TOPIC
12

Topic Review

TOPIC ESSENTIAL QUESTION

1. How can you find the probability of events and combinations of events?

Vocabulary Review

Choose the correct term to complete each sentence.

2. An arrangement of items in a specific order is called a(n) ________.
3. Two events are ________ if there is no outcome that is in both events.
4. Two events are ________ if the occurrence of one event affects the probability of the other event.
5. An arrangement of items in which order is not important is called a(n) ________.
6. The predicted average outcome of many trials in an experiment is called the ________.

- combination
- complement
- conditional probability
- dependent events
- expected value
- independent events
- mutually exclusive
- permutation
- probability distribution

Concepts & Skills Review

LESSON 12-1 Probability Events

Quick Review

Two events are **mutually exclusive** if and only if there is no outcome that lies in the sample space of both events. Two events are **independent events** if and only if the outcome of one event does not affect the probability of a second event.

Example

Let A represent the event "even number," or $A = \{2, 4, 6, 8\}$.

Let B represent the event "odd number," or $B = \{1, 3, 5, 7\}$.

Let C represent the event "divisible by 3," or $C = \{3, 6\}$.

Are A and B mutually exclusive? Explain.
Yes; all of their elements are different.

Are A and C mutually exclusive? Explain.
No; they both have a 3 in their sample space.

Practice & Problem Solving

Ten craft sticks lettered A through J are in a coffee cup. Consider the events "consonant," "vowel," "letter before D in the alphabet," "letter after A in the alphabet," and "letter after E in the alphabet." State whether each pair of events is mutually exclusive.

7. vowel, letter before D
8. letter before D, letter after E
9. letter after A, letter before D
10. **Communicate Precisely** Edward is rolling a number cube to decide on the new combination for his bicycle lock. If he only has one number to go, find the probability of each event. Use what you know about mutually exclusive events to explain your reasoning.

    a. Edward rolls a number that is both even and less than 2. Explain.

    b. Edward rolls a number that is even or less than 2. Explain.

LESSON 12-2 Conditional Probability

Quick Review

For any two events A and B, with $P(A) \neq 0$, $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$. Events A and B are independent if and only if $P(B \mid A) = P(B)$.

Example

The table shows the number of students on different teams by grade. One of these students is selected at random for an interview. Are selecting a sophomore and selecting a member of the track team independent events?

Team Enrollment by Year

	Sophomore	Junior
Cross Country	9	6
Track	12	23

$P(\text{Sophomore and Track}) = 0.24$

$P(\text{Sophomore}) = 0.42 \quad P(\text{Track} \mid \text{Sophomore}) \approx 0.57$

$P(\text{Soph and Track}) \neq P(\text{Soph}) \cdot P(\text{Track} \mid \text{Soph})$ because $0.24 \neq 0.42 \cdot 0.57$

No, selecting a sophomore and selecting a member of the track team are dependent events.

Practice & Problem Solving

Use the table in the Example for Exercises 11–15. All students are selected at random.

11. $P(\text{Junior})$

12. $P(\text{Cross Country})$

13. $P(\text{Junior} \mid \text{Cross Country})$

14. $P(\text{Cross Country} \mid \text{Junior})$

15. Are selecting a junior and selecting a cross country runner dependent or independent events?

16. Error Analysis One card is selected at random from five cards numbered 1–5. A student says that drawing an even number and drawing a prime number are dependent events because $P(\text{prime} \mid \text{even}) = 0.5$ and $P(\text{even}) = 0.4$. Describe and correct the error the student made.

17. Use Structure A person entered in a raffle has a 3% chance of winning a prize. A prize winner has a 25% chance of winning two theater tickets. What is the probability that a person entered in the raffle will win the theater tickets?

LESSON 12-3 Permutations and Combinations

Quick Review

The number of permutations of n items taken r at a time is ${}_nP_r = \frac{n!}{(n-r)!}$ for $0 \leq r \leq n$.

The number of combinations of n items chosen r at a time is ${}_nC_r = \frac{n!}{r!(n-r)!}$ for $0 \leq r \leq n$.

Example

A bag contains 4 blue tiles and 4 yellow tiles. Three tiles are drawn from the bag at random without replacement. What is the probability all three tiles are blue?

Use combinations since order does not matter. Select 3 blue from 4 blue tiles ${}_4C_3$, or 4, ways.

Select 0 yellow from 4 yellow tiles ${}_4C_0$, or 1, way. Select 3 blue tiles from 8 total tiles ${}_8C_3$, or 56, ways.

$P(\text{3 blue}) = \frac{4 \cdot 1}{56} = \frac{1}{14} \approx 0.07 \approx 7\%$

Practice & Problem Solving

In Exercises 18 and 19, determine whether the situation involves finding permutations or combinations. Then find the number.

18. How many ways can a team choose a captain and a substitute captain from 8 players?

19. How many ways can 3 numbers be selected from the digits 0–9 to set a lock code if the digits cannot be repeated?

20. Error Analysis A student computed ${}_5C_2$, and said that it is equal to 20. Describe and correct the error the student made.

21. Look for Relationships The formulas for permutations and combinations must always evaluate to a natural number. Explain why.

LESSON 12-4 Probability Distributions

Quick Review

For a binomial experiment consisting of n trials, the probability of r successes out of n trials is called the **binomial probability** and given by:

$$P(r) = {}_nC_r \cdot p^r(1 - p)^{n-r}$$

Example

Curtis scores a touchdown 24% of the time he receives the ball. If Curtis receives the ball 7 times, what is the probability he scores a touchdown 4 of those times?

$$\begin{aligned} P(4 \text{ touchdowns}) &= {}_7C_4 \cdot 0.24^4(1 - 0.24)^{7-4} \\ &= 35 \cdot 0.24^4(0.76)^3 \\ &\approx 0.051 \approx 5.1\% \end{aligned}$$

Practice & Problem Solving

Rhoda finds that every seed she plants has a 56% chance to grow to full height. If she plants 10 seeds, what is the probability each number of plants grows to full height? Round to the nearest hundredth of a percent.

22. 1 plant

23. 3 plants

24. 5 plants

25. 10 plants

26. Error Analysis Using the Example, Akasi tried to calculate Curtis' probable success rate of 3 touchdowns if he received the ball 5 times, but could not get an answer. Find and correct her mistake.

$$\begin{aligned} P(3) &= {}_3C_5 \cdot 0.24^5\,(1\text{-}0.24)^{3-5} \\ &= ? \end{aligned}$$

LESSON 12-5 Expected Value

Quick Review

The **expected value** $E(x)$ of a trial of an experiment is the sum of the value of each possible outcome x_n times its probability p_n or $E(x) = x_1p_1 + x_2p_2 + \ldots + x_np_n$.

Example

The outer ring on a dartboard is worth 10 points, the middle ring is worth 25 points, and the bullseye is worth 100 points. When throwing darts, Ravi has a 45% chance of hitting the outer ring, a 40% chance of hitting the inner ring, a 5% chance of hitting the bullseye, and a 10% chance of missing the board. What is the expected value of a single dart throw?

$$\begin{aligned} E(x) &= 10(0.45) + 25(0.4) + 100(0.05) + 0(0.1) \\ &= 4.5 + 10 + 5 + 0 \\ &= 19.5 \end{aligned}$$

Practice & Problem Solving

Use the information in the Example to find the expected value of 15 throws from each of the following people.

27. Rosa: 20% outer ring; 65% inner ring, 10% bullseye, 5% miss

28. Vicki: 60% outer ring; 20% inner ring, 12% bullseye, 8% miss

29. Higher Order Thinking A basketball player takes 2 shots from the 3-point line and misses them both. She calculates the expected value of taking a shot from the 3-point line is 0 points. Do you agree with the player's calculation? Her reasoning? How could the player improve the accuracy of her estimate?

LESSON 12-6 Probability and Decision Making

Quick Review

Combined with probability, expected value can be used to help make decisions.

Example

Frederica is playing a game tossing 20 beanbags from a choice of three lines. Frederica has a 90% chance of success from the 5-point line, a 65% chance of success from the 10-point line, and a 20% chance from the 20-point line. Frederica wants to toss every beanbag from the same line, and thinks she should toss from the 5-point line since it has the highest probability of success. Is Frederica correct?

Find the expected points per toss, or expected value.

5-point line:
5 points • 0.90 = 4.5 points per toss

10-point line:
10 points • 0.65 = 6.5 points per toss

20-point line:
20 points • 0.20 = 4 points per toss

Frederica should toss the beanbag from the 10-point line.

Practice & Problem Solving

Both situations have the same expected value. Find the missing information.

30. Situation 1: Paul hits a dart target worth 15 points 45% of the time.

Situation 2: He hits a dart target worth 10 points x% of the time.

31. Situation 1: Lenora has a success rate of 25% when selling bracelets at \$15 each.

Situation 2: She has a success rate of 20% when selling bracelets at \$$x$ each.

32. **Make Sense of Problems** Use the information from the Example. Frederica practices her shots and increases her chances from the 20-point line to 30%. Should she now toss the beanbag from the 20-point line? Explain your reasoning.