Grammar

Types:

$A, B ::= A \to B \mid A \times B \mid A + B \mid \mathbf{1} \mid \mathbf{0}$

Typing contexts:

$\Gamma ::= \cdot \mid \Gamma, x : A$

Terms:

$$
\begin{aligned}
e ::= \quad & x \mid (e : A) \mid \\
& \lambda x.e \mid e_1\ e_2 \mid \\
& (e_1, e_2) \mid \texttt{outl}\ e \mid \texttt{outr}\ e \mid \\
& \texttt{inl}\ e \mid \texttt{inr}\ e \mid \texttt{case}\ e\ \texttt{of}\ (e_1, e_2) \mid \\
& \texttt{unit} \mid \texttt{exfalso}\ e
\end{aligned}
$$

Declarative typing basics

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A}\textsc{Var}$$

$$\frac{\Gamma \vdash e : A}{\Gamma \vdash (e : A) : A}\textsc{Annot}$$

Declarative typing – type-directed rules

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x.e : A \to B} \qquad \frac{\Gamma \vdash f : A \to B \quad \Gamma \vdash a : A}{\Gamma \vdash f\ a : B}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B}{\Gamma \vdash (a, b) : A \times B} \qquad \frac{\Gamma \vdash e : A \times B}{\Gamma \vdash \mathtt{outl}\ e : A} \qquad \frac{\Gamma \vdash e : A \times B}{\Gamma \vdash \mathtt{outr}\ e : B}$$

$$\frac{\Gamma \vdash e : A}{\Gamma \vdash \mathtt{inl}\ e : A + B} \qquad \frac{\Gamma \vdash e : B}{\Gamma \vdash \mathtt{inr}\ e : A + B}$$

$$\frac{\Gamma \vdash e : A + B \quad \Gamma \vdash f : A \to C \quad \Gamma \vdash g : B \to C}{\Gamma \vdash \mathtt{case}\ e\ \mathtt{of}\ (f, g) : C}$$

$$\frac{}{\Gamma \vdash \mathtt{unit} : \mathbf{1}} \qquad \frac{\Gamma \vdash e : \mathbf{0}}{\Gamma \vdash \mathtt{exfalso}\ e : A}$$

Bidirectional typing

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x \Rightarrow A}\textsc{Var}$$

$$\frac{\Gamma \vdash x \Leftarrow A}{\Gamma \vdash (x : A) \Rightarrow A}\textsc{Annot}$$

$$\frac{\Gamma \vdash x \Rightarrow B \quad A = B}{\Gamma \vdash x \Leftarrow A}\textsc{Sub}$$

Bidirectional typing

$$\frac{\Gamma, x : A \vdash e \Leftarrow B}{\Gamma \vdash \lambda x.e \Leftarrow A \to B} \qquad \frac{\Gamma \vdash f \Rightarrow A \to B \quad \Gamma \vdash a \Leftarrow A}{\Gamma \vdash f\ a \Rightarrow B}$$

$$\frac{\Gamma \vdash a \Leftarrow A \quad \Gamma \vdash b \Leftarrow B}{\Gamma \vdash (a, b) \Leftarrow A \times B} \qquad \frac{\Gamma \vdash e \Rightarrow A \times B}{\Gamma \vdash \texttt{outl}\ e \Rightarrow A} \qquad \frac{\Gamma \vdash e \Rightarrow A \times B}{\Gamma \vdash \texttt{outr}\ e \Rightarrow B}$$

$$\frac{\Gamma \vdash e \Leftarrow A}{\Gamma \vdash \texttt{inl}\ e \Leftarrow A + B} \qquad \frac{\Gamma \vdash e \Leftarrow B}{\Gamma \vdash \texttt{inr}\ e \Leftarrow A + B}$$

$$\frac{\Gamma \vdash e \Rightarrow A + B \quad \Gamma \vdash f \Leftarrow A \to C \quad \Gamma \vdash g \Leftarrow B \to C}{\Gamma \vdash \texttt{case}\ e\ \texttt{of}\ (f, g) \Leftarrow C}$$

$$\frac{}{\Gamma \vdash \texttt{unit} \Rightarrow \mathbf{1}} \qquad \frac{\Gamma \vdash e \Leftarrow \mathbf{0}}{\Gamma \vdash \texttt{exfalso}\ e \Leftarrow A}$$