Zadanie 3

Wiktoria

Zmienna losowa podlega standardowemu rozkładowi normalnemu, tzn. $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, gdzie $x \in \mathbb{R}$. Znaleźć rozkład zmiennej $Y = X^2$.

$$F_Y(t) = P(Y < t) = P(X^2 < t) = P(-\sqrt{t} < X < \sqrt{t}) =$$
$$P(X < \sqrt{t}) - P(X < -\sqrt{t}) = F_X(\sqrt{t}) - F_X(-\sqrt{y})$$

$$f_Y(y) = (F_Y(y))' = (F_X(\sqrt{y}) - F_X(-\sqrt{y}))' =$$
$$f_X(\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} + f_X(-\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} =$$
$$\frac{1}{\sqrt{2\pi}} e^{-\frac{y}{2}} \cdot \frac{1}{2\sqrt{y}} + \frac{1}{\sqrt{2\pi}} e^{-\frac{y}{2}} \cdot \frac{1}{2\sqrt{y}} =$$
$$\frac{1}{\sqrt{2y\pi}} e^{-\frac{y}{2}} = \frac{1}{\sqrt{y}} f_X(\sqrt{y})$$