Lab 7
William Keilsohn
Section 32
4/24/2013

Question 1: Define the function a(n), then find numerical values to evalute the the convergence of its series using the ratio test with the large values of n given.

```
a[n_] = Factorial[n] / n^n
```

$n^{-n}\, n!$

```
A[n_] = a[n] / a[n + 1]
```

$$\frac{n^{-n}\,(1+n)^{1+n}\,n!}{(1+n)!}$$

```
A[1000.]
```

2.71692393224

```
A[10 000.]
```

2.718145927

```
A[100 000.]
```

2.71826824

Since all large values for a(n+1)/a(n) are greater than one, the series that corosponds to a(n) from n=0 to Infinity is mostlikely divergent. The value for the limit as n approaches infinity of a(n+1)/a(n) is most likely 2.7183. As stated, this value would imply that the series is divergent.

Question 2: Plot the cos(x) function and the Taylor series over the interval [-8,8] with the Tsylor series in degree 4. Use different colors for each. At what values is the Taylor series a good aproximation and at what values is it a bad one? Try using the Taylor function in degrees 8,12,and 16. When do these make good approximations?

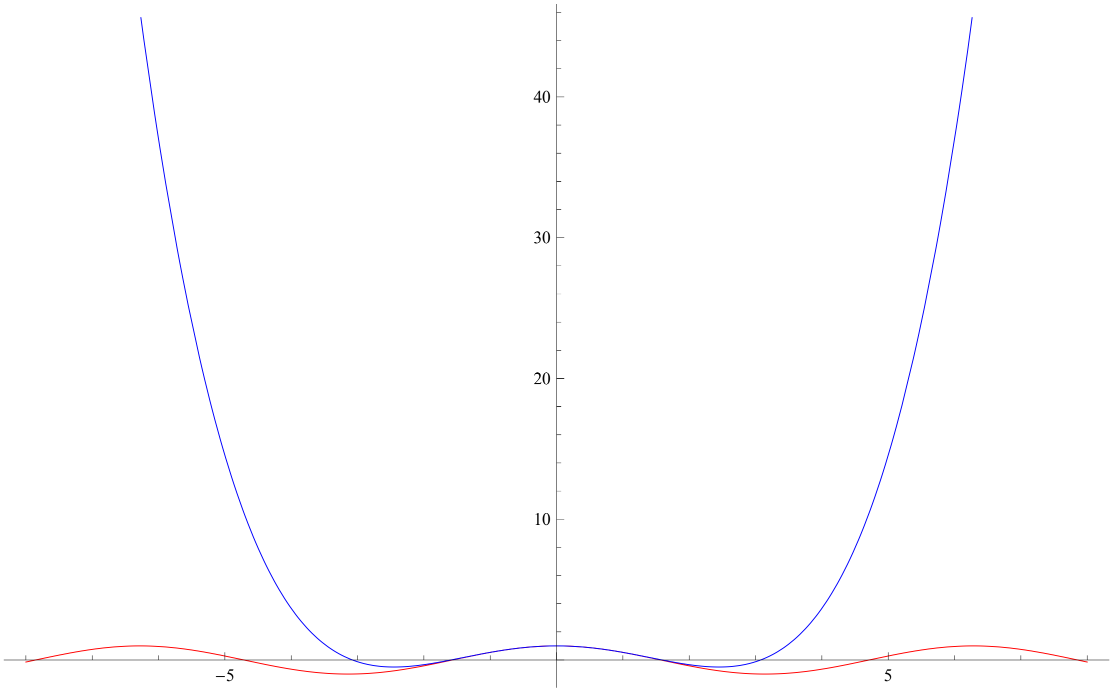
```
f[x_] = Cos[x]
```

Cos[x]

```
T4[x_] = Sum[(-1)^n / Factorial[2 * n] * x^(2 * n), {n, 0, 4 / 2.}]
```

$$1-\frac{x^2}{2}+\frac{x^4}{24}$$

```
Plot[{f[x], T4[x]}, {x, -8, 8}, PlotStyle → {Red, Blue}]
```

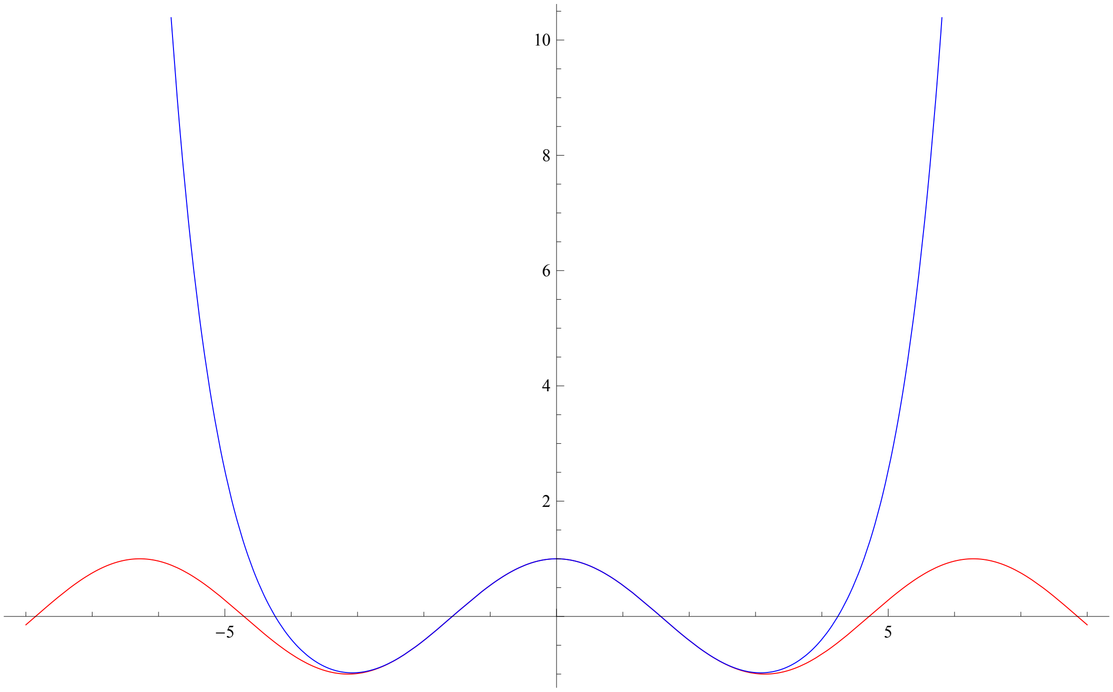


Based on the above graph, the Taylor series of degree 4 is a good aproximation of Cos(x) for x values [-2,2]. It is not a good approximation for (-Infinity, -2)U(2,Infinity).

```
T8[x_] = Sum[(-1)^n / Factorial[2 * n] * x^(2 * n), {n, 0, 8 / 2.}]
```

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320}$$

```
Plot[{f[x], T8[x]}, {x, -8, 8}, PlotStyle → {Red, Blue}]
```

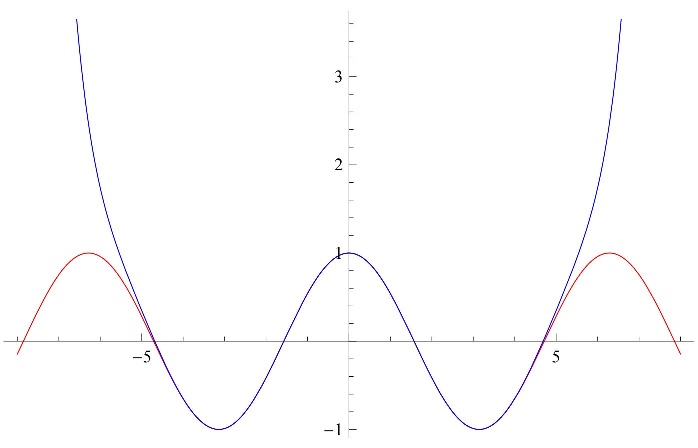


Based on the graph, one can conclude that the Taylor series with degree 8 is a good approximation of Cos(x) for x values in the interval [-3,3] and is a bad approximation for x values in (-Infinity,-3)U(3,Infinity).

```
T12[x_] = Sum[(-1)^n / Factorial[2 * n] * x^(2 * n), {n, 0, 12 / 2.}]
```

$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40\,320}-\frac{x^{10}}{3\,628\,800}+\frac{x^{12}}{479\,001\,600}$$

```
Plot[{f[x], T12[x]}, {x, -8, 8}, PlotStyle → {Red, Blue}]
```

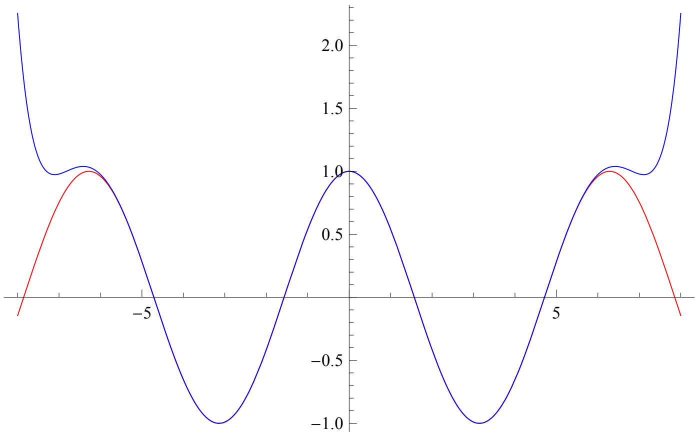


Based on the graph, one can conlude that the Taylor series with degree 12 is a good approximation for Cos(x) for x values in [-5,5] and is a bad approximation for x values in (-Infinity,-5)U(5,Infinity).

```
T16[x_] = Sum[(-1)^n / Factorial[2 * n] * x^(2 * n), {n, 0, 16 / 2.}]
```

$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}-\frac{x^{10}}{3628800}+\frac{x^{12}}{479001600}-\frac{x^{14}}{87178291200}+\frac{x^{16}}{20922789888000}$$

```
Plot[{f[x], T16[x]}, {x, -8, 8}, PlotStyle → {Red, Blue}]
```



Based on the graphn one can conclude that the Taylor series with degree 16 is a good approximation for Cos(x) for x values in [-6,6] and is a bad approximation for x values in (-infinity,-6)U(6,Infinity).