Module 4: The Noether Bridge

Validating G4: Derivation of Conserved Currents $\nabla\cdot\mathbf{T}=0$

G4S — Proof of Conservation (V.tex Compliant)

October 13, 2025

1 Input Node: Stationarity from Kinematic Closure (G2)

The conservation law relies on the *action principle* being stationary. The kinematic closure $\mathbf{U}^{(4)} = 0$ guarantees that the fundamental field $\mathbf{U}$ is a solution to the Euler–Lagrange equations in the sense needed to invoke stationarity $\delta\mathcal{S} = 0$.

Definition 1 (Action Functional $\mathcal{S}$)**.** *Let $\mathcal{L}$ be a local Lagrangian density encoding informational curvature. The action is*

$$\mathcal{S}[\mathbf{U}] \;=\; \int \mathcal{L}\big(\mathbf{U},\, \nabla\mathbf{U}\big)\, d\tau,$$

where the measure $d\tau$ is left abstract (index-free).

2 Theorem: Conservation Law (Index-Free Noether)

Theorem 1 (Noether Conservation)**.** *For every continuous symmetry Ξ that leaves the action invariant ($\delta\mathcal{S} = 0$), there exists an associated current* $\mathbf{J}$

whose divergence vanishes:

$$\nabla\cdot\mathbf{J} \;=\; 0.$$

G4 Proof Obligation Fulfillment (Translational Case). 1. **Hypothesis (stationarity).** The closure $\mathbf{U}^{(4)} = 0$ positions $\mathbf{U}$ at a stationary point: $\delta\mathcal{S} = 0$.

2. **Symmetry selection.** Choose translational symmetry Ξ_{trans}.

3. **Noether bridge (API).** Invoke the abstract construction

$$\mathbf{T} \;\leftarrow\; \mathsf{N}[\mathcal{L}, \mathbf{U}; \Xi_{\text{trans}}]\,.$$

4. **Conservation identity.** Symmetry implies the resulting current is conserved:

$$\nabla\cdot\mathbf{T} \;=\; 0.$$

Thus the Noether bridge carries the geometric constraint $\mathbf{U}^{(4)} = 0$ into the required conservation statement, furnishing the bookkeeping law for subsequent thermodynamic closure. □