PRACTICE & PROBLEM SOLVING

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UNDERSTAND

15. **Mathematical Connections** Evaluate the expression $(2 + i)(2 + i)(2 + i)(2 + i)$ using traditional multiplication. Compare the results with the results using polar coordinates and $(2 + i)^4$. Which method do you prefer and why?

16. **Look for Relationships** Explain how to apply an exponent as a power of an expression with a complex number.

17. **Error Analysis** Describe and correct the error a student made in writing the complex number $z = \sqrt{3} - i$ in polar form.

$$r = \sqrt{x^2 + y^2} = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$$

So, $\sin\theta = \frac{y}{r} = \frac{-1}{2}$ and $\cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{2}$,

for $0 \le \theta \le 2\pi$.

$\theta = \frac{5\pi}{6}$ and $r = 2$,

So $\sqrt{3} - i = 2\text{cis}\left(\frac{5\pi}{6}\right)$.

18. **Communicate Precisely** Explain how to graph the complex number $3 \text{ cis } \frac{5\pi}{6}$ in the complex plane.

19. **Use Structure** Similar to the product formula, there is a quotient formula for two complex numbers in polar form. This formula is given by $\frac{(r \text{ cis } \alpha)}{(s \text{ cis } \beta)} = \frac{r}{s} \text{ cis } (\alpha - \beta)$. Use this formula to find $\frac{24 \text{ cis } \frac{5\pi}{3}}{8 \text{ cis } \frac{5\pi}{12}}$. Find the quotient in rectangular form.

20. **Higher Order Thinking** Explain why the polar coordinates $\left(4, -\frac{\pi}{6}\right)$ and $\left(4, \frac{11\pi}{6}\right)$ correspond to each other on the complex plane.

PRACTICE

Graph each complex number on the complex plane. SEE EXAMPLE 1

21. $4 \text{ cis}\left(\frac{\pi}{3}\right)$

22. $2 \text{ cis}\left(\frac{5\pi}{6}\right)$

23. $3 \text{ cis}(-\pi)$

24. $1 \text{ cis}\left(\frac{2\pi}{3}\right)$

Express each complex number in rectangular form. SEE EXAMPLE 2

25. $2 \text{ cis}\left(\frac{\pi}{2}\right)$

26. $1 \text{ cis}\left(-\frac{\pi}{3}\right)$

27. $4 \text{ cis}\left(\frac{11\pi}{6}\right)$

28. $3 \text{ cis}\left(\frac{\pi}{4}\right)$

Express each complex number in polar form. SEE EXAMPLE 2

29. $-\sqrt{3} + i$

30. $2 - 3i$

31. $4 + 5i$

32. $-3 - 3i$

Find the product of each set of complex numbers in polar and rectangular form. SEE EXAMPLE 3

33. $3 \text{ cis}\left(\frac{7\pi}{6}\right)$ and $2 \text{ cis}\left(\frac{2\pi}{3}\right)$

34. $6 \text{ cis}\left(\frac{\pi}{3}\right)$ and $4 \text{ cis}\left(\frac{2\pi}{3}\right)$

Express each number in polar form. Then find the product $z_1 z_2$ in both polar and rectangular form. SEE EXAMPLE 4

35. $z_1 = 2 - 2i$; $z_2 = -3 + 3i$

36. $z_1 = \sqrt{3} + i$; $z_2 = 2 - 2i\sqrt{3}$

Use the polar form to find each power. Write the power in both polar and rectangular form. SEE EXAMPLE 5

37. $(-2 + 2i)^3$

38. $(1 + i\sqrt{3})^5$

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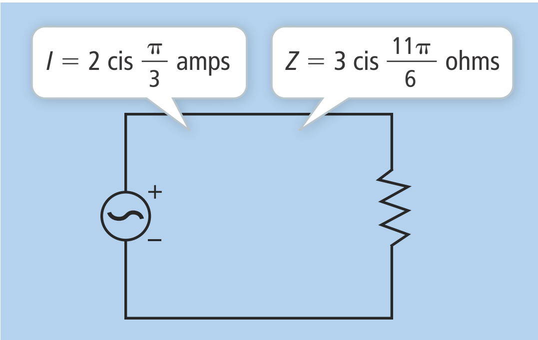
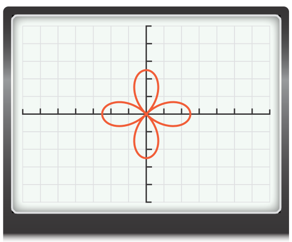
APPLY

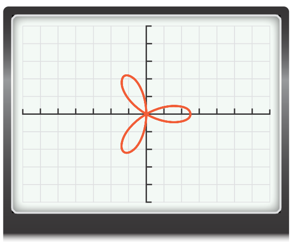
39. **Use Structure** Determine the voltage in a circuit when there is a current of 2 cis $\frac{\pi}{3}$ amps and an impedance of 3 cis $\frac{11\pi}{6}$ ohms. (*Hint*: Use $E = I \cdot Z$, where E is voltage, I is current, and Z is impedance.)

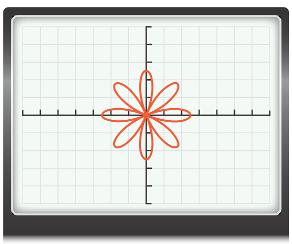


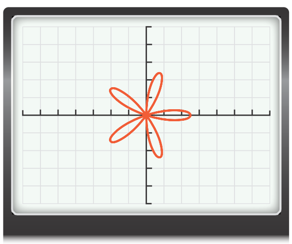
40. **Use Structure** Determine the current in a circuit when there is a voltage of 8 cis $\frac{7\pi}{6}$ volts and an impedance of 4 cis $\frac{\pi}{3}$ ohms.

41. **Use Appropriate Tools** A coding class is using the polar form of complex numbers to develop a program to draw a flower. Based on the graphs below, what is an equation a coder could use to draw a flower with nine petals? Explain your answer. (*Hint*: You may want to use graphing technology to check your answer.)


$r = 5 \cos(2\theta)$


$r = 5 \cos(3\theta)$


$r = 5 \cos(4\theta)$


$r = 5 \cos(5\theta)$

ASSESSMENT PRACTICE

42. The rectangular form of the complex number 4 cis$\left(\frac{7\pi}{4}\right)$ is ____ ____ ____i.

43. **SAT/ACT** Find the polar form of the complex number $4 - 4i$.

Ⓐ $4\sqrt{2}$ cis$\left(\frac{7\pi}{4}\right)$

Ⓑ $2\sqrt{2}$ cis$\left(\frac{7\pi}{4}\right)$

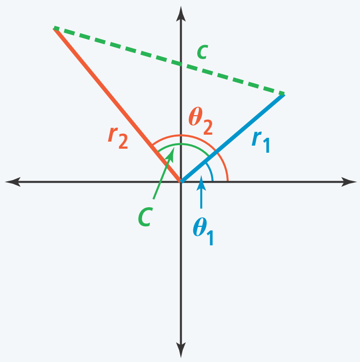
Ⓒ $4\sqrt{2}$ cis$\left(\frac{5\pi}{4}\right)$

Ⓓ 4 cis$\left(\frac{5\pi}{4}\right)$

44. **Performance Task** Either the Distance Formula or the Law of Cosines can be used to generate a formula to find the distance between two points in the complex plane.

Part A Use the Distance Formula to find the distance between two points r_1 cis θ_1 and r_2 cis θ_2 in the complex plane. (*Hint*: Remember that for $z = r \text{ cis } \theta = a + bi$, $a = r \cos \theta$ and $b = r \sin \theta$.)

Part B Use the Law of Cosines to calculate the same distance. Compare the result to the result for part (a).



Part C Use your formula to find the distance between 3 cis $\frac{\pi}{6}$ and 5 cis π. Round to the nearest hundredth.