

## Hinting for records and variants

# Types, hints and signatures

Signatures:

$$\Sigma ::= \emptyset \mid \Sigma, \ell : A$$

Types:

$$A, B ::= A \rightarrow B \mid \text{Record } \Sigma \mid \text{Variant } \Sigma$$

Hints:

$$H ::= ? \mid H_1 \rightarrow H_2 \mid \text{Record } \Sigma \mid \text{Variant } \Sigma$$

# Terms, records and variants

Terms:

$e ::=$

$x \mid (e : H) \mid$   
 $\lambda x. e \mid e_1 e_2 \mid$   
 $r \mid e.l$   
 $c \ e \mid \text{vcase } e_1 \text{ of } e_2$

Records:

$r ::= \{\} \mid \{r, \ell := e\}$

Note: the terms are, in order of appearance, variables, annotated terms, functions, applications, records, record projections, applied variant constructors and pattern matching expressions.

# Well-formed signatures

$$\frac{}{\emptyset \text{ ok}} \text{OK-EMPTY}$$

$$\frac{\Sigma \text{ ok} \quad l \notin \Sigma}{\Sigma, l : A \text{ ok}} \text{OK-EXTEND}$$

For now we will assume that signatures are finite partial maps from labels to types. A label may appear at most once.

# Signatures for pattern-matching

We define an operation on signatures which will be useful later.

$$\emptyset \rightarrow B = \emptyset$$

$$(\Sigma, \ell : A) \rightarrow B = (\Sigma \rightarrow B, \ell : A \rightarrow B)$$

# Declarative typing – basics

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \text{VAR}$$

$$\frac{\Gamma \vdash e : A \quad H \sqsubseteq A}{\Gamma \vdash (e : H) : A} \text{ANNOT}$$

# Declarative typing – type-directed rules

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : A \rightarrow B} \quad \frac{\Gamma \vdash f : A \rightarrow B \quad \Gamma \vdash a : A}{\Gamma \vdash f \ a : B}$$

$$\frac{}{\Gamma \vdash \{\} : \text{Record } \emptyset} \quad \frac{\Gamma \vdash r : \text{Record } \Sigma \quad \Gamma \vdash e : A}{\Gamma \vdash \{r, \ell := e\} : \text{Record } (\Sigma, \ell : A)}$$

$$\frac{\Gamma \vdash e : \text{Record } \Sigma \quad (\ell : A) \in \Sigma}{\Gamma \vdash e.\ell : A}$$

$$\frac{\Gamma \vdash e : A \quad (c : A) \in \Sigma}{\Gamma \vdash c \ e : \text{Variant } \Sigma}$$

$$\frac{\Gamma \vdash e_1 : \text{Variant } \Sigma \quad \Gamma \vdash e_2 : \text{Record } (\Sigma \rightarrow A)}{\Gamma \vdash \text{vcase } e_1 \text{ of } e_2 : A}$$

# Algorithmic typing – basic rules

$$\frac{(x : A) \in \Gamma \quad H \sqsubseteq A}{\Gamma \vdash x \leftarrow H \Rightarrow A} \text{VAR}$$

$$\frac{\Gamma \vdash e \leftarrow H_1 \sqcup H_2 \Rightarrow A}{\Gamma \vdash (e : H_1) \leftarrow H_2 \Rightarrow A} \text{ANNOT}$$



# Algorithmic typing – functions

$$\frac{\Gamma, x : A \vdash e \Leftarrow H \Rightarrow B}{\Gamma \vdash \lambda x. e \Leftarrow A \rightarrow H \Rightarrow A \rightarrow B}$$

$$\frac{\Gamma \vdash f \Leftarrow ? \rightarrow H \Rightarrow A \rightarrow B \quad \Gamma \vdash a \Leftarrow A \Rightarrow A}{\Gamma \vdash f \ a \Leftarrow H \Rightarrow B}$$

# Algorithmic typing – records

$$\frac{H \sqsubseteq \text{Record } \emptyset}{\Gamma \vdash \{\} \leftarrow H \Rightarrow \text{Record } \emptyset}$$

$$\frac{H \sqcup \text{Record } ? = H' \quad \Gamma \vdash \{r, \ell := e\} \leftarrow H' \Rightarrow A}{\Gamma \vdash \{r, \ell := e\} \leftarrow H \Rightarrow A}$$

$$\frac{\Gamma \vdash r \leftarrow \text{Record } H_{\Sigma} \Rightarrow \text{Record } \Sigma \quad \Gamma \vdash e \leftarrow H_A \Rightarrow A}{\Gamma \vdash \{r, \ell := e\} \leftarrow \text{Record } (H_{\Sigma}, \ell : H_A) \Rightarrow \text{Record } (\Sigma, \ell : A)}$$

$$\frac{\Gamma \vdash e \leftarrow \text{Record } (?, \ell : H) \Rightarrow \text{Record } (\Sigma, \ell : A)}{\Gamma \vdash e.\ell \leftarrow H \Rightarrow A}$$

# Algorithmic typing – variants

$$\frac{H \sqcup \text{Variant } (?, c : ?) = \text{Variant } (\Sigma, c : H_A) \quad \Gamma \vdash e \Leftarrow H_A \Rightarrow A}{\Gamma \vdash c \ e \Leftarrow H \Rightarrow \text{Variant } (\Sigma, c : A)}$$

$$\frac{\begin{array}{l} \Gamma \vdash e_1 \Leftarrow ? \Rightarrow \text{Variant } \Sigma \\ \Gamma \vdash e_2 \Leftarrow \text{Record } (\Sigma \rightarrow H) \Rightarrow \text{Record } (\Sigma \rightarrow A) \end{array}}{\Gamma \vdash \text{vcase } e_1 \text{ of } e_2 \Leftarrow H \Rightarrow A}$$