Types and judgements

Types:

$$A, B ::= \alpha \mid \forall \alpha. A \mid A \rightarrow B$$

Contexts:

$$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, \alpha$$

Judgements:

$$\Gamma \vdash e : A, \Gamma \vdash A \text{ type}$$

Valid type judgement

$$\frac{\alpha \in \Gamma}{\Gamma \vdash \alpha \text{ type}} \text{TyVar}$$

$$\frac{\Gamma, \alpha \vdash A \text{ type}}{\Gamma \vdash \forall \alpha. A \text{ type}} ALL$$

$$\frac{\Gamma \vdash A \text{ type} \quad \Gamma \vdash B \text{ type}}{\Gamma \vdash A \to B \text{ type}} F^{\text{UN}}$$

```
Terms: e := x \mid \lambda x : A. e \mid e_1 \mid e_2 \mid \Lambda \alpha. e \mid e \mid eA
```

Declarative typing

$$\frac{(x:A)\in\Gamma}{\Gamma\vdash x:A}\mathrm{Var}$$

$$\frac{\Gamma, x: A \vdash e: B}{\Gamma \vdash \lambda x: A. e: A \rightarrow B} \qquad \frac{\Gamma \vdash f: A \rightarrow B \quad \Gamma \vdash a: A}{\Gamma \vdash f \ a: B}$$

$$\frac{\Gamma, \alpha \vdash e : A}{\Gamma \vdash \Lambda \alpha. e : \forall \alpha. A}$$

$$\frac{\Gamma \vdash e : \forall \alpha. A}{\Gamma \vdash e @B : A[\alpha := B]}$$

Algorithmic typing

$$\frac{(x:A) \in \Gamma}{\Gamma \vdash x \Rightarrow A} VAR$$

$$\frac{1, x : A \vdash e \Rightarrow B}{\Gamma \vdash \lambda x : A. e \Rightarrow A \rightarrow B}$$

$$\frac{\Gamma, x : A \vdash e \Rightarrow B}{\Gamma \vdash \lambda x : A. e \Rightarrow A \rightarrow B} \qquad \frac{\Gamma \vdash f \Rightarrow A \rightarrow B \quad \Gamma \vdash a \Rightarrow A}{\Gamma \vdash f \ a \Rightarrow B}$$

$$\frac{\Gamma, \alpha \vdash e \Rightarrow A}{\Gamma \vdash \Lambda \alpha. \ e \Rightarrow \forall \alpha. \ A}$$

$$\frac{\Gamma \vdash e \Rightarrow \forall \alpha. A}{\Gamma \vdash e @B \Rightarrow A [\alpha := B]}$$

Terms

```
Terms: e := x \mid \lambda x. e \mid e_1 \mid e_2 \mid
```

Declarative typing

$$\frac{(x:A)\in\Gamma}{\Gamma\vdash x:A}\mathrm{Var}$$

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : A \rightarrow B} \qquad \frac{\Gamma \vdash f : A \rightarrow B \quad \Gamma \vdash a : A}{\Gamma \vdash f \ a : B}$$

$$\frac{\Gamma, \alpha \vdash e : A}{\Gamma \vdash e : \forall \alpha . A}$$

$$\frac{\Gamma \vdash e : \forall \alpha. A}{\Gamma \vdash e : A [\alpha := B]}$$