

Mortality and natality data for a cohort of Belding's ground squirrel, *Citellus beldingi* (after Zammuto and Sherman 1986, Can. J. Zool. 64 602-605). Squirrels were censused once per year in mid-summer, shortly after weaning; m_x is the average number of female young just weaned by a female of age x .

Age class x	Number alive N_x	Annual Survival S_x	Cumulative survival l_x	Expected life e_x	Fecundity m_x	Realized fecundity $l_x m_x$	Reproductive value V_x	Cohort size at SAD C_x
-----	-----	-----	-----	-----	-----	-----	-----	-----
0	238				0.0			
1	93				1.7			
2	49				2.1			
3	18				2.4			
4	12				3.0			
5	6				2.1			
6	0							

1. Fill in the life table.
2. What proportion of squirrels live to be 2 years old?
3. Of those squirrels that live to be 1 year old, what proportion survive to the 2nd year?
4. What is the expected future lifespan of a 1 year old squirrel?
5. Calculate R_0 . Write it with the correct units. If there are 100 squirrels in mid-summer this year, how many would you expect to find in mid-summer next year? How many the year after?
6. Write the correct general equation for calculating reproductive value V_x for a post-breeding model.
7. If you were managing this population of squirrels and a users group wanted to harvest as many squirrels as possible on a sustained yield basis, which age class would you recommend that they exploit? Explain.

BIO 21/51/120: EXAMPLE OF LIFE TABLE CALCULATIONS

Cohort life table for little brown bat *Myotis lucifugus* (based on annual pre-breeding censuses)*

x	N_x	S_x	l_x	e_x	m_x	$l_x m_x$	V_x	C_x	$\lambda^{-x} l_x$	$l_x m_x x$	v_x	v_{x+1}	v_{x+2}	v_{x+3}	v_{x+4}
1*	320	0.4	1	0.82	0	0	1.1	0.563	0.968	0	0	0.4	0.42	0.28	0
2	128	0.7	0.4	1.05	1	0.4	2.75	0.218	0.375	0.80	1	1.05	0.7	0	
3	90	0.5	0.28	0.5	1.5	0.42	2.5	0.148	0.254	1.26	1.5	1	0		
4	45	0	0.14	0	2	0.28	2	0.071	0.123	1.12	2	0			
5	0		0			0		0	0	0					
Σ						1.10		1.000	1.718	3.18					

* The value of x in the first row of the table is the approximate age at the time of sampling of the youngest individuals that are recognized (with a pre-breeding census, the table starts with $x = 1$; with a post-breeding census, the table starts with $x = 0$).

N_x = Number in cohort surviving to age x (measured; e.g., from marked individuals).

S_x = Annual probability of survival
 = Probability that an individual of age x will survive to age $x+1$.
 = N_{x+1} / N_x eq. 1

0.40 = 128 / 320

l_x = Cumulative Survivorship
 = Proportion of individuals in age class 0 that survive to age class x . Note that $l_0 = 1.0$ by definition.
 = $l_{x-1} \cdot S_{x-1}$; eq. 2

0.4 = $1 \cdot 0.4$ i.e., $l_1 = l_0 \cdot P_0$
 0.28 = $0.4 \cdot 0.7$

e_x = Expected life
 = Expectation of future life given survival to age x .
 = $\left(\sum_{i=x+1}^{\infty} l_i \right) \cdot \frac{1}{l_x}$ eq. 3

1.05 = $(0.28 + 0.14) / 0.4$

m_x = Fecundity at age x (average number of female offspring produced during the next year that survive to the time of the next census; e.g., average number of female bats produced last summer that survived to the next spring; measured in nature). Note that this is sometimes denoted as b_x in life table models. Note that we define m_x differently in a post-breeding model.

$l_x m_x$ = Realized fecundity
 = $l_x \cdot m_x$ eq. 4
 = Probability of survival to age x · Fecundity given survival.
 = Female offspring in year x per initial female of age 1

0.42 = $0.28 \cdot 1.5$ females / female

R_0 = Net reproductive rate (per capita progeny / lifetime, individuals · individual⁻¹ · lifetime⁻¹)
 = Average number of offspring produced by an average newborn offspring during its entire lifetime. Also equals the reproductive value for age class 0 (RV_0).

= $\sum_{x=0}^{\infty} l_x m_x$ eq. 5
 = 1.10

$$\begin{aligned}
G &= \text{Generation time (units = time step of life table; in this case, years)} \\
&= \text{Average difference between the birth of an individual and the birth of its own progeny} \\
&\approx \left(\sum_{x=0}^{\infty} l_x m_x x \right) / R_0. \quad \text{eq. 6}
\end{aligned}$$

$$2.89 = 3.18 / 1.10 .$$

$$\begin{aligned}
r &= \text{Intrinsic rate of increase (individuals} \cdot \text{individual}^{-1} \cdot \text{year}^{-1}) \\
&\approx \ln R_0 / G. \quad \text{eq. 7} \quad \text{An exact solution requires iteration with Euler's equation.} \\
0.033 &\approx \ln 1.10 / 2.89
\end{aligned}$$

$$\begin{aligned}
\lambda &= \text{Finite rate of increase. (pronounced lambda)} \\
&= e^r \quad \text{eq. 8} \\
1.034 &= e^{0.033}
\end{aligned}$$

$$\begin{aligned}
V_x &= \text{Reproductive value}^a \\
&= \text{Age-specific expectation of future offspring (females of age 1 / female of age } x) \\
&= \text{Expected reproduction during the remainder of its life for an organism of age } x \\
&= m_x + \sum_{i=x+1}^{\infty} \left(\frac{l_i}{l_x} \cdot m_i \right) \quad \text{eq. 9} \quad \text{Note that columns in the table labelled } v_0, v_1, \text{ etc., show values used in the summation for each } V_x
\end{aligned}$$

$$\begin{aligned}
2.75 &= 1 + .28 / .4 \cdot 1.5 + .14 / .4 \cdot 2 \\
&= 1 + 1.05 + 0.7
\end{aligned}$$

^a This equation can be different in alternative life table models. Derive it as Eq 10 for a post-breeding census model.

$$\begin{aligned}
C_x &= \text{Cohort size at stable age distribution} \\
&= \text{Proportion of total population of age } x \text{ at stable age distribution} \\
&= \frac{\lambda^{-x} \cdot l_x}{\sum_{x=0}^{\infty} (\lambda^{-x} \cdot l_x)}. \quad \text{eq. 11}
\end{aligned}$$

$$0.563 = 1 / 1.718$$

$$0.218 = 0.375 / 1.718$$

Given current population size, N_0 , future population size at time t can be projected using r or λ assuming that (1) mortality and natality schedules remain the same and (2) the population is at a stable age distribution.

$$N_t = N_0 \cdot e^{r \cdot t} \quad \text{eq. 12}$$

e.g., if $N_0 = 100$, and $r = 0.033$, $N_{10} = 139$

$$N_t = \lambda^t \cdot N_0 \quad \text{eq. 13}$$

e.g., if $N_0 = 100$, and $\lambda = 1.034$, $N_{10} = 139$