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Controlling anonymous mobile agents to form a circle formation in a plane without collision

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Abstract— **We study the circle formation problem for a group of anonymous mobile agents *in a plane*, in which we require that all the agents converge onto a desired circle surrounding a preset target point asymptotically as well as they maintain *any* desired relative distance from their neighbors. Each agent is modeled as a kinematic point and can merely sense the relative position information of the target and its neighbors. A distributed control law is designed to solve the problem. One feature of the proposed control law is that it guarantees that no collision between agents ever takes place throughout the system's evolution. Both theoretical analysis and numerical simulations are given to show the effectiveness and performance of the proposed formation control law.**

I. Introduction

In recent years, multi-robot systems have attracted much attention due to their wide applications, such as environmental monitoring [1] and exploration [2]. When carrying out such cooperative tasks, it is usually advantageous to keep the robots moving in a desired formation with certain geometric shapes in order to complete the tasks or improve their performance [3]. One of the challenges for such pattern-forming problem for robot teams lies in the fact that the robots can use only local information to implement their distributed control strategies without centralized coordination. Forming circle formations becomes a benchmark problem, since on one hand circle formations are one of the simplest classes of formations with geometric shapes and on the other they are natural choices of the geometric shapes for a robot team to exploit an area of interest [4], [5], [6]. In particular, the focus is how to lead the agents to distributed evenly on a circle.

In theoretical computer science, the circle formation problem becomes challenging since the *semi-synchronous* model is developed and introduced [5], [7], [8], [9]. This model has successfully gained insight into the autonomous agents' key characteristics which restrict the agents' recognition, sensing and communication capabilities and these restrictions make the problem formulation more attractive. To be more specific, the agents are assumed to be oblivious (without memories about past actions and observations), anonymous (not distinguishable from one another), unable to communicate directly, and can only interact through sensing other agents' positions. Based on the semi-synchronous model, Défago and Konagaya [8] have proposed algorithms to decompose the circle formation problem into two subproblems and solve them separately. The first is to form a circle in finite time, and the second is to guide the robots to the configuration where all of them are positioned evenly on the circle. Later on, the circle-forming problem has been further studied in [10] in a complete asynchronous setting but requiring that all the agents can only move on a circle.

In the systems and control community, research efforts have also been made on the circle formation problem [3]. Distributed control laws have been designed for teams of anonymous mobile agents to realize circle formations where agents are evenly distributed on a circle [3], [11], [12], [13], [14]. However, even distribution may not be the optimal configuration when carrying out some tasks [15]. Only a few works [15], [16], [17] have considered the problem of forming arbitrary formations on a circle. Wang et al. [16] have proposed distributed control laws to guide the agents to form the desired circle formation. They also discuss the corresponding sampled-data control laws and finite-time control laws, respectively, to make the control strategies more practical. Wang et al. [17] have further considered the scenario when the agents are under locomotion constraints. Note that their works [15], [16], [17] have restricted all the agents to move in the one-dimensional space of a circle.

The goal of this paper is to design distributed control laws that can guide a group of anonymous mobile agents to form *any* given circle formation *in a plane*. We pay special attention to the requirement of *collision avoidance* between agents, which makes the strategies more attractive when they are implemented in real robots. To be more specific, we consider a system consisting of multiple mobile agents modeled as kinematic points, all of which move in a plane. The agents are oblivious, anonymous, and unable to communicate directly; they can only sense the relative position information of the target and their neighbors. Then we design a distributed control law for the group of anonymous mobile agents in a plane to solve the circle formation problem asymptotically. We present theoretical analysis to show the effectiveness of our proposed control law. We further prove that the designed control law guarantees that no collision between agents ever takes place throughout the whole system's evolution.

The main contribution of the paper is twofold. First, we study the circle formation problem not only **without** the requirement that all the desired distances between neighboring agents are equal but also **without** the constraints that all the agents move in the one-dimensional space of a circle. Second, the group of agents under the proposed control law

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can converge asymptotically to the desired circle formation. Meanwhile, it guarantees that no collision between agents ever takes place throughout the system's evolution.

The rest of the paper is organized as follows. In Section II, we formulate the circle formation problem in a plane. Then we design a distributed control law and provide rigorous analysis on its performances in Section III. Simulation results are given in Section IV. Finally, Section V concludes this paper.

II. Problem formulation

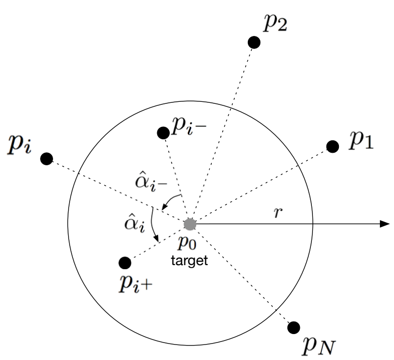

Fig. 1. Circle formation in a plane.

We consider a group of N, $N \geq 2$, agents $p_1, \cdots, p_N$ that are initially positioned in a plane (see Fig. 1). They are anonymous that they cannot distinguish one from another, and can move freely in the plane. In addition, suppose there is a preset target point p_0 in the plane. The agents' initial positions are NOT required to be distinguished from each other, whereas no agent occupies the same position with the target. For ease of expression, we label the agents based on their initial positions according to the following three rules: i) the labels are sorted firstly in ascending order in a counterclockwise manner around the target; ii) for the agents who lie on the same ray extending from the target, their labels are sorted in ascending order by the distance to the target point; and iii) for the agents who occupy the same position, their labels are chosen randomly. Then we consider the case when the agents' neighbor relationships are described by a ring $\mathbb{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{E} = \{(1, 2), (2, 3), \ldots, (N-1, N), (N, 1)\}$. In such a way, each agent only has two neighbors that are immediately in front of or behind itself. We denote the set of agent i's two neighbors by $\mathcal{N}_i = \{i^-, i^+\}$ where

$$i^+ = \begin{cases} i+1 & \text{when } i = 1, 2, \ldots, N-1 \\ 1 & \text{when } i = N \end{cases},$$

and

$$i^- = \begin{cases} N & \text{when } i = 1 \\ i-1 & \text{when } i = 2, 3, \ldots, N \end{cases}. \tag{1}$$

Let $p_i(t) = [x_i(t), y_i(t)]^T \in \mathbb{R}^2$ be the position of agent i, $1 \leq i \leq N$ at time t, and $p_0 = [x_0, y_0]^T \in \mathbb{R}^2$ is the target's position in the plane. Each agent is described as a kinematic point

$$\dot{p}_i(t) = u_i(t) \qquad i = 1, 2, \ldots, N, \tag{2}$$

where $u_i(t) \in \mathbb{R}^2$ is the control input to be designed.

Assume that each agent can only use the relative positions between the target and its two neighbors under the neighbor relationship $\mathbb{G}$. Note that by sensing and using the information about its two immediate neighbors, the agents do not need to track who its neighbors are. In fact, as to be proved later in the paper, under our control laws, the agents' spatial ordering (except the initial state) will always satisfies the first rule we just mentioned above; that is, the N agents are always arranged in such a way that their labels are sorted in ascending order in a counterclockwise manner around the target. So that the neighbor relationship $\mathbb{G}$ is guaranteed to be fixed, which greatly simplifies the analysis. In our paper, with such local information, it is expected that the agents asymptotically form a circle with a desired radius to keep the target p_0 as its centroid. The circle formation is required to rotate counterclockwise around the target, and to maintain a prescribed distribution pattern without the requirement that all the desired distances between neighboring agents are equal. This problem is called the *Circle Formation Problem in a Plane* in this paper.

To formulate the problem mathematically, the following variables are introduced. Let

$$\bar{p}_i(t) = p_i(t) - p_0 \qquad i = 1, 2, \ldots, N \tag{3}$$

be the relative position between agent i and the target measured by agent i at time t. The relative position between neighboring agents can be derived as

$$\hat{p}_i(t) = p_{i^+}(t) - p_i(t) \qquad i = 1, 2, \ldots, N \tag{4}$$

where $\hat{p}_i$ is the relative position between agents i and i^+. It follows that $\hat{p}_{i^-}$ denotes the relative positions between agents i^- and i. We further introduce the variables $\hat{\alpha}_i$ as the *angular distance* from agent i to i^+, which is formed by counterclockwise rotating the ray extending from the target to agent i until reaching agent i^+. It follows that $\hat{\alpha}_{i^-}$ is the angular distance from agent i^- to i. Let d_i denote the desired angular distance from agent i to i^+. Then the desired distribution pattern of the N agents is determined by the vector

$$d = [d_1, d_2, \ldots, d_N]^T. \tag{5}$$

We say a prescribed circle formation is *admissible* if $r > 0$, $d_i > 0$ and $\sum_{i=1}^N d_i = 2\pi$, where r is the desired radius of the circle. Moreover, we introduce some vector forms as $p = [p_1, p_2, \ldots, p_N]^T$, $\bar{p} = [\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_N]^T$, and $\hat{\alpha} = [\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_N]^T$.

We emphasize that, in our problem setting, each agent can only measure the relative position information of the target and its two neighbors. More precisely, agent i can only measure the relative positions $\bar{p}_i$, $\hat{p}_i$ and $\hat{p}_{i^-}$. Furthermore, it is easy to check that, agent i can calculate the angular distance $\hat{\alpha}_i$ just using $\bar{p}_i$ and $\hat{p}_i$ by the definition of inner product, since $\hat{\alpha}_i$ happened to be the angle between two vectors $\bar{p}_i$ and $\bar{p}_i + \hat{p}_i$. Similarly, agent i can calculate the angular distance $\hat{\alpha}_{i^-}$ just using $\bar{p}_i$ and $\hat{p}_{i^-}$. Thus, $\hat{\alpha}_i$ and $\hat{\alpha}_{i^-}$ can also be regarded as relative information obtained by agent i.

With the above preparation, we are ready to formulate the *Circle Formation Problem in a Plane* of interest. First of all, in order to represent mathematically the N anonymous agents' initial states with their labels, the following two definitions for the agents' spatial ordering are proposed.

Definition 1 (Counterclockwise order): The N agents are said to be arranged in a *counterclockwise order* if $\hat{\alpha}_i \in (0, 2\pi)$ for all $i = 1, 2, \ldots, N$ and $\sum_{i=1}^{N} \hat{\alpha}_i = 2\pi$.

Definition 2 (Almost Counterclockwise Order): The N agents are said to be arranged in an *almost counterclockwise order* if i) $\hat{\alpha}_i \in [0, 2\pi)$ for all $i = 1, 2, \ldots, N$ and $\sum_{i=1}^{N} \hat{\alpha}_i = 2\pi$; and ii) $\hat{\alpha}_i = 0$ implies $\|\bar{p}_{i^+}\| \geq \|\bar{p}_i\|$, where $\|\cdot\|$ represents the Euclidean norm.

Then one can check that, adopting the three rules when labels these anonymous agents based on their initial positions leads to the fact that the N agents are initially arranged in an almost counterclockwise order.

Now we are able to present the formal problem formulation.

Definition 3 (Circle Formation Problem in a Plane): Given an admissible circle formation characterized by r and d in a plane, design distributed control laws $u_i(t) = u_i(\bar{p}_i, \hat{p}_i, \hat{p}_{i^-}, r, d_i, d_{i^-})$, $i = 1, \ldots, N$, such that under any initial condition when the N agents arranged in an almost counterclockwise order, the solution to system (2) converges to some equilibrium point p^* satisfying

$$\|\bar{p}_i^*\| = r \qquad i = 1, 2, \ldots, N, \tag{6}$$

$$\hat{\alpha}^* = d. \tag{7}$$

Furthermore, considering the possible applications to multi-robot systems, two desired/required properties of the N-agent systems are defined as following.

Definition 4 (Order Preservation): For the N-agent system under consideration, we say the agents' spatial ordering is preserved under control laws $u_i(t)$ if the N agents initially are arranged in an almost counterclockwise order in the plane, the solution to system (2) can ensure the N agents remain in a counterclockwise order for all $t > 0$.

Definition 5 (Collision Avoidance): For the N-agent system under consideration, we say the agents' have the property of collision avoidance if the N agents initially are arranged in an almost counterclockwise order in the plane, the solution to system (2) satisfies $\|p_i - p_j\| > 0$ for any pair of i, j ($i \neq j$) for all $t > 0$.

Obviously, order preservation is sufficient (but NOT necessary) for collision avoidance. Nevertheless, we will show, by proving the spatial ordering of the agents are preserved, that our proposed control laws guarantee that no collision between agents ever takes place throughout the system's evolution.

III. Main results

In this section, we propose a control law to solve the Circle Formation Problem in a Plane with the requirement of collision avoidance and then give theoretical analysis.

A. Controller design

The proposed control law takes the following form:

$$u_i(t) = \lambda \begin{bmatrix} \gamma l_i(t) & -1 \\ 1 & \gamma l_i(t) \end{bmatrix} \bar{p}_i(t) f_i(t) \qquad i = 1, 2, \ldots, N, \tag{8}$$

where

$$f_i(t) = 1 + \frac{1}{2\pi} \left[\frac{d_{i^-}}{d_i + d_{i^-}} \hat{\alpha}_i(t) - \frac{d_i}{d_i + d_{i^-}} \hat{\alpha}_{i^-}(t) \right], \tag{9}$$

$\lambda > 0, \gamma > 0$ are constant, and

$$l_i(t) = r^2 - \|\bar{p}_i(t)\|^2 \tag{10}$$

shows the differences between current relative position and the desired one between agent i and the target.

B. Closed-loop dynamics of the N-agent system under the control law

In the previous subsection, distributed control protocols (8)(9)(10) have been designed. Before analyzing the performance of the proposed control law, we first specify the form of the N-agent system under the control law.

Substituting (8) into the dynamic equations of the system (2), we arrive at the resulting closed-loop dynamics of the N-agent system

$$\dot{p}_i(t) = \lambda \begin{bmatrix} \gamma l_i(t) & -1 \\ 1 & \gamma l_i(t) \end{bmatrix} \bar{p}_i(t) f_i(t) \qquad i = 1, 2, \ldots, N, \tag{11}$$

which can be rewritten equivalently using $\bar{p}_i$'s as

$$\dot{\bar{p}}_i(t) = \lambda \begin{bmatrix} \gamma l_i(t) & -1 \\ 1 & \gamma l_i(t) \end{bmatrix} \bar{p}_i(t) f_i(t) \qquad i = 1, 2, \ldots, N, \tag{12}$$

where $f_i(t) \in \mathbb{R}^1$ and $l_i(t) \in \mathbb{R}^1$ are given by (9) and (10), respectively.

Moreover, the second objective (7) in the circle formation problem in a plane implies that the angular distance $\hat{\alpha}_i$ between pairs of neighbors becomes the focus of attention. Thus we treat the variables $\hat{\alpha}_i$ as additional states of the N-agent system. It is known from the definition of the angular distance $\hat{\alpha}_i$ that

$$\dot{\hat{\alpha}}_i(t) = \dot{\alpha}_{i^+}(t) - \dot{\alpha}_i(t) \qquad i = 1, 2, \ldots, N, \tag{13}$$

where $\alpha_i(t)$ is denoted as the angle of the vector $\bar{p}_i(t)$ for agent i. Let $\rho_i(t) \triangleq \|\bar{p}_i\|$ be the radius of the vector $\bar{p}_i(t)$ for agent i. Then the system (12) can be represented in the polar coordinates

$$\bar{p}_i(t) = \rho_i(t) \begin{bmatrix} \cos\alpha_i(t) \\ \sin\alpha_i(t) \end{bmatrix}$$

by

$$\dot{\rho}_i(t) = \gamma\lambda\rho_i(r^2-\rho_i^2)f_i(t) \quad (14)$$
$$\dot{\alpha}_i(t) = \lambda f_i(t). \quad (15)$$

Consequently, we arrive at the dynamical equation of the additional states

$$\dot{\hat{\alpha}}_i(t) = \lambda[f_{i^+}(t) - f_i(t)] \qquad i = 1, 2, \ldots, N, \quad (16)$$

where $f_i(t)$ is given by (9). It is worth to emphasize that, $\alpha_i(t)$ here is only used for analysis purposes and is not known to the agents.

Now we have the overall closed-loop system in the polar coordinates with states $\rho_i(t)$ and $\hat{\alpha}_i(t)$ described by equations (14) and (16). We emphasize that the overall system is a time-invariant autonomous system.

C. Analysis of convergence and collision avoidance

In this section, we analyze the closed-loop system (14) and (16). Towards this end, we first list some useful matrix analysis results. For a positive integer n, we use $\mathcal{M}_n$ to denote the set of all n-by-n real matrices. We say a matrix A is *nonnegative* (resp. *positive*), denoted by $A \geq 0$ (resp. $A > 0$), if all its entries are nonnegative (resp. positive). The directed graph of a matrix $A \in \mathcal{M}_n$, denoted by $\mathbb{G}(A)$, is the directed graph with the vertex set $\{v_i\}$, $i \in \{1, 2, \ldots, n\}$, such that there is a directed edge in $\mathbb{G}(A)$ from v_j to v_i if and only if $a_{ij} \neq 0$. A directed graph is said to be *strongly connected* if there is a directed path between any pair of distinct vertices [18], [19].

Lemma 1 (Lemma 5 of [16]): It holds that

i) $L(d)$ is diagonalizable and $\lambda_i \in [0, 2]$, $i = 1, 2, \ldots, N$;

ii) 0 is a single eigenvalue;

iii) When N is even, 2 is an eigenvalue, while when N is odd, 2 is not.

Now we focus on system (14), and give the following lemma.

Lemma 2: For each agent i, under the control law (8), the solution to system (14) asymptotically converges to an equilibrium point ρ_i^* satisfying $\|\rho_i\| = r$ if $f_i(t) > 0$ and $f_i(t)$ is bounded for all $t \geq 0$.

Proof: From the equation

$$0 = \gamma\lambda\rho_i(r^2-\rho_i^2)f_i(t),$$

we can determine that the system has two equilibrium points, $\rho_i = 0$ and $\rho_i = r$.

We first check the stability of the equilibrium point $\rho_i = 0$. A Lyapunov function candidate may be taken as

$$V_1(\rho_i) = \rho_i^2.$$

One can check that $V_1(\rho_i)$ is continuously differentiable and positive definite. The derivative of $V_1(\rho_i)$ along the trajectories of the system is given by

$$\dot{V}_1(\rho_i) = 2\rho_i\dot{\rho}_i = 2\gamma\lambda(r^2-\rho_i^2)\rho_i^2 f_i(t).$$

In a certain neighborhood Ω of the origin ($\rho_i = 0$), both V_1 and $\dot{V}_1$ are positive definite and bounded, since $\gamma > 0, \lambda > 0$, $f_i(t) > 0$ and $f_i(t)$ is bounded for all $t \geq 0$. It follows the fact that the equilibrium point $\rho_i = 0$ is unstable.

In order to show the stability of the equilibrium point $\rho_i = r$, consider the Lyapunov function candidate

$$V_2(\rho_i) = (r^2-\rho_i^2)^2.$$

One can check that $V_2(\rho_i)$ is continuously differentiable, positive definite and radially unbounded. The derivative of $V_2(\rho_i)$ along the trajectories of the system is given by

$$\begin{aligned}\dot{V}_2(\rho_i) &= -4(r^2-\rho_i^2)\rho_i\dot{\rho}_i \\ &= -4\gamma\lambda(r^2-\rho_i^2)^2\rho_i^2 f_i(t) \leq 0.\end{aligned}$$

Since $f_i(t) > 0$ and $f_i(t)$ is bounded for all $t \geq 0$, one can further check that $\{\rho_i = 0\}\bigcup\{\rho_i = r\}$ is the largest invariant set in $\{\rho_i(t) \in \mathbb{R}^1 | \dot{V}_2 = 0\}$. From the LaSalle's theorem (Theorem 4.4 of [20]), we know that every solution starting in $\{\rho_i(t) \in \mathbb{R}^1\}$ approaches $\{\rho_i = 0\}\bigcup\{\rho_i = r\}$ as $t \to \infty$.

Furthermore, since the equilibrium point $\rho_i = 0$ is unstable, $\rho_i = r$ is asymptotically stable. So under the control law (8), for each agent i, the every solution starting in $\{\mathbb{R}^1 - 0\}$ approaches the equilibrium point $\{\rho_i = r\}$ as $t \to \infty$. ■

In view of Lemma 2, we have proven the first objective (6) in the problem that each agent i converge to a circle around the target with the desired radius r when $f_i(t) > 0$ and $f_i(t)$ is bounded for all $i = 1, 2, \ldots, N$ and all $t \geq 0$.

We go ahead analyzing the complete circle formation problem in a plane with two objectives (6), (7) for the N-agent system. For this purpose, we have to take into account the N agents' neighbor relationships and mainly focus on the introduced additional states $\hat{\alpha}_i(t)$.

The dynamical equation of $\hat{\alpha}_i(t)$ (16) can be summarized into a compact form

$$\dot{\hat{\alpha}}(t) = -\frac{\lambda}{2\pi}L(d)\hat{\alpha}(t) \quad (17)$$

where $L(d)$ is given by (18).

Now we prove the main result in this paper.

Theorem 1: Given an admissible circle formation characterized by r and d in a plane, the Circle Formation Problem in a Plane is solved with order preservation under the proposed control law (8).

Proof: We first prove the second objective (7) in the problem using the similar idea in Theorem 1 of [16]. Let $D = \text{diag}\{d_1, d_2, \ldots, d_N\}$, and then $D^{-1}L(d)D = L^T(d)$. Let $\delta = D^{-1}\hat{\alpha}$, and then we have

$$\dot{\delta}(t) = -\frac{\lambda}{2\pi}L^T(d)\delta(t).$$

Since $L^T(d)$ is the Laplacian matrix of $\mathbb{G}(L^T(d))$ which is strongly connected, we have

$$\lim_{t\to\infty}\delta(t) = c\mathbf{1}_N,$$

where c is a constant. It follows then from the definition of δ that

$$\lim_{t\to\infty}\hat{\alpha}(t) = cd.$$

$$
L(d)=\begin{bmatrix}
\frac{d_2}{d_2+d_1}+\frac{d_N}{d_1+d_N} & -\frac{d_1}{d_2+d_1} & 0 & \cdots & 0 & -\frac{d_1}{d_1+d_N} \\
-\frac{d_2}{d_2+d_1} & \frac{d_3}{d_3+d_2}+\frac{d_1}{d_2+d_1} & -\frac{d_2}{d_3+d_2} & \cdots & 0 & 0 \\
0 & -\frac{d_3}{d_3+d_2} & \frac{d_4}{d_4+d_3}+\frac{d_2}{d_3+d_2} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \frac{d_N}{d_N+d_{N-1}}+\frac{d_{N-2}}{d_{N-1}+d_{N-2}} & -\frac{d_{N-1}}{d_N+d_{N-1}} \\
-\frac{d_N}{d_1+d_N} & 0 & 0 & \cdots & -\frac{d_N}{d_N+d_{N-1}} & \frac{d_1}{d_1+d_N}+\frac{d_{N-1}}{d_N+d_{N-1}}
\end{bmatrix}. \tag{18}
$$

Furthermore, the solution to system (17) is

$$\hat{\alpha}(t)=e^{-\frac{\lambda}{2\pi}L(d)t}\hat{\alpha}(0), \qquad t\geq 0.$$

Since $\lambda>0$ and from Lemma 1 and Theorem 1 of [16], one can know that $e^{-\frac{\lambda}{2\pi}L(d)t}$ is positive for all $t\geq 0$. The initial condition when the N agents arranged in an almost counterclockwise order (Definition 2) ensures that $\hat{\alpha}(0)\geq 0, \hat{\alpha}(0)\neq 0$ and $\sum_{i=1}^{N}\hat{\alpha}_i(0)=2\pi$. So under the initial condition, any solution to system (17) satisfies $\hat{\alpha}(t)>0$ and $\sum_{i=1}^{N}\hat{\alpha}_i(t)=2\pi$ for all $t\geq 0$. Thus the N agents are arranged in a counterclockwise order (Definition 1) for all $t>0$.

Then we recall the fact we have got that

$$\lim_{t\to\infty}\hat{\alpha}(t)=cd.$$

where c is a constant. Noticing $\sum_{i=1}^{N}\hat{\alpha}_i(t)=2\pi$ for all t and $\sum_{i=1}^{N}d_i=2\pi$, it must be true that $c=1$ and thus

$$\lim_{t\to\infty}\hat{\alpha}(t)=d.$$

So we have proven the second objective in the problem that the requirement of desired distribution is achieved with order preservation.

Now we prove that the designed $f_i(t)$ satisfies $f_i(t)>0$ and $f_i(t)$ is bounded for all $t\geq 0$ to ensure that the result of Lemma 2 still works here. Since the N-agents have the property of order preservation, based on Definitions 1,2,4 one can check that $0<f_i(t)<2$ for all $i=1,2,\ldots,N$ and all $t\geq 0$. Further from Lemma 2, we know that the first objective in the problem is achieved.

Summarizing the above results, we conclude that the circle formation problem in a plane is solved with order preservation. ■

It is worth to point out that, order preservation is sufficient for collision avoidance because of Definitions 4 and 5. In the next section, we present simulation results that validate our theoretical analysis in this section.

IV. Simulations

In the simulations, we consider a system consisting of five agents. The target is set at the point of $(5,5)$ in the plane without loss of generality. The radius of the desired enclosing circle is $r=1.5$. Both the initial positions of the five agents and the desired distribution pattern d are generated randomly.

We choose the two parameters of the control laws (8) as $\lambda=1$ and $\gamma=0.2$. We run the simulations and show the results in Fig. 2-5. Fig. 2 and Fig. 3 show the trajectories of five agents in the plane at $t\in[0,2]$ and $t\in[0,50]$, respectively. Fig. 4 shows the distances between the agents to the desired circle for all the five agents, and the differences between current angular distance and the prescribed distances between all pairs of neighboring agents. In Fig. 2-4, the simulation results clearly indicate that the group of agents generates asymptotically the prescribed circle formation characterized by d and r around the given target under the control law (8). In addition, we show the angular distances between all pairs of neighboring agents and the distances between any two agents of the group in Fig. 5. On can see that the multi-agent system under the control law (8) has the properties of Order Preservation and Collision Avoidance.

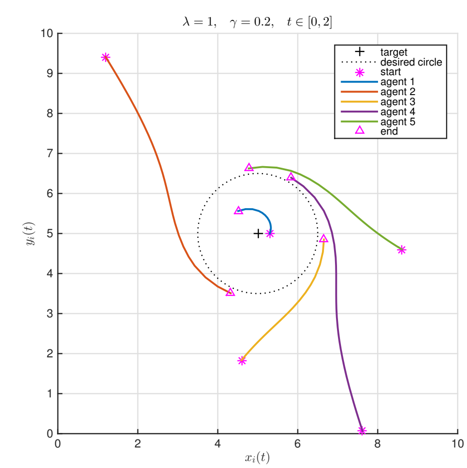

Fig. 2. Trajectories of five agents in the plane at $t\in[0,2]$ when $\lambda=1$ and $\gamma=0.2$.

V. Conclusions

In this paper, we have studies the circle formation problem for a group of anonymous mobile agents in a plane. A distributed control law is designed to guide the agents to converge to a circle surrounding a preset target as well as maintain any prescribed distribution pattern. It has been proven that our proposed control law can solve the circle formation problem in a plane asymptotically with the additional guarantee that no collision between agents ever takes place. From a theoretical point of view, the results of this paper provide an effective method to solve the circle formation problem in a plane, which complements existing results. From a practical point of view, our control law can prevent collision between agents such that it is easy to be used in real-robot systems.

We are conducting experiments using mobile robots to implement the designed control strategies. We are also interested in extending this work in three aspects: allowing for a moving target, modeling the agents as unicycles, and using the model representation in the Frenet-Serret frame[21].

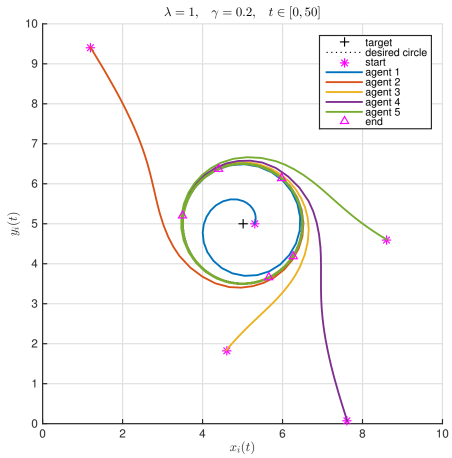

Fig. 3. Trajectories of five agents in the plane at $t \in [0, 50]$ when $\lambda = 1$ and $\gamma = 0.2$.

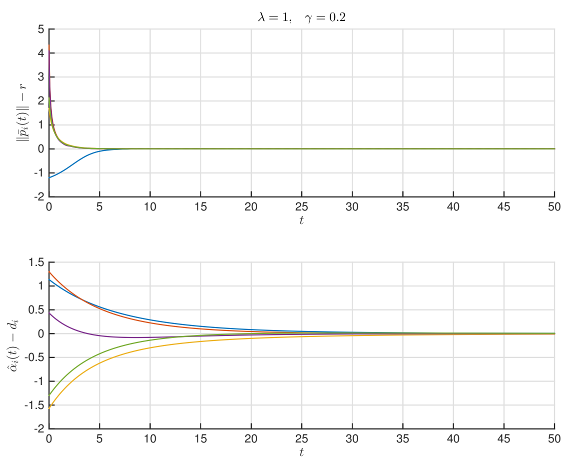

Fig. 4. The evolution of $\|\bar{p}_i(t)\| - r$, $\hat{\alpha}_i(t) - d_i$ for $i = 1, \ldots, 5$ when $\lambda = 1$ and $\gamma = 0.2$.

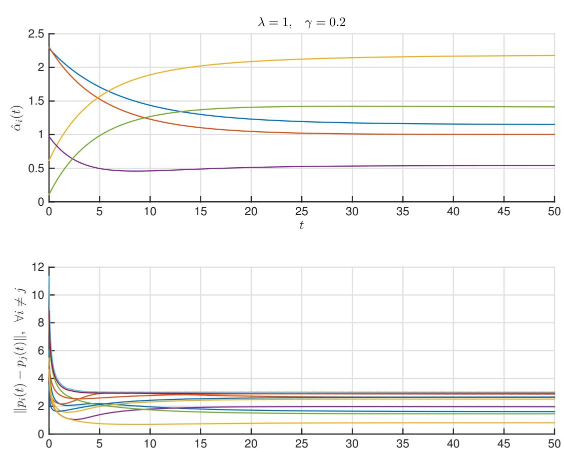

Fig. 5. The evolution of $\hat{\alpha}_i(t)$, $\|p_i(t) - p_j(t)\|, j \in \mathcal{N}_i$ for $i = 1, \ldots, 5$ when $\lambda = 1$ and $\gamma = 0.2$.

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