

Hint-based typing for polymorphism

Types and judgements

Types:

$$A, B ::= \alpha \mid \forall \alpha. A \mid A \rightarrow B$$

Contexts:

$$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, \alpha$$

Judgements:

$$\Gamma \vdash e : A, \Gamma \vdash A \text{ type}$$

Valid type judgement

$$\frac{\alpha \in \Gamma}{\Gamma \vdash \alpha \text{ type}}_{\text{TYVAR}}$$

$$\frac{\Gamma, \alpha \vdash A \text{ type}}{\Gamma \vdash \forall \alpha. A \text{ type}}_{\text{ALL}}$$

$$\frac{\Gamma \vdash A \text{ type} \quad \Gamma \vdash B \text{ type}}{\Gamma \vdash A \rightarrow B \text{ type}}_{\text{FUN}}$$

Terms

Terms:

$e ::=$

$x \mid$

$\lambda x : A. e \mid e_1 \ e_2 \mid$

$\Lambda \alpha. e \mid e @ A$

Declarative typing

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \text{VAR}$$

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x : A. e : A \rightarrow B} \quad \frac{\Gamma \vdash f : A \rightarrow B \quad \Gamma \vdash a : A}{\Gamma \vdash f a : B}$$

$$\frac{\Gamma, \alpha \vdash e : A}{\Gamma \vdash \Lambda \alpha. e : \forall \alpha. A}$$

$$\frac{\Gamma \vdash e : \forall \alpha. A}{\Gamma \vdash e @ B : A[\alpha := B]}$$

Algorithmic typing

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x \Rightarrow A} \text{VAR}$$

$$\frac{\Gamma, x : A \vdash e \Rightarrow B}{\Gamma \vdash \lambda x : A. e \Rightarrow A \rightarrow B} \quad \frac{\Gamma \vdash f \Rightarrow A \rightarrow B \quad \Gamma \vdash a \Rightarrow A}{\Gamma \vdash f a \Rightarrow B}$$

$$\frac{\Gamma, \alpha \vdash e \Rightarrow A}{\Gamma \vdash \Lambda \alpha. e \Rightarrow \forall \alpha. A}$$

$$\frac{\Gamma \vdash e \Rightarrow \forall \alpha. A}{\Gamma \vdash e @ B \Rightarrow A[\alpha := B]}$$

Terms

Terms:

$e ::=$

$x \mid$

$\lambda x. e \mid \text{app}_A e_1 e_2 \mid$

$e @ A$

Declarative typing

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \text{VAR}$$

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : A \rightarrow B} \quad \frac{\Gamma \vdash f : A \rightarrow B \quad \Gamma \vdash a : A}{\Gamma \vdash \text{app}_A f a : B}$$

$$\frac{\Gamma, \alpha \vdash e : A}{\Gamma \vdash e : \forall \alpha. A}$$

$$\frac{\Gamma \vdash e : \forall \alpha. A}{\Gamma \vdash e @ B : A[\alpha := B]}$$

Algorithmic typing

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x \Leftarrow A} \text{VAR}$$

$$\frac{\Gamma, x : A \vdash e \Leftarrow B}{\Gamma \vdash \lambda x. e \Leftarrow A \rightarrow B} \quad \frac{\Gamma \vdash f \Leftarrow A \rightarrow B \quad \Gamma \vdash a \Leftarrow A}{\Gamma \vdash \text{app}_A f a \Leftarrow B}$$

$$\frac{\Gamma, \alpha \vdash e \Leftarrow A}{\Gamma \vdash e \Leftarrow \forall \alpha. A}$$

$$\frac{\Gamma \vdash e \Leftarrow \forall \alpha. A}{\Gamma \vdash e @ B \Leftarrow A[\alpha := B]}$$

Terms

Terms:

$e ::=$

$x \mid$

$\lambda x. e \mid e_1 \ e_2 \mid$

Declarative typing

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \text{VAR}$$

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : A \rightarrow B} \quad \frac{\Gamma \vdash f : A \rightarrow B \quad \Gamma \vdash a : A}{\Gamma \vdash f a : B}$$

$$\frac{\Gamma, \alpha \vdash e : A}{\Gamma \vdash e : \forall \alpha. A}$$

$$\frac{\Gamma \vdash e : \forall \alpha. A}{\Gamma \vdash e : A[\alpha := B]}$$