Exercise Sheet 2

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Exercise 2

In order to prove that the causal effect X on Y is identifiable in G we need to show that $P(y|do(x))$ is defined. That is, it can be defined using only pre-intervention probabilities involving variables in R, or output that is not possible.

the idea is a very good!

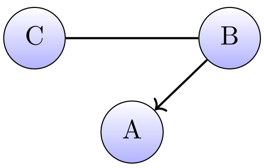
By definition:

not correct: you have to marginalize over v_i not in Y \cup X

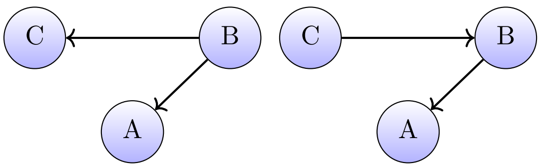
$$P(y|do(x)) = P(v_1, ..., v_n|do(x)) = \begin{cases} \prod_{i|V_i \notin X} P(v_i|pa_i) & \text{for all v consistent with x} \\ 0 & \text{otherwise} \end{cases}$$

Since all non-observable variables are sink nodes they cannot be parents (as they don't have any outgoing edges). Therefore, all parents in G are observable. This means that the causal effect X on Y is identifiable and the effect of the intervention is given by the definition above.

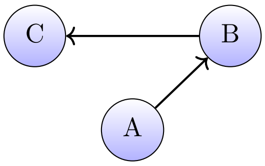
Exercise 3



Here's a partially directed graph. It describes graphs:



There is, however, one more Markov equivalent DAG:

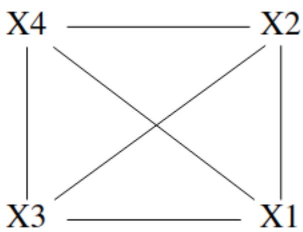


Therefore, proposed partially directed graph is not a CPDAG - it doesn't describe the whole Markov equivalence class.

Exercise 4

1. Find the CPDAG G representing the corresponding Markov equivalence class for P using the PC-algorithm

   The algorithm begins with a complete undirected graph, in our case it's:

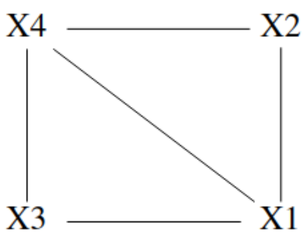
   

   (a) Finding the Skeleton

   In this phase we use the fact that P is Markovian and faithful and follow Skeleton Estimation lemma.

   We're going to iterate over every two adjacent vertices and separation sets each time sending the query of conditional independence.

   Since only one of them is true - $(X_2 \perp\!\!\!\perp X_3 | X_4)$. We'll remove the edge $X_2 - X_3$ and save X_4 in conditioning sets $S(2,3)$ and $S(3,2)$.

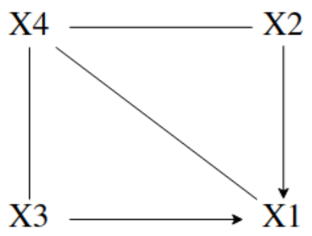
   

(b) Finding the v-Structures

Now, given that X_i, X_j are not adjacent, from the previous phase we know that the set $S(i,j)$ d-separates those nodes.

Then, each structure $X_i - X_k - X_j$ can be oriented as $X_i \rightarrow X_k \leftarrow X_j$ as long as $X_k \notin S(i,j)$.

After finding the skeleton we've obtained the graph above and two non-empty separation sets $S(2,3) = S(3,2) = \{X_4\}$. By applying the above, after finding the c-Structures we recieve the following graph.



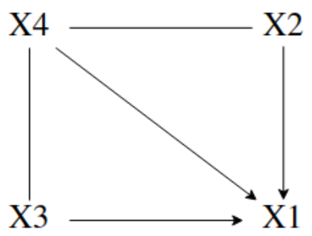
X_2, X_3 were the only pair of non-adjacent vertices, and X_4 d-separates them so we only found one V-structure that is $X_3 \rightarrow X_1 \leftarrow X_2$.

(c) Propagate Orientations

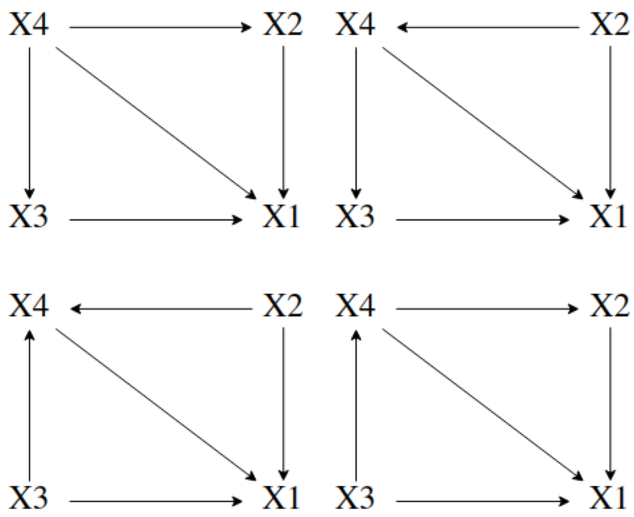
While propagating orientation we apply Meek's orientation rules.

First, we're going to use 3^{rd} rule - the quartet rule.

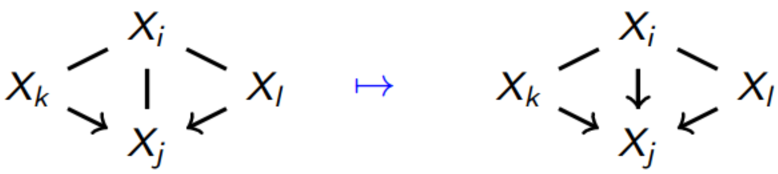
We've got $X_4 - X_3 \rightarrow X_1$ and $X_4 - X_2 \rightarrow X_1$ such that X_3 and X_2 are non-adjacent. Therefore, we're ought to orient the $X_4 - X_1$ to $X_4 \rightarrow X_1$. After that, no further rule to be applied.



2. List all DAGs which are in Markov equivalence class represented by the CPDAG G from part 1 of this task.



3. Using your insights from the previous tasks, prove the soundness of Meek's third rule.



Suppose the edge between X_i, X_j was oriented in the opposite direction. Then by using the $2nd$ rule twice we'd orient $X_l \to X_i$ and $X_k \to X_i$ edges thus giving us a new v-structure.