Module 1: Order Monotonicity and The Second Law

Validating G1: Thermodynamic Closure $\Delta\,\mathrm{S} \geq 0$S0

G1S — Module Contract Fulfillment (V.tex Compliant)

October 13, 2025

1 Axiomatic Foundation: Causal Order and Entropy

The analysis begins with the locally finite causal order $\mathcal{C}$ of distinguishable events, whose structure is preserved by the Axiom of Event Selection (Martin-like consistency).

Definition 1 (Causal Entropy S)**.** *The* **Entropy** *associated with a causal set $\mathcal{C}$ is defined via the number of admissible micro-orderings, $\Omega(\mathcal{C})$:*

$$\mathrm{S}_{\mathcal{C}} = k_{\mathrm{B}} \ln |\Omega(\mathcal{C})|.$$

S *quantifies the number of distinct internal configurations consistent with the field* **U***.*

2 Theorem: The Second Law of Causal Order ($\Delta\,\mathrm{S} \geq 0$)

Theorem 1 (Monotonicity of Causal Entropy)**.** *In any admissible extension of a finite causal order, the count of distinguishable states cannot decrease.*

$$\Delta\,\mathrm{S} \geq 0.$$

G1 Proof Obligation Fulfillment. Let $\mathcal{C}_n$ be a causal set and $\mathcal{C}_{n+1}$ be an admissible refinement. By the Axiom of Event Selection, the set of admissible orderings is monotonically non-decreasing:

$$\Omega(\mathcal{C}_n) \subseteq \Omega(\mathcal{C}_{n+1}).$$

Taking the logarithm yields the constraint on the informational field $\mathbf{U}$:

$$\mathrm{S}_{\mathcal{C}_{n+1}} - \mathrm{S}_{\mathcal{C}_n} = \Delta\mathrm{S} \geq 0.$$

The non-decreasing nature of S is a theorem of order consistency, establishing thermodynamic closure. $\square$