

The Limits of Representation

Volume I: The Conditions of $\Delta S \geq 0$

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Abstract

This work presents a constructive proof that the entropy of any causally consistent universe is non-decreasing, $\Delta S \geq 0$. Measurement itself defines the metric structure of physics: each distinction generates an increment of causal order. Within the axioms of ZFC with Choice, a finite causal order of distinguishable events is defined whose dual operations—measurement and variation—form a bijective pair under the Reciprocity Law of Physics. Requiring global coherence under an axiom of Event Selection enforces the fourth-order cancellation $\mathbf{U}^{(4)} = 0$, which identifies the cubic spline as the minimal analytic closure of the dual system. This closure produces the continuous calculus as the smooth limit of finite causal measurement. The theorem $\Delta S \geq 0$ emerges as the necessary condition that any universe consistent with its own record of distinctions must increase the count of what can be known.

Overview: The Logical Chain of Consistency

The constructive proof proceeds in six stages, mapping axiomatic necessity to physical law.

1. **I. Causal Primitives:** Defining the Ordered Set (\mathbf{U}).
2. **II. Global Consistency:** The Reciprocity Law (Duality and \mathbf{B}).
3. **III. Kinematic Closure:** Deriving Minimal Curvature ($\mathbf{U}^{(4)} = 0$).
4. **IV. Dynamics:** Noether's Theorem and Conserved Bookkeeping ($\nabla_\mu \mathbf{T}^{\mu\nu} = 0$).
5. **V. Thermodynamic Closure:** Proving the Arrow of Time ($\Delta S \geq 0$).
6. **VI. The Fixed Point:** Unifying the Ledger.

Chapter 1

I. The Calculus of Distinction

Core Idea: Time is the ordinal index of measurable events, defining the Universe Tensor \mathbf{U} .

Axiom 1 (The Axiom of Order). *Time is an ordinal rank (k) on the sequence of distinguishable events.*

Definition 1 (Universe Tensor). *The cumulative record of distinctions is the ordered fold $\mathbf{U}_n = \sum_{k=1}^n \mathbf{E}_k$.*

Chapter 2

II. The Reciprocal Duality

Core Idea: The Axiom of Event Selection enforces a bijective duality (B) between observable measurements and admissible variations.

Axiom 2 (Axiom of Event Selection (MA-like)). *Every countable family of admissible local causal choices admits a globally consistent extension.*

Equivalence 1 (The Reciprocity Law: Bilinear Symmetry). *The duality between observation and variation defines the symmetric **Bilinear Form** B , ensuring the consistency of measurable energy.*

Chapter 3

III. Kinematic Closure: The $\mathbf{U}^{(4)} = 0$ Fixed Point

Core Idea: Enforcing minimal information curvature ($\mathcal{A}[\mathbf{U}]$) requires the continuity provided by the unique analytic solution: $\mathbf{U}^{(4)} = 0$.

Theorem 1 (Kinematic Closure: Euler Fixed Point). *The minimizer of the **Informational Curvature Action** $\mathcal{A}[\mathbf{U}]$ yields the Euler–Lagrange condition $\mathbf{U}^{(4)} = 0$, defining the minimal analytic closure.*

Chapter 4

IV. Conservation: Dynamics from Consistency

Core Idea: Translational symmetry of the $\mathbf{U}^{(4)} = 0$ field yields the conserved stress tensor $\mathbf{T}^{\mu\nu}$, enforcing the bookkeeping identity $\nabla_\mu \mathbf{T}^{\mu\nu} = 0$.

Theorem 2 (Noether Conservation). *The translational symmetry of the $\mathbf{U}^{(4)} = 0$ kinematic field yields the conserved Noether current $\mathbf{T}^{\mu\nu}$, satisfying the bookkeeping identity $\nabla_\mu \mathbf{T}^{\mu\nu} = 0$.*

Chapter 5

V. Thermodynamic Closure: The $\Delta S \geq 0$ Theorem

Core Idea: Causal consistency requires that every admissible refinement increases the set of distinguishable classes, proving the Second Law ($\Delta S \geq 0$).

Theorem 3 (Second Law: Injective Count Monotonicity). *For every consistent refinement that maintains observable compatibility, there exists an **Injective Class Map** ι ensuring the cardinality of distinguishable classes is non-decreasing: $\Delta S \geq 0$.*

Chapter 6

VI. The Invariant: Unifying the Causal Fixed Point

Core Idea: The entire hierarchy of laws is unified as the unique, self-consistent structure of the Proof Tensor.

Quod erat demonstrandum. Order implies dynamics.