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Observer-based Time-varying Formation Tracking for One-sided Lipschitz Nonlinear Systems via Adaptive Protocol

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Abstract: In this paper, we investigate the problem of observer-based time-varying leader-following formation tracking for multi-agent systems with the one-sided Lipschitz and quadratic inner-boundedness nonlinear dynamics. An idea of the observer-based protocol with an edge-based adaptive law is designed for nonlinear systems, which enable agents to achieve a desired formation tracking when it cannot obtain the full state of the nonlinear system. In contrast to the previous nonlinear formation systems, the advantage of the developed method is that it is less conservative and more general for the nonlinear system with large Lipschitz constants. Besides, under the proposed adaptive law, the design of the protocol does not rely on the known communication topology. Finally, the simulation results for one leader and six followers are proposed to show the feasibility and effectiveness of the proposed method in the nonlinear time-varying formation system.

Keywords: Adaptive protocol, leader-following systems, one-sided Lipschitz systems, time-varying formation.

1. INTRODUCTION

In recent years, the research on cooperative control for multi-agents and complex networks has gained extensive attention from lots of scientific communities [1–3]. Cooperative control is widely used in many aspects, such as consensus control [4–8], formation control [9, 10], environment monitoring [11] and target tracking [12] in unmanned aerial vehicle, underwater vehicle and traffic network control [13–16]. Agents, in the cooperative control network, should communicate between their neighboring agents, and the information exchange between them can be location, speed, acceleration, or others.

With the development of the cooperative control of multi-agent systems, especially the consensus theory, lots of researchers found consensus theory can be applied to the formation control problems. For the consensus control, in [17], the consensus control of multi-agent linear systems have been studied, while in [18], the interest in the advancement of consensus of the multi-agent systems for nonlinear dynamics has been increased. It is worth noting that there are many methods to cancel nonlinearity, such as using sliding mode control [19], Lipschitz condition and neural networks. In [2], Li *et al.* studied using Lipschitz function to achieve consensus control for nonlinear multi-agent systems. Inspired by [20, 21], [22, 23] proposed to use one-side Lipschitz condition to deal with the consensus control of nonlinear multi-agent systems. Later, to avoid the continuous monitoring of the system states, Wang *et al.* [24] investigated the output consensus problem for uncertain nonlinear multi-agent systems via event-triggered communication.

On the other hand, formation control is a important issue in the field of cooperative control of multiple agents, which is developed in the multi-agent consensus theory. The issues of formation control can be transformed into some kinds of consensus issues with the tools of consensus theory. Formation control requires each agent to move to the desired position and form a specified formation geometry [25]. In recent years, an increasing number of researchers have studied formation control and have achieved a lot of research results. In [26], time-invariant formation for multi-agent linear system was discussed, while [27] studied the linear time-invariant multi-agent systems considering skidding and slipping effects. These are the studies for multi-agent time-invariant formation systems. However, considering the actual situation, each agent needs to reach a different state all the time and makes the formation achieve a desired geometry. Hence, the formation system with dynamic change is necessary in practical application. In [28], Dong *et al.* have proposed the time-varying formation control with one leader. And

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on the basis of one leader, they conducted on the time-varying formation of multi-agent systems with multiple leaders in [29]. Based on [29], Su *et al.* studied the formation system with sampling data delay under some suitable conditions in [1].

On the basis of linear formation theory, many researchers have studied nonlinear formation. In [30], Ran *et al.* studied distributed time-varying formation control for uncertain second-order nonlinear systems with external disturbance. [31] proposed the nonlinear multi-agent formation systems with Lipschitz dynamics using sliding mode control. Moreover, second-order formation control algorithms for nonlinear system has been discussed in [32], which proposes sufficient conditions to guarantee the multi-agent systems with time-varying delay to form a desired formation. In [33], practical formation control for second-order nonlinear systems have been proposed based on neural networks, which can process both the matched and mismatched nonlinearities and disturbances.

It should be noticed that most of the above-mentioned literature deal with the nonlinear problems of the leader-following formation system using full state feedback. However, in many circumstances, it is difficult to detect the full states of each agent. Therefore, designing an observer-based protocol to solve this formation control problem is a good method. The observer-based method has been applied in many fields [34], and many researchers have employed the observer to nonlinear systems and obtained good results [34–37]. Wang *et al.* [38] proposed an observer-based control method to handle with time-varying formation nonlinear systems with disturbances. Besides, [39] proposed an observer-based method to solve nonlinear formation system, which employs the distributed feedback input. From these researches for nonlinear formation systems in [38, 39], we find they prefer to use Lipschitz condition to address the nonlinear systems and these conditions they have used are the conservative Lipschitz condition, which can be replaced by a more general and less conservative condition called the one-sided Lipschitz continuity [40]. The one-sided Lipschitz constant has less conservativeness in comparison to the Lipschitz constant, which have been applied to the observer and other control design problems. Thus, as far as our knowledge, observer-based time-varying formation control for leader-following multi-agent nonlinear system using one-sided Lipschitz condition leaves a wide research field.

Inspired by the aforementioned works, this current paper studies the observer-based time-varying formation tracking problems for the leader-following multi-agent nonlinear system using adaptive protocol. The main contributions of the current paper have been discussed as follows. First, by employing the output feedback protocol with an edge-based adaptive law, all the agents can adjust control input more efficiently with appropriate unknown communication topology. In [30, 32, 38, 39], they did not consider the adaptive protocol in the time-varying formation systems, which will reduce the set of feasible solutions. Second, the control input is nonzero and all of the agents have nonlinear term. In [29], these multi-agent systems will achieve an expected formation tracking by requiring the control inputs to be zero. Besides, the method in the current paper is more general with all of the agents have nonlinear term compared with [39], which do not consider the nonlinear term of the leader. Third, we first proposed to use one-sided Lipschitz condition to the time-varying formation system to handle with the nonlinear problems, which will make the condition less conservative. In [2] and [22], distributed consensus control protocol under the nonlinear system were only used to the consensus problems, and it did not considering the formation control problems.

The reminder of this work is organized as follows: Section 2 has introduced some useful notations, definitions and formulation, and has proposed an adaptive coupling weight protocol. In Section 3, the conditions and proof process for the multi-agent to reach formation tracking is proposed. Then we will give two simulation examples to prove our theoretical results in Section 4. Finally, Section 5 gives the main conclusions of this paper.

2. PRELIMINARIES

2.1. Notation

In the this paper, $\otimes$ represents the Kronecker product. $\mathbb{R}^{n\times m}$ is defined as the set of $n\times m$ real matrices. For a symmetric matrix, "$*$" is represented as the symmetric entry. I_N represents the identity matrix of dimension N, and $\mathbf{1}_N$ denotes $[1,1,\dots,1]^T$ with dimension N.

2.2. Basic concepts

For a system containing N agents, it can be described as a weighted graph $G=\{\mathscr{V},\mathscr{E},\mathscr{W}\}$, $\mathscr{V}=\{v_0,v_1,v_2,\dots,v_N\}$ represents the set of nodes. The set of edges can be represented by $\mathscr{E}\subseteq\{(v_i,v_j):v_i,v_j\in\mathscr{V},i\neq j\}$. Meanwhile, $\mathscr{W}=[\omega_{ij}]\in\mathbb{R}^{n\times n}\,(\omega_{ij}\geq 0)$ represents the weighted adjacency matrix. The edges of G are denoted by $\varepsilon_{ij}=(v_i,v_j), i\neq j$. $\varepsilon_{ij}\in\mathscr{E}(\varepsilon_{ij}>0)$, it means there is an edge from agent j to agent i, and it can convert the information of agent from agent j to agent i. Otherwise, if $\varepsilon_{ij}\notin\mathscr{E}$, then $\omega_{ij}=0$. So the directed path from point to point can be represented by edges $(v_j,v_o),(v_o,v_p),\dots(v_y,v_i)$. The adjacency matrix of the network graph $G=\{\mathscr{V},\mathscr{E},\mathscr{W}\}$ for agents is expressed as $\mathscr{W}=[\omega_{ij}]_{N\times N}$, which is defined as

$$\omega_{ij}=\begin{cases}0, & \text{if} \quad (v_i,v_j)\notin\mathscr{E},\\ 0, & \text{if} \quad \text{i}=\text{j},\\ 1, & \text{if} \quad (v_i,v_j)\in\mathscr{E}.\end{cases}$$

Lemma 1 [32]**:** For any vector p,q with appropriate dimensions, there exists a suitable positive definite matrix $\mathscr{S} > 0$ satisfying the following inequality:

$$p^T \mathscr{S} p + q^T \mathscr{S}^{-1} q \geq \pm 2 p^T q. \tag{1}$$

2.3. Agent dynamics

For a multi-agent system consisting of $N+1$ agents, 1 and N represent the number of one leader and followers in the multi-agent system, respectively. And the set of N agents is described as $F = [1,2,\ldots,N]$. Then the Laplacian matrix of the network communication between agents can be described as $L = \begin{bmatrix} 0 & \mathbf{0}_{1\times N} \\ Q & \mathscr{L} \end{bmatrix} \in \mathbb{R}^{(N+1)\times(N+1)}$, where $Q = [\omega_{10}, \omega_{20}, \ldots, \omega_{N0}]^T \in \mathbb{R}^{N\times 1}$, and $\mathscr{L} = [\mathscr{L}_{ij}]_{N\times N}$,

$$\mathscr{L}_{ij} = \begin{cases} \omega_{i0} + \sum_{j=1, i\neq j}^{N} \omega_{ij}, & i = j, \\ -\omega_{ij}, & i \neq \mathrm{j}, \end{cases} \quad i,j = 1,2,...,N. \tag{2}$$

The multi-agent dynamics system with followers are defined as the following:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + D\psi(x_i(t),t), \\ y_i(t) = Cx_i(t), \end{cases} \quad i \in F, \tag{3}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector of the follower i, $y_i(t) \in \mathbb{R}^q$ is the output. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $C \in \mathbb{R}^{q\times n}$ and $D \in \mathbb{R}^{n\times n}$ are the given constant matrices. $\mathrm{Rank}(B) = q, q \leq n$. $u_i(t) \in \mathbb{R}^p$ is defined as the controller of ith follower. And $\psi(x_i(t),t) \in \mathbb{R}^n$ is the nonlinear dynamics for the ith follower.

Similarly, the dynamics system for the leader is described by the following:

$$\begin{cases} \dot{x}_0(t) = Ax_0(t) + Bu_0(t) + D\psi(x_0(t),t), \\ y_0(t) = Cx_0(t), \end{cases} \tag{4}$$

where $x_0(t) \in \mathbb{R}^n$ denotes the state of the leader, $y_0(t) \in \mathbb{R}^q$ is the output, and the $u_0 \in \mathbb{R}^p$ is the control input for the leader. $\psi(x_0(t),t) \in \mathbb{R}^n$ is a vector function that denote the leader's nonlinear dynamics.

Definition 1: The function $\psi(x_i,t)$ validates the *one-sided Lipschitz condition* as

$$\langle \psi(z_a,t) - \psi(z_b,t), z_a - z_b \rangle \leq \eta \|z_a - z_b\|^2, \tag{5}$$

where $z_a, z_b \in \mathbb{R}^n$ and the scalar $\eta \in \mathbb{R}$ is a constant, denoting the one-sided Lipschitz constant.

Definition 2: The function $\psi(x_i,t)$ is said to be satisfied the *quadratic inner-boundedness constraint*, if

$$\begin{aligned} &(\psi(z_a,t) - \psi(z_b,t))^T (\psi(z_a,t) - \psi(z_b,t)) \\ &\leq \gamma \|z_a - z_b\|^2 + \mu \langle \psi(z_a,t) - \psi(z_b,t), z_a - z_b \rangle, \end{aligned} \tag{6}$$

where $z_a, z_b \in \mathbb{R}^n$ and the scalars $\gamma \in \mathbb{R}$ and $\mu \in \mathbb{R}$ are known constants, denoting the quadratic inner-boundedness constants.

Remark 1: It is worth emphasizing that the one-sided Lipschitz property contains the well-known Lipschitz property, but the converse does not true. Besides, the property of quadratical inner-boundedness includes the Lipschitz condition, and the opposite is not true(see e.g., [41–43]).

Assumption 1: The nonlinear item is matched in the system, i.e., there exists a matrix $M \in \mathbb{R}^{p\times n}$ such that $D = BM$.

Remark 2: The matched condition in Assumption 1 is similar to the system with disturbance in [44]. It is used to offset the nonlinear term in the formation system such as turbulence by outputting the corresponding nonlinear term from the controller, which is based on the observed information of the agent. And if the nonlinearity in the system is not dealt with, the performance of the designed system will degrade. To the best of our knowledge, there exist a few methods to handle with the nonlinear formation systems such as using Lipschitz condition to cancel the nonlinearity by matched nonlinear items [31, 32, 39], and employing neural network [33].

Assumption 2: There exists a directed spanning tree for the communication network of the leader-following systems with the leader as the root.

Assumption 3: The connection between followers is bidirectional, and the communication from leader to followers is directed.

Assumption 4: The pair (A,B) is controllable, and the pair (A,C) is detectable.

Lemma 2 [33]**:** All the eigenvalues of the matrix $\mathscr{L}$ have positive real parts, if Assumption 2 holds.

Since the input matrix B has full rank q, there exists a pseudo inverse $\bar{B} \in \mathbb{R}^{q\times n}$ satisfied $\bar{B}B = I_q$. Then, there exists an appropriate matrix $\tilde{B} \in \mathbb{R}^{(n-q)\times n}$ which leads to $\tilde{B}B = 0$ and $\left[\bar{B}^T, \tilde{B}^T\right]^T$ is nonsingular matrix.

For the time-varying formation of $N+1$ agents, the leader is used as reference points for followers, and N followers need to refer to the points of the leader as the center point to change into the desired geometric shape as time changes. Therefore, for N followers, time-varying formation can be formed by a compensation set of column vectors $h_F(t) = \left[{h_1}^T(t), {h_2}^T(t), \ldots {h_N}^T(t)\right]^T \in \mathbb{R}^{Nn}$ with ${h_i}^T(t) \in \mathbb{R}^n$, and $h_F(t)$ is a continuous differentiable function with bounded segments, which is used to change the distance between each follower and the leader over time.

Definition 3 (See e.g. [1], [29])**:** For the given bounded states, the multi-agent dynamics formation system (4) is said to achieve the desired time-varying *formation track-*

ing, the following equation must be satisfied:

$$\lim_{t\to\infty}(x_i(t)-h_i(t)-x_0(t))=0, \tag{7}$$

where $i\in F$.

For any follower i, an adaptive time-varying formation tracking protocol is defined as follows:

$$\begin{aligned}
u_i(t)&=K\sum_{j=1}^{N}\omega_{ij}c_{ij}(t)\left((\hat{x}_i(t)-h_i(t))-(\hat{x}_j(t)-h_j(t))\right)\\
&\quad+K\omega_{i0}c_{i0}(t)\left(\hat{x}_i(t)-h_i(t)-\hat{x}_0(t)\right)\\
&\quad-M\psi(\hat{x}_i(t),t)+M\psi(\hat{x}_0(t),t)+u_0(t)+v_i(t)\\
\dot{c}_{ij}(t)&=\varsigma_{ij}\omega_{ij}[(\hat{x}_i(t)-h_i(t))-(\hat{x}_j(t)-h_j(t))]^T\mathscr{H}\\
&\quad\times[(\hat{x}_i(t)-h_i(t))-(\hat{x}_j(t)-h_j(t))],\\
\dot{c}_{i0}(t)&=\varsigma_{i0}\omega_{i0}(\hat{x}_i(t)-h_i(t)-\hat{x}_0(t))^T\mathscr{H}\\
&\quad\times(\hat{x}_i(t)-h_i(t)-\hat{x}_0(t)),\\
\dot{\hat{x}}_i(t)&=A\hat{x}_i(t)+Bu_i(t)+D\psi(\hat{x}_i(t),t)\\
&\quad+\vartheta T\left(y_i-C\hat{x}_i(t)\right),\\
\dot{\hat{x}}_0(t)&=A\hat{x}_0(t)+Bu_0(t)+D\psi(\hat{x}_0(t),t)\\
&\quad+\vartheta T(y_0-C\hat{x}_0(t)),
\end{aligned}\tag{8}$$

where $i,j\in F$, $u_0(t)=\varepsilon_0-M\psi(\hat{x}_0(t),t)$; $\hat{x}_i(t),\hat{x}_0(t)\in\mathbb{R}^n$. $\hat{x}_i(t)$ and $\hat{x}_0(t)$ are the estimated state for ith follower and the leader, respectively. $\varepsilon_0\in\mathbb{R}^p$ is a part of the control input for the leader. $c_{ij}(t)$ and $c_{i0}(t)$ are adaptive coupling weight. $\varsigma_{ij}(t)$ and $\varsigma_{i0}(t)$ represent the control factor of convergence rate of coupling weights for the communication between follower to follower and leader to follower, respectively. $K\in\mathbb{R}^{p\times n}$, $T\in\mathbb{R}^{n\times q}$ are the state feedback matrix and observer gain matrix, $\mathscr{H}\in\mathbb{R}^{n\times n}$ is the feedback gain matrix, $\vartheta>0$ is the adjustment weight of the observer. $\varsigma_{ij}(0)>0,\varsigma_{i0}(0)>0$ and $\mathscr{H}>0$, then we easily obtain $\dot{c}_{ij}(t)\geq 0,\dot{c}_{i0}(t)\geq 0$. $c_{ij}(t)$ and $c_{i0}(t)$ will change the controller $u_i(t)$ as the difference between each follower and the target position. $v_i(t)$ is the compensational signal function designed as $v_i(t)=-\bar{B}(Ah_i(t)-\dot{h}_i(t))$, it can widen the feasible conditions for ith follower to form time-varying formation.

By using the protocol (8) and Assumption 1, the multi-agent dynamic systems in (3) and (4) become

$$\begin{aligned}
\dot{x}_i(t)&=Ax_i(t)+BK\omega_{i0}c_{i0}(t)\delta_i(t)\\
&\quad+BK\sum_{j=1}^{N}\omega_{ij}c_{ij}(t)\left(\delta_i(t)-\delta_j(t)\right)\\
&\quad+BM\left(\psi(x_i(t),t)-\psi(\hat{x}_i(t),t)\right)+B\varepsilon_0+Bv_i(t),\\
\dot{\hat{x}}_i(t)&=A\hat{x}_i(t)+BK\omega_{i0}c_{i0}(t)\delta_i(t)\\
&\quad+BK\sum_{j=1}^{N}\omega_{ij}c_{ij}(t)\left(\delta_i(t)-\delta_j(t)\right)\\
&\quad+B\varepsilon_0+Bv_i(t)-\vartheta TCe_i(t),\\
\dot{e}_i(t)&=Ae_i(t)+D\left(\psi(x_i(t),t)-\psi(\hat{x}_i(t),t)\right)\\
&\quad-\vartheta TCe_i(t),\\
\dot{e}_0(t)&=Ae_0(t)+D\left(\psi(x_0(t),t)-\psi(\hat{x}_0(t),t)\right)\\
&\quad-\vartheta TCe_0(t),
\end{aligned}\tag{9}$$

where $\delta_i(t)=\hat{x}_i(t)-h_i(t)-\hat{x}_0(t)$. And $e_i(t)=x_i(t)-\hat{x}_i(t)$, $e_0(t)=x_0(t)-\hat{x}_0(t)$ denote the state error of ith follower and the leader, respectively.

3. MAIN RESULTS

In this section, we propose a sufficient condition for a distributed multi-agent system (3)-(4) with one leader to achieve time-varying formation tracking.

Theorem 1: Suppose that Assumption 1-4 hold. The multi-agent system (3) and (4) under protocol (8) achieves asymptotic stability of time-varying *formation tracking* if the following conditions are satisfied:

1) For any follower i $(i\in F)$ in the formation, it needs to satisfy the following condition:

$$\tilde{B}Ah_i(t)-\tilde{B}\dot{h}_i(t)=0. \tag{10}$$

2) The scalars $c_{ij}(0)=c_{ji}(0)>0$, $\varsigma_{ij}=\varsigma_{ji}>0$ $(i,j=0,1,...,N)$. There exist appropriate scalars $\tau_1,\tau_2,\tau_3>0,\rho_1,\rho_2>0$, and matrices $K=-B^TP^{-1}$, $\mathscr{H}=P^{-1}BB^TP^{-1}$, $T=S^{-1}C^T$, where $P>0$, $S>0$ are the solutions to the following LMI:

$$\begin{bmatrix}
\Xi & 0 & 0 & \sqrt{\tau_2}S^{-1}C^T\\
* & \Theta+(2\rho_1\eta+2\rho_2\gamma)I_N & SBM+(\rho_2\mu-\rho_1)I_N & 0\\
* & * & -2\rho_2 I_N & 0\\
* & * & * & -I_N
\end{bmatrix}<0, \tag{11}$$

where $\Xi=AP+PA^T-\tau_1BB^T$ and $\Theta=SA+A^TS-\tau_3C^TC$.

Proof: Recalling that $\delta_i(t)=\hat{x}_i(t)-h_i(t)-\hat{x}_0(t)$. From (9), the following transformation holds

$$\begin{aligned}
\dot{\delta}_i(t)&=A\delta_i(t)+BK\sum_{j=1}^{N}\omega_{ij}c_{ij}(t)\left(\delta_i(t)-\delta_j(t)\right)\\
&\quad+BK\omega_{i0}c_{i0}(t)\delta_i(t)-\vartheta TCe_i(t)\\
&\quad-\vartheta TCe_0(t)+Ah_i(t)-\dot{h}_i(t)+Bv_i(t),\\
\dot{c}_{ij}(t)&=\varsigma_{ij}a_{ij}(\delta_i(t)-\delta_j(t))^T\mathscr{H}\left(\delta_i(t)-\delta_j(t)\right),\\
\dot{c}_{i0}(t)&=\varsigma_{i0}q_i\delta_i^T(t)\mathscr{H}\delta_i(t).
\end{aligned}\tag{12}$$

Then, in view of the condition (10), we have $\tilde{B}B=0$ and $\bar{B}B=I_q$. Thus, it is not difficult to obtain that $\tilde{B}Ah_i(t)-$

$\tilde{B}\dot{h}_i(t) + \tilde{B}Bv_i(t) = 0$. and $\bar{B}Ah_i(t) - \bar{B}\dot{h}_i(t) + \bar{B}Bv_i(t) = 0$, where $v_i(t)$ is designed to be $v_i(t) = -\bar{B}(Ah_i(t) - \dot{h}_i(t))$. Recalling that $\left[\bar{B}^T, \tilde{B}^T\right]^T$ is nonsingular matrix, and then it is not difficult to obtain $Ah_i(t) - \dot{h}_i(t) + Bv_i(t) = 0$.

On the basis of this transformation, we obtain

$$\begin{aligned}\dot{\delta}_i(t) =& A\delta_i(t) + BK\sum_{j=1}^{N}\omega_{ij}c_{ij}(t)\left(\delta_i(t) - \delta_j(t)\right) \\ &+ BK\omega_{i0}c_{i0}(t)\delta_i(t) - \vartheta TCe_i(t) - \vartheta TCe_0(t).\end{aligned} \tag{13}$$

Accordingly, consider the Lyapunov function candidate as $V(t) = V_1(t) + V_2(t)$, where

$$\begin{aligned}V_1(t) =& \sum_{i=1}^{N}\delta_i^T(t)P^{-1}\delta_i(t) + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\frac{(c_{ij}(t) - \alpha)^2}{2\varsigma_{ij}} \\ &+ \sum_{i=1}^{N}\frac{(c_{i0}(t) - \alpha)^2}{\varsigma_{i0}}, \\ V_2(t) =& \sum_{i=1}^{N}e_i^T(t)Se_i(t) + e_0^T(t)Se_0(t),\end{aligned} \tag{14}$$

where $\alpha > 0$, $\varsigma_{ij} > 0$ and $\varsigma_{i0} > 0$ are scalars, P^{-1} and S are designed to be positive definite matrices. Differentiating $V_1(t)$ along the trajectory of (14) is

$$\begin{aligned}&\dot{V}_1(t) \\ =& \sum_{i=1}^{N}\delta_i^T(t)\left(P^{-1}A + A^TP^{-1}\right)\delta_i(t) \\ &+ 2\sum_{i=1}^{N}\delta_i^T(t)P^{-1}\left(BK\sum_{j=1}^{N}\omega_{ij}c_{ij}(t)\left(\delta_i(t) - \delta_j(t)\right)\right) \\ &+ 2\sum_{i=1}^{N}\delta_i^T(t)P^{-1}BK\omega_{i0}c_{i0}(t)\delta_i(t) \\ &+ \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\frac{(c_{ij}(t) - \alpha)}{\varsigma_{ij}}\varsigma_{ij}\omega_{ij}(\delta_i(t) - \delta_j(t))^T \\ &\times \mathscr{H}\left(\delta_i(t) - \delta_j(t)\right) \\ &+ 2\sum_{i=1}^{N}\frac{(c_{i0}(t) - \alpha)}{\varsigma_{i0}}\varsigma_{i0}\omega_{i0}\delta_i^T(t)\mathscr{H}\delta_i(t) \\ &- 2\vartheta\sum_{i=1}^{N}\delta_i^T(t)P^{-1}TCe_i(t) \\ &- 2\vartheta\sum_{i=1}^{N}\delta_i^T(t)P^{-1}TCe_0(t).\end{aligned} \tag{15}$$

Because $\forall t \geq 0$, $c_{ij}(0) = c_{ji}(0)$ and $\varsigma_{ij} = \varsigma_{ji} > 0$, it holds $c_{ij}(t) = c_{ji}(t)$. One gets $\sum\limits_{i=1}^{N}\sum\limits_{j=1, j\neq i}^{N}(c_{ij}(t) - \alpha)\omega_{ij}(\delta_i(t) - \delta_j(t))^T\mathscr{H}(\delta_i(t) - \delta_j(t)) = 2\sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N}(c_{ij}(t) - \alpha)\omega_{ij}\delta_i^T(t)\mathscr{H}(\delta_i(t) - \delta_j(t))$.

Substituting $K = -B^TP^{-1}$, $\mathscr{H} = P^{-1}BB^TP^{-1}$ into (15), it holds

$$\begin{aligned}\dot{V}_1(t) =& \sum_{i=1}^{N}\delta_i^T(t)\left(P^{-1}A + A^TP^{-1}\right)\delta_i(t) \\ &- 2\alpha\sum_{i=1}^{N}\sum_{j=1}^{N}\mathscr{L}_{ij}\delta_i^T(t)P^{-1}BB^TP^{-1}\delta_j(t) \\ &- 2\vartheta\sum_{i=1}^{N}\delta_i^T(t)P^{-1}TCe_i(t) \\ &- 2\vartheta\sum_{i=1}^{N}\delta_i^T(t)P^{-1}TCe_0(t).\end{aligned} \tag{16}$$

Then, by denoting $\delta(t) = [\delta_1^T(t), \delta_2^T(t), \ldots, \delta_n^T(t)]^T$, $e(t) = [e_1^T(t), e_2^T(t), \ldots, e_n^T(t)]^T$, and $E_0(t) = \mathbf{1}_N \otimes e_0(t)$, we can rewrite (16) as

$$\begin{aligned}\dot{V}_1(t) =& \delta^T(t)\left[I_N \otimes \left(P^{-1}A + A^TP^{-1}\right)\right]\delta(t) \\ &- 2\alpha\delta^T(t)\left(\mathscr{L} \otimes P^{-1}BB^TP^{-1}\right)\delta(t) \\ &- 2\vartheta\delta^T(t)\left(I_N \otimes P^{-1}TC\right)e(t) \\ &- 2\vartheta\delta^T(t)\left(I_N \otimes P^{-1}TC\right)E_0(t).\end{aligned} \tag{17}$$

In view of Lemma 1, we can obtain that

$$\begin{aligned}&- 2\delta^T(t)\left(I_N \otimes P^{-1}TC\right)e(t) \\ \leq& \frac{1}{\Delta_1}e^T(t)\left(I_N \otimes C^TC\right)e(t) \\ &+ \Delta_1\delta^T(t)\left(I_N \otimes P^{-1}TT^TP^{-1}\right)\delta(t), \\ &- 2\delta^T(t)\left(I_N \otimes P^{-1}TC\right)E_0(t) \\ \leq& \frac{N}{\Delta_2}e_0^T(t)C^TCe_0(t) \\ &+ \Delta_2\delta^T(t)\left(I_N \otimes P^{-1}TT^TP^{-1}\right)\delta(t),\end{aligned} \tag{18}$$

where Δ_1, Δ_2 are suitable positive constants. Then, choosing $T = S^{-1}C^T$, and substituting (18) into (17), we get

$$\begin{aligned}\dot{V}_1(t) \leq& \delta^T(t)\Big[I_N \otimes \left(P^{-1}A + A^TP^{-1}\right) \\ &- \mathscr{L} \otimes 2\alpha P^{-1}BB^TP^{-1}\Big]\delta(t) \\ &+ \delta^T(t)\Big(I_N \otimes (\vartheta\Delta_2 + \vartheta\Delta_1) \\ &\times P^{-1}S^{-1}CC^TS^{-1}P^{-1}\Big)\delta(t) \\ &+ \vartheta\frac{1}{\Delta_1}e^T(t)\left(I_N \otimes C^TC\right)e(t) \\ &+ \vartheta\frac{N}{\Delta_2}e_0^T(t)C^TCe_0(t).\end{aligned} \tag{19}$$

Choosing an appropriate matrix U to ensure $U^T\mathscr{L}U = \Lambda$, where the diagonal of $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_N)$ represents as the eigenvalues of the matrix $\mathscr{L}$, and it follows from Lemma 2 that the eigenvalues satisfy $0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$. Meanwhile, U is an unitary matrix with

$U^T = U^{-1}$. Then we get $\mathscr{L} = \left(U^T\right)^{-1}\Lambda U^{-1} = U\Lambda U^T$. Defining $\bar{\delta}^T(t) = \delta^T(t)(U\otimes I_N)$, $\bar{e}^T(t) = e^T(t)(U\otimes I_N)$, we have

$$\begin{aligned}
\dot{V}_1(t) \leq& \bar{\delta}^T(t)\Big[I_N\otimes\left(P^{-1}A + A^TP^{-1}\right) \\
& - \Lambda\otimes 2\alpha P^{-1}BB^TP^{-1}\Big]\bar{\delta}(t) \\
& + \bar{\delta}^T(t)\Big(I_N\otimes(\vartheta\Delta_1+\vartheta\Delta_2) \\
& \times P^{-1}S^{-1}CC^TS^{-1}P^{-1}\Big)\bar{\delta}(t) \\
& + \vartheta\frac{1}{\Delta_1}\bar{e}^T(t)\left(I_N\otimes C^TC\right)\bar{e}(t) \\
& + \vartheta\frac{N}{\Delta_2}e_0^T(t)C^TCe_0(t) \\
\leq& \bar{\delta}^T(t)\Big[I_N\otimes\Big(P^{-1}A + A^TP^{-1} \\
& - 2\alpha\lambda_1P^{-1}BB^TP^{-1}\Big)\Big]\bar{\delta}(t) \\
& + \bar{\delta}^T(t)\Big[I_N\otimes(\vartheta\Delta_2+\vartheta\Delta_1) \\
& \times P^{-1}S^{-1}CC^TS^{-1}P^{-1}\Big]\bar{\delta}(t) \\
& + \vartheta\frac{1}{\Delta_1}\bar{e}^T(t)\left(I_N\otimes C^TC\right)\bar{e}(t) \\
& + \vartheta\frac{N}{\Delta_2}e_0^T(t)C^TCe_0(t).
\end{aligned} \tag{20}$$

Predefine $\tilde{\psi}_i(t) = \psi(x_i(t),t) - \psi(\hat{x}_i(t),t)$, $\tilde{\psi}_0(t) = \psi(x_0(t),t) - \psi(\hat{x}_0(t),t)$. Letting $\bar{\psi} = \left[\bar{\psi}_1^T(t),\ldots\bar{\psi}_N^T(t)\right]^T$, where $\bar{\psi}_i^T(t) = \tilde{\psi}_i^T(t)U$. The derivative of $V_2(t)$ in (14) along (3) is

$$\begin{aligned}
\dot{V}_2(t) =& \bar{e}^T(t)\left[I_N\otimes\left(SA + A^TS - 2\vartheta C^TC\right)\right]\bar{e}(t) \\
& e_0{}^T(t)\left(SA + A^TS - 2\vartheta C^TC\right)e_0(t) \\
& + \bar{e}^T(t)\left(I_N\otimes SBM\right)\bar{\psi} + e_0{}^T(t)SBM\tilde{\psi}_0(t) \\
& + \bar{\psi}^T\left(I_N\otimes M^TB^TS\right)\bar{e}(t) \\
& + \tilde{\psi}_0^T(t)M^TB^TSe_0(t).
\end{aligned} \tag{21}$$

Combining (20) and (21), it gets

$$\begin{aligned}
&\dot{V}(t) \\
&\leq \bar{\delta}^T(t)\left[I_N\otimes\left(P^{-1}A + A^TP^{-1} - 2\alpha\lambda_1P^{-1}BB^TP^{-1}\right.\right. \\
&\quad \left.\left. + (\vartheta\Delta_2+\vartheta\Delta_1)P^{-1}S^{-1}CC^TS^{-1}P^{-1}\right)\right]\bar{\delta}(t) \\
&\quad + \bar{e}^T(t)\left[I_N\otimes\left(SA + A^TS - \vartheta(2-\frac{1}{\Delta_1})C^TC\right)\right]\bar{e}(t) \\
&\quad + e_0{}^T(t)\left(SA + A^TS - \vartheta(2-\frac{N}{\Delta_2})C^TC\right)e_0(t) \\
&\quad + \bar{e}^T(t)(I_N\otimes SBM)\bar{\psi}(t) + e_0{}^T(t)SBM\tilde{\psi}_0(t) \\
&\quad + \bar{\psi}^T(t)\left(I_N\otimes M^TB^TS\right)\bar{e}(t) + \tilde{\psi}_0^T(t)M^TB^TSe_0(t).
\end{aligned} \tag{22}$$

Define $\bar{E}^T(t) = [e_0{}^T(t), \bar{e}^T(t)]^T$ and $\bar{\Psi}^T = [\tilde{\psi}_0^T(t), \bar{\psi}^T(t)]^T$. Consequently, selecting appropriate scalars τ_1, τ_2, τ_3 such that $2\alpha\lambda_1 \geq \tau_1$, $\vartheta\Delta_2 + \vartheta\Delta_1 \leq \tau_2$ and $\min\left[\vartheta(2-\frac{1}{\Delta_1}), \vartheta(2-\frac{N}{\Delta_2})\right] \geq \tau_3$, respectively. Recalling that α can be any positive value, so τ_1 can be any positive value. Then, (22) can be written as

$$\begin{aligned}
\dot{V}(t) \leq& \bar{\delta}^T(t)\left[I_N\otimes\left(P^{-1}A + A^TP^{-1} - \tau_1P^{-1}BB^TP^{-1}\right.\right. \\
& \left.\left. + \tau_2P^{-1}S^{-1}CC^TS^{-1}P^{-1}\right)\right]\bar{\delta}(t) \\
& + \bar{E}^T(t)\left[I_{N+1}\otimes\left(SA + A^TS - \tau_3C^TC\right)\right]\bar{E}(t) \\
& + \bar{E}^T(t)\left(I_{N+1}\otimes SBM\right)\bar{\Psi} \\
& + \bar{\Psi}^T\left(I_{N+1}\otimes M^TB^TS\right)\bar{E}(t).
\end{aligned} \tag{23}$$

By using the condition of the one-sided Lipschitz and the quadratic inner-boundedness condition from Definition 1 and 2, we find

$$\begin{aligned}
0 \geq& 2\rho_1\left(\psi(x_k,t) - \psi(\hat{x}_i,t)\right)^T\left(x_k(t) - \hat{x}_k(t)\right) \\
& - 2\rho_1\eta\left(x_k(t) - \hat{x}_k(t)\right)^T\left(x_k(t) - \hat{x}_k(t)\right), \\
0 \geq& 2\rho_2\left(\psi(x_k(t),t) - \psi(\hat{x}_k(t),t)\right)^T \\
& \times\left(\psi(x_k(t),t) - \psi(\hat{x}_k(t),t)\right) \\
& - 2\rho_2\gamma\left(x_k(t) - \hat{x}_k(t)\right)^T\left(x_k(t) - \hat{x}_k(t)\right) \\
& - 2\rho_2\mu\left(x_k(t) - \hat{x}_k(t)\right)^T \\
& \times\left(\psi(x_k(t),t) - \psi(\hat{x}_k(t),t)\right),
\end{aligned} \tag{24}$$

where $\rho_1,\rho_2 > 0$, $k = 0,1,\ldots,N$. Subsequently, (24) can be converted to

$$\begin{aligned}
&0 \geq 2\rho_1\bar{\Psi}^T\bar{E}(t) - 2\rho_1\eta\bar{E}^T(t)\bar{E}(t), \\
&0 \geq 2\rho_2\bar{\Psi}^T\bar{\Psi} - 2\rho_2\gamma\bar{E}^T(t)\bar{E}(t) - 2\rho_2\mu\bar{E}^T(t)\bar{\Psi}.
\end{aligned} \tag{25}$$

Combining (23),(25) and rearranging, the upper bound on the derivative of $V(t)$ becomes

$$\begin{aligned}
\dot{V}(t) \leq& \bar{\delta}^T(t)\left[I_N\otimes\left(P^{-1}A + A^TP^{-1} - \tau_1P^{-1}BB^TP^{-1}\right.\right. \\
& \left.\left. + \tau_2P^{-1}S^{-1}CC^TS^{-1}P^{-1}\right)\right]\bar{\delta}(t) \\
& + \bar{E}^T(t)\left[I_{N+1}\otimes\left(SA + A^TS - \tau_3C^TC\right.\right. \\
& \left.\left. + 2\rho_1\eta I_N + 2\rho_2\gamma I_N\right)\right]\bar{E}(t) \\
& + \bar{E}^T(t)\left(I_{N+1}\otimes(SBM - \rho_1I_N + \rho_2\mu I_N)\right)\bar{\Psi} \\
& + \bar{\Psi}^T\left(I_{N+1}\otimes\left(M^TB^TS - \rho_1I_N + \rho_2\mu I_N\right)\right)\bar{E}(t) \\
& - 2\rho_2\bar{\Psi}^T\bar{\Psi}.
\end{aligned} \tag{26}$$

Taking $\Upsilon = \left[\bar{\delta}^T(t)\ \bar{E}^T(t)\ \bar{\Psi}^T\right]^T$ thus reveals that $\dot{V}(t) \leq \Upsilon^T\begin{bmatrix} I_N\otimes\Omega_1 & 0 \\ 0 & I_{N+1}\otimes\Omega_2\end{bmatrix}\Upsilon$, where

$$\begin{aligned}
\Omega_1 =& P^{-1}A + A^TP^{-1} - \tau_1P^{-1}BB^TP^{-1} \\
& + \tau_2P^{-1}S^{-1}CC^TS^{-1}P^{-1}, \\
\Omega_2 =& \begin{bmatrix} SA + A^TS - \tau_3C^TC + 2\rho_1\eta I_N + 2\rho_2\gamma I_N & SBM - \rho_1I_N + \rho_2\mu I_N \\ * & -2\rho_2I_N\end{bmatrix}.
\end{aligned} \tag{27}$$

If $\dot{V}(t) < 0$, we must have the following inequality as $\begin{bmatrix} I_N \otimes \Omega_1 & 0 \\ 0 & I_{N+1} \otimes \Omega_2 \end{bmatrix} < 0$, which holds if

$$\begin{bmatrix} \Omega_1 & 0 \\ 0 & \Omega_2 \end{bmatrix} < 0. \tag{28}$$

Then, multiplying both sides of (28) by $\mathrm{diag}[P, I_n, I_n]$ implies

$$\left[\begin{array}{cc} \Xi + \tau_2 S^{-1} C^T C S^{-1} & 0 \\ * & \Theta + (2\rho_1 \eta + 2\rho_2 \gamma) I_N \\ * & * \end{array} \right. \tag{29}$$

$$\left. \begin{array}{c} 0 \\ c \quad SBM + (\rho_2 \mu - \rho_1) I_N \\ -2\rho_2 I_N \end{array} \right] < 0, \tag{30}$$

where $\Xi = AP + PA^T - \tau_1 BB^T$, $\Theta = SA + A^T S - \tau_3 C^T C$. Then, we can have LMI in (11) from (29) by using the Schur complement lemma. Hence, it can be concluded that under the condition in (10) and (11), $\dot{V}(t)$ will be negative definite. When $\dot{V}(t) < 0$, $V(t)$ is bounded and so is in all of the c_{ij} and c_{i0}. And it indicates that c_{ij} and c_{i0} are monotonically increasing, and will converge to some finite steady value finally. Thus, multi-agent system with one leader achieves the desired time-varying formation tracking. This completes the proof. □

Remark 3: Several studies have been studied in the formation control for the nonlinear systems in [30, 31, 38, 39], which need to obtain the information from neighbors and the eigenvalues of Lapalacian matrix. In this paper, to help avoid using the information of communication topology, the novel edge-based adaptive protocol is proposed by employing relative output feedback with unknown communication topology. It is worth mentioning that since the nonlinearity of the leader in the formation system is not considered in [39], the nonlinear systems we proposed are more general with all agents contain nonlinear dynamics.

Remark 4: The equation (8) shows that a novel edge-based protocol to guarantee the time-varying formation tracking for the nonlinear systems. On contrast with existing consensus protocols in [2] and the formation protocol in [31, 38, 39], which can only deal with the nonlinear system with Lipschitz condition, the protocol we proposed can handle not only the Lipschitz system, but also the one-sided Lipschitz system. As mentioned in Remark 1, the one-sided Lipschitz system is a wide range of nonlinear plants which contains the Lipschitz system as a special case, and it is less conservativeness than the Lipschitz system proved in [41–43]. Furthermore, due to the smaller (or at most equal) one-sided Lipschitz constant than the Lipschitz constant, the constraints of the systems we proposed satisfied the one-sided Lipschitz and quadratic inner-boundedness condition are less conservativeness than the Lipschitz systems. Deserved to be mentioned, when the nonlinear function in [31, 38, 39] satisfies the one-side Lipschitz condition rather than Lipschitz condition, the system will be unstable, but the proposed method can solve this problem. Thus, in light of the property of the one-sided Lipschitz nonlinear system, we explore to design an adaptive edge-based protocol for the one-sided Lipschitz nonlinear systems to achieve formation tracking.

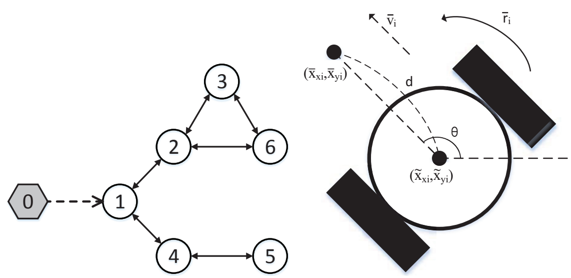
Remark 5: Based on the above-mentioned analysis for Theorem 1, we now begin to develop an algorithm to get the feasible feedback gain matrices of $K, \mathscr{H}$ and T.

Algorithm 1: The Calculative Procedure to Find the Feasible Feedback Gain Matrices for Nonlinear Constraints in Theorem 1.

Step 1: Using the conditions in (5) and (6) to choose the one-sided Lipschitz constant η and the quadratic inner-boundedness constant μ, γ, which satisfy the nonlinear function $\psi(x_i(t))$.
Step 2: Select Δ_1, Δ_2 in (18), choose ρ_1, ρ_2 from (25). (Note: Δ_1, Δ_2, ρ_1, ρ_2 are used to extend to the feasible set of the the solution, and can be any positive values.)
Step 3: Set adjustment weight ϑ of state observer from (8), then to specify the positive values τ_1, τ_2, τ_3 such that $\tau_1 > 0, \vartheta(\Delta_2 + \Delta_1) \leq \tau_2, \min\left[\vartheta(2 - \frac{1}{\Delta_1}), \vartheta(2 - \frac{N}{\Delta_2})\right] \geq \tau_3$.
Step 4: Solve the LMI in (11) to find the feasible set of solution. If there exist the feasible set of solution, jump to Step 5, otherwise return to Step 2.
Step 5: If the feasible set of the solution is received from (11), calculate the feedback matrices $K = -B^T P^{-1}$, $\mathscr{H} = P^{-1} BB^T P^{-1}$ and $T = S^{-1} C^T$, then terminate.

4. SIMULATION EXAMPLE

In this section, the proposed control strategy is validated by two examples. First, Let us consider a scenario in which seven nonholonomic mobile robots, one leader and six followers, engage in a multi-target surveillance operation, and the directed communication topology G we selected is represented in Fig. 1(a). As depicted in Fig. 1(b), all agents possess the same structure and motion model and are described by the following kinematic equations: $\dot{\tilde{x}}_{xi} = \bar{v}_i \cos\theta_i$, $\dot{\tilde{x}}_{yi} = \bar{v}_i \sin\theta_i$, $\dot{\theta}_i = \bar{r}_i$, where $(\tilde{x}_{xi}, \tilde{x}_{yi}), \bar{v}_i, \theta_i$ and $\bar{r}_i$ are the center position, linear velocities, heading angle and rotational velocity of the ith robot, respectively. Similar to the analysis of the nonholonomic mobile robot systems in [45], we use a fixed point $(\bar{x}_{xi}, \bar{x}_{yi})$ off the center of the wheel axis to denote the inertial position of the ith robot, with $\begin{bmatrix} \bar{x}_{xi} \\ \bar{x}_{yi} \end{bmatrix} = \begin{bmatrix} \tilde{x}_{xi} \\ \tilde{x}_{yi} \end{bmatrix} + d \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}$. Then,



(a) (b)

Fig. 1. (a) The communication topology of the simulation example. (b) The nonholonomic differentially driven wheeled mobile robot.

it is easy to obtain that $\begin{bmatrix}\dot{\bar{x}}_{xi}\\ \dot{\bar{x}}_{yi}\end{bmatrix} = \begin{bmatrix}\cos\theta_i & -d\sin\theta_i\\ \sin\theta_i & d\cos\theta_i\end{bmatrix}\begin{bmatrix}\bar{v}_i\\ \bar{r}_i\end{bmatrix}$, where

$$\begin{bmatrix}\bar{v}_i\\ \bar{r}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & \sin\theta_i\\ -\sin\theta_i/d & \cos\theta_i/d\end{bmatrix}\begin{bmatrix}\bar{v}_{xi}\\ \bar{v}_{yi}\end{bmatrix},\quad d\neq 0.$$

Later, we define $\dot{\bar{v}}_{xi} = u_{xi}$, $\dot{\bar{v}}_{yi} = u_{yi}$, where $\bar{v}_{xi}, \bar{v}_{yi}$ are the components of the linear velocities along the X, Y directions, respectively. Now, employing the feedback linearization method, we will have the linearized system without the nonlinear term $D\psi(x_i(t),t)$ in (3) and (4), where $x_i = [\bar{x}_{xi}^T, \bar{v}_{xi}^T, \bar{x}_{yi}^T, \bar{v}_{yi}^T]^T$, $u_i = [u_{xi}^T, u_{yi}^T]^T$ and the matrice A, B, C are given by the following:

$$A = I_2\otimes\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\ B = I_2\otimes\begin{bmatrix}0\\ 1\end{bmatrix},\ C = I_2\otimes\begin{bmatrix}1 & 1\end{bmatrix}.$$

In this example, our task is to cancel nonlinear term in the input channel. In order to verify the effectiveness of our proposed method, we now assume the nonlinear term for all of the agents is related to v_{xi} and v_{yi}, i.e., $\psi(x_i(t),t) = [0,\ -\bar{v}_{xi}(\bar{v}_{xi}^2+\bar{v}_{yi}^2),\ 0,\ -\bar{v}_{yi}(\bar{v}_{xi}^2+\bar{v}_{yi}^2)]^T$ and $D = BM = B\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$. As discussed in [21], $\psi(x_i(t),t)$ is global one-sided Lipschitz with respect to x_i as the one-sided Lipschitz constant is $\eta = 0$ under Definition 1. And the system is also the local Lipschitz in region $\mathcal{D} = \{x_i\in\mathbb{R}^2 : \|x_i\|\le\bar{r}\}$. However, compared with the literature in [32, 33, 38, 39], which they discussed is the global Lipschitz cannot be directly applied to the case of the local Lipschitz and one-sided Lipschitz system. Thus, according to [21], we now begin to select the value μ, γ as $\bar{r} = \min\left(\sqrt{-\frac{\mu}{4}}, \sqrt[4]{\gamma+\frac{\mu^2}{4}}\right)$, $\mu<0$, $\gamma+\frac{\mu^2}{4}>0$. Note that, since the globally one-sided Lipschitz nonlinearities, the choosing range of the quadratic innerboundedness constants can be capriciously large. Thus, we choose $\mu = -100$ and $\gamma = -99$. Then, we have $\bar{r} = \sqrt{\bar{v}_{xi}^2+\bar{v}_{yi}^2} = 5$, which means the value we chose can solve the nonlinearity when the total linear velocity of the agent is less than 5. Accordingly, we select the scalars as $\rho_1 = 1$, $\rho_2 = 0.05$, $\vartheta = 1$, $\tau_1 = 2$, $\tau_2 = 9$, $\tau_3 = 1$. By using the LMI Toolbox of MATLAB to obtain the solution of the LMI (11). Then, the three gain matrices are given as $K = I_2\otimes\begin{bmatrix}-0.1792 & -1.2750\end{bmatrix}$, $\mathscr{H} = I_2\otimes\begin{bmatrix}0.0321 & 0.2285\\ 0.2285 & 1.6256\end{bmatrix}$, $T = I_2\otimes\begin{bmatrix}0.1657\\ 0.1687\end{bmatrix}$. Furthermore, the time-varying formation function for ith follower to achieve expected formation tracking is described by $h_F(t) = [h_1{}^T(t), h_2{}^T(t), \ldots, h_6{}^T(t)]^T$, where $h_i(t) = [10\sin(0.1t+(i-1)\pi/3),\ \cos(0.1t+(i-1)\pi/3),\ 10\cos(0.1t+(i-1)\pi/3),\ -\sin(0.1t+(i-1)\pi/3)]^T$. And by choosing $\bar{B} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$, $\tilde{B} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$ such that $\bar{B}B = I_2$ and $\tilde{B}B = 0$, the compensational signal fuction $v_i(t)$ for ith follower can be described as $v_i(t) = [-0.1\sin(0.1t+(i-1)\pi/3),\ -0.1\cos(0.1t+(i-1)\pi/3)]^T$. Meanwhile, let all of the coupling weights as $c_{ij}(0) = c_{ji}(0) = 1$, $c_{i0}(0) = 1$. The control factor of convergence rate is $\varsigma = 0.5$. Then, we choose $\varepsilon_0 = \begin{cases}[0.01,\ 0.01]^T, & 0\le t\le 30,\\ [0,\ 0]^T, & 30<t\le 80.\end{cases}$

In Fig. 2, the trajectory curve of the leader and followers in $t = 0$s to $t = 80$s is given, and it reveals the snapshots graph of each agent in $t = 0$ s, $t = 40$ s, $t = 60$ s and $t = 80$ s in detail. We use "∇," "◯," and "□" to represent the positions of six followers in the formation, and pentagram to represent the positions of the leader. It needs to be specified that these six followers form the convex inclusion around the leader and will move over time, and the relative position between the follower and the leader is constantly changing, but the distance remains the same. Fig. 3(a) illustrates the value of coupling weights between

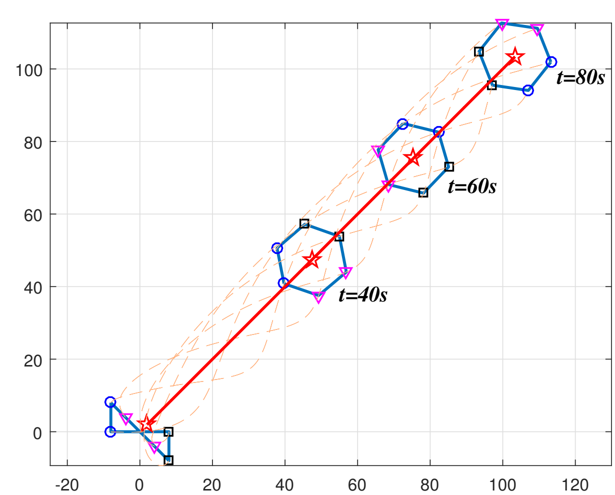
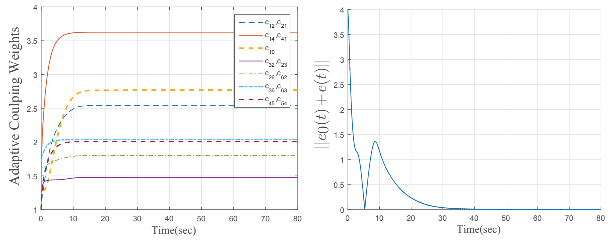
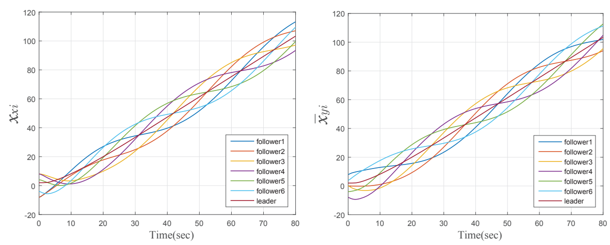


Fig. 2. The snapshots of the multi-agent formation.

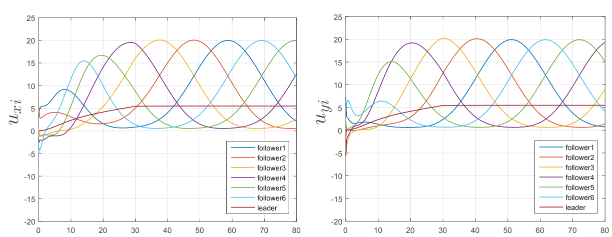


(a) (b)

Fig. 3. (a) The value of coupling weights $c_{ij}(t)$ and $c_{i0}(t)$. (b) Curve of states observation error $e(t)$+$e_0(t)$.



(a) (b)



(c) (d)

Fig. 4. Time evolvement of the positions and control inputs of agents. (a) Positions $\bar{x}_{xi}$. (b) Positions $\bar{x}_{yi}$. (c) The input signal u_{xi}. (d) The input signal u_{yi}.

each agent. It is clear to find that each coupling weight c_{ij} and c_{i0} eventually tends to a finite stable value. And from Fig. 3(b), we can attain that the estimate error of the distributed state observer $e(t)$+$e_0(t)$. As depicted in Figs. 4(a) and (b), the evolvement of the positions for all the agents are given, which will better show the detailed state of each parameter of robot motion. Figs. 4(c) and (d) show the input signals of the controller for ith agent.

In the next example, we will consider the robustness of the proposed method for the multi-agent systems in (3) and (4) with output disturbances. We consider a team of seven mobile robots, with six followers and one leader, as shown in Fig. 1(b). The output state y_i in (3) and (4) with output disturbances for all agents are described as $y_i(t) = Cx_i(t) + w_i(t)$, $i = 0$, 1, ..., 6, where w_i denotes the output disturbance of ith agent, is unknown but bounded. In this simulation exam-

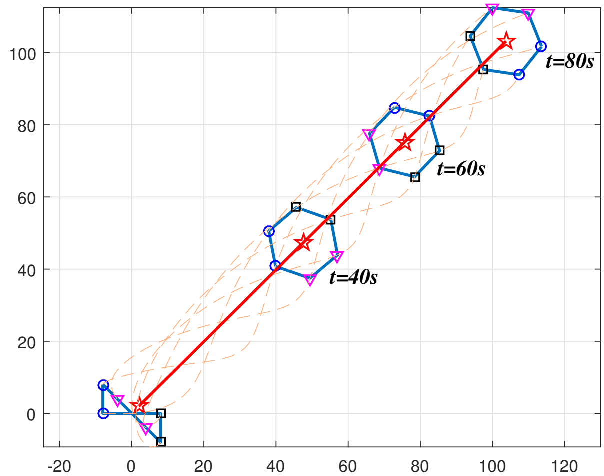
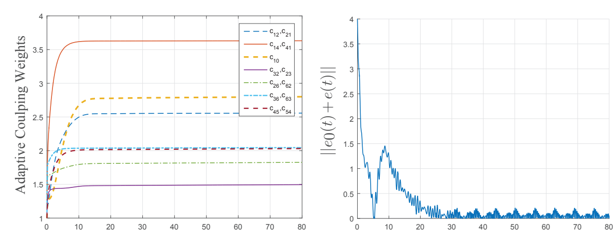


Fig. 5. The snapshots of the multi-agent formation with output disturbance.



(a) (b)

Fig. 6. (a) The value of coupling weights $c_{ij}(t)$ and $c_{i0}(t)$ with output disturbances. (b) Curve of states observation error $e(t)$+$e_0(t)$ with output disturbances.

ple, the output disturbances w_i for all agents are considered as $w_0^T = [1.2\cos(6t),\ 0.6\sin(8t)]$, $w_1^T = [0.4\cos(8t),\ 0.3\sin(4t)]$, $w_2^T = [0.5\cos(7t),\ 0.7\sin(9t)]$, $w_3^T = [0.8\cos(9t),\ 1.3\sin(6t)]$, $w_4^T = [0.7\cos(8t),\ 0.8\sin(9t)]$, $w_5^T = [0.4\cos(6t),\ 0.6\sin(8t)]$, $w_6^T = [1.1\cos(9t),\ 0.6\sin(8t)]$. Besides, other parameters are the same as the parameters in the aforementioned example where the output disturbances are not considered.

In Fig. 5, in the case of output disturbances in multi-agent systems, the trajectories of all agents at 0 to 80 seconds are displayed, and it shows the snapshots of formation in $t = 40$ s, $t = 60$ s and $t = 80$ s. We can see that all seven agents eventually reach the desired formation tracking though they are subject to different disturbances. Fig. 6(a) indicates the change of the adaptive coupling weights, and the curve of states observation error are plotted in Fig. 6(b). The evolvement of the positions of all agents are depicted in Figs. 7(a) and (b). Besides, Figs. 7(c) and (d) show the performance of the input signals u_i.

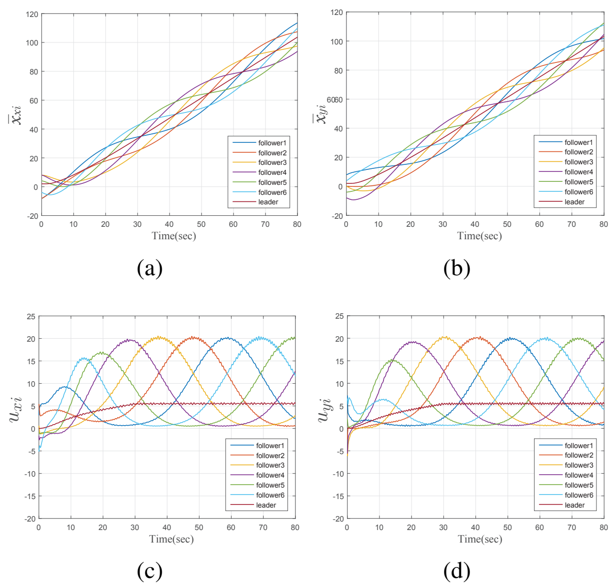


Fig. 7. Time evolvement of the positions and control inputs of agents with output disturbances. (a) Positions $\bar{x}_{xi}$. (b) Positions $\bar{x}_{yi}$. (c) The input signal u_{xi} with output disturbances. (d) The input signal u_{yi} with output disturbances.

5. CONCLUSION

We have investigated the observer-based time-varying leader-following formation problem for a class of nonlinear systems with the one-sided Lipschitz and quadratic inner-boundedness nonlinearities. The design of edge-based output feedback protocol for time-varying formation can achieve the desired formation tracking effectively with appropriate unknown communication information among agents. Besides, by employing the one-sided Lipschitz property, we proposed a less conservative condition to solve the problem of the nonlinear formation system compared with the existing article. In order to verify the accuracy of our theory, we illustrate the accuracy and validity of our theoretical analysis with examples. Based on the current study, our future works will aim at the observer-based distributed bipartite formation control with unknown input of the leader under directed topology.

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