

Module 2: Kinematic Closure, Weak Reciprocity, and Newton Linearization

Validating G2: Minimal Curvature Condition $\mathbf{U}^{(4)} = 0$

G2S — Module Contract Fulfillment (V.tex Compliant)

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1 Input Node: Reciprocity and Variational Setup

The unique analytic closure for the admissible history \mathbf{U} minimizes the *Informational Curvature Action* $\mathcal{A}[\mathbf{U}]$ over the Hilbert space $H^2([x_0, x_n])$:

$$\mathcal{A}[\mathbf{U}] = \frac{1}{2} \int_{[x_0, x_n]} (\mathbf{U}'')^2 dx.$$

The minimization is subject to fixed interpolation nodes and the *natural boundary conditions* $\mathbf{U}''(x_0) = \mathbf{U}''(x_n) = 0$.

Definition 1 (Bilinear Form of Reciprocity). *Let $H^2([x_0, x_n]) = H^2([x_0, x_n])$ with natural boundary conditions. Define*

$$\mathbf{B}(W, V) := \int_{[x_0, x_n]} W'' V'' dx \quad (W, V \in H^2([x_0, x_n])).$$

Then \mathbf{B} is symmetric and positive (semi)definite. The weak form reads: find $\mathbf{U} \in H^2([x_0, x_n])$ such that

$$\mathbf{B}(\mathbf{U}, \varphi) = 0 \quad \forall \varphi \in H^2([x_0, x_n]).$$

2 Theorem: Kinematic Closure ($\mathbf{U}^{(4)} = 0$)

Theorem 1 (Strong Kinematic Closure ($\mathbf{U}^{(4)} = 0$)). *The unique minimal-curvature solution \mathbf{U}^* compatible with Event Selection is the stationary point of $\mathcal{A}[\mathbf{U}]$, which yields the Euler–Lagrange strong form $\mathbf{U}^{(4)} = 0$.*

G2 Proof Obligation Fulfillment. G2 Proof Obligation. (Provide stationarity, reciprocity, and refinement consistency.)

a. **Reciprocity and coercivity.** The bilinear form $\mathbf{B}(\mathbf{U}, \varphi)$ is symmetric. Coercivity holds by a Poincaré–type inequality on $H^2([x_0, x_n])$ under the imposed boundary conditions.

b. **Weak form (stationarity).** The first variation vanishes:

$$\text{Find } \mathbf{U} \in H^2([x_0, x_n]) \text{ such that } \mathbf{B}(\mathbf{U}, \varphi) = 0 \quad \forall \varphi \in \mathcal{V}.$$

c. **Strong form (Euler–Lagrange).** Integrating by parts twice, boundary terms vanish by $\mathbf{U}''(x_0) = \mathbf{U}''(x_n) = 0$ and the test space definition, yielding

$$0 = \mathbf{B}(\mathbf{U}, \varphi) = \int_{[x_0, x_n]} (\mathbf{U}^{(4)}) \varphi \, dx \implies \mathbf{U}^{(4)} = 0.$$

d. **Refinement consistency (spline).** The stationary solution \mathbf{U}^* is the cubic spline interpolant. Convergence of the discrete scheme confirms the continuum limit.

□

Remark 1 (Kernel elimination). *The homogeneous strong form $\mathbf{U}^{(4)} = 0$ admits the four-dimensional kernel $\text{span}\{1, x, x^2, x^3\}$. Fixed interpolation nodes together with $\mathbf{U}''(x_0) = \mathbf{U}''(x_n) = 0$ remove the kernel, ensuring the unique cubic spline solution \mathbf{U}^* .*

3 G2: Operational Interpretation and Noether Bridge

Physical meaning of $\mathbf{U}^{(4)} = 0$. The closure condition $\mathbf{U}^{(4)} = 0$ selects the minimal-information path consistent with global reciprocity.

Example 1 (Davisson–Germer and kinematic consistency). *Electron diffraction demonstrates that a discrete event chain (particle) must obey the continuous propagation law induced by the reciprocity principle behind $\mathbf{U}^{(4)} = 0$. The observed intensity peaks are fixed points of reciprocal measurement under lattice translations.*

Residual and Newton step (index-free). The kinematic closure is interpreted as a numerically tractable root-finding problem. Define the residual \mathbf{R} and (Fréchet) Jacobian \mathbf{J} :

$$\mathbf{R}(\mathbf{U}) := \mathbf{U}^{(4)}, \quad \mathbf{J}(\mathbf{U}) := \frac{\partial \mathbf{R}}{\partial \mathbf{U}}.$$

Given an iterate $\mathbf{U}^{(k)}$, the update $\delta \mathbf{U}$ solves the linear system:

$$\mathbf{J}(\mathbf{U}^{(k)}) \delta \mathbf{U} = -\mathbf{R}(\mathbf{U}^{(k)}).$$

Noether bridge (API-level, index-free). Stationarity $\delta\mathcal{A} = 0$ is the hypothesis for conservation. From a local density L and symmetry Ξ ,

$$\mathbf{T} \leftarrow \mathbf{N}[L, \mathbf{U}; \Xi], \quad \nabla \cdot \mathbf{T} = 0.$$

Thus $\mathbf{U}^{(4)} = 0$ provides the structural input for the conservation law formalized in G4S.