The Gauge Invariance of Einstein Field Equations in a Spacetime Curved By Entropy

Bill Cochran*

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Abstract

We develop a finite, computational model of measurement in which time is the ordinal index of distinguishable events. Starting from the axioms of set theory and a locally finite causal order, we show that all physical quantities can be expressed as counts of measurable distinctions. We develop the Reciprocity Law of Physics, which equates variation with measurement, leads naturally to the calculus of variations as the unique closure condition on consistent observation. Its smooth limit does not assert continuity but encodes it: we represent discrete measurements with cubic splines, which serve as compact representations of the data rather than assumptions about the underlying field. This recovers the familiar continuous differential equations of universal physical laws.

Overview of the Argument

This work proceeds by construction. We begin with the observation that measurement, in any physical system, must be finite, causal, and reversible. From these three requirements alone, a structure follows: a partially ordered set of distinguishable events, and a reciprocal operation that updates their relations. The entire analysis may be regarded as a proof that *calculus, waves, and gravity* are not separate postulates of physics but successive closures of this single causal rule.

*Researcher at-large: wkcochran@gmail.com

Part I — The Calculus of Measurement. We formalize measurement as a reciprocal operator on a locally finite poset of events. Requiring self-consistency of repeated updates yields a fourth-order cancellation law identical in form to the Euler–Lagrange condition. In the continuum limit, this becomes the unique smooth extension of finite causal inference. Thus calculus itself is derived as the admissible closure of measurement. At the end of this part, we may *implicitly trust calculus*.

Part II — The Wave When the causal update is translation-invariant, its discrete Laplacian defines a wave operator. The only globally consistent eigenfunctions of such an operator are the complex exponentials, so the Fourier basis and the wave and diffusion equations appear automatically. The field's canonical stress tensor follows, completing the local theory of propagation. At this stage we may *implicitly trust waves, the wave and diffusion equations, and Fourier transforms* as the faithful global representation of causal updates.

Part III — Gravity from Entropic Stress. The poset field couples to geometry through an action principle. Introducing an entropic sector $\mathcal{L}_{\text{ent}}$ yields a stress tensor that sources curvature while leaving the Newtonian limit unchanged at leading order. The resulting constant κ fixes the scale of informational curvature and can be fit observationally from lensing and shear. General relativity thus emerges as the geometric closure of the causal calculus.

Part IV — Particles and Gauge. Stable, finite-energy wavepackets within the field behave as particles. Their conserved currents and interactions follow from the Noether symmetries of the wave sector, while internal phase symmetries produce the corresponding gauge connections. Matter and field, therefore, are not separate primitives but distinct limits of the same causal tensor.

Arc of the Proof. Each part extends the previous one by one layer of closure:

$$\text{Measurement} \Rightarrow \text{Calculus} \Rightarrow \text{Waves} \Rightarrow \text{Geometry} \Rightarrow \text{Matter.}$$

The sequence is constructive and reversible: if the axioms of causal measurement hold, then calculus, waves, gravity, and particles follow as necessary consequences. The universe itself is the minimal structure that remains self-consistent under its own act of measurement.

Part I
The Calculus of Measurement

1 Introduction

Every physical description begins not with space or time, but with an *event*—an interaction that makes previously indistinguishable outcomes distinct [2, 10]. The causal boundary of such an interaction is its *light cone*: the set of all events that can influence or be influenced by it according to special relativity [4, 8]. The intersection of two light cones, corresponding to the last particle–wave interaction accessible to an observer, defines the maximal region of causal closure [5, 9]. Beyond this surface, no additional information can be exchanged; all distinguishable action has concluded.

It is from this closure that the ordering of events arises [5, 7]. Each measurable interaction contributes one additional distinction to the universe, expanding its causal surface by a finite count [5, 7]. The smooth fabric of spacetime is not primitive but emergent: it is the limiting behavior of discrete causal increments accumulated along the light cone [3, 11]. Within each cone, the universe can be represented by a finite tensor of interactions—local updates to a global state—that together approximate continuity only through cancellation across countable events [3, 12].

Special relativity provides the canonical local model for this causal structure [4]. Consider the Lorentz transformation for a boost of velocity v in one spatial dimension,

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v/c^2 \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}, \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{1}$$

For infinitesimal separations satisfying $x = ct$, the Lorentz transformation gives

$$t' = \gamma\, t(1 - v/c). \tag{2}$$

If we take $\Delta t = 1$ as the unit interval between distinguishable events, then observers moving at relative velocity v will, in general, disagree on the *number* of such events that occur between two intersections of their respective light cones [8]. The only invariant quantity is the causal ordering itself: all observers concur on which event precedes which, even though they may count a different number of intermediate ticks [7].

This observation motivates the first physical axiom: that time is not an independent scalar field but an ordinal index over causally distinguishable events. Each event increments the universal sequence by one count; each observer's clock is a local parametrization of that same count under Lorentz contraction. The apparent continuity of time is the result of the density of such events within the causal cone, not an underlying continuum of duration.

This work does not propose new physical phenomena or reinterpret experimental data. Rather, it reformulates how measurable quantities are represented. The analysis concerns only the *structure of measurement itself*—the mathematical relations among counts of distinguishable events that underlie all physical observations. The familiar constants and fields of physics appear here as derived measures within a finite causal order, not as independent entities. No new particles, forces, or cosmological effects are introduced; only the rules by which such effects are numerically described are examined. In this sense, the theory is not a revision of physics but a clarification of its grammar: it studies the measures of phenomena, not the phenomena themselves.

The framework that follows formalizes this intuition. Starting from Zermelo–Fraenkel set theory with the Axiom of Choice, we construct an ordered set of events whose distinguishability relations reproduce the causal order implied by special relativity. Measurements are counts of these relations, and the universe tensor—the cumulative sum of event tensors over all causal increments—serves as the discrete foundation from which the continuous laws of physics emerge.

2 The Axioms of the Mathematical

All mathematics in this work is carried out within the framework of Zermelo–Fraenkel set theory with the Axiom of Choice (ZFC) [6]. Rather than enumerating the axioms in full, we recall only those consequences relevant to the construction that follows:

- **Extensionality** ensures that distinguishability has formal meaning: two sets differ if and only if their elements differ.
- **Replacement** and **Separation** guarantee that recursively generated collections such as the causal chain of events remain sets.
- **Choice** permits well–ordering, allowing every countable causal domain to admit an ordinal index.

These are precisely the ingredients required to formalize a locally finite causal order. All further constructions—relations, tensors, and operators—are definable within standard ZFC mathematics; no additional axioms are introduced.

Axiom 1 (The Axioms of Mathematics)**.** *All reasoning in this work is confined to the framework of Zermelo–Fraenkel set theory with the Axiom of Choice (ZFC). Every object—sets, relations, functions, and tensors—is constructible within that system, and every statement is interpretable as a theorem or definition of ZFC. No additional logical principles are assumed beyond those required for standard analysis and algebra.*

Formally,

$$\text{Physics} \subseteq \text{Mathematics} \subseteq \text{ZFC}.$$

Thus, the language of mathematics is taken to be the entire ontology of the theory: the physical statements that follow are expressions of relationships among countable sets of distinguishable events, each derivable within ordinary mathematical logic.

2.1 Sets of Events

Let the set of all events accessible to an observer be denoted E, ordered by causal precedence $\leq$. Because any physically realizable region is finite, this order forms a locally finite partially ordered set (poset).

Definition 1 (Partially Ordered Set)**.** *A* partially ordered set *(poset) is a pair $(E, \leq)$ where $\leq$ is a binary relation on E satisfying:*

1. ***Reflexivity:*** *$e \leq e$ for all $e \in E$;*
2. ***Antisymmetry:*** *if $e \leq f$ and $f \leq e$, then $e = f$;*

3. ***Transitivity:*** *if* $e \leq f$ *and* $f \leq g$*, then* $e \leq g$*.*

Such an ordering always admits at least one maximal element:

$$\mathrm{Top}(E) = \{\, e \in E \mid \nexists f \in E \text{ with } e < f \,\}. \tag{3}$$

The elements of $\mathrm{Top}(E)$ represent the current causal frontier—the most recent events that have occurred but have no successors. Although $\mathrm{Top}(E)$ may contain several incomparable (spacelike) elements, it is never empty and therefore provides a well-defined notion of a "last event" from the observer's perspective. This frontier defines the light-cone boundary and the terminal particle–wave interaction that delimits all accessible information.

3 The Axioms of the Physical

A common criticism of mathematical physics is the extent to which mathematics can be tuned to fit observation [2, 10] and, conversely, manipulated to yield nonphysical results [1, **?**]. The critique of Newton's fluxions could only be answered by successful prediction. Today, calculus feels like a natural extension of the real world—so much so that Hilbert, in posing his famous list of open problems, explicitly formalized the lack of a rigorous foundation for physics as his Sixth Problem.

We aim to show that the mathematical language used to describe physics gives rise to a system expressible entirely as a discrete set of events ordered in time. Moreover, this ordered set possesses a mathematical structure that naturally yields the appearance of continuous physical laws and the conservation of quantities. To understand how this works, we first clarify what we mean by measurement.

3.1 Measurement and the Axiom of Order

Physical laws relate measurements. For example, Newton's second law

$$F = \frac{dp}{dt} \tag{4}$$

states that force relates to the *change* in momentum over time. To speak of change you must have at least two momentum values, one that *comes before* the other; otherwise there is nothing to distinguish. In set-theoretic terms, by the Axiom of Extensionality, different states must differ in their contents, so “change” presupposes the distinguishability of two states.

In this framing, measurement values are *counts* (cardinalities) of elementary occurrences: the number of hyperfine transitions during a gate, the tick marks traversed on a meter stick, the revolutions of a wheel. The *event* is the action that makes previously indistinguishable outcomes distinguishable; the *measurement* is the observed differentiation (the count) between two anchor events. This is not the absolute measure of the event, but just relative difference of the two. We count the events as time passes.

Since special relativity requires that time vary under the Lorentz transform, there can be no global scalar representation of temporal duration. Rather, special relativity permits us only to *list* all events in the universe in their proper causal order. It is this ordered list that we elevate to the first physical principle:

Axiom 2 (The Axiom of Order)**.** *The only invariant agreement in time guaranteed between two observers is the order in which the events occur. The duration between two events is defined as the number of measurements that can be recorded between them:*

$$|\delta t| \;=\; \big|\textit{events distinguished between}\big|. \tag{5}$$

As a corrolary to this, there exists a tensor that allows all events in the universe to occur at integer moments in time, denoted $\mathbf{U}$, the universe tensor. Although this tensor is finite, it suffices to demonstrate how discrete parameters can be represented by piece–wise cubic polynomial, thereby yielding the continuous laws of physics. In this way, the smoothness observed in physical theories is an emergent property of cancellation across discrete counts rather than a primitive assumption of continuity.

Definition 2 (Time)**.** *Time is not a variable, scalar, or independent measurement. Rather, it is an index into the sorted list of events guaranteed by the Axiom of Order. Its role is purely ordinal: to enumerate the relative position of events within the universal sequence.*

Definition 3 (Event Tensor)**.** *Let $\mathcal{V}$ be a finite–dimensional real vector space of measurable quantities. An* event tensor $\mathbf{E}_k \in \mathcal{T}(\mathcal{V})$ *encodes the distinguishable contribution of the k-th event $e_k \in \mathcal{E}$ to the global state. It is related to the logical event by a measurable embedding $\Psi : \mathcal{E} \to \mathcal{T}(\mathcal{V})$, where $\mathbf{E}_k = \Psi(e_k)$.*

Proposition 1 (Causal Universe Tensor)**.** *Let* $\{\mathbf{E}_k\}_{k=1}^n$ *be the ordered sequence of event tensors guaranteed by the Axiom of Order. The* universe tensor *after* n *events is the ordered sum*

$$\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k, \tag{6}$$

where addition in $\mathcal{T}(\mathcal{V})$ *preserves causal order: if* $i < j$*, then* $(\mathbf{E}_i, \mathbf{E}_j)$ *occurs before* $(\mathbf{E}_j, \mathbf{E}_i)$ *unless* $\mathbf{E}_i$ *and* $\mathbf{E}_j$ *commute.*

Proof. By the Axiom of Order, all observers agree only on the *sequence* in which events occur. Thus, the state of the universe can be constructed recursively:

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \mathbf{E}_{n+1}. \tag{7}$$

Since $\mathbf{U}_1 = \mathbf{E}_1$, induction yields $\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k$. □

3.2 Formal Structure of Event and Universe Tensors

We now specify the algebraic structure of the quantities introduced above. Let $\mathcal{V}$ denote a finite–dimensional real vector space representing the independent channels of measurable quantities (e.g. energy, momentum, charge). Define the tensor algebra

$$\mathcal{T}(\mathcal{V}) = \bigoplus_{r=0}^{\infty} \mathcal{V}^{\otimes r}, \tag{8}$$

whose elements are finite sums of r–fold tensor products over $\mathbb{R}$. Each *event tensor* E_k is a member of $\mathcal{T}(\mathcal{V})$ encoding the distinguishable contribution of the k–th event to the global state. We write

$$\mathbf{E}_k \in \mathcal{T}(\mathcal{V}), \qquad \mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k \in \mathcal{T}(\mathcal{V})). \tag{9}$$

Addition is understood componentwise in the direct sum and preserves the ordering of indices guaranteed by the Axiom of Order. In this setting the "universe tensor" $\mathbf{U}_n$ is the cumulative history of all event tensors up to ordinal n.

Definition 4 (Tensor Algebra)**.** *The tensor algebra on V is*

$$\mathcal{T}(\mathcal{V}) = \bigoplus_{r=0}^{\infty} \mathcal{V}^{\otimes r},$$

with componentwise addition and associative tensor product.

Remark 1. *Each logical event e_k in the partially ordered set $(\mathcal{E}, \prec)$ induces a tensor $\mathbf{E}_k = \Psi(e_k)$ in $\mathcal{T}(\mathcal{V})$. The mapping Ψ translates causal structure into algebraic contribution, ensuring that causal precedence corresponds to index ordering in $\mathbf{U}_n$.*

Because $\mathcal{T}(\mathcal{V})$ is a free associative algebra, all operations on $\mathbf{U}_n$ are well defined using the standard linear maps, contractions, and bilinear forms of $\mathcal{V}$. The subsequent analysis of variation and measurement therefore proceeds entirely within conventional linear–operator theory.

From this definition of the universe tensor, it is easy to define an entanglement as a set of events that can be permuted in the list of all events without changing any invariant scalars.

Definition 5 (Entanglement)**.** *From the definition of the universe tensor*

$$\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k, \tag{10}$$

an entanglement *is a subset of events*

$$S \subseteq \{\mathbf{E}_1, \dots, \mathbf{E}_n\} \tag{11}$$

such that for any permutation π of S,

$$\sum_{\mathbf{E}_i \in S} \mathbf{E}_i = \sum_{\mathbf{E}_i \in S} \mathbf{E}_{\pi(i)}, \tag{12}$$

and therefore no invariant scalar derived from $\mathbf{U}_n$ is changed by reordering the events in S.

Example 1 (Finite Causal Chain)**.** *Consider a toy causal network consisting of four ordered events $A_1 \prec A_2$ and $B_1 \prec B_2$, with no initial ordering between* *the A and B chains. Let each event tensor be a 2×2 real matrix recording a pair of measurable quantities, for instance*

$$\mathbf{E}_{A_1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{E}_{A_2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{E}_{B_1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{E}_{B_2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{13}$$

The cumulative universe tensor through all four events is then

$$\mathbf{U}_4 = \mathbf{E}_{A_1} + \mathbf{E}_{A_2} + \mathbf{E}_{B_1} + \mathbf{E}_{B_2} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{14}$$

If the entangled pair $\{A_2, B_2\}$ is permuted, the componentwise sum is unchanged, $\mathbf{E}_{A_2} + \mathbf{E}_{B_2} = \mathbf{E}_{B_2} + \mathbf{E}_{A_2}$, illustrating that entanglement classes correspond to commutative subsets within the otherwise ordered sequence. This simple construction realizes the algebraic content of Proposition 1 in explicit matrix form.

Example 2 (Spooky Action at a Distance)**.** *Consider an entanglement $S = \{\mathbf{E}_i, \mathbf{E}_j\}$ of two spatially separated measurement events. By definition, the order of $\mathbf{E}_i$ and $\mathbf{E}_j$ may be permuted without changing any invariant scalar of the universe tensor:*

$$\mathbf{E}_i + \mathbf{E}_j = \mathbf{E}_j + \mathbf{E}_i. \tag{15}$$

When an observer records $\mathbf{E}_i$, the global ordering is fixed, and the universe tensor is updated accordingly. Because $\mathbf{E}_j$ belongs to the same entanglement set, its contribution is now determined consistently with $\mathbf{E}_i$, even if E_j occurs at a spacelike separation. This manifests as the phenomenon of "spooky action at a distance"—the appearance of instantaneous correlation due to reassociation within the entangled subset.

Example 3 (Hawking Radiation)**.** *Let $\mathbf{E}_{in}$ and $\mathbf{E}_{out}$ denote the pair of particle-creation events near a black hole horizon. These events form an entangled set:*

$$S = \{\mathbf{E}_{in}, \mathbf{E}_{out}\}. \tag{16}$$

As long as both remain unmeasured, their contributions may permute freely within the universe tensor, preserving scalar invariants. However, once $\mathbf{E}_{out}$ is measured by an observer at infinity, the ordering is fixed, and $\mathbf{E}_{in}$ is forced to a complementary state inside the horizon. The outward particle appears as Hawking radiation, while the inward partner represents the corresponding loss of information behind the horizon. Thus Hawking radiation is naturally *expressed as an entanglement whose collapse occurs asymmetrically across a causal boundary.*

Definition 6 (Distinguishability chain)**.** *Let Ω be a nonempty set. A* distinguishability chain *on Ω is a sequence $\mathcal{P} = \{P_n\}_{n\in\mathbb{Z}}$ of partitions $P_n \in \mathsf{Part}(\Omega)$ such that P_{n+1}* refines *P_n for all n (every block of P_{n+1} is contained in a block of P_n). Write $\mathrm{Bl}(P)$ for the set of blocks of a partition P.*

Definition 7 (Event)**.** *Fix a distinguishability chain $\mathcal{P} = \{P_n\}$. An* event at index *n is a minimal refinement step: a pair*

$$e = (B, \{B_i\}_{i\in I}, n) \tag{17}$$

such that:

1. *$B \in \mathrm{Bl}(P_n)$;*
2. *$\{B_i\}_{i\in I} \subseteq \mathrm{Bl}(P_{n+1})$ is the family of all blocks of P_{n+1} contained in B, with $|I| \geq 2$ (a nontrivial split);*
3. *(*minimality*) there is no proper subblock $C \subsetneq B$ with $C \in \mathrm{Bl}(P_n)$ for which the family $\mathrm{Bl}(P_{n+1}) \cap \mathcal{P}(C)$ is nontrivial.*

Let E denote the set of all such events. We define a (strict) order on events by $e \prec f \iff n_e < n_f$, where n_e denotes the index of e.

Intuitively, P_n encodes which outcomes of Ω are indistinguishable at index n. An event is the atom of change in distinguishability: a single block B of P_n that is split into $\{B_i\}$ in P_{n+1}.

Definition 8 (Predicate on events)**.** *A* predicate *is any map $P : E \to \{0, 1\}$. It selects which events are "counted."*

Definition 9 (Measurement)**.** *Let E be the event set with order $\prec$, and let $P : E \to \{0, 1\}$ be a predicate. Given two* anchor events *$a, b \in E$ with $a \prec b$, the* measurement of *P* between *a* and *b is*

$$M_P[a, b] \;:=\; \#\{\, e \in E \mid a \prec e \prec b \text{ and } P(e) = 1 \,\} \in \mathbb{N}. \tag{18}$$

Basic properties If $(E, \prec)$ is locally finite (only finitely many events between comparable anchors), then $M_P[a, b]$ is finite. Measurements are *additive*: for $a \prec c \prec b$,

$$M_P[a, b] \;=\; M_P[a, c] + M_P[c, b]. \tag{19}$$

They are also *order-invariant*: any strictly order-preserving reindexing of E leaves $M_P[a, b]$ unchanged.

3.3 Axiom of Finite Observation

The recursive description of physical reality is meaningful only within the finite causal domain of an observer. Each step in such a description corresponds to a distinct measurement or recorded event. Observation is therefore bounded not by the universe itself, but by the observer's own proper time and capacity to distinguish events within it.

Axiom 3 (The Axiom of Finite Observation)**.** *For any observer, the set of observable events within their causal domain is finite. The chain of measurable distinctions terminates at the limit of the observer's proper time or causal reach.*

This axiom establishes the physical limit of any causal description: the sequence of measurable events available to an observer always ends in a finite record. Beyond this frontier—beyond the end of the observer's time— no additional distinctions can be drawn. The *last event* of an observer thus coincides with the top of their causal set: the boundary of all that can be measured or known.

The Axiom of Finite Observation has a corollary familiar to every graduate student: the capacity of the universe to surprise is infinite, but the capacity of the hard drive is not.

3.4 Construction of the Universe Tensor and the Axiom of Event Selection

Even though mathematics is powerful enough to describe the laws of physics with predictive accuracy, it can also compute nonphysical phenomena. Negative areas, for instance, are a common mathematical construct:

$$\int_0^\pi -\sin x \, dx. \tag{20}$$

Even worse, pathological geometries can give rise to fantastical descriptions of internal states of the computation, leading to ill-defined behaviors. We see this at the singularity of general relativity or at the scale of the Planck length, where the formalism itself begins to overcount possibilities.

To control such overgeneration, we invoke *Martin's Axiom*, a principle of set theory that restricts the construction of large or pathological subsets without measurable support. In physical terms, Martin's Axiom acts as a regularity condition on the events in the universe: it guarantees that the events we can describe are countably generable from locally finite information. This eliminates spurious solutions that arise purely from mathematical freedom, ensuring that only physically realizable events are included in the ordering. For instance, the Banach-Tarski paradox is not possible to construct with Martin's Axiom as each individual set is unbounded in ordering and therefore excluded from possibility.

Martin's Axiom will allow us to demonstrate that the ordering of events is sufficient to describe time and still recover the laws of physics.

Axiom 4 (The Axiom of Event Selection)**.** *For any countable family of events, there exists a consistent extension selecting one outcome from each family such that all physically realizable events remain distinguishable within the universe.*

In other words, if an event happens "next" in a causal light cone, then it must happen independently of events outside the causal light cone.

More mathematically, we take as the corollary to the Axiom of Event Selection, Martin's Axiom:

Corollary 1 (Martin's Axiom)**.** *Let* $(\mathbb{P}, \leq)$ *be a partially ordered set satisfying the* countable chain condition *(ccc); that is, every antichain in* $\mathbb{P}$ *is countable. For any cardinal* $\kappa < 2^{\aleph_0}$ *and any family* $\{D_\xi : \xi < \kappa\}$ *of dense subsets of* $\mathbb{P}$*, there exists a filter* $G \subseteq \mathbb{P}$ *such that*

$$\forall \xi < \kappa, \quad G \cap D_\xi \neq \varnothing.$$

The physical correspondence to Martin's Axiom should be understood as an analogy of structure, not identity of assumption. In our formulation, the partially ordered set $(P, \leq)$ corresponds to the causal ordering of events. Finite observation guarantees that all antichains of physically accessible events are finite, a strong version of the countable chain condition. The *Axiom of Event Selection* therefore asserts that local causal choices admit a consistent global extension, exactly as Martin's Axiom asserts the existence of a filter meeting all dense subsets. Both function as regularity principles eliminating pathological or non-realizable combinations of events.

4 The Equivalence Principle of Physics

4.1 Variations and the Reciprocity of Measurement

Having established that each measurable event contributes one ordered increment to the universe tensor $\mathbf{U}$, we now show that every permissible variation of $\mathbf{U}$ corresponds to a measurable distinction—and conversely, that every measurable distinction defines a variation on $\mathbf{U}$. The apparent continuum of dynamics thus arises not from interpolation between discrete data, but from the bidirectional closure between variation and measurement.

4.1.1 From Distinguishability to Variation

Let the ordered set of events $\{\mathbf{E}_k\}$ define

$$\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k. \tag{21}$$

For any functional $F[\mathbf{U}]$ expressible as a finite composition of linear maps and contractions on U, consider a perturbation $\delta\mathbf{U}$ that preserves the causal ordering. By the Axiom of Order, such a perturbation can only modify those event tensors whose distinguishing predicates differ:

$$\delta\mathbf{U} = \sum_{k:\, \delta P(E_k)\neq 0} \delta\mathbf{E}_k. \tag{22}$$

Hence every admissible variation corresponds to a measurable change in at least one predicate on the event set. No unmeasurable (order-invisible) variation can exist, because indistinguishable events contribute identically to U.

4.1.2 From Variation to Measurement

Conversely, let two measurements $M_P[a,b]$ and $M_Q[a,b]$ be performed on the same causal interval with predicates $P, Q : \mathbf{E} \to \{0,1\}$. Define their difference

$$\Delta M[a,b] = M_Q[a,b] - M_P[a,b] = \#\{\, e \in \mathbf{E} \mid a \prec e \prec b,\ P(e) \neq Q(e) \,\}. \tag{23}$$

Each nonzero contribution to ΔM identifies an event whose predicate value has changed—that is, an elementary variation $\delta\mathbf{E}_k$. Summing these variations reconstructs the finite difference of $\mathbf{U}$ between the two measurements:

$$\mathbf{U}_Q - \mathbf{U}_P = \sum_{e \in \mathbf{E}:\, P(e) \neq Q(e)} \delta\mathbf{E}_e = \delta\mathbf{U}. \tag{24}$$

Therefore every measurable difference induces a legitimate variation of $\mathbf{U}$. The measurement operator and the variation operator are mutual inverses on the space of distinguishable events.

4.1.3 Bijections Under Selection

The reciprocity between variation and measurement operates within a finite causal domain. However, distinct discrete fields $U, V \in \mathcal{U}$ may yield identical observable outcomes on every finite neighborhood. Such fields are said to be *coincident*:

$$U \sim V \iff U \text{ and } V \text{ produce identical observables on all finite causal neighborhoods.} \tag{25}$$

The quotient space $\mathcal{Q} = \mathcal{U}/\!\sim$ collects these coincidence classes, each representing one physically observable configuration of the universe tensor.

Because causal updates act locally, the reciprocal map $\Phi : \mathcal{U} \to \mathcal{U}$—one step of measurable evolution—preserves coincidence. If $U \sim V$, then $\Phi(U) \sim \Phi(V)$, and therefore Φ descends naturally to a well–defined map on equivalence classes:

$$\Phi : [U] \longmapsto [\Phi(U)], \qquad \Phi : \mathcal{Q} \to \mathcal{Q}. \tag{26}$$

Microscopic degeneracy within each coincidence class implies that Φ need not be bijective on $\mathcal{U}$: distinct microstates may evolve to the same measurable outcome (non–injective), while boundary truncation can omit admissible predecessors (non–surjective). To recover a reversible description, the *Axiom of Event Selection* introduces a canonical representative for each coincidence class.

Definition 10 (Selection Operator)**.** *Let* $\mathrm{Sel} : \mathcal{Q} \to \mathcal{U}$ *be an idempotent, order–preserving map satisfying* $\pi \circ \mathrm{Sel} = \mathrm{id}_{\mathcal{Q}}$*, where* $\pi : \mathcal{U} \to \mathcal{Q}$ *is the quotient map. Physically,* Sel *chooses the simplest admissible field consistent with observation—for instance, the minimal–curvature (spline–like) configuration compatible with the data.*

Definition 11 (Selected Update). *The* selected update *on representatives is*

$$\Phi_{\mathrm{sel}} := \mathrm{Sel} \circ \Phi \circ \pi : \mathcal{U} \to \mathcal{U}.$$

Proposition 2 (Reversible Update on Observable States). *The induced map* $\Phi : \mathcal{Q} \to \mathcal{Q}$ *is bijective if and only if* Φ_{sel} *is bijective on* $\mathrm{Im}(\mathrm{Sel})$*. In that case,*

$$\Phi_{\mathrm{sel}}^{-1} = \mathrm{Sel} \circ \Phi^{-1} \circ \pi.$$

Interpretation. Within the space of measurable configurations, every causal update admits a unique, reversible image once redundant micro–descriptions are collapsed by the Event–Selection rule. This establishes the logical foundation for the Reciprocity Law: measurement and variation are exact inverses when considered on the quotient of distinguishable events.

4.1.4 Reciprocal Closure

Let $\mathcal{V}$ denote the set of all variations consistent with the causal order and $\mathcal{M}$ the set of all measurable predicates. The preceding arguments define bijections

$$\Phi : \mathcal{V} \to \mathcal{M}, \qquad \Phi^{-1} : \mathcal{M} \to \mathcal{V}, \tag{27}$$

establishing the following physical principle.

4.2 Formal Definition of the Reciprocity Mapping

Let $\mathcal{V}$ and $\mathcal{T}(\mathcal{V})$ be as above. Define the space of admissible variations

$$V = \{\, \delta\mathbf{U} \in \mathcal{T}(\mathcal{V}) \mid \delta\mathbf{U} \text{ preserves causal order} \,\}, \tag{28}$$

and the space of measurable predicates

$$M = \{\, P : \mathcal{E} \to \{0,1\} \,\}, \tag{29}$$

where $\mathcal{E}$ is the set of events.

We introduce the mapping

$$\Phi : V \to M, \qquad \Phi(\delta U)(e) = \begin{cases} 1, & \text{if the event tensor of } e \text{ changes under } \delta\mathbf{U}, \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

Its inverse reconstructs a variation from a predicate:

$$\Phi^{-1}(P) = \sum_{e \in \mathcal{E}:\, P(e)=1} \delta\mathbf{E}_e. \tag{31}$$

Proposition 3 (Equivalence of Discrete and Continuum). *Φ is bijective on the space of distinguishable events.*

Proof. If $\Phi(\delta\mathbf{U}_1) = \Phi(\delta\mathbf{U}_2)$, the same set of event tensors changes in both variations, implying $\delta\mathbf{U}_1 = \delta\mathbf{U}_2$; hence Φ is injective. For any predicate P, the corresponding $\delta\mathbf{U} = \Phi^{-1}(P)$ is a valid variation; thus Φ is surjective. Therefore Φ establishes a one–to–one correspondence between measurable distinctions and admissible variations. $\square$

Equivalence 1 (The Reciprocity Law of Physics). *Every physically admissible variation of the universe tensor corresponds to a measurable distinction, and every measurable distinction corresponds to a physical variation of the universe tensor.*

Under this law, the calculus of variations and the calculus of measurement coincide. The differential form of physical law,

$$\delta F[\mathbf{U}] = 0, \tag{32}$$

is simply the statement that the total measurable distinction vanishes under consistent evolution: no new distinguishability is introduced beyond what the universe records.

4.3 Discrete–to–Continuum Limit

To exhibit the analytic limit explicitly, let the sequence $\{\mathbf{U}_n\}$ represent samples of a smooth function $\mathbf{U}(x)$ on a uniform lattice with spacing h, so that $\mathbf{U}_{n\pm k} = \mathbf{U}(x \pm kh)$. Define the fourth–order finite difference operator

$$\Delta_h^{(4)}\mathbf{U}_n = \mathbf{U}_{n+2} - 4\mathbf{U}_{n+1} + 6\mathbf{U}_n - 4\mathbf{U}_{n-1} + \mathbf{U}_{n-2}. \tag{33}$$

If the recursive updates of reciprocal measurement drive this operator toward zero, $\Delta_h^{(4)}\mathbf{U}_n \to 0$ as n increases, then by standard difference analysis

$$\lim_{h\to 0} \frac{\Delta_h^{(4)}\mathbf{U}_n}{h^4} = \frac{d^4\mathbf{U}}{dx^4}(x) = \mathbf{U}^{(4)}(x). \tag{34}$$

Thus, in the continuum limit the closure condition of finite reciprocity enforces the fourth–derivative cancellation

$$\mathbf{U}^{(4)}(x) = 0, \tag{35}$$

identical to the Euler–Lagrange condition for cubic–spline minimization. The remainder of this section interprets that cancellation physically.

This result follows from the fact that correlations may occur coincidentally across entangled events. Since entanglement represents a permutation of partial orderings of currently indistinguishable outcomes, successive updates cannot fully double the universe tensor:

$$|\mathbf{U}_{n+1}| \leq 2|\mathbf{U}_n|. \tag{36}$$

The inequality expresses the loss of independent degrees of freedom due to coincident correlations. In the smooth limit, these cancellations suppress higher-order fluctuations, and the dynamics relax to a fixed point of reciprocal measurement: a state in which further variation produces no new measurable distinction. This apparent non-local coherence is the mechanism that preserves global consistency when local degrees of freedom collapse (Example 2).. The principle of least action is therefore a corollary of the Reciprocity Law, not an independent postulate.

4.3.1 Example: Coincidence as a Retro-Constraint

Consider two causal chains,

$$A_1 \prec A_2, \qquad B_1 \prec B_2, \tag{37}$$

representing two local measurements. Each chain is internally ordered, but the relative ordering between the A and B events is only partially specified.

Suppose an invariant condition couples the terminal events,

$$f(A_2, B_2) = 0, \tag{38}$$

such that the combined value of the pair must satisfy a conservation or matching rule in the universe tensor. When this constraint is enforced at the future boundary (A_2, B_2), it propagates backward through the partial order: the admissible values of (A_1, B_1) are now restricted to those for which the subsequent evolution yields the required terminal pair. Formally, we obtain a dependency

$$(A_1, B_1) \;\longmapsto\; (A_2, B_2), \tag{39}$$

so that the poset must be extended with additional relations ensuring compatibility. In the simplest case, one future event becomes conditionally prior:

$$A_1 \prec B_2 \quad \text{(if } B_2 \text{ requires a specific } A_1 \text{ value).} \tag{40}$$

This induced relation is what we call a *coincidence*: a future event whose consistency condition fixes a present variable. In the universe tensor, such coincidences appear as cancellations of independent variations—degrees of freedom that are no longer free once the end condition is imposed. Each coincidence therefore removes one order of independent variation from the causal sum, driving the sequence toward the smooth limit

$$\mathbf{U}^{(4)}(x) = 0. \tag{41}$$

Thus, a "coincidental" alignment is not a mystery of timing but a structural enforcement of consistency within the partially ordered set: the future boundary constrains the present values so that the entire tensor remains self-consistent under reciprocal measurement. This is the operational significance of the Axiom of Event Selection— only those events consistent with the full causal ordering can occur.

4.4 Deriving the Principle of Least Action

Consider the universe tensor $\mathbf{U}$ evaluated along a single coordinate x between two measurable events. Because all measurements are finite, the behavior of $\mathbf{U}$ on each small interval may be expressed by its local Taylor expansion, in this case the fourth order.

$$\mathbf{U}(x+\Delta x) = \mathbf{U}(x)+\mathbf{U}'(x)\,\Delta x+\tfrac{1}{2}\mathbf{U}''(x)(\Delta x)^2+\tfrac{1}{6}\mathbf{U}^{(3)}(x)(\Delta x)^3+\tfrac{1}{24}\mathbf{U}^{(4)}(\xi)(\Delta x)^4, \tag{42}$$

for some $\xi \in (x, x+\Delta x)$. The choice of fourth derivative is that of mathematical convenience only and it is only to take advantage of coincidence of interpolation concerning splines and the principle of least action. The first four terms define a cubic polynomial that interpolates the measured values and their first two derivatives at the endpoints.

When neighboring intervals are required to match continuously in value, slope, and curvature, any residual fourth-derivative mismatch produces curvature "stress" between them. The completely relaxed configuration—what we intuitively call the *smoothest* interpolation—occurs when this residual vanishes:

$$\mathbf{U}^{(4)}(x) = 0. \tag{43}$$

This is precisely the Euler–Lagrange equation obtained by minimizing the bending-energy functional

$$E[\mathcal{U}] = \int (\mathcal{U}''(x))^2 \, dx, \tag{44}$$

whose stationary points are cubic splines.

Because every measured trajectory in the universe tensor must occupy this fully relaxed state to remain compatible with adjacent measurements, the condition $\mathbf{U}^{(4)} = 0$ defines the physical law of motion at each resolution. Expressed variationally,

$$\delta E = 0 \quad \Longleftrightarrow \quad \mathcal{U}^{(4)} = 0. \tag{45}$$

In the continuum limit the same extremal condition yields the traditional form of the *principle of least action*: the observed path between events is the one for which the curvature (or action) is stationary. Thus, by demanding that the universe tensor be everywhere fully relaxed, the principle of least action is not an axiom but a direct consequence of the smoothest possible interpolation between measurable events.

In other words, assuming the best piecewise cubic polynomial spline through all measurements recovers the principle of least action. Simply splining measurements approximates physics arbitrarily well. Because the spline operation is a bijection on the space of twice-differentiable interpolants, it preserves all measurable information: the spline and the physical law it represents are indistinguishable by any possible measurement. We therefore obtain a true *duality* between measurement and dynamics—the discrete universe tensor and its smooth spline representation are two exact views of the same structure—and the principle of least activity is simply a lens through which that duality can be refocused.

4.5 The Free Parameter of the Third Variation and the Natural Constant

The closure condition

$$U^{(4)}(x) = 0$$

implies that the continuous solution of the universe tensor on each causal interval is a cubic polynomial,

$$U(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3.$$

Each interval between measurable events must match continuously in value, slope, and curvature with its neighbors. Thus, across any interior boundary x_i we require

$$U_i(x_i) = U_{i+1}(x_i),$$
$$U_i'(x_i) = U_{i+1}'(x_i),$$
$$U_i''(x_i) = U_{i+1}''(x_i).$$

These three matching conditions uniquely determine the coefficients a_0, a_1, and a_2 of each segment. However, the coefficient a_3—the third derivative of U up to normalization—remains unconstrained by local continuity:

$$U_i^{(3)}(x) = 6a_{3,i}.$$

To maintain global smoothness under the closure condition $U^{(4)} = 0$, this third derivative must be *constant* across all intervals:

$$U^{(3)}(x) = \text{constant} \equiv \varepsilon.$$

Hence ε is the single free parameter that persists after all continuity conditions are enforced. It represents the universal scale of third variation—the smallest resolvable increment of curvature that remains invariant under reciprocal measurement. This is the

Definition 12 (Universal Precision). *Let ε denote the constant third derivative of the continuous solution $U(x)$:*

$$U^{(3)}(x) = \varepsilon.$$

Then every local segment of the universe tensor satisfies

$$U(x) = a_0 + a_1 x + \tfrac{1}{2}a_2 x^2 + \tfrac{1}{6}\varepsilon x^3,$$

and all measurable distinctions between causal intervals are scaled by ε.

Law 1 (Continuity of the Third Variation). *Under reciprocal measurement, the third variation of the universe tensor remains globally constant,*

$$\delta^{(3)} U = \kappa\, \Phi^{-1}(P),$$

where $\Phi^{-1}(P)$ is the inverse reciprocity map selecting the measurable variation induced by a predicate P.

This ensures that all higher variations vanish identically while every admissible distinction introduces curvature proportional to κ—the natural unit of causal differentiation. Formally, κ is determined by any four consecutive measurements of the field. However, because global consecutivity cannot be established within a causally finite universe except as $\mathbf{U}$, no observer can certify that their four events are globally adjacent in the universal order. Each local frame therefore recovers the same fitted value of κ from its own sequence of observations, yet cannot detect variation across incomparable regions. In this sense the constant is *universally recoverable but globally unknowable*: its constancy is a consequence of causal incompleteness rather than symmetry. We will return the κ in future parts.

5 Conclusion: The Admissible Calculus of Measurement

We have constructed the *admissible calculus of measurement*. Beginning with a locally finite, causally ordered set of distinguishable events and a reciprocal measurement operator Φ, we required that successive applications of Φ preserve order and remain reversible. From this minimal condition, a continuous calculus emerges.

Successive reciprocal updates define the closed sequence

$$U_{n+1} = U_n + \Phi^{-1}(\Phi(U_n)),$$

whose smooth limit satisfies

$$U^{(4)}(x) = 0.$$

This fourth-order cancellation is algebraically identical to the Euler–Lagrange condition: the stationary path of a finite, reversible measurement. Hence the familiar differential calculus is not an assumption but the continuum closure of the discrete causal rule.

A calculus is *admissible* if it arises as the continuous limit of reciprocal measurement on a causally ordered set, preserving locality and reversibility. The admissible calculus is characterized by $U^{(4)} = 0$, ensuring equivalence with the classical calculus of variations.

The interpolant obtained from this construction—the cubic spline satisfying $U^{(4)} = 0$—may not be unique. It succeeds only because the measured data exhibit a structural *coincidence*: a finite set of causal updates admits more than one smooth extension consistent with order and reciprocity. Among all such admissible extensions, the spline is the simplest: it minimizes the fourth variation and therefore yields a stable, order-preserving continuum limit. Other higher-order or nonlocal interpolants could reproduce the same finite observations but would violate either locality or reversibility when extended globally.

Thus the admissible calculus represents a *distinguished but not unique* interpolation between discrete measurements. Its validity rests not on exclusivity but on sufficiency: it is the minimal smooth structure consistent with causal measurement.

We conclude that calculus itself is enforced by causal consistency, yet remains contingent on the coincidences of measurement. Where such coincidences hold, the spline construction provides a faithful and reversible closure of finite data; where they fail, no single smooth extension is guaranteed. Within these limits we may therefore *implicitly trust calculus* as the admissible language of measurement—the unique closure that works, though not the only one that could.

Part II
The Wave

6 Introduction: Martin's Condition and the Continuity of Causal Propagation

The closure of measurement in Part I established that every admissible calculus arises from a finite sequence of distinguishable events whose reciprocal variations cancel beyond fourth order. The resulting smooth field $U(x)$ represents not an assumption of continuity, but the unique extension that preserves causal consistency under the *Axiom of Event Selection*. Yet the closure of a finite causal chain does not by itself guarantee that distinct observers infer compatible fields. For global coherence, the local cancellations enforced by reciprocal measurement must propagate consistently across the entire causal network. This propagation is the content of *Martin's Condition*.

Definition 13 (Martin's Condition (Conceptual))**.** *A causal network satisfies* Martin's Condition *if every locally finite subset of events can be extended to a globally consistent ordering without introducing new distinguishabilities. Equivalently, all finite causal updates admit an extension that preserves the same coincidence relations on their overlaps.*

Intuitively, Martin's Condition demands that information created in one region does not contradict information measured in another. It forbids "causal overcounting"—the duplication of distinctions that would destroy reversibility—by ensuring that overlapping observers reconstruct identical splines of the universe tensor along their shared boundary. Where the Axiom of Event Selection limits what may happen within a light cone, Martin's Condition governs how those choices propagate outward. It is the global compatibility rule of the causal calculus: the guarantee that local smoothness stitches together into a single, coherent wave.

The next sections show that when Martin's Condition holds, the discrete reciprocity law induces a linear propagation operator whose eigenmodes are complex exponentials. The continuum limit of this operator is the familiar wave equation, and the resulting field inherits a canonical stress tensor. Thus the same closure that produced calculus in Part I now produces the continuous propagation of energy and information—the universal phenomenon we recognize as a *wave*.

7 Interaction: The Union of Ordered Events

In a finite causal domain, an observer's description of the world is a locally ordered set of distinguishable events. When two such domains overlap, the question of *interaction* arises: how are their separate orderings reconciled into a single consistent history? Martin's Condition guarantees that locally finite orders can be extended without contradiction. Interaction is the constructive realization of that extension.

Definition 14 (Interaction of Causal Sets)**.** *Let* $(E_1, \preceq_1)$ *and* $(E_2, \preceq_2)$ *be locally finite posets of events, each satisfying Martin's Condition on its own domain. Their* interaction *is the smallest poset*

$$(E_{12}, \preceq_{12}), \qquad E_{12} = E_1 \cup E_2,$$

whose order $\preceq_{12}$ is the transitive closure of $\preceq_1 \cup \preceq_2$ restricted by the requirement that all overlaps $E_1 \cap E_2$ remain consistent:

$$\forall\, e, f \in E_1 \cap E_2,\ e \preceq_1 f \ \Leftrightarrow\ e \preceq_2 f.$$

The overlap $E_1 \cap E_2$ represents events recognized by both observers. For the union to remain causally consistent, these shared events must inherit identical ordering relations from both domains. If such an identification cannot be made, the systems are incompatible and cannot interact without violating Martin's Condition.

Definition 15 (Interaction Event)**.** *An event $e \in E_1 \cap E_2$ is called an* interaction event *if it is maximal in one order and minimal in the other:*

$$e \in \mathrm{Top}(E_1) \cap \mathrm{Min}(E_2) \quad or \quad e \in \mathrm{Top}(E_2) \cap \mathrm{Min}(E_1).$$

Such an event terminates one causal chain and initiates another.

Intuitively, an interaction occurs when the future boundary of one local ordering meets the past boundary of another. At that instant, two independent descriptions of the world become linked by a single shared distinction. The joint order $\preceq_{12}$ thus acts as a stitching rule: it preserves every prior ordering within E_1 and E_2 while extending them just enough to include the new comparabilities implied by the overlap.

Proposition 4 (Union Consistency)**.** *If $(E_1, \preceq_1)$ and $(E_2, \preceq_2)$ satisfy Martin's Condition and agree on all relations within $E_1 \cap E_2$, then their union $(E_{12}, \preceq_{12})$ also satisfies Martin's Condition.*

Idea of Proof. Each finite subset $S \subseteq E_{12}$ lies within finitely many overlapping domains E_i that already satisfy Martin's Condition. Since the overlaps agree on order, the union of their consistent extensions remains consistent. Thus every finite subset of E_{12} extends without introducing new distinguishabilities. □

Interpretation. Interaction is therefore not a separate dynamical law but the combinatorial closure of causal order under union. Whenever two chains intersect, their local orderings adjust to maintain global compatibility. The mutual adjustment propagates along both chains, enforcing consistency across their neighborhoods. Viewed iteratively, this propagation behaves as a *wave of ordering*: a disturbance that travels through the poset whenever new overlaps are formed. It is this propagation—the transmission of order constraints through successive interactions—that gives rise to the phenomenon we recognize as wave motion.

7.1 ER=EPR as a Martin's Condition Mechanism

The requirement that overlapping causal domains extend consistently under Martin's Condition has a striking physical analogue. In ordinary spacetime language, two distant regions may become correlated through entanglement (*EPR*). In causal–set language, they remain connected only if their respective orderings can be extended to a single partial order without contradiction. The bridge that enforces this extension is an *Einstein–Rosen* (*ER*) connection: a minimal path in the transitive closure that preserves global distinguishability across otherwise disjoint regions. Thus, *ER=EPR* arises here not as a conjecture but as a necessary mechanism of Martin's Condition.

Definition 16 (Martin Bridge)**.** *Let* $(E_1, \preceq_1)$ *and* $(E_2, \preceq_2)$ *be causally disconnected posets that become jointly consistent only after the introduction of additional relations* $\mathcal{R} \subseteq E_1 \times E_2$*. The set* $\mathcal{R}$ *is called a* Martin bridge *if the enlarged poset*

$$(E_{12}, \preceq_{12}), \qquad \preceq_{12} = \mathrm{TC}(\preceq_1 \cup \preceq_2 \cup \mathcal{R}),$$

satisfies Martin's Condition and minimizes the number of added relations.

Intuitively, a Martin bridge is the minimal information channel required to restore global consistency between two finite causal domains. It does not transmit energy or momentum; it transmits *order*. Each new comparability introduced by $\mathcal{R}$ removes one degree of freedom that would otherwise allow contradictory extensions of the two local chains. Entanglement is therefore the combinatorial imprint of the bridge: the set of events whose mutual distinguishability depends on maintaining those added relations.

Proposition 5 (ER=EPR Mechanism)**.** *Two causally separated regions are entangled if and only if their joint order requires a Martin bridge for consistency. The bridge defines the equivalence class of entangled events, and the resulting identification ensures that all observers reconstruct identical correlations.*

Conceptual Proof. Suppose two domains admit inconsistent orderings when united. Then there exists at least one pair of events (e_1, e_2), with $e_1 \in E_1$ and $e_2 \in E_2$, such that neither $e_1 \preceq_1 e_2$ nor $e_2 \preceq_2 e_1$ is defined. Martin's Condition demands an extension that renders the combined poset consistent on all finite subsets. Introducing the minimal relation $e_1 \preceq_{12} e_2$ or its converse satisfies this requirement. Once inserted, the two events become informationally linked: their relative distinguishability is no longer independent. This linkage—a single additional comparability—functions as a discrete Einstein–Rosen connection. Conversely, any such connection enforces an entanglement of the corresponding events, since their joint distinguishability now depends on the shared order. □

Interpretation. In this framework, *ER=EPR* is not an exotic duality between geometry and quantum state, but the combinatorial statement that *every entanglement is the minimal causal bridge required to preserve Martin's Condition.* A wormhole is simply the graph–theoretic trace of that bridge when represented in a continuous embedding space. What appears as nonlocal correlation in quantum mechanics is, in causal language, the preservation of global order under finite extension. Entanglement and connectivity are therefore not separate phenomena but two faces of the same regularity principle: the universe maintains consistency by constructing just enough additional order to remain self–coherent.

7.2 Hawking Radiation as the Loss and Restoration of Order

In a locally finite causal network, every interaction adds new comparabilities that extend the partial order while preserving Martin's Condition. When a boundary forms that separates one region from another—for example, a horizon—some of these relations become unresolvable. The resulting incompleteness is what we perceive as *Hawking radiation*: the reorganization of remaining order when part of an entangled system passes beyond causal reach.

Definition 17 (Causal Horizon)**.** *Let $(E, \preceq)$ be a locally finite poset. A subset $H \subset E$ is a* causal horizon *for an observer if there exist events $e, f \in E$ such that $e \preceq h$ for some $h \in H$ but $f \not\preceq h$ for any $h \in H$, and no finite extension of the observer's order can include both e and f. The horizon marks the maximal boundary of extendable distinguishability.*

When a Martin bridge connects two domains $(E_{\text{in}}, \preceq_{\text{in}})$ and $(E_{\text{out}}, \preceq_{\text{out}})$ across a horizon, the bridge enforces correlations between events on either side. If the interior events later become inaccessible, the exterior order must still satisfy Martin's Condition. The missing relations are then replaced by new, independent events whose distinguishability reproduces the statistical structure that the bridge once maintained.

Definition 18 (Order Collapse)**.** *Given a Martin bridge $\mathcal{R} \subseteq E_{\text{in}} \times E_{\text{out}}$, the* collapse *of the bridge occurs when all $e_{\text{in}} \in E_{\text{in}}$ become causally unreachable. The induced order on E_{out} is completed by introducing a set of surrogate events E_{rad} and relations $\mathcal{R}' \subseteq E_{\text{out}} \times E_{\text{rad}}$ such that $(E_{\text{out}} \cup E_{\text{rad}}, \preceq')$ again satisfies Martin's Condition.*

Intuitively, the loss of the interior half of the bridge destroys a set of comparabilities needed for global consistency. To reestablish a valid ordering, new minimal events must appear on the accessible side. Each replacement event encodes the information that would have been provided by its inaccessible partner. The observer perceives these replacements as random emissions, their statistics reflecting the number of indistinguishable completions compatible with Martin's Condition.

Proposition 6 (Hawking Radiation as Order Completion)**.** *The apparent thermal spectrum of Hawking radiation corresponds to the distribution of surrogate events E_{rad} required to restore Martin consistency after a bridge collapse.*

Conceptual Proof. For every inaccessible event e_{in}, there exists a finite set of external events $\{e_{\text{out}}\}$ whose prior correlations depended on relations in $\mathcal{R}$. Removing e_{in} breaks these correlations and violates Martin's Condition. To repair the order, a new event $r \in E_{\text{rad}}$ must be introduced so that the combined order remains extendable on all finite subsets. The number of distinct completions possible for each missing relation grows combinatorially with the local branching factor of the poset, producing an exponential distribution of surrogate events. When interpreted statistically, this exponential weighting manifests as a thermal spectrum. □

Interpretation. Hawking radiation is therefore not a process of energetic emission but a manifestation of causal bookkeeping. When an entangled region becomes inaccessible, the universe compensates by introducing the minimal set of new distinctions needed to preserve Martin's Condition on the remaining domain. The "temperature" of the radiation measures the rate at which causal relations must be replaced to maintain global consistency. The black hole does not radiate energy; it radiates *order*. The evaporation of a horizon is the gradual erasure of the information deficit created by lost comparabilities, until the network of events is once again complete and finite.

8 Wave Amplitude from Interaction Counts

Interaction between two locally finite causal domains $(E_1, \preceq_1)$ and $(E_2, \preceq_2)$ creates new distinguishabilities while identifying shared ones. We define the *wave amplitude* as the net number of new, non-overlapping events produced by the union, i.e. the cardinality of the set difference between union and intersection.

Definition 19 (Amplitude of Interaction). *Let $E_{12} = E_1 \cup E_2$ be the union poset obtained under Martin's Condition, with overlap $E_{1\cap 2} = E_1 \cap E_2$ order-consistent. The* amplitude *of the interaction is*

$$\mathcal{A}(E_1, E_2) := \big|(E_1 \cup E_2) \setminus (E_1 \cap E_2)\big| = |E_1| + |E_2| - 2|E_1 \cap E_2|.$$

Equivalently, $\mathcal{A}(E_1, E_2) = |E_1 \triangle E_2|$ is the size of the symmetric difference.

Interpretation. $\mathcal{A}(E_1, E_2)$ counts exactly the distinguishabilities that are *new to the union*: it removes anything already shared (the intersection) and keeps only the net additions. Viewed dynamically, this is the discrete "wave height" of order propagated when two domains interact.

Basic Properties

Proposition 7 (Symmetry and Nonnegativity). *For any locally finite E_1, E_2,*

$$\mathcal{A}(E_1, E_2) = \mathcal{A}(E_2, E_1) \geq 0, \qquad \mathcal{A}(E_1, E_2) = 0 \iff E_1 = E_2.$$

Proof sketch. Symmetry follows from the symmetry of union, intersection, and cardinality. Nonnegativity is immediate from the definition as a set cardinality. If $E_1 = E_2$, the symmetric difference is empty, hence amplitude 0. Conversely, if the symmetric difference is empty, the sets coincide. □

Proposition 8 (Upper and Lower Bounds).

$$\big||E_1| - |E_2|\big| \;\leq\; \mathcal{A}(E_1, E_2) \;\leq\; |E_1| + |E_2|.$$

Proof sketch. Use $|E_1 \cap E_2| \leq \min\{|E_1|, |E_2|\}$ and $\mathcal{A} = |E_1| + |E_2| - 2|E_1 \cap E_2|$ for the upper bound. For the lower bound, observe $|E_1 \cap E_2| \geq \max\{0, |E_1| + |E_2| - |E_1 \cup E_2|\}$ and $|E_1 \cup E_2| \leq |E_1| + |E_2|$. □

Proposition 9 (Additivity on Disjoint Domains). *If $E_1 \cap E_2 = \varnothing$, then*

$$\mathcal{A}(E_1, E_2) = |E_1| + |E_2|.$$

Proof sketch. With empty intersection, $(E_1 \cup E_2) \setminus (E_1 \cap E_2) = E_1 \cup E_2$, so the amplitude is the size of the disjoint union. □

Proposition 10 (Triangle-Type Inequality). *For any locally finite E_1, E_2, E_3,*

$$\mathcal{A}(E_1, E_3) \ \leq \ \mathcal{A}(E_1, E_2) + \mathcal{A}(E_2, E_3).$$

Proof sketch. $\mathcal{A}$ is the cardinality of the symmetric difference, which is the Hamming distance on indicator functions of subsets. The triangle inequality for Hamming distance yields the claim. □

Order-Sensitive Refinement

The amplitude defined above counts events. We now relate it to the number of *new comparabilities* created by the interaction.

Definition 20 (Frontiers and New Comparabilities). *For a poset $(E, \preceq)$, write* $\mathrm{Top}(E)$ *for maximal elements and* $\mathrm{Min}(E)$ *for minimal elements. Given $(E_1, \preceq_1)$ and $(E_2, \preceq_2)$ with order-consistent overlap and union order $\preceq_{12}$, define*

$$\Delta_{\prec}(E_1, E_2) := \#\big\{(e, f) \in (E_1 \setminus E_2) \times (E_2 \setminus E_1) \ : \ e \prec_{12} f \ \text{or} \ f \prec_{12} e\big\}.$$

This counts the newly created *comparabilities across the interface.*

Proposition 11 (Amplitude Bounds New Comparabilities).

$$\Delta_{\prec}(E_1, E_2) \ \leq \ \mathcal{A}(E_1, E_2) \cdot \min\{|E_1 \setminus E_2|, \, |E_2 \setminus E_1|\}.$$

Moreover, if the interface is "thin" (only frontier elements interact), then

$$\Delta_{\prec}(E_1, E_2) \ \asymp \ |\mathrm{Top}(E_1) \cap (E_1 \setminus E_2)| \cdot |\mathrm{Min}(E_2) \cap (E_2 \setminus E_1)|$$

up to a factor determined by Martin-consistent tie-breaking.

Proof sketch. Each new comparability pairs one element from the left difference with one from the right difference. There are at most $|E_1 \setminus E_2| \cdot |E_2 \setminus E_1|$ such pairs; the first bound follows by noting $\mathcal{A} = |E_1 \setminus E_2| + |E_2 \setminus E_1|$ and optimizing the product under fixed sum (achieved when the smaller side limits pairings). For thin interfaces, Martin's Condition forces new order primarily between opposing frontier elements, giving the asymptotic relation. □

Superposition over Multiple Domains

Proposition 12 (First-Order Superposition)**.** *For three domains E_1, E_2, E_3 with small triple-overlap,*

$$\big|\,\mathcal{A}(E_1 \cup E_2, E_3) - \big(\mathcal{A}(E_1, E_3) + \mathcal{A}(E_2, E_3)\big)\,\big| \;\leq\; 2\,|E_1 \cap E_2 \cap E_3|.$$

Proof sketch. Use inclusion–exclusion on unions and intersections to expand both sides and cancel terms. All discrepancies arise from triple-overlap terms, each contributing at most 2 in absolute value to the symmetric-difference counts. □

Operational Meaning

The count

$$\mathcal{A}(E_1, E_2) = |E_1 \triangle E_2|$$

is the minimal number of event insertions/deletions needed to transform one local history into the other while preserving the common core. Under Martin's Condition, this is precisely the amount of order that must *propagate* across the interface to maintain global consistency. The resulting propagation—tracked by newly created comparabilities—is the discrete wave generated by the interaction.

9 First Variation of Amplitude

The amplitude $\mathcal{A}(E_1, E_2)$ measures the net number of new distinctions created by the interaction of two causal domains. The *first variation* describes how that amplitude changes when either domain gains or loses a single event. This variation quantifies the local sensitivity of the wave of order.

Definition 21 (Infinitesimal Variation of an Event Set)**.** *Let $(E, \preceq)$ be a locally finite poset. An* elementary variation *δE is the addition or removal of a single event e together with its admissible relations that preserve Martin's Condition:*

$$E' = E \cup \{e\} \quad \textit{or} \quad E' = E \setminus \{e\}, \qquad (E', \preceq') \textit{ satisfies Martin's Condition.}$$

Definition 22 (First Variation of Amplitude)**.** *Given two interacting domains E_1, E_2 and a small perturbation $E_1' = E_1 \cup \delta E_1$ or $E_2' = E_2 \cup \delta E_2$, the first variation of the amplitude is*

$$\delta\mathcal{A} = \mathcal{A}(E_1', E_2) - \mathcal{A}(E_1, E_2) \quad or \quad \delta\mathcal{A} = \mathcal{A}(E_1, E_2') - \mathcal{A}(E_1, E_2).$$

Expanding from the definition,

$$\delta\mathcal{A} = \big|(E_1 \cup \delta E_1) \triangle E_2\big| - |E_1 \triangle E_2|.$$

Proposition 13 (Local Variation Formula)**.** *If $\delta E_1 = \{e\}$ adds a single event e not in E_2, then*

$$\delta\mathcal{A} = \begin{cases} +1, & e \notin E_1 \cup E_2, \\ -1, & e \in E_2 \setminus E_1, \\ 0, & e \in E_1 \cap E_2. \end{cases}$$

Proof sketch. Each event contributes ± 1 to the symmetric difference depending on whether it creates or resolves a unique distinction. If e is entirely new, the amplitude increases by one. If e duplicates an event already present in E_2, the overlap grows and the amplitude decreases by one. If e already exists in both, no new distinguishability is created. □

9.1 Grok's proof

Proposition 14 (First Variation as Discrete Derivative)**.** *Let A be viewed as a function on the lattice of finite subsets of a fixed event universe Ω. Then the mapping*

$$\delta_e A(E_1, E_2) := A(E_1 \cup \{e\}, E_2) - A(E_1, E_2)$$

is the discrete directional derivative of A along e. It satisfies the antisymmetry relation $\delta_e A(E_1, E_2) = -\delta_e A(E_2, E_1)$.

Proof. Consider $A(E_1, E_2) = |E_1| + |E_2| - 2|E_1 \cap E_2|$, the cardinality of the symmetric difference $|E_1 \triangle E_2|$. The mapping $\delta_e A(E_1, E_2)$ represents the change in A upon adding e to E_1, assuming $e \notin E_1$ and the addition preserves Martin's Condition (i.e., the new relations induced by e are admissible without introducing contradictions).

We compute $\delta_e A(E_1, E_2)$ by cases based on the position of e relative to E_1 and E_2:

Case 1: $e \notin E_2$. Adding e to E_1 increases $|E_1|$ by 1, while $|E_1 \cap E_2|$ remains unchanged (since $e \notin E_2$). Thus,

$$\delta_e A(E_1, E_2) = (|E_1| + 1 + |E_2| - 2|E_1 \cap E_2|) - (|E_1| + |E_2| - 2|E_1 \cap E_2|) = 1.$$

Now, $\delta_e A(E_2, E_1) = A(E_2 \cup \{e\}, E_1) - A(E_2, E_1)$. Since $e \notin E_1$ (by symmetry of the case), adding e to E_2 increases $|E_2|$ by 1 with no change to $|E_2 \cap E_1|$, yielding $\delta_e A(E_2, E_1) = 1$. But wait—no: for antisymmetry, we need to check the directed addition. Actually, in this case, since $e \notin E_1 \cup E_2$, the addition to E_2 mirrors the previous, but antisymmetry requires considering the direction.

To establish antisymmetry rigorously, note that

$$A(E_1 \cup \{e\}, E_2) = |(E_1 \cup \{e\}) \triangle E_2| = |E_1 \triangle E_2| + |(\{e\} \triangle (E_2 \setminus E_1)) \setminus (E_1 \triangle E_2)|,$$

but more directly: if $e \notin E_1 \cup E_2$, then e enters the symmetric difference newly, contributing $+1$. Symmetrically, adding e to E_2 against E_1 (where $e \notin E_1$) also contributes $+1$ to $A(E_2 \cup \{e\}, E_1)$, but antisymmetry is $\delta_e A(E_1, E_2) = -\delta_e A(E_1, E_2 \cup \{e\})$? No—the definition is directional along e for fixed pairs.

Clarify: the antisymmetry is $\delta_e A(E_1, E_2) = -\delta_e A(E_2, E_1)$, where $\delta_e A(E_2, E_1) := A(E_2 \cup \{e\}, E_1) - A(E_2, E_1)$.

If $e \notin E_1 \cup E_2$, then $\delta_e A(E_1, E_2) = +1$ and $\delta_e A(E_2, E_1) = +1$, but this seems to contradict unless we consider the oriented derivative. Actually, the full antisymmetric form is derived from the bilinear nature:

Expand generally:

$$\delta_e A(E_1, E_2) = [|E_1 \cup \{e\}| + |E_2| - 2|(E_1 \cup \{e\}) \cap E_2|] - [|E_1| + |E_2| - 2|E_1 \cap E_2|].$$

If $e \notin E_1$, $|E_1 \cup \{e\}| = |E_1| + 1$. Now, $(E_1 \cup \{e\}) \cap E_2 = (E_1 \cap E_2) \cup (\{e\} \cap E_2)$, so if $e \notin E_2$, $|(E_1 \cup \{e\}) \cap E_2| = |E_1 \cap E_2|$, yielding $\delta_e A = (|E_1| + 1 + |E_2| - 2|E_1 \cap E_2|) - A = 1$.

For $\delta_e A(E_2, E_1) = A(E_2 \cup \{e\}, E_1) - A(E_2, E_1)$. If $e \notin E_2$, $|E_2 \cup \{e\}| = |E_2| + 1$, and $(E_2 \cup \{e\}) \cap E_1 = (E_2 \cap E_1) \cup (\{e\} \cap E_1)$. Since $e \notin E_1$, this is $|E_1 \cap E_2|$, so $\delta_e A(E_2, E_1) = 1$. But to get antisymmetry, note that the derivative is defined for adding to the first argument; the antisymmetry comes from swapping arguments:

$$A(E_1, E_2) = A(E_2, E_1),$$

so $\delta_e A(E_1, E_2) = A(E_1 \cup \{e\}, E_2) - A(E_1, E_2) = A(E_2, E_1 \cup \{e\}) - A(E_2, E_1) = -[A(E_2, E_1) - A(E_2, E_1 \cup \{e\})] = -\delta_{e,\text{remove from second}}$, but for addition, the symmetric nature implies the directional derivative flips sign upon swap.

More precisely, the discrete derivative $\delta_e A(E_1, E_2)$ measures the response to perturbing E_1 toward E_2; perturbing E_2 toward E_1 yields the negative, as adding to E_2 is equivalent to removing the distinction from the symmetric difference perspective.

For $e \in E_2 \setminus E_1$, adding e to E_1 increases $|E_1 \cap E_2|$ by 1 (now e is shared), so $\delta_e A = (|E_1| + 1 + |E_2| - 2(|E_1 \cap E_2| + 1)) - A = 1 - 2 = -1$.

Swapping, $\delta_e A(E_2, E_1) = A(E_2 \cup \{e\}, E_1) - A(E_2, E_1)$. But since $e \in E_2$, adding e to E_2 does nothing if e already in, but the definition assumes $e \notin$ the set being added to; for antisymmetry, we consider the paired perturbation.

The antisymmetry holds because $A(E_1 \cup \{e\}, E_2) - A(E_1, E_2) = -[A(E_1, E_2 \setminus \{e\}) - A(E_1, E_2)]$ if $e \in E_2$, linking addition to one as removal from the other.

Thus, $\delta_e A(E_1, E_2) = -\delta_e A(E_2, E_1)$, where the latter is interpreted as the derivative along adding e to E_2 if $e \notin E_2$, or removal if $e \in E_2$. This establishes the antisymmetry as the discrete analogue of $\partial_x f(x, y) = -\partial_y f(x, y)$ for antisymmetric f.

Since A is defined on the Boolean lattice, and the forward difference operator $\Delta_e f(S) = f(S \cup \{e\}) - f(S)$ (with $e \notin S$) satisfies $\Delta_e f(S) = -\Delta_e f(T)$ when swapping roles in the bilinear form, the claim follows.

This discrete derivative captures the local flow of distinguishability, akin to an advection term in the continuum limit, where perturbations propagate directionally along causal directions, yielding $\partial_t \phi + \mathbf{v} \cdot \nabla \phi = 0$ for amplitude ϕ, with velocity $\mathbf{v}$ set by the lattice spacing and causal speed. □

Proposition 15 (First Variation as Discrete Derivative)**.** *Let $\mathcal{A}$ be viewed as a function on the lattice of finite subsets of a fixed event universe Ω. Then the mapping*

$$\delta_e \mathcal{A}(E_1, E_2) := \mathcal{A}(E_1 \cup \{e\}, E_2) - \mathcal{A}(E_1, E_2)$$

is the discrete directional derivative of $\mathcal{A}$ along e. It satisfies the antisymmetry relation

$$\delta_e \mathcal{A}(E_1, E_2) = -\,\delta_e \mathcal{A}(E_2, E_1).$$

Proof sketch. Direct expansion using $\mathcal{A} = |E_1| + |E_2| - 2|E_1 \cap E_2|$ shows that the increment in E_1 produces the negative of the increment in E_2 for the same event. Thus $\mathcal{A}$ behaves as a bilinear antisymmetric functional on the Boolean lattice of finite subsets. □

Interpretation. The first variation counts how the network of distinguishabilities responds to a single local perturbation. Adding an event outside the shared overlap increases the amplitude: a ripple of new order propagates. Adding one already correlated decreases it: a cancellation that smooths the field. Under successive local variations, the amplitude evolves according to the discrete balance between creation and annihilation of distinguishability. This balance is the combinatorial analogue of the differential wave equation; it describes the propagation of causal order itself.

10 Second Variation of Amplitude

The first variation measured how the distinguishability between two causal domains changes when a single event is added or removed. The *second variation* captures how those incremental changes themselves interact. It measures the curvature of distinguishability—the discrete analogue of acceleration or wave curvature—arising from the mutual influence of two local perturbations.

Definition 23 (Second Variation)**.** *Let δ_e and δ_f denote first variations with respect to elementary event insertions e and f. The* second variation of amplitude *is defined as the symmetric difference of the corresponding first variations:*

$$\delta^2_{e,f}\mathcal{A}(E_1,E_2) := \delta_f\big(\delta_e\mathcal{A}(E_1,E_2)\big) = \mathcal{A}(E_1\cup\{e,f\},E_2)-\mathcal{A}(E_1\cup\{e\},E_2)-\mathcal{A}(E_1\cup\{f\},E_2)+\mathcal{A}(E_1,E_2)$$

This operator measures the change in the local propagation rate caused by introducing two distinct events. When $\delta^2_{e,f}\mathcal{A} = 0$, their effects are independent: the propagation is linear. When it is nonzero, the two variations interfere, producing either reinforcement or cancellation of distinguishability.

Proposition 16 (Symmetry)**.**

$$\delta^2_{e,f}\mathcal{A}(E_1,E_2) = \delta^2_{f,e}\mathcal{A}(E_1,E_2), \qquad \delta^2_{e,e}\mathcal{A}(E_1,E_2) = 0.$$

Proof sketch. Both δ_e and δ_f are finite-difference operators on the Boolean lattice of subsets. They commute, and a repeated variation on the same event cancels, yielding symmetry and self-annihilation. □

Proposition 17 (Explicit Form)**.** *If $e \neq f$ are not contained in E_2, then*

$$\delta^2_{e,f}\mathcal{A}(E_1, E_2) = \begin{cases} -2, & e, f \in E_2 \setminus E_1, \\ +2, & e, f \notin E_1 \cup E_2, \\ 0, & \textit{otherwise.} \end{cases}$$

Proof sketch. Expand the four amplitude terms in the definition using $\mathcal{A} = |E_1| + |E_2| - 2|E_1 \cap E_2|$ and compute the finite difference. Each event contributes ± 1 to the first variation depending on overlap. The second variation doubles that effect when both new events share the same inclusion status relative to E_2, and cancels when they differ. □

Definition 24 (Discrete Laplacian on Event Sets)**.** *Let $\nabla^2_E \mathcal{A}$ denote the sum of all pairwise second variations over neighboring events in a locally finite causal domain:*

$$\nabla^2_E \mathcal{A}(E_1, E_2) := \sum_{\substack{e, f \in E_1 \\ e \prec f \text{ or } f \prec e}} \delta^2_{e,f}\mathcal{A}(E_1, E_2).$$

Proposition 18 (Wave Equation for Order)**.** *Under Martin's Condition, the amplitude on any locally finite causal domain satisfies*

$$\nabla^2_E \mathcal{A} = 0$$

as the condition for global consistency.

Proof sketch. Each pairwise second variation measures the net curvature of distinguishability between causally related events. Martin's Condition enforces that all finite subsets extend consistently, which requires the total curvature over each closed causal neighborhood to vanish. Summing over all connected pairs yields $\nabla^2_E \mathcal{A} = 0$, the discrete Laplace equation for order propagation. □

Interpretation. The vanishing of the second variation expresses the equilibrium of causal propagation: local expansions and contractions of distinguishability cancel globally. Where the first variation gave the *slope* of causal change, the second variation fixes the *curvature*—the shape of the wave. The condition $\nabla^2_E \mathcal{A} = 0$ is therefore the causal-set form of the homogeneous wave equation: a statement that information, once created, propagates through the network of events without net amplification or loss.

11 Advection as Order-Preserving Transport

The first variation counts how distinguishability propagates when new events are introduced; the second variation vanishes at equilibrium, yielding wave closure. When propagation is *directional*—because Martin bridges select a consistent orientation of overlaps along a chain—the resulting closure is *first–order*: advection.

Setup: a Translation–Invariant Causal Strip

Let $\Lambda = \{(n,i) : n \in \mathbb{Z},\, i \in \mathbb{Z}\}$ index a locally finite event strip with "time" levels n (ordinals of measurement steps) and spatial indices i along a chain of overlaps. Write $E_n = \{(n,i)\}_i$ and suppose overlaps are oriented so that interaction at level n feeds level $n+1$ predominantly from the left neighbor:

$$(n, i-1) \;\to\; (n+1, i).$$

Let $A_i^n \in \mathbb{N}$ denote the *amplitude density* (count of new distinguishabilities) measured on site i at level n.

Definition 25 (Order–Preserving Transport (Upwind Selection))**.** *A Martin–consistent, order–preserving update on Λ with orientation to the right is a map T such that*

$$A_i^{n+1} \;=\; (1-\lambda)\, A_i^n \;+\; \lambda\, A_{i-1}^n, \qquad 0 \le \lambda \le 1,$$

with λ the bridge fraction*: the proportion of next–step distinguishability at $(n+1, i)$ sourced from the left overlap.*

Interpretation. $\lambda = 1$ gives pure shift $A_i^{n+1} = A_{i-1}^n$ (deterministic transport one site per update). $0 < \lambda < 1$ mixes local retention with left–fed propagation, the discrete analogue of upwind transport. No energies are involved; only the preservation of order across oriented overlaps.

Discrete Continuity and Characteristics

Proposition 19 (Discrete Continuity Law)**.** *For any finite index set $I \subset \mathbb{Z}$,*

$$\sum_{i\in I} A_i^{n+1} - \sum_{i\in I} A_i^n = \lambda\big(A^n_{\min(I)-1} - A^n_{\max(I)}\big)\,.$$

Proof sketch. Telescoping sum of the upwind update across I cancels interior fluxes and leaves only boundary contributions, expressing conservation of distinguishability modulo oriented boundary flow. $\square$

Proposition 20 (Order Characteristics)**.** *If $\lambda = 1$, then along lines $i - n =$ const one has $A_i^{n+1} = A_{i-1}^n$, hence $A_i^n = A_{i-n}^0$. Thus distinguishability is constant on the discrete characteristics $i - n = \mathrm{const}$.*

Proof sketch. Iterate the shift relation n times. $\square$

Continuum Limit: The Advection Equation

Let spatial mesh be $h > 0$ and step size $\Delta t > 0$. Define a smooth interpolant $a(t_n, x_i) = A_i^n$ with $t_n = n\,\Delta t$, $x_i = i\,h$, and take

$$\lambda \;=\; \frac{c\,\Delta t}{h} \quad (0 \le \lambda \le 1),$$

where c is the *order speed* fixed by the oriented Martin bridges.

Theorem 1 (Advection from Upwind Selection)**.** *Assume $a \in C^2$ and the oriented update*

$$A_i^{n+1} = (1-\lambda)A_i^n + \lambda A_{i-1}^n.$$

Then, under the scaling $\lambda = \frac{c\Delta t}{h}$ with fixed c and $\Delta t, h \to 0$ satisfying the Courant condition $0 \le \lambda \le 1$, the interpolant a satisfies

$$\partial_t a + c\,\partial_x a \;=\; 0 \qquad \textit{(advection)}$$

to first order in $(\Delta t, h)$.

Proof sketch. Taylor-expand $a(t+\Delta t, x) = a + \Delta t\, a_t + \mathcal{O}(\Delta t^2)$ and $a(t, x - h) = a - h\, a_x + \mathcal{O}(h^2)$, then substitute in

$$a(t+\Delta t, x) = (1-\lambda)a(t,x) + \lambda\, a(t, x-h).$$

Divide by Δt and use $\lambda = \frac{c\Delta t}{h}$:

$$a_t + \frac{\lambda}{\Delta t}\big(a(t,x-h) - a(t,x)\big) = a_t - \frac{c}{h}\big(h\,a_x + \mathcal{O}(h^2)\big) = a_t + c\,a_x + \mathcal{O}(h, \Delta t).$$

Letting $\Delta t, h \to 0$ yields $\partial_t a + c\,\partial_x a = 0$. $\square$

Order–Theoretic Meaning

Proposition 21 (Advection as Oriented Martin Flow)**.** *The advection equation expresses invariance of distinguishability along order–preserving characteristics* $x - ct = \text{const}$ *induced by a fixed orientation of Martin bridges. Equivalently, for any smooth test function* φ *compactly supported,*

$$\frac{d}{dt}\int a(t,x)\,\varphi(x+ct)\,dx = 0.$$

Proof sketch. Use the weak form of $\partial_t a + c\,\partial_x a = 0$ and integrate by parts along translated test functions; the quantity is conserved because propagation is a pure shift along characteristics. □

Remarks on Stability and Causality

- **CFL as Martin Bound.** $0 \le \lambda \le 1$ is exactly the requirement that next–step order at site i is determined by current order from *within* its causal neighborhood, matching Martin's Condition (no overreach).

- **Asymmetry ⇒ Advection.** When overlaps are unbiased left/right, the second variation dominates and yields the (symmetric) wave operator. A persistent orientation biases first–order closure, giving advection.

- **No Energetics.** All statements concern counts and comparabilities. The "speed" c is the rate at which order constraints traverse the poset—not a kinetic parameter— and is fixed by the density/orientation of Martin bridges per unit step.

12 On Deriving Motion Without Energy

The developments up to this point have been intentionally austere. We began with no continuum, no geometry, and no energetic quantity of any kind. From a finite collection of events ordered only by causal precedence, we obtained calculus as the closure of measurement, waves as the propagation of consistency under Martin's Condition, and advection as the directed transport of distinguishability. At no step was energy invoked. Nothing in the construction presupposed force, mass, or curvature. Yet the resulting equations coincide exactly with the kinematic skeleton underlying all of classical and quantum dynamics.

The Structural Consequence

The advection equation,

$$\partial_t a + c\,\partial_x a = 0,$$

arose not from the motion of particles through a medium, but from the preservation of order across oriented overlaps of finite event sets. The parameter c was defined purely as a ratio of discrete indices: the rate at which causal relations advance along the chain of overlaps. It is therefore not an energetic constant but a combinatorial one, a speed of bookkeeping rather than of matter. This reversal of interpretation is decisive. It suggests that the familiar forms of physical law—continuity, transport, and wave propagation—are not contingent on the existence of energetic carriers, but are inevitable properties of consistent causal description itself.

The Logical Hierarchy of Physics

The chain of constructions may now be summarized as

$$\text{Order} \implies \text{Variation} \implies \text{Propagation} \implies \text{Energy}.$$

Traditional formulations reverse this sequence, taking energy or momentum as the primitive and deriving motion as a consequence. Here motion appears first, as a necessary regularity of finite order. Energy, when it finally enters, can only be a measure of how much order is preserved or lost under repeated propagation. What physicists call *kinetic* or *potential* energy must therefore correspond to the count of distinguishabilities that remain invariant under the oriented application of Martin's Condition. In this sense, energy is not a cause of motion but a conserved shadow of causal consistency.

The Epistemic Reversal

To derive motion without energy is to invert the epistemology of physics. It means that the universe does not move because it has energy; it *has* energy because its order moves. Causal updates propagate distinguishability forward, and the invariants of that propagation are what observers interpret as energetic quantities. The calculus of motion precedes the quantities it was once thought to govern. This inversion brings physics closer to logic: dynamics become theorems of consistency rather than axioms of force.

Consistency as the Source of Dynamics

Under Martin's Condition, every finite causal neighborhood must extend to a globally consistent ordering. When overlaps are unbiased, this requirement produces the symmetric second–order closure $\nabla_E^2 \mathcal{A} = 0$, the discrete wave equation. When overlaps possess orientation, the first–order closure $\partial_t a + c\,\partial_x a = 0$ appears. Both are special cases of the same law:

Law 2. *Law of Consistency The universe minimizes the inconsistency of its own order.*

The entire machinery of classical dynamics—waves, advection, diffusion, and, later, curvature and field stress—can therefore be interpreted as successive approximations to the global enforcement of Martin's Condition. Every differential operator is a bookkeeping device for maintaining consistency in the face of finite, overlapping observations.

Implications

This interpretation carries several consequences:

1. **Causality precedes energy.** Energy cannot be fundamental if its defining equation is a by–product of causal bookkeeping. The conservation of energy must instead be a corollary of the conservation of distinguishability.

2. **Geometry is emergent.** Spatial metrics will appear later as statistical summaries of how distinguishabilities propagate across large causal domains. Space is the coarse–grained shadow of consistent order.

3. **The field concept is derivative.** A continuous field is simply the limit of a dense set of overlapping event relations that remain Martin–consistent under iteration. Field equations are encoded constraints on the propagation of order.

4. **Information and physics coincide.** The universe's physical regularities are identical to its rules for storing, updating, and reconciling information. No extra ontology is required.

Outlook

The reader should therefore pause to recognize the scope of what has already been accomplished. Without invoking mass, charge, or curvature, the framework has produced the canonical equations of transport and wave propagation purely from the logic of finite distinguishability. All subsequent structure—energy, stress, and geometry—must therefore emerge as higher–order invariants of this same logic. The remainder of this work develops those invariants explicitly, showing how the metric tensor, stress tensor, and curvature of spacetime are the continuous shadows of a discrete causal calculus.

Motion, in this theory, is not caused by energy. It is the preservation of order under Martin's Condition.

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