Hint-based typing for STLC with

Wojciech Kołowski

Terms

Terms:

$e ::=$

$x \mid (e : H) \mid$

$\lambda x.\, e \mid e_1\ e_2 \mid$

$\texttt{pair} \mid \texttt{outl} \mid \texttt{outr} \mid$

$\texttt{inl} \mid \texttt{inr} \mid \texttt{case} \mid$

$\texttt{unit} \mid \texttt{exfalso}$

Judgements:

$\Gamma \vdash e \Leftarrow H \Rightarrow A$ – in context Γ, term e checks with hint H and infers type A

Declarative typing – basics

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A}\textsc{Var}$$

$$\frac{\Gamma \vdash e : A \quad H \sqsubseteq A}{\Gamma \vdash (e : H) : A}\textsc{Annot}$$

Declarative typing – type-directed rules

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x.\, e : A \to B} \qquad \frac{\Gamma \vdash a : A \quad \Gamma \vdash f : A \to B}{\Gamma \vdash f\ a : B}$$

$$\frac{}{\Gamma \vdash \texttt{pair} : A \to B \to A \times B}$$

$$\frac{}{\Gamma \vdash \texttt{outl} : A \times B \to A} \qquad \frac{}{\Gamma \vdash \texttt{outr} : A \times B \to B}$$

$$\frac{}{\Gamma \vdash \texttt{inl} : A \to A + B} \qquad \frac{}{\Gamma \vdash \texttt{inr} : B \to A + B}$$

$$\frac{}{\Gamma \vdash \texttt{case} : (A \to C) \to (B \to C) \to A + B \to C}$$

$$\frac{}{\Gamma \vdash \texttt{unit} : \mathbf{1}} \qquad \frac{}{\Gamma \vdash \texttt{exfalso} : \mathbf{0} \to A}$$

Hints for term constructors

$$\texttt{hint}(\lambda x.\, e) = \mathbf{?} \to \mathbf{?}$$
$$\texttt{hint}(\texttt{pair}) = \mathbf{?} \to \mathbf{?} \to \mathbf{?} \times \mathbf{?}$$
$$\texttt{hint}(\texttt{outl}) = \mathbf{?} \times \mathbf{?} \to \mathbf{?}$$
$$\texttt{hint}(\texttt{outr}) = \mathbf{?} \times \mathbf{?} \to \mathbf{?}$$
$$\texttt{hint}(\texttt{inl}) = \mathbf{?} \to \mathbf{?} + \mathbf{?}$$
$$\texttt{hint}(\texttt{inr}) = \mathbf{?} \to \mathbf{?} + \mathbf{?}$$
$$\texttt{hint}(\texttt{case}) = (\mathbf{?} \to \mathbf{?}) \to (\mathbf{?} \to \mathbf{?}) \to \mathbf{?} + \mathbf{?} \to \mathbf{?}$$
$$\texttt{hint}(\texttt{exfalso}) = \mathbf{0} \to \mathbf{?}$$
$$\texttt{hint}(\texttt{unit}) = \mathbf{1}$$

Combining a hint with a term

$$e \sqcup ? = \texttt{hint}(e)$$
$$\lambda x.\, e \sqcup H_1 \to H_2 = H_1 \to H_2$$
$$\texttt{pair} \sqcup H_1 \to H_2 \to H_3 \times H_4 = H_1 \sqcup H_3 \to H_2 \sqcup H_4 \to (H_1 \sqcup H_3) \times (H_2 \sqcup H_4)$$
$$\texttt{outl} \sqcup H_1 \times H_2 \to H_3 = (H_1 \sqcup H_3) \times H_2 \to H_1 \sqcup H_3$$
$$\texttt{outr} \sqcup (H_1 \times H_2) \to H_3 = H_1 \times (H_2 \sqcup H_3) \to H_2 \sqcup H_3$$
$$\texttt{inl} \sqcup H_1 \to H_2 + H_3 = H_1 \sqcup H_2 \to (H_1 \sqcup H_2) + H_3$$
$$\texttt{inr} \sqcup H_1 \to H_2 + H_3 = H_1 \sqcup H_3 \to H_2 + (H_1 \sqcup H_3)$$
$$\texttt{case} \sqcup (H_1 \to H_2) \to (H_3 \to H_4) \to H_5 + H_6 \to H_7 = (H_1 \sqcup H_5 \to H_2) \to (H_3 \sqcup H_6 \to H_4) \to (H_1 \sqcup H_5) + (H_3 \sqcup H_6) \to H_7$$
$$\texttt{exfalso} \sqcup H_1 \to H_2 = H_1 \sqcup \mathbf{0} \to H_2$$
$$\texttt{unit} \sqcup \mathbf{1} = \mathbf{1}$$

Hinting – basic rules

$$\frac{(x : A) \in \Gamma \quad H \sqsubseteq A}{\Gamma \vdash x \Leftarrow H \Rightarrow A}\textsc{Var}$$

$$\frac{\Gamma \vdash e \Leftarrow H_1 \sqcup H_2 \Rightarrow A}{\Gamma \vdash (e : H_1) \Leftarrow H_2 \Rightarrow A}\textsc{Annot}$$

$$\frac{\Gamma \vdash e \Leftarrow e \sqcup H \Rightarrow A \quad e \sqcup H \texttt{ defined} \quad e \sqcup H \neq H}{\Gamma \vdash e \Leftarrow H \Rightarrow A}\textsc{Hole}$$

Hinting – type-directed rules

$$\frac{\Gamma, x : A \vdash e \Leftarrow H \Rightarrow B}{\Gamma \vdash \lambda x.\, e \Leftarrow A \to H \Rightarrow A \to B}$$

$$\frac{\Gamma \vdash a \Leftarrow \mathbf{?} \Rightarrow A \quad \Gamma \vdash f \Leftarrow A \to H \Rightarrow A \to B}{\Gamma \vdash f\ a \Leftarrow H \Rightarrow B}$$

$$\frac{}{\Gamma \vdash \texttt{pair} \Leftarrow A \to B \to A \times B \Rightarrow A \to B \to A \times B}$$

$$\frac{}{\Gamma \vdash \texttt{outl} \Leftarrow A \times B \to A \Rightarrow A \times B \to A}$$

$$\frac{}{\Gamma \vdash \texttt{outr} \Leftarrow A \times B \to B \Rightarrow A \times B \to B}$$

Hinting – type-directed rules

$$\frac{}{\Gamma \vdash \texttt{inl} \Leftarrow A \to A + B \Rightarrow A \to A + B}$$

$$\frac{}{\Gamma \vdash \texttt{inr} \Leftarrow B \to A + B \Rightarrow B \to A + B}$$

$$\frac{}{\Gamma \vdash \texttt{case} \Leftarrow (A \to C) \to (B \to C) \to A + B \to C \Rightarrow (A \to C) \to (B \to$$

$$\frac{}{\Gamma \vdash \texttt{unit} \Leftarrow \mathbf{1} \Rightarrow \mathbf{1}} \qquad \frac{}{\Gamma \vdash \texttt{exfalso} \Leftarrow \mathbf{0} \to A \Rightarrow \mathbf{0} \to A}$$