

Grammar

Types:

$$A, B ::= A \rightarrow B \mid A \times B \mid A + B \mid \mathbf{1} \mid \mathbf{0}$$

Typing contexts:

$$\Gamma ::= \cdot \mid \Gamma, x : A$$

Terms:

$$e ::=$$
$$\begin{aligned} & x \mid (e : A) \mid \\ & \lambda x. e \mid e_1 \ e_2 \mid \\ & (e_1, e_2) \mid \text{outl } e \mid \text{outr } e \mid \\ & \text{inl } e \mid \text{inr } e \mid \text{case } e \text{ of } (e_1, e_2) \mid \\ & \text{unit} \mid \text{exfalse } e \end{aligned}$$

Declarative typing basics

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \text{VAR}$$

$$\frac{\Gamma \vdash e : A}{\Gamma \vdash (e : A) : A} \text{ANNOT}$$

Declarative typing – type-directed rules

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : A \rightarrow B} \quad \frac{\Gamma \vdash f : A \rightarrow B \quad \Gamma \vdash a : A}{\Gamma \vdash f \ a : B}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B}{\Gamma \vdash (a, b) : A \times B} \quad \frac{\Gamma \vdash e : A \times B}{\Gamma \vdash \text{outl } e : A} \quad \frac{\Gamma \vdash e : A \times B}{\Gamma \vdash \text{outr } e : B}$$

$$\frac{\Gamma \vdash e : A}{\Gamma \vdash \text{inl } e : A + B} \quad \frac{\Gamma \vdash e : B}{\Gamma \vdash \text{inr } e : A + B}$$

$$\frac{\Gamma \vdash e : A + B \quad \Gamma \vdash f : A \rightarrow C \quad \Gamma \vdash g : B \rightarrow C}{\Gamma \vdash \text{case } e \text{ of } (f, g) : C}$$

$$\frac{}{\Gamma \vdash \text{unit} : \mathbf{1}} \quad \frac{\Gamma \vdash e : \mathbf{0}}{\Gamma \vdash \text{exfalse } e : A}$$