Hint-based typing for polymorphism

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Terms, contexts and judgements

Types:

$A, B ::= \alpha \mid \forall \alpha.\, A \mid A \to B$

Terms:

$$
\begin{aligned}
e ::= \quad & \\
& x \mid \\
& \lambda x : A.\, e \mid e_1\ e_2 \mid \\
& \Lambda \alpha.\, e \mid e\ @A
\end{aligned}
$$

Contexts:

$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, \alpha$

Judgements:

$\Gamma \vdash e : A$, $\Gamma \vdash A\ \texttt{type}$

Valid type judgement

$$\frac{\alpha \in \Gamma}{\Gamma \vdash \alpha \ \texttt{type}}\textsc{TyVar}$$

$$\frac{\Gamma, \alpha \vdash A \ \texttt{type}}{\Gamma \vdash \forall \alpha.\, A \ \texttt{type}}\textsc{All}$$

$$\frac{\Gamma \vdash A \ \texttt{type} \quad \Gamma \vdash B \ \texttt{type}}{\Gamma \vdash A \to B \ \texttt{type}}\textsc{Fun}$$

Declarative typing – basics

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A}\,\textsc{Var}$$

Declarative typing – type-directed rules

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x : A.\, e : A \to B} \qquad \frac{\Gamma \vdash f : A \to B \quad \Gamma \vdash a : A}{\Gamma \vdash f\ a : B}$$

$$\frac{\Gamma, \alpha \vdash e : A}{\Gamma \vdash \Lambda\alpha.\, e : \forall\alpha.\, A}$$

$$\frac{\Gamma \vdash e : \forall\alpha.\, A}{\Gamma \vdash e\ @B : A\,[\alpha := B]}$$