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Adaptive Bipartite Time-Varying Formation Control for Multi-Agent Systems on Directed Graph

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Abstract: Distributed bipartite time-varying formation control for multi-agent systems with a leader of unknown input on directed graphs is investigated. An adaptive non-smooth protocol proposed in this paper only utilizes the local output feedback information among neighboring agents and thus can avoid using eigenvalue information of the Laplacian matrix of the graph. It is shown that if the directed interaction network containing a spanning tree with the leader as the root is structurally balanced, then the bipartite time-varying formation tracking can be achieved with a leader of bounded input via the proposed scheme. A convergence analysis of the proposed protocol for multi-agent systems is reflected by the Lyapunov method. Finally, a validly numerical simulation is illustrated to show the performance of the proposed scheme.

Key Words: time-varying formation, multi-agent system, bipartite graph, distributed control, cooperative control

1 Introduction

Over the past two decades, with the rapid development of the cooperative control for multi-agent systems (MAS), numbers of researchers have focused on the theoretical research and innovation of MAS, such as consensus control [1], formation control, containment control. Among these researches, formation control of MAS is the one of the most meaningful domain with its extensive practical application in industry and military, for instance, unmanned aerial vehicle formation, unmanned surface craft formation, etc.

It is wildly recognized that consensus theory can be well applied to solve the problems of formation control in MAS. In many practical circumstances, the formation is required to be dynamic so as to achieve obstacle avoidance or defense. Thus, the shape of the formation need to change over time in some cases. In order to realize this purpose, Dong *et al.* studied the time-varying formation for linear systems in [2]. And later, more studies were aimed at the time-varying formation system in [3, 4]. Note that these researches were discussed about the formation systems using full state information. However, in some cases, it is not easy to obtain the full state information. Thus, using observer-based protocol to handle with the time-varying formation systems without accurate state feedback is a good method.

Recently, the output feedback technology has received much attention [5, 6], and many studies have extensively discussed about the formation systems with output feedback. In [7], the distributed adaptive time-varying formation tracking using output feedback has been studied. Besides, adaptive output-feedback formation tracking for ensuring connectivity preservation and collision avoidance are proposed in [8]. And the formation control for nonlinear MAS have been concerned in [9].

It is worthy mentioned that all the aforementioned studies have discussed about the formation systems in the unsigned interaction networks, whose communication among agents are usually described as graph in which the nodes in the graph denote agents, the edges mean the cooperative relation. Nevertheless, in some special cases, the relationships among these agents can not only be the cooperative but also be competitive. Therefore, [10–12] studied the bipartite consensus for linear systems among agents in signed interaction networks. In [10, 11], it expounded that if the system applies the Laplacian-like control strategy under the structurally balanced communication topology, the agents will achieve bipartite consensus, which implies that the states of all agents will eventually converge to two kinds of states with equal magnitude but opposite sign. Compared with the existing classical consensus theory, in which the agents will achieve the same states, the MAS in [14] employing the bipartite consensus strategy contains two different clusters of agents, which are either cooperative or competitive. In addition, [15] studies leader-follower bipartite consensus of MAS under a signed directed graph.

In view of these bipartite consensus studies above, to the best of our knowledge, only few researchers have considered to utilize this bipartite consensus strategy to formation control. [16] studies formation control scheme of multi-robot systems under communication delays. Moreover, bipartite antagonistic time-varying formation tracking in undirect communication topology have been discussed in [17]. These works in [16, 17] are required to obtain the information of communication topology in terms of the nonzero eigenvalues of Laplacian matrix, which is less conservative to get the feasible solution in practical application. Thus, the problems of how to avoid to use the global information and how to achieve desired formation by employing output feedback are leaving a wild field.

In this brief, inspired by the works of [18], we address the distributed bipartite time-varying formation for linear systems via adaptive output feedback law in signed digraph. The main contributions are twofold. (1) The problem of bipartite time-varying formation (BTVF) for MAS has been considered by utilizing output feedback among agents with the leader of unknown input. Different from previous work in [16], which considers neither the systems with leader or

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using output feedback method to solve the problem of the formation. Moreover, in this literature, we utilize a novel observer-based non-smooth protocol to achieve desired BTVF with unknown input of the leader such that can be better applied to practice. (2) The adaptive control protocol is used for MAS under directed communication topology without the eigenvalue information of the interaction networks. Unlike the existing study in [17], the proposed control strategy can be full-distributed and less conservative for the formation systems in directed topology.

The reminder of this work proceeds as follows. Section 2 expressed some basic notations, definitions and formulation, and the problem statement. In Section 3, the conditions and proof process for MAS to reach the desired BTVF is proposed. Then, a valid numerical simulation to prove our theoretical is given in Section 4. Finally, Section 5 concludes the main works of this paper and future research directions.

In order to simplify the notation, we define I_N as an identify matrix with dimension N, and 1_N denotes $[1, 1, \dots, 1]^T$ with dimension N. This form $\|\cdot\|$ represents the 2-norm of a vector. $\otimes$ means the Kronecker product. Besides, $A > 0, A < 0$ denote the matrix A is positive definite, negative definite, respectively.

2 Preliminaries and Problem Description

2.1 Preliminaries on Graph Theory

Define $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ as a weighted *directed signed digraph* in the communication network, with a set of nodes $\mathscr{V} = \{v_1, v_2, ..., v_N\}$, a set of edges $\mathscr{E} \subseteq \{(v_i, v_j) : v_i, v_j \subseteq \mathscr{V}\}$ and $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ as the adjacency matrix. Note that, in a *signed digraph* $\mathscr{G}$, an edge (v_i, v_j) is expressed as $v_{ij} = (v_i, v_j), i \neq j$, and it implies there exists a direct edge from j-th agent to i-th agent, and j-th agent can send the information to i-th agent. The signed weight $a_{ij}(\neq 0)$ can be either negative or positive with $i \neq j$. And when $i = j$, $a_{ij} = 0$. However, when in unsigned digraph, the weight a_{ij} can only be nonnegative. Furthermore, the digraph is said to contain a *directed spanning tree* in the communication network, if there exist at least one node (named root) has a path to all the agents in the digraph $\mathscr{G}$. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$associated with the signed digraph $\mathscr{G}$ is structured as following:

$$l_{ij} = \begin{cases} \sum_{k=1}^{N} ||a_{ik}|| & i = j \\ -a_{ij} & i \neq j \end{cases}. \tag{1}$$

Definition 1 *A signed digraph $\mathscr{G}$ is structurally balanced if there exists a node set $\mathscr{V}$ with two subsets $\mathscr{V}_1$ and $\mathscr{V}_2$, satisfying the following two conditions:*
(1) $\mathscr{V}_1 \cup \mathscr{V}_2 = \mathscr{V}$ and $\mathscr{V}_1 \cap \mathscr{V}_2 = \emptyset$.
(2) $\begin{cases} a_{ij} \geq 0, \ \forall i, j \in \mathscr{V}_l \ (l = 1, 2) \\ a_{ij} \leq 0, \ \forall i \in \mathscr{V}_l, j \in \mathscr{V}_k, l \neq k \ (l, k = 1, 2) \end{cases}.$

To employ the *gauge transformation* [11, 12], we now introduce a matrix $D = diag\{d_1, d_2, ..., d_n\} \in \mathbb{R}^{N\times N}$, $d_i \in \{1, -1\}$. More specifically, if there exists a node set $\mathscr{V}$ containing two subsets $\mathscr{V}_1 = \{i|d_i > 0\}, \mathscr{V}_2 = \{i|d_i < 0\}$, then there exists a matrix $D = diag\{d_1, d_2, ..., d_n\}$, where $d_i = 1$ when $i \in \mathscr{V}_1$, and $d_i = -1$ when $i \in \mathscr{V}_2$. Then, we give the equivalent algebraic condition below.

Lemma 1 *[12] A signed digraph $\mathscr{G}$ is said to be structurally balanced if and only if there exists a matrix D such that the entries of off-diagonal in DLD is nonpositive and the entries of $D\mathscr{A}D$ is nonnegative.*

2.2 Problem Formulation

Consider the MAS formed by $N+1$ agents with one leader and N followers, which have the following dynamics:

$$\begin{array}{l} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t) \end{array}, i = 0, 1, 2, ..., N, \tag{2}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times q}$, $C \in \mathbb{R}^{p\times n}$. And $x_i(t) \in \mathbb{R}^n$, $y_i(t) \in \mathbb{R}^p$, and $u_i(t) \in \mathbb{R}^q$ represent state vector, measured output vector and input vector of i-th agent, respectively. Assume that $rank(B) = q$, and $q \leq n$. The agent labeled 0 is called leader. The rest of the agents are named as followers and we define the set of them as $F = \{1, 2, ..., N\}$.

Assumption 1 *There exists a signed digraph $\mathscr{G}$ containing a* directed spanning tree *with at least one leader as the root, and $\mathscr{G}$ is structurally balanced and connected.*

Without lost of generality, we define both the leader and it's neighbors are in the same subset $\mathscr{V}_1$, which implies $a_{i0} > 0$, $d_i = 1$ if the leader is connected with i-th agent ($i \in F$). After that, the Laplacian matrix L corresponding to the signed digraph $\mathscr{G}$ can be expressed as $L = \begin{bmatrix} 0 & \mathbf{0}_{1\times N} \\ \mathscr{H} & \mathscr{L} \end{bmatrix} \in \mathbb{R}^{(N+1)\times(N+1)}$, where $\mathscr{L} \in \mathbb{R}^{N\times N}$, $\mathscr{H} \in \mathbb{R}^{N\times 1}$. Then, under Lemma 1 and Assumption 1, there exists a matrix D such that $\hat{\mathscr{L}} = D\mathscr{L}D > 0$ (see more details in [11]).

For the BTVF including $N + 1$ agents, the one leader is the reference for followers, and N followers need to take the position of the leader as the reference point and change their position relative to the leader over time. Therefore, the BTVF is specified by a set of compensation vectors $h(t) = [h_1^T(t), h_2^T(t), \dots h_N^T(t)]^T \in \mathbb{R}^{Nn}$ with $h_i(t) \in \mathbb{R}^n$. $h_i(t)$ is piecewise continuous differentiable function, which can be denoted as the dynamic distance between the position of the leader and the desired position of i-th follower.

Definition 2 *The BTVF is said to be achieved for the multi-agent system in (2) if all the states of followers are bounded, satisfying the following equations:*

$$\begin{cases} \lim\limits_{t\to\infty} \|x_i(t) - h_i(t) - x_0(t)\| = 0, \ \forall i \in \mathscr{V}_1 \\ \lim\limits_{t\to\infty} \|x_i(t) - h_i(t) + x_0(t)\| = 0, \ \forall i \in \mathscr{V}_2 \end{cases} \tag{3}$$

Remark 1 *Based on the above-mentioned definition of d_i, for simplify, we can obtain the equivalent expression of (3) as*

$$\lim_{t\to\infty} \|x_i(t) - h_i(t) - d_i x_0(t)\| = 0. \tag{4}$$

Before proceeding the main results in the next section, some necessary assumptions and lemmas are as follows.

Assumption 2 *The pair (A,B) is controllable, and the pair (A,C) is detectable.*

Assumption 3 *The control input u_0 of the leader is unknown for all the followers in signed graph $\mathscr{G}$, satisfies $||u_0|| \leq \varrho$, where $\varrho > 0$ is a bounded constant.*

Lemma 2 *[18] If a matrix $\hat{\mathscr{L}}$ satisfies det($\hat{\mathscr{L}}$) > 0, then there exists a diagonal matrix $Z = diag\{z_1, z_2, ..., z_N\} > 0$ so that $Z\hat{\mathscr{L}} + \hat{\mathscr{L}}^T Z > 0$.*

The input matrix $B \in \mathbb{R}^{n\times q}$ satisfies $rank(B) = q \leq n$, and there exists a pseudo inverse matrix $\bar{B} \in \mathbb{R}^{q\times n}$ such that $\bar{B}B = I_q$. Meanwhile, there also exists appropriate matrix $\hat{B} \in \mathbb{R}^{(n-q)\times n}$ satisfies $\hat{B}B = 0$ and $[\bar{B}^T, \hat{B}^T]^T$ is nonsingular matrix.

In order to achieve desired BTVF under signed digraph, we consider the following non-smooth control protocol:

$$\begin{aligned} u_i =& K\left(\delta_i - m_i\right) + \eta_i - d_i\beta\phi_i(B^T P\bar{\varphi}_i) \\ \dot{\delta}_0 =& A\delta_0 + Bu_0 + F\left(C\delta_0 - y_0\right) \\ \dot{\delta}_i =& A\delta_i + BK\left(\delta_i - m_i\right) + F\left(C\delta_i - y_i + Ch_i\right) \\ &- d_i\beta B\phi_i(B^T P\bar{\varphi}_i) \\ \dot{m}_i =& Am_i + \left(c_i + g_i\right)FC\omega_i - d_i\beta B\phi_i(B^T S\bar{\omega}_i) \\ \dot{c}_i =& \bar{\omega}_i^T C^T C\bar{\omega}_i \end{aligned} \tag{5}$$

where $i \in F$, $\delta_0, \delta_i \in \mathbb{R}^n$ denote the observed state of the leader and the observed state of the i-th follower, respectively. $\bar{\omega}_i = d_i\omega_i$, $\bar{\varphi}_i = d_i\varphi_i$, $m_0 = \delta_0$, and $c_i(0) > 0$ represents the coupling weight associated with the relative i-th agent, and K, F, P, S, η_i are the control parameters that will be determined later. Besides, the parameters ω_i, φ_i are designed as the following:

$$\begin{aligned} \omega_i &= \sum\nolimits_{j=1}^{N} \|a_{ij}\|\left(m_i - \mathrm{sgn}(a_{ij})m_j\right) + a_{i0}\left(m_i - d_i m_0\right), \\ \varphi_i &= \sum\nolimits_{j=1}^{N} \|a_{ij}\|\left(\delta_i - \mathrm{sgn}(a_{ij})\delta_j\right) + a_{i0}\left(\delta_i - d_i\delta_0\right). \end{aligned} \tag{6}$$

Furthermore, the values g_i, β are constructed as $g_i = \bar{\omega}_i^T S\bar{\omega}_i$ and $\beta > 0$. $\phi_i(\cdot)$ is an nonlinear function, which is constructed below

$$\phi_i(\kappa) = \begin{cases} \frac{\kappa}{\|\kappa\|} & \text{when } \|\kappa\| \neq 0 \\ 0 & \text{when } \|\kappa\| = 0 \end{cases}, \quad \kappa \in \mathbb{R}^q. \tag{7}$$

Subsequently, the closed-loop systems dynamic in (2) under the protocol (5) can be expressed as

$$\dot{x}_i = Ax_i + BK\left(\delta_i - m_i\right) - d_i\beta B\phi_i(B^T P\bar{\varphi}_i) + B\eta_i. \tag{8}$$

For simplicity, we now let $m = [m_1^T, m_2^T, ..., m_N^T]^T$, $\delta = [\delta_1^T, \delta_2^T, ..., \delta_N^T]^T$, $\omega = [\omega_1^T, \omega_2^T, ..., \omega_N^T]^T$, $\varphi = [\varphi_1^T, \varphi_2^T, ..., \varphi_N^T]^T$, $d = [d_1^T, d_2^T, ..., d_N^T]^T$. Then, we can transform (6) into the following compact form:

$$\omega = \left(\mathscr{L} \otimes I_n\right)\left(m - d \otimes m_0\right), \varphi = \left(\mathscr{L} \otimes I_n\right)\left(\delta - d \otimes \delta_0\right). \tag{9}$$

In order to achieve the BTVF as described in Definition 2, we introduce a parameter ε_i as:

$$\begin{aligned} \varepsilon_i =& \sum\nolimits_{j=1}^{N} \|a_{ij}\|\left[\left(x_i - h_i\right) - sgn(a_{ij})(x_j - h_j)\right] \\ &+ a_{i0}\left(x_i - h_i - d_i x_0\right). \end{aligned} \tag{10}$$

Let $r_i = x_i - h_i - d_i x_0$, $\tilde{\delta}_i = \delta_i - d_i\delta_0$, $\tilde{m}_i = m_i - d_i m_0$, and then define $r = [r_1^T, r_2^T, ..., r_N^T]^T$, $\tilde{\delta} = [\tilde{\delta}_1^T, \tilde{\delta}_2^T, ..., \tilde{\delta}_N^T]^T$, $\tilde{m} = [\tilde{m}_1^T, \tilde{m}_2^T, ..., \tilde{m}_N^T]^T$ and $\varepsilon = [\varepsilon_1^T, \varepsilon_2^T, ..., \varepsilon_N^T]^T$. In view of (9) and (10), it is easy to obtain that

$$\varepsilon = \left(\mathscr{L} \otimes I_n\right)r, \ \varphi = \left(\mathscr{L} \otimes I_n\right)\tilde{\delta}, \ \omega = \left(\mathscr{L} \otimes I_n\right)\tilde{m}. \tag{11}$$

Furthermore, by substituting $c = diag\{c_1, c_2, ..., c_N\}$, $\phi(B^T S\bar{\varphi}) = [\phi_1(B^T S\bar{\varphi}_1)^T, ..., \phi_N(B^T S\bar{\varphi}_N)^T]^T$, $\phi(B^T S\bar{\omega}) = [\phi_1(B^T S\bar{\omega}_1)^T, ..., \phi_N(B^T S\bar{\omega}_N)^T]^T$, $\eta = [\eta_1^T, \eta_2^T, ..., \eta_N^T]^T$, $g = diag\{g_1, g_2, ..., g_N\}$, and $e_0 = \delta_0 - x_0$, from (5) and (8), we will get the compact form below

$$\begin{aligned} \dot{\varepsilon} =& \left(I_N \otimes A\right)\varepsilon + \left(I_N \otimes BK\right)\varphi - \left(I_N \otimes BK\right)\omega \\ &- \left(\mathscr{L}D \otimes B\right)\left(\beta\phi(B^T P\bar{\varphi}) + 1_N \otimes u_0\right) \\ &+ \left(\mathscr{L} \otimes B\right)\eta + \left(\mathscr{L} \otimes A\right)\dot{h} - \left(\mathscr{L} \otimes I_n\right)h \\ \dot{\varphi} =& \left(I_N \otimes (A + BK)\right)\varphi - \left(I_N \otimes BK\right)\omega \\ &+ \left(I_N \otimes FC\right)\left(\varphi - \varepsilon\right) \\ &- \left(\mathscr{L}D \otimes B\right)\left(\beta\phi(B^T P\bar{\varphi}) + 1_N \otimes u_0\right) \\ \dot{\omega} =& \left[I_N \otimes A + \mathscr{L}\left(c + g\right) \otimes FC\right]\omega \\ &- \left(\mathscr{L}D \otimes B\right)\left(\beta\phi(B^T S\bar{\omega}) + 1_N \otimes u_0\right) \\ &- \left(\mathscr{L}D \otimes FC\right)\left(1_N \otimes e_0\right) \end{aligned} \tag{12}$$

Let $\bar{\varepsilon} = \left(D \otimes I_n\right)\varepsilon, \bar{\omega} = \left(D \otimes I_n\right)\omega, \bar{\varphi} = \left(D \otimes I_n\right)\varphi$, $\bar{\psi} = \bar{\varphi} - \bar{\varepsilon}$, and employ the fact that $\hat{\mathscr{L}} = D\mathscr{L}D$, we have

$$\begin{aligned} \dot{\bar{\varepsilon}} =& \left(I_N \otimes A\right)\bar{\varepsilon} + \left(I_N \otimes BK\right)\bar{\varphi} - \left(I_N \otimes BK\right)\bar{\omega} \\ &+ \left(\hat{\mathscr{L}} \otimes B\right)\left(\beta\phi(B^T P\bar{\varphi}) + 1_N \otimes u_0\right) + \left(D\mathscr{L} \otimes I_n\right)\Upsilon \\ \dot{\bar{\psi}} =& \left(I_N \otimes (A + FC)\right)\bar{\psi} - \left(D\mathscr{L} \otimes I_n\right)\Upsilon \\ \dot{\bar{\varphi}} =& \left(I_N \otimes (A + BK)\right)\bar{\varphi} - \left(I_N \otimes BK\right)\bar{\omega} + \left(I_N \otimes FC\right)\bar{\psi} \\ &- \left(\hat{\mathscr{L}} \otimes B\right)\left(\beta\phi(B^T P\bar{\varphi}) + 1_N \otimes u_0\right) \\ \dot{\bar{\omega}} =& \left[I_N \otimes A + \hat{\mathscr{L}}\left(c + g\right) \otimes FC\right]\bar{\omega} \\ &- \left(\hat{\mathscr{L}} \otimes B\right)\left(\beta\phi(B^T S\bar{\omega}) + 1_N \otimes u_0\right) \\ &- \left(\hat{\mathscr{L}} \otimes FC\right)\left(1_N \otimes e_0\right) \end{aligned} \tag{13}$$

where $\Upsilon = \left(I_N \otimes B\right)\eta + \left(I_N \otimes A\right)h - \left(I_N \otimes I_n\right)\dot{h}$.

3 MAIN RESULTS

Theorem 1 *Suppose that Assumptions 1-3 hold and consider the MAS in (2) contains $N + 1$ agents with one leader and N followers. Then, the desired BTVF is achieved under the distributed protocol (5), if the parameters satisfy $K = -B^T P$, $F = -S^{-1}C^T$, $\beta \geq \varrho$, $\eta_i = -\bar{B}(Ah_i(t) - \dot{h}_i(t))$, and the following conditions are satisfied:*

$$\hat{B}Ah_i - \hat{B}\dot{h}_i = 0, \tag{14}$$

$$SA + A^T S - 2C^T C < 0, \tag{15}$$

$$AP^{-1} + P^{-1}A^T - 2BB^T < 0, \tag{16}$$

where P, S are designed to be positive definite matrices.

Proof : We consider the following Lyapunov function candidate:

$$\begin{aligned} V =& \mu V_1 + V_2 \\ V_1 =& \frac{1}{2}\sum_{i=1}^{N} z_i(2c_i + g_i)\bar{\omega}_i^T S\bar{\omega}_i + \frac{1}{2}\sum_{i=1}^{N} z_i(c_i - \alpha)^2 + \rho_1 {e_0}^T S e_0 \\ V_2 =& \bar{\omega}^T\left(I_N \otimes P\right)\bar{\omega} + \rho_2\bar{\psi}^T\left(I_N \otimes S\right)\bar{\psi} \end{aligned} \tag{17}$$

where α can be any positive value, and ρ_1, ρ_2, μ are positive values that will be considered later. The derivative of V_1 along the trajectory in (17) is that

$$\begin{aligned}\dot{V}_1 =& \sum_{i=1}^{N} z_i(2c_i + g_i)\bar{\omega}_i^T S\dot{\bar{\omega}}_i + \sum_{i=1}^{N} z_i(\dot{c}_i + \bar{\omega}_i^T S\dot{\bar{\omega}}_i)\bar{\omega}_i^T S\bar{\omega}_i \\ &+ \sum_{i=1}^{N} z_i\left(c_i - \alpha\right)\dot{c}_i + 2\rho_1 {e_0}^T S\dot{e}_0 \\ =& 2\sum_{i=1}^{N} z_i(c_i + g_i)\bar{\omega}_i^T S\dot{\bar{\omega}}_i + \sum_{i=1}^{N} z_i g_i \dot{c}_i \\ &+ \sum_{i=1}^{N} z_i\left(c_i - \alpha\right)\bar{\omega}_i^T C^T C\bar{\omega}_i \\ &+ \rho_1 {e_0}^T \left[S\left(A + FC\right) + \left(A + FC\right)^T S\right] e_0.\end{aligned} \tag{18}$$

For the sake of simplicity, one has the following compact form:

$$\begin{aligned}\dot{V}_1 =& \bar{\omega}^T \left[Z\left(c+g\right) \otimes \left(SA + A^T S\right)\right]\bar{\omega} \\ &- \bar{\omega}^T \left[\left(c+g\right)\left(Z\hat{\mathscr{L}} + \hat{\mathscr{L}}Z\right)\left(c+g\right) \otimes C^T C\right]\bar{\omega} \\ &- 2\bar{\omega}^T \left[\left(c+g\right)Z\mathscr{L} \otimes SB\right]\left(\beta\phi\left(B^T S\bar{\omega}\right) + 1_N \otimes u_0\right) \\ &- \bar{\omega}^T \left[Z\left(c+g-\alpha 1_N\right) \otimes C^T C\right]\bar{\omega} \\ &- 2\bar{\omega}^T \left[\left(c+g\right)Z\hat{\mathscr{L}} \otimes C^T C\right]\left(1_N \otimes e_0\right) \\ &+ \rho_1 {e_0}^T \left(SA + A^T S - 2C^T C\right) e_0.\end{aligned} \tag{19}$$

From (7), it is not difficult to get that

$$\begin{aligned}\bar{\omega}_i^T SB\phi_i\left(B^T S\bar{\omega}_i\right) &= \left\|B^T S\bar{\omega}_i\right\|, \\ \bar{\omega}_i^T SB\phi_i\left(B^T S\bar{\omega}_j\right) &\leq \left\|B^T S\bar{\omega}_i\right\|\left\|\phi\left(B^T S\bar{\omega}_j\right)\right\| \\ &\leq \left\|B^T S\bar{\omega}_i\right\|, \\ -\bar{\omega}_i^T SBu_0 &\leq \left\|B^T S\bar{\omega}_i\right\|\left\|u_0\right\| \leq \varrho\left\|B^T S\bar{\omega}_i\right\|.\end{aligned} \tag{20}$$

Then, we can obtain

$$\begin{aligned}&- 2\bar{\omega}^T \left[\left(c+g\right)Z\hat{\mathscr{L}} \otimes SB\right]\left(\beta\phi\left(B^T S\bar{\omega}\right) + 1_N \otimes u_0\right) \\ \leq& - 2\sum_{i=1}^{N}\left(c_i + g_i\right)z_i a_{i0}\bar{\omega}_i^T SB\left(\beta\phi_i\left(B^T S\bar{\omega}_i\right) + u_0\right) \\ &- 2\sum_{i=1}^{N}\left(c_i + g_i\right)z_i\left\|a_{ij}\right\|\bar{\omega}_i^T SB \\ &\quad\times\left(\beta\phi_i\left(B^T S\bar{\omega}_i\right) - \beta\phi_i\left(B^T S\bar{\omega}_j\right)\right) \\ \leq& 2\left(\varrho - \beta\right)\sum_{i=1}^{N}\left(c_i + g_i\right)z_i a_{i0}\left\|B^T S\bar{\omega}_i\right\| \leq 0.\end{aligned} \tag{21}$$

According to Lemma 2, we can construct $\bar{\mathscr{L}} = Z\hat{\mathscr{L}} + \hat{\mathscr{L}}Z$. Then, by defining $\lambda_\sigma = \lambda_{min}(\bar{\mathscr{L}})$, one gets

$$\begin{aligned}&-\bar{\omega}^T \left[\left(c+g\right)\left(Z\hat{\mathscr{L}} + \hat{\mathscr{L}}Z\right)\left(c+g\right) \otimes C^T C\right]\bar{\omega} \\ =& -\bar{\omega}^T \left[\left(c+g\right)\bar{\mathscr{L}}\left(c+g\right) \otimes C^T C\right]\bar{\omega} \\ \leq& - \lambda_\sigma\bar{\omega}^T \left[\left(c+g\right)^2 \otimes C^T C\right]\bar{\omega}.\end{aligned} \tag{22}$$

Substituting (21) and (22) into (19), one obtains

$$\begin{aligned}\dot{V}_1 \leq& \bar{\omega}^T \left[Z\left(c+g\right) \otimes \left(SA + A^T S\right)\right]\bar{\omega} \\ &- \lambda_\sigma\bar{\omega}^T \left[\left(c+g\right)^2 \otimes C^T C\right]\bar{\omega} \\ &- \bar{\omega}^T \left[Z\left(c+g-\alpha 1_N\right) \otimes C^T C\right]\bar{\omega} \\ &+ \rho_1 {e_0}^T \left(SA + A^T S - 2C^T C\right) e_0.\end{aligned} \tag{23}$$

By employing that $\Omega = -\left(SA + A^T S - 2C^T C\right) > 0$, we can convert (23) into the following

$$\begin{aligned}\dot{V}_1 \leq& \bar{\omega}^T \left[\left(c+g\right)Z \otimes \left(SA + A^T S + C^T C\right)\right]\bar{\omega} \\ &- \lambda_\sigma\bar{\omega}^T \left[\left(\left(c+g\right)^2 + \alpha Z\right) \otimes C^T C\right]\bar{\omega} \\ &- 2\bar{\omega}^T \left[\left(c+g\right)Z\hat{\mathscr{L}} \otimes C^T C\right]\left(1_N \otimes e_0\right) \\ &- \rho_1 {e_0}^T \Omega e_0.\end{aligned} \tag{24}$$

Recalling that α can be any positive value, so there exists a value α can obtain that $\alpha Z \geq 3\left(c+g\right)Z$. Later, by applying the Young's inequality, we can derive that

$$\begin{aligned}&- 2\bar{\omega}^T \left[\left(c+g\right)Z\hat{\mathscr{L}} \otimes C^T C\right]\left(1_N \otimes e_0\right) \\ \leq& \lambda_\sigma\bar{\omega}^T \left[\left(c+g\right)^2 \otimes C^T C\right]\bar{\omega} + \frac{N}{\lambda_\sigma}\left(\hat{\mathscr{L}}^T ZZ\hat{\mathscr{L}}\right){e_0}^T C^T C e_0 \\ \leq& \lambda_\sigma\bar{\omega}^T \left[\left(c+g\right)^2 \otimes C^T C\right]\bar{\omega} \\ &+ \frac{N\lambda_{\max}\left(C^T C\right)Z\hat{\mathscr{L}}\hat{\mathscr{L}}^T Z}{\lambda_\sigma\lambda_{\min}\left(\Omega\right)}{e_0}^T \Omega e_0,\end{aligned} \tag{25}$$

and $-\bar{\omega}^T\left(\alpha Z \otimes C^T C\right)\bar{\omega} \leq -\bar{\omega}^T\left[\left(c+g\right)Z \otimes 3C^T C\right]\bar{\omega}$. Furthermore, employ $\rho_1 = \frac{N\lambda_{\max}\left(C^T C\right)Z\hat{\mathscr{L}}\hat{\mathscr{L}}^T Z}{\lambda_\sigma\lambda_{\min}(\Omega)} + 1$, one gets that

$$\dot{V}_1 \leq -\bar{\omega}^T\left[\left(c+g\right)Z \otimes \Omega\right]\bar{\omega} - {e_0}^T \Omega e_0. \tag{26}$$

Then, the derivative of V_2 according to (17) can be converted to the following:

$$\begin{aligned}\dot{V}_2 =& -\bar{\varphi}^T\left(I_N \otimes \Xi\right)\bar{\varphi} - 2\bar{\varphi}^T\left(I_N \otimes PBK\right)\bar{\omega} \\ &+ 2\bar{\varphi}^T\left(I_N \otimes PFC\right)\bar{\psi} - \rho_2\bar{\psi}^T(I_N \otimes \Omega)\bar{\psi} \\ &- 2\bar{\varphi}^T\left(\hat{\mathscr{L}} \otimes PB\right)\left(\beta\phi(B^T P\bar{\varphi}) + 1_N \otimes u_0\right) \\ &- 2\rho_2\bar{\varphi}^T\left(D\mathscr{L} \otimes S\right)\Upsilon,\end{aligned} \tag{27}$$

where $\Xi = -\left(PA + A^T P - 2PBB^T P\right) > 0$.

In light of the condition in (14), one attains $\hat{B}B\eta_i + \hat{B}Ah_i - \hat{B}\dot{h}_i = 0$, where $\hat{B}B = 0$. Then, since $\bar{B}B = I_q$ and $\eta_i = -\bar{B}(Ah_i - \dot{h}_i)$, it is not difficult to find $\bar{B}B\eta_i + \bar{B}Ah_i - \bar{B}\dot{h}_i = 0$. By employing the fact that $[\bar{B}^T, \hat{B}^T]^T$ is nonsingular matrix, it implies that $B\eta_i + Ah_i - \dot{h}_i = 0$, which can be rewritten as follows

$$\left(I_N \otimes B\right)\eta + \left(I_N \otimes A\right)h - \left(I_N \otimes I_n\right)\dot{h} = \Upsilon = 0. \tag{28}$$

Furthermore, similar to the analysis of (20), one holds that

$$-2\bar{\varphi}^T\left(\hat{\mathscr{L}} \otimes PB\right)\left(\beta\phi(B^T P\bar{\varphi}) + 1_N \otimes u_0\right) \leq 0. \tag{29}$$

Combining (28) and (29), the inequality in (27) can be simplified as $\dot{V}_2 \leq -\bar{\varphi}^T\left(I_N \otimes \Xi\right)\bar{\varphi} - 2\bar{\varphi}^T\left(I_N \otimes PBK\right)\bar{\omega}$

$+\, 2\bar{\varphi}^T \left(I_N \otimes PFC\right)\bar{\psi} - \rho_2 \bar{\psi}^T \left(I_N \otimes \Omega\right)\bar{\psi}$. By using the theory of Young's inequality, we can get

$$\begin{aligned}
&-2\bar{\varphi}^T \left(I_N \otimes PBK\right)\bar{\omega} \\
\leq &\frac{1}{4}\bar{\varphi}^T \left(I_N \otimes \Xi\right)\bar{\varphi} \\
&+ \frac{4\lambda_{\max}^2 \left(PBB^T P\right)\lambda_{\max}\left((c+g)\,Z\right)}{\lambda_{\min}\left(\Xi\right)\lambda_{\min}\left(\Omega\right)}\bar{\omega}^T \left[(c+g)\,Z\otimes\Omega\right]\bar{\omega},
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
&2\bar{\varphi}^T \left(I_N \otimes PFC\right)\bar{\psi} \\
\leq &\frac{1}{4}\bar{\varphi}^T \left(I_N \otimes \Xi\right)\bar{\varphi} + \frac{4\lambda_{\max}\left(PFCC^T F^T P\right)}{\lambda_{\min}\left(\Omega\right)\lambda_{\min}\left(\Xi\right)}\bar{\psi}^T \left(I_N \otimes \Omega\right)\bar{\psi}.
\end{aligned} \tag{31}$$

Let $\rho_2 = 1 + \frac{4\lambda_{\max}\left(PFCC^T F^T P\right)}{\lambda_{\min}(\Omega)\lambda_{\min}(\Xi)}$, one gets

$$\begin{aligned}
\dot{V}_2 \leq &-\bar{\varphi}^T \left(I_N \otimes \Xi\right)\bar{\varphi} - \bar{\psi}^T \left(I_N \otimes \Omega\right)\bar{\psi} \\
&+ \frac{4\lambda_{\max}^2 \left(PBB^T P\right)\lambda_{\max}\left((c+g)\,Z\right)}{\lambda_{\min}\left(\Xi\right)\lambda_{\min}\left(\Omega\right)}\bar{\omega}^T \left(I_N \otimes \Omega\right)\bar{\omega}.
\end{aligned} \tag{32}$$

Choose $\mu = 1 + \frac{4\lambda_{\max}^2\left(PBB^T P\right)\lambda_{\max}\left((c+g)Z\right)}{\lambda_{\min}(\Xi)\lambda_{\min}(\Omega)}$, we have

$$\begin{aligned}
\dot{V} =&\mu\dot{V}_1 + \dot{V}_2 \\
\leq &-\bar{\omega}^T \left[(c+g)\,Z\otimes\Omega\right]\bar{\omega} - \mu {e_0}^T \Omega e_0 \\
&-\frac{1}{2}\bar{\varphi}^T \left(I_N \otimes \Xi\right)\bar{\varphi} - \bar{\psi}^T \left(I_N \otimes \Omega\right)\bar{\psi} \\
<&0.
\end{aligned} \tag{33}$$

Note that if $\Omega > 0$ and $\Xi > 0$, which are equivalent to (15), (16), respectively, we will have $\dot{V} < 0$. In view of Lyapunov stability theory, since $V > 0$ and $\dot{V} < 0$, we find that $\bar{\omega}, e_0, \bar{\varphi}, \bar{\psi}$ will finally converge to 0, which implies that

$$\lim_{t\to\infty}\left\|x_i(t) - h_i(t) - d_i x_0(t)\right\| = \lim_{t\to\infty}\left\|r_i(t)\right\| = 0. \tag{34}$$

Thus, under the conditions in (14)-(16), the BTVF control problem is solved. □

Remark 2 *Under the adaptive controller in (5), the BTVF tracking is accomplished by using relative output states among its neighboring agents, and the uses of leader's control input and the smallest eigenvalue of Laplacian matrix of the graph are avoided. Nevertheless, this adaptive controller is non-smooth, and in practical application, the non-smooth controller will lead to the chattering phenomenon. The boundary layer concept and modification method can be employ to reconstruct controller to a continuous controller. Future works will take efforts on this concern.*

4 Numerical Simulation

In this section, we utilize the proposed control protocol in (5) to deal with the BTVF problem of multi-target surveillance. We consider the multi-agent system (2) with one leader and six followers, whose system parameters are described

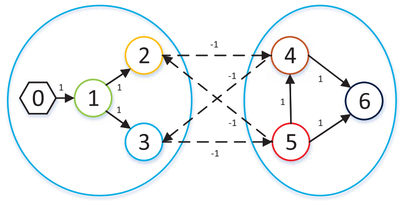

Fig. 1: The communication topology in the signed digraph.

as follows [7]:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, x_i = \begin{bmatrix} x_{xi} \\ v_{xi} \\ x_{yi} \\ v_{xi} \end{bmatrix}, u_i = \begin{bmatrix} u_{xi} \\ u_{yi} \end{bmatrix}.$$

Then, we give $\|u_0\| \leq \varrho = 0.1$, $\beta = 0.2$. And the control input of the leader is structured as

$$u_0 = \begin{cases} \begin{bmatrix} 0.05 & 0 \end{bmatrix}^T, & t \leq 30s \\ \begin{bmatrix} -0.05 & -0.06 \end{bmatrix}^T, & 30s < t \leq 60s \\ \begin{bmatrix} 0 & 0 \end{bmatrix}^T, & 60s < t \leq 80s \end{cases}.$$

The structurally balanced communication topology is displayed in Fig.1, in which the communication weight a_{ij} are shown. Besides, two different node sets $\mathscr{V}_1 = \{0,1,2,3\}$, $\mathscr{V}_2 = \{4,5,6\}$ can be obtained from Fig.1, which implies the matrix D is described as $D = diag\{1,1,1,-1,-1,-1\}$. Then, let us select the compensation vector $h_i(t)$ for i-th follower as

$$h_i(t) = \begin{cases} \begin{bmatrix} r_1\sin(\theta_1 t + (i-1)\pi/n_1) \\ r_1\theta_1\cos(\theta_1 t + (i-1)\pi/n_1) \\ r_1\cos(\theta_1 t + (i-1)\pi/n_1) \\ -r_1\theta_1\sin(\theta_1 t + (i-1)\pi/n_1) \end{bmatrix}, i \in \mathscr{V}_1 \\ \begin{bmatrix} r_2\sin(\theta_2 t + (i-1)\pi/n_2) \\ r_2\theta_2\cos(\theta_2 t + (i-1)\pi/n_2) \\ r_2\cos(\theta_2 t + (i-1)\pi/n_2) \\ -r_2\theta_2\sin(\theta_2 t + (i-1)\pi/n_2) \end{bmatrix}, i \in \mathscr{V}_2 \end{cases}$$

where we choose $r_1 = r_2 = 10, \theta_1 = 0.2, \theta_2 = 0.3$. And $n_1 = 3, n_2 = 3$ represent the number of the followers in $\mathscr{V}_1, \mathscr{V}_2$, respectively. In the cause of satisfying the equation in (14), we choose the matrix $\bar{B} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\hat{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ so that $\hat{B}B = 0$ and $\bar{B}B = I_2$. Furthermore, η_i can be chosen as

$$\eta_i = \begin{cases} \begin{bmatrix} -0.4\sin(0.2t + (i-1)\pi/3) \\ -0.4\cos(0.2t + (i-1)\pi/3) \end{bmatrix}, i \in \mathscr{V}_1 \\ \begin{bmatrix} -0.9\sin(0.3t + (i-1)\pi/3) \\ -0.9\cos(0.3t + (i-1)\pi/3) \end{bmatrix}, i \in \mathscr{V}_2 \end{cases}.$$

By solving the linear matrix inequalities in (15), (16), we have

$$S = \begin{bmatrix} 2.2324 & -0.2324 & 0 & 0 \\ -0.2324 & 2.6973 & 0 & 0 \\ 0 & 0 & 2.2324 & -0.2324 \\ 0 & 0 & -0.2324 & 2.6973 \end{bmatrix},$$

$$P = \begin{bmatrix} 0.7480 & 0.7480 & 0 & 0 \\ 0.7480 & 2.2442 & 0 & 0 \\ 0 & 0 & 0.7480 & 0.7480 \\ 0 & 0 & 0.7480 & 2.2442 \end{bmatrix}.$$

Then, based on the matrices S, P, the following control parameters are given as $F = \begin{bmatrix} -0.4909 & 0 \\ -0.4130 & 0 \\ 0 & -0.4909 \\ 0 & -0.4130 \end{bmatrix}$, $K = \begin{bmatrix} -0.7480 & -2.2442 & 0 & 0 \\ 0 & 0 & -0.7480 & -2.2442 \end{bmatrix}$.

The trajectories of all robots with time between 0 and 80 seconds are plotted in Fig.2, in which the snapshots of the BTVF in 50s and 80s are shown and two antagonistic sub-formations are formed in 50s and 80s, respectively (different colors are used to distinguish the snapshots in different times). It reveals that when the systems is stable, the centers of the sub-formations are symmetric about the coordinates origin $(0, 0)$ at each moment. Fig.3 (a) depicts the adaptive coupling weight c_i for i-th agent, which indicates that the coupling weight c_i for each agent converge to steady-state value. Besides, the curve of the observation error for all agents is shown in Fig.3 (b).

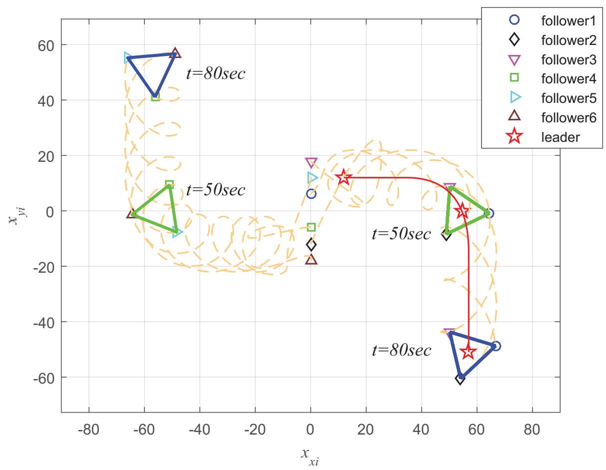

Fig. 2: The snapshots for bipartite formation.

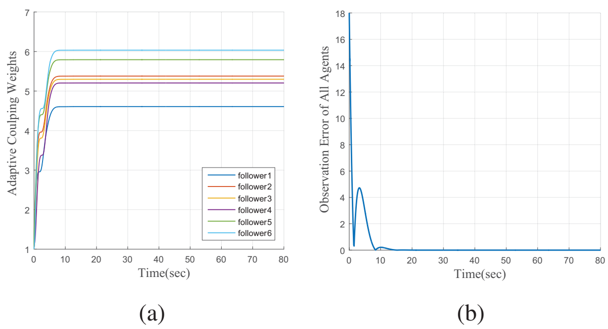

Fig. 3: (a) The adaptive coupling weight c_{ij} for i-th agent. (b) The curve of observation error of all agents.

5 Conclusion

This paper addressed the bipartite time-varying formation control for linear multi-agent systems based on the output feedback protocol under directed graphs. This novel adaptive protocol only uses relative output information among agents and does not rely on the leader's control input, so that the desired BTVF tracking can be full-distributed without any global information. Unlike previous bipartite formation works whose protocols should consider the eigenvalues of the Laplacian matrix of communication network, the protocol in this brief can avoid to use global information and thus can be less conservative to get feasible solution. Our future works will explore to the bipartite formation control with uncertain output information in directed graphs.

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