Module 4: The Noether Bridge

Validating G4: Derivation of Conserved Currents $\nabla\cdot\mathbf{T} = 0$

G4S — Proof of Conservation $\nabla\cdot\mathbf{T} = 0$

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1 Input Node: Stationarity from G2

The conservation law rests on the action principle. From the **Kinematic Closure** (G2), the admissible history $\mathbf{U}$ satisfies the stationarity condition, $\delta\mathcal{S} = 0$.

Definition 1 (Action Functional)**.** *The action functional $\mathcal{S}[\mathbf{U}]$ is the integral of the local Lagrangian density $\mathcal{L}$ that encodes informational curvature:*

$$\mathcal{S}[\mathbf{U}] = \int \mathcal{L}(\mathbf{U}, \nabla\mathbf{U}, g)\, d\tau.$$

2 Conservation as Translational Symmetry

Conservation of energy and momentum is a direct consequence of the field $\mathbf{U}$ being invariant under translations.

Theorem 1 (Noether's Theorem (Abstract Form))**.** *For every continuous symmetry Ξ that leaves the action $\mathcal{S}$ invariant ($\delta\mathcal{S} = 0$), there exists a conserved current $\mathbf{J}$ such that the divergence vanishes:*

$$\nabla\cdot\mathbf{J} = 0.$$

G4 Proof Obligation Fulfillment (Translational Invariance).

1. **Hypothesis (G2 Closure):** The field $\mathbf{U}$ is a stationary point of $\mathcal{S}$ ($\delta\mathcal{S} = 0$), a condition guaranteed by the **Kinematic Closure** $\mathbf{U}^{(4)} = 0$.

2. **Symmetry:** We consider the continuous translational symmetry, Ξ^{trans}, which leaves the underlying structure of $\mathbf{U}$ unchanged.

3. **Current Derivation (API Call):** The abstract Noether current associated with translational symmetry is the **Stress-Energy Tensor**, $\mathbf{T}$. This tensor is constructed by the abstract API primitive:

$$\mathbf{T} \leftarrow \mathsf{N}\left[\mathcal{L}, \mathbf{U}; \Xi^{\text{trans}}\right].$$

4. **Divergence Identity:** Applying the Noether identity to this current yields the conservation law for energy and momentum:

$$\nabla\cdot\mathbf{T} = 0.$$

The **Noether bridge** is complete: the kinematic closure is the logical engine that produces the conserved current required for dynamics. □

3 Output Node: Chain Completion

The derived conservation law, $\nabla\cdot\mathbf{T} = 0$, is the necessary intermediate structural constraint that links the local analytical consistency ($\mathbf{U}^{(4)} = 0$) to the final **global thermodynamic consistency** ($\Delta\,\mathrm{S} \geq 0$). The logical proof chain is now ready for its final step.