Module 3: Discrete Kinematics and Consistency

Validating G3: Discrete–Continuum Reciprocity

Discrete–continuum reciprocity via B

G3S — Module Contract Fulfillment (V.tex Compliant)

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1 Input Node: The Discrete Energy Functional

The continuum action $\mathcal{A}[\mathbf{U}]$ is the smooth limit of the discrete action $[\mathbf{U}_\mathbf{h}]$, whose stationarity yields the discrete Euler-Lagrange equation $\Delta_\mathbf{h}^{(4)}\mathbf{U}_\mathbf{h} = 0$.

2 Theorem: The Kinematic Closure Chain (G3.Chain)

Theorem 1 (Discrete Kinematic Convergence)**.** *The discrete Euler-Lagrange operator converges to the continuous solution:*

$$\Delta_\mathbf{h}^{(4)}\mathbf{U}_\mathbf{h} = 0 \quad \Longrightarrow \quad \lim_{\mathsf{h}\to 0} \frac{\Delta_\mathbf{h}^{(4)}\mathbf{U}_\mathbf{h}}{\mathsf{h}^2} = \mathbf{U}^{(4)}.$$

G3: Discrete–Continuum Reciprocity

Summary. The validation of this module confirms the self-consistent closure chain by asserting that the discrete scheme accurately reflects the continuum system:

$$\text{Discrete–continuum reciprocity via } \mathsf{B}.$$

Compatibility with conservation (index-free). The validity of the discrete scheme hinges on its use of a pairing ($\mathsf{B}(\cdot,\cdot)$ or $(\cdot,\cdot)$) that ensures **reciprocity** holds for the field $\mathbf{U}$. This reciprocity guarantees that the conservation law derived from Noether's theorem is preserved across the numerical limit. Specifically, the abstract conservation current $\mathbf{J}$ must satisfy:

$$\nabla\cdot\mathbf{J} = 0,$$

and this identity is invariant under the continuous interpolation and discrete sampling defined by the core reciprocity pairing. The consistent convergence of the field $\mathbf{U}$ means that the continuity laws for $\mathbf{T}$ are preserved.

Comment. The convergence proof ensures that the flow of distinguishability—the source of $\mathbf{T}$ and $\mathbf{J}$—is numerically stable and topologically invariant as the mesh $\mathsf{h} \to 0$. This sets the necessary foundation for subsequent derivations of metric structure and field dynamics.