remember which minimum was chosen.

7.5.2 Causal Interpretation

In the causal framework, symmetry breaking represents the loss of perfect order propagation. The gauge can no longer be reconciled exactly between neighboring domains, and a residual phase difference accumulates. That phase difference behaves as inertia: a tendency of the causal structure to resist change in its internal configuration. The quantity we call *mass* measures the curvature of causal order in the informational direction—the degree to which a system's internal symmetry lags behind the propagation of light.

Thus the Higgs mechanism appears as a natural bookkeeping adjustment. The scalar field ϕ provides an additional column in the ledger that can absorb the mismatch of local phase conventions. When the ledger cannot close exactly, the residual correction manifests as a finite mass term. Mass is therefore not a separate entity but the universe's accounting of imperfect causal synchronization.

7.5.3 Statistical View

In the statistical mechanics of causal order, mass quantifies the variance of the action around its stationary value:

$$m^2 \;\propto\; \big\langle (\delta \mathcal{S})^2 \big\rangle.$$

Lightlike propagation corresponds to zero variance: every observer's record of order agrees. Massive propagation corresponds to finite variance: local histories differ slightly, and the ensemble average restores consistency only statistically. The rest energy $E = mc^2$ measures the informational cost of maintaining a coherent description across those variations.

Remark 35. *Mass is the finite residue of broken symmetry—the price the universe pays for keeping its causal books consistent when perfect gauge balance cannot be sustained. Where light moves without lag, massive matter hesitates, accumulating phase in time. The rest mass of any field is thus the measure of its informational inertia: how much causal order must bend to preserve consistency within a finite universe.*

Thought Experiment 7.5.2 (Semiconductors as Partially Broken Informational Lattices)**.** *In a crystalline solid, the atoms form a periodic causal network—a lattice of distinguishable sites linked by local order relations. Within this structure, electrons occupy quantized informational states whose distinguishability depends on both lattice symmetry and the observer's partition of measurement.*

At zero temperature, all available states up to the Fermi level are filled, and the partition $\mathcal{P}_n$ groups occupied and unoccupied states into two disjoint causal classes. In a perfect insulator these classes are fully separated by a forbidden bandgap: no variation in the universe tensor can map one class into the other without violating order preservation. In a metal the classes overlap completely, forming a continuous manifold of accessible distinctions.

A semiconductor occupies the intermediate regime. Its informational lattice is nearly symmetric but not fully resolved; there exists a narrow causal boundary between filled and unfilled states. Thermal or dopant-induced perturbations refine the partition from $\mathcal{P}_n$ to $\mathcal{P}_{n+1}$, enabling limited causal transitions across the bandgap. The carrier density

$$n \;\propto\; e^{-E_g/k_{\mathrm{B}}T}$$

measures the probability that such a refinement occurs—an exponential suppression of distinguishability transitions with increasing gap energy E_g.

In this view, conduction arises when the partition between causal classes of electron states becomes permeable under variation. Doping, temperature, *and illumination are operations that adjust the informational curvature of the lattice, controlling how easily one class of distinguishability flows into another. Semiconductors are thus macroscopic examples of causal fuzziness under controlled refinement: a solid-state realization of partition dynamics between measurement and variation.*

7.6 Conclusion: Quantization as Finite Consistency

The classical universe is the ledger of perfect causal balance: every distinction is matched, every event accounted for, every observer's record consistent with the next. Quantum mechanics emerges when that perfection is relaxed—when the bookkeeping of order is carried out on a finite register. Each quantum of action, each exchange of $\hbar$, is a discrete adjustment in the causal gauge: the smallest step by which the universe can preserve consistency without infinite precision.

From this point of view, the quantum field is not a separate ontology but the statistical completion of the same calculus that defines the geometry of spacetime. The field amplitudes are probability weights for maintaining order across overlapping causal neighborhoods. Their phases encode the orientation of the gauge, and their interference expresses the collective effort of all observers to remain mutually consistent. The path integral is thus the partition function of causal order.

Mass, spin, and charge are the residues of that consistency process. Mass records temporal lag, spin records the rotational structure of labeling, and charge records the bookkeeping of internal symmetries. None are primitive; all arise from the same principle that distinguishes light: the demand that order be preserved even when the universe must correct itself locally.

In the causal formalism, conservation laws, gauge interactions, and quantization share a single origin. They are not independent laws written into nature but emergent regularities of a self-consistent informational network. The Causal Universe Tensor provides the grammar of that network; its contractions yield spacetime geometry, its variations yield fields, and its statistical extension yields the quantum.

Remark 36. *The universe is not made of matter or of energy, but of consistency. What we call physics is the continuous reconciliation of local descriptions of order, carried out one quantum at a time. Quantization is simply the discreteness of that reconciliation—the finite resolution of cause.*

Thought Experiment 7.6.1 (Thought Experiment: The Echo Chamber Maze and Curvature Residue)**. *N.B.*** *This experiment translates geometric curvature into informational inconsistency.*

Setup. *Navigate a maze by clapping; echoes trace causal paths. Straight corridors (flat metric) return clean echoes—perfect parallel transport. Curved passages distort the return, producing phase residue.*

Demonstration. *Walk a closed loop and compare the echoed rhythm. Any mismatch measures curvature* $R \neq 0$*: the difference between expected and returned distinction. When total residue cancels (*$U^{(4)} = 0$*), the maze is globally consistent.*

Interpretation. *Curvature is the informational stress of maintaining closure in a finite domain. Echo intensity corresponds to entropy: more paths, higher distinguishability. Einstein's equation emerges as the balancing condition between geometric residue and informational flux.*

Epilogue. When the calculus of variations meets the calculus of observation, they become one and the same. The least action principle is not a rule imposed from outside; it is the expression of the universe's preference for maximal consistency within finite means. Light traces the paths where this consistency is perfect. Matter records where it is not. And the quantum is the measure of how the universe keeps its books.

Chapter 8

The Second Law of Causal Order

8.1 Statement of the Law

Theorem 4 (Monotonicity of Causal Entropy). *For any sequence of Martin–consistent causal sets*

$$\mathcal{C}_1 \subseteq \mathcal{C}_2 \subseteq \cdots ,$$

the associated entropies

$$S[\mathcal{C}_n] = k_{\mathrm{B}} \ln |\Omega(\mathcal{C}_n)|$$

satisfy

$$\Delta S_n \equiv S[\mathcal{C}_{n+1}] - S[\mathcal{C}_n] \geq 0,$$

with equality only for informationally complete partitions.

Proof. Each causal refinement $\mathcal{C}_n \to \mathcal{C}_{n+1}$ corresponds to an enlargement of the observer's partition of distinguishable events. By the Axiom of Finite

Observation, refinement cannot reduce the set of admissible micro–orderings:

$$\Omega(\mathcal{C}_n) \subseteq \Omega(\mathcal{C}_{n+1}).$$

Taking logarithms gives $S[\mathcal{C}_{n+1}] \geq S[\mathcal{C}_n]$. The inequality is strict whenever the refinement exposes previously indistinguishable configurations. □

Thought Experiment 8.1.1 (Thought Experiment: The Library Catalog and the Arrow of Distinction)**. *N.B.*** *This experiment illustrates Theorem 1 as a theorem of causal order, not a postulate of thermodynamics. It shows how monotonic distinguishability (*$\Delta S \geq 0$*) arises naturally from the structure of consistent extension.*

Setup. *Imagine a vast library whose books represent events* $\{e_i\}$*. Each measurement attaches finer tags—subject, author, edition—refining the causal order. By the Axiom of Event Selection, no tag can be removed without creating inconsistency among shelves (e.g., merging sci-fi and history). Hence, the total number of distinguishable configurations* N *can only increase or remain constant.*

Demonstration. *Attempting to "un-tag" a shelf merges incompatible categories, breaking bijection with prior distinctions. Thus time's arrow emerges as the monotonic count of consistent refinements:*

$$S = \ln N, \qquad \Delta S \geq 0.$$

Interpretation. *Entropy here is not disorder but bookkeeping: the log of consistent distinctions maintained through observation. The irreversible direction of measurement follows directly from order preservation, not energy dissipation.*

8.2 Entropy as Informational Curvature

In differential form, the same statement appears as the non-negativity of informational curvature:

$$\nabla_i \nabla_j S \geq 0.$$

Flat informational geometry corresponds to equilibrium ($\Delta S = 0$), while positive curvature indicates the growth of accessible micro-orderings. The flux of this curvature defines the *entropy current*

$$J_S^\mu = k_\mathrm{B}\, \partial^\mu S,$$

whose divergence measures local entropy production:

$$\nabla_\mu J_S^\mu = k_\mathrm{B}\, \Box S \geq 0.$$

Thus $\Delta S > 0$ is equivalent to the statement that the informational Laplacian $\Box S$ is positive definite under Martin–consistent transport.

Thought Experiment 8.2.1 (Maxwell's Demon as Non-commutative Selection)**.** *Consider a classical gas divided by a partition with a single gate controlled by a demon who measures particle velocities and opens the gate selectively. Let M denote the demon's measurement operator and U the physical evolution of the gas. If M and U commute—$[M, U] = 0$—the demon's observation does not alter the causal order: measurement and evolution can be exchanged without changing the macrostate. But in reality $[M, U] \neq 0$: the act of measurement refines the partition of distinguishable states, altering the subsequent evolution. This non-commutativity forces the entropy balance*

$$\Delta S_{gas} + \Delta S_{demon} \;=\; k_\mathrm{B} \ln |\Omega_{joint}| \;>\; 0,$$

because the demon's internal record adds new causal distinctions to the universe tensor even as it reduces them locally.

Operationally, the demon cannot perform a measurement without joining the measured system's causal order; the refinement of its internal partition $P_n \to P_{n+1}$ *increases the global count of distinguishable configurations. The apparent violation of the Second Law disappears: the measurement and evolution operators fail to commute, and that failure* is *the entropy production term. Thus Maxwell's demon exemplifies the theorem* $\Delta S \geq 0$*: informational refinement in one domain demands compensating coarsening in another so that the global order remains consistent.*

8.3 Statistical Interpretation

From the causal partition function

$$Z = \int \exp\left(\frac{i}{\hbar} S[T]\right) DT,$$

the ensemble average of the informational gradient obeys

$$\langle \nabla_\mu J_S^\mu \rangle = k_\mathrm{B} \, \langle \nabla_\mu \nabla^\mu S \rangle \geq 0.$$

The equality $\Delta S = 0$ corresponds to detailed balance of causal fluxes; any deviation yields positive entropy production.

8.4 Physical Consequences

1. **Arrow of Time.** Causal order expands in one direction only—toward increasing distinguishability of events. Time is the parameter labeling this monotonic refinement.

2. **Thermodynamic Limit.** In the continuum limit, $\Delta S > 0$ reproduces the classical second law, but here the law is not statistical: it is a theorem of consistency. No causal evolution that decreases S can remain

Martin–consistent.

3. **Gravitational Coupling.** From Chapter 4, curvature couples to gradients of S through the entropic stress tensor:

$$G_{\mu\nu} = 8\pi \left(T_{\mu\nu} + T^{(S)}_{\mu\nu}\right), \quad T^{(S)}_{\mu\nu} = \frac{1}{k_{\mathrm{B}}}\nabla_\mu \nabla_\nu S.$$

Hence $\Delta S > 0$ corresponds to a net positive contribution of informational curvature to spacetime geometry—a causal analogue of energy influx.

8.5 Conclusion

The Second Law of Causal Order may be stated succinctly:

$$\boxed{\Delta S \geq 0 \quad \text{for every Martin–consistent refinement of causal structure.}}$$

Entropy is not a measure of disorder but of latent order yet unresolved. Every act of measurement refines the universe's partition, and each refinement enlarges the count of admissible configurations. The universe evolves by distinguishing itself.

8.6 Epiloge

We began with the observation that every act of physics is an act of distinction: to measure is to separate one possibility from another. Within ZFC, such distinctions are represented as finite subsets of a causal order, and the act of measurement is the enumeration of their admissible refinements. Nothing else is assumed.

Martin's Axiom enters only to ensure that these refinements can be extended consistently—that the space of distinguishable events admits countable dense families without contradiction. This single assumption is the logical equivalent of σ-additivity in measure theory, the minimal condition required for any self-consistent calculus of observation.

From this, the Second Law follows as a theorem of order: each consistent extension of the causal set increases the number of distinguishable configurations, and therefore

$$\Delta S \geq 0.$$

Entropy is not a statistical tendency but a logical necessity—the price of consistency within a self-measuring universe.

No new forces, particles, or cosmologies are introduced; only the rule by which distinction propagates. What began as a grammar of measurement closes as the unique structure of physical law.

Theorem 5 (The Second Law of Causal Order)**.** *In any finite, causally consistent ordering of distinguishable events, the number of measurable distinctions cannot decrease. Every admissible extension of order produces at least one new differentiation, and therefore every universe consistent with its own record of events obeys the inequality*

$$\Delta S \geq 0.$$

Conclusion. We are left with but one conclusion:

$$\boxed{\text{Order implies dynamics.}}$$

A universe that preserves its own causal record must, by necessity, increase the count of what can be distinguished. □

Quod erat demonstrandum.

Coda: The Causal Universe as a "White Hole"

N.B. This coda is a conceptual reflection, not a cosmological claim. It extends the logic of causal order one step beyond the proof: if a black hole represents a local failure of informational closure—an *informational sink*—then a universe that must always increase its distinguishability ($\Delta S \geq 0$) behaves as its formal converse, an *informational source.* No inference about classical white-hole solutions, singularities, or cosmic acceleration is intended [39, 96, 64, 101, 31, 10].

Setup. In the causal framework developed above, a black hole corresponds to a horizon of informational saturation: beyond it, further distinctions cannot be reconciled without contradiction. From the external observer's perspective, the stream of incoming updates exceeds the capacity of the causal record; order collapses into opacity. It is the local end of distinguishability—a finite boundary of the universe's bookkeeping.

Invariant to Enforce. The Second Law of Causal Order forbids the universe as a whole from entering such a state. A global informational sink would halt the count of distinguishable events, violating the monotonic condition $\Delta S \geq 0$. Therefore, the universe must remain an *informational source*: a domain that can always emit new, Martin-consistent refinements of causal order [10, 54].

Formal Analogy. In general relativity, a white hole is the time-reversal of a black hole: a region from which events emerge but into which none can enter. Under the logic of causal measurement, the same symmetry arises abstractly. If measurement always increases distinguishability, the global causal field must behave as a continual emitter of information—a formal white hole in informational phase space. Its "expansion" is not a dynamic expansion of matter, but a logical expansion of the record of distinctions.

Interpretation. The outward "pressure" observed as dark-energy expansion can thus be viewed, purely formally, as the informational tension that maintains the universe's role as a source. Each new distinction contributes to the curvature of the causal field; each increment of ΔS is an act of emission that preserves global consistency. In this sense, the universe is not merely growing in size but in *resolution*: its geometry expands because the space of distinguishable configurations must.

Thought Experiment 8.6.1 (Extrapolation: Leavitt's Ladder and the Hubble Constant)**.** ***N.B.*** *This extrapolation is purely formal. It illustrates how the causal requirement $\Delta S \geq 0$ manifests observationally through Leavitt's period–luminosity law and the Hubble constant. No cosmological claim beyond formal analogy is intended.*

Setup. *In 1912, Henrietta Swan Leavitt discovered that the luminosity L of Cepheid variable stars increases monotonically with their oscillation period P, obeying*

$$M = a \log_{10} P + b.$$

This law defined the first invariant mapping from local temporal oscillation to global metric scale. By calibrating redshift z against Leavitt's ladder, Edwin Hubble (1929) derived the proportionality $v = H_0 d$, establishing the expansion rate of the universe.

Formal Interpretation. *Within the causal framework, Leavitt's relation is the archetype of order expansion. Each measured Cepheid adds a consistent distinction between temporal frequency and spatial magnitude. The mapping $P \mapsto L$ is an order-preserving bijection between local oscillation and global extension. Its monotonicity enforces the same logical law as the Second Law of Causal Order: each refinement increases distinguishability, so $\Delta S \geq 0$.*

Analogy. *Just as Leavitt's ladder converts periodic variation into distance, the causal universe converts informational differentiation into metric expansion. The Hubble constant H_0 expresses the global rate at which new distinctions become measurable—an informational expansion, not a mechanical one.*

Scope. *This extrapolation is formal. It demonstrates that cosmic expansion, when viewed through Leavitt's law, reflects the universe's role as a* *causal source maintaining* $\Delta S \geq 0$*, not a dynamical explosion of matter in space.*

Scope. This reflection is purely formal. It demonstrates how a universe obeying the logical law $\Delta S \geq 0$ shares the causal-source structure of a white hole, not that it *is* one. The correspondence is informational, not physical, and serves only to illuminate the symmetry between causal emission and causal measurement that underlies the theorem just proved.

□

Proofs

8.7 The Calculus of Measurement

8.7.1 Proposition 1

Proof goes here.

Glossary

distinguishability chain A sequence $\mathcal{P} = \{P_n\}$ of finite partitions of an observational domain, where each P_{n+1} strictly refines P_n.

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event A minimal refinement step in a distinguishability chain $\mathcal{P} = \{P_n\}$, represented by a pair $(B, \{B_i\}_{i \in I}, n)$ where a block $B \in \mathrm{Bl}(P_n)$ splits into a family of subblocks $\{B_i\} \subseteq \mathrm{Bl}(P_{n+1})$ with $|I| \geq 2$.

event tensor An element $\mathbf{E}_k \in \mathcal{T}(\mathcal{V})$ encoding the measurable contribution of an event $e_k \in \mathcal{E}$ to the global state via an embedding $\Psi : \mathcal{E} \to \mathcal{T}(\mathcal{V})$.

ordered fold An associative left fold over a totally ordered sequence of event tensors, preserving the order of composition in a non-commutative algebraic structure.

partially ordered set A pair $(E, \leq)$ where $\leq$ is a binary relation on E that is reflexive, antisymmetric, and transitive.

predicate A map $P : E \to \{0, 1\}$ assigning a truth value to each event, used to indicate which events satisfy a specified property.

rank time Order-embedding $\tau : E \to \mathrm{Ord}$ assigning an ordinal rank to each event in a locally finite poset.

tensor algebra The direct sum $\mathcal{T}(\mathcal{V}) = \bigoplus_{r=0}^{\infty} \mathcal{V}^{\otimes r}$ with componentwise addition and associative tensor product on a vector space $\mathcal{V}$.

time An ordinal index into the ordered list of events guaranteed by the Axiom of Order.

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