Hint-based unification for STLC

Wojciech Kołowski

Terms, contexts and judgements

Terms:

$e ::=$

$\quad x \mid (e : H) \mid$

$\quad \lambda x. e \mid e_1\ e_2 \mid$

$\quad (e_1, e_2) \mid \texttt{outl}\ e \mid \texttt{outr}\ e \mid$

$\quad \texttt{inl}\ e \mid \texttt{inr}\ e \mid \texttt{case}\ e\ \texttt{of}\ (e_1, e_2) \mid$

$\quad \texttt{unit} \mid \texttt{exfalso}\ e$

Contexts:

$\Gamma ::= \cdot \mid \Gamma, x : H$

Judgements:

$\Gamma \vdash e \Leftarrow H \Rightarrow H' \dashv \Gamma'$ – in context Γ, term e checks with hint H and infers hint H' in output context Γ'

Output contexts – basic rules

$$\overline{\Gamma_1, x : A, \Gamma_2 \vdash x \Leftarrow B \Rightarrow A \sqcup B \dashv \Gamma_1, x : A \sqcup B, \Gamma_2}\textsc{Var}$$

$$\frac{\Gamma \vdash e \Leftarrow A \sqcup B \Rightarrow C \dashv \Gamma'}{\Gamma \vdash (e : A) \Leftarrow B \Rightarrow C \dashv \Gamma'}\textsc{Annot}$$

$$\frac{\Gamma \vdash e \Leftarrow \texttt{hint}(e) \Rightarrow A \dashv \Gamma' \quad e \texttt{ constructor}}{\Gamma \vdash e \Leftarrow \textbf{?} \Rightarrow A \dashv \Gamma'}\textsc{Hole}$$

Note that the rule $\textsc{Hole}$ can only be applied once, because $\texttt{hint}(e)$ can never be $\textbf{?}$. After applying $\textsc{Hole}$, the only applicable rules are the type-directed ones.

Output contexts – type-directed rules

$$\frac{\Gamma, x : A \vdash e \Leftarrow B \Rightarrow B' \dashv \Gamma', x : A'}{\Gamma \vdash \lambda x.\, e \Leftarrow A \to B \Rightarrow A' \to B' \dashv \Gamma'}$$

$$\frac{\Gamma \vdash f \Leftarrow \mathbf{?} \to B \Rightarrow A \to B' \dashv \Gamma' \quad \Gamma' \vdash a \Leftarrow A \Rightarrow A' \dashv \Gamma''}{\Gamma \vdash f\ a \Leftarrow B \Rightarrow B' \dashv \Gamma''}$$

$$\frac{\Gamma \vdash a \Leftarrow A \Rightarrow A' \dashv \Gamma' \quad \Gamma' \vdash b \Leftarrow B \Rightarrow B' \dashv \Gamma''}{\Gamma \vdash (a, b) \Leftarrow A \times B \Rightarrow A' \times B' \dashv \Gamma''}$$

$$\frac{\Gamma \vdash e \Leftarrow A \times \mathbf{?} \Rightarrow A' \times B \dashv \Gamma'}{\Gamma \vdash \mathtt{outl}\ e \Leftarrow A \Rightarrow A' \dashv \Gamma'} \qquad \frac{\Gamma \vdash e \Leftarrow \mathbf{?} \times B \Rightarrow A \times B' \dashv \Gamma'}{\Gamma \vdash \mathtt{outr}\ e \Leftarrow B \Rightarrow B' \dashv \Gamma'}$$

Output contexts – type-directed rules

$$\frac{\Gamma \vdash e \Leftarrow A \Rightarrow A' \dashv \Gamma'}{\Gamma \vdash \texttt{inl } e \Leftarrow A + B \Rightarrow A' + B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash e \Leftarrow B \Rightarrow B' \dashv \Gamma'}{\Gamma \vdash \texttt{inr } e \Leftarrow A + B \Rightarrow A + B' \dashv \Gamma'}$$

$$\frac{\Gamma \vdash e \Leftarrow \mathbf{?} + \mathbf{?} \Rightarrow A + B \dashv \Gamma_1 \qquad \begin{array}{l} \Gamma_1 \vdash f \Leftarrow A \rightarrow C_1 \Rightarrow A' \rightarrow C_2 \dashv \Gamma_2 \\ \Gamma_2 \vdash g \Leftarrow B \rightarrow C_2 \Rightarrow B' \rightarrow C_3 \dashv \Gamma_3 \end{array}}{\Gamma \vdash \texttt{case } e \texttt{ of } (f, g) \Leftarrow C_1 \Rightarrow C_3 \dashv \Gamma_3}$$

$$\frac{}{\Gamma \vdash \texttt{unit} \Leftarrow \mathbf{1} \Rightarrow \mathbf{1} \dashv \Gamma'} \qquad \frac{\Gamma \vdash e \Leftarrow \mathbf{0} \Rightarrow \mathbf{0} \dashv \Gamma'}{\Gamma \vdash \texttt{exfalso } e \Leftarrow A \Rightarrow A \dashv \Gamma'}$$

Output contexts – abridged basic rules

$$\overline{\Gamma_1, x : A, \Gamma_2 \vdash x \Leftarrow B \Rightarrow A \sqcup B \dashv \Gamma_1, x : A \sqcup B, \Gamma_2}^{\text{VAR}}$$

$$\frac{e \Leftarrow A \sqcup B \Rightarrow C}{(e : A) \Leftarrow B \Rightarrow C}^{\text{ANNOT}}$$

$$\frac{e \Leftarrow \text{hint}(e) \Rightarrow A \quad e \text{ constructor}}{e \Leftarrow ? \Rightarrow A}^{\text{HOLE}}$$

Note that the rule HOLE can only be applied once, because $\text{hint}(e)$ can never be **?**. After applying HOLE, the only applicable rules are the type-directed ones.

Output contexts – abridged type-directed rules

$$\frac{\Gamma, x : A \vdash e \Leftarrow B \Rightarrow B' \dashv \Gamma', x : A'}{\Gamma \vdash \lambda x.\, e \Leftarrow A \rightarrow B \Rightarrow A' \rightarrow B' \dashv \Gamma'}$$

$$\frac{\Gamma \vdash f \Leftarrow \mathbf{?} \rightarrow B \Rightarrow A \rightarrow B' \dashv \Gamma' \quad \Gamma' \vdash a \Leftarrow A \Rightarrow A' \dashv \Gamma''}{\Gamma \vdash f\ a \Leftarrow B \Rightarrow B' \dashv \Gamma''}$$

$$\frac{\Gamma \vdash a \Leftarrow A \Rightarrow A' \dashv \Gamma' \quad \Gamma' \vdash b \Leftarrow B \Rightarrow B' \dashv \Gamma''}{\Gamma \vdash (a, b) \Leftarrow A \times B \Rightarrow A' \times B' \dashv \Gamma''}$$

$$\frac{e \Leftarrow A \times \mathbf{?} \Rightarrow A' \times B}{\mathtt{outl}\ e \Leftarrow A \Rightarrow A'} \qquad \frac{e \Leftarrow \mathbf{?} \times B \Rightarrow A \times B'}{\mathtt{outr}\ e \Leftarrow B \Rightarrow B'}$$

Output contexts – abridged type-directed rules

$$\frac{e \Leftarrow A \Rightarrow A'}{\texttt{inl } e \Leftarrow A + B \Rightarrow A' + B}$$

$$\frac{e \Leftarrow B \Rightarrow B'}{\texttt{inr } e \Leftarrow A + B \Rightarrow A + B'}$$

$$\frac{\Gamma \vdash e \Leftarrow \textbf{?} + \textbf{?} \Rightarrow A + B \dashv \Gamma_1 \qquad \begin{array}{l} \Gamma_1 \vdash f \Leftarrow A \to C_1 \Rightarrow A' \to C_2 \dashv \Gamma_2 \\ \Gamma_2 \vdash g \Leftarrow B \to C_2 \Rightarrow B' \to C_3 \dashv \Gamma_3 \end{array}}{\Gamma \vdash \texttt{case } e \texttt{ of } (f, g) \Leftarrow C_1 \Rightarrow C_3 \dashv \Gamma_3}$$

$$\frac{}{\texttt{unit} \Leftarrow \textbf{1} \Rightarrow \textbf{1}} \qquad \frac{e \Leftarrow \textbf{0} \Rightarrow \textbf{0}}{\texttt{exfalso } e \Leftarrow A \Rightarrow A}$$

Types, holes, terms and contexts

Holes:
$$H ::= \alpha \mid H_1 \to H_2 \mid H_1 \times H_2 \mid H_1 + H_2 \mid \mathbf{1} \mid \mathbf{0}$$

Terms:
$e ::=$
$\quad x \mid (e : H) \mid$
$\quad \lambda x.\, e \mid e_1\ e_2 \mid$
$\quad (e_1, e_2) \mid \texttt{outl}\ e \mid \texttt{outr}\ e \mid$
$\quad \texttt{inl}\ e \mid \texttt{inr}\ e \mid \texttt{case}\ e\ \texttt{of}\ (e_1, e_2) \mid$
$\quad \texttt{unit} \mid \texttt{exfalso}\ e$

Note: the terms are the same as in STLC with Hints.

Contexts and judgements

Contexts:
$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, \alpha \mid \Gamma, \alpha := H$

We introduce unification variables, denoted with Greek letter
$(\alpha, \beta, \gamma, \dots)$. When extending the context with a unification
variable, it is set to some hint, which at the beginning will be just
?.

Judgements:
$\Gamma \vdash e \Leftarrow H \Rightarrow H' \dashv \Gamma'$ – in context Γ, term e checks with hint H
and infers hint H' in output context Γ'