

Hint-based typing for STLC with

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Terms

Terms:

$e ::=$

$x \mid (e : H) \mid$
 $\lambda x. e \mid e_1 \ e_2 \mid$
 $\text{pair} \mid \text{outl} \mid \text{outr} \mid$
 $\text{inl} \mid \text{inr} \mid \text{case} \mid$
 $\text{unit} \mid \text{exfalse}$

Judgements:

$\Gamma \vdash e \Leftarrow H \Rightarrow A$ – in context Γ , term e checks with hint H and infers type A

Declarative typing – basics

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \text{VAR}$$

$$\frac{\Gamma \vdash e : A \quad H \sqsubseteq A}{\Gamma \vdash (e : H) : A} \text{ANNOT}$$

Declarative typing – type-directed rules

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : A \rightarrow B} \quad \frac{\Gamma \vdash a : A \quad \Gamma \vdash f : A \rightarrow B}{\Gamma \vdash f \ a : B}$$

$$\overline{\Gamma \vdash \text{pair} : A \rightarrow B \rightarrow A \times B}$$

$$\overline{\Gamma \vdash \text{outl} : A \times B \rightarrow A} \quad \overline{\Gamma \vdash \text{outr} : A \times B \rightarrow B}$$

$$\overline{\Gamma \vdash \text{inl} : A \rightarrow A + B} \quad \overline{\Gamma \vdash \text{inr} : B \rightarrow A + B}$$

$$\overline{\Gamma \vdash \text{case} : (A \rightarrow C) \rightarrow (B \rightarrow C) \rightarrow A + B \rightarrow C}$$

$$\overline{\Gamma \vdash \text{unit} : \mathbf{1}} \quad \overline{\Gamma \vdash \text{exfalse} : \mathbf{0} \rightarrow A}$$

Hints for term constructors

```
hint( $\lambda x. e$ ) = ?  $\rightarrow$  ?  
hint(pair) = ?  $\rightarrow$  ?  $\rightarrow$  ?  $\times$  ?  
hint(outl) = ?  $\times$  ?  $\rightarrow$  ?  
hint(outr) = ?  $\times$  ?  $\rightarrow$  ?  
hint(inl) = ?  $\rightarrow$  ? + ?  
hint(inr) = ?  $\rightarrow$  ? + ?  
hint(case) = (?  $\rightarrow$  ?)  $\rightarrow$  (?  $\rightarrow$  ?)  $\rightarrow$  ? + ?  $\rightarrow$  ?  
hint(exfalse) = 0  $\rightarrow$  ?  
hint(unit) = 1
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Combining a hint with a term

$$\begin{aligned}
& e \sqcup ? = \text{hint}(e) \\
& \lambda x. e \sqcup H_1 \rightarrow H_2 = H_1 \rightarrow H_2 \\
& \text{pair} \sqcup H_1 \rightarrow H_2 \rightarrow H_3 \times H_4 = H_1 \sqcup H_3 \rightarrow H_2 \sqcup H_4 \rightarrow \\
& \quad (H_1 \sqcup H_3) \times (H_2 \sqcup H_4) \\
& \text{outl} \sqcup H_1 \times H_2 \rightarrow H_3 = (H_1 \sqcup H_3) \times H_2 \rightarrow H_1 \sqcup H_3 \\
& \text{outr} \sqcup (H_1 \times H_2) \rightarrow H_3 = H_1 \times (H_2 \sqcup H_3) \rightarrow H_2 \sqcup H_3 \\
& \text{inl} \sqcup H_1 \rightarrow H_2 + H_3 = H_1 \sqcup H_2 \rightarrow (H_1 \sqcup H_2) + H_3 \\
& \text{inr} \sqcup H_1 \rightarrow H_2 + H_3 = H_1 \sqcup H_3 \rightarrow H_2 + (H_1 \sqcup H_3) \\
& \text{case} \sqcup (H_1 \rightarrow H_2) \rightarrow (H_3 \rightarrow H_4) \rightarrow H_5 + H_6 \rightarrow H_7 = (H_1 \sqcup H_5 \rightarrow \\
& \quad H_2) \rightarrow (H_3 \sqcup H_6 \rightarrow H_4) \rightarrow (H_1 \sqcup H_5) + (H_3 \sqcup H_6) \rightarrow H_7 \\
& \text{exfalse} \sqcup H_1 \rightarrow H_2 = H_1 \sqcup \mathbf{0} \rightarrow H_2 \\
& \text{unit} \sqcup \mathbf{1} = \mathbf{1}
\end{aligned}$$

Hinting – basic rules

$$\frac{(x : A) \in \Gamma \quad H \sqsubseteq A}{\Gamma \vdash x \leftarrow H \Rightarrow A} \text{VAR}$$

$$\frac{\Gamma \vdash e \leftarrow H_1 \sqcup H_2 \Rightarrow A}{\Gamma \vdash (e : H_1) \leftarrow H_2 \Rightarrow A} \text{ANNOT}$$

$$\frac{\Gamma \vdash e \leftarrow e \sqcup H \Rightarrow A \quad e \sqcup H \text{ defined} \quad e \sqcup H \neq H}{\Gamma \vdash e \leftarrow H \Rightarrow A} \text{HOLE}$$

Hinting – type-directed rules

$$\frac{\Gamma, x : A \vdash e \leftarrow H \Rightarrow B}{\Gamma \vdash \lambda x. e \leftarrow A \rightarrow H \Rightarrow A \rightarrow B}$$

$$\frac{\Gamma \vdash a \leftarrow ? \Rightarrow A \quad \Gamma \vdash f \leftarrow A \rightarrow H \Rightarrow A \rightarrow B}{\Gamma \vdash f \ a \leftarrow H \Rightarrow B}$$

$$\overline{\Gamma \vdash \text{pair} \leftarrow A \rightarrow B \rightarrow A \times B \Rightarrow A \rightarrow B \rightarrow A \times B}$$

$$\overline{\Gamma \vdash \text{outl} \leftarrow A \times B \rightarrow A \Rightarrow A \times B \rightarrow A}$$

$$\overline{\Gamma \vdash \text{outr} \leftarrow A \times B \rightarrow B \Rightarrow A \times B \rightarrow B}$$

Hinting – type-directed rules

$$\overline{\Gamma \vdash \text{inl} \leftarrow A \rightarrow A + B \Rightarrow A \rightarrow A + B}$$

$$\overline{\Gamma \vdash \text{inr} \leftarrow B \rightarrow A + B \Rightarrow B \rightarrow A + B}$$

$$\overline{\Gamma \vdash \text{case} \leftarrow (A \rightarrow C) \rightarrow (B \rightarrow C) \rightarrow A + B \rightarrow C \Rightarrow (A \rightarrow C) \rightarrow (B \rightarrow C) \rightarrow A + B \rightarrow C}$$

$$\overline{\Gamma \vdash \text{unit} \leftarrow \mathbf{1} \Rightarrow \mathbf{1}} \quad \overline{\Gamma \vdash \text{exfalse} \leftarrow \mathbf{0} \rightarrow A \Rightarrow \mathbf{0} \rightarrow A}$$