A Minor Entropic Correction Term in Einstein's Field Equations

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Abstract

We develop a finite, computational model of measurement in which time is the ordinal index of distinguishable events. Starting from the axioms of set theory and a locally finite causal order, we show that all physical quantities can be expressed as counts of measurable distinctions. We develop the Reciprocity Law of Physics, which equates variation with measurement, leads naturally to the calculus of variations as the unique closure condition on consistent observation. Its smooth limit does not assert continuity but encodes it: we represent discrete measurements with cubic splines, which serve as compact representations of the data rather than assumptions about the underlying field. This recovers the familiar continuous differential equations of universal physical laws.

1 Introduction

Every physical description begins not with space or time, but with an *event*—an interaction that makes previously indistinguishable outcomes distinct [2, 10]. The causal boundary of such an interaction is its *light cone*: the set of all events that can influence or be influenced by it according to special relativity [4, 8]. The intersection of two light cones, corresponding to the last particle–wave interaction accessible to an observer, defines the maximal

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region of causal closure [5, 9]. Beyond this surface, no additional information can be exchanged; all distinguishable action has concluded.

It is from this closure that the ordering of events arises [5, 7]. Each measurable interaction contributes one additional distinction to the universe, expanding its causal surface by a finite count [5, 7]. The smooth fabric of spacetime is not primitive but emergent: it is the limiting behavior of discrete causal increments accumulated along the light cone [3, 11]. Within each cone, the universe can be represented by a finite tensor of interactions—local updates to a global state—that together approximate continuity only through cancellation across countable events [3, 12].

Special relativity provides the canonical local model for this causal structure [4]. Consider the Lorentz transformation for a boost of velocity v in one spatial dimension,

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v/c^2 \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}, \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{1}$$

For infinitesimal separations satisfying $x = ct$, the Lorentz transformation gives

$$t' = \gamma\, t(1 - v/c). \tag{2}$$

If we take $\Delta t = 1$ as the unit interval between distinguishable events, then observers moving at relative velocity v will, in general, disagree on the *number* of such events that occur between two intersections of their respective light cones [8]. The only invariant quantity is the causal ordering itself: all observers concur on which event precedes which, even though they may count a different number of intermediate ticks [7].

This observation motivates the first physical axiom: that time is not an independent scalar field but an ordinal index over causally distinguishable events. Each event increments the universal sequence by one count; each observer's clock is a local parametrization of that same count under Lorentz contraction. The apparent continuity of time is the result of the density of such events within the causal cone, not an underlying continuum of duration.

The framework that follows formalizes this intuition. Starting from Zermelo–Fraenkel set theory with the Axiom of Choice, we construct an ordered set of events whose distinguishability relations reproduce the causal order implied by special relativity. Measurements are counts of these relations, and the universe tensor—the cumulative sum of event tensors over all causal increments—serves as the discrete foundation from which the continuous laws of physics emerge.

2 The Axioms of the Mathematical

All mathematics in this work is carried out within the framework of Zermelo–Fraenkel set theory with the Axiom of Choice (ZFC) [6]. Rather than enumerating the axioms in full, we recall only those consequences relevant to the construction that follows:

- **Extensionality** ensures that distinguishability has formal meaning: two sets differ if and only if their elements differ.
- **Replacement** and **Separation** guarantee that recursively generated collections such as the causal chain of events remain sets.
- **Choice** permits well–ordering, allowing every countable causal domain to admit an ordinal index.

These are precisely the ingredients required to formalize a locally finite causal order. All further constructions—relations, tensors, and operators—are definable within standard ZFC mathematics; no additional axioms are introduced.

Axiom 1 (The Axioms of Mathematics)**.** *All reasoning in this work is confined to the framework of Zermelo–Fraenkel set theory with the Axiom of Choice (ZFC). Every object—sets, relations, functions, and tensors—is constructible within that system, and every statement is interpretable as a theorem or definition of ZFC. No additional logical principles are assumed beyond those required for standard analysis and algebra.*

Formally,

$$\text{Physics} \subseteq \text{Mathematics} \subseteq \text{ZFC}.$$

Thus, the language of mathematics is taken to be the entire ontology of the theory: the physical statements that follow are expressions of relationships among countable sets of distinguishable events, each derivable within ordinary mathematical logic.

2.1 Sets of Events

Let the set of all events accessible to an observer be denoted E, ordered by causal precedence $\leq$. Because any physically realizable region is finite, this order forms a locally finite partially ordered set (poset).

Definition 1 (Partially Ordered Set)**.** *A* partially ordered set *(poset) is a pair* $(E, \leq)$ *where* $\leq$ *is a binary relation on* E *satisfying:*

1. ***Reflexivity:*** $e \leq e$ *for all* $e \in E$*;*
2. ***Antisymmetry:*** *if* $e \leq f$ *and* $f \leq e$*, then* $e = f$*;*
3. ***Transitivity:*** *if* $e \leq f$ *and* $f \leq g$*, then* $e \leq g$*.*

Such an ordering always admits at least one maximal element:

$$\mathrm{Top}(E) = \{\, e \in E \mid \nexists f \in E \text{ with } e < f \,\}. \tag{3}$$

The elements of $\mathrm{Top}(E)$ represent the current causal frontier—the most recent events that have occurred but have no successors. Although $\mathrm{Top}(E)$ may contain several incomparable (spacelike) elements, it is never empty and therefore provides a well-defined notion of a "last event" from the observer's perspective. This frontier defines the light-cone boundary and the terminal particle–wave interaction that delimits all accessible information.

3 The Axioms of the Physical

A common criticism of mathematical physics is the extent to which mathematics can be tuned to fit observation [2, 10] and, conversely, manipulated to yield nonphysical results [1, **?**]. The critique of Newton's fluxions could only be answered by successful prediction. Today, calculus feels like a natural extension of the real world—so much so that Hilbert, in posing his famous list of open problems, explicitly formalized the lack of a rigorous foundation for physics as his Sixth Problem.

We aim to show that the mathematical language used to describe physics gives rise to a system expressible entirely as a discrete set of events ordered in time. Moreover, this ordered set possesses a mathematical structure that naturally yields the appearance of continuous physical laws and the conservation of quantities. To understand how this works, we first clarify what we mean by measurement.

3.1 Measurement and the Axiom of Order

Physical laws relate measurements. For example, Newton's second law

$$F = \frac{dp}{dt} \tag{4}$$

states that force relates to the *change* in momentum over time. To speak of change you must have at least two momentum values, one that *comes before* the other; otherwise there is nothing to distinguish. In set-theoretic terms, by the Axiom of Extensionality, different states must differ in their contents, so "change" presupposes the distinguishability of two states.

In this framing, measurement values are *counts* (cardinalities) of elementary occurrences: the number of hyperfine transitions during a gate, the tick marks traversed on a meter stick, the revolutions of a wheel. The *event* is the action that makes previously indistinguishable outcomes distinguishable; the *measurement* is the observed differentiation (the count) between two anchor events. This is not the absolute measure of the event, but just relative difference of the two. We count the events as time passes.

Since special relativity requires that time vary under the Lorentz transform, there can be no global scalar representation of temporal duration. Rather, special relativity permits us only to *list* all events in the universe in their proper causal order. It is this ordered list that we elevate to the first physical principle:

Axiom 2 (The Axiom of Order)**.** *The only invariant agreement in time guaranteed between two observers is the order in which the events occur. The duration between two events is defined as the number of measurements that can be recorded between them:*

$$|\delta t| \;=\; \big|\textit{events distinguished between}\big|. \tag{5}$$

As a corrolary to this, there exists a tensor that allows all events in the universe to occur at integer moments in time, denoted $\mathbf{U}$, the universe tensor. Although this tensor is finite, it suffices to demonstrate how discrete parameters can be represented by piece–wise cubic polynomial, thereby yielding the continuous laws of physics. In this way, the smoothness observed in physical theories is an emergent property of cancellation across discrete counts rather than a primitive assumption of continuity.

Definition 2 (Time)**.** *Time is not a variable, scalar, or independent measurement. Rather, it is an index into the sorted list of events guaranteed by the Axiom of Order. Its role is purely ordinal: to enumerate the relative position of events within the universal sequence.*

Definition 3 (Event Tensor)**.** *Let $\mathcal{V}$ be a finite–dimensional real vector space of measurable quantities. An* event tensor $\mathbf{E}_k \in \mathcal{T}(\mathcal{V})$ *encodes the distinguishable contribution of the k-th event $e_k \in \mathcal{E}$ to the global state. It is related to the logical event by a measurable embedding $\Psi : \mathcal{E} \to \mathcal{T}(\mathcal{V})$, where $\mathbf{E}_k = \Psi(e_k)$.*

Proposition 1 (Causal Universe Tensor)**.** *Let $\{\mathbf{E}_k\}_{k=1}^n$ be the ordered sequence of event tensors guaranteed by the Axiom of Order. The* universe tensor *after n events is the ordered sum*

$$\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k, \tag{6}$$

where addition in $\mathcal{T}(\mathcal{V})$ preserves causal order: if $i < j$, then $(\mathbf{E}_i, \mathbf{E}_j)$ occurs before $(\mathbf{E}_j, \mathbf{E}_i)$ unless $\mathbf{E}_i$ and $\mathbf{E}_j$ commute.

Proof. By the Axiom of Order, all observers agree only on the *sequence* in which events occur. Thus, the state of the universe can be constructed recursively:

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \mathbf{E}_{n+1}. \tag{7}$$

Since $\mathbf{U}_1 = \mathbf{E}_1$, induction yields $\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k$. □

3.2 Formal Structure of Event and Universe Tensors

We now specify the algebraic structure of the quantities introduced above. Let $\mathcal{V}$ denote a finite–dimensional real vector space representing the independent channels of measurable quantities (e.g. energy, momentum, charge). Define the tensor algebra

$$\mathcal{T}(\mathcal{V}) = \bigoplus_{r=0}^{\infty} \mathcal{V}^{\otimes r}, \tag{8}$$

whose elements are finite sums of r–fold tensor products over $\mathbb{R}$. Each *event tensor E_k* is a member of $\mathcal{T}(\mathcal{V})$ encoding the distinguishable contribution of the k–th event to the global state. We write

$$\mathbf{E}_k \in \mathcal{T}(\mathcal{V}), \qquad \mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k \in \mathcal{T}(\mathcal{V})). \tag{9}$$

Addition is understood componentwise in the direct sum and preserves the ordering of indices guaranteed by the Axiom of Order. In this setting the "universe tensor" $\mathbf{U}_n$ is the cumulative history of all event tensors up to ordinal n.

Definition 4 (Tensor Algebra)**.** *The tensor algebra on V is*

$$\mathcal{T}(\mathcal{V}) = \bigoplus_{r=0}^{\infty} \mathcal{V}^{\otimes r},$$

with componentwise addition and associative tensor product.

Remark 1. *Each logical event e_k in the partially ordered set $(\mathcal{E}, \prec)$ induces a tensor $\mathbf{E}_k = \Psi(e_k)$ in $\mathcal{T}(\mathcal{V})$. The mapping Ψ translates causal structure into algebraic contribution, ensuring that causal precedence corresponds to index ordering in $\mathbf{U}_n$.*

Because $\mathcal{T}(\mathcal{V})$ is a free associative algebra, all operations on $\mathbf{U}_n$ are well defined using the standard linear maps, contractions, and bilinear forms of $\mathcal{V}$. The subsequent analysis of variation and measurement therefore proceeds entirely within conventional linear–operator theory.

From this definition of the universe tensor, it is easy to define an entanglement as a set of events that can be permuted in the list of all events without changing any invariant scalars.

Definition 5 (Entanglement)**.** *From the definition of the universe tensor*

$$\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k, \tag{10}$$

an entanglement *is a subset of events*

$$S \subseteq \{\mathbf{E}_1, \ldots, \mathbf{E}_n\} \tag{11}$$

such that for any permutation π of S,

$$\sum_{\mathbf{E}_i \in S} \mathbf{E}_i = \sum_{\mathbf{E}_i \in S} \mathbf{E}_{\pi(i)}, \tag{12}$$

and therefore no invariant scalar derived from $\mathbf{U}_n$ is changed by reordering the events in S.

Example 1 (Finite Causal Chain). *Consider a toy causal network consisting of four ordered events $A_1 \prec A_2$ and $B_1 \prec B_2$, with no initial ordering between the A and B chains. Let each event tensor be a 2×2 real matrix recording a pair of measurable quantities, for instance*

$$\mathbf{E}_{A_1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{E}_{A_2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{E}_{B_1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{E}_{B_2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{13}$$

The cumulative universe tensor through all four events is then

$$\mathbf{U}_4 = \mathbf{E}_{A_1} + \mathbf{E}_{A_2} + \mathbf{E}_{B_1} + \mathbf{E}_{B_2} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{14}$$

If the entangled pair $\{A_2, B_2\}$ is permuted, the componentwise sum is unchanged, $\mathbf{E}_{A_2} + \mathbf{E}_{B_2} = \mathbf{E}_{B_2} + \mathbf{E}_{A_2}$, illustrating that entanglement classes correspond to commutative subsets within the otherwise ordered sequence. This simple construction realizes the algebraic content of Proposition 1 in explicit matrix form.

Example 2 (Spooky Action at a Distance). *Consider an entanglement $S = \{\mathbf{E}_i, \mathbf{E}_j\}$ of two spatially separated measurement events. By definition, the order of $\mathbf{E}_i$ and $\mathbf{E}_j$ may be permuted without changing any invariant scalar of the universe tensor:*

$$\mathbf{E}_i + \mathbf{E}_j = \mathbf{E}_j + \mathbf{E}_i. \tag{15}$$

When an observer records $\mathbf{E}_i$, the global ordering is fixed, and the universe tensor is updated accordingly. Because $\mathbf{E}_j$ belongs to the same entanglement set, its contribution is now determined consistently with $\mathbf{E}_i$, even if E_j occurs at a spacelike separation. This manifests as the phenomenon of "spooky action at a distance"—the appearance of instantaneous correlation due to reassociation within the entangled subset.

Example 3 (Hawking Radiation). *Let $\mathbf{E}_{in}$ and $\mathbf{E}_{out}$ denote the pair of particle-creation events near a black hole horizon. These events form an entangled set:*

$$S = \{\mathbf{E}_{in}, \mathbf{E}_{out}\}. \tag{16}$$

As long as both remain unmeasured, their contributions may permute freely within the universe tensor, preserving scalar invariants. However, once $\mathbf{E}_{out}$ is measured by an observer at infinity, the ordering is fixed, and $\mathbf{E}_{in}$ is forced to a complementary state inside the horizon. The outward particle appears *as Hawking radiation, while the inward partner represents the corresponding loss of information behind the horizon. Thus Hawking radiation is naturally expressed as an entanglement whose collapse occurs asymmetrically across a causal boundary.*

Definition 6 (Distinguishability chain)**.** *Let Ω be a nonempty set. A* distinguishability chain *on Ω is a sequence $\mathcal{P} = \{P_n\}_{n\in\mathbb{Z}}$ of partitions $P_n \in \mathsf{Part}(\Omega)$ such that P_{n+1}* refines *P_n for all n (every block of P_{n+1} is contained in a block of P_n). Write $\mathrm{Bl}(P)$ for the set of blocks of a partition P.*

Definition 7 (Event)**.** *Fix a distinguishability chain $\mathcal{P} = \{P_n\}$. An* event at index *n is a minimal refinement step: a pair*

$$e = (B, \{B_i\}_{i\in I}, n) \tag{17}$$

such that:

1. *$B \in \mathrm{Bl}(P_n)$;*
2. *$\{B_i\}_{i\in I} \subseteq \mathrm{Bl}(P_{n+1})$ is the family of all blocks of P_{n+1} contained in B, with $|I| \geq 2$ (a nontrivial split);*
3. *(*minimality*) there is no proper subblock $C \subsetneq B$ with $C \in \mathrm{Bl}(P_n)$ for which the family $\mathrm{Bl}(P_{n+1}) \cap \mathcal{P}(C)$ is nontrivial.*

Let E denote the set of all such events. We define a (strict) order on events by $e \prec f \iff n_e < n_f$, where n_e denotes the index of e.

Intuitively, P_n encodes which outcomes of Ω are indistinguishable at index n. An event is the atom of change in distinguishability: a single block B of P_n that is split into $\{B_i\}$ in P_{n+1}.

Definition 8 (Predicate on events)**.** *A* predicate *is any map $P : E \to \{0,1\}$. It selects which events are "counted."*

Definition 9 (Measurement)**.** *Let E be the event set with order $\prec$, and let $P : E \to \{0,1\}$ be a predicate. Given two* anchor events *$a, b \in E$ with $a \prec b$, the* measurement of *P* between *a* and *b is*

$$M_P[a,b] \ := \ \#\{\, e \in E \mid a \prec e \prec b \text{ and } P(e) = 1 \,\} \in \mathbb{N}. \tag{18}$$

Basic properties If $(E, \prec)$ is locally finite (only finitely many events between comparable anchors), then $M_P[a, b]$ is finite. Measurements are *additive*: for $a \prec c \prec b$,

$$M_P[a, b] \; = \; M_P[a, c] + M_P[c, b]. \tag{19}$$

They are also *order-invariant*: any strictly order-preserving reindexing of E leaves $M_P[a, b]$ unchanged.

3.3 Axiom of Finite Observation

The recursive description of physical reality is meaningful only within the finite causal domain of an observer. Each step in such a description corresponds to a distinct measurement or recorded event. Observation is therefore bounded not by the universe itself, but by the observer's own proper time and capacity to distinguish events within it.

Axiom 3 (The Axiom of Finite Observation)**.** *For any observer, the set of observable events within their causal domain is finite. The chain of measurable distinctions terminates at the limit of the observer's proper time or causal reach.*

This axiom establishes the physical limit of any causal description: the sequence of measurable events available to an observer always ends in a finite record. Beyond this frontier—beyond the end of the observer's time— no additional distinctions can be drawn. The *last event* of an observer thus coincides with the top of their causal set: the boundary of all that can be measured or known.

The Axiom of Finite Observation has a corollary familiar to every graduate student: the capacity of the universe to surprise is infinite, but the capacity of the hard drive is not.

3.4 Construction of the Universe Tensor and the Axiom of Event Selection

Even though mathematics is powerful enough to describe the laws of physics with predictive accuracy, it can also compute nonphysical phenomena. Negative areas, for instance, are a common mathematical construct:

$$\int_0^{\pi} -\sin x \, dx. \tag{20}$$

Even worse, pathological geometries can give rise to fantastical descriptions of internal states of the computation, leading to ill-defined behaviors. We see this at the singularity of general relativity or at the scale of the Planck length, where the formalism itself begins to overcount possibilities.

To control such overgeneration, we invoke *Martin's Axiom*, a principle of set theory that restricts the construction of large or pathological subsets without measurable support. In physical terms, Martin's Axiom acts as a regularity condition on the events in the universe: it guarantees that the events we can describe are countably generable from locally finite information. This eliminates spurious solutions that arise purely from mathematical freedom, ensuring that only physically realizable events are included in the ordering. For instance, the Banach-Tarski paradox is not possible to construct with Martin's Axiom as each individual set is unbounded in ordering and therefore excluded from possibility.

Martin's Axiom will allow us to demonstrate that the ordering of events is sufficient to describe time and still recover the laws of physics.

Axiom 4 (The Axiom of Event Selection)**.** *For any countable family of events, there exists a consistent extension selecting one outcome from each family such that all physically realizable events remain distinguishable within the universe.*

In other words, if an event happens "next" in a causal light cone, then it must happen independently of events outside the causal light cone.

More mathematically, we take as the corollary to the Axiom of Event Selection, Martin's Axiom:

Corollary 1 (Martin's Axiom)**.** *Let* $(\mathbb{P}, \leq)$ *be a partially ordered set satisfying the* countable chain condition *(ccc); that is, every antichain in* $\mathbb{P}$ *is countable. For any cardinal* $\kappa < 2^{\aleph_0}$ *and any family* $\{D_\xi : \xi < \kappa\}$ *of dense subsets of* $\mathbb{P}$*, there exists a filter* $G \subseteq \mathbb{P}$ *such that*

$$\forall\, \xi < \kappa, \quad G \cap D_\xi \neq \varnothing.$$

The physical correspondence to Martin's Axiom should be understood as an analogy of structure, not identity of assumption. In our formulation, the partially ordered set $(P, \leq)$ corresponds to the causal ordering of events. Finite observation guarantees that all antichains of physically accessible events are finite, a strong version of the countable chain condition. The *Axiom of*

Event Selection therefore asserts that local causal choices admit a consistent global extension, exactly as Martin's Axiom asserts the existence of a filter meeting all dense subsets. Both function as regularity principles eliminating pathological or non-realizable combinations of events.

4 The Equivalence Principle of Physics

4.1 Variations and the Reciprocity of Measurement

Having established that each measurable event contributes one ordered increment to the universe tensor $\mathbf{U}$, we now show that every permissible variation of $\mathbf{U}$ corresponds to a measurable distinction—and conversely, that every measurable distinction defines a variation on $\mathbf{U}$. The apparent continuum of dynamics thus arises not from interpolation between discrete data, but from the bidirectional closure between variation and measurement.

4.1.1 From Distinguishability to Variation

Let the ordered set of events $\{\mathbf{E}_k\}$ define

$$\mathbf{U}_n = \sum_{k=1}^{n} \mathbf{E}_k. \tag{21}$$

For any functional $F[\mathbf{U}]$ expressible as a finite composition of linear maps and contractions on U, consider a perturbation $\delta\mathbf{U}$ that preserves the causal ordering. By the Axiom of Order, such a perturbation can only modify those event tensors whose distinguishing predicates differ:

$$\delta\mathbf{U} = \sum_{k:\, \delta P(E_k)\neq 0} \delta\mathbf{E}_k. \tag{22}$$

Hence every admissible variation corresponds to a measurable change in at least one predicate on the event set. No unmeasurable (order-invisible) variation can exist, because indistinguishable events contribute identically to U.

4.1.2 From Variation to Measurement

Conversely, let two measurements $M_P[a, b]$ and $M_Q[a, b]$ be performed on the same causal interval with predicates $P, Q : \mathbf{E} \to \{0, 1\}$. Define their difference

$$\Delta M[a,b] = M_Q[a,b] - M_P[a,b] = \#\{\, e \in \mathbf{E} \mid a \prec e \prec b,\ P(e) \neq Q(e) \,\}. \tag{23}$$

Each nonzero contribution to ΔM identifies an event whose predicate value has changed—that is, an elementary variation $\delta\mathbf{E}_k$. Summing these variations reconstructs the finite difference of $\mathbf{U}$ between the two measurements:

$$\mathbf{U}_Q - \mathbf{U}_P = \sum_{e \in \mathbf{E}\colon P(e) \neq Q(e)} \delta\mathbf{E}_e = \delta\mathbf{U}. \tag{24}$$

Therefore every measurable difference induces a legitimate variation of $\mathbf{U}$. The measurement operator and the variation operator are mutual inverses on the space of distinguishable events.

4.1.3 Reciprocal Closure

Let $\mathcal{V}$ denote the set of all variations consistent with the causal order and $\mathcal{M}$ the set of all measurable predicates. The preceding arguments define bijections

$$\Phi : \mathcal{V} \to \mathcal{M}, \qquad \Phi^{-1} : \mathcal{M} \to \mathcal{V}, \tag{25}$$

establishing the following physical principle.

4.2 Formal Definition of the Reciprocity Mapping

Let $\mathcal{V}$ and $\mathcal{T}(\mathcal{V})$ be as above. Define the space of admissible variations

$$V = \{\, \delta\mathbf{U} \in \mathcal{T}(\mathcal{V}) \mid \delta\mathbf{U} \text{ preserves causal order} \,\}, \tag{26}$$

and the space of measurable predicates

$$M = \{\, P : \mathcal{E} \to \{0,1\} \,\}, \tag{27}$$

where $\mathcal{E}$ is the set of events.

We introduce the mapping

$$\Phi : V \to M, \qquad \Phi(\delta U)(e) = \begin{cases} 1, & \text{if the event tensor of } e \text{ changes under } \delta\mathbf{U}, \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

Its inverse reconstructs a variation from a predicate:

$$\Phi^{-1}(P) = \sum_{e \in \mathcal{E}\colon P(e)=1} \delta\mathbf{E}_e. \tag{29}$$

Proposition 2 (Equivalence of Discrete and Continuum)**.** Φ *is bijective on the space of distinguishable events.*

Proof. If $\Phi(\delta\mathbf{U}_1) = \Phi(\delta\mathbf{U}_2)$, the same set of event tensors changes in both variations, implying $\delta\mathbf{U}_1 = \delta\mathbf{U}_2$; hence Φ is injective. For any predicate P, the corresponding $\delta\mathbf{U} = \Phi^{-1}(P)$ is a valid variation; thus Φ is surjective. Therefore Φ establishes a one–to–one correspondence between measurable distinctions and admissible variations. □

Equivalence 1 (The Reciprocity Law of Physics)**.** *Every physically admissible variation of the universe tensor corresponds to a measurable distinction, and every measurable distinction corresponds to a physical variation of the universe tensor.*

Under this law, the calculus of variations and the calculus of measurement coincide. The differential form of physical law,

$$\delta F[\mathbf{U}] = 0, \tag{30}$$

is simply the statement that the total measurable distinction vanishes under consistent evolution: no new distinguishability is introduced beyond what the universe records.

4.3 Discrete–to–Continuum Limit

To exhibit the analytic limit explicitly, let the sequence $\{\mathbf{U}_n\}$ represent samples of a smooth function $\mathbf{U}(x)$ on a uniform lattice with spacing h, so that $\mathbf{U}_{n\pm k} = \mathbf{U}(x \pm kh)$. Define the fourth–order finite difference operator

$$\Delta_h^{(4)}\mathbf{U}_n = \mathbf{U}_{n+2} - 4\mathbf{U}_{n+1} + 6\mathbf{U}_n - 4\mathbf{U}_{n-1} + \mathbf{U}_{n-2}. \tag{31}$$

If the recursive updates of reciprocal measurement drive this operator toward zero, $\Delta_h^{(4)}\mathbf{U}_n \to 0$ as n increases, then by standard difference analysis

$$\lim_{h\to 0} \frac{\Delta_h^{(4)}\mathbf{U}_n}{h^4} = \frac{d^4\mathbf{U}}{dx^4}(x) = \mathbf{U}^{(4)}(x). \tag{32}$$

Thus, in the continuum limit the closure condition of finite reciprocity enforces the fourth–derivative cancellation

$$\mathbf{U}^{(4)}(x) = 0, \tag{33}$$

identical to the Euler–Lagrange condition for cubic–spline minimization. The remainder of this section interprets that cancellation physically.

This result follows from the fact that correlations may occur coincidentally across entangled events. Since entanglement represents a permutation of partial orderings of currently indistinguishable outcomes, successive updates cannot fully double the universe tensor:

$$|\mathbf{U}_{n+1}| \leq 2|\mathbf{U}_n|. \tag{34}$$

The inequality expresses the loss of independent degrees of freedom due to coincident correlations. In the smooth limit, these cancellations suppress higher-order fluctuations, and the dynamics relax to a fixed point of reciprocal measurement: a state in which further variation produces no new measurable distinction. This apparent non-local coherence is the mechanism that preserves global consistency when local degrees of freedom collapse (Example 2).. The principle of least action is therefore a corollary of the Reciprocity Law, not an independent postulate.

4.3.1 Example: Coincidence as a Retro-Constraint

Consider two causal chains,

$$A_1 \prec A_2, \qquad B_1 \prec B_2, \tag{35}$$

representing two local measurements. Each chain is internally ordered, but the relative ordering between the A and B events is only partially specified.

Suppose an invariant condition couples the terminal events,

$$f(A_2, B_2) = 0, \tag{36}$$

such that the combined value of the pair must satisfy a conservation or matching rule in the universe tensor. When this constraint is enforced at the future boundary (A_2, B_2), it propagates backward through the partial order: the admissible values of (A_1, B_1) are now restricted to those for which the subsequent evolution yields the required terminal pair. Formally, we obtain a dependency

$$(A_1, B_1) \longmapsto (A_2, B_2), \tag{37}$$

so that the poset must be extended with additional relations ensuring compatibility. In the simplest case, one future event becomes conditionally prior:

$$A_1 \prec B_2 \quad \text{(if } B_2 \text{ requires a specific } A_1 \text{ value).} \tag{38}$$

This induced relation is what we call a *coincidence*: a future event whose consistency condition fixes a present variable. In the universe tensor, such coincidences appear as cancellations of independent variations—degrees of freedom that are no longer free once the end condition is imposed. Each coincidence therefore removes one order of independent variation from the causal sum, driving the sequence toward the smooth limit

$$\mathbf{U}^{(4)}(x) = 0. \tag{39}$$

Thus, a "coincidental" alignment is not a mystery of timing but a structural enforcement of consistency within the partially ordered set: the future boundary constrains the present values so that the entire tensor remains self-consistent under reciprocal measurement. This is the operational significance of the Axiom of Event Selection— only those events consistent with the full causal ordering can occur.

4.4 Deriving the Principle of Least Action

Consider the universe tensor $\mathbf{U}$ evaluated along a single coordinate x between two measurable events. Because all measurements are finite, the behavior of $\mathbf{U}$ on each small interval may be expressed by its local Taylor expansion,

$$\mathbf{U}(x+\Delta x) = \mathbf{U}(x)+\mathbf{U}'(x)\,\Delta x+\tfrac{1}{2}\mathbf{U}''(x)(\Delta x)^2+\tfrac{1}{6}\mathbf{U}^{(3)}(x)(\Delta x)^3+\tfrac{1}{24}\mathbf{U}^{(4)}(\xi)(\Delta x)^4, \tag{40}$$

for some $\xi \in (x, x + \Delta x)$. The first four terms define a cubic polynomial that interpolates the measured values and their first two derivatives at the endpoints.

When neighboring intervals are required to match continuously in value, slope, and curvature, any residual fourth-derivative mismatch produces curvature "stress" between them. The completely relaxed configuration—what we intuitively call the *smoothest* interpolation—occurs when this residual vanishes:

$$\mathbf{U}^{(4)}(x) = 0. \tag{41}$$

This is precisely the Euler–Lagrange equation obtained by minimizing the bending-energy functional

$$E[\mathcal{U}] = \int (\mathcal{U}''(x))^2\,dx, \tag{42}$$

whose stationary points are cubic splines.

Because every measured trajectory in the universe tensor must occupy this fully relaxed state to remain compatible with adjacent measurements, the condition $\mathbf{U}^{(4)} = 0$ defines the physical law of motion at each resolution. Expressed variationally,

$$\delta E = 0 \quad \Longleftrightarrow \quad \mathcal{U}^{(4)} = 0. \tag{43}$$

In the continuum limit the same extremal condition yields the traditional form of the *principle of least action*: the observed path between events is the one for which the curvature (or action) is stationary. Thus, by demanding that the universe tensor be everywhere fully relaxed, the principle of least action is not an axiom but a direct consequence of the smoothest possible interpolation between measurable events.

In other words, assuming the best piecewise cubic polynomial spline through all measurements recovers the principle of least action. Simply splining measurements approximates physics arbitrarily well. Because the spline operation is a bijection on the space of twice-differentiable interpolants, it preserves all measurable information: the spline and the physical law it represents are indistinguishable by any possible measurement. We therefore obtain a true *duality* between measurement and dynamics—the discrete universe tensor and its smooth spline representation are two exact views of the same structure—and the principle of least activity is simply a lens through which that duality can be refocused.

4.5 The Free Parameter of the Third Variation and the Natural Constant

The closure condition

$$U^{(4)}(x) = 0$$

implies that the continuous solution of the universe tensor on each causal interval is a cubic polynomial,

$$U(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3.$$

Each interval between measurable events must match continuously in value, slope, and curvature with its neighbors. Thus, across any interior boundary

x_i we require

$$U_i(x_i) = U_{i+1}(x_i),$$
$$U_i'(x_i) = U_{i+1}'(x_i),$$
$$U_i''(x_i) = U_{i+1}''(x_i).$$

These three matching conditions uniquely determine the coefficients a_0, a_1, and a_2 of each segment. However, the coefficient a_3—the third derivative of U up to normalization—remains unconstrained by local continuity:

$$U_i^{(3)}(x) = 6a_{3,i}.$$

To maintain global smoothness under the closure condition $U^{(4)} = 0$, this third derivative must be *constant* across all intervals:

$$U^{(3)}(x) = \text{constant} \equiv \varepsilon.$$

Hence ε is the single free parameter that persists after all continuity conditions are enforced. It represents the universal scale of third variation—the smallest resolvable increment of curvature that remains invariant under reciprocal measurement.

Definition 10 (The Natural Constant of the Third Variation). Let ε denote the constant third derivative of the continuous solution $U(x)$:

$$U^{(3)}(x) = \varepsilon.$$

Then every local segment of the universe tensor satisfies

$$U(x) = a_0 + a_1 x + \tfrac{1}{2}a_2 x^2 + \tfrac{1}{6}\varepsilon x^3,$$

and all measurable distinctions between causal intervals are scaled by ε.

Physical Interpretation. The constant ε defines the conversion between discrete distinguishability and continuous curvature. Because it multiplies the third derivative, its units are those of *action*—the product of energy and time—establishing a direct correspondence to the Planck constant:

$$\varepsilon = \hbar_{\text{eff}}.$$

In the discrete regime, each new distinction contributes curvature of magnitude ε to the universe tensor; in the smooth limit, these increments accumulate to reproduce continuous dynamics. The parameter ε therefore sets the absolute scale of measurable differentiation: it is the universal constant that links the finite logic of event counting to the continuous fabric of physical law.

Proposition 3 (Continuity of the Third Variation). Under reciprocal measurement, the third variation of the universe tensor remains globally constant,

$$\delta^{(3)}U = \varepsilon\, \Phi^{-1}(P),$$

where $\Phi^{-1}(P)$ is the inverse reciprocity map selecting the measurable variation induced by a predicate P. This ensures that all higher variations vanish identically while every admissible distinction introduces curvature proportional to ε—the natural unit of causal differentiation.

5 Gauge Invariance of the Einstein Field Equations on the Image of the Universe Tensor

Having established that the universe tensor U encodes all measurable distinctions and that its smooth limit reproduces the calculus of variations, we now demonstrate that the Einstein field equations arise as the unique gauge–invariant contraction of U on its image in the measurable vector space V. This shows that general relativity is not an independent postulate but a consequence of the symmetry structure inherent in reciprocal measurement.

5.1 Tensor Action on the Measurable Space

Let V denote the finite–dimensional real vector space of measurable quantities (energy, momentum, charge, etc.), and let

$$U : V \to T(V)$$

be the universe tensor constructed from the ordered sum of event tensors. Every element $v \in V$ corresponds to a measurable vector; its image $U(v)$ represents the cumulative effect of all causal updates acting on v.

The measurable image of U is therefore the subspace

$$\mathrm{Im}(U) = \{\, U(v) \in T(V) \mid v \in V \,\},$$

which contains all physically realizable tensor states accessible to observation. Any physical law must be invariant under transformations that leave this image unchanged.

Definition 11 (Gauge Transformation). A gauge transformation is a local automorphism

$$G : V \to V, \qquad v \mapsto Gv,$$

that preserves inner products and causal order within the image of U:

$$U(Gv) = GU(v)G^{-1}.$$

Gauge transformations therefore form a group of local symmetries of the universe tensor; they represent relabelings of distinguishable events that do not alter measurable invariants.

5.2 The Metric as a Second Variation of U

From the Reciprocity Law, every measurable distinction corresponds to a variation of U. The second variation defines a symmetric bilinear form:

$$g_{\mu\nu} = \frac{\partial^2 U}{\partial x^\mu \partial x^\nu},$$

which measures the curvature of distinguishability between infinitesimally separated events. This form is invariant under permutations of indices and serves as the metric tensor on the image of U. It satisfies

$$\delta^{(2)} U = g_{\mu\nu}\, \delta x^\mu \delta x^\nu,$$

linking the discrete count of distinctions to the continuous geometry of spacetime.

Because the metric arises from a second variation, any further transformation that preserves $g_{\mu\nu}$ leaves all measurable intervals invariant. Such transformations form the local Lorentz group, the infinitesimal gauge group of U.

5.3 Curvature as a Third Variation

The third variation of U,

$$\delta^{(3)}U = \varepsilon\,\Delta^{(3)}U,$$

measures the failure of parallel transport around a closed infinitesimal loop within the causal manifold. Expressed in differential form, this variation defines the Riemann curvature tensor on the image of U:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}.$$

Here, the connection coefficients $\Gamma^{\rho}{}_{\mu\nu}$ are the first derivatives of the metric derived from U,

$$\Gamma^{\rho}{}_{\mu\nu} = \tfrac{1}{2} g^{\rho\lambda}(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}),$$

so curvature is itself a measurable consequence of the third variation, scaled by the universal constant ε.

ectionGauge Invariance and the Vanishing of Curvature

The derivation of the cubic closure condition

$$U^{(4)}(x) = 0, \qquad U^{(3)}(x) = \varepsilon,$$

shows that all measurable curvature within the universe tensor has been absorbed into the constant third variation. Once this constant is fixed, every higher variation vanishes identically, and the universe occupies its fully relaxed state. In this state, the apparent curvature of spacetime is no longer a physical field, but the residual bookkeeping of coordinate choice. What we call "force" is therefore not an external quantity acting on U but a local gauge of its image on the measurable space V.

5.4 Curvature as a Gauge Artifact

In the continuum picture, curvature arises from non-commutativity of parallel transport. In the discrete causal model, non-commutativity appears when the order of distinguishable events is ambiguous. When the universe tensor reaches the limit of consistent measurement, every ambiguity has been resolved:

$$[\nabla_\mu, \nabla_\nu]U = 0.$$

This is equivalent to a flat connection on the image of U. The Riemann tensor that would normally measure curvature,

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma},$$

now vanishes identically:

$$R^{\rho}{}_{\sigma\mu\nu} = 0.$$

The universe tensor in its relaxed configuration is globally flat—every apparent bending of trajectories is absorbed into the gauge of measurement itself.

Proposition 4 (Gauge Flatness of the Universe Tensor). On the image of U acting on V, the Einstein tensor identically vanishes:

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = 0.$$

All apparent gravitational effects are therefore encoded not in curvature but in the local choice of gauge on V—the mapping that relates one consistent frame of measurement to another.

5.5 Force as a Gauge of Measurement

If curvature is eliminated, what remains to produce acceleration? Within this framework, what we call "force" is the transformation that preserves all measurable invariants of U while relabeling its local coordinates. Formally, a gauge transformation

$$G : V \to V, \qquad U \mapsto GUG^{-1},$$

redefines the basis of measurement without altering any scalar quantity derived from U. The apparent force felt by an observer arises from the rate of change of this local transformation:

$$F_{\mu} = \frac{dG_{\mu}^{\ \nu}}{dx^{\nu}}.$$

Force is therefore not a new tensor fi

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