Exercise Sheet 1

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Exercise 1

(a) Probability tables

	$P(A,B,C)$
$a^0b^0c^0$	0
$a^0b^0c^1$	$\frac{1}{4}$
$a^0b^1c^0$	$\frac{1}{4}$
$a^0b^1c^1$	0
$a^1b^0c^0$	$\frac{1}{4}$
$a^1b^0c^1$	0
$a^1b^1c^0$	0
$a^1b^1c^1$	$\frac{1}{4}$

	c^0	c^1	$P(A,B)$
a^0b^0	0	$\frac{1}{4}$	$\frac{1}{4}$
a^0b^1	$\frac{1}{4}$	0	$\frac{1}{4}$
a^1b^0	$\frac{1}{4}$	0	$\frac{1}{4}$
a^1b^1	0	$\frac{1}{4}$	$\frac{1}{4}$
$P(C)$	$\frac{1}{2}$	$\frac{1}{2}$	

	b^0	b^1	$P(A,C)$
a^0c^0	0	$\frac{1}{4}$	$\frac{1}{4}$
a^0c^1	$\frac{1}{4}$	0	$\frac{1}{4}$
a^1c^0	$\frac{1}{4}$	0	$\frac{1}{4}$
a^1c^1	0	$\frac{1}{4}$	$\frac{1}{4}$
$P(B)$	$\frac{1}{2}$	$\frac{1}{2}$	

	a^0	a^1	$P(B,C)$
b^0c^0	0	$\frac{1}{4}$	$\frac{1}{4}$
b^0c^1	$\frac{1}{4}$	0	$\frac{1}{4}$
b^1c^0	$\frac{1}{4}$	0	$\frac{1}{4}$
b^1c^1	0	$\frac{1}{4}$	$\frac{1}{4}$
$P(A)$	$\frac{1}{2}$	$\frac{1}{2}$	

	$P(A,B\|C=0)$		$P(A,B\|C=1)$
a^0b^0	$\frac{1}{4}$	a^0b^0	$\frac{1}{4}$
a^0b^1	$\frac{1}{4}$	a^0b^1	$\frac{1}{4}$
a^1b^0	$\frac{1}{4}$	a^1b^0	$\frac{1}{4}$
a^1b^1	$\frac{1}{4}$	a^1b^1	$\frac{1}{4}$

(b) Analizying independencies

- $(A \perp\!\!\!\perp B)_p$
  For the independence to hold the following must be true: $P(A|B) = P(A)$.

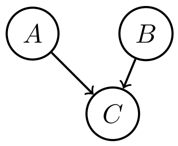
  Since $P(A) = \frac{1}{2}, P(B) = \frac{1}{2}$, and from the table above,

  $$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

  for $A, B \in \{0,1\}$.
  Therefore, random variables A, B are independent.

- $(A \perp\!\!\!\perp C)_p$
  For A, C to be independent the $P(A|C) = P(A)$.
  $$P(A|C) = \frac{P(A,C)}{P(C)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$
  Therefore, the independence holds.
- $(A \perp\!\!\!\perp B|C)_p$
  $$P(A|B,C) = \frac{P(A,B,C)}{P(B,C)}$$
  Now, i.e., for $A = 0, B = 0, C = 0$ we would get:
  $$P(A=0|B=0,C=0) = \frac{P(A=0,B=0,C=0)}{P(B=0,C=0)} = \frac{0}{P(B=0,C=0)} \neq \frac{1}{2} = P(A|C)$$
  Therefore, A is not indepenent of B given C.

(c) Joint probability distribution as Bayesian network



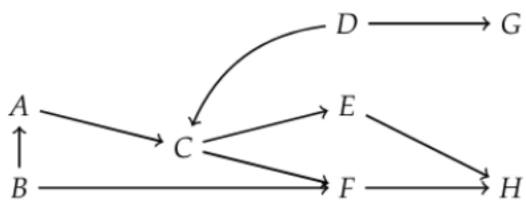
(d) Analazing independencies using d-separation

- $(A \perp\!\!\!\perp B)_G$
  A and B are d-separated (by the set $Z = \emptyset$); A and B are marginally independent.
- $(A \perp\!\!\!\perp C)_G$
  There's unblocked path (A, C) by Z (where $Z = \emptyset$), therefore A and C are d-connected. This means that A and C are marginally dependent.
- $(A \perp\!\!\!\perp B|C)_G$
  Here, C would normally be a collider on a path from A to B. Because C is a member of conditioning set it is no longer blocking the path between A and B. And therefore, A and B are d-connected and the following holds: $(A \not\!\perp\!\!\!\perp B|C)_G$

(e) Given $(X \perp\!\!\!\perp Y|Z)_p$ does not imply $(X \perp\!\!\!\perp Y|Z)_G$.

Exercise 2

(a) $X = \{A, B\}$, $Y = \{G, H\}$. Find all d-separators Z relative to (X, Y)

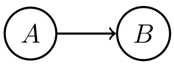


Let's consider all possible subsets of vertices $V_G \backslash \{A, B, G, H\} = \{C, D, E, F\}$ and check if they're the d-separators relative to (X, Y).

Z	Is d-separator
$\emptyset$	No, i.e $(A, C)(C, E)(E, H)$
C	No, i.e. $(A, C)(C, D)(D, G)$
D	No, i.e $(A, C)(C, E)(E, H)$
E	No, i.e $(A, C)(C, F)(F, H)$
F	No, i.e $(A, C)(C, E)(E, H)$
C, D	No, i.e $(B, F)(F, H)$
C, E	No, i.e $(B, F)(F, H)$
C, F	No, i.e. $(A, C)(C, D)(D, G)$
D, E	No, i.e $(B, F)(F, H)$
D, F	No, i.e $(A, C)(C, E)(E, H)$
E, F	No, i.e. $(A, C)(C, D)(D, G)$
C, D, E	No, i.e $(B, F)(F, H)$
C, D, F	Yes
C, E, F	No, i.e. $(A, C)(C, D)(D, G)$
D, E, F	Yes
C, D, E, F	Yes

(b) Is there for every DAG G and disjoint sets $X, Y \subseteq V$ a set $Z \subseteq V$ which d-separates X and Y?

Let's consider following DAG:



The path between A and B is just an edge between them. Regardless of Z, the A, B cannot be d-separated (since there's no node between to block the path).

(c) Minimal d-separators are $Z = \{C, D, F\}$ and $Z = \{D, E, F\}$. $Z = \{C, D, E, F\}$ is not minimal because for $W = \{E\}$, $Z \backslash W = \{C, D, F\}$ d-separates X and Y.