



UNDERSTAND

13. **Construct Arguments** Consider the polynomial $P(x) = 5x^3 + mx^2 + nx + 6$, where m and n are rational coefficients. Is 3 *sometimes*, *always*, or *never* a root? Explain.
14. **Use Structure** Write a fourth-degree polynomial function Q with roots -1 , 0 , and $2i$.
15. **Error Analysis** A student says that a fifth-degree polynomial equation with rational coefficients has roots -5 , -3 , 1 , 2 , and $\sqrt{3}$. Describe possible errors the student may have made.
16. **Reason** Write a third-degree polynomial with rational coefficients that has the following possible roots. Explain your reasoning.
 $\pm \frac{1}{1}, \pm \frac{1}{2}, \pm \frac{2}{1}, \pm \frac{2}{2}, \pm \frac{5}{1}, \pm \frac{5}{2}, \pm \frac{10}{1}, \pm \frac{10}{2}$
17. **Error Analysis** Describe and correct the error a student made in finding the roots of the polynomial equation $2x^3 - x^2 - 10x + 5 = 0$.

List all possible rational roots.

$$\pm 1, \pm \frac{1}{2}, \pm 5, \pm \frac{5}{2}$$

Testing reveals that $\frac{1}{2}$ is a root.

Dividing the polynomial by the binomial $x - \frac{1}{2}$ results in the factored form

$$f(x) = (x - \frac{1}{2})(2x^2 - 10)$$

The equation $2x^2 - 10 = 0$ has two irrational roots, $\sqrt{10}$ and $-\sqrt{10}$.

The complete set of roots is $\{\frac{1}{2}, \sqrt{10}, -\sqrt{10}\}$.



18. **Higher Order Thinking** What is the least number of terms a fifth-degree polynomial with root $3i$ can have? Give an example of such a polynomial equation. Explain.
19. **Use Structure** Show that the Fundamental Theorem of Algebra is true for all quadratic equations with real coefficients. (*Hint:* Use the Quadratic Formula and examine the possibilities for the value of the discriminant.)

PRACTICE

List all the possible rational solutions for each equation. SEE EXAMPLE 1

20. $0 = x^3 - 3x^2 + 4x - 12$

21. $0 = 2x^4 + 13x^3 - 47x^2 - 13x + 45$

22. $0 = 4x^3 + 64x^2 - x - 16$

23. $0 = 8x^3 + 11x^2 - 13x - 6$

24. A closet in the shape of a rectangular prism has the measurements shown. What is the height of the closet, in feet, if its volume is 220 ft^3 ?

SEE EXAMPLE 2



What are all real and complex roots of the following functions? SEE EXAMPLE 3

25. $0 = x^3 - 3x - 52$

26. $0 = x^3 + 9x^2 - 7x - 63$

27. $0 = x^4 + 34x^2 - 72$

28. $0 = x^6 + 4x^4 - 41x^2 + 36$

29. Suppose a cubic polynomial f has one rational zero c and two irrational zeros which are a conjugate pair $a + \sqrt{b}$ and $a - \sqrt{b}$, where a and b are rational numbers. Does f have rational coefficients? SEE EXAMPLE 4

Find a polynomial function $P(x)$ such that P has the degree and $P(x) = 0$ has the root(s) listed.

SEE EXAMPLE 5

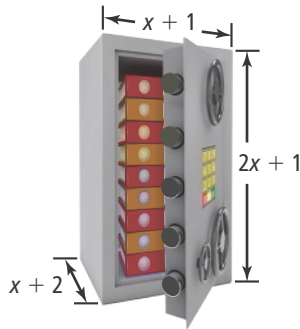
30. degree of $P = 2$;
zero: $1 + 6i$

31. degree of $P = 4$;
zeros: $3 - \sqrt{11}$ and $-9i$

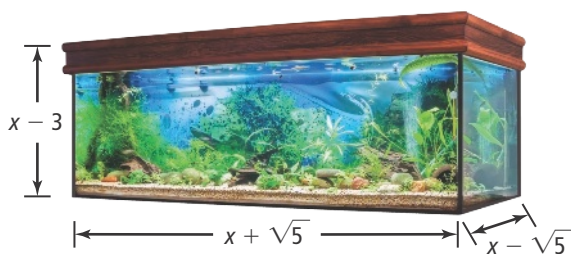
32. degree of $P = 3$;
zeros: -5 and $4 - 8i$

APPLY

- 33. Make Sense and Persevere** A fireproof safe has the measurements shown.



- Write an equation to represent the situation when the volume of the fireproof safe is 270 in.³. Rewrite the equation in the form $P(x) = 0$.
 - List all of the possible factors of the polynomial expression.
 - What are the real roots of the equation? Explain how you know these are the only real roots.
 - What are the length, width, and height of the fireproof safe?
- 34. Make Sense and Persevere** What are the dimensions of the fish tank, in feet, if its volume is 176 ft³?



- 35. Reason** The cost of producing x video game consoles is modeled by the function $C(x) = x^4 - 5x^3 - 12x^2 - 22x - 40$. If a company spent \$1,706 to produce video game consoles, how many consoles were made?

ASSESSMENT PRACTICE

- 36.** A fifth-degree polynomial equation with rational coefficients has the roots 3, $8i$, and $7 - \sqrt{5}$. Which are also roots of the polynomial equation? Select all that apply.
- -3
 - $-8i$
 - $1 - 8i$
 - $-7 - \sqrt{5}$
 - $7 + \sqrt{5}$
- 37. SAT/ACT** Which is a third-degree polynomial equation with rational coefficients that has roots -2 and $6i$?
- $x^3 + 2x^2 + 36x + 72$
 - $x^3 - 2x^2 + 36x - 72$
 - $x^3 + 2x^2 - 36x - 72$
 - $x^2 + (6i - 2)x - 12$
 - $x^2 - (6i - 2)x - 12$
- 38. Performance Task** The table shows the number of possible real and imaginary roots for an n th degree polynomial equation with rational coefficients.

Degree	Real Roots	Imaginary Roots
3	3	0
3	1	2
5	5	0
5	3	2
5	1	4

Part A List all of the possible combinations of real and imaginary roots for a seventh-degree polynomial equation.

Part B What do you notice about the number of real roots of a polynomial equation with an odd degree?