The value of information for conservation auctions

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Abstract

Conservation auctions are tools for natural resource managers to cost-efficiently protect biodiversity and ecosystem services. Benefits particularly induce uncertainty in the value to be maximised. Therefore learning about benefits is valuable to decide which bids will be successful. Here we use the concept of expected value of sample information to assess how an agency should allocate a limited learning budget among bids to conduct a conservation auction that yields the most cost-efficient return on investment. We propose a simple model where an agent has two or more assets among which they must pick the single most cost-efficient to invest in with an auction. The cost-efficiency of each asset is more or less uncertain and there is some budget allocated to reduce the uncertainty of the assets. The agent must decide how to allocate the learning budget among the assets to maximize the expected benefit of the auction. Using a mix of analytical, heuristic and simulation solutions to the model we glean a number of rules-of-thumb that may be helpful to agencies conducting conservation auctions. When learning budgets are small, then it is best to learn about the most marginal assets. Then, as the budget increases it becomes more optimal to learn about assets which have uncertain cost-efficiency in the more general sense. Finally, as learning budgets become large enough, resources can be allocated evenly across all uncertain assets. Our findings imply that a naive even allocation to learning among assets in a conservation auction may lead to less than cost-efficient outcomes.

Introduction

Conservation auctions have become a widespread and established market mechanism aimed at achieving public environmental good by contracting with landholders and managers cost-efficiently (Schomers and

Matzdorf, 2013). Conservation auctions include payment for ecosystem services (PES) schemes (Engel et al., 2008), environmental stewardship (Ribaudo et al., 2008) and conservation easement programs (Brown et al., 2011). Some recent examples include the US Water Quality Incentives Program (Kraft et al., 1996), the English Countryside Stewardship Scheme (Lobley and Potter, 1998), and Bush Tender in Victoria, Australia (Stoneham et al., 2003).

Uncertainty about the benefits resulting from a conservation auction investment is not often addressed. For example, in implementing a conservation auction to facilitate vegetation regeneration, the Goulburn-Broken Catchment Management Authority (Miles, 2008) used what they call a restoration benefit index (RBI) to rank auction bids. While the RBI included multiple components, including conservation significance, regeneration potential and landholder management action, it did not include any measure of uncertainty. Ignoring uncertainty, one may forego untapped benefit in the implementation of a conservation auction. If the uncertainty in a conservation auction was first characterized and then reduced, then it could potentially be implemented more cost-efficiently, because one could better identify the superior conservation assets.

Here we consider learning to reduce uncertainty in a conservation auction and how resources should be allocated among the assets available for investment. We use the concept of expected value of sample information (EVSI) (Raiffa and Schlaifer, 1961) and a simple auction model to discover the best strategy to allocate resources to learning in a conservation auction. With our model and the solutions we provide below, we derive three guiding principles that will aid an agency undertaking a conservation auction in allocating resources to learning about the cost-efficiency of the assets in their auction pool. The rest of the manuscript is structured as follows: first we describe in more detail how conservation auctions work and then briefly introduce the reader to theory of expected value of sample information. We then formalise a problem of information valuing for conservation auctions, describing how uncertainty is pertinent in this context. We then demonstrate our solutions to the problem presented before finally summarizing the findings with a set of principles that practitioners should heed when implementing a conservation auction with uncertain benefits. We have kept our final discussion brief as in opening up a diversity of fronts for thinking about learning for conservation auctions, this work is necessarily preliminary and exploratory.

Reverse auctions

Conservation auctions often take the form of a reverse auction (though sometimes conservation auctions are standard auctions see e.g., Tóth et al., 2013). In a reverse auction, the agency conducting the auction, usually a branch of a government, is the buyer. The bidders are the landholders or land-managers who compete for the pool of funding available by offering to sell some prespecified environmental good desired by the auction-conducting agency (McAfee and McMillan, 1987). The environmental good may take the form of land title or a contract to conduct particular management actions. In some cases the outcome of management is the good specified in the contract and the management action is left up to the managers (Hanley et al., 2014). Unlike a standard auction where the buyers are competing and price is maximized, reverse auctions are aimed at reducing the price of the goods being purchased, as it is the sellers who are in competition with one another. In facilitating the competition for a single pool of funding, the auction conducting agent seeks to maximize the environmental benefit it can get for the lowest cost and thus maximize the cost-efficiency of the environmental scheme (Latacz-Lohmann and Van der Hamsvoort, 1997).

Ranking auction bids by cost-efficiency

In a typical reverse auction, aimed at getting some public environmental good, the conducting agency will assess the bids by their cost-efficiency. The bids will be ranked from highest to lowest in terms of cost-efficiency (Stoneham et al., 2003). The winning bids will be the most cost-efficient down to some cutoff. The cutoff may be the cumulative total cost of the most cost efficient bids measured against a fixed budget, or it could be a prespecified level of cost-efficiency (Latacz-Lohmann and Van der Hamsvoort, 1997). Time-limited reverse auctions will often use the budget exhaustion method whereas longer-term schemes with repeated rounds might use a cost-efficiency-based cutoff. The rank-by-cost-efficiency strategy is usually near optimal, though in some circumstances it can produce non-optimal results and more sophisticated portfolio methods can be used to maximize the total benefit. When there are many cheap assets in the auction pool the method tends to work well, but in cases where the bids consist of a small number of expensive assets, overall performance may be reduced considerably (Hajkowicz et al., 2007).

Conservation auctions and uncertainty

There are multiple sources of uncertainty in a conservation auction that can affect the outcome in a number of ways. Moreover, there are multiple perspectives from which to view uncertainty within the framework of a conservation auction, and the different actors may be affected by different uncertainties about different aspects of the auction. Here we focus only one aspect of uncertainty: uncertainty about the cost-efficiency of bids and only from the perspective of the agency conducting the auction. Other important facets of uncertainty in reverse-auctions, are the knowledge bidders have of one another's circumstances and intentions, and the circumstances of the seller. There is a vast game-theoretic literature dealing with these types of uncertainty in auctions (see for example Hailu and Schilizzi, 2004), but we don't deal with them further here.

After the bids are submitted in an auction, that component of the auction cost will have no uncertainty. So in most conservation auctions the bulk of the uncertainty about cost-efficiency is due to uncertainty about benefits. Conservation benefit of the kind sought after in a reverse auction is inherently uncertain, as it is often only realized at some point far in future, long after the auction scheme is implemented. Land regeneration, water or soil quality improvement, or other ecosystem services are some examples of the type of benefit that is paid for at one point in time, while the payoff is not expected until much later on (Vesk et al., 2008). This time-lag in return on investment is one issue that makes the cost-efficiency of auction bids uncertain. Uncertainty in the individual auction bids then leads to a necessary uncertainty in their ranking. And therefore, the uncertainty in benefits flows through to the cost-efficiency and to the total benefit realized for the conservation auction scheme.

The value of information

Given that the outcome of many conservation auctions maybe uncertain, it may be wise for an auction conducting agency to invest resources in learning about the benefits before they rank the assets and determine the winning bids. If they can increase the probability of correctly ranking the bids in order of cost-efficiency, they could avoid investing in unwarranted assets and increase the total benefits realized after the auction is completed. The performance gain one might expect after learning and a subsequent reduction in uncertainty is known as the expected value of information (EVI) (Raiffa and Schlaifer, 1961). Decision makers can use an EVI analysis to predetermine the worth of learning about the outcome a decision problem such as a conservation auction. A type of EVI is the expected value of sample information (EVSI), which is the value of reducing uncertainty by some degree, by collecting a sample of data, as opposed to eliminating uncertainty completely, which is the expected value of perfect information (EVPI) (Yokota and Thompson, 2004).

Using the concept of EVSI, a conservation auctioneer could work out if it would be worthwhile collecting data to learn about the benefits of a conservation auction and even how much data would be most to appropriate to collect. Here we extend the idea of expected value of sample information for a conservation auction and consider how to allocate the learning effort among the different assets available in the auction. The naive solution to problem is simply to allocate learning evenly among the assets and reducing the uncertainty about the benefits of each by the same degree. However, depending on the particular circumstances of the initial levels of uncertainty, this may not be the most optimal allocation of learning resources.

Analysis

The Model

Here we describe a simple model of a conservation auction where the cost-efficiency of each asset in the auction pool is uncertain. In our model, there are $n > 1$, assets. The i^{th} $(i = 1...n)$ asset's, A_i, cost-efficiency, c_i, is described by a normal distribution with mean, μ_i and standard deviation, σ_i. There are two separate budgets, one for investing in assets in the reverse auction and a second budget that can be used to reduce the uncertainty about the assets' cost-efficiencies. The first budget is large enough to invest in any one of the assets, while the second budget is variable and can be used to collect a sample of data about the cost-efficiency of each individual asset. In the hypothetical case we consider here, the agency has an oppurtunity to collect new information after bids are made but before the successful bids are decided. The total budget for data collection M, may be divided between each of the n assets in the auction with a different proportion, p_i, allocated to each asset, where Mp_i is proportional to the sample-size of data collected about the cost-efficiency of the i^{th} asset. Here, M can be considered to be the total sample-size assuming that the sampling variance is one. An increase in sample-size or reduction in sampling variance would both increase the effective budget size.

Expected value under uncertainty

Under the initial uncertainty, a risk-neutral auctioneer would simply rank the assets in order of their expected cost-efficiency and invest in the asset with the highest expected cost-efficiency. This is the expected value under uncertainty or also known as the expected value with original information (EVWOI). More formally,

$$\mathrm{EVWOI} = \max_i(\mu_i) \tag{1}$$

Expected value of sample information

Now we turn to the allocation of sampling among the assets in the auction pool and the calculation of the EVSI. In the following sections we outline three solutions to the problem of calculating EVSI for the model above, given a sampling budget and an allocation of the budget among the assets in the auction pool. The first is an analytical solution which applies when $n = 2$. For $n = 3$, the analytical solution does not apply, so we have formulated a heuristic definition. The heuristic definition of EVSI is based on valuing the rank order of cost-efficiency rather than actual benefit achieved. We then compare these solutions to a general solution using Monte Carlo simulation. All analysis has been implemented in the programming language R (R Core Team, 2017) unless otherwise stated.

Analytical solution for $n = 2$

When $n = 2$ an analytical solution exists. Its derivation can be found in Moore et al. (2017) where EVSI is defined as:

$$\mathrm{EVSI} = \frac{1}{2}\left(\Theta\sqrt{\frac{2}{\pi}}e^{-\frac{\mu_1-\mu_2}{2\Theta^2}} + (\mu_1 - \mu_2)\,\mathrm{erf}\left(\frac{\mu_1 - \mu_2}{\Theta\sqrt{2}}\right) - |\mu_1 - \mu_2|\right) \tag{2}$$

where,

$$\Theta = \sqrt{\sigma_1^2 \frac{Mp_1\sigma_1^2}{Mp_1\sigma_1^2 + 1} + \sigma_2^2 \frac{Mp_2\sigma_2^2}{Mp_2\sigma_2^2 + 1}} \tag{3}$$

Note that to calculate the EVPI one can replace equation (3) with $\Theta = \sqrt{\sigma_1^2 + \sigma_2^2}$. With the above definition we can find the value of p_1 (where $p_2 = 1 - p_1$) that maximizes EVSI for a given budget, M. To find the optimal solutions we used a combination of golden section search and successive parabolic interpolation (as implemented in Forsythe et al., 1977).

In figure 1 we show the results of such an optimization for a case where one asset has high expected cost-efficiency and high uncertainty and the other asset has relatively lower expected cost-efficiency and uncertainty. Using a heuristic solution (discussed below and in Appendix C) we arrive at exactly the same solution.

In examining either the analytic or heuristic solutions for $n = 2$, we find that when the assets have different amounts of uncertainty about their respective cost-efficiencies the optimal strategy is to learn only about the more uncertain asset when the budget is low. Then, if the budget is increased, allocation to learning can gradually switch to learning about both assets. When the budget is sufficiently large, then learning can be allocated evenly between the assets. Figure 2 shows how this allocation depends on the ratio of σ_1 and σ_2 for a fixed ratio means. The ratio of the means though, does not affect the optimal allocation (Appendix C).

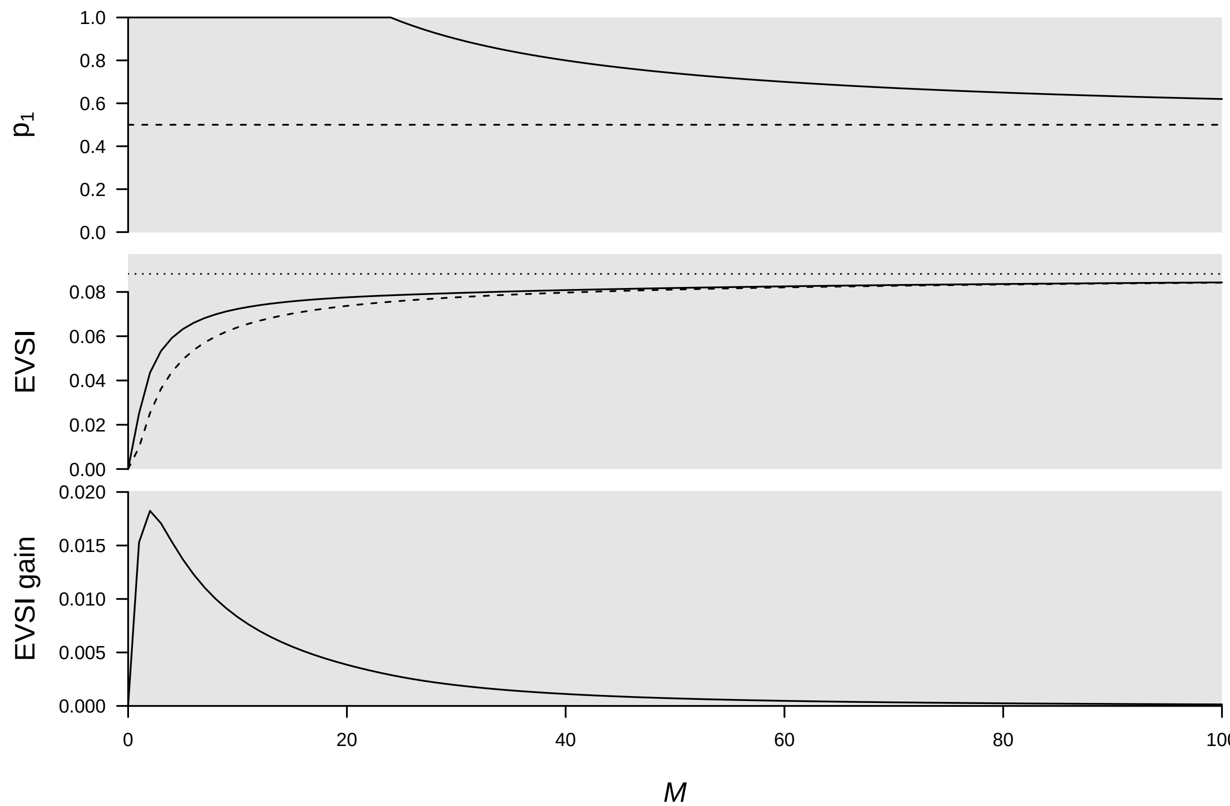


Figure 1: Analytical solution to optimal allocation of sampling between two assets. In this case, the two assets have prior distributions $\mathrm{N}(1, 1)$ and $\mathrm{N}(0, 0.2)$ respectively. The top panel indicates the optimal proportional allocation of sampling between the two assets (solid line) as well as the naive allocation with even sampling between two assets (dashed line). Here p_1 is the proportion allocated to the first asset and M is the total number of samples (the budget). The middle panel shows the EVSI for the optimal and naive sampling strategies of the panel above. The dotted line is the EVPI (0.09). The solid line indicates the EVSI for the optimal allocation while the dashed line is the naive allocation. The bottom panel indicates the gain in EVSI of using the optimal strategy over the naive solution (the difference between the two curves in the middle panel).

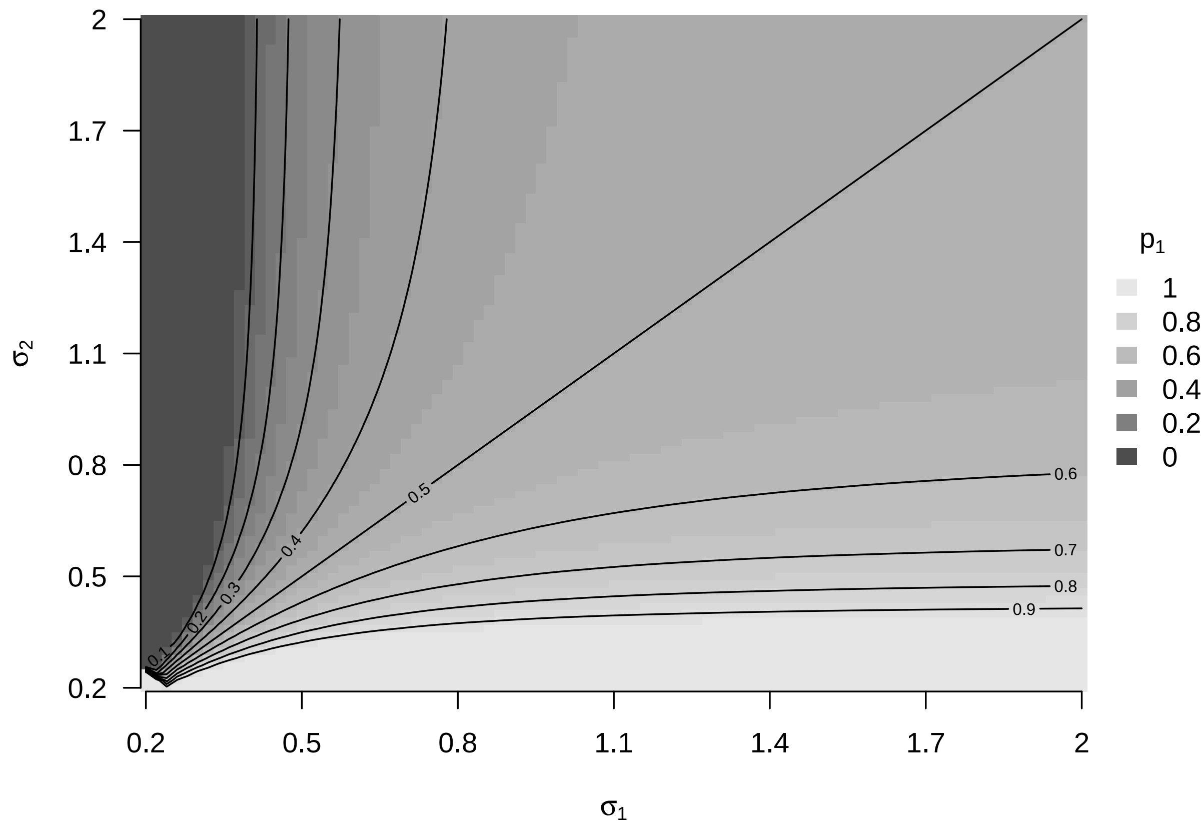


Figure 2: Relationship between optimal allocation to asset 1, p_1, and the parameters σ_1 and σ_2, according to both the analytical and heuristic solutions. Here $M = 7$, $\mu_1 = 1$ and $\mu_2 = 0$.

Heuristic solution for $n = 3$

As the analytical solution above does not hold for $n > 2$ we propose the following heuristic solution (when applied to $n = 2$ the heuristic solution allocates learning in exactly the same manner as the analytical solution, see Appendix C) based on valuing the rank order of cost-efficiency of assets in the auction pool. To elaborate, in valuing the assets by rank, we mean we assign utilities to choosing a single asset that is ranked first, second or third in terms of cost-efficiency. In this sense, utility is indifferent to how much better, for example, the first-ranked asset is than the second and only concerned that it is the better of the two assets. With this principle we assign utilities, u, to each combination of asset choice, A_i, and true rank order of c_i such that:

$$\begin{gathered}
u(A_1, c_1 > c_2 > c_3) = 1 \\
u(A_1, c_1 > c_3 > c_2) = 1 \\
u(A_1, c_2 > c_1 > c_3) = 0.5 \\
u(A_1, c_3 > c_1 > c_2) = 0.5 \\
u(A_1, c_2 > c_3 > c_1) = 0 \\
u(A_1, c_3 > c_2 > c_1) = 0 \\
u(A_2, c_2 > c_1 > c_3) = 1 \\
\text{etc...}
\end{gathered} \tag{4}$$

So here, one receives utility 1 when the chosen asset is truly top ranked, but only utility 0.5 when the chosen asset is in fact the second ranked. Note that the choice of utilities is arbitrary and that a different set of values will change the solution. However, as long as the order of the utilities is the same, the general shape of the solution remains. It is this that makes the solution heuristic rather than exact.

To determine EVSI given the above utilities we need only determine the probability of each rank order and calculate the expected utility of choosing each action with either the original or updated knowledge of the asset cost-efficiencies.

We can express the probability of the assets being in a given rank order as the probability of two differences being less than zero. Such that, for example,

$$\Pr(c_1 > c_2 > c_3) = \Pr(c_2 - c_1 < 0, c_3 - c_2 < 0) \tag{5}$$

Following this we define two new variables, z_1 and z_2 where

$$\begin{aligned} z_1 &= c_2 - c_1, \\ z_2 &= c_3 - c_2 \end{aligned} \tag{6}$$

When c_i are uncorrelated then the covariance of z_1 and z_2 is $-\sigma_2^2$ and they will have a joint distribution defined as:

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} \mu_2 - \mu_1 \\ \mu_3 - \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 + \sigma_2^2 & -\sigma_2^2 \\ -\sigma_2^2 & \sigma_2^2 + \sigma_3^2 \end{bmatrix} \right) \tag{7}$$

Given this joint distribution we can calculate $\Pr(c_1 > c_2 > c_3)$ by evaluating the multivariate normal cumulative distribution function, $\Phi(z_1, z_2)$ within the limits, $-\infty$ and 0, using the algorithm of Genz (1992). With the above, we can calculate the EVWOI, which is the maximum of the expected utilities of choosing the i^{th} asset:

$$\mathrm{EVWOI} = \max(\mathrm{E}[u(A_i)]) \tag{8}$$

where each expected value is the sum of the utilities assigned for that choice of asset, multiplied by the rank order probabilities defined above. For example,

$$\begin{aligned} \mathrm{E}[u(A_1)] =& 1 \times \Pr(c_1 > c_2 > c_3) + 1 \times \Pr(c_1 > c_3 > c_2) + \\ & 0.5 \times \Pr(c_2 > c_1 > c_3) + 0.5 \times \Pr(c_3 > c_1 > c_2) + \\ & 0 \times \Pr(c_2 > c_3 > c_1) + 0 \times \Pr(c_3 > c_2 > c_1) \end{aligned} \tag{9}$$

To calculate the EVSI we need not only know the EVWOI, but also the expected value with sample information (EVWSI), for EVSI is the magnitude of their difference:

$$\mathrm{EVSI} = \max(\mathrm{E}[u(A_i')]) - \max(\mathrm{E}[u(A_i)]) \tag{10}$$

Where EVWOI relied on the expected utilities under the prior knowledge of cost efficiency ranking, the EVWSI relies on the expected utility under the posterior (after knowledge of cost efficiency ranking has been improved). To go from the expected utility under the prior, $\mathrm{E}[u(A_i)]$, to expected utility under the posterior, $\mathrm{E}[u(A_i')]$, we need to adjust the variances in equation (7) from σ_i^2 to $\sigma_i'^2$, where

$$\sigma_i'^2 = \frac{\sigma_i^2}{Mp_i\sigma_i^2 + 1} \tag{11}$$

which accounts for the new information given the sampling allocation Mp_i.

With equation (11), we can find the optimal values of p_i for any given learning budget, M, and set of prior distributions describing uncertainty in cost-efficiency, c_i. To find the optimal allocation of M we performed a constrained optimization using the algorithm of Nelder and Mead (1965). Figure 3 shows such an optimal allocation of M for a case where the expected cost-efficiency and level of uncertainty varies across the three assets. Appendix D contains an examination of the optimal allocation of learning to three assets over increasing M and for different combinations of uncertainty in the three asset's cost-efficiencies of which figure 3 is one example.

The important difference between the $n = 3$ cases and the simpler version where $n = 2$ is that adding another asset now means that having a heterogeniety prior means now has an effect on optimal allocation. Where before ($n = 2$), having different means, but the same degree of unceratinty across assets, meant that the optimal allocation was always to allocate learning evenly, when $n = 3$ in a case with equal uncertainty across assets, different prior means alone will lead to a uneven optimal allocation of learning (see e.g. Appendix D, case-study 2b). Here we summarise the findings of the case-studies in Appendix C and D with set of principles that may be applied to a conservation auction during a pre-auction learning phase (see section:

principles for allocating resources to learning).

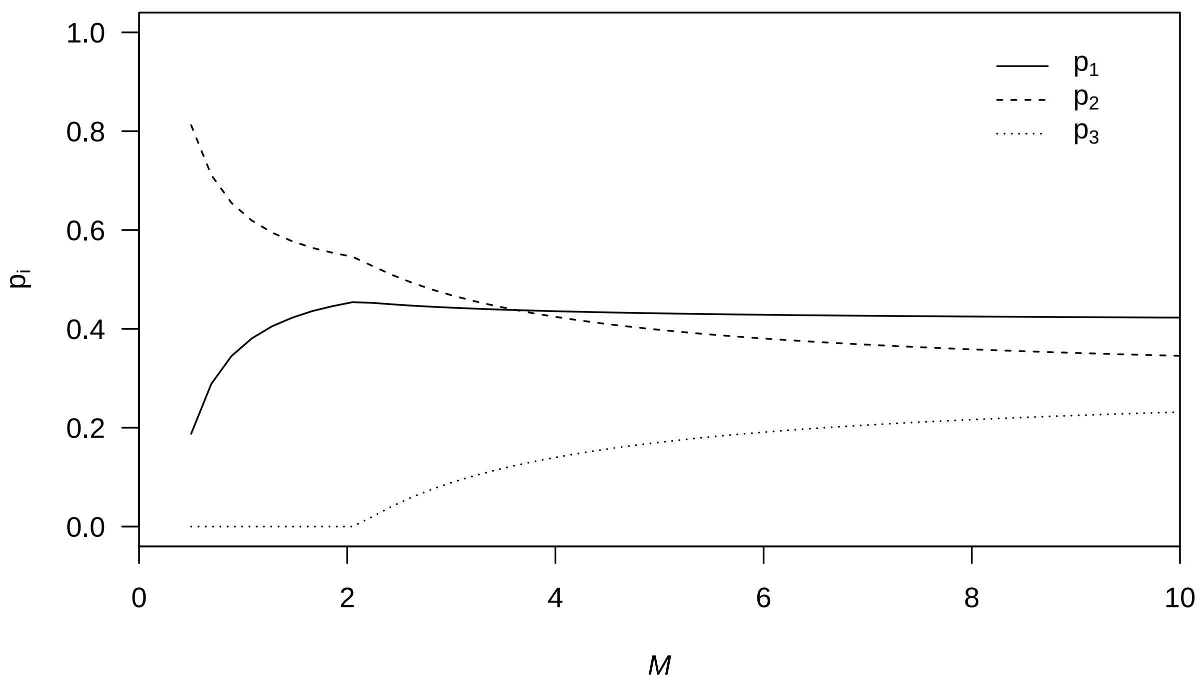


Figure 3: Heuristic solution to the optimal allocation of sampling among three assets. In this case the assets have prior distributions of $N(1, 0.8)$, $N(0.5, 1.25)$ and $N(0.5, 0.75)$ describing the uncertainty in the cost-efficiency respectively. The curves show the optimal allocation, p_i, of the sampling budget, M, to the first (solid line), second (dashed line) and third (dotted line) assets.

Monte Carlo Simulation

Finally we present a general solution to calculate the EVSI for the model with Monte Carlo simulation. The simulation uses an algorithm we have implemented in the programming language Julia (Bezanson et al., 2017) and presented in the pseudo code below.

Begin outer loop: for s of 1 to S simulations

Begin inner loop: for i of 1 to n assets

1. Draw a *true* value, $c^*_{s,i}$, at random from prior distribution $\mathrm{N}(\mu_i, \sigma_i)$

2. Draw a sample mean, $y_{s,i}$, at random from $\mathrm{N}(c^*_{s,i}, \sqrt{\frac{1}{Mp_i}})$

3. Calculate a posterior mean μ'_i as weighted sum of prior and sample means $\mu_i \frac{\frac{1}{\sigma_i}}{Mp_i + \frac{1}{\sigma_i}} + y_{s,i} \frac{Mp_i}{Mp_i + \frac{1}{\sigma_i}}$

End inner loop

4. Calculate value given sample information, v_s, as *true* value of asset with largest posterior mean $c^*_{s,\mathrm{argmax}_i(\mu'_i)}$

End outer loop

5. Calculate EVSI as expected value given sample information, $\frac{1}{S}\sum_{s=1}^{S} v_s$, minus expected value given prior information, $\max_i(\mu_i)$

While the algorithm is relatively simple to implement and gives unbiased estimates of EVSI, it is computationally expensive and the estimates are relatively imprecise. Moreover the impact of this imprecision increases with M, as changes in EVSI in response to changes in p_i are more subtle for larger budgets. Therefore, we use the simulation as a tool to validate assertions about optimal allocation of learning resources based on the heuristic solutions above. Figures 4 and 5 illustrate the application of the simulation solution to the same case studies outlined in figures 1 and 3 respectively.

Principles for allocating resources to learning

From the solutions to our simple model we can glean a number of rules of thumb that agencies conducting conservation auctions should consider. By examining the analytical solution to the two asset problem we can learn a number of things, some of which hold when we increase the complexity by adding a third asset and some of which do not. In examining both the two-asset and three-asset solutions we have elucidated the following principles guiding the allocation of learning resources in a conservation auction.

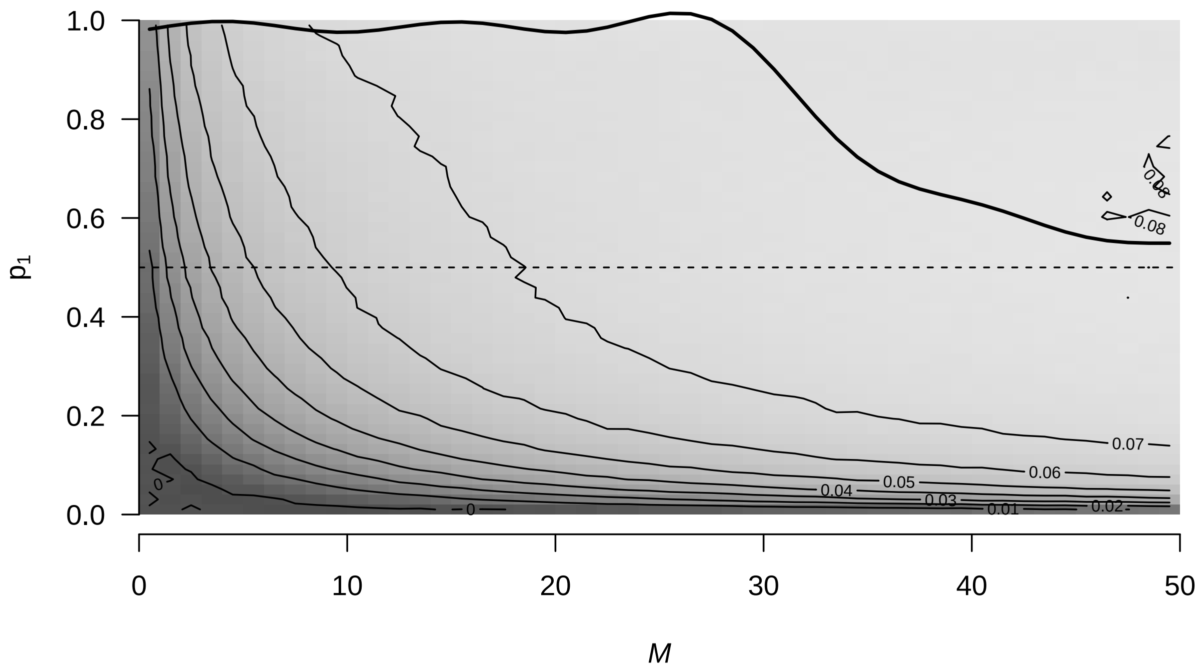


Figure 4: Simulation of EVSI for different allocations of sampling effort among two assets with increasing budget. Again, the two assets have prior distributions N$(1, 1)$ and N$(0, 0.2)$ respectively. Contours and shading indicates the estimated EVSI for the given allocation and budget. Solid line is a smoothed curve fit to the optimal (maximum EVSI) value of p_1 for each budget. Note that this curve has a similar shape to analytical solution in the top panel of figure 1.

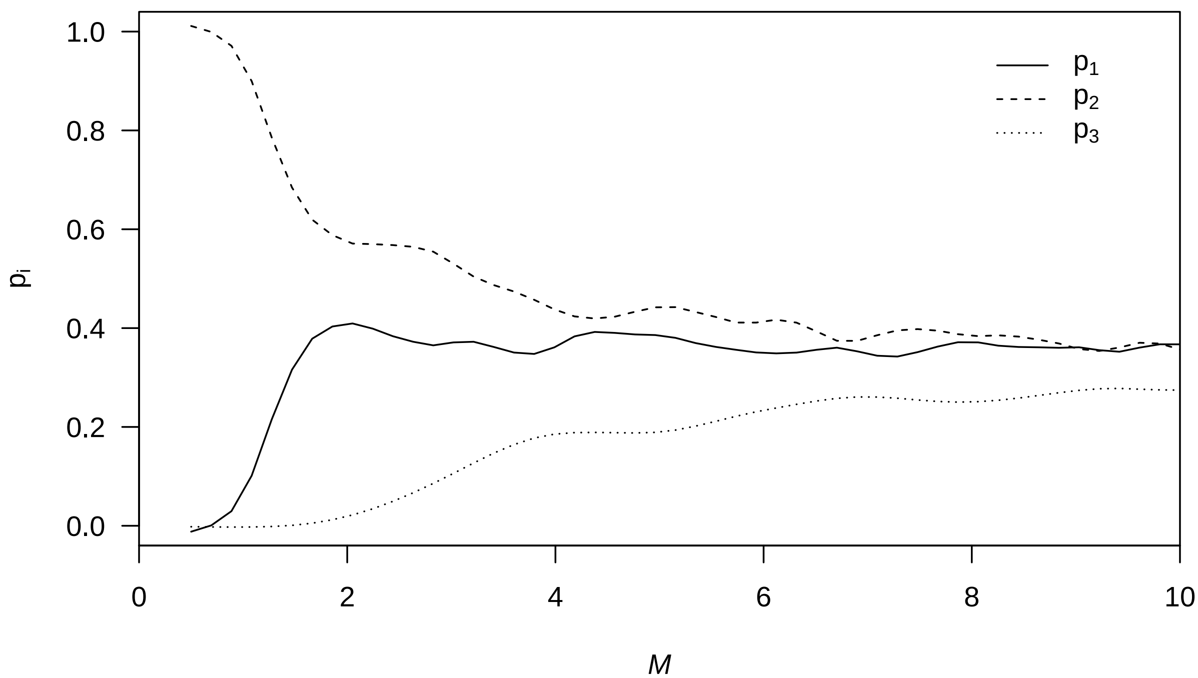


Figure 5: Simulation of optimal allocation of sampling among three assets. Prior distributions of the three assets are as in figure 3. Curves are smooth splines of the value of p_i that maximizes the simulated EVSI for a given budget level of M. Note that the three curves are broadly similar to the heuristic solution of figure 3.

Principle 1: Unequal sampling allocation

In general an optimized unequal allocation of sampling among the assets in an auction will have greater EVSI and result in a more cost-effective auction than simply allocating sampling equally among the assets. However, the larger the budget for learning, the less having an optimal allocation matters. For example, in the case study of figure 1 we see that the peak of expected gain from sampling optimally is for a small budget that is expected to return an EVSI about half the EVPI. As the budget increases and with it EVSI approaches EVPI asymptotically, the difference between an optimal allocation of learning and a naive, even allocation becomes negligible. When considering two assets, this principle only applies when the uncertainty around each asset's cost-efficiency is unequal. Even if each asset has a different expected cost-efficiency it is only optimal to unevenly allocate learning if the variance of their prior cost-efficiencies is unequal. However, when we consider a case with three assets, then it is only optimal to allocate evenly when all the prior means and all the prior variances are the same. That is to say, we should only allocate learning equally when we are completely in the dark about the rank order of asset cost efficiencies.

Principle 2: Learn first at the margin

Knowing that it is probably sub-optimal to allocate learning equally among assets is only useful if one knows in what way they should otherwise distribute sampling effort. Our second principle is that given a small to moderate budget and some uncertainty about the cost-efficiency of auction assets, it is wisest to allocate to assets on the margin. By on the margin, we mean assets that are borderline cases for potential investment. These are important cases for learning about because new knowledge can impact whether those assets should be invested in or not. By contrast, we can identify two other classes of asset: those likely to be included among winning bids, and those unlikely to be among the winners. For each of these classes, learning is less preferential than the more marginal cases. In figure 3 this principle is illustrated by the fact that assets 1 and 2 demand greater allocation of sampling than asset 3, as asset 3 has the lowest prior mean cost-efficiency as well as the greatest certainty. Further, for small budgets investment in learning about asset 2 is preferred over asset 1.

Principle 3: Learn about the more uncertain assets

Again, figure 3 highlights the final principle. Given the choice of allocating learning among assets with similar cost-efficiency, it is more optimal to learn about the more uncertain. This third principle however, interacts with the second, as it is only more preferential to learn about asset 2 (the most uncertain cost-efficiency) when the budget is small. But, when the budget is large enough the allocation to learning about asset 1 approaches the allocation to asset 2.

Conclusion

This work begins to formalise a problem of information valuing for conservation auctions. We have addressed this problem using a blend of analytical, heuristic and simulation-based approaches, necessitated by the absence of a closed-form solution for $n > 2$ assets. Clearly, this work only scratches the surface of the value of learning in conservation auctions. Yet the model, solutions and principles we outline above have the potential to change the way information is used when implementing conservation auctions. In the past information gathering to inform conservation auctions has been either minimal, or when substantive, allocated evenly across bids in the auction (see e.g., Miles, 2008). Now, even if an auction conducting agency did not wish to apply value of information formally, they may be able to apply the principles we outline here to their pre-auction learning phase and save learning resources, leading to more cost-effective auctions. This could fundementally change the design of past conservation auctions, such as Bush Tender (Stoneham et al., 2003), Bush Returns (Miles, 2008) and others (e.g., Lobley and Potter, 1998; Hajkowicz et al., 2007; Hanley et al., 2014), as it demonstrates a benefit of emphasising learning in the period between auction bids arriving, and the decision to invest in them. However, of course some caveats apply. In our model we only consider cases where a single asset is purchased at the end of the auction, and we assume that uncertainty is normal distributed. Changing these assumptions may lead to different results. Future work would help to verify and consolidate the principles we outline above. Such work might include finding analytical solutions for $n > 2$ and even $n > 3$, as well as auctions with multiple successful bids and multiple auction rounds.

References

Bezanson, J., A. Edelman, S. Karpinski, and V. B. Shah. 2017. Julia: A fresh approach to numerical computing. SIAM Review **59**:65–98.

Brown, L. K., E. Troutt, C. Edwards, B. Gray, and W. Hu. 2011. A uniform price auction for conservation easements in the Canadian prairies. Environmental and Resource Economics **50**:49–60.

Engel, S., S. Pagiola, and S. Wunder. 2008. Designing payments for environmental services in theory and practice: An overview of the issues. Ecological economics **65**:663–674.

Forsythe, G. E., C. B. Moler, and M. A. Malcolm. 1977. Computer methods for mathematical computations. Prentice-Hall.

Genz, A. 1992. Numerical computation of multivariate normal probabilities. Journal of computational and graphical statistics **1**:141–149.

Hailu, A., and S. Schilizzi. 2004. Are auctions more efficient than fixed price schemes when bidders learn? Australian Journal of Management **29**:147–168.

Hajkowicz, S., A. Higgins, K. Williams, D. P. Faith, and M. Burton. 2007. Optimisation and the selection of conservation contracts. Australian Journal of Agricultural and Resource Economics **51**:39–56.

Hanley, N., B. White, et al. 2014. Incentivizing the provision of ecosystem services. International Review of Environmental and Resource Economics **7**:299–331.

Kraft, S. E., C. Lant, and K. Gillman. 1996. WQIP: an assessment of its chances for acceptance by farmers. Journal of Soil and Water Conservation **51**:494–498.

Latacz-Lohmann, U., and C. Van der Hamsvoort. 1997. Auctioning conservation contracts: a theoretical analysis and an application. American Journal of Agricultural Economics **79**:407–418.

Lobley, M., and C. Potter. 1998. Environmental stewardship in UK agriculture: a comparison of the environmentally sensitive area programme and the countryside stewardship scheme in South East England. Geoforum **29**:413–432.

McAfee, R. P., and J. McMillan. 1987. Auctions and bidding. Journal of economic literature **25**:699–738.

Miles, C., 2008. Testing market-based instruments for conservation in northern Victoria. Pages 149–163 *in* Biodiversity: Integrating Conservation and Production—case studies from Australian farms, forests and fisheries. CSIRO Publishing, Collingwood.

Moore, A. L., L. Walker, M. C. Runge, E. McDonald-Madden, and M. A. McCarthy. 2017. Two-step Adaptive Management for choosing between two management actions. Ecological Applications .

Nelder, J. A., and R. Mead. 1965. A simplex method for function minimization. The Computer Journal **7**:308–313.

R Core Team, 2017. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

Raiffa, H., and R. Schlaifer. 1961. Applied statistical decision theory. John Wiley & Sons Inc., New York.

Ribaudo, M., L. Hansen, D. Hellerstein, and C. Greene, 2008. The use of markets to increase private investment in environmental stewardship. Technical report, U.S. Department of Agriculture.

Schomers, S., and B. Matzdorf. 2013. Payments for ecosystem services: A review and comparison of developing and industrialized countries. Ecosystem Services **6**:16–30.

Stoneham, G., V. Chaudhri, A. Ha, and L. Strappazzon. 2003. Auctions for conservation contracts: an empirical examination of Victoria's BushTender trial. Australian Journal of Agricultural and Resource Economics **47**:477–500.

Tóth, S. F., G. J. Ettl, N. Könnyű, S. S. Rabotyagov, L. W. Rogers, and J. M. Comnick. 2013. ECOSEL: multi-objective optimization to sell forest ecosystem services. Forest policy and economics **35**:73–82.

Vesk, P. A., R. Nolan, J. R. Thomson, J. W. Dorrough, and R. Mac Nally. 2008. Time lags in provision of habitat resources through revegetation. Biological Conservation **141**:174–186.

Yokota, F., and K. M. Thompson. 2004. Value of information analysis in environmental health risk management decisions: past, present, and future. Risk analysis **24**:635–650.

Appendix C: VOI for conservation auctions heuristic solution $n = 2$

Priors

Let A_1 and A_2 be two assets. Each has some cost efficiency c_1 and c_2, both of which are uncertain with prior probability distributions,

$$c_1 \sim \mathcal{N}(\mu_1, \sigma_1) \tag{12}$$

and

$$c_2 \sim \mathcal{N}(\mu_2, \sigma_2) \tag{13}$$

where μ_1 and μ_2 are the prior means and σ_1 and σ_2 are the prior standard deviations.

Let $\pi = \Pr(c_1 > c_2)$ and therefore,

$$\pi = \Phi\left(\frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right) \tag{14}$$

Utilities

If $\pi > 1 - \pi$ then the optimal action is to purchase asset A_1, otherwise it is to purchase A_2.

We can assign utilities to each combination of the condition of c_1 and c_2, and each action such that,

$$
\begin{aligned}
u(c_1 > c_2, A_1) &= 1 \\
u(c_1 < c_2, A_1) &= 0 \\
u(c_1 > c_2, A_2) &= 0 \\
u(c_1 < c_2, A_2) &= 1
\end{aligned}
\tag{15}
$$

Therefore the expected values of taking each action are $\mathrm{E}[u(A_1)] = \pi$ and $\mathrm{E}[u(A_2)] = 1 - \pi$.

Value of perfect information

If we could reduce the uncertainty in the prior probabilites of c_1 and c_2 such that π approached either limit, then the expected value of perfect information is,

$$
\mathrm{EVPI} = 1 - \max(\pi, 1 - \pi) = \min(\pi, 1 - \pi) \tag{16}
$$

Preposterior analysis

Now let's assume we can sample from some process and learn about c_1 and c_2.

Let X_1 and X_2 be observations from a process described by the following sampling distributions,

$$
X_1 \sim \mathcal{N}(c_1, 1) \tag{17}
$$

and

$$
X_2 \sim \mathcal{N}(c_2, 1) \tag{18}
$$

where for simplicity the standard deviation is one.

Furthermore, we can make Mp and $M(p-1)$ observations of X_1 and X_2 respectively. Where M is the total number of samples the budget allows and p and $p-1$ are proportions of that budget allocated to each assets. Given the observations of X_1 and X_1 with sample sizes Mp and $M(p-1)$ we can update the prior beliefs in

c_1 and c_2 and arrive at posterior distributions,

$$\begin{aligned} c_1' &\sim \mathcal{N}(\mu_1', \sigma_1') \\ c_2' &\sim \mathcal{N}(\mu_2', \sigma_2') \end{aligned} \tag{19}$$

where,

$$\begin{aligned} \mu_1' &= \frac{\mu_1 + X_1 M p \sigma_1^2}{M p \sigma_1^2 + 1} \\ \sigma_1' &= \sqrt{\frac{\sigma_1^2}{M p \sigma_1^2 + 1}} \\ \mu_2' &= \frac{\mu_2 + X_2 M (p-1) \sigma_2^2}{M (p-1) \sigma_2^2 + 1} \\ \sigma_2' &= \sqrt{\frac{\sigma_2^2}{M (p-1) \sigma_2^2 + 1}} \end{aligned} \tag{20}$$

Value of sample information

Now we make the simplifying assumptions that $\mathrm{E}[X_1] = \mathrm{E}[c_1] = \mu_1$ and $\mathrm{E}[X_1] = \mathrm{E}[c_1] = \mu_1 = 0$.

In this scenario the prior probability of $c_1 > c_2$ is,

$$\pi = \Phi\left(\frac{\mu_1}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right) \tag{21}$$

, and the posterior is

$$\pi' = \Phi\left(\frac{\mu_1}{\sqrt{\frac{\sigma_1^2}{M p \sigma_1^2 + 1} + \frac{\sigma_2^2}{M (p-1) \sigma_2^2 + 1}}}\right) \tag{22}$$

We can now calculate the expected value of sample information,

$$\mathrm{EVSI} = \max(\pi', 1 - \pi') - \max(\pi, 1 - \pi) \tag{23}$$

Optimal sampling allocation

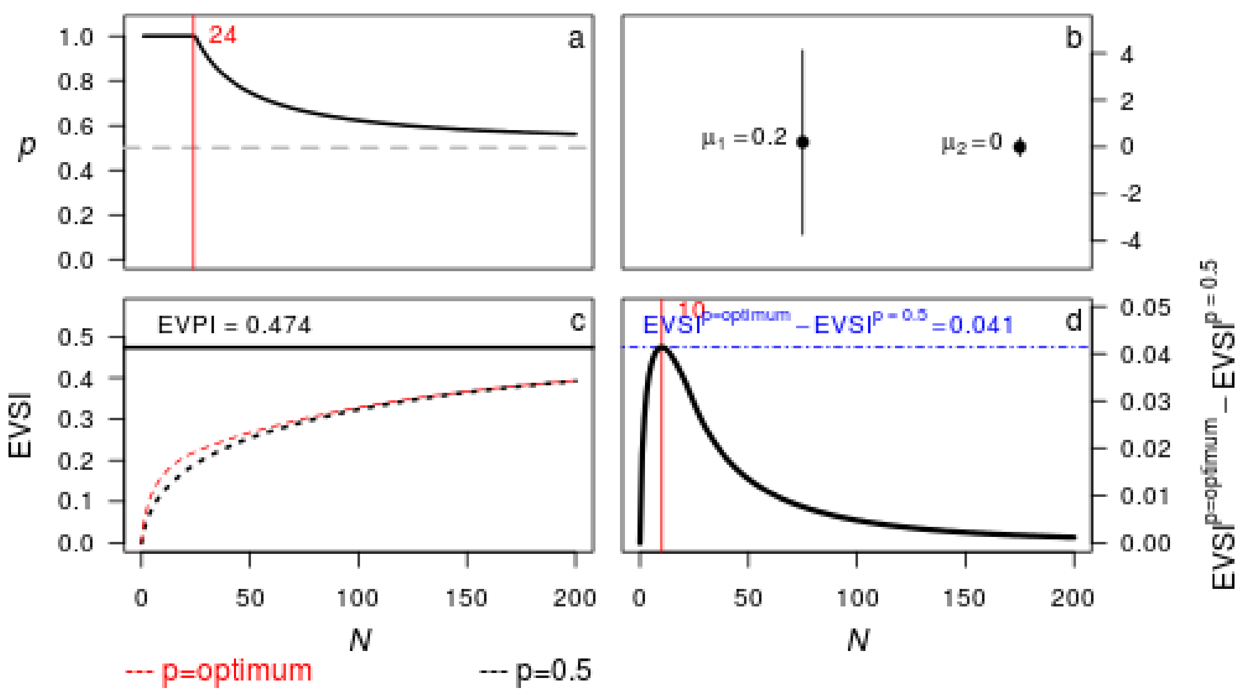
With equations 10-12, for any given set of μ_1, σ_1, μ_2, and M we can find the optimal allocation to sample for asset A_1, p to maximise the value of sample information.

Summary

- Optimal allocation insensitive to ratio of μ_a to μ_b.
- Optimal allocation sensitive to ratio of σ_1 to σ_2.
- Always preferential to sample asset with greater uncertainty.
- Solution is symmetrical.
- The M at which you start to allocate sampling to both assets is proportional to the ratio of σ_1 to σ_2.
- The M at which you start to allocate sampling to both assets is insensitive to whether $\sigma_1 < \sigma_2$ or $\sigma_1 > \sigma_2$.
- Optimal allocation always has greater EVSI when $\sigma_1 \neq \sigma_2$.

$\mu_1 > \mu_2, \sigma_1 \gg \sigma_2$

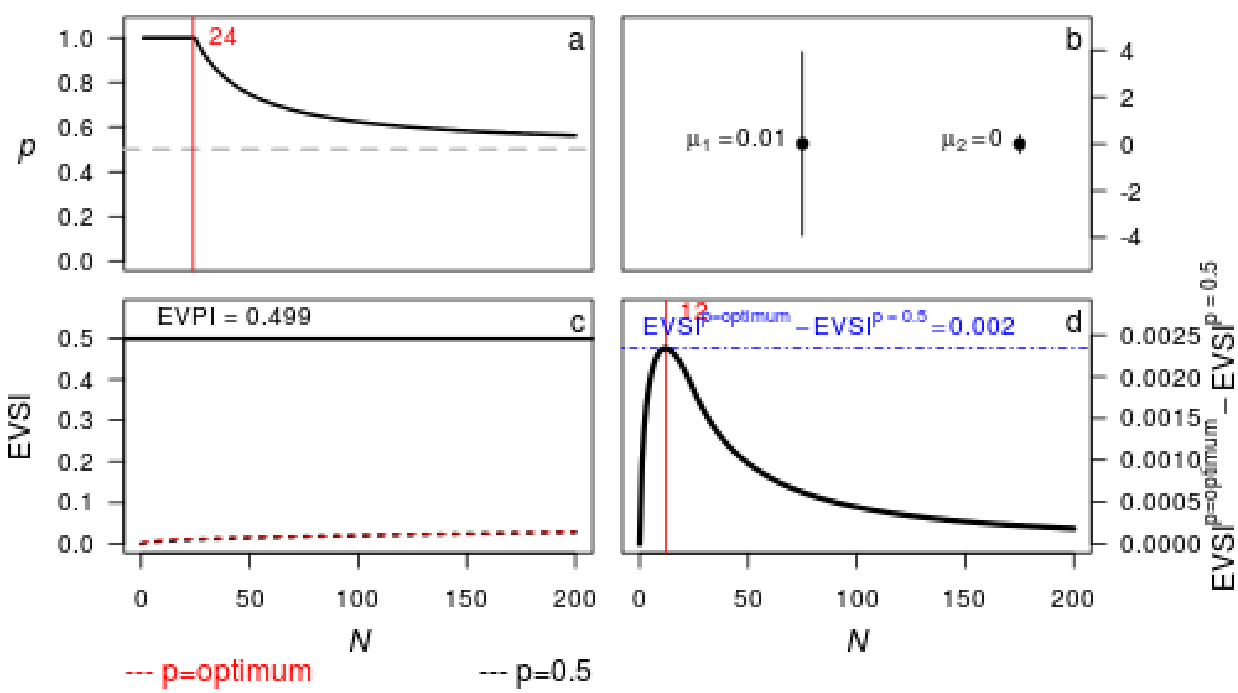
First let's examine the case where our prior belief is that c_1 is somewhat greater than c_2, where $\mu_1 = .2$ and $\mu_2 = 0$ but the uncertainty in θ_1 is far greater than in θ_2, $\sigma_1 = 2$ and $\sigma_2 = .2$.



In this case, until we can take more than a total of 24 samples ($N = 24$) the optimal allocation is to allocate all sampling effort to asset A_1 (a). With a total budget greater than 24 samples, we start allocating sampling to asset A_2 at a diminishing rate with total sample size and asymptoting at $p = 0.5$. The expected value of sample information (EVSI) increases with sample size with diminishing returns asymptoting below the EVPI (c). At all sample sizes, the optimal allocation has greater expected value than naive assumption of constant allocation of $p = 0.5$. The greatest benefit of allocating optimally is when the sample size is $N = 10$. As sample size increases the additional benefit of allocating optimally declines asymptotically as the optimal allocation approaches $p = 0.5$ (d).

$\mu_1 \simeq \mu_2, \sigma_1 \gg \sigma_2$

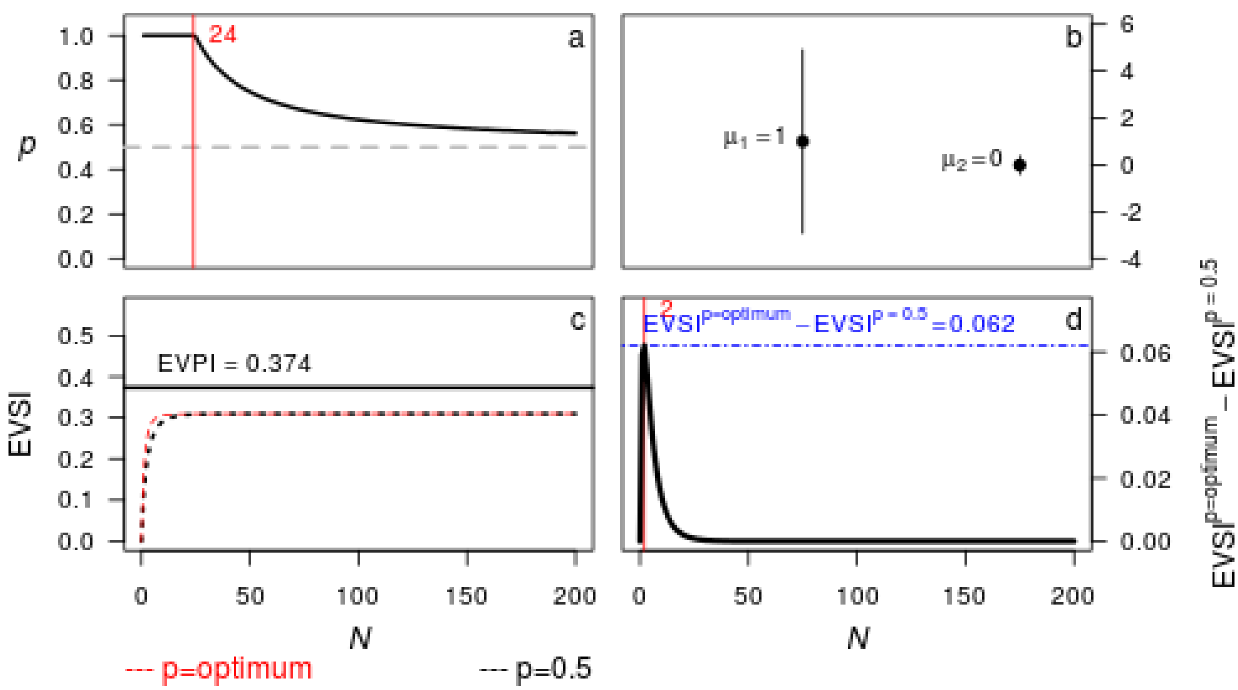
Holding all the other parameters constant, let's examine a scenario where the prior expection of c_1 is only marginally better than c_2 (i.e., reduce μ_1 to .01).



We still allocate in the same way as before (a). The EVPI approaches its theoretical maxmimum of 0.5. But the value of sample information achievable for anything less than 200 samples is reduced to near zero (c). The shape of the additional benefit from optimal allocation is the same but the scale has reduced and the optimal value of N has increased meaning more samples must be taken to maxmise the additional gain in EVSI by using the optimal allocation vs the naive allocation of $p = 0.5$ (d).

$\mu_1 \gg \mu_2, \sigma_1 \gg \sigma_2$

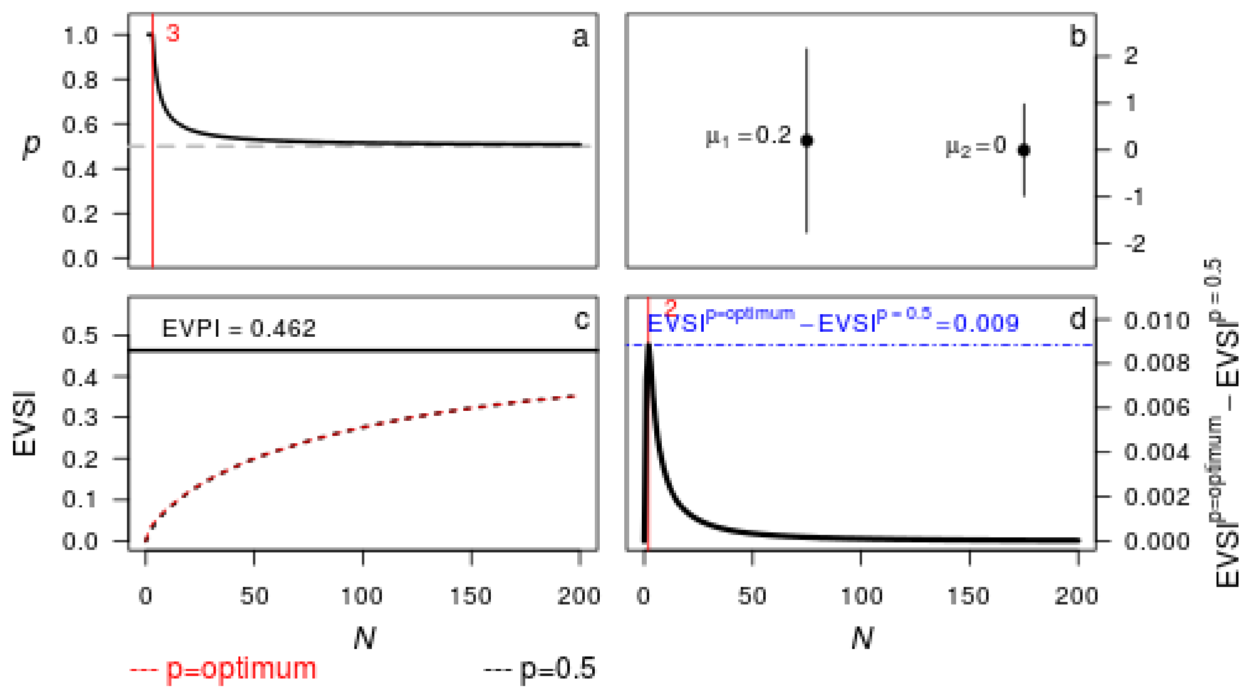
All other parameters still the same, but now with $\mu_1 = 1$ much greater than μ_2.



Still same optimal allocation (a). EVPI reduced as we start already more certain that A_1 is a better asset than A_2. The return on investing in each additional sample diminishes quicker and sooner as EVSI asymptotes at nearer EVPI (c). The additional benefit in EVSI seen by sampling optimally has increased by peaks earlier at $N = 2$ (d).

$\mu_1 > \mu_2, \sigma_1 > \sigma_2$

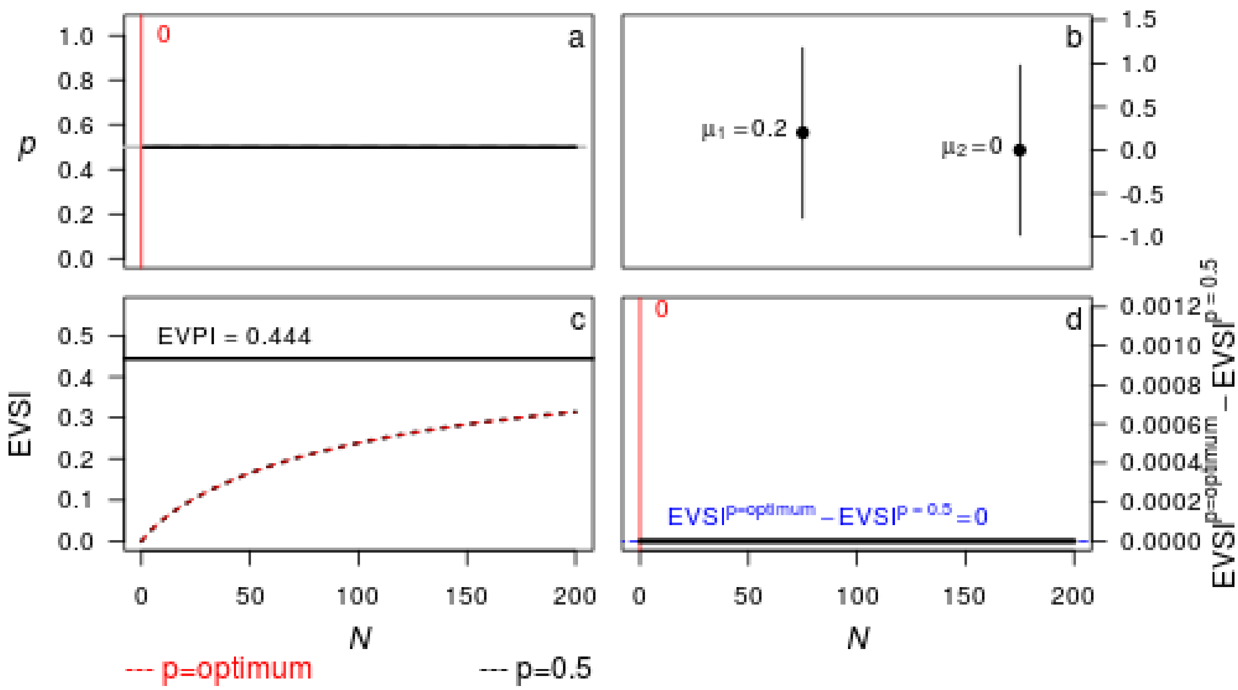
Resetting μ_1 now we examine what happens when σ_1 is double σ_2.



The optimal allocation has the same shape as before but now we allocate sampling to asset A_2 sooner than before. Now if we take more than 3 samples we will allocate an increasing amount of them to asset A_2 (a). The EVPI has been reduced as we have started off more certain that A_1 is greater A_2 (c). There is less advantage to sampling optimally rather than the naive allocation but the point at which the additional benefit in EVSI peaks is at higher N ($N = 2$) than when σ_1 was much more than σ_2 (d).

$\mu_1 > \mu_2, \sigma_1 = \sigma_2$

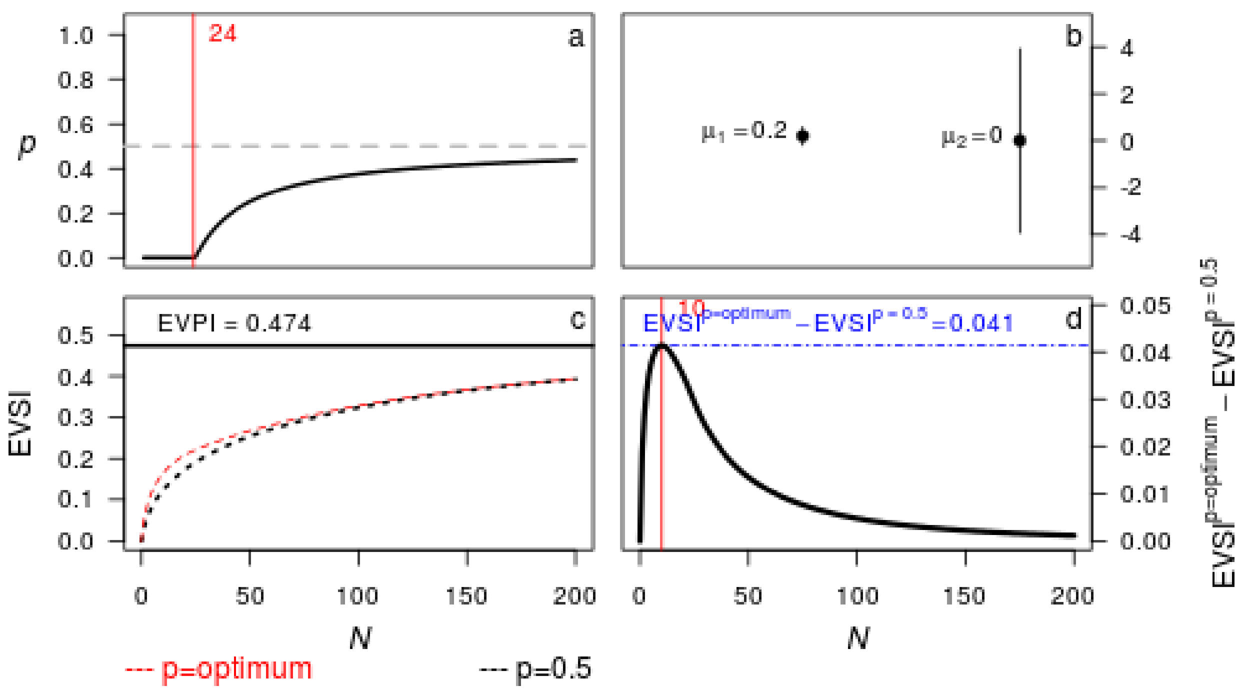
Now increase the precision of c_1 so that $\sigma_1 = \sigma_2$.



Now the optimal allocation is to just allocate evenly between the assets (i.e., the naive allocation is now optimal and there is effectively no advantage to optimize) (a). The allocation is still insentive to the value of μ_1 and the value of σ's. Decreasing the prior precisions increases EVPI, and with it the point at which EVSI vs N asymptotes, but without changing the rate EVPI increases with N. Increaseing the value of μ_1 decreases EVPI (but less sensitively than changing the precision) and also increases the rate at which EVPI vs N approaches EVPI (c).

$\mu_1 > \mu_2, \sigma_1 \ll \sigma_2$

Returning to the original paramterisation now reverse the prior precisions so that τ'_a is much greater than τ'_b.



The allocation is the same but now reflected so that we still allocate to the asset with more uncertainty (a). This applies no matter what the other parameter values are (i.e., the problem is symetrical).

Appendix D: VOI for conservation auctions heuristic solution $n = 3$

Priors

Let A_1, A_2, A_3 be assets. Each has some value c_1, c_2, and c_3 which are uncertain with a joint prior probability distribution,

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}\right) \tag{24}$$

where μ_1, μ_2, and μ_3 are means and σ_1, σ_2 and σ_3 are the prior standard deviations.

Utilities

We assign utilities to each rank order of c_1, c_2, and c_3 in combination with each action of purchasing one of A, B or C,

$$\begin{gathered} u(A_1, c_1 > c_2 > c_3) = 1 \\ u(A_1, c_1 > c_3 > c_2) = 1 \\ u(A_1, c_2 > c_1 > c_3) = 0.5 \\ u(A_1, c_3 > c_1 > c_2) = 0.5 \\ u(A_1, c_2 > c_3 > c_1) = 0 \\ u(A_1, c_3 > c_2 > c_1) = 0 \\ u(A_2, c_2 > c_1 > c_3) = 1 \\ \text{etc...} \end{gathered} \tag{25}$$

such that utility is maximised when we purchase the highest ranked asset, zero when we purchase the lowest ranked and somewhere inbetween when we purchase the middle ranked asset.

Ranking probabilties

We can express the probability of any assets being in a given rank order as the probabililty of two differences being less than zero. Such that, for example,

$$\Pr(c_1 > c_2 > c_3) = \Pr(c_2 - c_1 < 0, c_3 - c_2 < 0) \tag{26}$$

Given this we define two new variables, z_1 and z_2 where

$$\begin{aligned} z_1 &= c_2 - c_1, \\ z_2 &= c_3 - c_2 \end{aligned} \tag{27}$$

Even if c_1, c_2, and c_3 are all uncorrelated z_1 and z_2 will not be, where

$$\mathrm{cov}(z_1, z_2) = \mathrm{cov}(c_1, c_2) - \mathrm{var}(c_2) - \mathrm{cov}(c_1, c_3) + \mathrm{cov}(c_2, c_3) \tag{28}$$

Which, when c_1, c_2, and c_3 are all uncorrelated simplifies to

$$\mathrm{cov}(z_1, z_2) = -\mathrm{var}(c_2) \tag{29}$$

Therefore the joint distribution of z_1 and z_2 is,

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} \mu_2 - \mu_1 \\ \mu_3 - \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 + \sigma_2^2 & -\sigma_2^2 \\ -\sigma_2^2 & \sigma_2^2 + \sigma_3^2 \end{bmatrix} \right) \tag{30}$$

To obtain $\Pr(c_1 > c_2 > c_3)$ we evalute the multivariate cumulative distribution function of z_1 and z_2,

$$\Phi(z_1, z_2) \tag{31}$$

within the limits, $-\infty$ and 0.

Expected value of perfect information

To calculate the prior expected utility of purchasing any asset, we weight the utilities for that action (eqn. 2) by the relevant probablities calculated from eqns. 3–9. For instance;

$$\begin{aligned}\mathrm{E}[u(A_1)] =& 1 \times \Pr(c_1 > c_2 > c_3) + 1 \times \Pr(c_1 > c_3 > c_2) + \\ & 0.5 \times \Pr(c_2 > c_1 > c_3) + 0.5 \times \Pr(c_3 > c_1 > c_2) + \\ & 0 \times \Pr(c_2 > c_3 > c_1) + 0 \times \Pr(c_3 > c_2 > c_1)\end{aligned} \tag{32}$$

The expected value of perfect information then is,

$$\mathrm{EVPI} = 1 - \max(\mathrm{E}[u(A_1)], \mathrm{E}[u(A_2)], \mathrm{E}[u(A_3)]) \tag{33}$$

Updating

Now suppose we can update the priors for c_1, c_2, and c_3 by taking M samples from sampling distrubutions with, for simplicity, some fixed variance of 1 and centered on μ_1, μ_2 and μ_3 respectively. Further, we can define p_1 and p_2 as the proportion of the M samples allocated to sampling for A_1 and A_2 respectively with $1 - p_1 - p_2$ being allocated to A_3. We can then use these samples to update the priors for c_1, c_2, and c_3 to obtain preposterior estimates, c'_1, c'_2, and c'_3, where,

$$\begin{bmatrix} c'_1 \\ c'_2 \\ c'_3 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}, \begin{bmatrix} \frac{\sigma_1^2}{Mp_1\sigma_1^2+1} & 0 & 0 \\ 0 & \frac{\sigma_2^2}{Mp_2\sigma_2^2+1} & 0 \\ 0 & 0 & \frac{\sigma_3^2}{M(1-p_1-p_2)\sigma_3^2+1} \end{bmatrix}\right) \tag{34}$$

Expected value of sample information

For any given new rank order, based on the updated preposterior distributions, we can again calculate a probablity by defining new variables (i.e., z'_1 and z'_2) and evalute their multivariate cumulative distribution as in eqns. 3–9. Therefore we can obtain the preposterior expected utilities for each purchase action by weighting the preposterior probablites by their respective utilities as in eqn. 10. Accordingly the expected vale of sample information is

$$\mathrm{EVSI} = \max(\mathrm{E}'[u(A_1)], \mathrm{E}'[u(A_2)], \mathrm{E}'[u(A_3)]) - \max(\mathrm{E}[u(A_1)], \mathrm{E}[u(A_2)], \mathrm{E}[u(A_3)]) \tag{35}$$

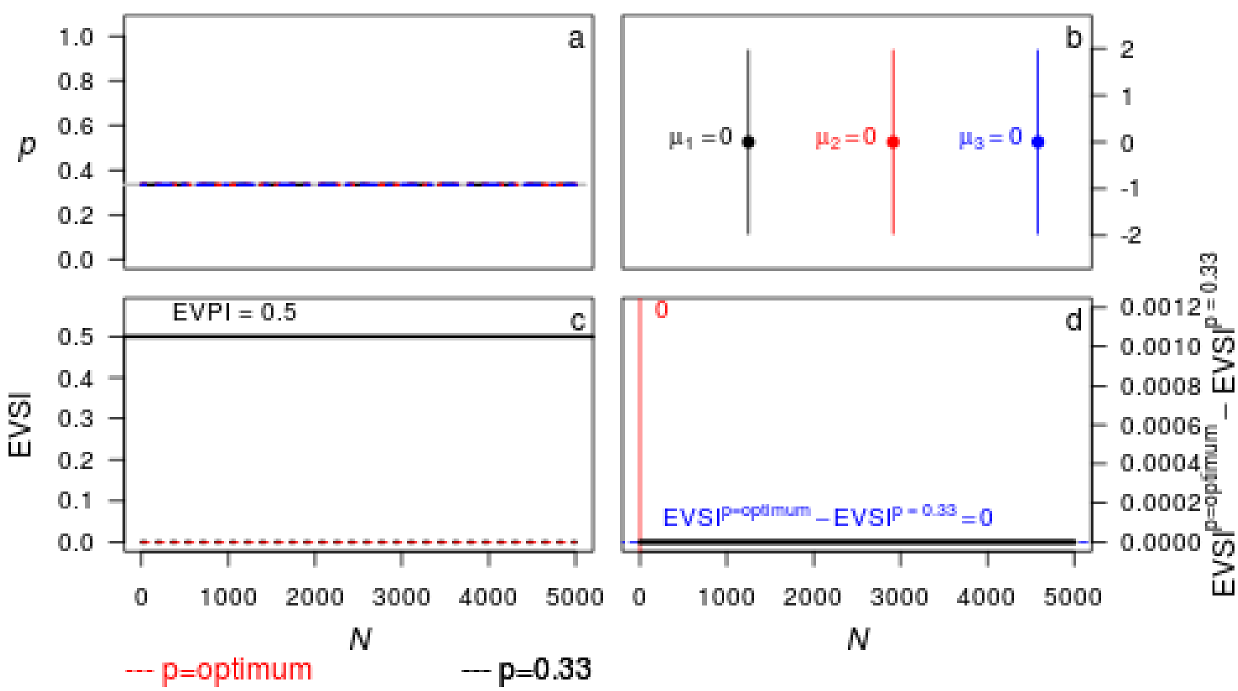
Optimisation

Using eqns 8-13 we can find the optimal values of p_1 and p_2 for any given M that will maximise the EVSI. Below we examine a number of ___Case studies for different sets of prior distrbutions for c_1, c_2 and c_3.

Summary

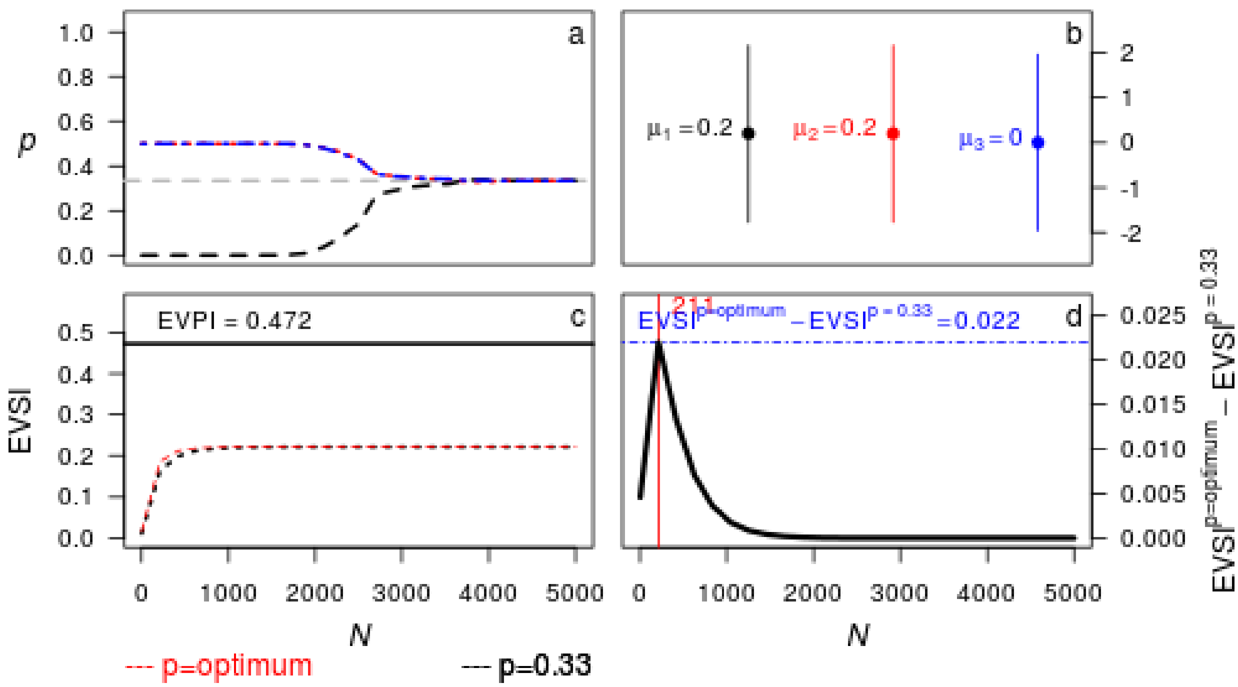
- Optimal allocation sensitive to ratios of μ's.
- Optimal allocation sensitive to ratio of σ's.
- Not always preferential to sample asset with greater uncertainty.
- Solution is symmetrical.

Case study 1: homogenous prior σ's and homogenous prior means



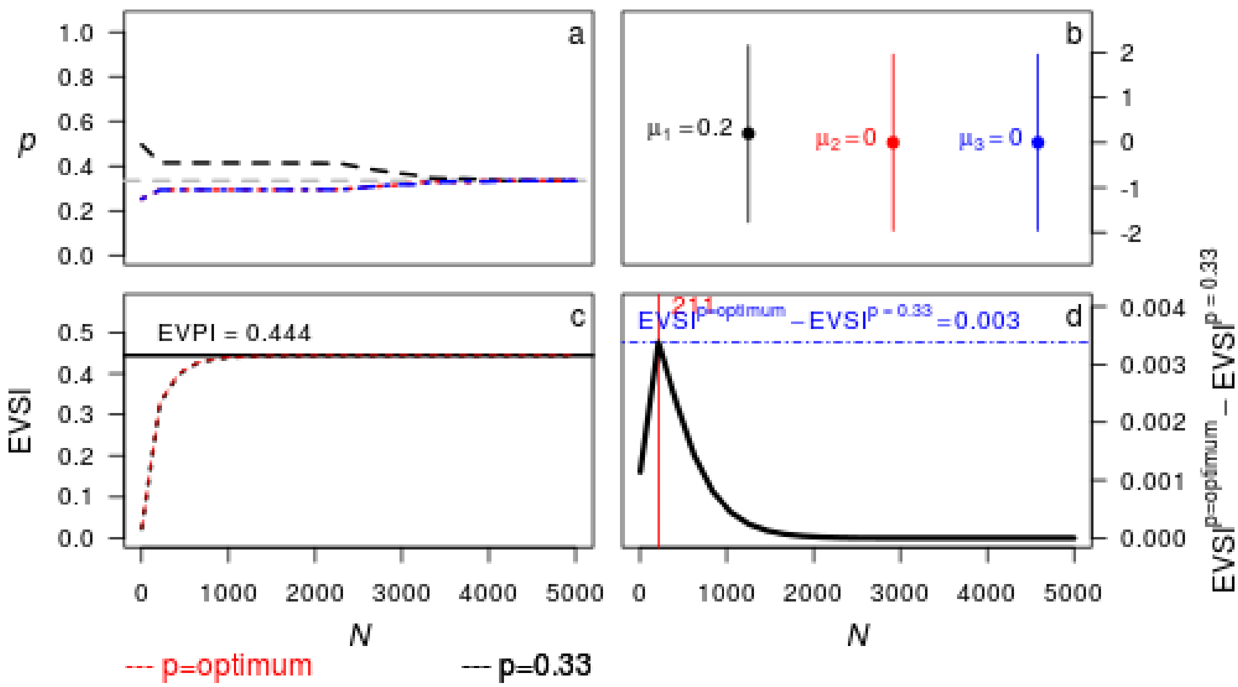
Case study 2a: homogenous prior σ's and heterogenous prior means

$\mu_1 = \mu_2 > \mu_3$



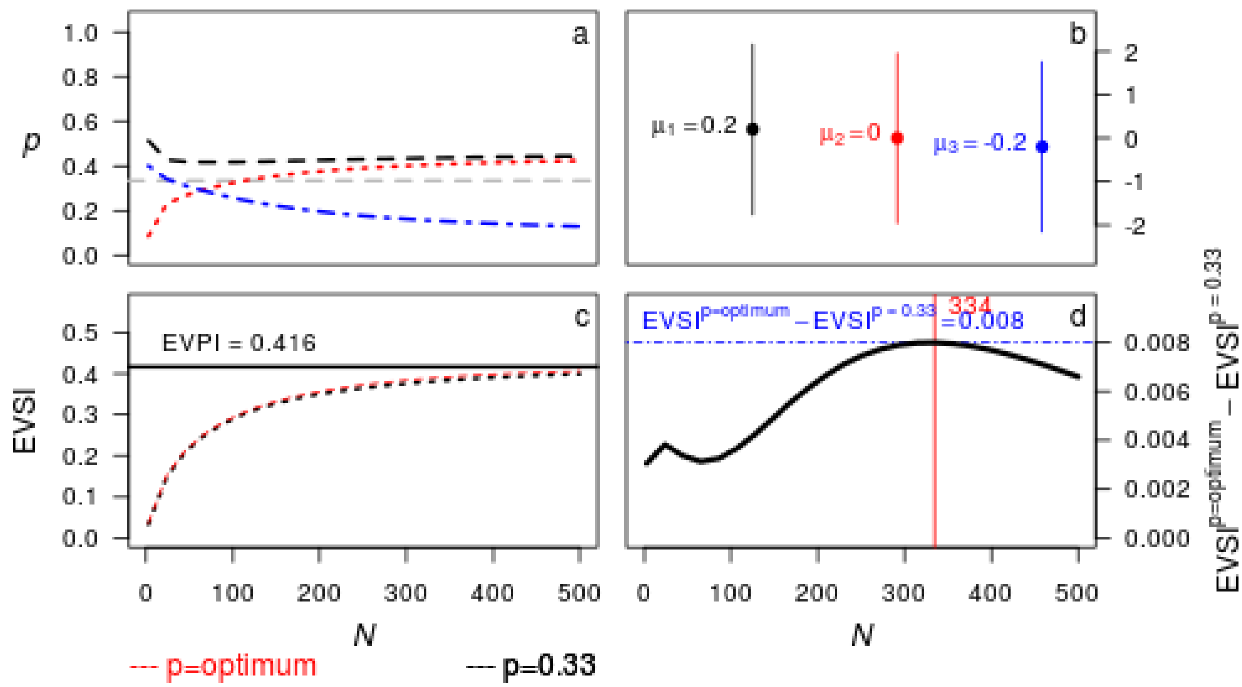
Case study 2b: homogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 = \mu_3$



Case study 2c: homogenous prior σ's and heterogenous prior means

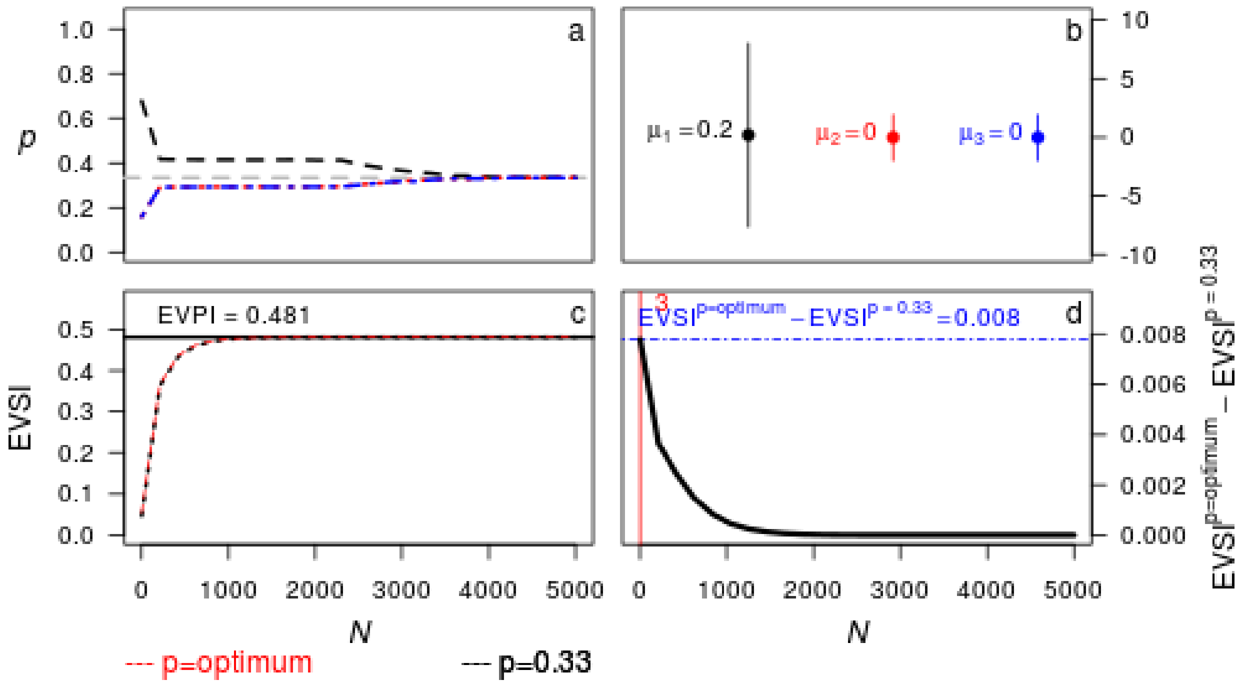
$\mu_1 > \mu_2 > \mu_3$



Case study 3a: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 = \mu_3$

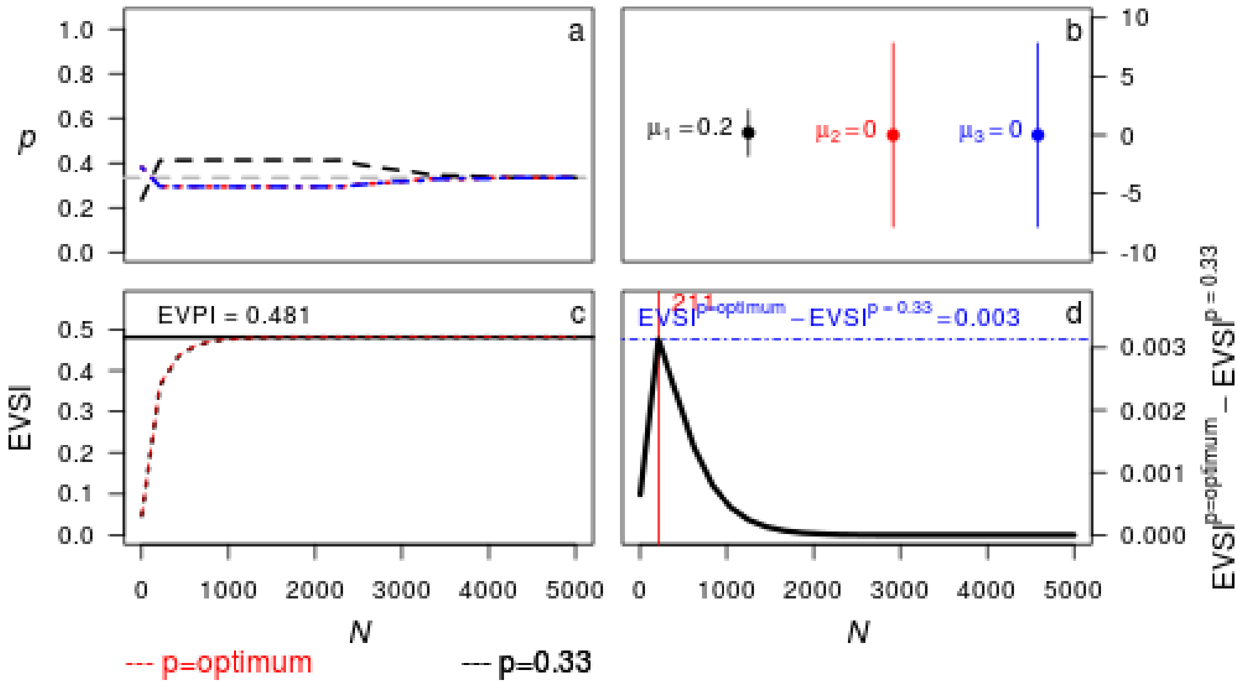
$\sigma_1 > \sigma_2 = \sigma_3$



Case study 3b: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 = \mu_3$

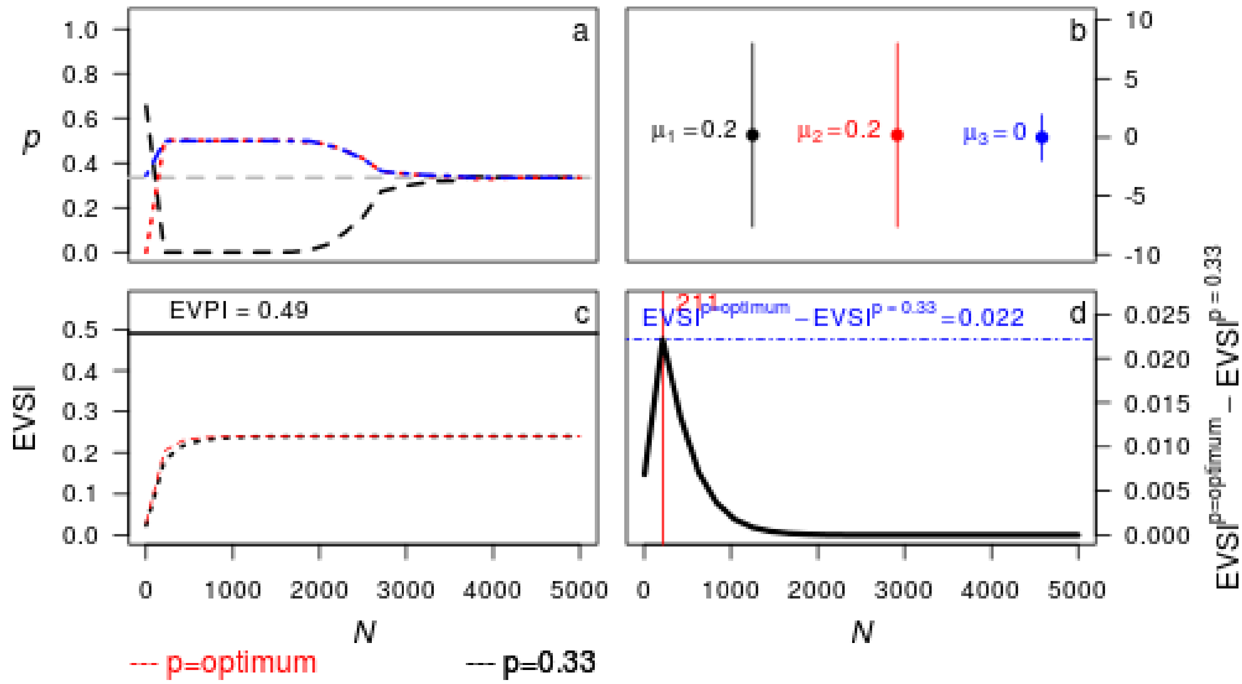
$\sigma_1 < \sigma_2 = \sigma_3$



Case study 3c: heterogenous prior σ's and heterogenous prior means

$\mu_1 = \mu_2 > \mu_3$

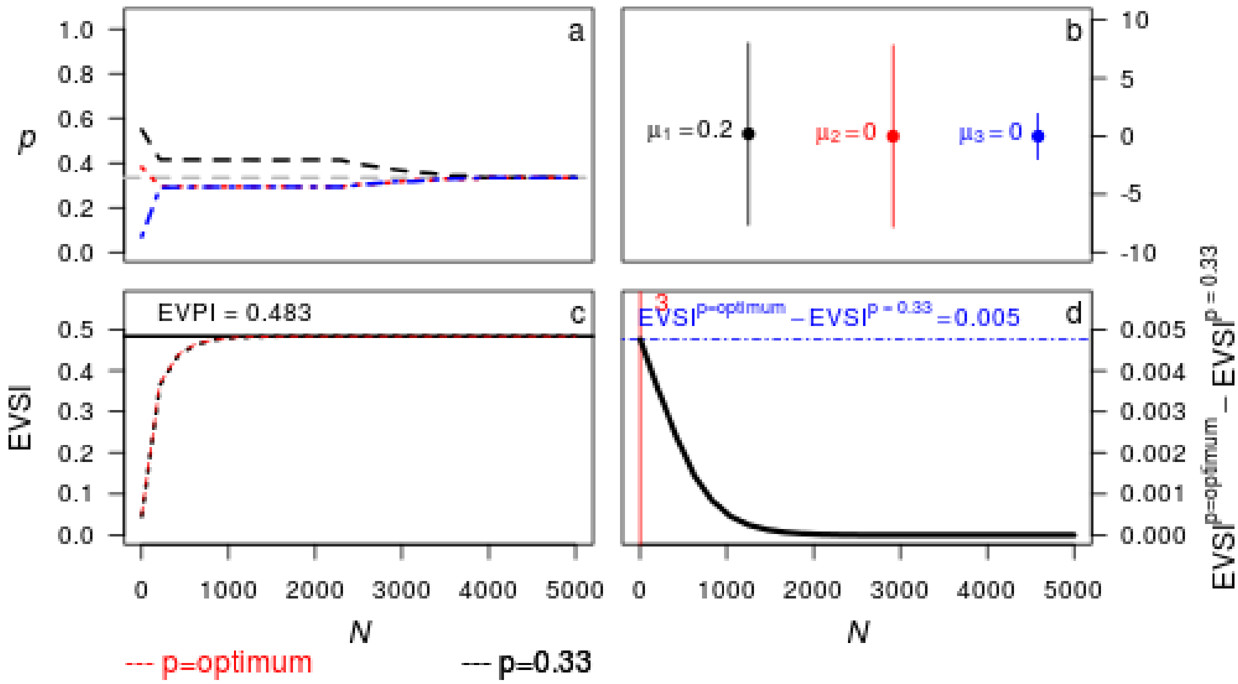
$\sigma_1 = \sigma_2 < \sigma_3$



Case study 3d: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 = \mu_3$

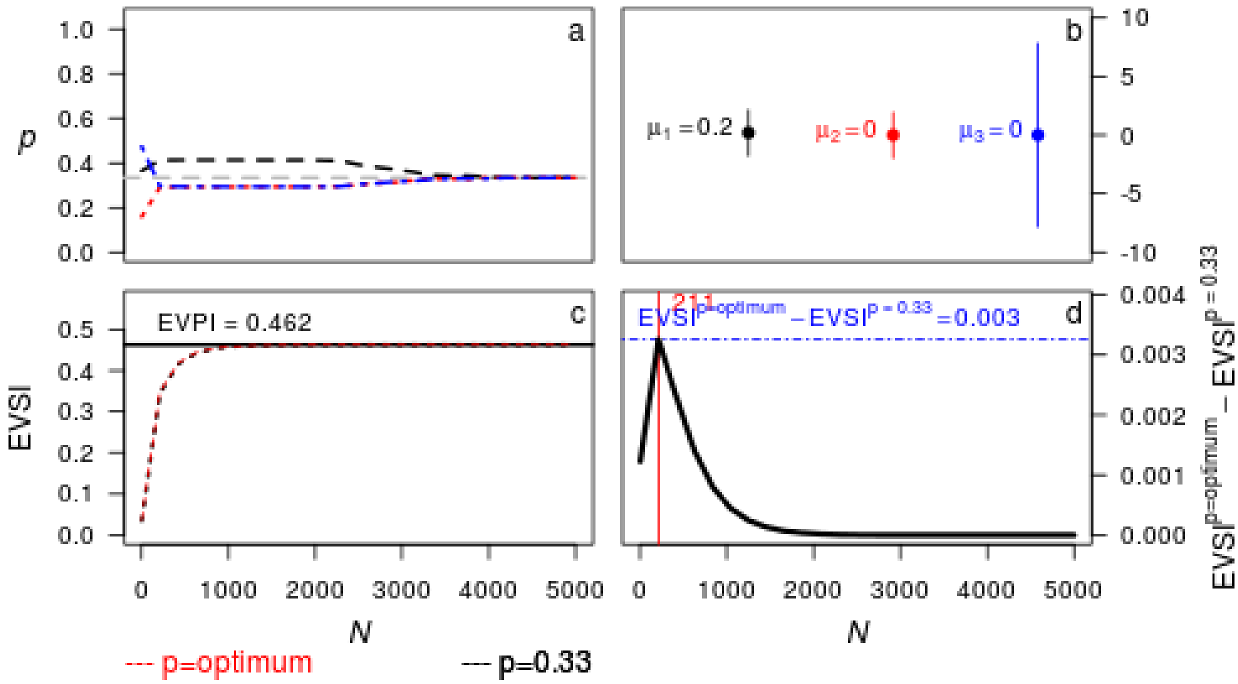
$\sigma_1 = \sigma_2 > \sigma_3$



Case study 3e: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 = \mu_3$

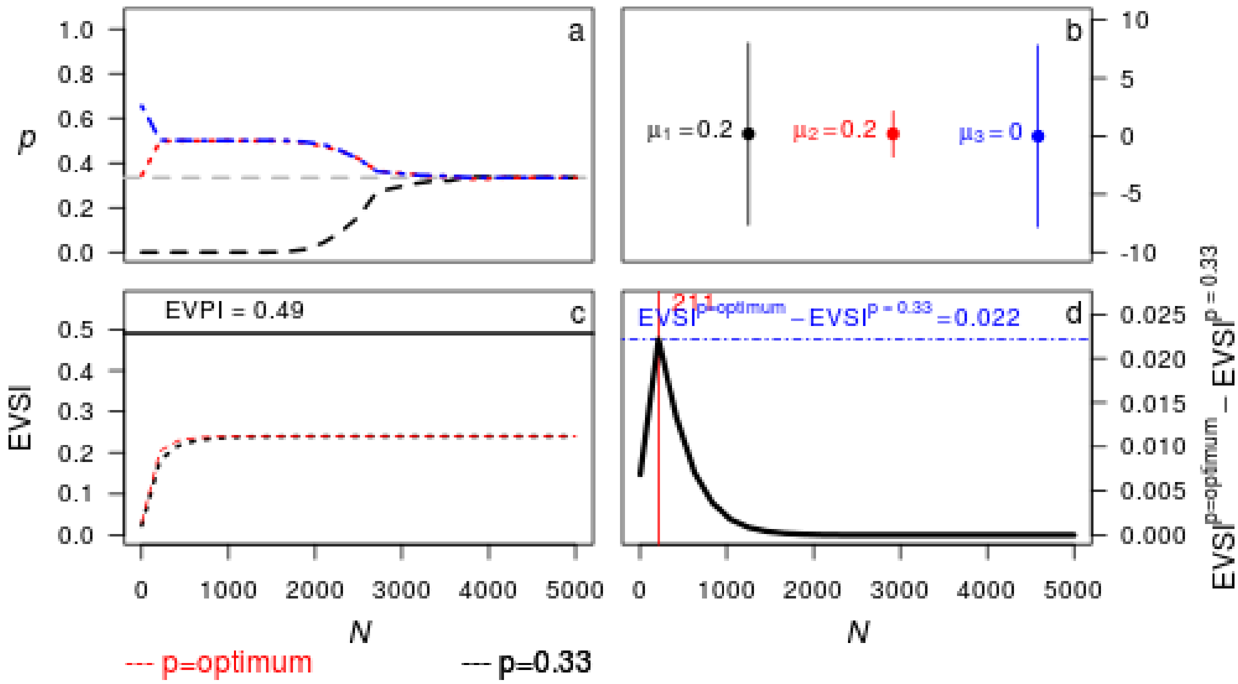
$\sigma_1 = \sigma_2 < \sigma_3$



Case study 3f: heterogenous prior σ's and heterogenous prior means

$\mu_1 = \mu_2 > \mu_3$

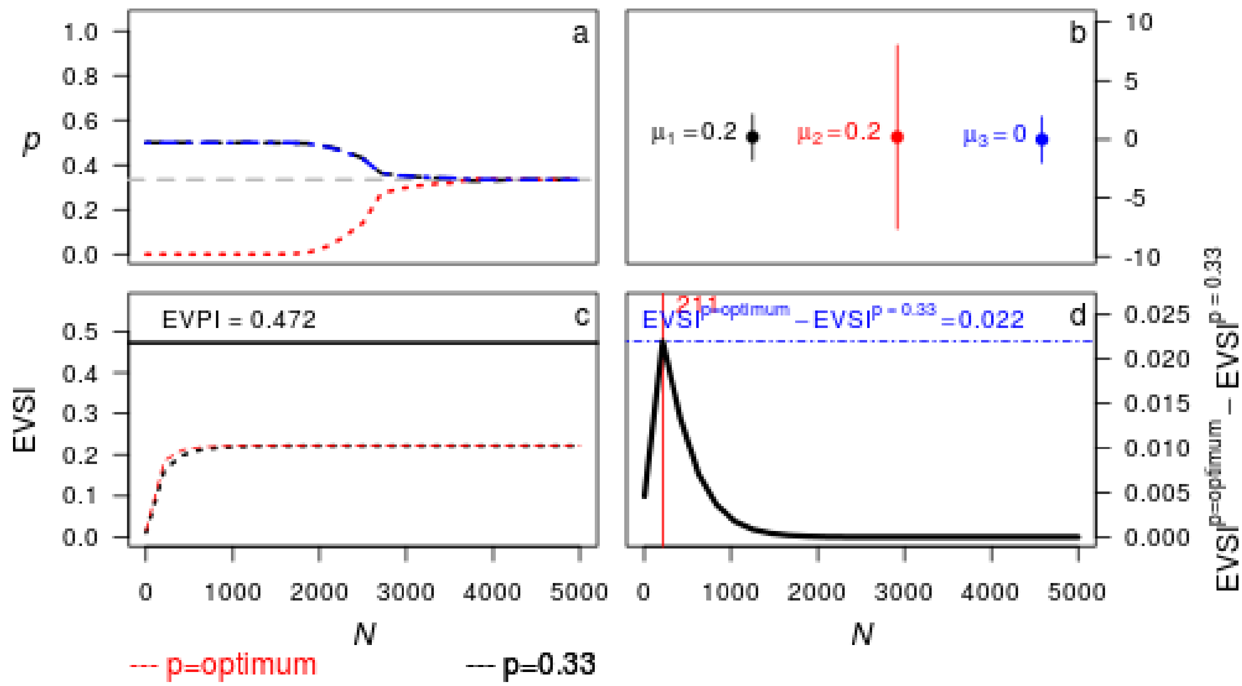
$\sigma_1 = \sigma_3 > \sigma_2$



Case study 3g: heterogenous prior σ's and heterogenous prior means

$\mu_1 = \mu_2 > \mu_3$

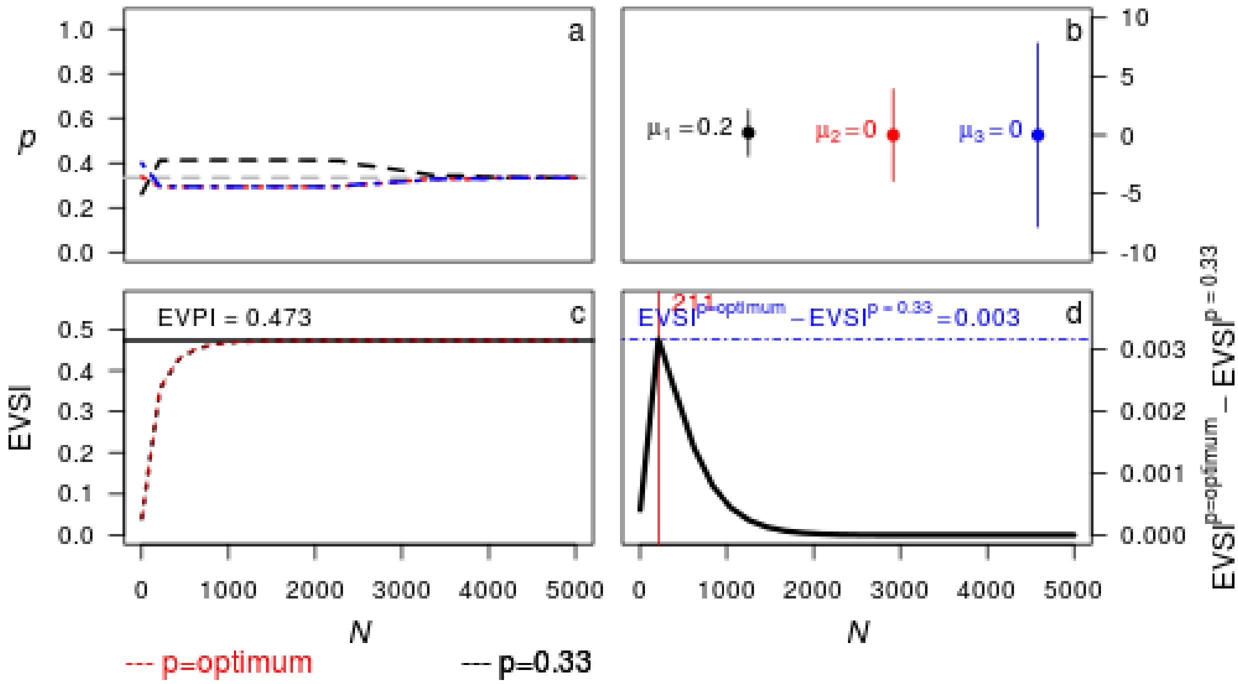
$\sigma_1 = \sigma_3 < \sigma_2$



541 **Case study 3h: heterogenous prior σ's and heterogenous prior means**

542 $\mu_1 > \mu_2 = \mu_3$

543 $\sigma_1 < \sigma_2 < \sigma_3$



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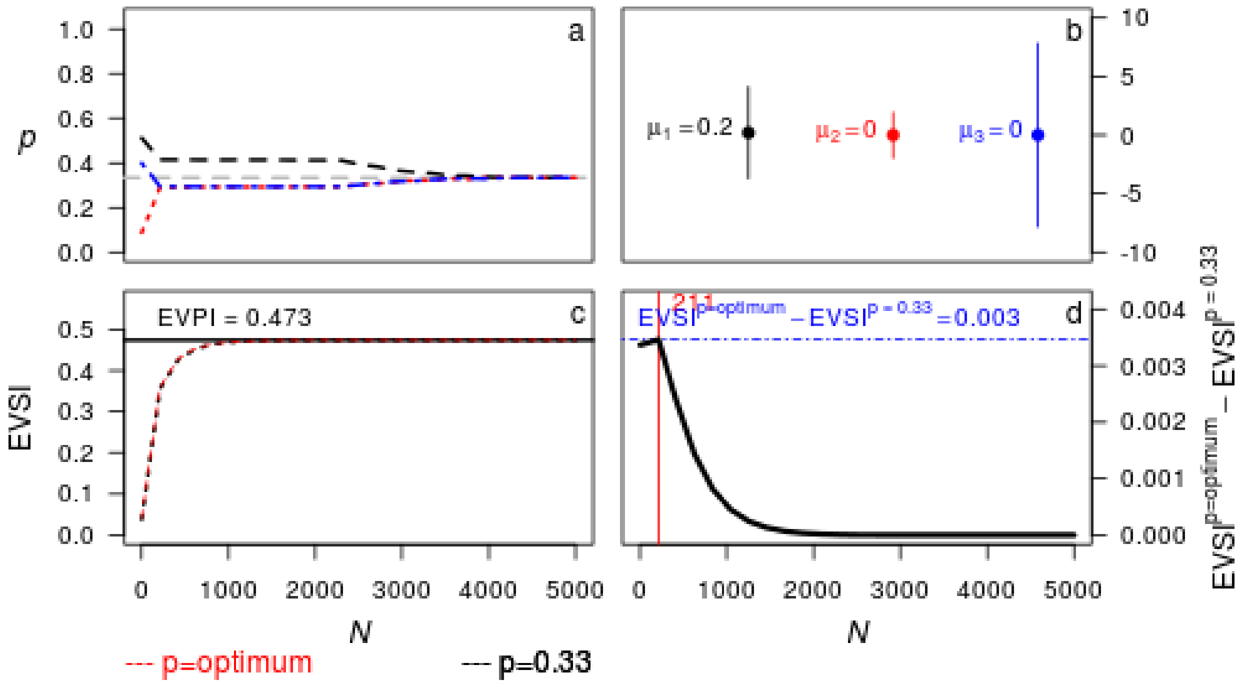
Case study 3i: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 = \mu_3$

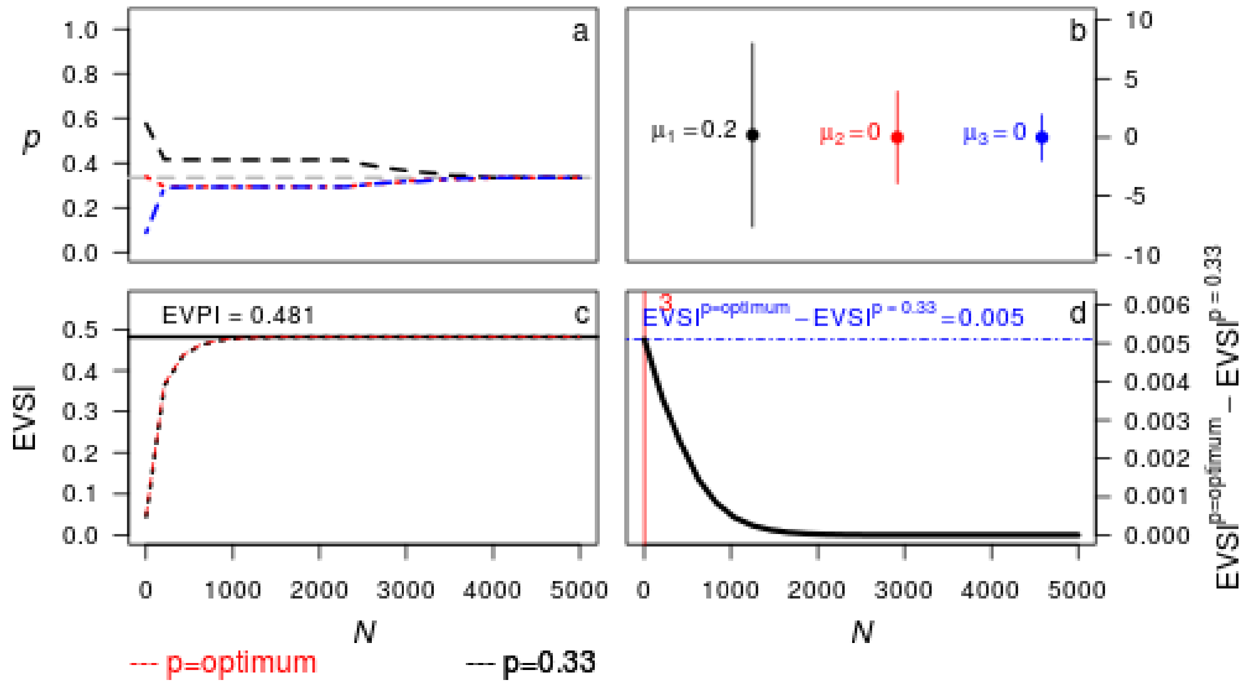
$\sigma_2 < \sigma_1 < \sigma_3$



Case study 3j: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 = \mu_3$

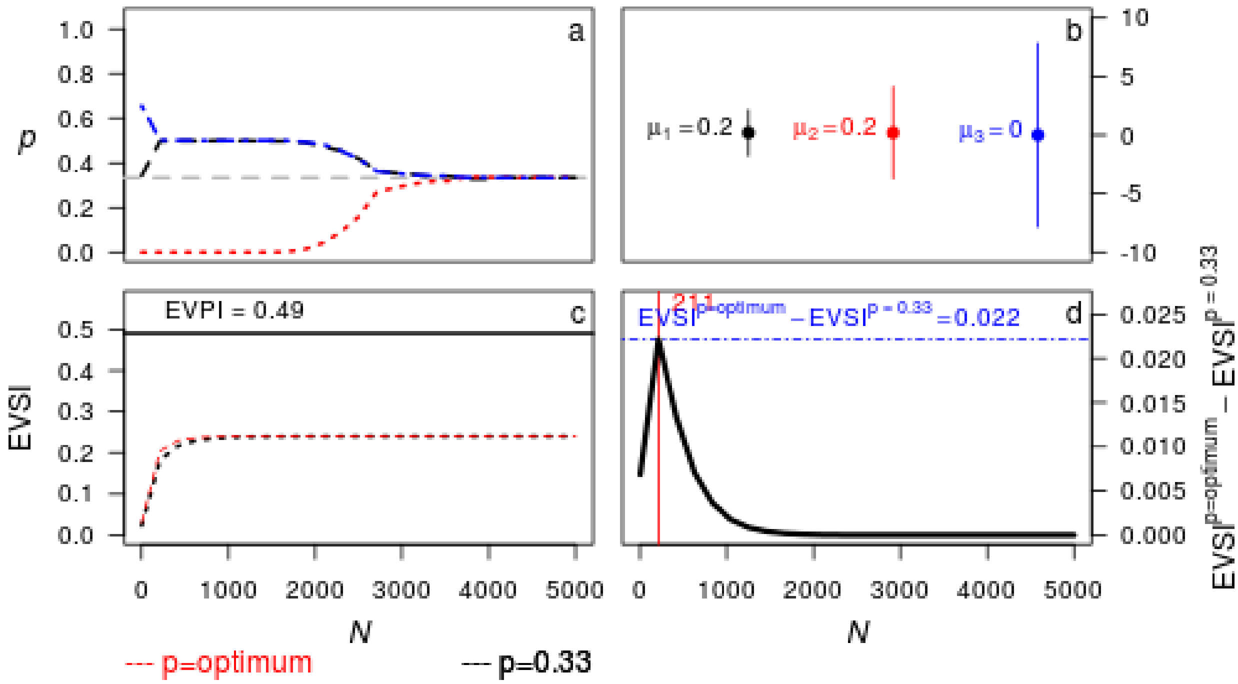
$\sigma_1 > \sigma_2 > \sigma_3$



Case study 3k: heterogenous prior σ's and heterogenous prior means

$\mu_1 = \mu_2 > \mu_3$

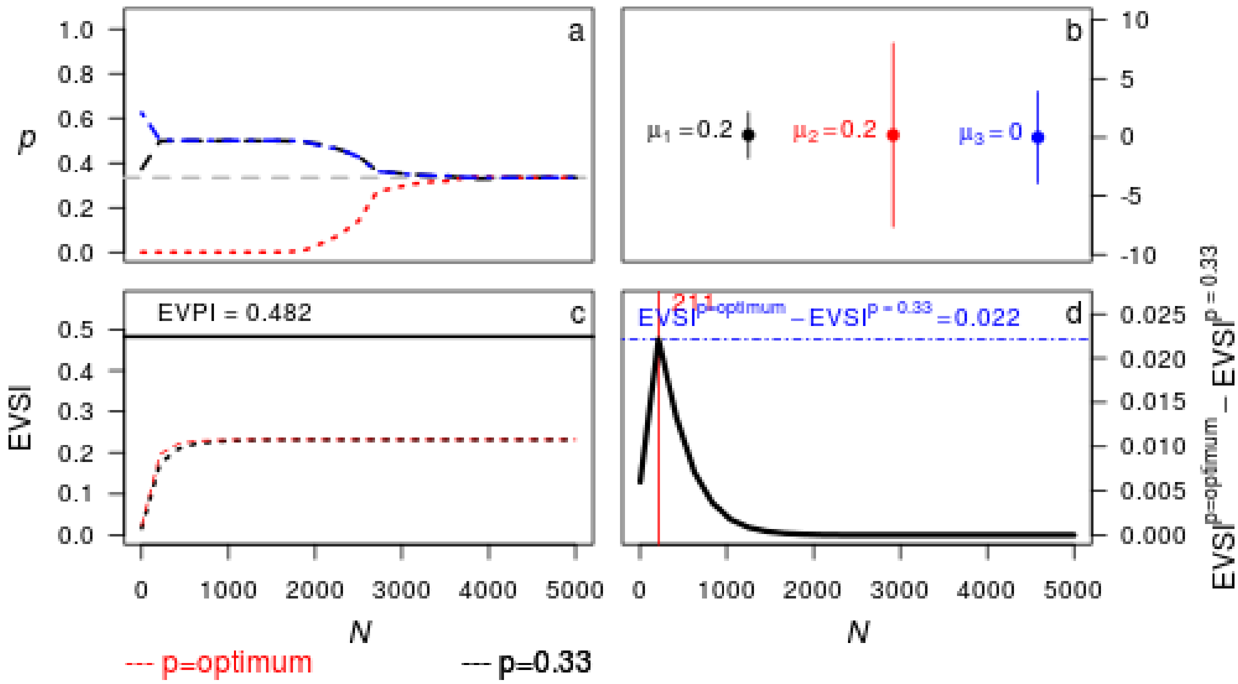
$\sigma_1 < \sigma_2 < \sigma_3$



Case study 3l: heterogenous prior σ's and heterogenous prior means

$\mu_1 = \mu_2 > \mu_3$

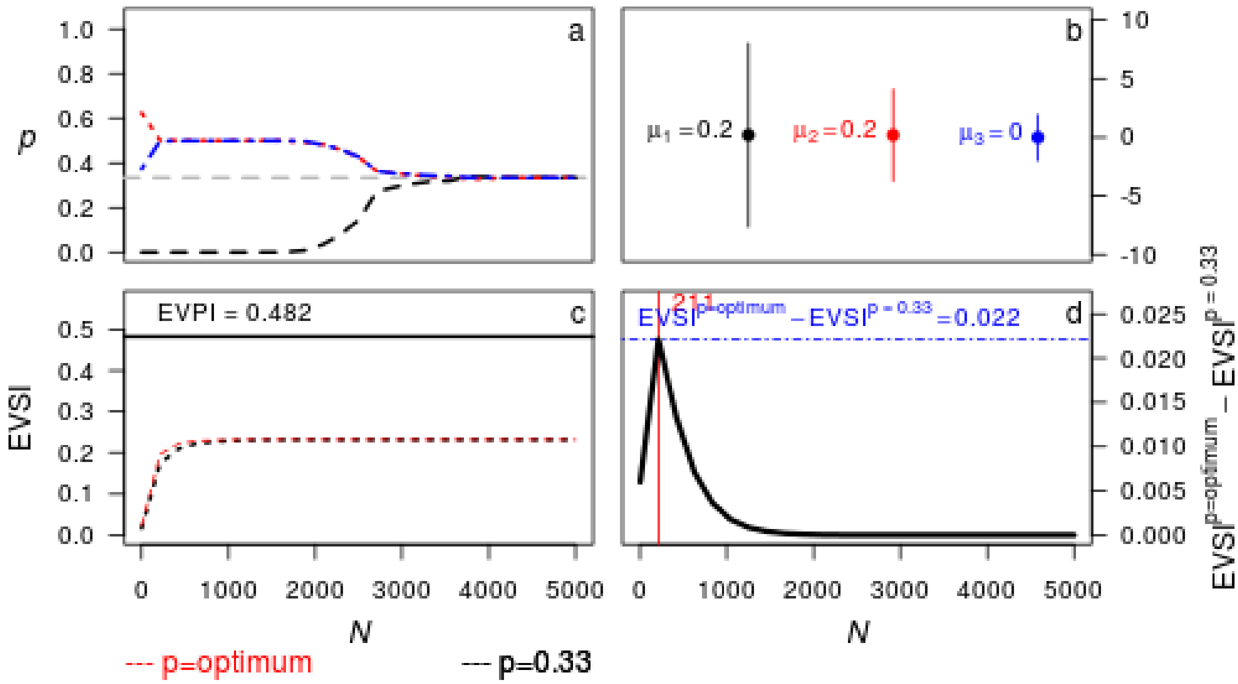
$\sigma_1 < \sigma_3 < \sigma_2$



Case study 3m: heterogenous prior σ's and heterogenous prior means

$\mu_1 = \mu_2 > \mu_3$

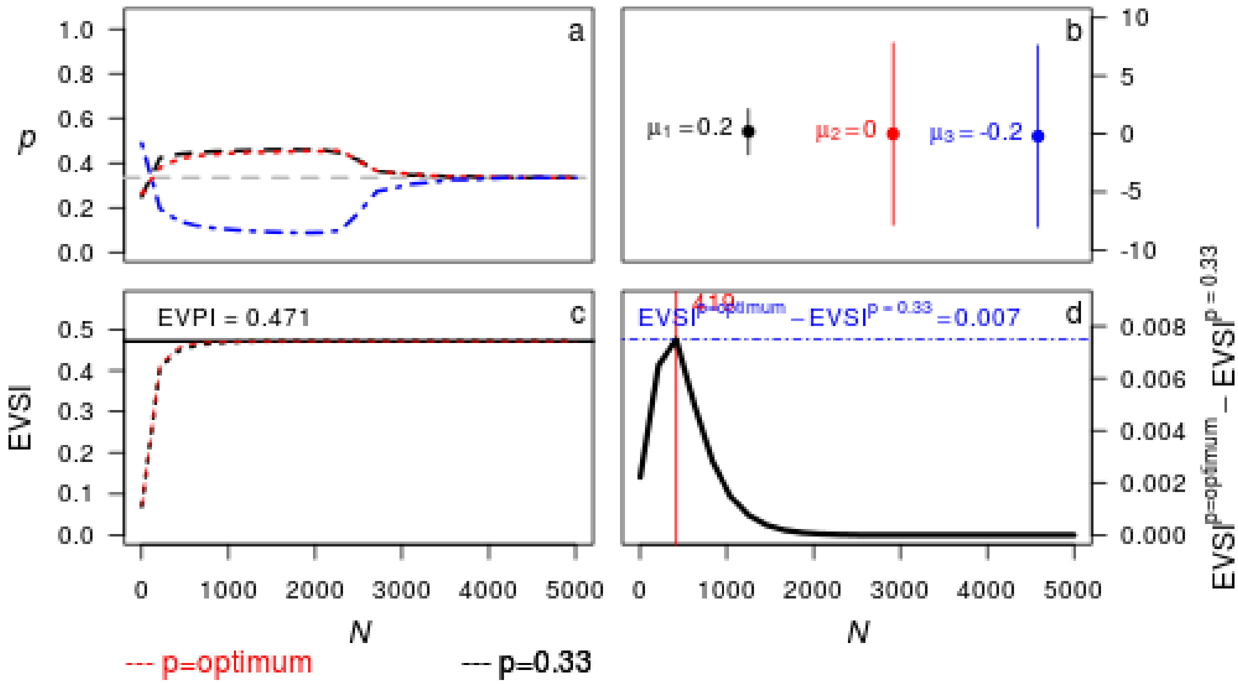
$\sigma_1 > \sigma_2 > \sigma_3$



Case study 3n: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 > \mu_3$

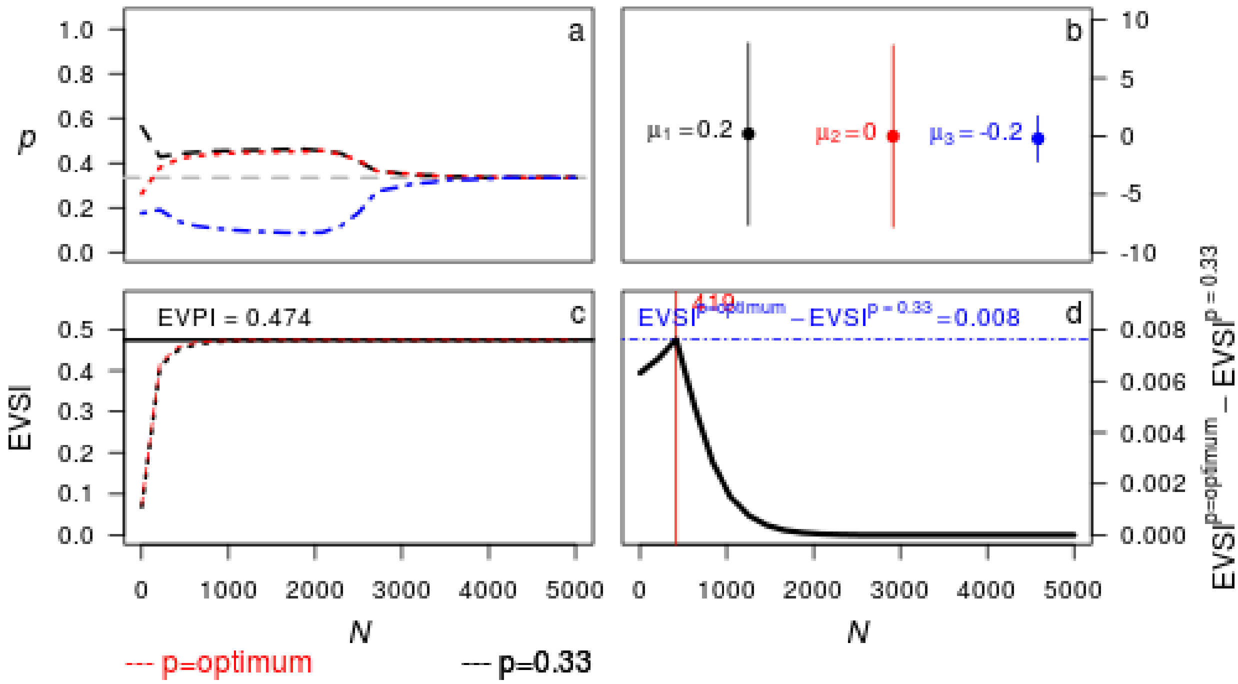
$\sigma_1 < \sigma_2 = \sigma_3$



Case study 3o: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 > \mu_3$

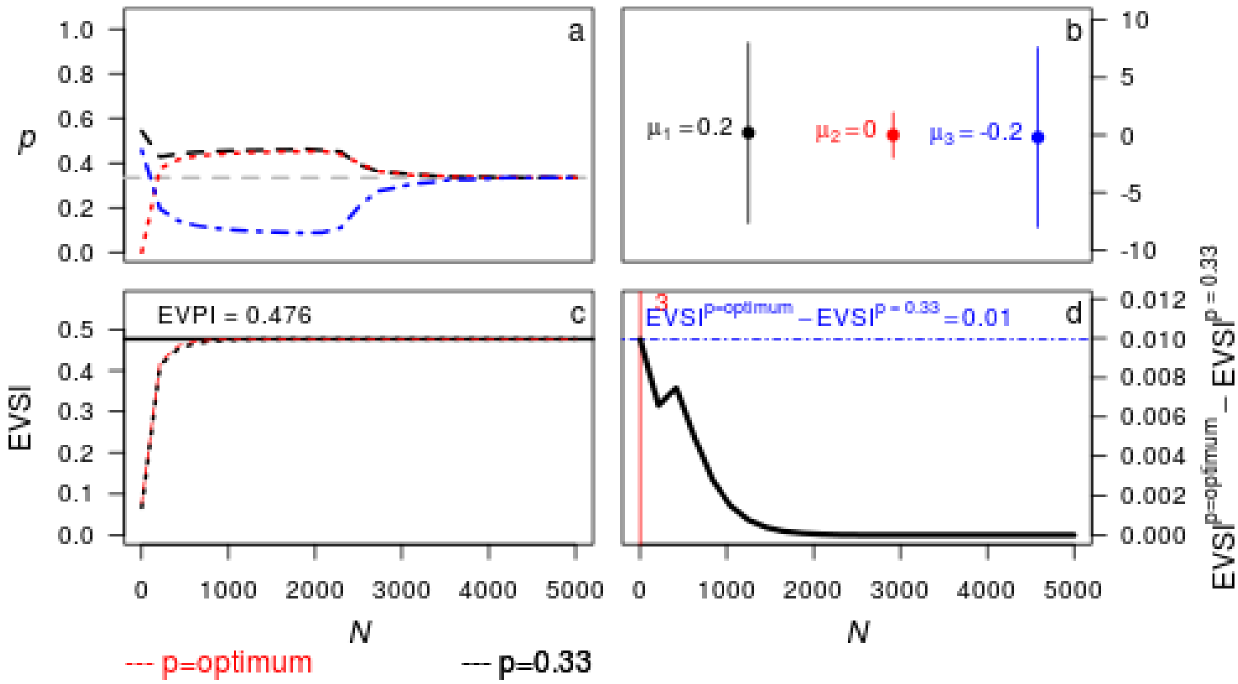
$\sigma_1 = \sigma_2 < \sigma_3$



Case study 3p: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 > \mu_3$

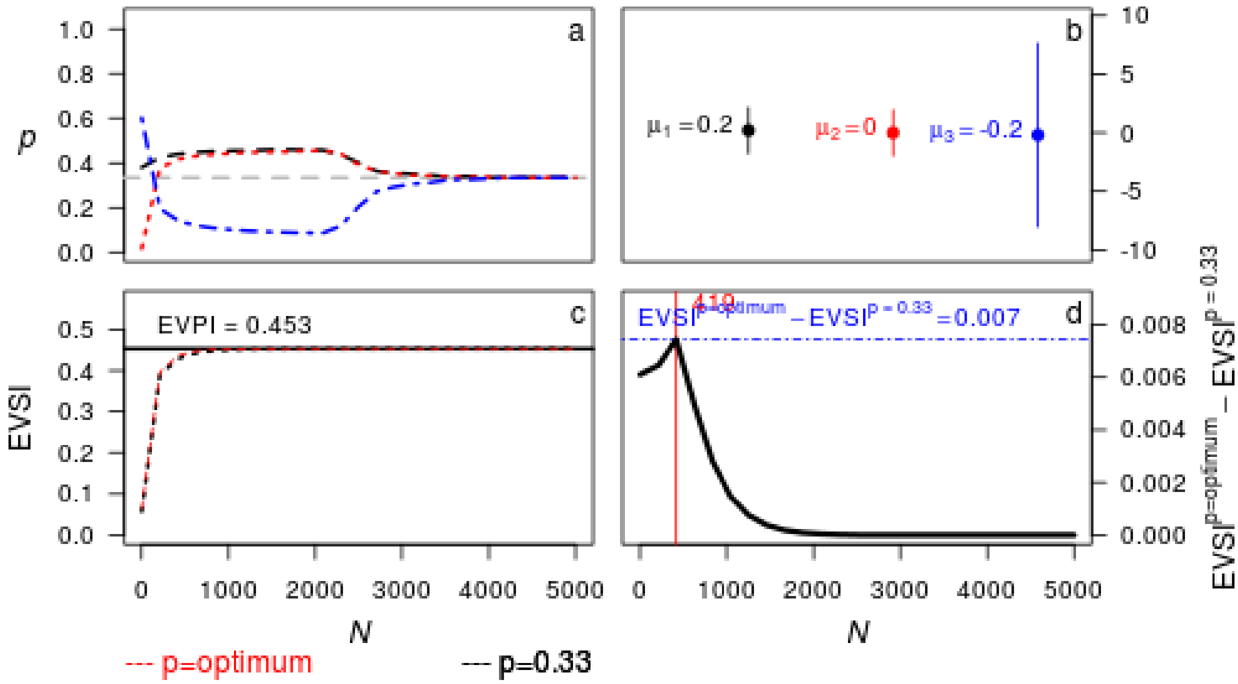
$\sigma_1 = \sigma_3 > \sigma_2$



Case study 3q: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 > \mu_3$

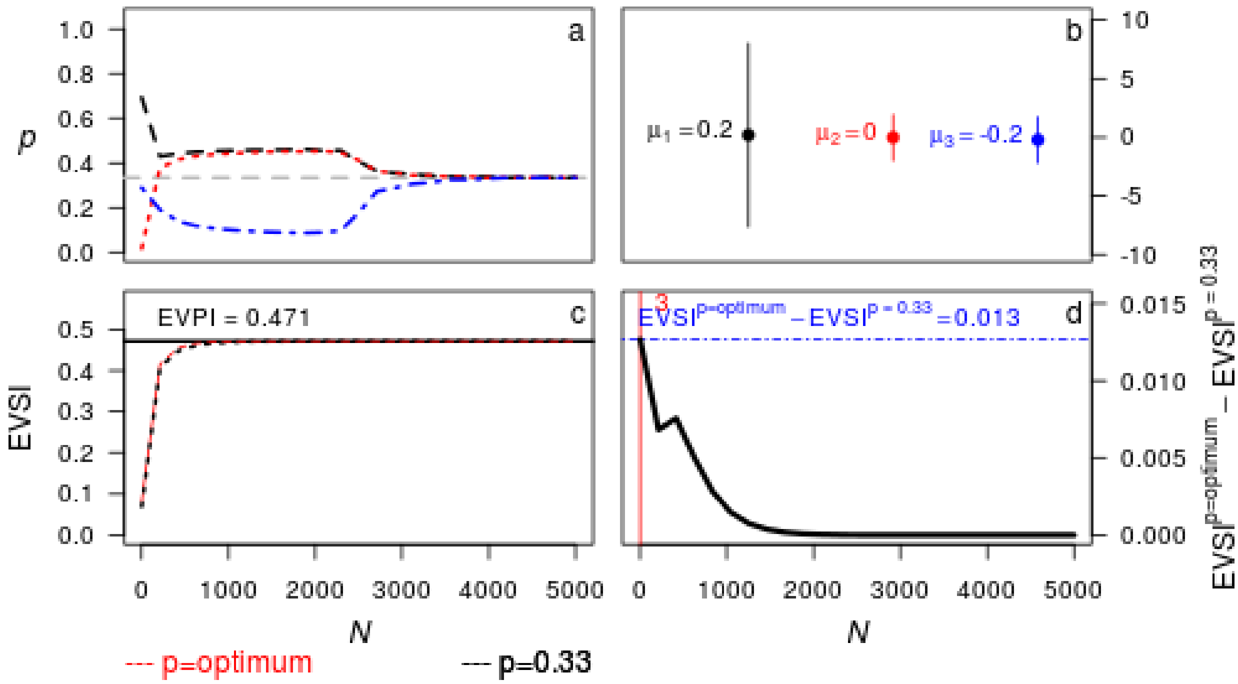
$\sigma_1 = \sigma_2 < \sigma_3$



581 **Case study 3r: heterogenous prior σ's and heterogenous prior means**

582 $\mu_1 > \mu_2 > \mu_3$

583 $\sigma_1 > \sigma_2 = \sigma_3$

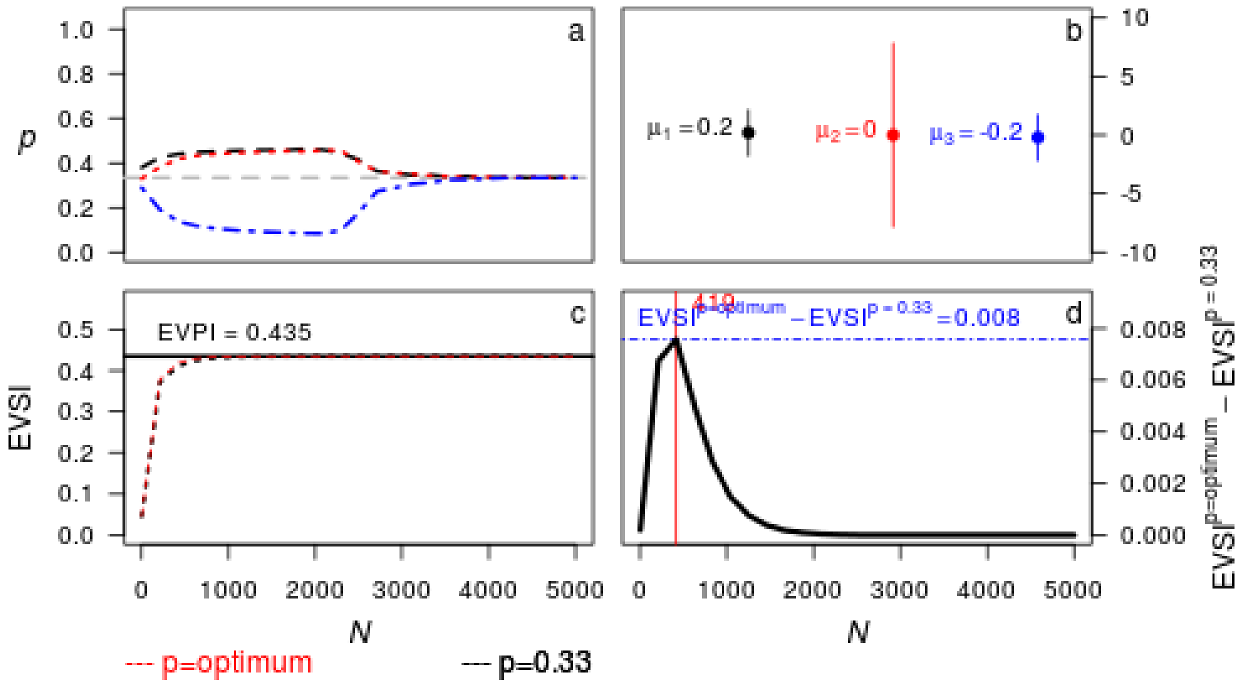


584

Case study 3s: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 > \mu_3$

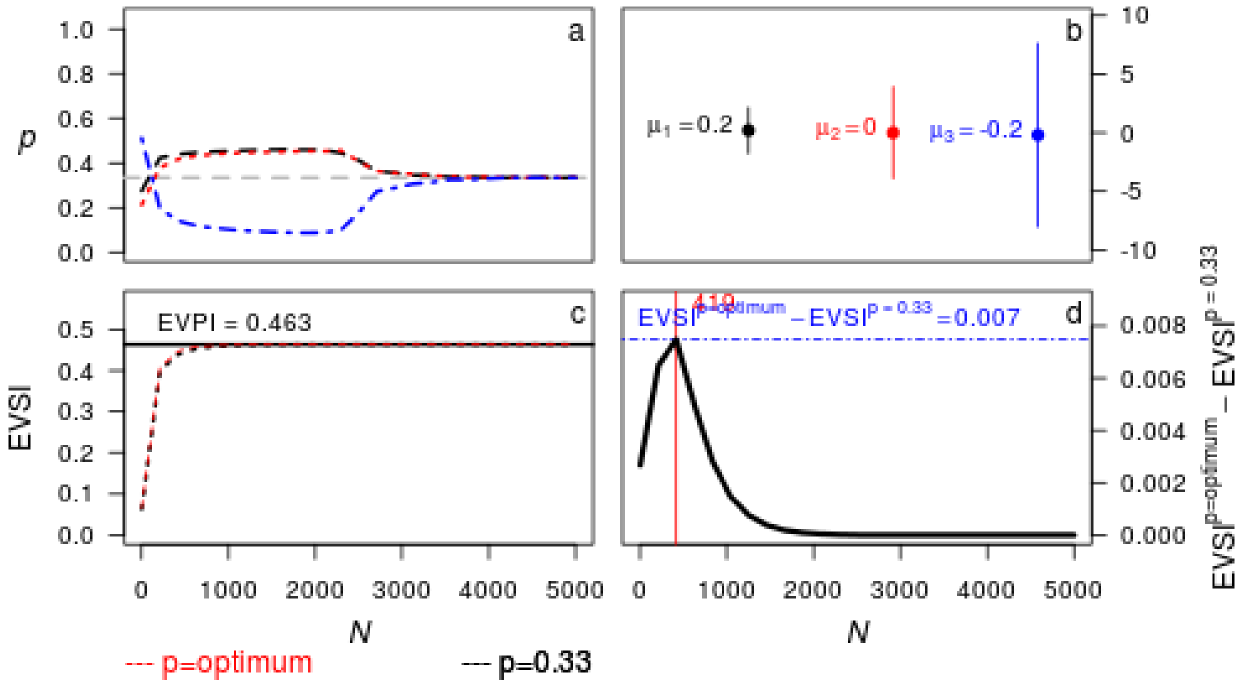
$\sigma_1 = \sigma_3 < \sigma_2$



Case study 3t: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 > \mu_3$

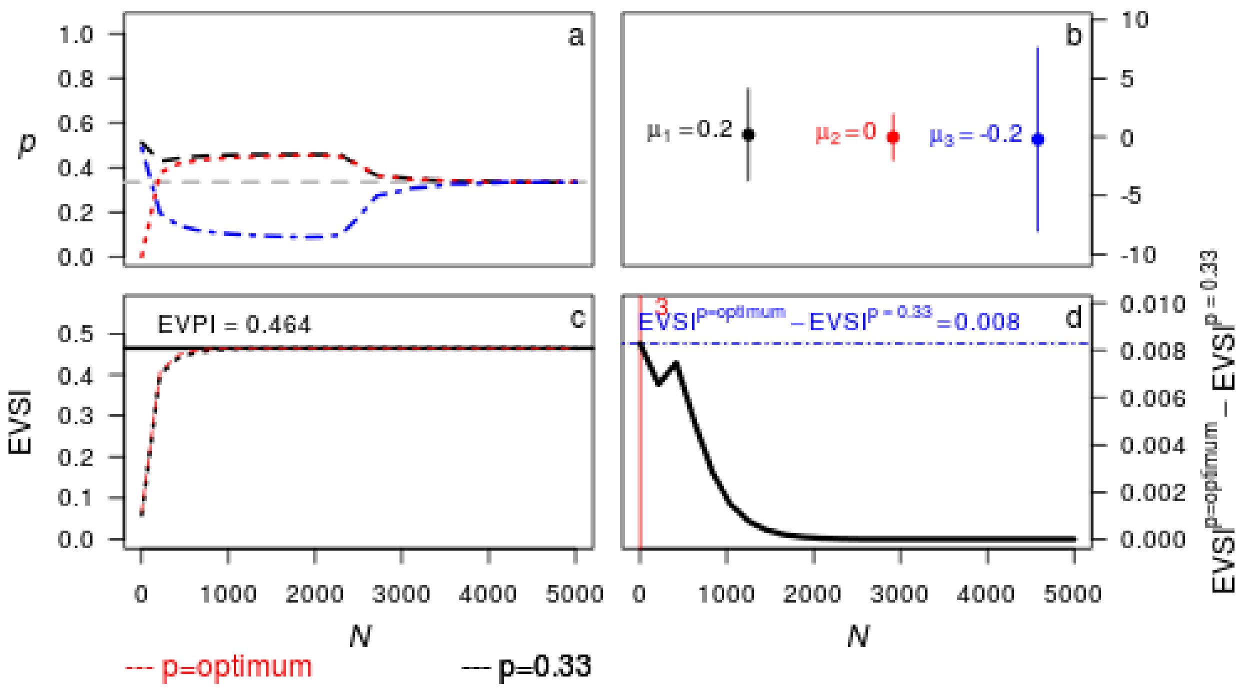
$\sigma_1 < \sigma_2 < \sigma_3$



Case study 3u: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 > \mu_3$

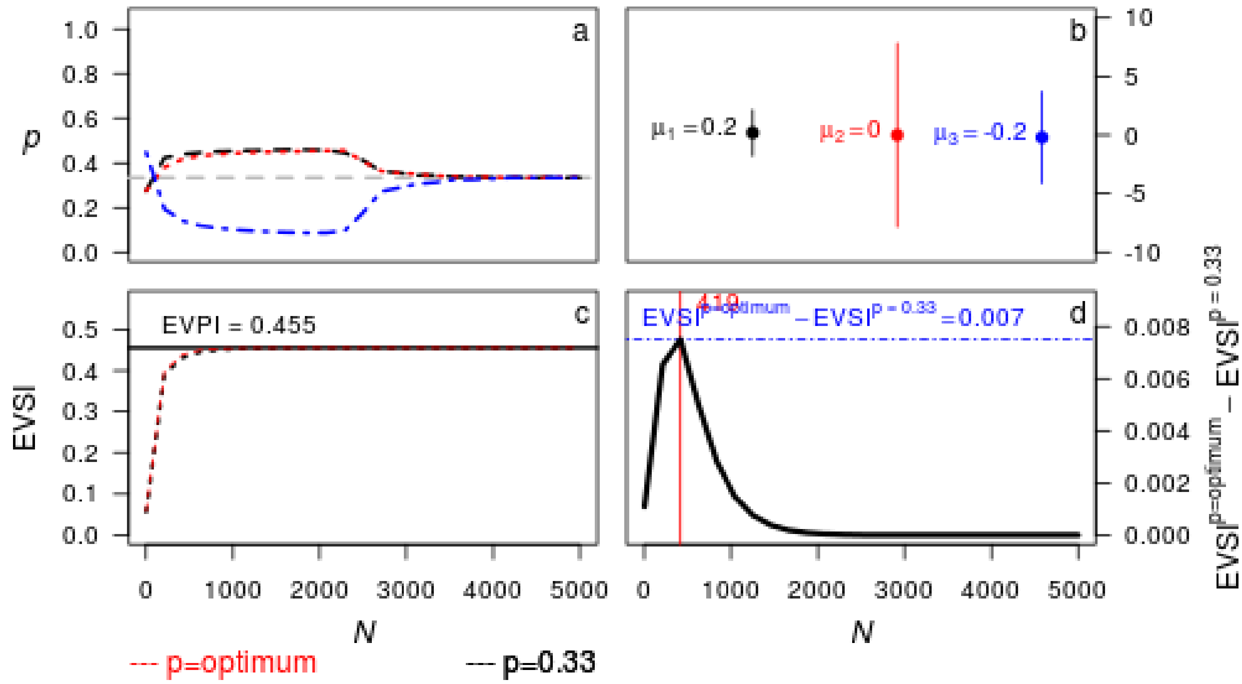
$\sigma_2 < \sigma_1 < \sigma_3$



597 **Case study 3v: heterogenous prior σ's and heterogenous prior means**

598 $\mu_1 > \mu_2 > \mu_3$

599 $\sigma_1 < \sigma_3 < \sigma_2$

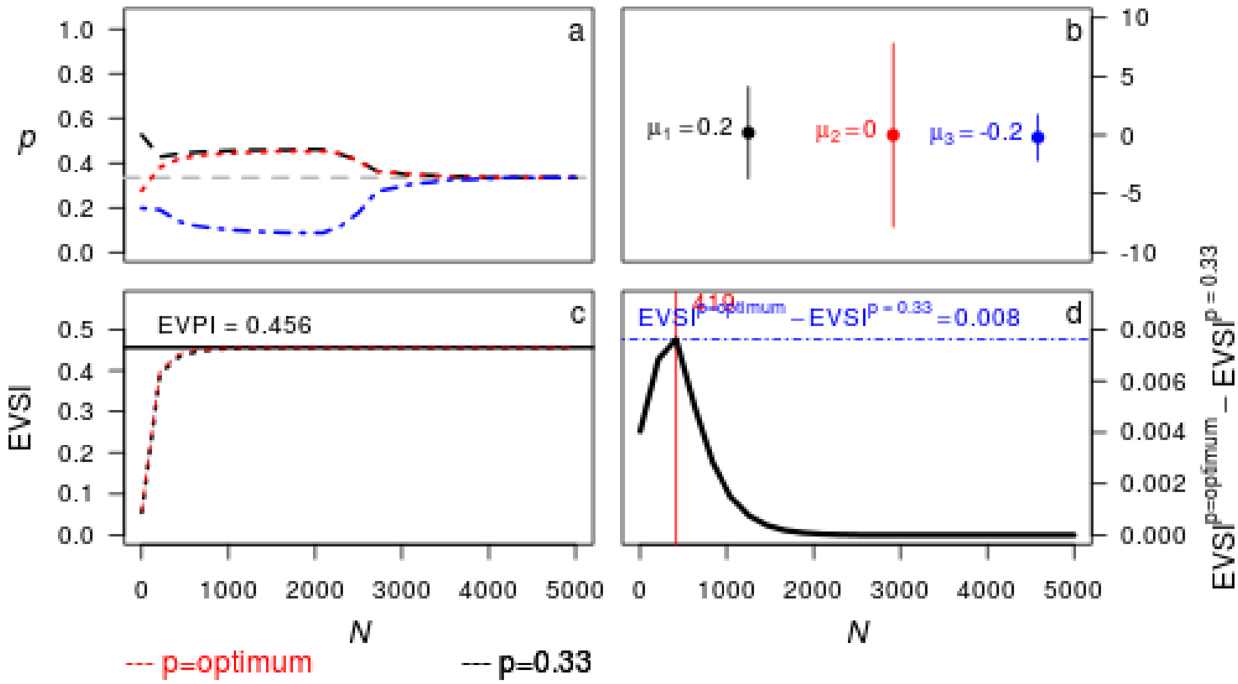


600

Case study 3w: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 > \mu_3$

$\sigma_1 > \sigma_3 > \sigma_2$



Case study 3x: heterogenous prior σ's and heterogenous prior means

$\mu_1 > \mu_2 > \mu_3$

$\sigma_1 > \sigma_2 > \sigma_3$