TOPIC 10

Topic Review

TOPIC ESSENTIAL QUESTION

1. How can you use matrices to help you solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The __________ has one column that represents all the variables in the system of equations.
3. The __________ is a square matrix with ones on the main diagonal and zeros for all other elements.
4. __________ means the multiplication of each element of a matrix by a single real number.
5. The __________ is the length of the vector.
6. The product of a matrix and its __________ is the identity matrix.
7. The __________ has one column that contains the constants from the right-hand side of the system of equations.
8. A(n) __________ is a matrix that has the same number of rows as columns.

- constant matrix
- identity matrix
- inverse matrix
- magnitude
- scalar multiplication
- square matrix
- variable matrix
- vector
- zero matrix

Concepts & Skills Review

LESSON 10-1 Operations With Matrices

Quick Review

To multiply a matrix by a scalar, multiply each element in the matrix by the scalar.

To add (or subtract) matrices, add (or subtract) the corresponding elements.

Example

Add matrices A and B.

$$A = \begin{bmatrix} 9 & 2 & 11 \\ -3 & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -1 & 7 \\ 8 & 12 & 0 \end{bmatrix}$$

Add corresponding elements of the two matrices.

$$A + B = \begin{bmatrix} 9 & 2 & 11 \\ -3 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 4 & -1 & 7 \\ 8 & 12 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9+4 & 2+(-1) & 11+7 \\ -3+8 & 5+12 & 6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 1 & 18 \\ 5 & 17 & 6 \end{bmatrix}$$

Practice & Problem Solving

Given matrices $C = \begin{bmatrix} 9 & -5 \\ 3 & 6 \end{bmatrix}$ and $D = \begin{bmatrix} -7 & 1 \\ 8 & 2 \end{bmatrix}$, calculate each of the following.

9. $D - C$
10. $5D$
11. A segment has endpoints $A(5, -3)$ and $B(2, 4)$. Use matrices to represent a translation of $\overline{AB}$ to $\overline{YZ}$ by 3 units right and 7 units down. What are the coordinates of Y and Z?
12. **Communicate Precisely** Suppose N is a 3×3 matrix. Explain how to find matrix P so that $N + P$ is the zero matrix.
13. **Make Sense and Persevere** A seminar has 6 women and 8 men register early. Then 18 women and 12 men register in class. Use matrix addition to find the total number of men and women in the seminar.

TOPIC 10 REVIEW

LESSON 10-2 Matrix Multiplication

Quick Review

The product of two matrices is a new matrix with the sums of the products of corresponding row and column elements.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{bmatrix}$$

For an $n \times n$ matrix A, the multiplicative **identity matrix** I is an $n \times n$ square matrix with 1s on the main diagonal and 0s for all other elements: $AI = IA = A$.

Example

Multiply matrices A and B.

$A = \begin{bmatrix} 3 & -2 \\ 1 & -4 \end{bmatrix}$ $\quad B = \begin{bmatrix} -1 & 6 \\ 0 & 5 \end{bmatrix}$

Find the sums of products of corresponding row and column elements.

$$AB = \begin{bmatrix} 3 & -2 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} -1 & 6 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (3)(-1) + (-2)(0) & (3)(6) + (-2)(5) \\ (1)(-1) + (-4)(0) & (1)(6) + (-4)(5) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 8 \\ -1 & -14 \end{bmatrix}$$

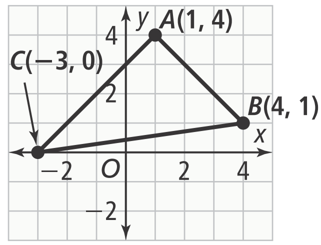
Practice & Problem Solving

Given matrices $A = \begin{bmatrix} 4 & -3 \\ 0 & 9 \end{bmatrix}$, $B = \begin{bmatrix} -7 & 8 \\ -5 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 6 & -1 \\ 2 & -2 \end{bmatrix}$, find each of the following.

14. AB **15.** AC **16.** BC

17. BA **18.** CA **19.** CB

20. Represent the coordinates of the triangle as a matrix. Then multiply by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ to find the coordinates of the image of triangle ABC after a reflectionacross the x-axis.



21. **Look for Relationships** Explain how to determine whether two matrices can be multiplied.

22. **Make Sense and Persevere** At Store X, Television A costs \$800 and Television B costs \$500. At Store Y, Television A costs \$750 and Television B costs \$550. Last month, each store sold 25 of Television A and 20 of Television B. Write and solve a matrix equation to find the total amount in sales at each store.

LESSON 10-3 Vectors

Quick Review

For vectors $\vec{u} = \langle a, b \rangle$ and $\vec{v} = \langle c, d \rangle$, the magnitude of $\vec{u}$ is $|\vec{u}| = \sqrt{a^2 + b^2}$, and the direction of $\vec{u}$ is $\theta = \tan^{-1}\left(\frac{b}{a}\right)$.

For a scalar k, $k \bullet \vec{u} = \langle k \bullet a, k \bullet b \rangle$, $|k \bullet \vec{u}| = |k| \bullet |\vec{u}|$.

$\vec{u} + \vec{v} = \langle a + c, b + d \rangle$ and

$\vec{u} - \vec{v} = \vec{u} + (-\vec{v}) = \langle a - c, b - d \rangle$.

Example

Add vectors $\overrightarrow{AB} = \langle 6, -2 \rangle$ and $\overrightarrow{CD} = \langle 3, 7 \rangle$.

$$\overrightarrow{AB} + \overrightarrow{CD} = \langle 6, -2 \rangle + \langle 3, 7 \rangle$$

$$= \langle 6 + 3, -2 + 7 \rangle$$

$$= \langle 9, 5 \rangle$$

Practice & Problem Solving

Add and subtract each vector pair.

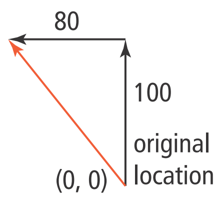
23. $\overrightarrow{AB} = \langle 8, 10 \rangle$ and $\overrightarrow{CD} = \langle -3, 2 \rangle$

24. $\overrightarrow{RS} = \langle -7, 9 \rangle$ and $\overrightarrow{TU} = \langle 11, -5 \rangle$

25. Multiply the vector $\vec{t} = \langle 13, -3 \rangle$ by the scalar 4.

26. **Communicate Precisely** Describe how $\overrightarrow{MN} = \langle -2, 9 \rangle$ is transformed when it is multiplied by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

27. **Reason** A blimp flying due north is pushed off course by a crosswind blowing west. By how many degrees did the crosswind change the blimp's original course?



LESSON 10-4 Inverses and Determinants

Quick Review

The determinant of a **2 × 2 matrix** $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is denoted det A and is equal to $ad - bc$.

The **inverse matrix** is $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Example

Find the inverse of matrix $A = \begin{bmatrix} 4 & 8 \\ 2 & 6 \end{bmatrix}$.

$\det A = ad - bc = (4)(6) - (2)(8) = 24 - 16 = 8$

Because the determinant does not equal 0, there is an inverse.

$$A^{-1} = \frac{1}{8}\begin{bmatrix} 6 & -8 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & -1 \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

Practice & Problem Solving

Find the determinant of each matrix.

28. $\begin{bmatrix} 12 & -6 \\ 8 & -3 \end{bmatrix}$ **29.** $\begin{bmatrix} 14 & -3 \\ 2 & 0 \end{bmatrix}$

Does each given matrix have an inverse? If so, find it.

30. $A = \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix}$ **31.** $B = \begin{bmatrix} 1 & 3 & 2 \\ -2 & -4 & 0 \\ -1 & -3 & 5 \end{bmatrix}$

32. Error Analysis Carla said the inverse of matrix $A = \begin{bmatrix} 8 & 2 \\ 2 & 1 \end{bmatrix}$ is $\begin{bmatrix} 2 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$. Describe and correct Carla's error.

33. Use Structure Find the area of the triangle defined by vectors $\langle 8, 6\rangle$ and $\langle 2, -4\rangle$.

LESSON 10-5 Inverse Matrices and Systems of Equations

Quick Review

Matrices can be used to solve systems of equations.

The coefficient matrix has rows which contain the coefficients from a single equation. Each column contains all coefficients of a single variable.

The **variable matrix** has one column that represents all the variables in the system of equations.

The constant values from the right-hand side of the equations are used to make the **constant matrix**.

$$\begin{matrix} ax + by + cz = k \\ dx + ey + fz = m \\ gx + hy + jz = n \end{matrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ m \\ n \end{bmatrix}$$

Example

Solve the system of equations $\begin{cases} 3x + 6y = 0 \\ -2x + 3y = -7 \end{cases}$ **using matrices.**

$$\begin{matrix} 3x + 6y = 0 \\ -2x + 3y = -7 \end{matrix} \Rightarrow \begin{bmatrix} 3 & 6 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ -2 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 3 & 6 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ -2 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & -\frac{2}{7} \\ \frac{2}{21} & \frac{1}{7} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Practice & Problem Solving

Solve the following systems of equations using inverse matrices.

34. $\begin{cases} 2x + 4y = 4 \\ -3x - 7y = -4 \end{cases}$ **35.** $\begin{cases} -2x + 3y + 3z = 6 \\ 6x - 8y - 2z = -4 \\ 2x - 2y - 3z = -13 \end{cases}$

36. Communicate Precisely Explain how to write a system of equations given the matrix equation $\begin{bmatrix} 4 & 9 & 1 \\ 8 & -2 & 0 \\ -7 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 6 \end{bmatrix}$.

37. Reason Two students visited the school store to buy supplies for the school year. One student purchased 8 folders and 6 notebooks for a total price of \$38. The other student purchased 2 folders and 9 notebooks for a total of \$47. If each folder is the same price and each notebook is the same price, how much does each folder and each notebook cost?