

# 0. Setting

```
In [1]: # import library
import torch
import torch.nn as nn
import matplotlib.pyplot as plt
import math
from pandas import Series, DataFrame
import pandas as pd
import numpy as np
import torch.nn.functional as F
import math
```

```
torch.__version__
```

```
Out[1]: '1.7.0+cu101'
```

```
In [2]: # using gpu
use_cuda = True
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
print(torch.cuda.is_available())
print(device)
```

```
True
cuda
```

## 1. Data

```
In [3]: from torchvision import transforms, datasets

transform = transforms.Compose([
    transforms.ToTensor(),
    transforms.Normalize((0.1307,), (0.3081,)), # mean value = 0.1307, standard deviation value = 0.3081
])
```

```
In [4]: data_path = './MNIST'

data_test = datasets.MNIST(root = data_path, train= True, download=True, transform= transform)
data_train = datasets.MNIST(root = data_path, train= False, download=True, transform= transform)

print("the number of your training data (must be 10,000) = ", data_train.__len__())
print("hte number of your testing data (must be 60,000) = ", data_test.__len__())
```

Downloading <http://yann.lecun.com/exdb/mnist/train-images-idx3-ubyte.gz> to ./MNIST/MNIST/raw/train-images-idx3-ubyte.gz

Extracting ./MNIST/MNIST/raw/train-images-idx3-ubyte.gz to ./MNIST/MNIST/raw

Downloading <http://yann.lecun.com/exdb/mnist/train-labels-idx1-ubyte.gz> to ./MNIST/MNIST/raw/train-labels-idx1-ubyte.gz

Extracting ./MNIST/MNIST/raw/train-labels-idx1-ubyte.gz to ./MNIST/MNIST/raw

Downloading <http://yann.lecun.com/exdb/mnist/t10k-images-idx3-ubyte.gz> to ./MNIST/MNIST/raw/t10k-images-idx3-ubyte.gz

Extracting ./MNIST/MNIST/raw/t10k-images-idx3-ubyte.gz to ./MNIST/MNIST/raw

Downloading <http://yann.lecun.com/exdb/mnist/t10k-labels-idx1-ubyte.gz> to ./MNIST/MNIST/raw/t10k-labels-idx1-ubyte.gz

Extracting ./MNIST/MNIST/raw/t10k-labels-idx1-ubyte.gz to ./MNIST/MNIST/raw

Processing...

Done!

the number of your training data (must be 10,000) = 10000

hte number of your testing data (must be 60,000) = 60000

/usr/local/lib/python3.6/dist-packages/torchvision/datasets/mnist.py:480: UserWarning: The given NumPy array is not writeable, and PyTorch does not support non-writeable tensors. This means you can write to the underlying (supposedly non-writeable) NumPy array using the tensor. You may want to copy the array to protect its data or make it writeable before converting it to a tensor. This type of warning will be suppressed for the rest of this program. (Triggered internally at /pytorch/torch/csrc/utils/tensor\_numpy.cpp:141.)

... return torch.from\_numpy(parsed.astype(m[2], copy=False)).view(\*s)

## 2. Model

```
In [18]: class MyModel(nn.Module):

    def __init__(self, num_classes=10, size_kernel=5):

        super(MyModel, self).__init__()

        # *****
        # input parameter
        #
        # data size:
        # mnist : 28 * 28
        #
        # the tensor given to the model should be of shape [batch_size, 1, height, width]
        # because first convolution has in_channels = 1
        # *****
        self.number_class = num_classes
        self.size_kernel = size_kernel

        # *****
        # feature layer
        # *****
        self.conv1 = nn.Conv2d(1, 20, kernel_size=size_kernel, stride=1, padding=int((size_kernel-1)/2), bias=True)
        self.conv2 = nn.Conv2d(20, 50, kernel_size=size_kernel, stride=1, padding=int((size_kernel-1)/2), bias=True)

        self.conv_layer1 = nn.Sequential(self.conv1, nn.MaxPool2d(kernel_size=2), nn.ReLU(True))
        self.conv_layer2 = nn.Sequential(self.conv2, nn.MaxPool2d(kernel_size=2), nn.ReLU(True))

        self.feature = nn.Sequential(self.conv_layer1, self.conv_layer2)
```

```

# *****
# classifier layer
# *****
self.fc1      = nn.Linear(50*7*7, 50, bias=True)
self.fc2      = nn.Linear(50, num_classes, bias=True)

self.fc_layer1 = nn.Sequential(self.fc1, nn.ReLU(True))
self.fc_layer2 = nn.Sequential(self.fc2, nn.LogSoftmax(dim=1))

self.classifier = nn.Sequential(self.fc_layer1, self.fc_layer2)

# *****
# dropout
# *****
self.dropout1 = nn.Dropout(0.25)
self.dropout2 = nn.Dropout(0.5)

self._initialize_weight()

def _initialize_weight(self):
    for m in self.modules():
        if isinstance(m, nn.Conv2d):
            nn.init.xavier_uniform_(m.weight, gain=math.sqrt(2))
            nn.init.kaiming_normal_(m.weight.data, a=0, mode='fan_in')

            if m.bias is not None:
                m.bias.data.zero_()

        elif isinstance(m, nn.Linear):
            nn.init.xavier_uniform_(m.weight, gain=math.sqrt(2))
            nn.init.kaiming_normal_(m.weight.data, a=0, mode='fan_in')

            if m.bias is not None:
                m.bias.data.zero_()

    def forward(self, x):
        x = x.to(device)
        x = self.feature(x)
        x = self.dropout1(x)
        x = x.view(x.size(0), -1)
        x = self.dropout2(x)
        x = self.classifier(x)

    return x

```

### 3. Loss Function

```

In [6]: model = MyModel(10, 5).to(device)
criterion = nn.CrossEntropyLoss()
train_y_pred = model.forward(data_train.data.unsqueeze(dim=1).float())
train_y = data_train.targets.to(device)
temp_loss = criterion(train_y_pred, train_y)
print(temp_loss.data.item())

```

776.919677734375

### 4. Optimization

Define Train Function

```

In [9]: def train(model, criterion, train_loader, optimizer, batch_size):

    model.train()
    loss_sum = 0
    acc_sum = 0
    iteration = 0
    for xs, ts in iter(train_loader):

        iteration = iteration + 1
        optimizer.zero_grad()
        y_pred = model(xs)
        ts = ts.to(device)
        loss = criterion(y_pred, ts)
        loss.backward()
        optimizer.step()

        loss_sum = loss_sum + float(loss)
        zs = y_pred.max(1, keepdim=True)[1] # first column has actual prob
        acc_sum = acc_sum + zs.eq(ts.view_as(zs)).sum().item()/batch_size

    loss_avg = round(loss_sum/iteration, 5)
    acc_avg = round(acc_sum/iteration, 5)

    return loss_avg, acc_avg

```

Define Test Function

```

In [10]: def test(model, criterion, test_loader, batch_size):
    model.eval()
    loss_sum = 0
    acc_sum = 0
    iteration = 0

```

```

with torch.no_grad():
    for xs, ts in iter(test_loader):
        iteration = iteration + 1
        ts = ts.to(device)
        y_pred = model(xs)
        loss_sum = loss_sum + criterion(y_pred, ts).data.item()
        zs = y_pred.max(1, keepdim=True)[1]
        acc_sum = acc_sum + zs.eq(ts.view_as(zs)).sum().item()/batch_size

loss_avg = round(loss_sum/iteration, 5)
acc_avg = round(acc_sum/iteration, 5)

return loss_avg, acc_avg

```

#### Define Gradient Descent Function

```

In [11]: def gradient_descent(model, optimizer, criterion, batch_size, num_epochs):

    # batching
    train_loader = torch.utils.data.DataLoader(
        data_train,
        batch_size=batch_size,
        num_workers=4,
        shuffle=True,
        drop_last=True)

    test_loader = torch.utils.data.DataLoader(
        data_test,
        batch_size=batch_size,
        num_workers=4,
        shuffle=False,
        drop_last=True)

    # return variables
    train_loss_list, train_acc_list = [], []
    test_loss_list, test_acc_list = [], []

    # run training & testing
    for epoch in range(num_epochs + 1):

        train_loss_avg, train_acc_avg = train(model, criterion, train_loader, optimizer, batch_size)
        test_loss_avg, test_acc_avg = test(model, criterion, test_loader, batch_size)

        # add loss and accuracy data
        train_loss_list.append(train_loss_avg)
        train_acc_list.append(train_acc_avg)
        test_loss_list.append(test_loss_avg)
        test_acc_list.append(test_acc_avg)

        # print
        if epoch % 10 != 0 :
            continue

        print("epoch : ", epoch, " ----- ")
        print("train loss : {}      accuracy = {}".format(train_loss_avg, train_acc_avg))
        print("test loss : {}      accuracy = {}".format(test_loss_avg, test_acc_avg))

    return train_loss_list, train_acc_list, test_loss_list, test_acc_list

```

```

In [12]: def gradient_descent_with_scheduler(scheduler, model, optimizer, criterion, batch_size, num_epochs, train_loss_list, train_acc_list, test_loss_list, test_acc_list):

    # batching
    train_loader = torch.utils.data.DataLoader(
        data_train,
        batch_size=batch_size,
        num_workers=4,
        shuffle=True,
        drop_last=True)

    test_loader = torch.utils.data.DataLoader(
        data_test,
        batch_size=batch_size,
        num_workers=4,
        shuffle=False,
        drop_last=True)

    # return variables
    train_loss_list, train_acc_list = [], []
    test_loss_list, test_acc_list = [], []

    # run training & testing
    for epoch in range(num_epochs + 1):

        train_loss_avg, train_acc_avg = train(model, criterion, train_loader, optimizer, batch_size)
        test_loss_avg, test_acc_avg = test(model, criterion, test_loader, batch_size)
        scheduler.step(train_loss_avg)

        # add loss and accuracy data
        train_loss_list.append(train_loss_avg)
        train_acc_list.append(train_acc_avg)
        test_loss_list.append(test_loss_avg)
        test_acc_list.append(test_acc_avg)

        # print
        if epoch % 10 != 0 :
            continue

        print("epoch : ", epoch, " ----- ")
        print("train loss : {}      accuracy = {}".format(train_loss_avg, train_acc_avg))
        print("test loss : {}      accuracy = {}".format(test_loss_avg, test_acc_avg))

```

## 5. Plot Function

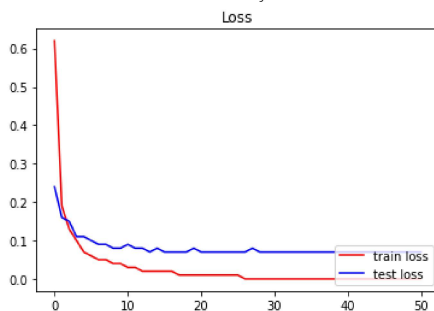
```
In [64]: def plot_loss(train_loss_list, test_loss_list):  
    plt.title("Loss")  
    plt.plot(train_loss_list, c = 'red', label = 'train loss')  
    plt.plot(test_loss_list, c = 'blue', label = 'test loss')  
    plt.legend(loc = 'upper right')  
    plt.show()
```

```
In [14]: def plot_accuracy(train_acc_list, test_acc_list):  
    plt.title("Accuracy")  
    plt.plot(train_acc_list, c = 'red', label = 'train accuracy')  
    plt.plot(test_acc_list, c = 'blue', label = 'test accuracy')  
    plt.legend(loc = 'lower right')  
    plt.show()
```

## 6. Run

```
In [ ]: # model  
num_classes=10  
size_kernel=5  
model1 = MyModel(num_classes, size_kernel).to(device)  
  
# mini-batch size  
batch_size = 32  
  
# num of epochs  
num_epochs = 50  
  
# learning rate  
learning_rate = 0.01  
  
# optimizer  
optimizer = torch.optim.SGD(model1.parameters(), lr = learning_rate, weight_decay=0.0001)  
scheduler = torch.optim.lr_scheduler.ReduceLROnPlateau(optimizer, 'min', factor=0.5, patience = 5, verbose=True)  
  
# loss function  
criterion = nn.CrossEntropyLoss()  
  
# run  
train_loss_list1, train_acc_list1, test_loss_list1, test_acc_list1 = [], [], [], []  
gradient_descent_with_scheduler(scheduler, model1, optimizer, criterion, batch_size, num_epochs, train_loss_list1, train_acc_list1, test_loss_list1, test_a  
  
# plot  
plot_loss(train_loss_list1, test_loss_list1)
```

```
epoch : 0 -----  
train loss : 0.62 ..... accuracy = 0.8  
test loss : 0.24 ..... accuracy = 0.92  
epoch : 10 -----  
train loss : 0.03 ..... accuracy = 0.98  
test loss : 0.09 ..... accuracy = 0.97  
epoch : 20 -----  
train loss : 0.01 ..... accuracy = 0.99  
test loss : 0.07 ..... accuracy = 0.97  
Epoch ... 24: reducing learning rate of group 0 to 5.0000e-03.  
epoch : 30 -----  
train loss : 0.0 ..... accuracy = 0.99  
test loss : 0.07 ..... accuracy = 0.97  
Epoch ... 33: reducing learning rate of group 0 to 2.5000e-03.  
Epoch ... 39: reducing learning rate of group 0 to 1.2500e-03.  
epoch : 40 -----  
train loss : 0.0 ..... accuracy = 0.99  
test loss : 0.07 ..... accuracy = 0.97  
Epoch ... 45: reducing learning rate of group 0 to 6.2500e-04.  
Epoch ... 51: reducing learning rate of group 0 to 3.1250e-04.  
epoch : 50 -----  
train loss : 0.0 ..... accuracy = 0.99  
test loss : 0.07 ..... accuracy = 0.97
```



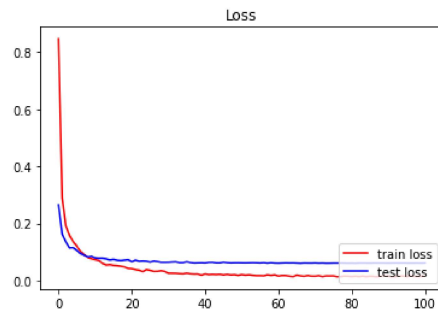
```
In [19]: # model  
num_classes=10  
size_kernel=5  
model2 = MyModel(num_classes, size_kernel).to(device)  
  
# mini-batch size  
batch_size = 32  
  
# num of epochs  
num_epochs = 100  
  
# learning rate  
learning_rate = 0.01  
  
# optimizer  
optimizer = torch.optim.SGD(model2.parameters(), lr = learning_rate, weight_decay=0.0001)  
scheduler = torch.optim.lr_scheduler.ReduceLROnPlateau(optimizer, 'min', factor=0.5, patience = 5, verbose=True)
```

```
# loss function
criterion = nn.CrossEntropyLoss()

# run
train_loss_list2, train_acc_list2, test_loss_list2, test_acc_list2 = [], [], [], []
gradient_descent_with_scheduler(scheduler, model2, optimizer, criterion, batch_size, num_epochs, train_loss_list2, train_acc_list2, test_loss_list2, test_a

# plot
plot_loss(train_loss_list2, test_loss_list2)
```

```
epoch : 0
train loss : 0.84726 ..... accuracy = 0.72436
test loss : 0.26487 ..... accuracy = 0.91923
epoch : 10
train loss : 0.0737 ..... accuracy = 0.97646
test loss : 0.0795 ..... accuracy = 0.97597
epoch : 20
train loss : 0.04198 ..... accuracy = 0.98718
test loss : 0.06594 ..... accuracy = 0.9801
Epoch : 30: reducing learning rate of group 0 to 5.0000e-03.
epoch : 30
train loss : 0.02578 ..... accuracy = 0.99069
test loss : 0.06463 ..... accuracy = 0.98165
epoch : 40
train loss : 0.02356 ..... accuracy = 0.99239
test loss : 0.06199 ..... accuracy = 0.98247
Epoch : 46: reducing learning rate of group 0 to 2.5000e-03.
epoch : 50
train loss : 0.02139 ..... accuracy = 0.99149
test loss : 0.06239 ..... accuracy = 0.98253
epoch : 60
train loss : 0.01587 ..... accuracy = 0.99479
test loss : 0.0612 ..... accuracy = 0.98315
Epoch : 71: reducing learning rate of group 0 to 1.2500e-03.
epoch : 70
train loss : 0.01546 ..... accuracy = 0.99429
test loss : 0.06202 ..... accuracy = 0.98293
epoch : 80
train loss : 0.01429 ..... accuracy = 0.99509
test loss : 0.06129 ..... accuracy = 0.983
Epoch : 83: reducing learning rate of group 0 to 6.2500e-04.
epoch : 90
train loss : 0.01523 ..... accuracy = 0.99519
test loss : 0.06138 ..... accuracy = 0.98313
Epoch : 98: reducing learning rate of group 0 to 3.1250e-04.
epoch : 100
train loss : 0.01534 ..... accuracy = 0.99479
test loss : 0.06177 ..... accuracy = 0.98318
```



```
In [34]: # model
num_classes=10
size_kernel=7
model3 = MyModel(num_classes, size_kernel).to(device)

# mini-batch size
batch_size = 32

# num of epochs
num_epochs = 100

# learning rate
learning_rate = 0.01

# optimizer
optimizer = torch.optim.SGD(model3.parameters(), lr = learning_rate, weight_decay=0.0001)
scheduler = torch.optim.lr_scheduler.ReduceLROnPlateau(optimizer, 'min', factor=0.5, patience = 5, verbose=True)

# loss function
criterion = nn.CrossEntropyLoss()

# run
train_loss_list3, train_acc_list3, test_loss_list3, test_acc_list3 = [], [], [], []
gradient_descent_with_scheduler(scheduler, model3, optimizer, criterion, batch_size, num_epochs, train_loss_list3, train_acc_list3, test_loss_list3, test_a

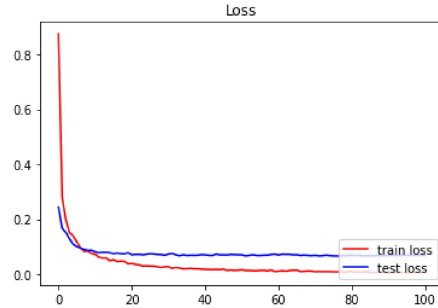
# plot
plot_loss(train_loss_list3, test_loss_list3)
```

```
epoch : 0
train loss : 0.87445 ..... accuracy = 0.70833
test loss : 0.24402 ..... accuracy = 0.93017
epoch : 10
train loss : 0.07214 ..... accuracy = 0.97486
test loss : 0.0816 ..... accuracy = 0.97485
epoch : 20
train loss : 0.03971 ..... accuracy = 0.98688
test loss : 0.07057 ..... accuracy = 0.97907
epoch : 30
train loss : 0.02691 ..... accuracy = 0.99219
test loss : 0.07332 ..... accuracy = 0.97922
epoch : 40
train loss : 0.01742 ..... accuracy = 0.99439
test loss : 0.07023 ..... accuracy = 0.98115
epoch : 50
train loss : 0.0164 ..... accuracy = 0.99479
test loss : 0.0702 ..... accuracy = 0.98198
```

```

epoch : 60 -----
train loss : 0.01049 ..... accuracy = 0.997
test loss : 0.06953 ..... accuracy = 0.9824
Epoch 66: reducing learning rate of group 0 to 5.0000e-03.
epoch : 70 -----
train loss : 0.00873 ..... accuracy = 0.9973
test loss : 0.06852 ..... accuracy = 0.983
epoch : 80 -----
train loss : 0.00945 ..... accuracy = 0.9974
test loss : 0.06948 ..... accuracy = 0.9827
Epoch 85: reducing learning rate of group 0 to 2.5000e-03.
epoch : 90 -----
train loss : 0.00486 ..... accuracy = 0.999
test loss : 0.06863 ..... accuracy = 0.9832
Epoch 97: reducing learning rate of group 0 to 1.2500e-03.
epoch : 100 -----
train loss : 0.00734 ..... accuracy = 0.9974
test loss : 0.0687 ..... accuracy = 0.9834

```



```

In [82]: # model
num_classes=10
size_kernel=7
model4 = MyModel(num_classes, size_kernel).to(device)

# mini-batch size
batch_size = 32

# num of epochs
num_epochs = 100

# learning rate
learning_rate = 0.01

# optimizer
optimizer = torch.optim.Adam(model4.parameters(), lr = learning_rate, weight_decay=0.0001)
scheduler = torch.optim.lr_scheduler.ReduceLROnPlateau(optimizer, 'min', factor=0.5, patience = 5, verbose=True)

# loss function
criterion = nn.CrossEntropyLoss()

# run
train_loss_list4, train_acc_list4, test_loss_list4, test_acc_list4 = [], [], [], []
gradient_descent_with_scheduler(scheduler, model4, optimizer, criterion, batch_size, num_epochs, train_loss_list4, train_acc_list4, test_loss_list4, test_a

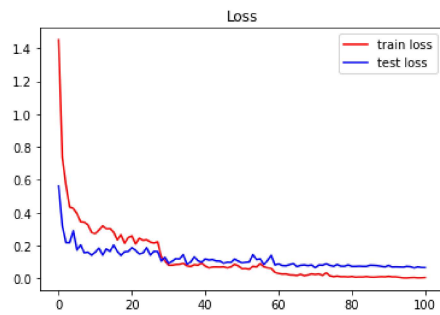
# plot
plot_loss(train_loss_list4, test_loss_list4)

```

```

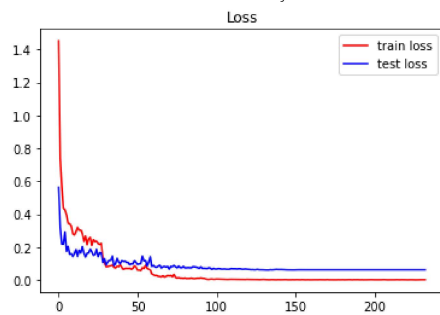
epoch : 0 -----
train loss : 1.45152 ..... accuracy = 0.52764
test loss : 0.56167 ..... accuracy = 0.8397
epoch : 10 -----
train loss : 0.27288 ..... accuracy = 0.92678
test loss : 0.16063 ..... accuracy = 0.95632
epoch : 20 -----
train loss : 0.25856 ..... accuracy = 0.94171
test loss : 0.18744 ..... accuracy = 0.96023
Epoch 28: reducing learning rate of group 0 to 5.0000e-03.
epoch : 30 -----
train loss : 0.08073 ..... accuracy = 0.97897
test loss : 0.09166 ..... accuracy = 0.97795
epoch : 40 -----
train loss : 0.07598 ..... accuracy = 0.97967
test loss : 0.11809 ..... accuracy = 0.97568
epoch : 50 -----
train loss : 0.05975 ..... accuracy = 0.98387
test loss : 0.09481 ..... accuracy = 0.97922
Epoch 59: reducing learning rate of group 0 to 2.5000e-03.
epoch : 60 -----
train loss : 0.03079 ..... accuracy = 0.99179
test loss : 0.0886 ..... accuracy = 0.9817
epoch : 70 -----
train loss : 0.02438 ..... accuracy = 0.99229
test loss : 0.06565 ..... accuracy = 0.98393
Epoch 74: reducing learning rate of group 0 to 1.2500e-03.
epoch : 80 -----
train loss : 0.01178 ..... accuracy = 0.99549
test loss : 0.07367 ..... accuracy = 0.984
epoch : 90 -----
train loss : 0.01253 ..... accuracy = 0.99539
test loss : 0.07908 ..... accuracy = 0.98203
Epoch 93: reducing learning rate of group 0 to 6.2500e-04.
epoch : 100 -----
train loss : 0.00571 ..... accuracy = 0.9985
test loss : 0.06625 ..... accuracy = 0.98513

```



```
[In [111]: num_epochs = 30
gradient_descent_with_scheduler(scheduler, model4, optimizer, criterion, batch_size, num_epochs, train_loss_list4, train_acc_list4, test_loss_list4, test_a
plot_loss(train_loss_list4, test_loss_list4)
```

```
epoch : 0 .....
train loss : 0.00191 ..... accuracy = 0.9994
test loss : 0.06209 ..... accuracy = 0.98573
Epoch 206: reducing learning rate of group 0 to 1.5259e-07.
Epoch 212: reducing learning rate of group 0 to 7.6294e-08.
epoch : 10 .....
train loss : 0.00137 ..... accuracy = 0.9997
test loss : 0.0621 ..... accuracy = 0.9857
Epoch 218: reducing learning rate of group 0 to 3.8147e-08.
epoch : 20 .....
train loss : 0.00156 ..... accuracy = 0.9996
test loss : 0.0621 ..... accuracy = 0.9857
Epoch 226: reducing learning rate of group 0 to 1.9073e-08.
epoch : 30 .....
train loss : 0.0017 ..... accuracy = 0.9995
test loss : 0.0621 ..... accuracy = 0.9857
```



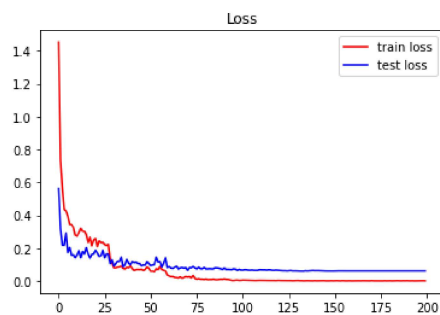
```
[In [117]: length = len(test_acc_list4)-33
print(length)
```

200

## 7. Output

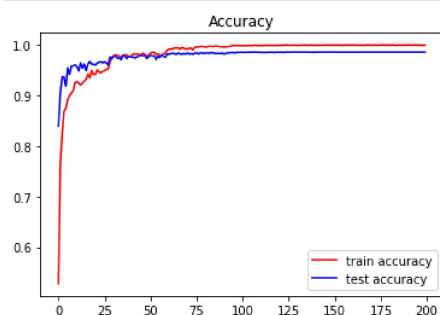
1. Plot the training and testing losses over epochs [2pt]

```
[In [130]: plot_loss(train_loss_list4[0:length], test_loss_list4[0:length])
```



1. Plot the training and testing accuracies over epochs [2pt]

```
[In [131]: plot_accuracy(train_acc_list4[0:length], test_acc_list4[0:length])
```



1. Print the final training and testing losses at convergence [2pt]

```
In [132]: data1 = {'': [train_loss_list4[length-1], test_loss_list4[length-1]]}
index1 = ['training', 'testing']
frame1 = DataFrame(data1, index = index1)
frame1.columns.name = 'loss'
frame1
```

```
Out[132]:
```

	loss
training	0.0015
testing	0.0621

1. Print the final training and testing accuracies at convergence [20pt]

```
In [133]: data2 = {'': [str(train_acc_list4[length-1]) + '0', str(test_acc_list4[length-1])]}
index2 = ['training', 'testing']
frame2 = DataFrame(data2, index = index2)
frame2.columns.name = 'accuracy'
frame2
```

```
Out[133]:
```

	accuracy
training	0.99960
testing	0.98575

1. Print the testing accuracies within the last 10 epochs [5pt]

```
In [134]: for i in range(10):
idx = length - 10 + i
print("[epoch = {0}] {1:0.5f}".format(idx, test_acc_list4[idx]))
```

```
[epoch = 190] 0.98570
[epoch = 191] 0.98570
[epoch = 192] 0.98570
[epoch = 193] 0.98572
[epoch = 194] 0.98572
[epoch = 195] 0.98572
[epoch = 196] 0.98572
[epoch = 197] 0.98573
[epoch = 198] 0.98573
[epoch = 199] 0.98575
```

```
In [ ]:
```