V: Reader's Guide & Contract Layer (v1.2)

thermodynamic $\Delta\,\mathrm{S} \geq 0$, kinematic $\mathbf{U}^{(4)} = 0$, reciprocity $\mathsf{B}(\mathbf{U}, \varphi) = 0\ \forall\,\varphi \in \mathcal{V}$, conservation $\nabla\cdot\mathbf{T} = 0$

Verification Tensor (V)

October 13, 2025

Abstract

This note declares the shared *contracts* (G1–G5), an index-free *Numerics Rosetta* for matrix-free Newton–Krylov with a multigrid preconditioner, and a compact *interpretation layer* for physical meaning. All handles are title-safe (`\ensuremath`) and may be used in text, headings, and captions.

Contracts (G1–G5)

- **G1 thermodynamic closure:** $\Delta\,\mathrm{S} \geq 0$.
- **G2 kinematic closure (strong):** $\mathbf{U}^{(4)} = 0$.
- **G2 weak/reciprocity:** $\mathsf{B}(\mathbf{U}, \varphi) = 0\,\forall\,\varphi \in \mathcal{V}$; summarized by Discrete–continuum reciprocity via B.
- **G4 conservation (index-free):** $\nabla\cdot\mathbf{J} = 0$; translational case via $\nabla\cdot\mathbf{T} = 0$.
- **G5 statistical invariants:** D_{KL} monotone under admissible coarsegraining and $\mathbb{E}[\mathcal{S}] = 0$.

Shared symbols (continuum + statistics)

$\mathbf{U}$, $\mathbf{V}$, $\varphi \in \mathcal{V}$, ∇, $\mathcal{C}$, B, S, $\mathbf{U}^{(4)}$, Ξ, $\mathsf{N}[L, \mathbf{U}; \Xi]$, $\mathbf{J}$, $\mathbf{T}$, $\nabla\mathbf{J}$, $\mathbb{E}$, $\mathrm{Var}[\cdot]$, $\widehat{\cdot}$, $\mathcal{S}$, $\mathcal{I}$, $\mathrm{D}_{\mathrm{KL}}(P\,\|\,Q)$, $\mathsf{Inv}[\cdot]$, R.

Numerics Rosetta (JFNK + MG, index-free)

Abstract Newton–Krylov with a multigrid preconditioner is referenced via:

$$\mathsf{R}(\mathbf{U}) := \mathbf{U}^{(4)}, \qquad \mathsf{J}(\mathbf{U})\,v \approx \frac{\mathsf{R}(\mathbf{U} + \varepsilon v) - \mathsf{R}(\mathbf{U})}{\varepsilon}, \qquad \delta\mathbf{U} \text{ from } \mathsf{K}\Big[\mathsf{J},\ \mathsf{MG}\Big].$$

Discrete alignment (G3) supplies the hierarchy:

$$\Delta_h^{(2)},\ \Delta_h^{(4)},\ \mathsf{B}_h,\ \mathsf{R},\ \mathsf{P},\ \mathsf{S},\ \mathsf{MG},\ h.$$

(Handles only; no stencil/indices are fixed in V.)

Interpretation layer (structure $\to$ meaning)

- **Coercivity $\to$ stability:** The symmetric, positive bilinear form B guarantees existence and uniqueness of the weak solution. Operationally, this implies *stability and well-posedness*: small perturbations in the input or data induce small, predictable changes in the solution.

- **Kernel significance:** The four-dimensional kernel $\mathrm{span}\{1, x, x^2, x^3\}$ represents gauge freedom (position, slope, and constant acceleration) in the absence of external constraints. Fixed nodes together with the natural boundary conditions lock this gauge, ensuring the physical solution $\mathbf{U}^\star$ is unique and non-drifting.

- **DPI & irreversibility:** The monotonicity of the KL divergence $\mathrm{D}_{\mathrm{KL}}(P \,\|\, Q)$ under admissible coarse-graining is the statistical statement of thermodynamic irreversibility. Since measurement is an admissible map of the causal class $\mathcal{C}$, it can only collapse distinctions; thus predictive power cannot increase merely by running forward in time, cohering with $\Delta\,\mathrm{S} \geq 0$.

- **Discrete $\to$ continuum stability:** Discrete coercivity and consistency imply convergence (Lax equivalence). Physically, the numerical scheme is stable: finite resolution effects decay under refinement $h \to 0$, recovering the continuum behavior without spurious modes.

Usage notes (do & don't)

- Use statement handles directly in text: "By ($\mathbf{U}^{(4)} = 0$) and ($\mathsf{B}(\mathbf{U}, \varphi) = 0\ \forall\, \varphi \in \mathcal{V}$) ..."

- Avoid `\left...\right` unless both sides are present.

- Do not redeclare V handles in modules; add local aliases only when necessary.

Smoke test (text-mode safety)

In text: $\mathsf{B}(\mathbf{U}, \varphi) = 0\ \forall\, \varphi \in \mathcal{V} \Rightarrow$ IBP $\times 2 \Rightarrow \mathbf{U}^{(4)} = 0$; solve via $\mathsf{J}(\mathbf{U})\,\delta\mathbf{U} = -\mathsf{R}(\mathbf{U})$ with $\delta\mathbf{U} = \mathsf{K}[\mathsf{J}, \mathsf{MG}]$; conclude $\nabla\cdot\mathbf{T} = 0$.