HL Klemove

Prediction

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Contents

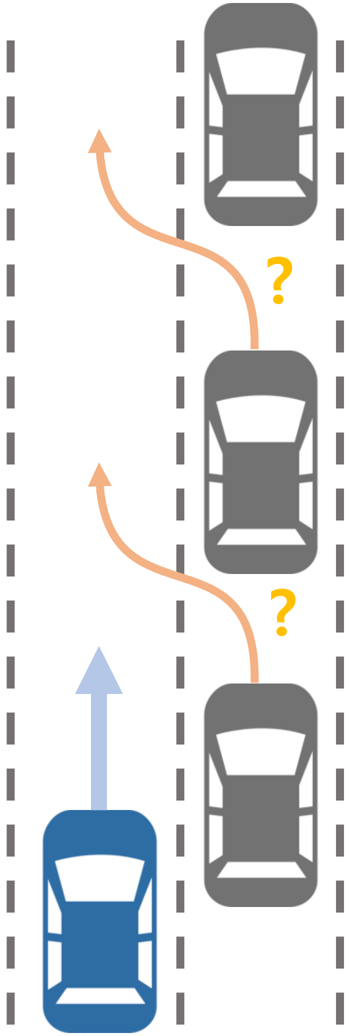
Introduction

- Prediction
  - 주변 Object의 Future trajectory나 Intention 을 예측

내가 최적한 경로를 잡아도, 상대방이 지랄맞으면 어떡함 ?

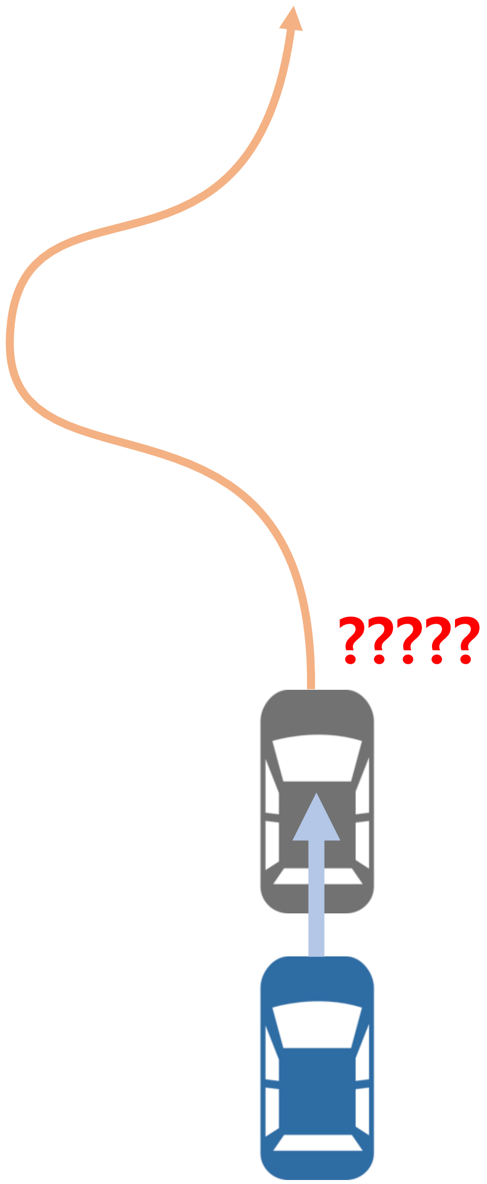
Introduction

- Prediction
  - 초보 운전 vs 숙련된 운전자



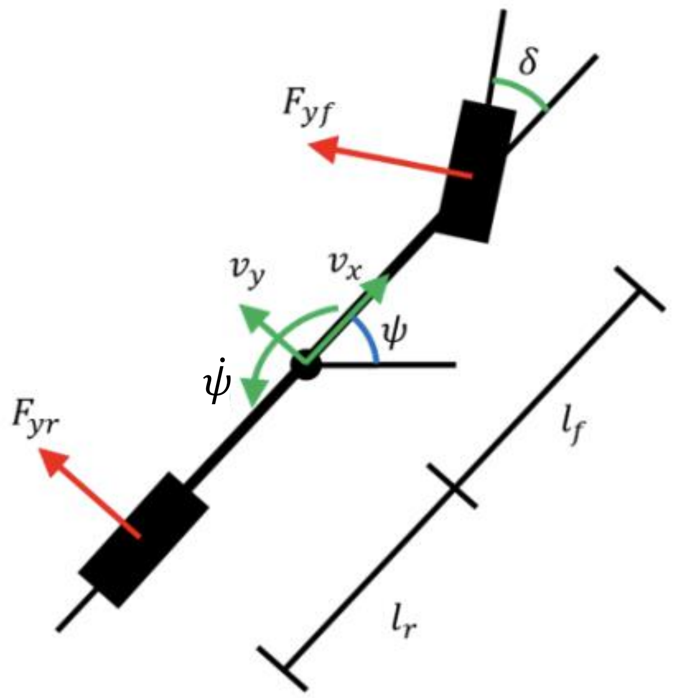
Introduction

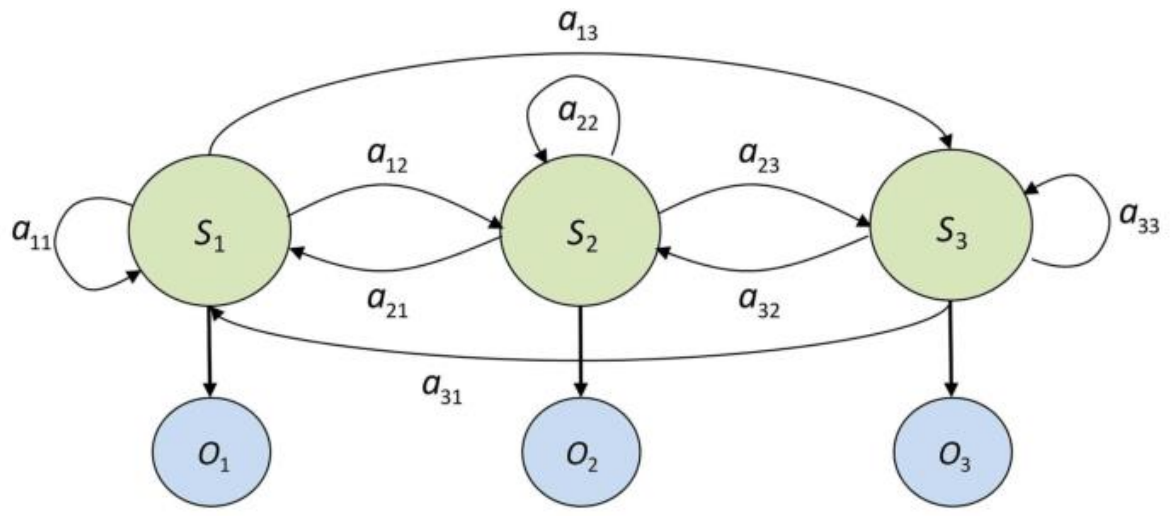
- Prediction
  - 초보 운전 vs 숙련된 운전자



Introduction

- Model based prediction
  - Physics based prediction
  - Maneuver based prediction

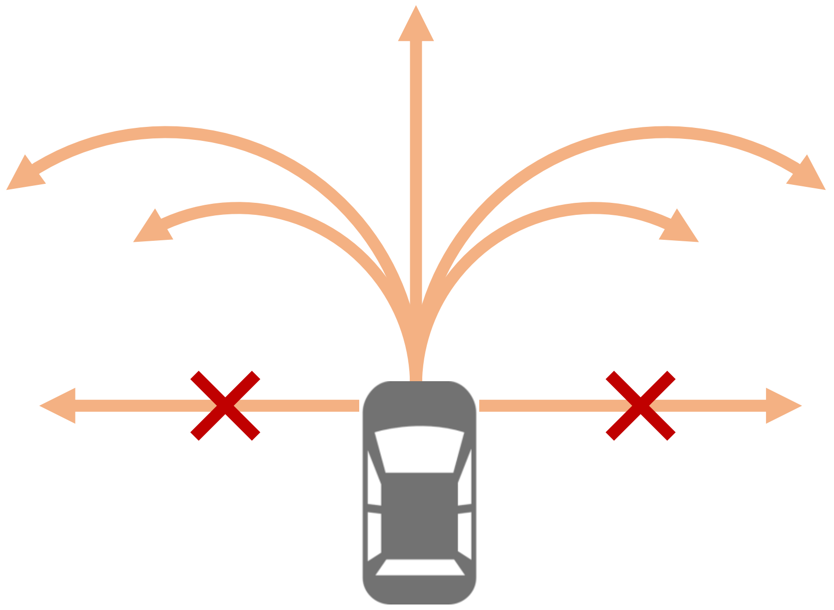




Physics based prediction

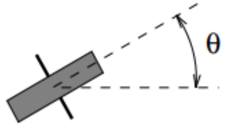
holonomic : 방향 전환에 제한이 없는 것.
ex) 로봇청소기

- Introduction
  - 자동차는 정해진 물리 법칙 상에서 움직임
  - 짧은 시간 예측에 유리
  - 주변 환경을 고려하지 않음
  - CV / CA / CTRV / CTRA model



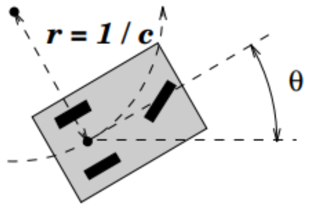
Nonholonomic Systems

Unicycle



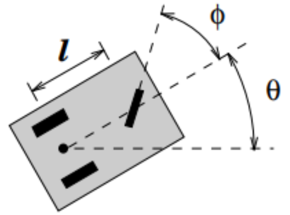
$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} v \\ v_\theta \end{pmatrix}$$

Car with fast steering



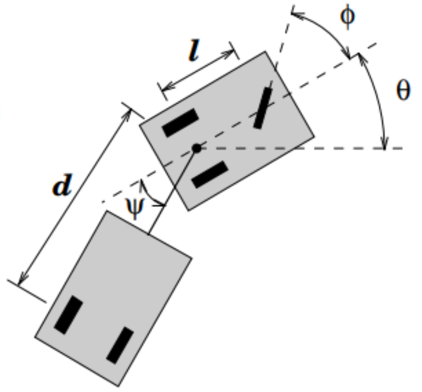
$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ \theta \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} v \\ v\,c \end{pmatrix}$$

Car with slow steering



$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ \theta \\ \phi \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ \tan(\phi)/l & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} v \\ v_\phi \end{pmatrix}$$

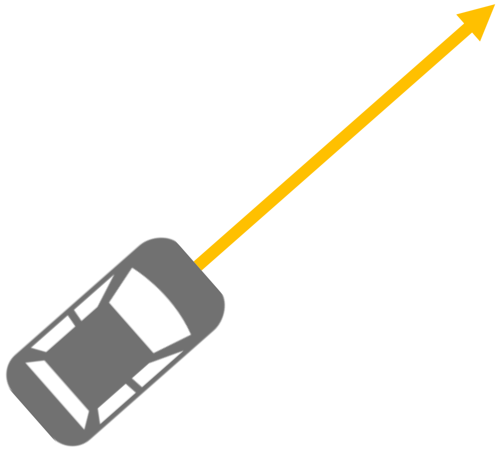
Car with trailer



$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ \theta \\ \psi \\ \phi \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ \tan(\phi)/l & 0 \\ \sin(\psi)/d & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} v \\ v_\phi \end{pmatrix}$$

Physics based prediction

- Constant velocity model (CV model)
  - 현재의 주행 속도를 유지한다는 가정
  - 가장 간단한 모델



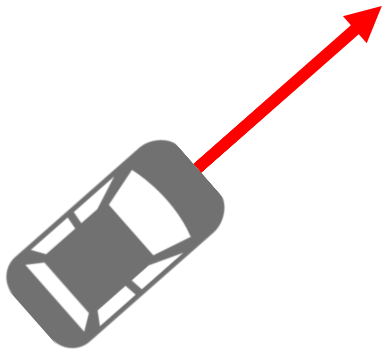
State : $X = \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}$

$$\begin{pmatrix} x_{k+1} = x_k + v_{x,k}\Delta t \\ y_{k+1} = y_k + v_{y,k}\Delta t \\ v_{x,k+1} = v_{x,k} \\ v_{y,k+1} = v_{y,k} \end{pmatrix}$$

$$\boxed{\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ v_{x,k+1} \\ v_{y,k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ v_{x,k} \\ v_{y,k} \end{bmatrix}}$$

Physics based prediction

- Constant acceleration model (CA model)
  - 현재의 가속도를 유지한다는 가정
  - 주의해야 할 점..?

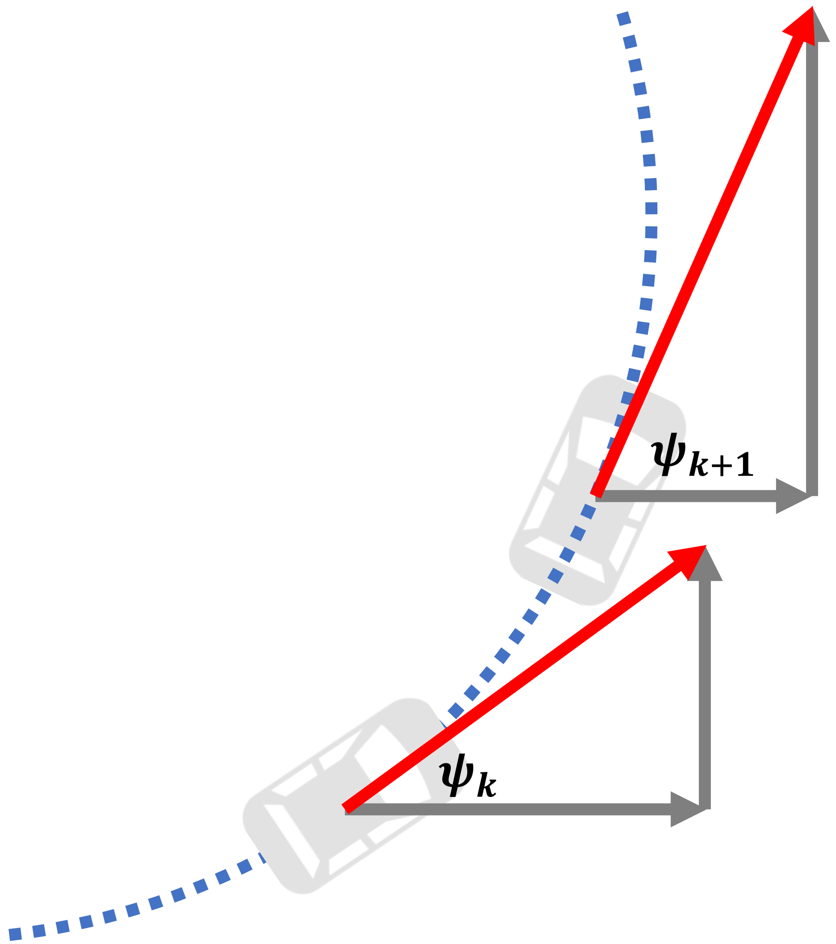


$$State: \ X = \begin{bmatrix} x \\ v \\ a \end{bmatrix} \qquad \left(\begin{array}{c} x_{k+1} = x_k + v_k \Delta t + \frac{1}{2} a_k \Delta t^2 \\ v_{k+1} = v_k + a_k \Delta t \\ a_{k+1} = a_k \end{array} \right.$$

$$\begin{bmatrix} \boldsymbol{x_{k+1}} \\ \boldsymbol{v_{k+1}} \\ \boldsymbol{a_{k+1}} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \boldsymbol{\Delta t} & \boldsymbol{\Delta t^2} \\ \mathbf{0} & \mathbf{1} & \boldsymbol{\Delta t} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_k} \\ \boldsymbol{v_k} \\ \boldsymbol{a_k} \end{bmatrix}$$

Physics based prediction

- Constant turn rate & velocity model (CTRV model) 핸들은 한 번은 꺾는다고 생각
  - 현재의 yaw rate(turn rate) 와 속도를 유지한다는 가정
  - If (yaw rate << 1) ?



$$State: X = \begin{bmatrix} x \\ y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$$

$$\left(\begin{aligned} x_{k+1} &= x_k + \int_0^{\Delta t} v_{k,\tau} \cos(\psi_{k,\tau}) d\tau \\ y_{k+1} &= y_k + \int_0^{\Delta t} v_{k,\tau} \sin(\psi_{k,\tau}) d\tau \\ v_{k,\tau} &= v_k(const) \\ \psi_{k,\tau} &= \psi_k + \dot{\psi}_k \tau \end{aligned} \right. \Rightarrow \begin{aligned} x_{k+1} &= x_k + \int_0^{\Delta t} v_k \cos(\psi_k + \dot{\psi}_k \tau) d\tau \\ y_{k+1} &= y_k + \int_0^{\Delta t} v_k \sin(\psi_k + \dot{\psi}_k \tau) d\tau \end{aligned}$$

$$\boldsymbol{x_{k+1} = x_k + \frac{v_k}{\dot{\psi}_k}\left(\sin(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k)\right)}$$
$$\boldsymbol{y_{k+1} = y_k + \frac{v_k}{\dot{\psi}_k}\left(-\cos(\psi_k + \dot{\psi}_k \Delta t) + \cos(\psi_k)\right)}$$
$$\boldsymbol{v_{k+1} = v_k}$$
$$\boldsymbol{\psi_{k+1} = \psi_k + \dot{\psi}_k \Delta t}$$
$$\boldsymbol{\dot{\psi}_{k+1} = \dot{\psi}_k}$$

Physics based prediction

turn + 가속

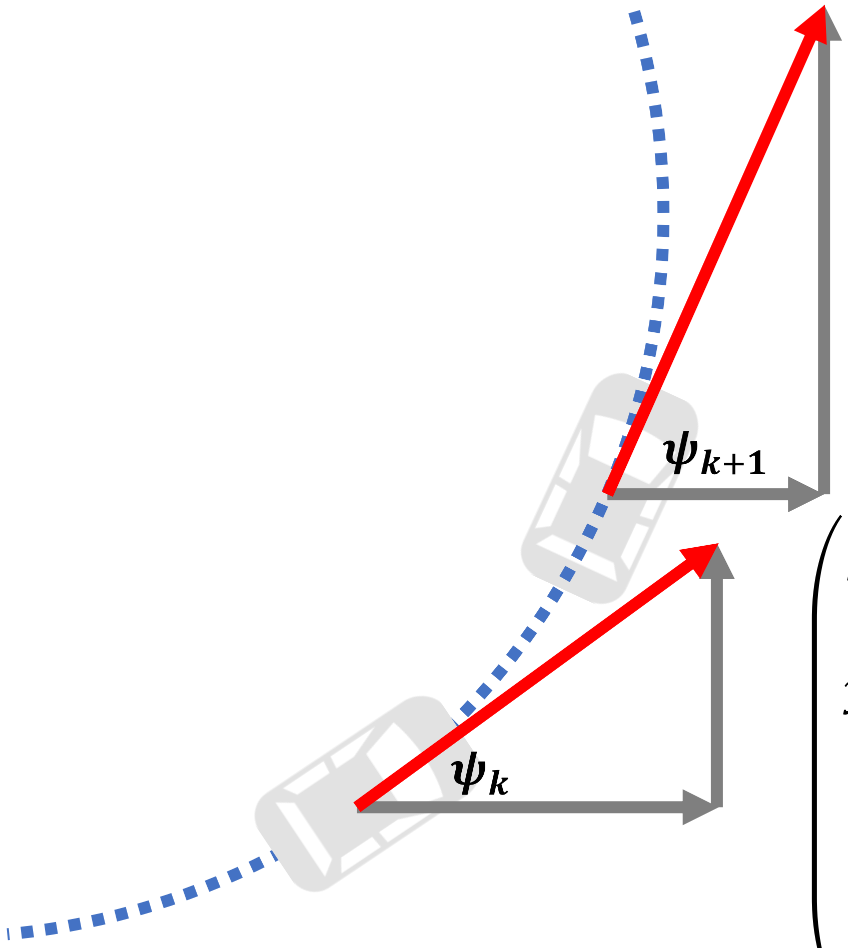
4개 physics
그런 모델들을 왜 배울까?
Prediction의 Candidates들을 표현한 방법

- Constant turn rate & acceleration model (CTRA model)
  - 현재의 yaw rate(turn rate) 와 가속도를 유지한다는 가정

당연히 얘도 비선형
→ 상태방정식 X

$$X = \begin{bmatrix} x \\ y \\ v \\ a \\ \psi \\ \dot{\psi} \end{bmatrix}$$

$$\begin{cases} v_{k+1} = v_k + a_k \Delta t \\ a_{k+1} = a_k \\ \psi_{k+1} = \psi_k + \dot{\psi}_k \Delta t \\ \dot{\psi}_{k+1} = \dot{\psi}_k \\ x_{k+1} = x_k + \int_0^{\Delta t} v_{k,\tau} \cos(\psi_{k,\tau}) d\tau \\ y_{k+1} = y_k + \int_0^{\Delta t} v_{k,\tau} \sin(\psi_{k,\tau}) d\tau \end{cases}$$



$$\begin{cases} x_{k+1} = x_k + \frac{v_k}{\dot{\psi}_k}\left(\sin(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k)\right) + \frac{a_k}{\dot{\psi}_k^2}\left(\cos(\psi_k + \dot{\psi}_k \Delta t) + \dot{\psi}_k \Delta t \sin(\psi_k + \dot{\psi}_k \Delta t) - \cos(\psi_k)\right) \\ y_{k+1} = y_k - \frac{v_k}{\dot{\psi}_k}\left(\cos(\psi_k + \dot{\psi}_k \Delta t) - \cos(\psi_k)\right) + \frac{a_k}{\dot{\psi}_k^2}\left(\sin(\psi_k + \dot{\psi}_k \Delta t) - \dot{\psi}_k \Delta t \cos(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k)\right) \\ v_{k+1} = v_k + a_k \Delta t \\ a_{k+1} = a_k \\ \psi_{k+1} = \psi_k + \dot{\psi}_k \Delta t \\ \dot{\psi}_{k+1} = \dot{\psi}_k \end{cases}$$

Physics based prediction

- Process noise
  - 그럼 모델대로 움직이는가?

Physics based prediction

- Basic Kalman filter

① Prediction

$$\tilde{x}_k = A\hat{x}_{k-1} + Bu_k$$

$$\tilde{y}_k = C\tilde{x}_k$$

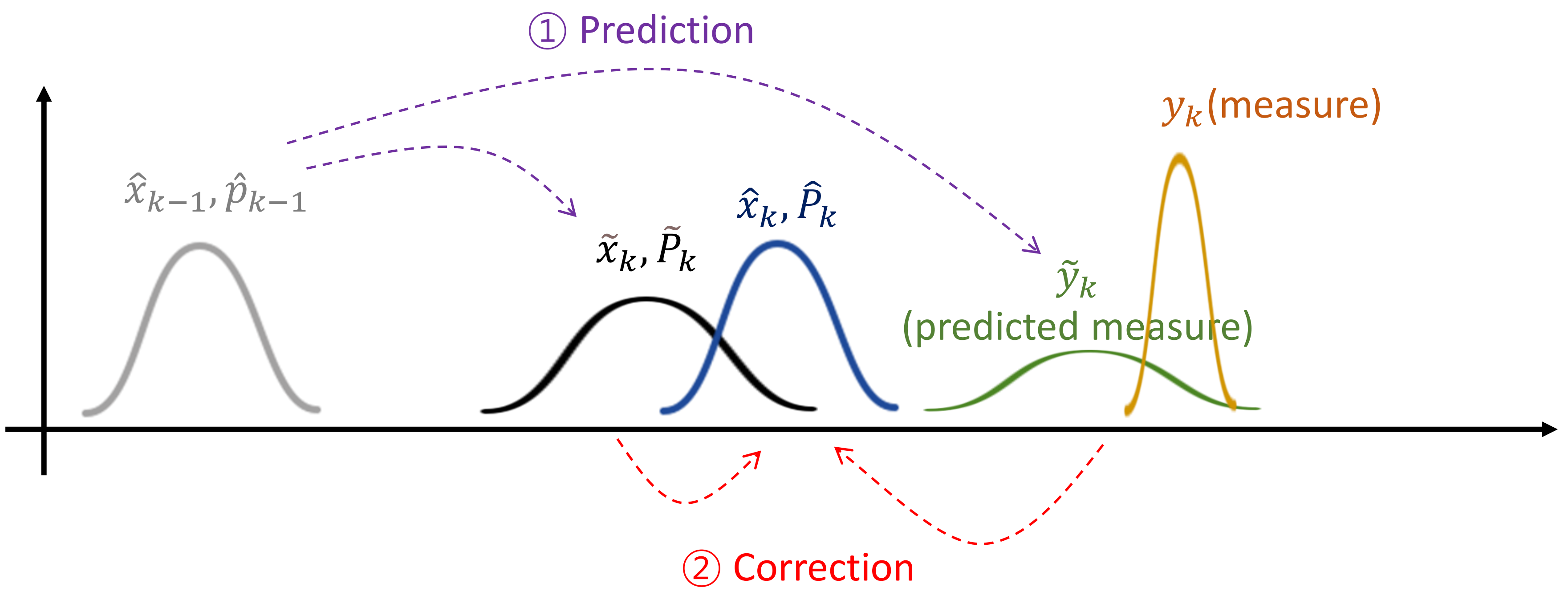
$$\tilde{P}_k = AP_{k-1}A^T + \underline{Q_k}$$

② Correction

$$\hat{x}_k = \tilde{x}_k + \underline{K_k}(y_k - \tilde{y}_k)$$

$$K_k = \frac{\tilde{P}_k C^T}{C\tilde{P}_k C^T + \underline{R_k}}$$

$$P_k = (I - K_k C)\tilde{P}_k$$



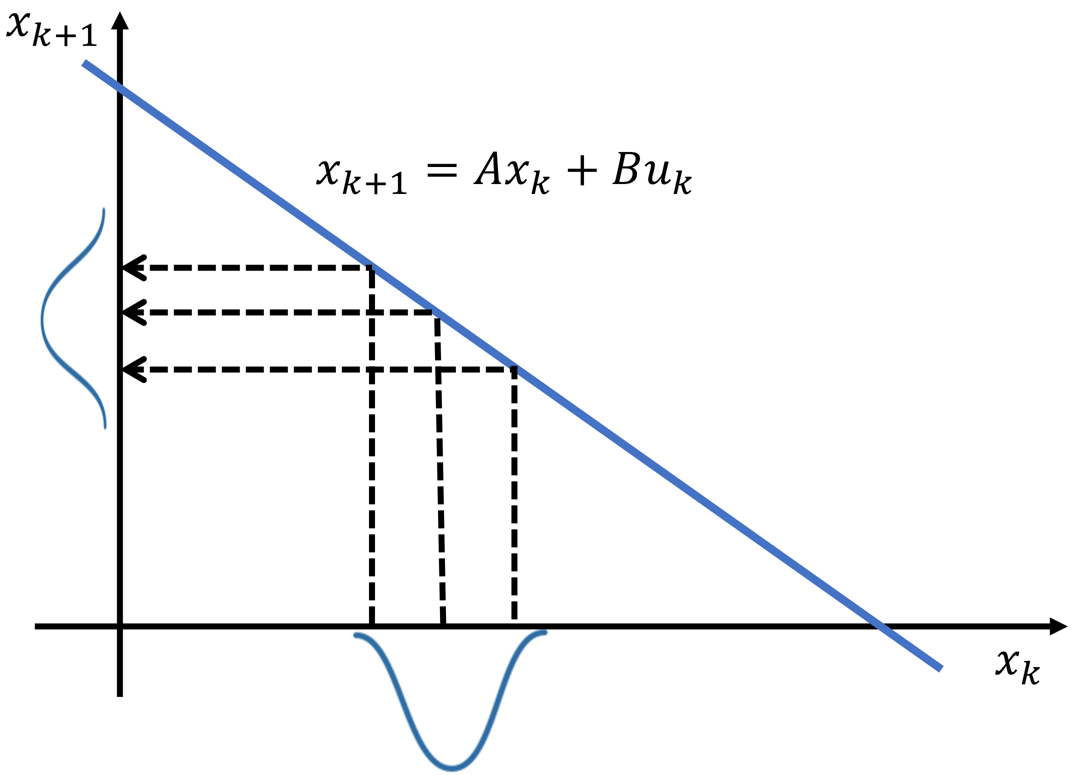
K_k(Kalman gain) : 예측된 값에 측정값과의 차이를 얼마나 반영할 것인지에 대한 값

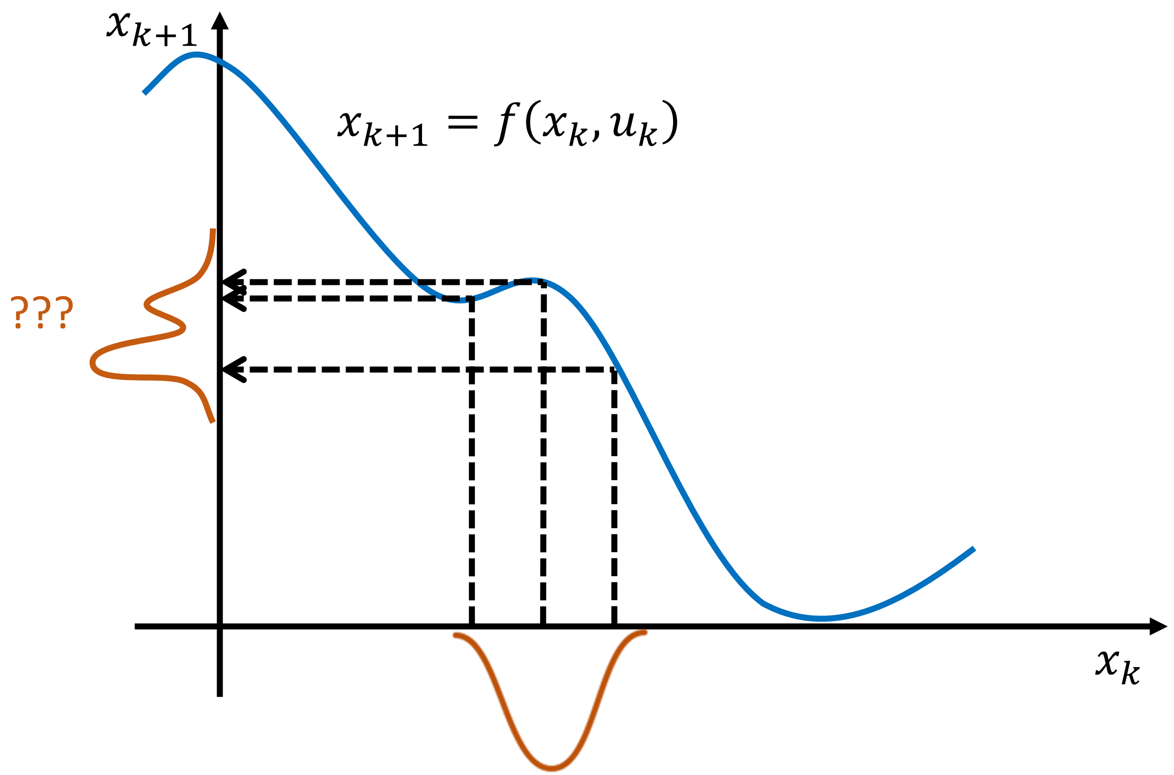
Q_k(Model noise) : Kalman gain 을 크게 하는 요소(Tuning parameter)

R_k(Sensor noise) : Kalman gain 을 작게 하는 요소 (Tuning parameter)

Physics based prediction

- Nonlinear Kalman filter
  - Linear vs Nonlinear model





Physics based prediction

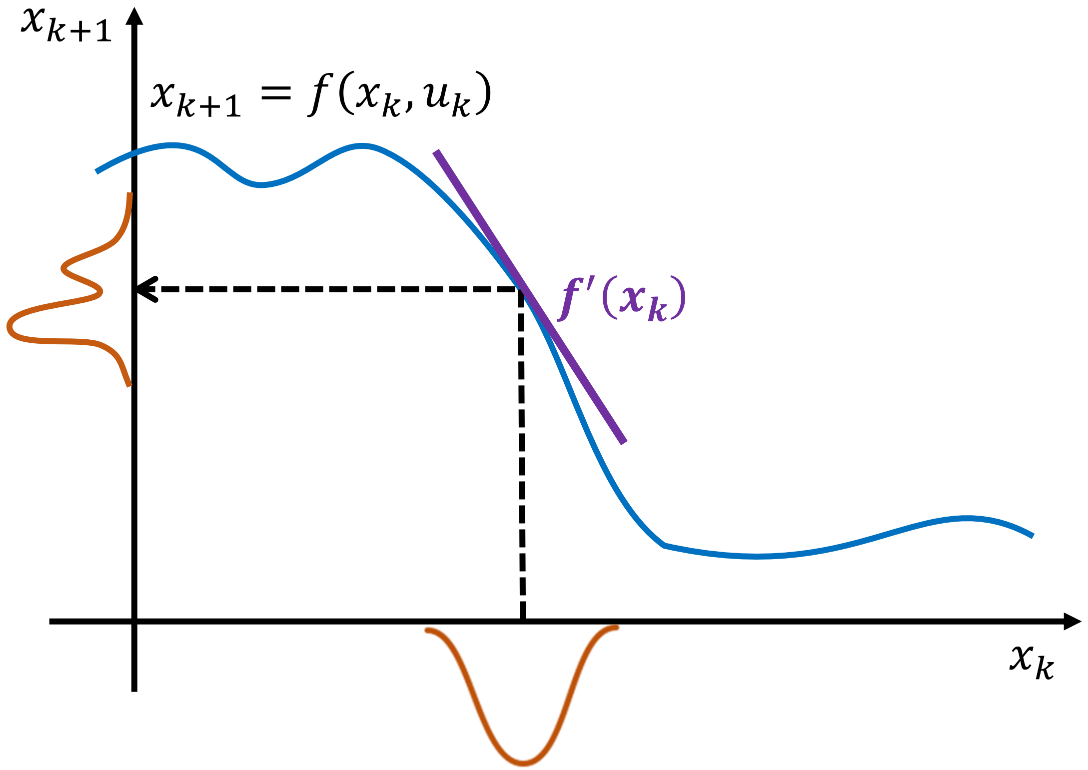
- Nonlinear Kalman filter
  - Idea 1 : Taylor series
    - 특정 함수는 한 점에서의 도함수 값들과 다항식의 무한 합으로 나타낼 수 있음

$$f(x) = \underline{f(a) + f'(a)(x-a)} + \frac{1}{2}f''(a)(x-a)^2 + \dots$$

$$f(x,y,z) = f(x_i,y_i,z_i) + (x-x_i)\left.\frac{\partial f}{\partial x}\right|_{x_i,y_i,z_i}$$

$$+(y-y_i)\left.\frac{\partial f}{\partial y}\right|_{x_i,y_i,z_i}$$

$$+(z-z_i)\left.\frac{\partial f}{\partial z}\right|_{x_i,y_i,z_i}$$



Physics based prediction

- Nonlinear Kalman filter
  - Example : CTRV Model

$$x_{k+1} = x_k + \frac{v_k}{\dot{\psi}_k}\left(\sin(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k)\right) = \boldsymbol{f_1(x_k, y_k, \dots)}$$

$$y_{k+1} = y_k + \frac{v_k}{\dot{\psi}_k}\left(-\cos(\psi_k + \dot{\psi}_k \Delta t) + \cos(\psi_k)\right) = \boldsymbol{f_2(x_k, y_k, \dots)}$$

$$v_{k+1} = v_k = \boldsymbol{f_3(x_k, y_k, \dots)}$$

$$\psi_{k+1} = \psi_k + \dot{\psi}_k \Delta t = \boldsymbol{f_4(x_k, y_k, \dots)}$$

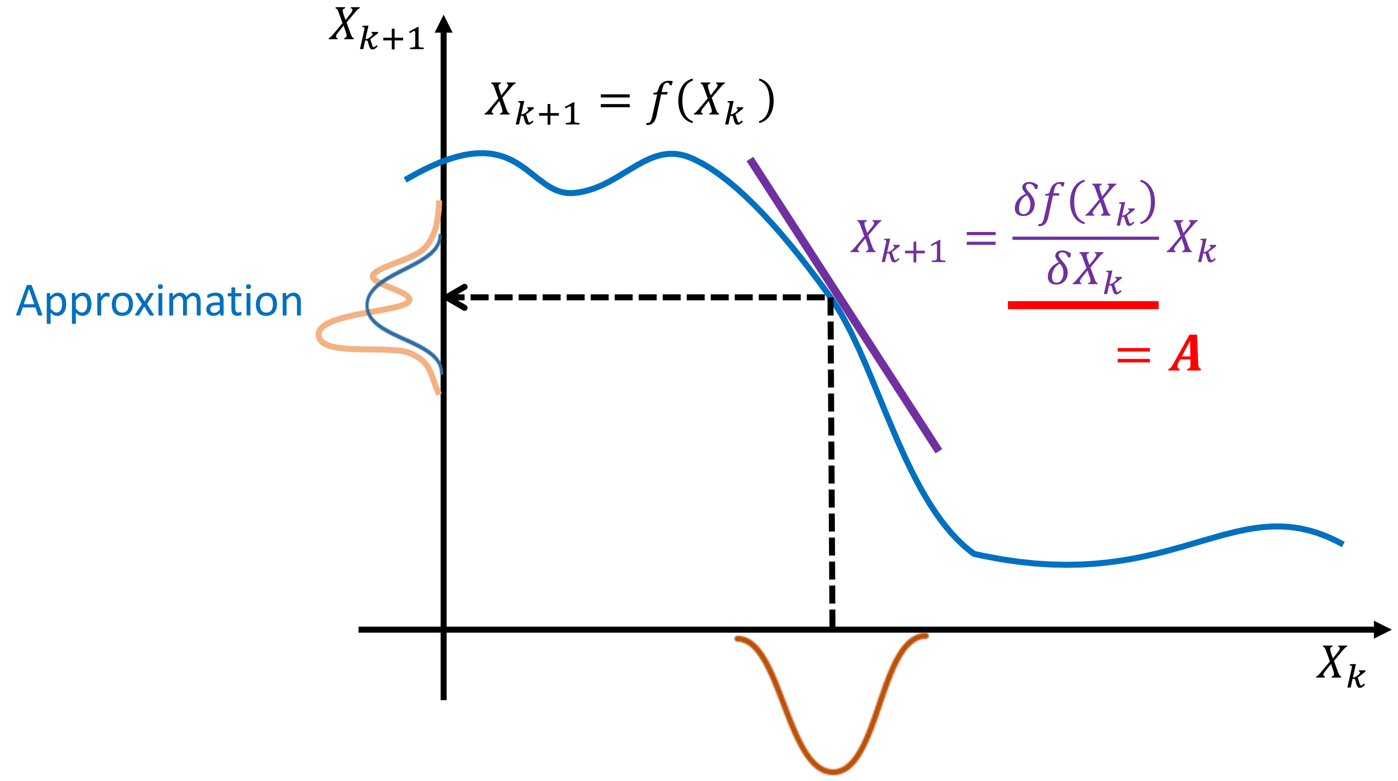
$$\dot{\psi}_{k+1} = \dot{\psi}_k = \boldsymbol{f_5(x_k, y_k, \dots)}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ v_{k+1} \\ \psi_{k+1} \\ \dot{\psi}_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\delta f_1}{\delta x} & \frac{\delta f_1}{\delta y} & \frac{\delta f_1}{\delta v} & \frac{\delta f_1}{\delta \psi} & \frac{\delta f_1}{\delta \dot{\psi}} \\ \frac{\delta f_2}{\delta x} & \frac{\delta f_2}{\delta y} & \frac{\delta f_2}{\delta v} & \frac{\delta f_2}{\delta \psi} & \frac{\delta f_2}{\delta \dot{\psi}} \\ \frac{\delta f_3}{\delta x} & \frac{\delta f_3}{\delta y} & \frac{\delta f_3}{\delta v} & \frac{\delta f_3}{\delta \psi} & \frac{\delta f_3}{\delta \dot{\psi}} \\ \frac{\delta f_4}{\delta x} & \frac{\delta f_4}{\delta y} & \frac{\delta f_4}{\delta v} & \frac{\delta f_4}{\delta \psi} & \frac{\delta f_4}{\delta \dot{\psi}} \\ \frac{\delta f_5}{\delta x} & \frac{\delta f_5}{\delta y} & \frac{\delta f_5}{\delta v} & \frac{\delta f_5}{\delta \psi} & \frac{\delta f_5}{\delta \dot{\psi}} \end{bmatrix}}_{\text{Jacobian matrix}} \begin{bmatrix} x_k \\ y_k \\ v_k \\ \psi_k \\ \dot{\psi}_k \end{bmatrix} + \xi$$

$$= \frac{\delta f(X_k)}{\delta X_k}$$

Physics based prediction

- Nonlinear Kalman filter
  - Extended Kalman filter



① Prediction

$$\tilde{x}_k = A\hat{x}_{k-1} + Bu_k$$

$$\tilde{y}_k = C\tilde{x}_k$$

$$\tilde{P}_k = AP_{k-1}A^T + Q_k$$

② Correction

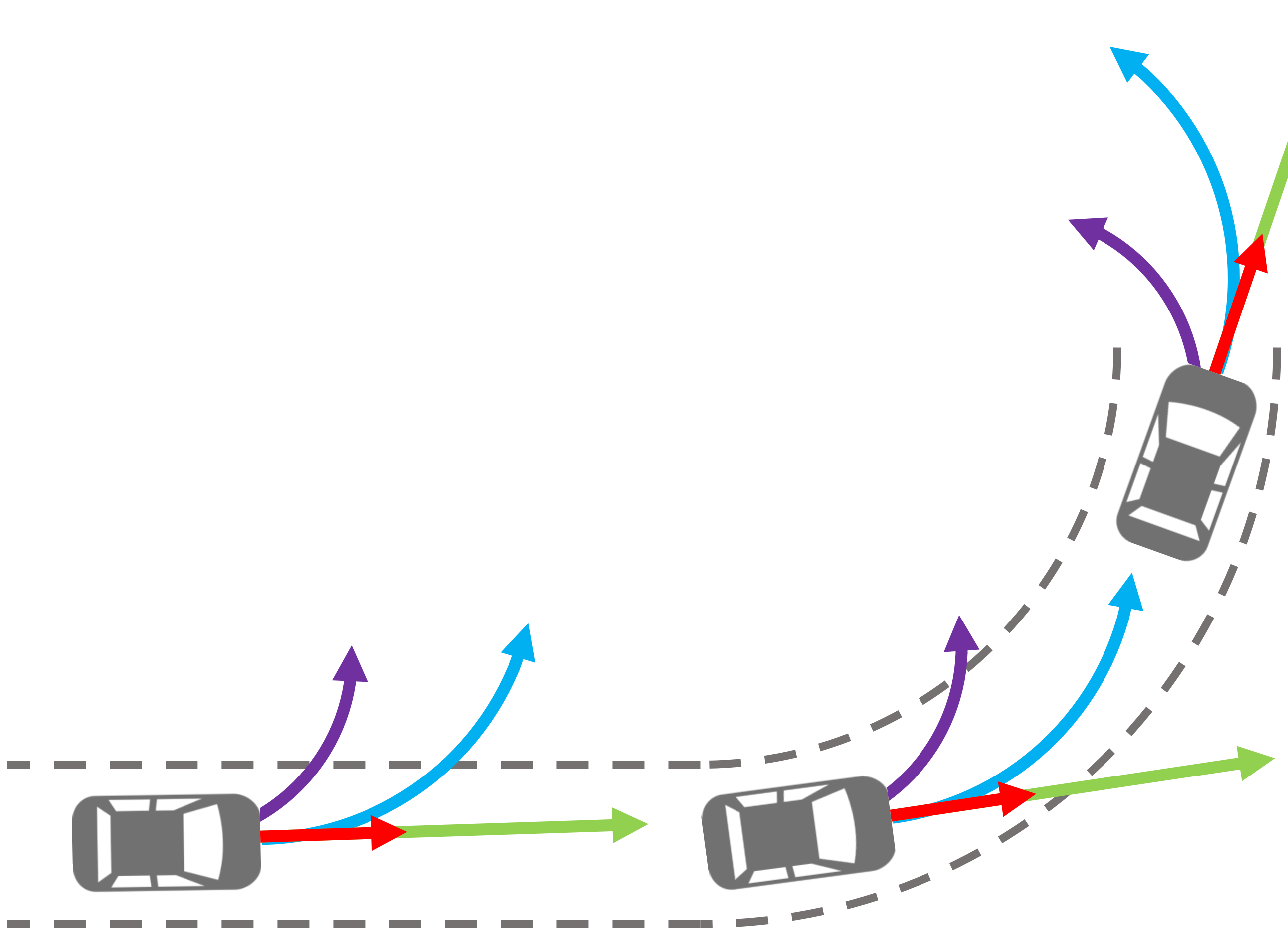
$$\hat{x}_k = \tilde{x}_k + K_k(y_k - \tilde{y}_k)$$

$$K_k = \frac{\tilde{P}_k C^T}{C\tilde{P}_k C^T + R_k}$$

$$P_k = (I - K_k C)\tilde{P}_k$$

Physics based prediction

- Which model is the best?
    - CV, CA, CTRV, CTRA

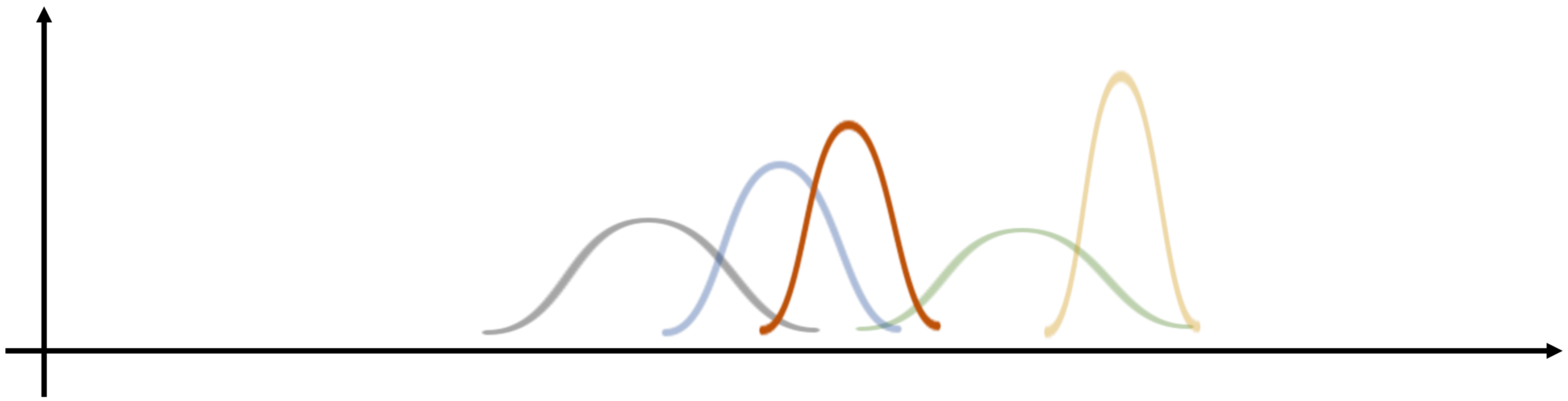


Physics based prediction

- Interacting multiple model (IMM)
  - Measure 가 나올 확률이 높은 모델 사용

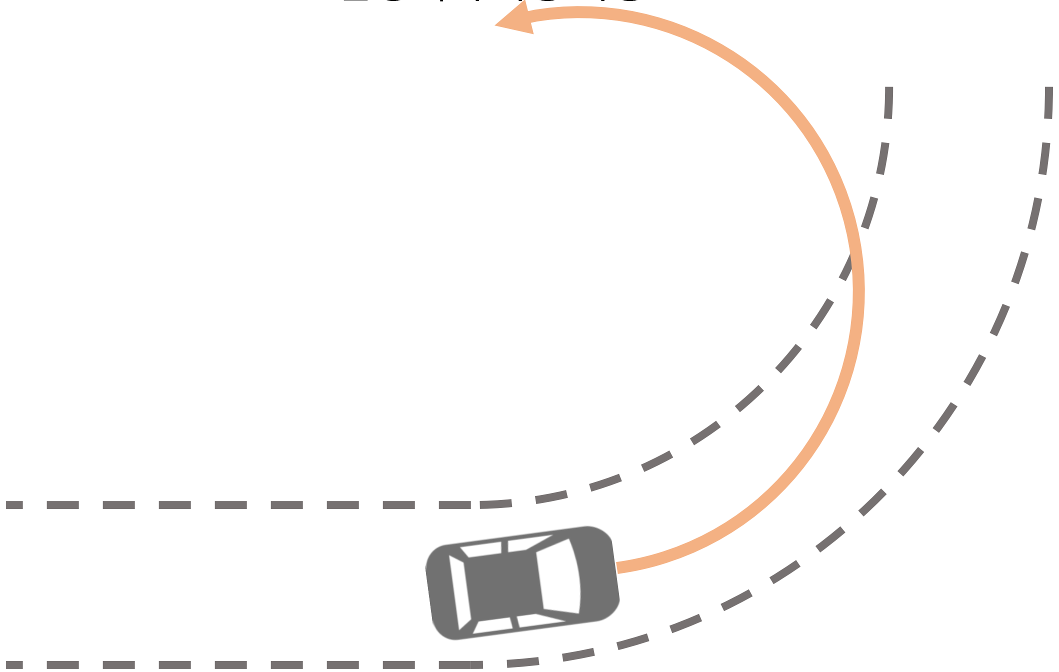
$$P(x_k|y_k) = \sum_{i}^{Model} P_i(x_k|y_k) \cdot \mu_{i,k}$$

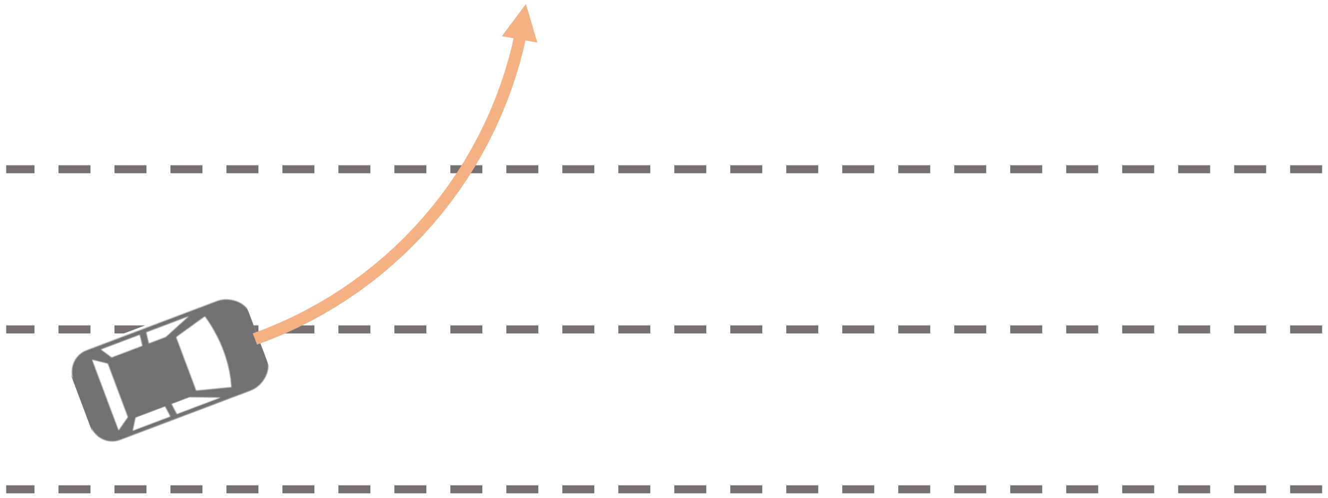
$$, \mu_{i,k} = \frac{P(y_{(1:k)}|M_i)P(M_i)}{P(y_{(1:k)})}$$



Physics based prediction

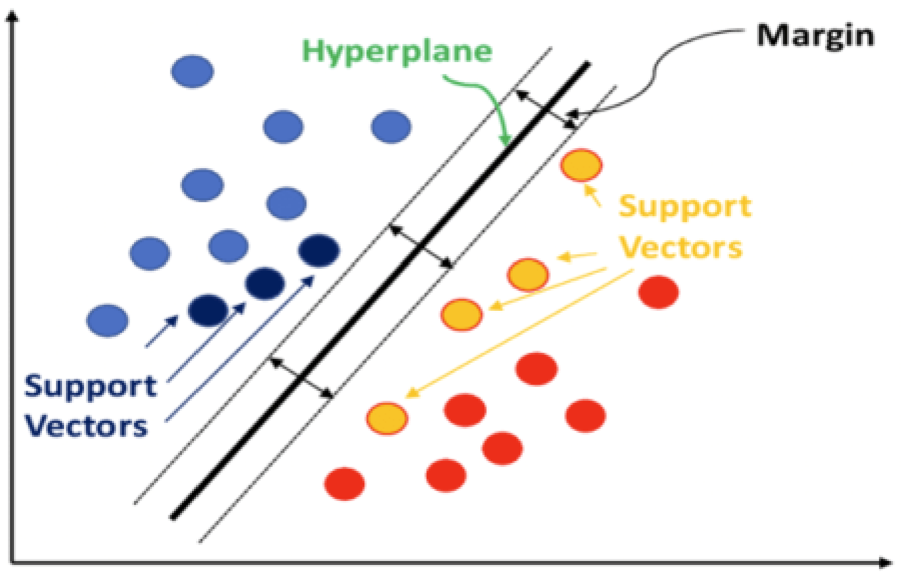
- Limits of physics-based prediction
  - Long-term prediction 에는 적합하지 않음
  - 도로 환경(차선 등) 을 고려할 수 없음
  - Frenet 환경에서 사용 가능?

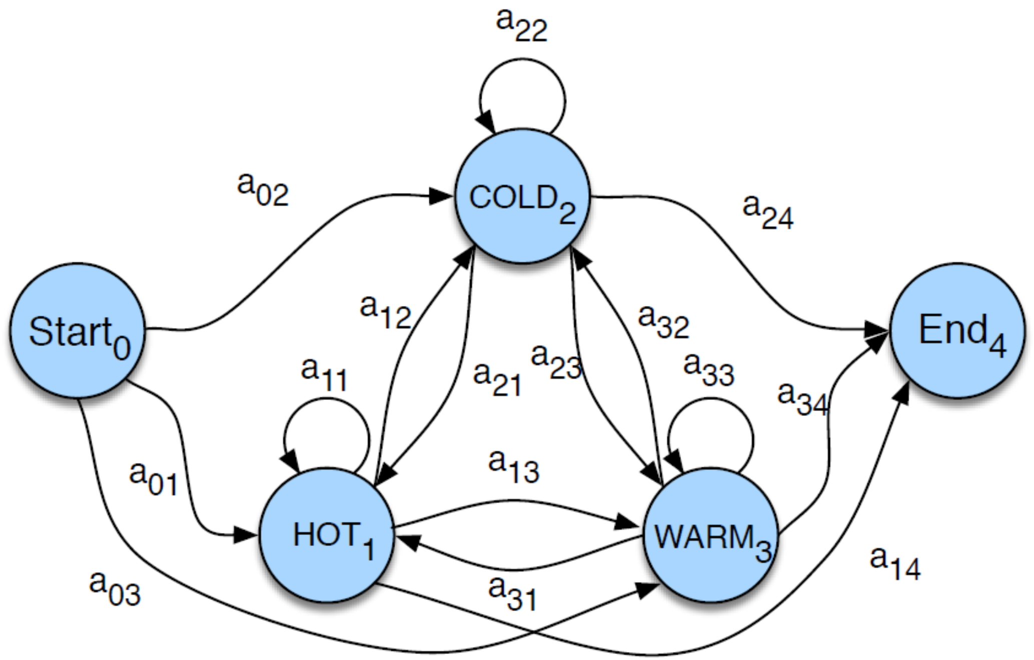




Maneuver based prediction

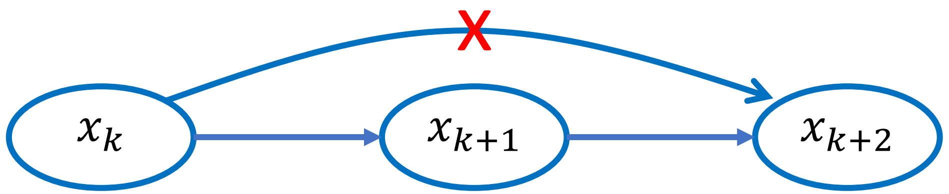
- Introduction
  - 다소 긴 시간 예측에 유리
  - 매우 긴 시간에 대한 예측은 불가
  - 주변 환경을 고려하도록 설계
  - Bayesian, SVM, HMM, …

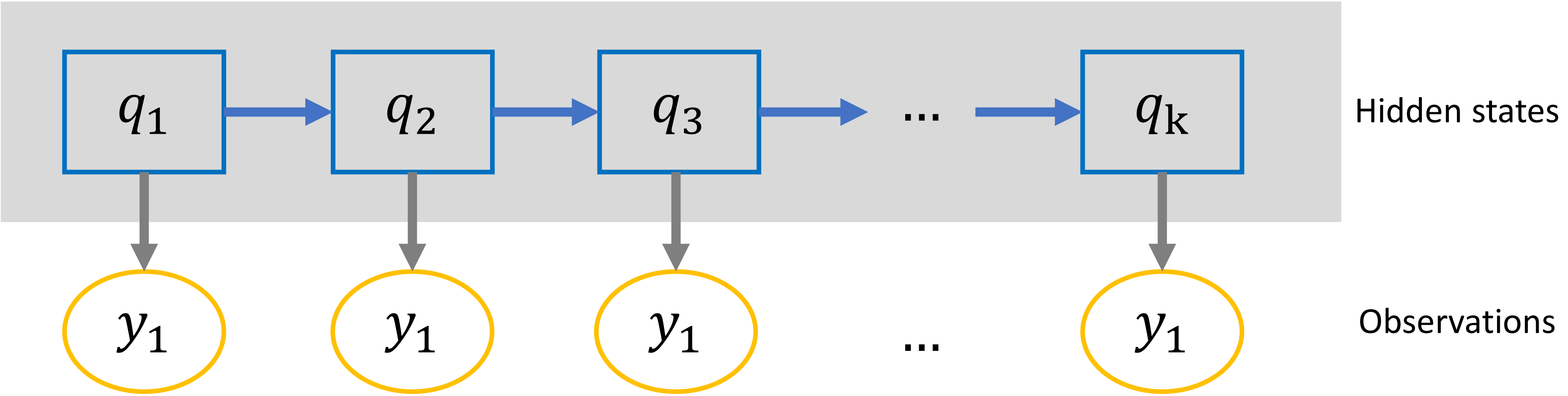




Maneuver based prediction

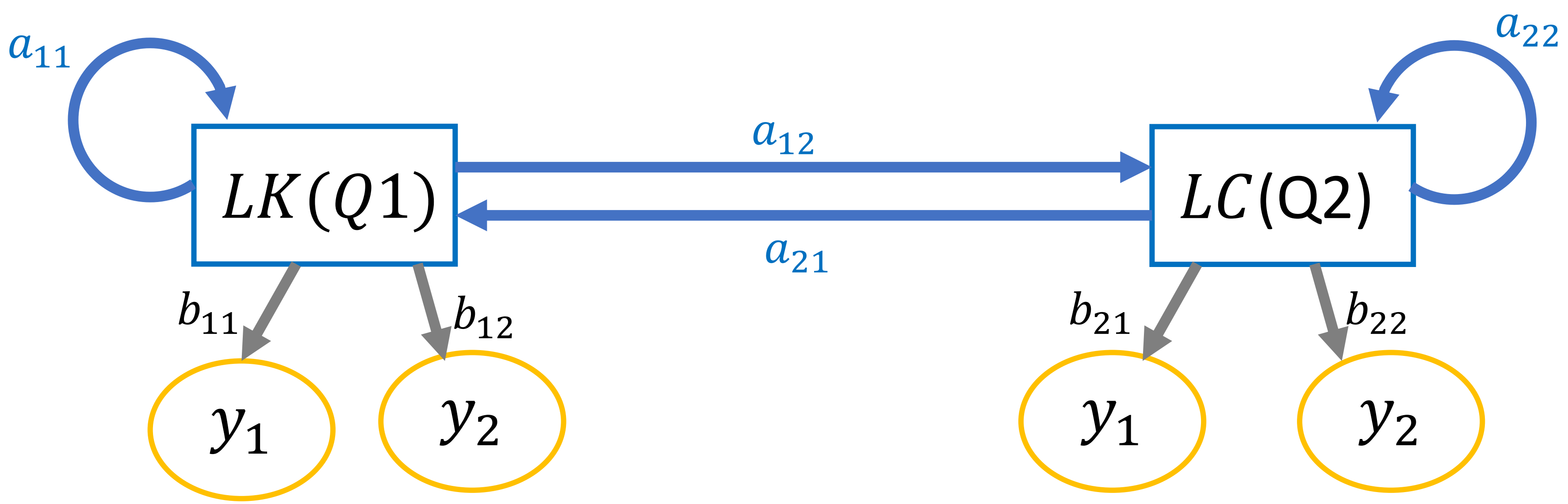
- Hidden Markov model
  - Main idea : 관측된 데이터는 숨겨진 State (Hidden state) 에서 비롯됨
  - Hidden state (Q) : State which we cannot observe. (Intension, Tendency, ⋯)
  - Observation (Y) : Value which we are able to observe. (Distance from centerline, acceleration, velocity, ⋯)
  - Assumption : Markov chain(model)





Maneuver based prediction

- Hidden Markov model
  - Transition matrix (A)
  - Emission matrix (B)
  - Initial probability (π)

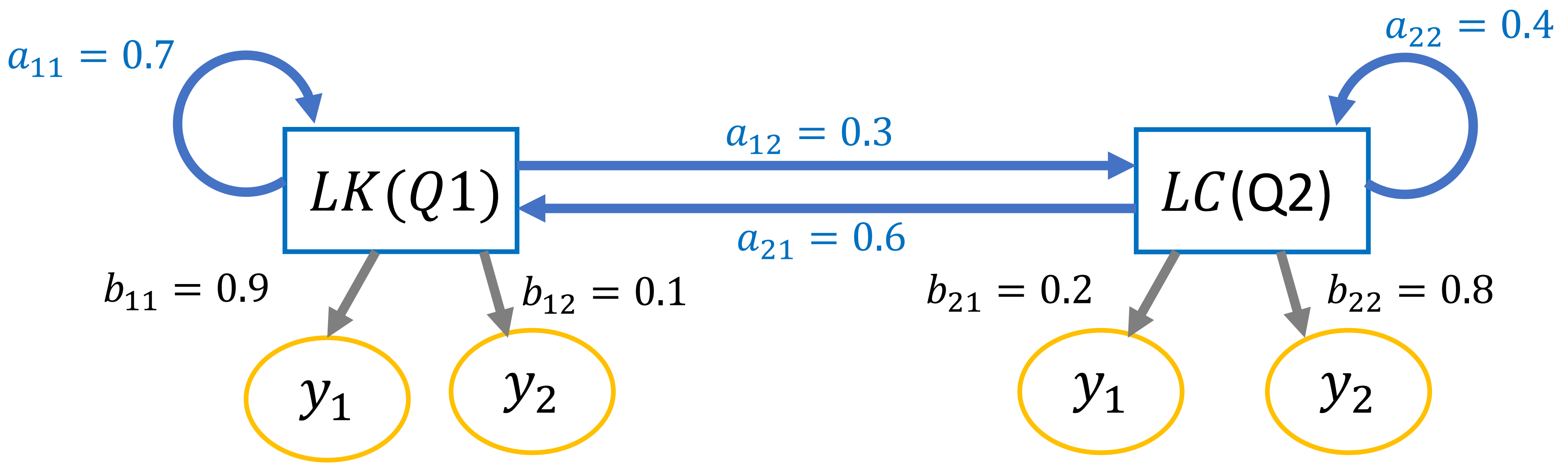


$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Maneuver based prediction

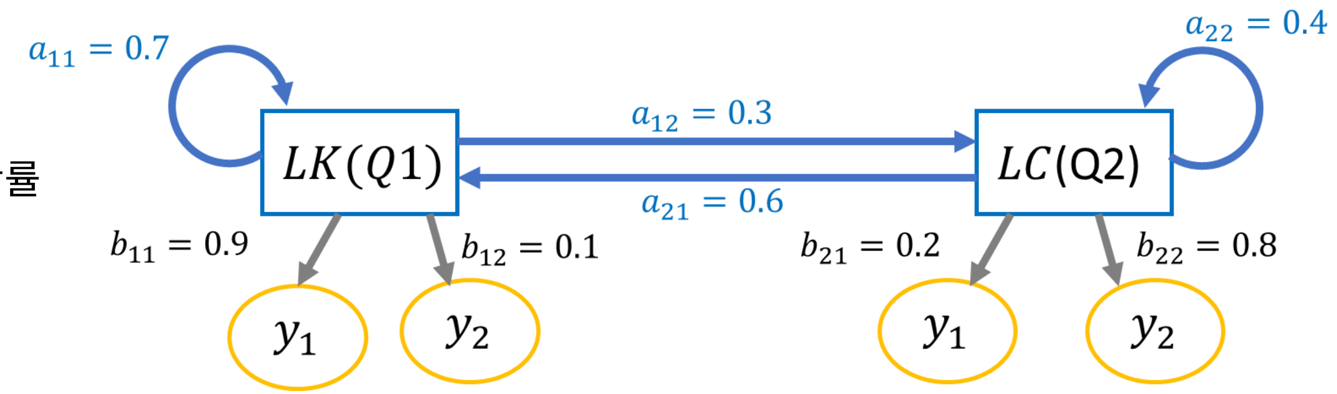
- Hidden Markov model
  - Example : P(y = {y1, y1, y2})



Maneuver based prediction

- Hidden Markov model
  - DP Problem : 반복되는 계산 값을 저장해서 사용
  - Forward probability (α) : 관측값이 주어졌을 때(1:k) 어떤 State(k)에 있을 확률

$$\alpha_{k,j} = \sum_{q} \alpha_{k-1} a_{qj}\, b_{jy}$$



$LK(Q1)$

$$\alpha_1 = \pi_1 b_{11} = 0.9 \text{ x } 0.9 = 0.81$$

$LC(\mathrm{Q2})$

$$\alpha_2 = \pi_2 b_{21} = 0.1 \text{ x } 0.1 = 0.01$$

y_1

$LK(Q1)$

$$\alpha_1 = 0.81 \text{ x } 0.7 \text{ x } 0.1 + 0.01 \text{ x } 0.6 \text{ x } 0.1$$

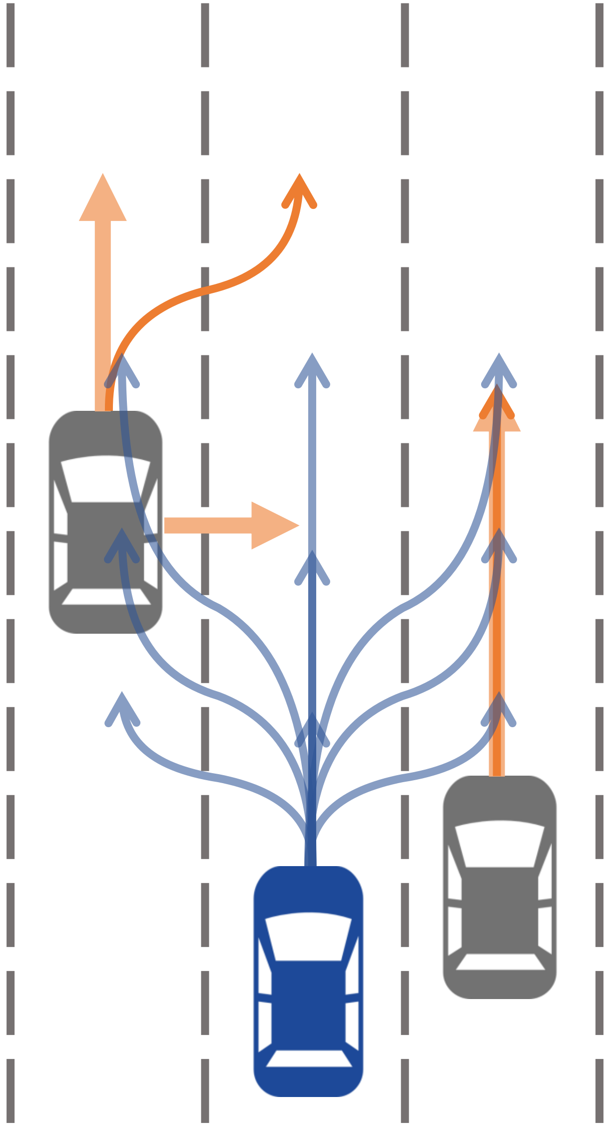
$LC(\mathrm{Q2})$

$$\alpha_2 = 0.81 \text{ x } 0.3 \text{ x } 0.8 + 0.01 \text{ x } 0.4 \text{ x } 0.8$$

y_2

Prediction

- Prediction on frenet frame
  - 선회 모델이 불필요
  - 모든 차들은 기본적으로 차선을 따라 주행한다!
  - 종방향 : Physical prediction
    - CV / CA 모델
  - 횡방향 : Maneuver prediction or physical prediction
    - Lane keeping / Lane change / CV / CA
  - 자차의 각 Trajectory candidate 별 Cost 계산 가능!



HL Klemove

Thank You