Hint-based typing for polymorphism

Types and judgements

Types:
$A, B ::= \alpha \mid \forall \alpha.\, A \mid A \to B$

Contexts:
$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, \alpha$

Judgements:
$\Gamma \vdash e : A$, $\Gamma \vdash A\ \mathtt{type}$

Valid context judgement

$$\frac{}{\cdot\ \texttt{ctx}}\textsc{Ctx-Empty}$$

$$\frac{\Gamma\ \texttt{ctx} \quad x \notin \Gamma \quad \Gamma \vdash A\ \texttt{type}}{\Gamma, x : A\ \texttt{ctx}}\textsc{Ctx-Extend}$$

$$\frac{\Gamma\ \texttt{ctx} \quad \alpha \notin \Gamma}{\Gamma, \alpha\ \texttt{ctx}}\textsc{Ctx-ExtendType}$$

Valid type judgement

$$\frac{\Gamma\ \texttt{ctx} \quad \alpha \in \Gamma}{\Gamma \vdash \alpha\ \texttt{type}}\textsc{TyVar}$$

$$\frac{\Gamma, \alpha \vdash A\ \texttt{type}}{\Gamma \vdash \forall \alpha.\, A\ \texttt{type}}\textsc{All}$$

$$\frac{\Gamma \vdash A\ \texttt{type} \quad \Gamma \vdash B\ \texttt{type}}{\Gamma \vdash A \to B\ \texttt{type}}\textsc{Fun}$$

Trivial theorems

- If $\Gamma \vdash A\ \texttt{type}$, then $\Gamma\ \texttt{ctx}$
- If $\Gamma\ \texttt{ctx}$ and $(x : A) \in \Gamma$, then $\Gamma \vdash A\ \texttt{type}$
- We will set up all system so that if $\Gamma \vdash e : A$, then $\Gamma \vdash A\ \texttt{type}$

Terms

Terms:

$e ::=$

$\quad x \mid$

$\quad \lambda x : A.\, e \mid e_1 \; e_2 \mid$

$\quad \Lambda\alpha.\, e \mid e \; \texttt{@} A$

Declarative typing

$$\frac{\Gamma\ \texttt{ctx} \quad (x : A) \in \Gamma}{\Gamma \vdash x : A}\textsc{Var}$$

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x : A.\, e : A \to B} \qquad \frac{\Gamma \vdash f : A \to B \quad \Gamma \vdash a : A}{\Gamma \vdash f\ a : B}$$

$$\frac{\Gamma, \alpha \vdash e : A}{\Gamma \vdash \Lambda\alpha.\, e : \forall\alpha.\, A}$$

$$\frac{\Gamma \vdash e : \forall\alpha.\, A \quad \Gamma \vdash B\ \texttt{type}}{\Gamma \vdash e\ \texttt{@}B : A[\alpha := B]}$$

Algorithmic typing

$$\frac{\Gamma\ \texttt{ctx} \qquad (x : A) \in \Gamma}{\Gamma \vdash x \Rightarrow A}\textsc{Var}$$

$$\frac{\Gamma, x : A \vdash e \Rightarrow B}{\Gamma \vdash \lambda x : A.\, e \Rightarrow A \to B} \qquad \frac{\Gamma \vdash f \Rightarrow A \to B \qquad \Gamma \vdash a \Rightarrow A}{\Gamma \vdash f\ a \Rightarrow B}$$

$$\frac{\Gamma, \alpha \vdash e \Rightarrow A}{\Gamma \vdash \Lambda\alpha.\, e \Rightarrow \forall\alpha.\, A}$$

$$\frac{\Gamma \vdash e \Rightarrow \forall\alpha.\, A \qquad \Gamma \vdash B\ \texttt{type}}{\Gamma \vdash e\ \texttt{@}B \Rightarrow A[\alpha := B]}$$

Terms

Terms:

$e ::=$

$x \mid$

$\lambda x.\, e \mid \mathrm{app}_A\ e_1\ e_2 \mid$

$e\ @A$

Declarative typing

$$\frac{\Gamma\ \mathtt{ctx} \quad (x : A) \in \Gamma}{\Gamma \vdash x : A}\textsc{Var}$$

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$$\frac{\Gamma, \alpha \vdash e : A}{\Gamma \vdash e : \forall \alpha.\, A}$$

$$\frac{\Gamma \vdash e : \forall \alpha.\, A \quad \Gamma \vdash B\ \mathtt{type}}{\Gamma \vdash e\ \mathtt{@}B : A[\alpha := B]}$$

Algorithmic typing

$$\frac{\Gamma\ \texttt{ctx} \quad (x : A) \in \Gamma}{\Gamma \vdash x \Leftarrow A}\ \textsc{Var}$$

$$\frac{\Gamma, x : A \vdash e \Leftarrow B}{\Gamma \vdash \lambda x.\, e \Leftarrow A \to B} \qquad \frac{\Gamma \vdash f \Leftarrow A \to B \quad \Gamma \vdash a \Leftarrow A}{\Gamma \vdash \mathsf{app}_A\ f\ a \Leftarrow B}$$

$$\frac{\Gamma, \alpha \vdash e \Leftarrow A}{\Gamma \vdash e \Leftarrow \forall \alpha.\, A}$$

$$\frac{\Gamma \vdash e \Leftarrow \forall \alpha.\, A \quad \Gamma \vdash B\ \texttt{type}}{\Gamma \vdash e\ \texttt{@}B \Leftarrow A[\alpha := B]}$$

Terms

Terms:

$e ::=$

$\quad x \mid$

$\quad \lambda x.\, e \mid e_1\ e_2 \mid$

Declarative typing

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