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#### 1 Misc

#### 1.1 Default Code [a78860]

```
i #include <bits/stdc++.h>
2 #define F first
3 #define S second
4 #define pb push back
 #define all(x) begin(x), end(x)
 #define FOR(i,a,b) for (int i = (a); i <= (b); i++)
 #define HEHE ios_base::sync_with_stdio(0), cin.tie(0);
  using namespace std;
10 #define debug(a...) cerr << #a << " = ", dout(a)</pre>
11 void dout() { cerr << "\n"; }</pre>
12 template <class T, class... U>
13 void dout(T t, U... u) { cerr << t << ' ', dout(u...); }</pre>
15 #define int long long
17 signed main() {
      HEHE
```

#### 1.2 Run

```
i from os import *
 f = "pA"
 while 1:
     i = input("input: ")
     system("clear")
     p = listdir(".")
     if i != "":
         f = i
     print(f"file = {f}")
     if system(f"g++ {f}.cpp -std=c++17 -Wall -Wextra -Wshadow
           -02 -D LOCAL -g -fsanitize=undefined, address -o {f}
          "):
         print("CE")
         continue
     for x in sorted(p):
         if f in x and ".in" in x:
             print(x)
             if system(f"./\{f\} < \{x\}"):
                  print("RE")
             print()
```

# 1.3 Custom Set PQ Sort [d4df55]

```
1 | // 所有自訂的結構體·務必檢查相等的 case·給所有元素一個排序
      的依據
2 struct my_struct{
     int val;
     my struct(int val) : val( val) {}
 };
 auto cmp = [](my_struct a, my_struct b) {
     return a.val > b.val;
set<my_struct, decltype(cmp)> ss({1, 2, 3}, cmp);
```

```
12 priority_queue<my_struct, vector<my_struct>, decltype(cmp)>
       pq(cmp, \{1, 2, 3\});
map<my_struct, my_struct, decltype(cmp)> mp({{1, 4}, {2, 5},
       {3, 6}}, cmp);
```

# 1.4 Dynamic Bitset [c78aa8]

```
const int MAXN = 2e5 + 5;
template <int len = 1>
void solve(int n) {
    if (n > len) {
        solve<min(len*2, MAXN)>(n);
        return;
    bitset<len> a;
```

#### 1.5 Enumerate Subset [a13e46]

```
1 // 時間複雜度 O(3^n)
2 / / 枚舉每個 mask 的子集
 for (int mask=0 ; mask<(1<<n) ; mask++){</pre>
     for (int s=mask ; s>=0 ; s=(s-1)&m){
         // s 是 mask 的子集
         if (s==0) break;
```

#### 1.6 Fast Input [6f8879]

```
1 // fast IO
2 // 6f8879
3 inline char readchar(){
      static char buffer[BUFSIZ], * now = buffer + BUFSIZ, *
           end = buffer + BUFSIZ;
      if (now == end)
          if (end < buffer + BUFSIZ)</pre>
              return EOF;
          end = (buffer + fread(buffer, 1, BUFSIZ, stdin));
          now = buffer:
      return *now++;
 inline int nextint(){
      int x = 0, c = readchar(), neg = false;
      while(('0' > c | | c > '9') && c!='-' && c!=EOF) c =
           readchar();
      if(c == '-') neg = true, c = readchar();
      while ('0' \le c \&\& c \le '9') x = (x << 3) + (x << 1) + (c^'0')
           , c = readchar();
      if(neg) x = -x;
      return x; // returns 0 if EOF
  1.7 OEIS [ec45dc]
```

```
ı | // 若一個線性遞迴有 k 項·給他恰好 2*k 個項可以求出線性遞迴
2 // f915c2
3 template <typename T>
4 vector<T> BerlekampMassey(vector<T> a) {
     auto scalarProduct = [](vector<T> v, T c) {
        for (T &x: v) x *= c;
```

```
return v;
      };
      vector<T> s, best;
      int bestPos = 0;
      for (int i=0 ; i<a.size() ; i++){</pre>
          T = a[i];
          for (int j=0 ; j<s.size() ; j++) error -= s[j] * a[i</pre>
          if (error == 0) continue;
          if (s.empty()) {
               s.resize(i + 1);
               bestPos = i;
               best.push_back(1 / error);
               continue;
          vector<T> fix = scalarProduct(best, error);
          fix.insert(fix.begin(), i - bestPos - 1, 0);
          if (fix.size() >= s.size()) {
               best = scalarProduct(s, - 1 / error);
               best.insert(best.begin(), 1 / error);
               bestPos = i;
               s.resize(fix.size());
          for (int j = 0; j < fix.size(); j++) s[j] += fix[j];</pre>
29
      reverse(s.begin(), s.end());
31
32
      return s;
33
```

#### 1.8 Pragma [09d13e]

```
1 #pragma GCC optimize("03,unroll-loops")
2 #pragma GCC target("avx,avx2,sse,sse2,sse3,sse4,popcnt")
```

#### 1.9 Xor Basis [840136]

```
vector<int> basis:
  void add vector(int x){
     for (auto v : basis){
         x=min(x, x^v);
     if (x) basis.push back(x);
  // 給一數字集合 S · 求能不能 XOR 出 x
  bool check(int x){
     for (auto v : basis){
         x=min(x, x^v);
     return 0:
17 // 給一數字集合 S, 求能 XOR 出多少數字
18 // 答案等於 2^{basis 的大小}
  // 給一數字集合 S · 求 XOR 出最大的數字
  int get_max(){
     int ans=0:
     for (auto v : basis){
         ans=max(ans, ans^v);
25
26
     return ans;
```

6 sys.setrecursionlimit(100000)

#### 1.10 random int [9cc603] 7 sys.set\_int\_max\_str\_digits(10000) 2 g++ ac.cpp -o ac g++ wa.cpp -o wa i | mt19937 seed(chrono::steady clock::now().time since epoch(). 4 for ((i=0;;i++)) from turtle import \* count()); echo "\$i" 2 int rng(int 1, int r){ 12 N = 3000000010return uniform int distribution<int>(1, r)(seed); python3 gen.py > input setworldcoordinates(-N, -N, N, N) ./ac < input > ac.out hideturtle() ./wa < input > wa.out speed(100) diff ac.out wa.out || break 1.11 **OEIS** 11 done def draw line(a, b, c, d): | from fractions import Fraction teleport(a, b) 1.14 disable ASLR goto(c, d) def BerlekampMassey(a: list[Fraction]) -> list[Fraction]: def scale(v: list[Fraction], c: Fraction) -> list[ def write dot(x, y, text, diff=1): # diff = 文字的偏移 Fraction]: teleport(x, y) return [x \* c for x in v] 3 setarch \$(uname -m) -R dot(5, "red") s: list[Fraction] = [] teleport(x+N/100\*diff, y+N/100\*diff) 1.15 hash command best: list[Fraction] = [] write(text, font=("Arial", 5, "bold")) bestPos = 0 for i in range(len(a)): | md5sum | cut -c-6 29 draw line(\*a[i], \*(a[i-1])) error: Fraction = a[i] 30 write\_dot(\*a[i], str(a[i])) for j in range(len(s)): 1.16 hash windows 31 # 以自己左方 100 單位為圓心向前畫一個 90 度的弧 / 正五邊形 error -= s[j] \* a[i - 1 - j](參數可以是負的) if error == 0: 32 circle(100, extent = 90) def get\_hash(path): continue 33 circle(100, steps = 5) from hashlib import md5 34 left(90) # 左轉 90 度 if not s: s = [Fraction(0)] \* (i + 1)35 fd(-100) # 倒退 100 單位 bestPos = i36 penup() p = run( best = [Fraction(1, error)] pendown() stdout = PIPE, continue stderr = PIPE, 39| tracer(0, 0) # IO 優化·必須在程式碼最後手動 update() text = True fix = scale(best, error) 40 pos() # 回傳目前座標 fix = [Fraction(0)] \* (i - bestPos - 1) + fix41 | heading() # 回傳目前方向: 0 是右方、90 是上方 color("red") # 改變顏色,可以用文字或是 #000000 的格式 if p.returncode != 0: if len(fix) >= len(s): best = scale(s, Fraction(-1, error)) # 00P best.insert(0, Fraction(1, error)) class Point: bestPos = idef \_\_init\_\_(self, x, y): if len(s) < len(fix):</pre> self.x = xs += [Fraction(0)] \* (len(fix) - len(s))return ret[:6] self.y = yfor j in range(len(fix)): def add (self, o): # use dir(int) to know operator s[j] += fix[j] 2 Convolution return Point(self.x+o.x, self.y+o.y) return list(reversed(s)) 2.1 FFT [16591a] @property 39 n = int(input()) def distance(self): 40 1 = list(map(Fraction, input().split())) typedef complex<double> cd; return (self.x\*\*2 + self.y\*\*2)\*\*(0.5) 41 for i in range(len(1)): coeffs = BerlekampMassey(l[:i+1]) // 778272 a = Point(3, 4)for x in coeffs: void FFT(vector<cd> &a) { 58 print(a.distance) print(x, end=" ") print() vector<cd> rt(2, 1); from fractions import Fraction 1.12 Python for (int k=2; k<n; k\*=2){</pre> 62 a = Fraction(Decimal(1.1)) 63 a.numerator # 分子 1 # Decimal 64 a.denominator # 分母 2 from decimal import \* /2]\*x : R[i/2]); getcontext().prec = 6 1.13 diff 12 5 # system setting 13

ı set -e

```
1 # Disable randomization of memory addresses
 setarch `uname -m` -R ./yourProgram
```

```
1 cat file.cpp | cpp -dD -P -fpreprocessed | tr -d "[:space:]"
```

```
from subprocess import run, PIPE
          ["cpp", "-dD", "-P", "-fpreprocessed", path],
          raise RuntimeError(p.stderr)
      s = ''.join(p.stdout.split())
      ret = md5(s.encode()).hexdigest()
print(get hash("Suffix Array.cpp"))
```

```
int n = a.size(), L = 31- builtin clz(n);
vector<complex<long double>> R(2, 1);
    R.resize(n), rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    for (int i=k; i<2*k; i++) rt[i] = R[i] = (i&1 ? R[i</pre>
vector<int> rev(n);
```

for (int k=1 ; k<n ; k\*=2){</pre>

```
for (int i=0; i<n; i++) rev[i] = (rev[i/2] | (i&1)<<L) 24
                                                                             for (int i=0 ; i<n ; i+=2*k){</pre>
                                                                                                                                      21 };
                                                                                  for (int j=0 ; j<k ; j++){</pre>
      for (int i=0 ; i<n ; i++) if (i<rev[i]) swap(a[i], a[rev[ 26</pre>
                                                                                      auto x = (double *)&rt[j+k];
                                                                                                                                        2.4 Min Convolution Concave Concave Iffb28dl
                                                                                      auto y = (double *)&a[i+j+k];
           i]]);
                                                                                      cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*
      for (int k=1; k<n; k*=2){</pre>
                                                                                                                                      11//需要增加註解
          for (int i=0 ; i<n ; i+=2*k){</pre>
                                                                                          y[0]);
                                                                                                                                        // min convolution
              for (int j=0 ; j<k ; j++){</pre>
                                                                                      a[i+j+k] = a[i+j]-z;
                                                                                                                                        vector<int> mkk(vector<int> a, vector<int> b) {
                  // cd z = rt[j+k] * a[i+j+k];
                                                                                      a[i+j] += z;
                                                                                                                                            vector<int> slope:
                  auto x = (double *)&rt[j+k], y = (double *)&a 31
                                                                                                                                            FOR (i, 1, ssize(a) - 1) slope.pb(a[i] - a[i - 1]);
                        [i+j+k];
                                                                                                                                            FOR (i, 1, ssize(b) - 1) slope.pb(b[i] - b[i - 1]);
                   cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]* 33
                                                                                                                                            sort(all(slope));
                       y[0]);
                                                                                                                                            slope.insert(begin(slope), a[0] + b[0]);
                  a[i+j+k] = a[i+j]-z;
                                                                                                                                            partial_sum(all(slope), begin(slope));
                  a[i+j] += z;
                                                                     // c3dcf6
                                                                                                                                            return slope;
                                                                     vector<int> PolyMul(vector<int> a, vector<int> b){
                                                                         if (a.empty() || b.empty()) return {};
                                                                          vector<int> res(a.size()+b.size()-1);
                                                                         int B = 32-__builtin_clz(res.size()), n = (1<<B), cut = (</pre>
                                                                                                                                        2.5 NTT mod 998244353 [f9fed4]
                                                                              int)(sart(MOD));
                                                                          vector<cd> L(n), R(n), outs(n), outl(n);
31 vector<double> PolyMul(const vector<double> a, const vector
                                                                                                                                        const int MOD = (119 << 23) + 1, ROOT = 62; // = 998244353</pre>
       double> b){
                                                                          for (int i=0; i<a.size(); i++) L[i] = cd((int) a[i]/cut 3</pre>
                                                                                                                                        // For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
                                                                               , (int)a[i]%cut);
      if (a.empty() || b.empty()) return {};
      vector<double> res(a.size()+b.size()-1);
                                                                          for (int i=0; i<b.size(); i++) R[i] = cd((int) b[i]/cut</pre>
                                                                                                                                        // and 483 << 21 (same root). The last two are > 10^9.
      int L = 32 - builtin clz(res.size()), n = 1<<L;</pre>
                                                                               , (int)b[i]%cut);
      vector<cd> in(n), out(n);
                                                                          FFT(L), FFT(R);
                                                                                                                                         // e169db
      copy(a.begin(), a.end(), in.begin());
                                                                          for (int i=0 ; i<n ; i++){</pre>
                                                                                                                                        void NTT(vector<int> &a) {
      for (int i=0; i < b.size(); i++) in[i].imag(b[i]);</pre>
                                                                             int j = -i&(n-1);
                                                                                                                                            int n = a.size();
      FFT(in);
                                                                             outl[j] = (L[i]+conj(L[j])) * R[i]/(2.0*n);
                                                                                                                                            int L = 31- builtin clz(n);
                                                                             outs[j] = (L[i]-conj(L[j])) * R[i]/(2.0*n)/1i;
      for (cd& x : in) x *= x;
                                                                                                                                            vector<int> rt(2, 1);
      for (int i=0; i<n; i++) out[i] = in[-i & (n - 1)] -</pre>
                                                                                                                                            for (int k=2, s=2; k< n; k*=2, s++){
           conj(in[i]);
                                                                          FFT(out1), FFT(outs);
                                                                                                                                                 rt.resize(n):
                                                                          for (int i=0 ; i<res.size() ; i++){</pre>
      FFT(out);
                                                                                                                                                 int z[] = {1, qp(ROOT, MOD>>s)};
      for (int i=0; i<res.size(); i++) res[i] = imag(out[i])</pre>
                                                                             int av = (int)(real(outl[i])+0.5), cv = (int)(imag(
                                                                                                                                                for (int i=k; i<2*k; i++) rt[i] = rt[i/2]*z[i&1]%
           / (4 * n);
                                                                                  outs[i])+0.5);
                                                                                                                                                      MOD:
                                                                             int bv = (int)(imag(outl[i])+0.5) + (int)(real(outs[i is))
      return res;
                                                                             res[i] = ((av%MOD*cut+bv) % MOD*cut+cv) % MOD;
                                                                                                                                            vector<int> rev(n);
                                                                                                                                            for (int i=0; i<n; i++) rev[i] = (rev[i/2]|(i&1)<<L)/2;</pre>
  2.2 FFT any mod [412000]
                                                                          return res;
                                                                                                                                            for (int i=0 ; i<n ; i++){</pre>
                                                                                                                                                 if (i<rev[i]) swap(a[i], a[rev[i]]);</pre>
2 // n=524288, ~=400ms
                                                                     2.3 FWT [e50788]
  const int MOD = 998244353;
                                                                                                                                            for (int k=1; k < n; k*=2){
  typedef complex<double> cd;
                                                                                                                                                 for (int i=0 ; i<n ; i+=2*k){</pre>
                                                                   11// 已經把 mint 刪掉,需要增加註解
                                                                                                                                                     for (int j=0 ; j<k ; j++){</pre>
6 // bb6d94
                                                                   2 vector<int> xor_convolution(vector<int> a, vector<int> b, int 26
                                                                                                                                                         int z = rt[j+k]*a[i+j+k]%MOD, &ai = a[i+j];
  void FFT(vector<cd> &a) {
                                                                                                                                                         a[i+i+k] = ai-z+(z>ai ? MOD : 0):
      int n = a.size(), L = 31-__builtin_clz(n);
                                                                          if (k==0) return vector<int>{a[0]*b[0]};
                                                                                                                                                         ai += (ai+z)=MOD ? z-MOD : z);
      vector<complex<long double>> R(2, 1);
                                                                          vector < int > aa(1 << (k - 1)), bb(1 << (k - 1));
      vector<cd> rt(2, 1);
                                                                          for (int i=0; i<(1<<(k-1)); i++){
      for (int k=2; k < n; k*=2){
                                                                              aa[i] = a[i] + a[i + (1 << (k - 1))];
          R.resize(n):
                                                                             bb[i] = b[i] + b[i + (1 << (k - 1))];
          rt.resize(n);
          auto x = polar(1.0L, acos(-1.0L) / k);
                                                                          vector<int> X = xor convolution(aa, bb, k - 1);
          for (int i=k ; i<2*k ; i++){</pre>
                                                                          for (int i=0; i < (1 < (k-1)); i++){
                                                                                                                                        vector<int> polyMul(vector<int> &a, vector<int> &b){
                                                                              aa[i] = a[i] - a[i + (1 << (k - 1))];
                                                                                                                                            if (a.empty() || b.empty()) return {};
              rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
                                                                             bb[i] = b[i] - b[i + (1 << (k - 1))];
                                                                                                                                            int s = a.size()+b.size()-1, B = 32-__builtin_clz(s), n =
                                                                                                                                                  1<<B:
                                                                          vector<int> Y = xor_convolution(aa, bb, k - 1);
                                                                                                                                            int inv = qp(n, MOD-2);
      vector<int> rev(n);
                                                                          vector<int> c(1 << k);</pre>
                                                                                                                                            vector<int> L(a), R(b), out(n);
      for (int i=0; i<n; i++) rev[i] = (rev[i/2] | (i&1)<<L)</pre>
                                                                          for (int i=0 ; i<(1<<(k-1)) ; i++){</pre>
                                                                                                                                            L.resize(n), R.resize(n);
                                                                                                                                            NTT(L), NTT(R);
                                                                                                  ] = (X[i] + Y[i]) / 2;
      for (int i=0 ; i<n ; i++) if (i<rev[i]) swap(a[i], a[rev[ 18</pre>
                                                                             c[i + (1 << (k - 1))] = (X[i] - Y[i]) / 2;
                                                                                                                                            for (int i=0 ; i<n ; i++) out[-i&(n-1)] = L[i]*R[i]%MOD*</pre>
                                                                                                                                                 inv%MOD;
```

return c;

NTT(out);

```
44 out.resize(s);
45 return out;
48 3 Data-Structure
3.1 BIT [7ef3a9]
```

```
| vector<int> BIT(MAX_SIZE);
| // const int MAX_N = (1<<20)
| int k_th(int k){ // 回傳 BIT 中第 k 小的元素 (based-1) int res = 0;
| for (int i=MAX_N>1; i>=1; i>>=1) if (BIT[res+i]<k) k -= BIT[res+=i];
| return res+1;
```

# 3.2 Disjoint Set Persistent [447002]

```
| struct Persistent Disjoint Set{
     Persistent_Segment_Tree arr, sz;
     void init(int n){
         arr.init(n);
         vector<int> v1;
         for (int i=0 ; i<n ; i++){</pre>
             v1.push back(i);
         arr.build(v1, 0);
         sz.init(n);
         vector<int> v2;
         for (int i=0 ; i<n ; i++){</pre>
             v2.push_back(1);
         sz.build(v2, 0);
     int find(int a){
         int res = arr.guery version(a, a+1, arr.version.size
              ()-1).val;
         if (res==a) return a:
         return find(res);
     bool unite(int a, int b){
         a = find(a):
         b = find(b);
         if (a!=b){
             int sz1 = sz.query version(a, a+1, arr.version.
                  size()-1).val;
             int sz2 = sz.query_version(b, b+1, arr.version.
                  size()-1).val;
             if (sz1<sz2){
                  arr.update version(a, b, arr.version.size()
                 sz.update version(b, sz1+sz2, arr.version.
                       size()-1);
             }else{
```

17

18

19 20

74

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# 3.3 PBDS GP Hash Table [866cf6]

```
i #include <ext/pb_ds/assoc_container.hpp>
  using namespace __gnu_pbds;
  typedef tree<int, null_type, less<int>, rb_tree_tag,
       tree order statistics node update> order set;
  struct custom hash {
      static uint64 t splitmix64(uint64 t x) {
           // http://xorshift.di.unimi.it/splitmix64.c
           x += 0x9e3779b97f4a7c15;
          x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
           return x ^ (x >> 31):
       size_t operator()(uint64_t x) const {
           static const uint64 t FIXED RANDOM = chrono::
                steady clock::now().time since epoch().count();
           return splitmix64(x + FIXED RANDOM);
16
17
  };
19 gp_hash_table<int, int, custom_hash> ss;
```

# 3.4 PBDS Order Set [231774]

```
| // .find_by_order(k) 回傳第 k 小的值(based-0)
| // .order_of_key(k) 回傳有多少元素比 k 小
| // 不能在 #define int Long Long 後 #include 檔案
| #include <ext/pb_ds/assoc_container.hpp>
| #include <ext/pb_ds/tree_policy.hpp>
| using namespace __gnu_pbds;
| typedef tree<int,null_type,less<int>,rb_tree_tag, tree_order_statistics_node_update> order_set;
```

# 3.5 Segment Tree Add Set [bb1898]

```
SegmentTree(int n){
    arr.resize(n<<2);</pre>
node pull(node A, node B){
    node C:
    C.sum = A.sum+B.sum;
    C.ma = max(A.ma, B.ma):
    return C:
}
void push(int idx, int ll, int rr){
    if (arr[idx].set tag!=0){
        arr[idx].sum = (rr-11)*arr[idx].set_tag;
        arr[idx].ma = arr[idx].set tag;
        if (rr-ll>1){
            arr[idx*2+1].add_tag = 0;
            arr[idx*2+1].set tag = arr[idx].set tag;
            arr[idx*2+2].add_tag = 0;
            arr[idx*2+2].set_tag = arr[idx].set_tag;
        arr[idx].set tag = 0;
    if (arr[idx].add tag!=0){
        arr[idx].sum += (rr-ll)*arr[idx].add_tag;
        arr[idx].ma += arr[idx].add tag;
        if (rr-ll>1){
            arr[idx*2+1].add_tag += arr[idx].add_tag;
            arr[idx*2+2].add tag += arr[idx].add tag;
        arr[idx].add_tag = 0;
}
void build(vector<int> &v, int idx = 0, int ll = 0, int
    if (rr-ll==1){
        arr[idx].sum = v[11];
        arr[idx].ma = v[ll];
        int mid = (11+rr)/2;
        build(v, idx*2+1, ll, mid);
        build(v, idx*2+2, mid, rr);
        arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
void add(int ql, int qr, int val, int idx = 0, int ll =
    0, int rr =n){
    push(idx, ll, rr);
    if (rr<=ql || qr<=ll) return;
    if (ql<=ll && rr<=qr){
        arr[idx].add tag += val;
        push(idx, ll, rr);
        return;
    int mid = (11+rr)/2;
    add(ql, qr, val, idx*2+1, ll, mid);
    add(ql, qr, val, idx*2+2, mid, rr);
    arr[idx]=pull(arr[idx*2+1], arr[idx*2+2]);
void set(int ql, int qr, int val, int idx=0, int ll=0,
    int rr=n){
    push(idx, 11, rr);
```

```
if (rr<=ql || qr<=ll) return;</pre>
           if (ql<=ll && rr<=qr){
                                                                    37
               arr[idx].add tag = 0:
               arr[idx].set_tag = val;
               push(idx, ll, rr);
               return:
           int mid = (11+rr)/2:
           set(ql, qr, val, idx*2+1, ll, mid);
           set(ql, qr, val, idx*2+2, mid, rr);
           arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
       node query(int ql, int qr, int idx = 0, int ll = 0, int
           push(idx, ll, rr);
           if (rr<=ql || qr<=ll) return node();</pre>
           if (ql<=11 && rr<=qr) return arr[idx];</pre>
           int mid = (11+rr)/2;
           return pull(query(ql, qr, idx*2+1, ll, mid), query(ql
                , qr, idx*2+2, mid, rr));
100 } ST;
```

## 3.6 Segment Tree Li Chao Line [45b8ba]

```
1 // 全部都是 0-based
2 // 宣告: LC Segment Tree st(n);
 4 // update({a, b}):插入一條 y=ax+b 的全域直線
5 // query(x): 查詢所有直線在位置 x 的最小值
 6 const int MAX V = 1e6+10; // 值域最大值
  struct LC Segment Tree{
     struct Node{ // v = ax+b
          int a = 0;
          int b = INF:
         int y(int x){
             return a*x+b:
      };
      vector<Node> arr;
      LC Segment Tree(int n = 0){
         arr.resize(4*n):
21
      void update(Node val, int idx = 0, int ll = 0, int rr =
          MAX V){
         if (rr-ll==0) return;
         if (rr-ll==1){
             if (val.y(l1) < arr[idx].y(l1)){</pre>
                 arr[idx] = val;
             return:
         }
         int mid = (11+rr)/2;
         if (arr[idx].a > val.a) swap(arr[idx], val); // 原本
              的線斜率要比較小
          if (arr[idx].y(mid) < val.y(mid)){ // 交點在左邊
             update(val, idx*2+1, ll, mid);
```

```
}else{ // 交點在右邊
        swap(arr[idx], val); // 在左子樹中,新線比舊線還
        update(val, idx*2+2, mid, rr);
    return:
}
int query(int x, int idx = 0, int 11 = 0, int rr = MAX_V) 41
    if (rr-ll==0) return INF:
    if (rr-ll==1){
        return arr[idx].y(ll);
    int mid = (11+rr)/2;
    if (x<mid){</pre>
        return min(arr[idx].y(x), query(x, idx*2+1, ll,
    }else{
        return min(arr[idx].y(x), query(x, idx*2+2, mid,
```

# 3.7 Segment Tree Li Chao Segment [2cb0a4]

1 / / 全部都是 0-based

3 // 函式:

2 // 宣告: LC\_Segment\_Tree st(n);

```
4 // update segment({a, b}, ql, qr):在 [ql, qr) 插入一條 y=ax+
5 | // query(x): 查詢所有直線在位置 x 的最小值
 const int MAX V = 1e6+10; // 值域最大值
 struct LC Segment Tree{
     struct Node{ // y = ax+b
         int a = 0;
         int b = INF;
         int y(int x){
             return a*x+b:
     vector<Node> arr;
     LC Segment Tree(int n = 0){
         arr.resize(4*n);
     }
     void update(Node val, int idx = 0, int ll = 0, int rr =
          MAX V){
         if (rr-11==0) return;
         if (rr-ll<=1){</pre>
             if (val.y(ll)<arr[idx].y(ll)){</pre>
                 arr[idx] = val:
             return;
         int mid = (11+rr)/2:
         if (arr[idx].a > val.a) swap(arr[idx], val); // 原本
              的線斜率要比較小
```

```
}else{ // 交點在右邊
              swap(arr[idx], val); // 在左子樹中,新線比舊線還
              update(val, idx*2+2, mid, rr);
          return;
      // 在 [al, ar) 加上一條 val 的線段
      void update segment(Node val, int ql, int qr, int idx =
          0, int 11 = 0, int rr = MAX_V){
          if (rr-11==0) return;
          if (rr<=ql || qr<=ll) return;</pre>
          if (q1<=11 && rr<=qr){
              update(val, idx, ll, rr);
              return:
          int mid = (11+rr)/2;
          update segment(val, ql, qr, idx*2+1, ll, mid);
          update segment(val, ql, qr, idx*2+2, mid, rr);
      int query(int x, int idx = 0, int ll = 0, int rr = MAX V)
          if (rr-ll==0) return INF;
          if (rr-ll==1){
              return arr[idx].y(11);
          int mid = (11+rr)/2;
          if (x<mid){</pre>
              return min(arr[idx].y(x), query(x, idx*2+1, ll,
                  mid)):
          }else{
              return min(arr[idx].y(x), query(x, idx*2+2, mid,
                  rr));
71 };
```

if (arr[idx].y(mid) < val.y(mid)){ // 交點在左邊

update(val, idx\*2+1, ll, mid);

37

56

57

70

#### 3.8 Segment Tree Persistent [3b5aa9]

```
1 // 全部都是 0-based
2 // Persistent Segment Tree st(n+g);
3 // st.build(v. 0):
4 // 函式:
5 // update_version(pos, val, ver): 對版本 ver 的 pos 位置改成
6 // query_version(ql, qr, ver): 對版本 ver 查詢 [ql, qr) 的區
7 // clone version(ver):複製版本 ver 到最新的版本
 struct Persistent_Segment_Tree{
     int node cnt = 0;
     struct Node{
         int lc = -1;
         int rc = -1;
         int val = 0;
     };
     vector<Node> arr:
     vector<int> version;
```

tag[x] += V;

```
Persistent Segment Tree(int sz){
                                                                                                                             20
                                                                                                                                   total[x] += V*sz[x];
    arr.resize(32*sz):
                                                                                                                            21 }
    version.push_back(node_cnt++);
                                                                      pull(ret, query(ql, qr, now.lc, ll, mid), query(ql,
                                                                          gr, now.rc, mid, rr));
                                                                                                                               void push(int x){
}
                                                                      return ret:
                                                                                                                                   if (tag[x]){
                                                                 }
                                                                                                                                       if (lc[x]) apply(lc[x], tag[x]);
void pull(Node &c, Node a, Node b){
                                                                                                                                       if (rc[x]) apply(rc[x], tag[x]);
    c.val = a.val+b.val;
                                                                  Node query version(int ql, int qr, int ver){
    return;
                                                                      return query(ql, qr, version[ver]);
                                                                                                                             28
                                                                                                                                   tag[x] = 0;
}
                                                                                                                             29
                                                                                                                               int pull(int x){
void build(vector<int> &v, int idx, int ll = 0, int rr =
                                                                  void clone_version(int ver){
                                                                                                                                   if (x){
                                                                      version.push back(node cnt);
                                                                                                                                       fa[x] = 0;
    auto &now = arr[idx];
                                                                      arr[node_cnt] = arr[version[ver]];
                                                                                                                                       sz[x] = 1+sz[lc[x]]+sz[rc[x]];
                                                                      node cnt++;
                                                                                                                                       total[x] = val[x]+total[lc[x]]+total[rc[x]];
    if (rr-ll==1){
                                                                                                                                       if (lc[x]) fa[lc[x]] = x;
                                                                                                                                       if (rc[x]) fa[rc[x]] = x;
        now.val = v[11];
                                                           96 };
        return:
                                                                                                                             38
                                                                                                                                   return x;
                                                             3.9 Sparse Table [31f22a]
    int mid = (11+rr)/2:
                                                            1 struct SparseTable{
    now.lc = node cnt++;
                                                                                                                               int merge(int a, int b){
                                                                  vector<vector<int>> st;
    now.rc = node cnt++;
                                                                                                                                   if (!a or !b) return a|b;
                                                                  void build(vector<int> v){
    build(v, now.lc, ll, mid);
                                                                                                                                   push(a), push(b);
                                                                     int h = lg(v.size());
    build(v, now.rc, mid, rr);
    pull(now, arr[now.lc], arr[now.rc]);
                                                                      st.resize(h+1);
                                                                                                                                   if (pri[a]>pri[b]){
    return:
                                                                      st[0] = v;
                                                                                                                                       rc[a] = merge(rc[a], b);
                                                                                                                                       return pull(a);
                                                                      for (int i=1 ; i<=h ; i++){</pre>
                                                                                                                                   }else{
                                                                          int gap = (1 << (i-1));
void update(int pos, int val, int idx, int ll = 0, int rr
                                                                                                                                       lc[b] = merge(a, lc[b]);
                                                                          for (int j=0 ; j+gap<st[i-1].size() ; j++){</pre>
                                                                                                                                       return pull(b);
                                                                              st[i].push_back(min(st[i-1][j], st[i-1][j+gap 51
    auto &now = arr[idx];
                                                                                  1));
    if (rr-ll==1){
        now.val = val;
                                                                     }
                                                                                                                               // [1, k] [k+1, n]
                                                                                                                               void split(int x, int k, int &a, int &b) {
        return;
                                                                                                                                   if (!x) return a = b = 0, void();
                                                                  // 回傳 [ll, rr) 的最小值
    int mid = (11+rr)/2;
                                                                 int query(int 11, int rr){
                                                                                                                                   if (sz[lc[x]] >= k) {
    if (pos<mid){</pre>
                                                                      int h = lg(rr-ll);
                                                                                                                                       split(lc[x], k, a, lc[x]);
        arr[node_cnt] = arr[now.lc];
                                                                      return min(st[h][ll], st[h][rr-(1<<h)]);</pre>
        now.lc = node cnt;
                                                                                                                                       pull(a); pull(b);
        node cnt++;
                                                                                                                                   }else{
        update(pos, val, now.lc, ll, mid);
                                                                                                                                       split(rc[x], k - sz[lc[x]] - 1, rc[x], b);
    }else{
                                                             3.10 Treap2 [3b0cca]
        arr[node cnt] = arr[now.rc];
                                                                                                                                       pull(a); pull(b);
        now.rc = node cnt;
                                                                                                                             66
                                                            ı|// 1-based · 請注意 MAX N 是否足夠大
        node cnt++:
                                                                                                                             67
                                                            2 int root = 0;
        update(pos, val, now.rc, mid, rr);
                                                             int lc[MAX_N], rc[MAX_N];
                                                                                                                               // functions
    pull(now, arr[now.lc], arr[now.rc]);
                                                            4 int pri[MAX_N], val[MAX_N];
                                                                                                                             70 // 回傳 x 在 Treap 中的位置
                                                            int sz[MAX_N], tag[MAX_N], fa[MAX_N], total[MAX_N];
    return;
                                                                                                                             71 int get pos(int x){
                                                             // tag 為不包含自己(僅要給子樹)的資訊
                                                                                                                                   vector<int> sta;
                                                             int nodeCnt = 0;
                                                                                                                                   while (fa[x]){
void update version(int pos, int val, int ver){
                                                            8 int& new node(int v){
                                                                                                                                       sta.push back(x);
    update(pos, val, version[ver]);
                                                                 nodeCnt++;
                                                                                                                             75
                                                                                                                                       x = fa[x];
                                                                  val[nodeCnt] = v;
                                                                  total[nodeCnt] = v;
                                                                                                                                   while (sta.size()){
Node query(int ql, int qr, int idx, int ll = 0, int rr =
                                                                  sz[nodeCnt] = 1;
                                                                                                                                       push(x);
                                                                  pri[nodeCnt] = rand();
                                                                                                                                       x = sta.back();
    auto &now = arr[idx];
                                                                  return nodeCnt:
                                                                                                                                       sta.pop back();
    if (ql<=ll && rr<=qr) return now;</pre>
                                                                                                                                   push(x);
    if (rr<=ql || qr<=ll) return Node();</pre>
                                                              void apply(int x, int V){
                                                                 val[x] += V;
```

int mid = (11+rr)/2;

```
int res = sz[x] - sz[rc[x]];
                                                                     }
                                                                                                                                     return 0;
                                                                                                                               53 }
      while (fa[x]){
          if (rc[fa[x]]==x){
                                                               40 } tr;
              res += sz[fa[x]]-sz[x];
                                                                                                                                 4.2 Knaspack On Tree [df69b1]
                                                                  4 Dynamic-Programming
          x = fa[x];
                                                                                                                               1 / / 需要重構、需要增加註解
                                                                  4.1 Digit DP [133f00]
                                                                                                                                 #include <bits/stdc++.h>
      return res;
                                                                                                                                 #define F first
                                                                | #include <bits/stdc++.h>
                                                                                                                                 #define S second
                                                                  using namespace std;
                                                                                                                                 #define all(x) begin(x), end(x)
94 // 1-based <前 [1, L-1] 個元素, [L, r] 個元素, [r+1, n] 個元
                                                                                                                                 using namespace std:
                                                                  long long l, r;
  array<int, 3> cut(int x, int 1, int r){
                                                                s | long long dp[20][10][2][2]; // dp[pos][pre][limit] = 後 pos
                                                                                                                                 #define chmax(a, b) (a) = (a) < (b) ? (b) : (a)
      array<int, 3> ret;
                                                                                                                                 #define chmin(a, b) (a) = (a) < (b) ? (a) : (b)
                                                                       位 pos 前一位是 pre (是/否)有上界 (是/否)有前綴零
      split(x, r, ret[1], ret[2]);
                                                                      的答案數量
      split(ret[1], 1-1, ret[0], ret[1]);
                                                                                                                                 #define ll long long
      return ret;
                                                                  long long memorize_search(string &s, int pos, int pre, bool
100
                                                                                                                                 #define FOR(i, a, b) for (int i = a; i <= b; i++)
                                                                      limit, bool lead){
101
102 void print(int x){
                                                                                                                                 int N, W, cur;
                                                                      // 已經被找過了,直接回傳值
      push(x);
                                                                                                                                 vector<int> w, v, sz;
                                                                      if (dp[pos][pre][limit][lead]!=-1) return dp[pos][pre][
      if (lc[x]) print(lc[x]);
                                                                                                                                 vector<vector<int>> adi. dp:
                                                                           limit][lead];
      cerr << val[x] << " "</pre>
105
      if (rc[x]) print(rc[x]);
                                                                                                                                  void dfs(int x) {
                                                                      // 已經搜尋完畢,紀錄答案並回傳
                                                                                                                                     sz[x] = 1;
                                                                      if (pos==(int)s.size()){
                                                                                                                                     for (int i : adj[x]) dfs(i), sz[x] += sz[i];
                                                                          return dp[pos][pre][limit][lead] = 1;
                                                                                                                                     cur++:
  3.11 Trie [b6475c]
                                                                                                                                     // choose x
                                                                                                                                     for (int i=w[x]; i<=W; i++){</pre>
 | struct Trie{
                                                                      // 枚舉目前的位數數字是多少
                                                                                                                                         dp[cur][i] = dp[cur - 1][i - w[x]] + v[x];
      struct Data{
                                                                      long long ans = 0:
          int nxt[2]={0, 0};
                                                                      for (int now=0 ; now<=(limit ? s[pos]-'0' : 9) ; now++){</pre>
                                                                                                                                     // not choose x
                                                                          if (now==pre){
                                                                                                                                     for (int i=0 ; i<=W ; i++){</pre>
                                                                                                                                         chmax(dp[cur][i], dp[cur - sz[x]][i]);
      int sz=0:
                                                                              // 1~9 絕對不能連續出現
                                                                                                                               30
      vector<Data> arr;
                                                                              if (pre!=0) continue;
                                                                                                                               31
      void init(int n){
                                                                              // 如果已經不在前綴零的範圍內·Ø 不能連續出現
                                                                                                                               33
                                                                                                                                  signed main() {
          arr.resize(n);
                                                                                                                                     cin >> N >> W;
                                                                              if (lead==false) continue:
                                                                                                                                     adj.resize(N + 1);
                                                                                                                                     w.assign(N + 1, 0);
      void insert(int n){
                                                                                                                                     v.assign(N + 1, 0);
                                                                          ans += memorize search(s, pos+1, now, limit&(now==(s[
          int now=0;
                                                                                                                                     sz.assign(N + 1, 0);
                                                                              pos[-'0'], lead&(now==0);
          for (int i=N ; i>=0 ; i--){
                                                                                                                                     dp.assign(N + 2, vector<int>(W + 1, 0));
              int v=(n>>i)&1;
                                                                                                                                     for (int i=1 ; i<=N ; i++){</pre>
              if (!arr[now].nxt[v]){
                                                                                                                                         int p; cin >> p;
                                                                      // 已經搜尋完畢,紀錄答案並回傳
                  arr[now].nxt[v]=++sz;
                                                                                                                                         adj[p].push_back(i);
                                                                      return dp[pos][pre][limit][lead] = ans;
              now=arr[now].nxt[v];
                                                                                                                                     for (int i=1; i<=N; i++) cin >> w[i];
      }
                                                                  // 回傳 [0, n] 有多少數字符合條件
                                                                                                                                     for (int i=1 ; i<=N ; i++) cin >> v[i];
                                                                  long long find answer(long long n){
                                                                      memset(dp, -1, sizeof(dp));
      int query(int n){
                                                                                                                                     cout << dp[N + 1][W] << ' \ '';
                                                                      string tmp = to string(n);
          int now=0, ret=0;
          for (int i=N ; i>=0 ; i--){
              int v=(n>>i)&1;
                                                                      return memorize_search(tmp, 0, 0, true, true);
                                                                                                                                 4.3 SOS DP [8dfa8b]
              if (arr[now].nxt[1-v]){
                  ret+=(1<<i);
                  now=arr[now].nxt[1-v];
                                                                  int main(){
                                                                                                                               1 // 總時間複雜度為 O(n 2^n)
                                                                                                                               2 // 計算 dp[i] = i 所有 bit mask 子集的和
              }else if (arr[now].nxt[v]){
                                                                      // input
                  now=arr[now].nxt[v];
                                                                                                                                 for (int i=0 ; i<n ; i++){</pre>
              }else{
                                                                      cin >> 1 >> r;
                                                                                                                                     for (int mask=0; mask<(1<<n); mask++){</pre>
                                                                                                                                         if ((mask>>i)&1){
                  return ret;
                                                                      // output - 計算 [L. r] 有多少數字任意兩個位數都不相同
                                                                                                                                             dp[mask] += dp[mask^(1<<i)];</pre>
                                                                      cout << find answer(r)-find answer(l-1) << "\n";</pre>
          return ret;
```

```
9 }
 4.4 Integer Partition
 dp[i][x] = 要將整數 x 拆成 i 堆的「組合數」
 dp[i+1][x+1]+=dp[i][x] ( 創造新的一堆 )
 dp[i][x+i]+=dp[i][x] (把每一堆都增加 1)
                                                                     friend ld Arg(point b) {
 5 Geometry
 5.1 Geometry Struct [d9966f]
using ld = double;
                                                                 template<typename T>
                                                                 struct line {
3 // 判斷數值正負: {1:正數,0:零,-1:負數}
                                                                     point<T> p1, p2;
4 int sign(long long x) {return (x \ge 0) ? ((bool)x) : -1; }
                                                                     // ax + by + c = 0
5 int sign(ld x) {return (abs(x) < 1e-9) ? 0 : (x>0 ? 1 : -1);}
 template<tvpename T>
8 struct point {
     T x, y;
                                                                         build();
     point() {}
                                                                     void build() {
     point(const T &x, const T &y) : x(x), y(y) {}
                                                                         a = p1.y - p2.y;
     explicit operator point<ld>() {return point<ld>(x, y); }
                                                                         b = p2.x - p1.x;
     // A [6357c4], Line 9 ~ 13
     point operator+(point b) {return {x+b.x, y+b.y}; }
                                                                         c = (-a*p1.x)-b*p1.y;
     point operator-(point b) {return {x-b.x, y-b.y}; }
     point operator*(T b) {return {x*b, y*b}; }
     point operator/(T b) {return {x/b, y/b}; }
                                                                     int ori(point<T> &p) {
     bool operator==(point b) {return x==b.x && y==b.y; }
                                                               83
     T operator*(point b) {return x * b.x + y * b.y; }
                                                                     // 判斷直線斜率是否相同
     T operator^(point b) {return x * b.y - y * b.x; }
                                                                     bool parallel(line &1) {
     // B [c415da], Line 14 ~ 22
     // 逆時針極角排序
     bool side() const{return (y == 0) ? (x > 0) : (y < 0); }
                                                                     // 兩直線交點
     bool operator<(const point &b) const {</pre>
         return side() == b.side() ?
                                                                         using P = point<ld>;
             (x*b.v > b.x*v) : side() < b.side():
     friend ostream& operator<<(ostream& os, point p) {</pre>
         return os << "(" << p.x << ", " << p.y << ")";
                                                                 };
     // 判斷 ab 到 ac 的方向: {1:逆時鐘,0:重疊,-1:順時鐘}
                                                                 template<typename T>
     friend int ori(point a, point b, point c) {
                                                                 struct polygon {
         return sign((b-a)^(c-a));
                                                                     vector<point<T>> v;
                                                                     polygon() {}
     friend bool btw(point a, point b, point c) {
         return ori(a, b, c) == 0 && sign((a-c)*(b-c)) <= 0;
     // 判斷線段 ab. cd 是否相交
     friend bool banana(point a, point b, point c, point d) {
         if (btw(a, b, c) || btw(a, b, d)
             || btw(c, d, a) || btw(c, d, b)) return true;
                                                                         simple = (bool)simple;
         int u = ori(a, b, c) * ori(a, b, d);
         int v = ori(c, d, a) * ori(c, d, b);
         return u < 0 && v < 0;
                                                              109
                                                                         vector<point<T>> hull;
     } // C [09fd7c], only this function
     // 判斷 "射線 ab" 與 "線段 cd" 是否相交
                                                              112
     friend bool rayHitSeg(point a, point b, point c, point d) {
                                                                             for (auto &i:v) {
         if (a == b) return btw(c, d, a); // Special case
                                                              113
         if (((a - b) ^ (c - d)) == 0) {
             return btw(a, c, b) || btw(a, d, b) || banana(a,
                  b, c, d);
                                                              116
```

```
point u = b - a, v = d - c, s = c - a;
                                                                           hull.push back(i);
    return sign(s ^{\circ} v) * sign(u ^{\circ} v) >= 0 && sign(s ^{\circ} u) 119
         * sign(u ^ v) >= 0 && abs(s ^ u) <= abs(u ^ v); 120
                                                                       hull.pop back():
} // D [db541a], only this function
                                                                       reverse(v.begin(), v.end());
// 旋轉 Arg(b) 的角度(小心溢位)
                                                                   swap(hull, v);
point rotate(point b){return {x*b.x-y*b.y, x*b.y+y*b.x};} 123
                                                               } // F [2bb3ef], only this function
// 回傳極座標角度·值域:[-\pi, +\pi]
                                                         125 // 可以在有 n 個點的簡單多邊形內,用 O(n) 判斷一個點:
    return (b.x != 0 || b.y != 0) ? atan2(b.y, b.x) : 0; 126 |// {1 : 在多邊形內, 0 : 在多邊形上, -1 : 在多邊形外}
                                                               int in polygon(point<T> a){
friend T abs2(point b) {return b * b; }
                                                                   const T MAX POS = 1e9 + 5; // [記得修改] 座標的最大值
                                                                   point<T> pre = v.back(), b(MAX_POS, a.y + 1);
                                                                   int cnt = 0;
                                                         132
                                                                   for (auto &i:v) {
                                                                       if (btw(pre, i, a)) return 0;
                                                                       if (banana(a, b, pre, i)) cnt++;
T a, b, c; //|a|, |b| \le 2C, |c| \le 8C^2
                                                         135
                                                                       pre = i:
line(const point\langle T \rangle &x, const point\langle T \rangle &y) : p1(x), p2(y){ _{137}
                                                                   return cnt%2 ? 1 : -1:
                                                               } // G [f11340], only this function
                                                         140 / / / 警告:以下所有凸包專用的函式都只接受逆時針排序且任三點不
                                                                共線的凸包 ///
                                                         141 // 可以在有 n 個點的凸包內,用 O(Log n)判斷一個點:
                                                         142 // {1: 在凸包內, 0: 在凸包邊上, -1: 在凸包外}
} // E [683239], Line 68 ~ 79
                                                               int in_convex(point<T> p) {
// 判斷點和有向直線的關係: {1:左邊,0:在線上,-1:右邊}
                                                                   int n = v.size();
                                                                   int a = ori(v[0], v[1], p), b = ori(v[0], v[n-1], p);
                                                         145
    return sign((p2-p1) ^ (p-p1));
                                                                   if (a < 0 || b > 0) return -1;
                                                                   if (btw(v[0], v[1], p)) return 0;
                                                         147
                                                         148
                                                                   if (btw(v[0], v[n - 1], p)) return 0;
                                                                   int l = 1, r = n - 1, mid;
    return ((p1-p2) ^ (l.p1-l.p2)) == 0;
                                                                   while (1 + 1 < r) {
                                                         150
                                                         151
                                                                       mid = (1 + r) >> 1;
                                                                       if (ori(v[0], v[mid], p) >= 0) 1 = mid;
                                                         152
point<ld> line_intersection(line &l) {
                                                         153
                                                                       else r = mid:
                                                         154
    point < T > u = p2-p1, v = l.p2-l.p1, s = l.p1-p1;
                                                                   int k = ori(v[1], v[r], p);
                                                         155
    return P(p1) + P(u) * ((ld(s^v)) / (u^v));
                                                                   if (k <= 0) return k;</pre>
                                                                   return 1;
                                                               } // H [e64f1e], only this function
                                                            // 凸包專用的環狀二分搜,回傳 0-based index
                                                               int cycle search(auto &f) {
                                                                   int n = v.size(), l = 0, r = n;
                                                         161
                                                         162
                                                                   if (n == 1) return 0;
                                                         163
                                                                   bool rv = f(1, 0);
polygon(const vector<point<T>> &u) : v(u) {}
                                                                   while (r - 1 > 1) {
// simple 為 true 的時候會回傳任意三點不共線的凸包
                                                         165
                                                                       int m = (1 + r) / 2;
void make_convex_hull(int simple) {
                                                                       if (f(0, m) ? rv: f(m, (m + 1) % n)) r = m;
                                                         166
    auto \overline{cmp} = [\overline{\&}](point<T> \&p, point<T> \&q) {
                                                         167
                                                                       else 1 = m;
        return (p.x == q.x)? (p.y < q.y): (p.x < q.x);
                                                                   return f(1, r % n) ? 1 : r % n;
                                                               } // I [fe2f51], only this function
    sort(v.begin(), v.end(), cmp);
                                                         171 // 可以在有 n 個點的凸包內 · 用 O(Log n) 判斷一條直線:
    v.resize(unique(v.begin(), v.end()) - v.begin());
                                                         172 // {1: 穿過凸包, 0: 剛好切過凸包, -1: 沒碰到凸包}
    if (v.size() <= 1) return;</pre>
                                                         173
                                                               int line cut convex(line<T> L) {
                                                                   L.build():
                                                         174
    for (int t = 0; t < 2; ++t){
                                                         175
                                                                   point<T> p(L.a, L.b);
        int sz = hull.size();
                                                                   auto gt = [&](int neg) {
                                                                       auto f = [&](int x, int y) {
            while (hull.size() >= sz+2 && ori(hull[hull.
                                                         178
                                                                           return sign((v[x] - v[y]) * p) == neg;
                  size()-2], hull.back(), i) < simple) {</pre>
                                                         179
                hull.pop back();
                                                                       return -(v[cycle search(f)] * p);
```

```
182
          T x = gt(1), y = gt(-1);
                                                              242
183
          if (L.c < x || y < L.c) return -1;
          return not (L.c == x || L.c == y);
      } // J [b6a4c8], only this function
                                                              245
      可以在有 n 個點的凸包內 · 用 O(Log n) 判斷一個線段:
                                                              247
      {1:存在一個凸包上的邊可以把這個線段切成兩半。
       0: 有碰到凸包但沒有任何凸包上的邊可以把它切成兩半,
189 // -1: 沒碰到凸包}
190 | /// 除非線段兩端點都不在凸包邊上·否則此函數回傳 Ø 的時候不
       定表示線段沒有通過凸包內部 ///
      int segment_across_convex(line<T> L) {
          L.build();
193
          point<T> p(L.a, L.b);
          auto gt = [&](int neg) {
194
195
              auto f = [\&](int x, int y) {
                  return sign((v[x] - v[y]) * p) == neg;
196
              };
                                                              256
198
              return cycle_search(f);
                                                              257
199
                                                              258
200
          int i = gt(1), j = gt(-1), n = v.size();
          T x = -(v[i] * p), y = -(v[j] * p);
201
                                                              260
          if (L.c < x || y < L.c) return -1;
202
                                                              261
          if (L.c == x || L.c == y) return 0;
203
                                                              262
204
          if (i > j) swap(i, j);
205
206
          auto g = [\&](int x, int lim) {
207
              int now = 0, nxt;
              for (int i = 1 << __lg(lim); i > 0; i /= 2) {
                  if (now + i > lim) continue;
209
                                                              267
                  nxt = (x + i) % n;
210
                  if (L.ori(v[x]) * L.ori(v[nxt]) >= 0) {
211
                                                              269
212
                     x = nxt;
                                                              270
213
                     now += i;
                                                              271
214
              } // ↓ BE CAREFUL
215
              return -(ori(v[x], v[(x + 1) % n], L.p1) * ori(v[ 274
                  x], v[(x + 1) % n], L.p2));
          return max(g(i, j - i), g(j, n - (j - i)));
                                                              277
      } // K [b4f073], only this function
     可以在有 n 個點的凸包內·用 O(Log n) 判斷一個線段:
     {1:線段上存在某一點位於凸包內部(邊上不算),
      0: 線段上存在某一點碰到凸包的邊但線段上任一點均不在凸包 281
     -1: 線段完全在凸包外面}
                                                              283
224
      int segment_pass_convex_interior(line<T> L) {
                                                              284
225
          if (in convex(L.p1) == 1 | in convex(L.p2) == 1)
          L.build();
227
          point<T> p(L.a, L.b);
228
          auto gt = [&](int neg) {
              auto f = [\&](int x, int y) {
                                                              288
                  return sign((v[x] - v[y]) * p) == neg;
230
                                                              289
231
                                                              290
232
              return cycle_search(f);
                                                              291
233
                                                              292
          int i = gt(1), j = gt(-1), n = v.size();
                                                              293
235
          T x = -(v[i] * p), y = -(v[j] * p);
                                                              294
          if (L.c < x || y < L.c) return -1;
                                                              295
237
          if (L.c == x || L.c == y) return 0;
                                                              296
238
                                                              297
239
          if (i > j) swap(i, j);
                                                              298
          auto g = [&](int x, int lim) {
```

```
int now = 0, nxt;
        for (int i = 1 << __lg(lim); i > 0; i /= 2) {
                                                            301
             if (now + i > lim) continue;
                                                            302
            nxt = (x + i) % n;
                                                            303
            if (L.ori(v[x]) * L.ori(v[nxt]) > 0) {
                 x = nxt;
                 now += i;
        } // ↓ BE CAREFUL
                                                            308 };
        return -(ori(v[x], v[(x + 1) % n], L.p1) * ori(v[ 309
             x], v(x + 1) % n, L.p2);
    int ret = \max(g(i, j - i), g(j, n - (j - i)));
                                                            312
    return (ret == 0) ? (in_convex(L.p1) == 0 &&
                                                            313
         in_convex(L.p2) == 0) : ret;
} // L [5f45ca], only this function
回傳點過凸包的兩條切線的切點的 0-based index (不保證兩條
                                                            315
 切線的順逆時針關係)
                                                            316
pair<int,int> convex_tangent_point(point<T> p) {
                                                            317
    int n = v.size(), z = -1, edg = -1;
                                                            318
    auto gt = [&](int neg) {
                                                            319
        auto check = [&](int x) {
            if (v[x] == p) z = x;
                                                            320
             if (btw(v[x], v[(x + 1) % n], p)) edg = x;
                                                            321
            if (btw(v[(x + n - 1) % n], v[x], p)) edg = (
                 x + n - 1) % n;
        auto f = [\&](int x, int y) {
            check(x); check(y);
                                                            325
            return ori(p, v[x], v[y]) == neg;
                                                            326
        };
                                                            327
        return cycle_search(f);
                                                            328
                                                            329
    int x = gt(1), y = gt(-1);
                                                            330
                                                            331
        return \{(z + n - 1) \% n, (z + 1) \% n\};
                                                            332
                                                            333
    else if (edg != -1) {
                                                            334
        return {edg, (edg + 1) % n};
                                                            335
                                                            336
    else {
        return {x, y};
                                                            337
                                                            338
} // M [a6f66b], only this function
                                                            339
friend int halfplane intersection(vector<line<T>> &s,
     polygon<T> &P) {
                                                            341
    auto angle cmp = [&](line<T> &A, line<T> &B) {
                                                            342 };
        point < T > a = A.p2-A.p1, b = B.p2-B.p1;
        return (a < b);</pre>
    sort(s.begin(), s.end(), angle_cmp); // 線段左側為該
         線段半平面
    int L, R, n = s.size();
    vector<point<T>> px(n);
    vector<line<T>> q(n);
    q[L = R = 0] = s[0];
    for(int i = 1; i < n; ++i) {</pre>
        while (L < R \&\& s[i].ori(px[R-1]) <= 0) --R;
        while(L < R && s[i].ori(px[L]) <= 0) ++L;</pre>
        q[++R] = s[i];
        if(q[R].parallel(q[R-1])) {
            if(q[R].ori(s[i].p1) > 0) q[R] = s[i];
        if(L < R) px[R-1] = q[R-1].line_intersection(q[R]);
```

```
while(L \langle R \& q[L].ori(px[R-1]) \langle = 0 \rangle --R;
           P.v.clear();
           if(R - L <= 1) return 0;
           px[R] = q[R].line_intersection(q[L]);
           for(int i = L; i <= R; ++i) P.v.push back(px[i]);</pre>
           return R - L + 1;
       } // N [102d48], only this function
310 struct Cir {
       point<ld> o; ld r;
       friend ostream& operator<<(ostream& os, Cir c) {</pre>
           return os \langle\langle "(x" \langle\langle "+-"[c.o.x \rangle= 0] \langle\langle abs(c.o.x) \rangle
                 ((")^2 + (y") < "+-"[c.o.y] >= 0] < abs(c.o.y)
                 <<")^2 = "<< c.r * c.r;
       bool covers(Cir b) {
           return sqrt((ld)abs2(o - b.o)) + b.r <= r;</pre>
       vector<point<ld>>> Cir_intersect(Cir c) {
           1d d2 = abs2(o - c.o), d = sqrt(d2);
           if (d < max(r, c.r) - min(r, c.r) || d > r + c.r)
                 return {};
           auto sqdf = [\&](1d x, 1d y) \{ return x*x - y*y; \};
           point < ld > u = (o + c.o) / 2 + (o - c.o) * (sqdf(c.r, ))
                 r) / (2 * d2));
           1d A = sqrt(sqdf(r + d, c.r) * sqdf(c.r, d - r));
           point<ld> v = (c.o - o).rotate({0,1}) * A / (2 * d2);
           if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
           return \{u - v, u + v\};
       } // 0 [330a1c], only this function
       auto point_tangent(point<ld> p) {
           vector<point<ld>> res;
           1d d_sq = abs2(p - o);
           if (sign(d sq - r * r) <= 0) {
                res.pb(p + (p - o).rotate(\{0, 1\}));
           } else if (d_sq > r * r) {
                1d s = d_sq - r * r;
                point < ld > v = p + (o - p) * s / d_sq;
                point < ld > u = (o - p).rotate({0, 1}) * sqrt(s) *
                     r / d_sq;
                res.pb(v + u);
                res.pb(v - u);
           return res;
       } // P [0067e6], only this function
  5.2 Geometry Struct 3D [4a50c9]
```

```
using ld = long double;
 // 判斷數值正負: {1:正數,0:零,-1:負數}
 int sign(long long x) {return (x \ge 0) ? ((bool)x) : -1; }
 int sign(ld x) {return (abs(x) < 1e-9) ? 0 : (x>0 ? 1 : -1);}
 template<typename T>
 struct pt3 {
     T x, y, z;
     pt3(){}
     pt3(const T &x, const T &y, const T &z):x(x),y(y),z(z){}
     explicit operator pt3<ld>() {return pt3<ld>(x, y, z); }
     pt3 operator+(pt3 b) {return {x+b.x, y+b.y, z+b.z}; }
```

```
pt3 operator-(pt3 b) {return {x-b.x, y-b.y, z-b.z}; }
      pt3 operator*(T b) {return {x * b, y * b, z * b}; }
      pt3 operator/(T b) {return {x / b, y / b, z / b}; }
      bool operator==(pt3 b){return x==b.x&&y==b.y&&z==b.z;}
      T operator*(pt3 b) {return x*b.x+y*b.y+z*b.z; }
      pt3 operator^(pt3 b) {
          return pt3(y * b.z - z * b.y,
                    z * b.x - x * b.z,
                    x * b.v - v * b.x);
      friend T abs2(pt3 b) {return b * b; }
      friend T len (pt3 b) {return sqrt(abs2(b)); }
      friend ostream& operator<<((ostream& os, pt3 p) {</pre>
          return os << "(" << p.x << ", " << p.y << ", " << p.z << ")";
32 };
34 template<typename T>
35 struct face {
      int a, b, c; // 三角形在 vector 裡面的 index
      pt3<T> q; // 面積向量(朝外)
40 /// 警告; v 在過程中可能被修改,回傳的 face 以修改後的為準
41 // O(n²),最多只有 2n-4 個面
42 // 當凸包退化時會回傳空的凸包,否則回傳凸包上的每個面
43 template<typename T>
44 vector<face<T>> hull3(vector<pt3<T>> &v) {
      int n = v.size();
      if (n < 3) return {};</pre>
      // don't use "==" when you use ld
      sort(all(v), [&](pt3<T> &p, pt3<T> &q) {
          return sign(p.x - q.x)? (p.x < q.x):
           (sign(p.y - q.y) ? p.y < q.y : p.z < q.z);
      v.resize(unique(v.begin(), v.end()) - v.begin());
      for (int i = 2; i <= n; ++i) {</pre>
          if (i == n) return {};
          if (sign(len(((v[1] - v[0]) ^ (v[i] - v[0]))))) {
              swap(v[2], v[i]);
              break;
      pt3<T> tmp q = (v[1] - v[0]) ^ (v[2] - v[0]);
      for (int i = 3; i <= n; ++i) {</pre>
          if (i == n) return {};
          if (sign((v[i] - v[0]) * tmp_q)) {
              swap(v[3], v[i]);
              break;
      }
      vector<face<T>> f:
      vector<vector<int>> dead(n, vector<int>(n, true));
      auto add face = [&](int a, int b, int c) {
          f.emplace_back(a, b, c, (v[b] - v[a]) ^ (v[c] - v[a])
          dead[a][b] = dead[b][c] = dead[c][a] = false;
      };
      add_face(0, 1, 2);
      add_face(0, 2, 1);
      for (int i = 3; i < n; ++i) {</pre>
```

#### 5.3 Pick's Theorem

給定頂點坐標均是整點的簡單多邊形,面積 = 內部格點數 + 邊上格點數/2 - 1

#### 6 Graph

# 6.1 2-SAT [5a6317]

```
1 struct TWO_SAT {
      int n, N;
      vector<vector<int>> G, rev_G;
      deque<bool> used;
      vector<int> order, comp;
      deque<bool> assignment;
      void init(int n) {
          n = n;
          N = n * 2;
          G.resize(N + 5);
          rev_G.resize(N + 5);
      void dfs1(int v) {
          used[v] = true;
          for (int u : G[v]) {
              if (!used[u])
                  dfs1(u);
          order.push back(v);
      void dfs2(int v, int cl) {
          comp[v] = cl;
          for (int u : rev_G[v]) {
              if (comp[u] == -1)
                  dfs2(u, c1);
27
      bool solve() {
          order.clear():
          used.assign(N, false);
          for (int i = 0; i < N; ++i) {
              if (!used[i])
                  dfs1(i);
          comp.assign(N, -1);
          for (int i = 0, j = 0; i < N; ++i) {
              int v = order[N - i - 1];
              if (comp[v] == -1)
                  dfs2(v, j++);
```

```
assignment.assign(n, false);
          for (int i = 0; i < N; i += 2) {
              if (comp[i] == comp[i + 1])
                  return false:
              assignment[i / 2] = (comp[i] > comp[i + 1]);
          return true:
      // A or B 都是 0-based
      void add_disjunction(int a, bool na, int b, bool nb) {
          // na is true => ~a, na is false => a
          // nb is true => ~b, nb is false => b
          a = 2 * a ^ na;
          b = 2 * b ^ nb;
          int neg_a = a ^ 1;
          int neg b = b ^ 1;
          G[neg a].push back(b);
          G[neg_b].push_back(a);
          rev G[b].push back(neg a);
          rev G[a].push back(neg b);
          return;
61
63
      void get result(vector<int>& res) {
          res.clear();
          for (int i = 0; i < n; i++)</pre>
              res.push back(assignment[i]);
66
67
```

## 6.2 Augment Path [f8a5dd]

```
1 struct AugmentPath{
      int n, m;
      vector<vector<int>> G;
      vector<int> mx, my;
      vector<int> visx, visy;
      int stamp;
      AugmentPath(int _n, int _m) : n(_n), m(_m), G(n), mx(n,
           -1), my(m, -1), visx(n), visy(n){
          stamp = 0;
      void add(int x, int y){
          G[x].push_back(y);
13
      // bb03e2
      bool dfs1(int now){
          visx[now] = stamp;
          for (auto x : G[now]){
              if (my[x]==-1){
                  mx[now] = x;
                  my[x] = now;
                  return true;
          for (auto x : G[now]){
              if (visx[my[x]]!=stamp && dfs1(my[x])){
                  mx[now] = x;
29
                  my[x] = now;
30
31
                  return true;
```

```
return false;
vector<pair<int, int>> find_max_matching(){
    vector<pair<int, int>> ret;
    while (true){
        stamp++;
        int tmp = 0;
        for (int i=0 ; i<n ; i++){</pre>
            if (mx[i]==-1 && dfs1(i)) tmp++;
        if (tmp==0) break;
    for (int i=0 ; i<n ; i++){</pre>
        if (mx[i]!=-1){
            ret.push_back({i, mx[i]});
    return ret;
// 645577
void dfs2(int now){
    visx[now] = true;
    for (auto x : G[now]){
        if (my[x]!=-1 && visy[x]==false){
            visy[x] = true;
            dfs2(my[x]);
// 要先執行 find max matching 一次
vector<pair<int, int>> find_min_vertex_cover(){
    fill(visx.begin(), visx.end(), false);
    fill(visy.begin(), visy.end(), false);
    vector<pair<int, int>> ret;
    for (int i=0 ; i<n ; i++){</pre>
        if (mx[i]==-1) dfs2(i);
    for (int i=0 ; i<n ; i++){</pre>
        if (visx[i]==false) ret.push back({1, i});
    for (int i=0 ; i<m ; i++){</pre>
        if (visy[i]==true) ret.push back({2, i});
    return ret;
```

# 6.3 Bounded Flow [7ba6b1]

```
| struct BoundedFlow : Dinic {
   vector<ll> tot:
   void init(int _n) {
     Dinic::init(_n + 2);
     tot.assign(n, 0);
```

```
void add_edge(int u, int v, ll lcap, ll rcap) {
      tot[u] -= lcap, tot[v] += lcap;
      g[u].pb(edge{v, rcap, lcap, SZ(g[v])});
     g[v].pb(edge{u, 0, 0, SZ(g[u]) - 1});
   bool feasible() {
     11 \text{ sum } = 0;
     int vs = n - 2, vt = n - 1;
      for(int i = 0; i < n - 2; ++i)
        if(tot[i] > 0)
          add_edge(vs, i, 0, tot[i]), sum += tot[i];
        else if(tot[i] < 0) add_edge(i, vt, 0, -tot[i]);</pre>
      if(sum != maxflow(vs, vt)) sum = -1;
      for(int i =0; i < n - 2; i ++){
        if(tot[i] > 0)
          g[vs].pop_back(), g[i].pop_back();
        else if(tot[i] < 0)</pre>
          g[i].pop_back(), g[vt].pop_back();
      return sum != -1;
   11 boundedflow(int _s, int _t) {
      add_edge( _t, _s, 0, INF);
     if(!feasible()) return -1;
     ll x = g[t].back().flow;
      g[_t].pop_back(), g[_s].pop_back();
      return x - maxflow( t, s);
 6.4 C3C4 [d00465]
1 // 0-based
```

```
void C3C4(vector<int> deg, vector<array<int, 2>> edges){
      int N = deg.size();
      int M = deges.size();
      vector<int> ord(N), rk(N);
      iota(ord.begin(), ord.end(), 0);
      sort(ord.begin(), ord.end(), [&](int x, int y) { return
           deg[x] > deg[y]; });
      for (int i=0 ; i<N ; i++) rk[ord[i]] = i;</pre>
      vector<vector<int>>> D(N), adj(N);
      for (auto [u, v] : e) {
           if (rk[u] > rk[v]) swap(u, v);
          D[u].emplace back(v);
          adj[u].emplace_back(v);
          adj[v].emplace_back(u);
      vector<int> vis(N);
      int c3 = 0, c4 = 0;
      for (int x : ord) { // c3
           for (int y : D[x]) vis[y] = 1;
           for (int y : D[x]) for (int z : D[y]){
               c3 += vis[z]; // xyz is C3
27
          for (int y : D[x]) vis[y] = 0;
      for (int x : ord) { // c4
           for (int y : D[x]) for (int z : adj[y])
               if (rk[z] > rk[x]) c4 += vis[z]++;
           for (int y : D[x]) for (int z : adj[y])
```

```
if (rk[z] > rk[x]) --vis[z];
      } // both are O(M*sqrt(M)), test @ 2022 CCPC guangzhou
      cout << c4 << "\n":
36 }
```

#### 6.5 Cut BCC [2af809]

```
| #include <bits/stdc++.h>
  using namespace std;
  const int N = 200005;
  vector <int> G[N];
  int low[N], depth[N];
  bool vis[N];
  vector <vector <int>> bcc;
  stack <int> stk;
  void dfs(int v, int p) {
      stk.push(v);
      vis[v] = true;
      low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
      for (int u : G[v]) {
          if (u == p) continue;
          if (!vis[u]) {
17
              /// (v, u) 是樹邊
              dfs(u, v);
              low[v] = min(low[v], low[u]);
              /// u 無法在不經過父邊的情況走到 v 的祖先
              if (low[u] >= depth[v]) {
                  bcc.emplace back();
                  while (stk.top() != u) {
                      bcc.back().push_back(stk.top());
                      stk.pop();
                  bcc.back().push back(stk.top());
                  stk.pop();
30
                  bcc.back().push_back(v);
          } else {
              /// (v, u) 是回邊
33
              low[v] = min(low[v], depth[u]);
34
35
36
```

#### 6.6 Dinic [170617]

```
1 // 一般圖: O(EV2)
2 // 二分圖: O(EVV)
  const 11 INF = 1 << 30;</pre>
  struct Dinic{
    struct edge{
      11 to, cap, flow, rev;
    int n,s,t;
    vector<vector<edge> > g;
    vector<int> dis,ind;
    void init(int _n){
13
      n = _n;
      g.assign(n,vector<edge>());
14
15
    void add edge(int u,int v,ll cap){
      g[u].pb(edge{v,cap,0,SZ(g[v])});
```

int root;

int operator [] (int x){

if (sdom[x]!=dom[x]) dom[x] = dom[dom[x]];

```
g[v].pb(edge{u,0,0,SZ(g[u])-1});
                                                                          return idom[x];
                                                                      }
   bool bfs() {
     dis.assign(n, -1);
     queue<int> q;
     q.push(s), dis[s] = 0;
     while(!q.empty()){
       int cur = q.front(); q.pop();
       for(auto &e : g[cur]) {
         if(dis[e.to] == -1 && e.flow != e.cap) {
                                                                      {
           q.push(e.to);
                                                                          stamp = 0:
           dis[e.to] = dis[cur] + 1;
                                                                          root = _root;
       }
     return dis[t] != -1;
   11 dfs(int u, ll cap) {
     if(u == t || !cap) return cap;
                                                                      void dfs(int x){
     for(int &i = ind[u]; i < SZ(g[u]); ++i) {</pre>
       edge & e = g[u][i];
       if(dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                                                                          stamp++:
         il df = dfs(e.to, min(e.cap - e.flow, cap));
         if(df){
           e.flow += df;
           g[e.to][e.rev].flow -= df;
                                                                                  dfs(u);
           return df;
     dis[u] = -1;
                                                                      }
     return 0;
   11 maxflow(int _s, int _t) {
     s = _s; t = _t;
     11 \text{ flow} = 0, \text{ df};
     while(bfs()) {
       ind.assign(n, 0);
       while((df = dfs(s, INF))) flow += df;
                                                                          fa[x] = p;
     return flow;
 6.7 Dominator Tree [52b249]
                                                                          fa[x] = y;
il // 全部都是 0-based
2 // G 要是有向無權圖
                                                                      void build(){
3 // 一開始要初始化 G(N, root)
                                                                          dfs(root);
4 // 用完之後要 build
5 // G[i] = 從 root 走到 i 時,一定要走到的點且離 i 最近
6 struct DominatorTree{
     int N:
     vector<vector<int>> G;
     vector<vector<int>> buckets, rg;
     // dfn[x] = the DFS otder of x
     // rev[x] = the vertex with DFS order x
     // par[x] = the parent of x
     vector<int> dfn, rev, par;
     vector<int> sdom, dom, idom;
     vector<int> fa, val;
     int stamp;
```

```
for (int i=1; i<stamp; i++) idom[rev[i]] = rev[dom[</pre>
DominatorTree(int _N, int _root) :
                                                                           i]];
                                                           88
    G(N), buckets(N), rg(N),
                                                           89 };
    dfn(N, -1), rev(N, -1), par(N, -1),
    sdom(N, -1), dom(N, -1), idom(N, -1),
                                                             6.8 EdgeBCC [d09eb1]
    fa(N, -1), val(N, -1)
                                                            1 // d09eb1
                                                            2 // 0-based, 支援重邊
                                                             struct EdgeBCC{
                                                                  int n, m, dep, sz;
                                                                  vector<vector<pair<int, int>>> G;
void add_edge(int u, int v){
    G[u].push_back(v);
                                                                  vector<vector<int>> bcc;
                                                                  vector<int> dfn, low, stk, isBridge, bccId;
                                                                  vector<pair<int, int>> edge, bridge;
    rev[dfn[x] = stamp] = x;
                                                                  EdgeBCC(int_n): n(n), m(0), sz(0), dfn(n), low(n), G(n)
    fa[stamp] = sdom[stamp] = val[stamp] = stamp;
                                                                      ), bcc(n), bccId(n) {}
                                                                  void add_edge(int u, int v) {
    for (int u : G[x]){
                                                                      edge.push back({u, v});
        if (dfn[u]==-1){
                                                                      G[u].push_back({v, m});
                                                                      G[v].push_back({u, m++});
            par[dfn[u]] = dfn[x];
                                                                  void dfs(int now, int pre) {
        rg[dfn[u]].push_back(dfn[x]);
                                                                      dfn[now] = low[now] = ++dep;
                                                                      stk.push back(now);
int eval(int x, bool first){
                                                                      for (auto [x, id] : G[now]){
    if (fa[x]==x) return !first ? -1 : x;
                                                                          if (!dfn[x]){
    int p = eval(fa[x], false);
                                                                              dfs(x, id);
                                                                              low[now] = min(low[now], low[x]);
    if (p==-1) return x;
                                                                          }else if (id!=pre){
    if (sdom[val[x]]>sdom[val[fa[x]]]) val[x] = val[fa[x
                                                                              low[now] = min(low[now], dfn[x]);
    return !first ? p : val[x];
                                                                      if (low[now]==dfn[now]){
                                                                          if (pre!=-1) isBridge[pre] = true;
                                                                          int u;
void link(int x, int y){
                                                                              u = stk.back():
                                                                              stk.pop_back();
                                                                              bcc[sz].push back(u);
                                                                              bccId[u] = sz;
                                                                          } while (u!=now);
    for (int x=stamp-1 ; x>=0 ; x--){
        for (int y : rg[x]){
            sdom[x] = min(sdom[x], sdom[eval(y, true)]);
                                                                  void get_bcc() {
        if (x>0) buckets[sdom[x]].push_back(x);
                                                                      isBridge.assign(m, 0);
        for (int u : buckets[x]){
            int p = eval(u, true);
                                                                      for (int i=0 ; i<n ; i++){</pre>
            if (sdom[p]==x) dom[u] = x;
                                                                          if (!dfn[i]) dfs(i, -1);
            else dom[u] = p;
        if (x>0) link(x, par[x]);
                                                                      for (int i=0 ; i<m ; i++){</pre>
                                                           51
                                                                          if (isBridge[i]){
                                                           52
                                                                              bridge.push_back({edge[i].first , edge[i].
                                                           53
    idom[root] = root;
                                                                                   second});
    for (int x=1; x<stamp; x++){</pre>
```

```
return ret;
                                                                                                                                       vector<vector<int>>> G, inv G, result;
                                                                                                                                       vector<pair<int, int>> edges;
                                                                61 };
                                                                                                                                       vector<bool> vis:
                                                                                                                                       vector<int> order;
                                                                  6.10 HLD [f57ec6]
 6.9 EnumeratePlanarFace [e70ee1]
                                                                                                                                       vector<vector<int>> SCC:
                                                                                                                                       vector<int> SCC id;
                                                                 | #include <bits/stdc++.h>
1 // 0-based
                                                                  #define int long long
2 struct PlanarGraph{
                                                                                                                                       SCC compress(int N):
                                                                  using namespace std;
     int n, m, id;
                                                                                                                                       N(N), M(0), sz(0),
     vector<point<int>> v;
                                                                                                                                       G(N), inv G(N),
                                                                  const int N = 100005;
     vector<vector<pair<int, int>>> G;
                                                                                                                                       vis(N), SCC_id(N)
                                                                  vector <int> G[N];
     vector<int> conv, nxt, vis;
                                                                                                                                       {}
                                                                  struct HLD {
                                                                      vector<int> pa, sz, depth, mxson, topf, id;
     PlanarGraph(int n, int m, vector<point<int>> v) :
                                                                                                                                       vector<int> operator [] (int x){
                                                                      int n, idcnt = 0;
                                                                                                                                           return result[x];
     n(n), m(m), id(0),
                                                                      HLD(int _n) : n(_n), pa(_n + 1), sz(_n + 1), depth(_n +
     v(v), G(n),
                                                                           1), mxson((n + 1), topf((n + 1), id((n + 1)))
     conv(2*m), nxt(2*m), vis(2*m) {}
                                                                                                                                 26
                                                                      void dfs1(int v = 1, int p = -1) {
                                                                                                                                 27
                                                                                                                                       void add edge(int u, int v){
                                                                          pa[v] = p; sz[v] = 1; mxson[v] = 0;
                                                                                                                                           G[u].push_back(v);
     void add_edge(int x, int y){
                                                                          depth[v] = (p == -1 ? 0 : depth[p] + 1);
         G[x].push_back({y, 2*id});
                                                                                                                                           inv_G[v].push_back(u);
                                                                          for (int u : G[v]) {
                                                                                                                                           edges.push_back({u, v});
         G[y].push_back({x, 2*id+1});
                                                                              if (u == p) continue;
                                                                                                                                           M++;
         conv[2*id] = x;
                                                                              dfs1(u, v);
         conv[2*id+1] = y;
                                                                                                                                 32
                                                                              sz[v] += sz[u];
                                                                                                                                 33
         id++;
                                                                              if (sz[u] > sz[mxson[v]]) mxson[v] = u;
                                                                                                                                       void dfs1(vector<vector<int>> &G, int now){
     }
                                                                                                                                           vis[now] = 1:
                                                                                                                                           for (auto x : G[now]) if (!vis[x]) dfs1(G, x);
     vector<int> enumerate_face(){
                                                                      void dfs2(int v = 1, int top = 1) {
                                                                                                                                           order.push_back(now);
         for (int i=0; i<n; i++){
                                                                          id[v] = ++idcnt;
             sort(G[i].begin(), G[i].end(), [&](pair<int, int>
                                                                          topf[v] = top;
                   a, pair<int, int> b){
                                                                          if (mxson[v]) dfs2(mxson[v], top);
                 return (v[a.first]-v[i])<(v[b.first]-v[i]);</pre>
                                                                                                                                       void dfs2(vector<vector<int>> &G, int now){
                                                                          for (int u : G[v]) {
                                                                                                                                           SCC id[now] = SCC.size()-1;
                                                                              if (u == mxson[v] || u == pa[v]) continue;
                                                                                                                                           SCC.back().push_back(now);
                                                                              dfs2(u, u);
             int sz = G[i].size(), pre = sz-1;
                                                                                                                                           vis[now] = 1;
                                                                                                                                           for (auto x : G[now]) if (!vis[x]) dfs2(G, x);
             for (int j=0 ; j<sz ; j++){</pre>
                 nxt[G[i][pre].second] = G[i][j].second^1;
                                                                      // query 為區間資料結構
                 pre = j;
                                                                      int path_query(int a, int b) {
             }
                                                                                                                                       void compress(){
                                                                          int res = 0:
                                                                                                                                           fill(vis.begin(), vis.end(), 0);
                                                                                                                                           for (int i=0; i<N; i++) if (!vis[i]) dfs1(G, i);</pre>
                                                                          while (topf[a] != topf[b]) { /// 若不在同一條鍊上
         vector<int> ret;
                                                                              if (depth[topf[a]] < depth[topf[b]]) swap(a, b);</pre>
         for (int i=0 ; i<2*m ; i++){</pre>
                                                                                                                                           fill(vis.begin(), vis.end(), 0);
                                                                              res = max(res, 011); // query : L = id[topf[a]],
                                                                                                                                 51
             if (vis[i]==false){
                                                                                                                                           reverse(order.begin(), order.end());
                                                                                   r = id[a]
                                                                                                                                           for (int i=0 ; i<N ; i++){</pre>
                 int area = 0, now = i;
                                                                              a = pa[topf[a]];
                                                                                                                                               if (!vis[order[i]]){
                 vector<int> pt;
                                                                                                                                 54
                                                               37
                                                                                                                                                   SCC.push back(vector<int>());
                                                                          /// 此時已在同一條鍊上
                                                                                                                                                   dfs2(inv G, order[i]);
                 while (!vis[now]){
                                                                          if (depth[a] < depth[b]) swap(a, b);</pre>
                     vis[now] = true;
                                                                          res = \max(\text{res}, 011); // query : l = id[b], r = id[a]
                     pt.push back(conv[now]);
                                                                          return res:
                     now = nxt[now]:
                                                                42
                                                                                                                                           result.resize(SCC.size());
                                                                43 };
                                                                                                                                           sz = SCC.size();
                                                                                                                                           for (auto [u, v] : edges){
                 pt.push back(pt.front());
                                                                  6.11 Kosaraju [c7d5aa]
                                                                                                                                               if (SCC id[u]!=SCC id[v]) result[SCC id[u]].
                 for (int i=0 ; i+1<pt.size() ; i++){</pre>
                     area -= (v[pt[i]]^v[pt[i+1]]);
                                                                                                                                                    push_back(SCC_id[v]);
                                                                1// 給定一個有向圖, 迴回傳縮點後的圖、SCC 的資訊
                                                                                                                                           for (int i=0 ; i<SCC.size() ; i++){</pre>
                                                                2 // 所有點都以 based-0 編號
                 // pt = face boundary
                                                                                                                                               sort(result[i].begin(), result[i].end());
                                                                3 // 函式:
                                                                                                                                               result[i].resize(unique(result[i].begin(), result
                 if (area>0){
                                                                4 // .compress: O(n Log n) 計算 G3、SCC、SCC_id 的資訊,並把縮
                                                                                                                                                    [i].end())-result[i].begin());
                     ret.push back(area);
                                                                       點後的結果存在 result 裡
                 }else{
                                                                5 // SCC[i] = 某個 SCC 中的所有點
                     // pt is outer face
                                                                                                                                 69
                                                                6 // SCC id[i] = 第 i 個點在第幾個 SCC
                                                                                                                                 70 };
                                                                7 struct SCC compress{
                                                                      int N, M, sz;
```

```
6.12 Kuhn Munkres [e66c35]
                                                                                                                                                    for (int j=0 ; j<N ; j++){</pre>
                                                                        int solve(){
                                                                                                                                                        LCA[i][j] = LCA[i-1][LCA[i-1][j]];
11 // O(n^3) 找到最大權匹配
                                                                             for (int i=0 ; i<n ; i++){</pre>
                                                                                                                                     33
                                                                                                                                               }
2 struct KuhnMunkres{
                                                                                 lx[i] = 0;
                                                                                                                                           }
     int n; // max(n, m)
                                                                                 for (int j=0 ; j<n ; j++){</pre>
     vector<vector<int>> G;
                                                                                     lx[i] = max(lx[i], G[i][j]);
                                                                                                                                           int jump(int u, int step){
     vector<int> match, lx, ly, visx, visy;
                                                                                }
                                                                                                                                               for (int i=0 ; i<H ; i++){</pre>
     vector<int> slack;
                                                                                                                                                   if (step&(1<<i)) u = LCA[i][u];</pre>
     int stamp = 0;
                                                                            fill(ly.begin(), ly.end(), 0);
                                                                                                                                               return u;
     KuhnMunkres(int n): n(n), G(n, vector(int)(n)), lx(n),
                                                                            fill(match.begin(), match.end(), -1);
          ly(n), slack(n), match(n), visx(n), visy(n) {}
                                                                             for(int i = 0; i < n; i++) {</pre>
                                                                                                                                           int get_LCA(int u, int v){
     void add(int x, int y, int w){
                                                                                fill(slack.begin(), slack.end(), INF);
                                                                                                                                               if (dep[u]<dep[v]) swap(u, v);</pre>
         G[x][y] = max(G[x][y], w);
                                                                                                                                               u = jump(u, dep[u]-dep[v]);
                                                                                 stamp++:
                                                                                 if(dfs(i, true)) continue;
                                                                                                                                               if (u==v) return u;
                                                                                                                                               for (int i=H-1; i>=0; i--){
     bool dfs(int i, bool aug){ // aug = true 表示要更新 match
                                                                                 while(augment()==false) relabel();
                                                                                                                                                   if (LCA[i][u]!=LCA[i][v]){
                                                                                 stamp++;
                                                                                                                                                       u = LCA[i][u];
         if (visx[i]==stamp) return false;
         visx[i] = stamp;
                                                                                 dfs(i, true);
                                                                                                                                                       v = LCA[i][v];
         for (int j=0 ; j<n ; j++){</pre>
                                                                            int ans = 0:
                                                                                                                                               return parent[u];
             if (visy[j]==stamp) continue;
                                                                                                                                     53
                                                                            for (int j=0 ; j<n ; j++){</pre>
             int d = lx[i]+ly[j]-G[i][j];
                                                                                                                                     54
                                                                                 if (match[j]!=-1){
                                                                                                                                     55 };
                                                                                     ans += G[match[j]][j];
             if (d==0){
                  visy[j] = stamp;
                                                                                                                                       6.14 MCMF [0d5244]
                  if (match[j]==-1 || dfs(match[j], aug)){
                      if (aug){
                                                                            return ans;
                          match[i] = i;
                                                                                                                                     | struct Flow {
                                                                                                                                         struct Edge {
                      return true;
                                                                                                                                           int u, rc, k, rv;
                                                                    6.13 LCA [5b6a5b]
             }else{
                  slack[j] = min(slack[j], d);
                                                                                                                                         vector<vector<Edge>> G;
                                                                  ı | // 1-based · 可以支援森林 · Ø 是超級源點 · 所有樹都要跟他建邊
                                                                                                                                         vector<int> par, par eid;
                                                                    struct Tree{
                                                                                                                                         Flow(int n) : G(n+1), par(n+1), par_eid(n+1) {}
         return false;
                                                                        int N, M = 0, H;
                                                                        vector<int> parent, dep;
                                                                                                                                         // v->u, capcity: c, cost: k
                                                                        vector<vector<int>> G, LCA;
                                                                                                                                         void add(int v, int u, int c, int k){
     bool augment(){
                                                                                                                                           G[v].push_back({u, c, k, G[u].size()});
         for (int j=0 ; j<n ; j++){</pre>
                                                                        Tree(int_N): N(_N+1), H(__lg(N)+1), parent(N, -1), dep( 13
                                                                                                                                           G[u].push_back({v, 0, -k, G[v].size()-1});
             if (visy[j]!=stamp && slack[j]==0){
                                                                             N), G(N)
                  visy[j] = stamp;
                                                                             LCA.resize(H, vector<int>(N, 0));
                  if (match[j]==-1 || dfs(match[j], false)){
                                                                                                                                         // 6d1140
                      return true;
                                                                                                                                         int spfa(int s, int t){
                                                                        void add edge(int u, int v){
                                                                                                                                           fill(par.begin(), par.end(), -1);
             }
                                                                                                                                           vector<int> dis(par.size(), INF);
                                                                            G[u].push_back(v);
                                                                                                                                           vector<bool> in_q(par.size(), false);
                                                                            G[v].push_back(u);
                                                                                                                                           queue<int> Q;
         return false;
                                                                                                                                           dis[s] = 0;
                                                                                                                                           in q[s] = true;
     void relabel(){
                                                                        void dfs(int now = 0, int pre = 0){
                                                                                                                                           Q.push(s);
         int delta = INF;
                                                                            dep[now] = dep[pre]+1;
                                                                                                                                           while (!Q.empty()){
         for (int j=0 ; j<n ; j++){</pre>
                                                                             parent[now] = pre;
             if (visy[j]!=stamp) delta = min(delta, slack[j]); 20
                                                                             for (auto x : G[now]){
                                                                                                                                             int v = 0.front();
                                                                                 if (x==pre) continue;
                                                                                                                                             Q.pop();
                                                                                 dfs(x, now);
         for (int i=0 ; i<n ; i++){</pre>
                                                                                                                                             in_q[v] = false;
             if (visx[i]==stamp) lx[i] -= delta;
                                                                                                                                             for (int i=0 ; i<G[v].size() ; i++){</pre>
                                                                                                                                     31
         for (int j=0 ; j<n ; j++){</pre>
                                                                                                                                     32
                                                                                                                                               auto [u, rc, k, rv] = G[v][i];
             if (visy[j]==stamp) ly[j] += delta;
                                                                                                                                               if (rc>0 && dis[v]+k<dis[u]){</pre>
                                                                        void build_LCA(int root = 0){
                                                                                                                                     33
             else slack[j] -= delta;
                                                                                                                                                 dis[u] = dis[v]+k;
                                                                 27
                                                                            dfs();
                                                                             for (int i=0 ; i<N ; i++) LCA[0][i] = parent[i];</pre>
                                                                                                                                     35
                                                                                                                                                 par[u] = v;
```

for (int i=1 ; i<H ; i++){</pre>

par\_eid[u] = i;

```
if (!in_q[u]) Q.push(u);
            in_q[u] = true;
      }
      return dis[t];
    // return <max flow, min cost>, d7e7ad
    pair<int, int> flow(int s, int t){
      int fl = 0, cost = 0, d;
      while ((d = spfa(s, t))<INF){</pre>
        int cur = INF;
        for (int v=t; v!=s; v=par[v])
          cur = min(cur, G[par[v]][par_eid[v]].rc);
        cost += d*cur;
        for (int v=t; v!=s; v=par[v]){
          G[par[v]][par_eid[v]].rc -= cur;
          G[v][G[par[v]][par_eid[v]].rv].rc += cur;
      return {fl, cost};
62 };
```

#### 6.15 Tarjan [8b2350]

```
| struct tarjan SCC {
     int now T, now SCCs;
     vector<int> dfn, low, SCC;
     stack<int> S;
     vector<vector<int>> E;
     vector<bool> vis, in stack;
     tarjan_SCC(int n) {
         init(n);
     void init(int n) {
         now_T = now_SCCs = 0;
         dfn = low = SCC = vector<int>(n);
         E = vector<vector<int>>(n);
         S = stack<int>();
         vis = in stack = vector<bool>(n);
     void add(int u, int v) {
         E[u].push_back(v);
     void build() {
         for (int i = 0; i < dfn.size(); ++i) {</pre>
             if (!dfn[i]) dfs(i);
     void dfs(int v) {
         now T++;
         vis[v] = in_stack[v] = true;
         dfn[v] = low[v] = now_T;
         S.push(v);
         for (auto &i:E[v]) {
             if (!vis[i]) {
                 vis[i] = true;
                 dfs(i);
                 low[v] = min(low[v], low[i]);
```

#### 6.16 Tarjan Find AP [1daed6]

```
vector<int> dep(MAX_N), low(MAX_N), AP;
bitset<MAX N> vis;
void dfs(int now, int pre){
    int cnt = 0;
    bool ap = 0;
    vis[now] = 1;
    low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
    for (auto x : G[now]){
        if (x==pre){
            continue;
        }else if (vis[x]==0){
            cnt++;
            dfs(x, now);
            low[now] = min(low[now], low[x]);
            if (low[x]>=dep[now]) ap=1;
            low[now] = min(low[now], dep[x]);
    if ((now==pre && cnt>=2) || (now!=pre && ap)){
        AP.push back(now);
```

#### 6.17 Tree Isomorphism [cd2bbc]

```
#include <bits/stdc++.h>
#pragma GCC optimize("03,unroll-loops")
#define fastio ios::sync_with_stdio(0), cin.tie(0), cout.tie
(0)

#define dbg(x) cerr << #x << " = " << x << endl
#define int long long
using namespace std;

// declare
const int MAX_SIZE = 2e5+5;
const int INF = 9e18;
const int MOD = 1e9+7;
const double EPS = 1e-6;
typedef vector<vector<int>> Graph;
typedef map
typedef map
int n, a, b;
```

```
17 int id1, id2;
18 pair < int , int > c1 , c2;
  vector<int> sz1(MAX SIZE), sz2(MAX SIZE);
  vector<int> we1(MAX_SIZE), we2(MAX_SIZE);
  Graph g1(MAX_SIZE), g2(MAX_SIZE);
  Hash m1, m2:
  int testcase=0;
  void centroid(Graph &g, vector<int> &s, vector<int> &w, pair
       int, int> &rec, int now, int pre){
      w[now]=0;
      for (auto x : g[now]){
          if (x!=pre){
               centroid(g, s, w, rec, x, now);
              s[now]+=s[x];
              w[now]=max(w[now], s[x]);
      w[now]=max(w[now], n-s[now]);
      if (w[now]<=n/2){</pre>
          if (rec.first==0) rec.first=now;
          else rec.second=now:
41
  int dfs(Graph &g, Hash &m, int &id, int now, int pre){
      vector<int> v;
      for (auto x : g[now]){
          if (x!=pre){
               int add=dfs(g, m, id, x, now);
              v.push back(add);
      sort(v.begin(), v.end());
      if (m.find(v)!=m.end()){
          return m[v];
      }else{
          m[v]=++id;
          return id;
59
  void solve1(){
      // init
      id1=0;
      id2=0;
      c1={0, 0};
      c2={0, 0};
      fill(sz1.begin(), sz1.begin()+n+1, 0);
      fill(sz2.begin(), sz2.begin()+n+1, 0);
      fill(we1.begin(), we1.begin()+n+1, 0);
      fill(we2.begin(), we2.begin()+n+1, 0);
      for (int i=1; i<=n; i++){</pre>
          g1[i].clear();
          g2[i].clear();
      m1.clear();
      m2.clear();
      // input
      cin >> n;
```

```
for (int i=0 ; i<n-1 ; i++){</pre>
                                                                    12 #define spc cerr << ' ';
           cin >> a >> b;
                                                                    13 #define BAE(x) (x).begin(), (x).end()
           g1[a].push back(b);
                                                                    14 #define STL(x) cerr << #x << " : ": for(auto &gwe:x) cerr <<
                                                                                                                                                        low[v] = min(low[v], dfn[i]);
           g1[b].push_back(a);
                                                                            qwe << ''; cerr << endl;</pre>
                                                                                                                                                        if(dfn[i] < dfn[v]){ // edge i--v will be visited</pre>
                                                                    #define deb1 cerr << "deb1" << endl;</pre>
                                                                                                                                                              twice at here, but we only need one.
                                                                     16 #define deb2 cerr << "deb2" << endl:
       for (int i=0; i<n-1; i++){
                                                                                                                                                            S.push(ne);
                                                                       #define deb3 cerr << "deb3" << endl;</pre>
           cin >> a >> b;
           g2[a].push back(b);
                                                                       #define deb4 cerr << "deb4" << endl:
           g2[b].push_back(a);
                                                                       #define deb5 cerr << "deb5" << endl;</pre>
                                                                                                                                        83
                                                                       #define bye exit(0);
                                                                       using namespace std;
       // get tree centroid
                                                                                                                                           int dep[mxn], imp[mxn][mxlg];
       centroid(g1, sz1, we1, c1, 1, 0);
                                                                       const int mxn = (int)(2e5) + 10;
                                                                                                                                           void dfs_lca(int v, int par, int depth){
       centroid(g2, sz2, we2, c2, 1, 0);
                                                                       const int mxlg = 17;
                                                                                                                                               dep[v] = depth;
                                                                       int last_special_node = (int)(1e5) + 1;
                                                                                                                                               for(auto &i:F[v]){
                                                                       vector<int> E[mxn], F[mxn];
                                                                                                                                                   if(i == par) continue;
       // process
       int res1=0, res2=0, res3=0;
                                                                                                                                                   jmp[i][0] = v;
       if (c2.second!=0){
                                                                                                                                                   dfs_lca(i, v, depth + 1);
                                                                       struct edg{
           res1=dfs(g1, m1, id1, c1.first, 0);
                                                                           int fr. to:
101
           m2=m1:
                                                                           edg(int _fr, int _to){
102
           id2=id1;
                                                                               fr = _fr;
103
           res2=dfs(g2, m1, id1, c2.first, 0);
                                                                               to = _to;
                                                                                                                                           inline void build lca(){
           res3=dfs(g2, m2, id2, c2.second, 0);
                                                                                                                                               imp[1][0] = 1;
       }else if (c1.second!=0){
                                                                                                                                               dfs lca(1, -1, 1);
                                                                       ostream& operator<<(ostream& os, edg x){os << x.fr << "--" << 99
           res1=dfs(g2, m1, id1, c2.first, 0);
                                                                                                                                               lp(j,1,mxlg){
107
           m2=m1;
                                                                             x.to;}
                                                                                                                                                   lp(i,1,mxn){
           id2=id1:
                                                                       vector<edg> EV:
                                                                                                                                                        jmp[i][j] = jmp[jmp[i][j-1]][j-1];
           res2=dfs(g1, m1, id1, c1.first, 0);
                                                                                                                                        102
           res3=dfs(g1, m2, id2, c1.second, 0);
                                                                       void tarjan(int v, int par, stack<int>& S){
110
                                                                                                                                        103
                                                                           static vector<int> dfn(mxn), low(mxn);
       }else{
111
                                                                                                                                        104
112
           res1=dfs(g1, m1, id1, c1.first, 0);
                                                                           static vector<bool> to_add(mxn);
           res2=dfs(g2, m1, id1, c2.first, 0);
                                                                           static int nowT = 0;
113
                                                                                                                                           inline int lca(int x, int y){
                                                                                                                                               if(dep[x] < dep[y]){ swap(x, y); }</pre>
114
                                                                           int childs = 0;
115
       // output
                                                                           nowT += 1:
                                                                                                                                               int diff = dep[x] - dep[y];
116
       cout << (res1==res2 || res1==res3 ? "YES" : "NO") << endl</pre>
                                                                           dfn[v] = low[v] = nowT;
                                                                                                                                               lp(j,0,mxlg){
                                                                           for(auto &ne:E[v]){
                                                                                                                                                   if((diff >> j) & 1){
                                                                                                                                        111
                                                                               int i = EV[ne].to;
                                                                                                                                                        x = jmp[x][j];
                                                                                                                                        112
                                                                               if(i == par) continue;
119
       return;
                                                                                                                                        113
                                                                               if(!dfn[i]){
120
                                                                                                                                        114
                                                                                   S.push(ne);
                                                                                                                                               if(x == y) return x;
121
                                                                                                                                        115
                                                                                   tarjan(i, v, S);
   signed main(void){
123
       fastio:
                                                                                   childs += 1;
                                                                                                                                               for(int j = mxlg - 1; j >= 0; j--){
                                                                                                                                        117
                                                                                   low[v] = min(low[v], low[i]);
                                                                                                                                                   if(jmp[x][j] != jmp[y][j]){
                                                                                                                                        118
       int t=1;
125
                                                                                                                                                       x = jmp[x][j];
                                                                                    if(par >= 0 \&\& low[i] >= dfn[v]){
126
       cin >> t;
                                                                                                                                        120
                                                                                                                                                        y = jmp[y][j];
       while (t--){
                                                                                        vector<int> bcc:
127
                                                                                                                                        121
           solve1();
                                                                                        int tmp;
128
                                                                                                                                        122
                                                                                                                                               return jmp[x][0];
129
                                                                                        do{
                                                                                                                                        123
       return 0;
                                                                                            tmp = S.top(); S.pop();
                                                                                                                                        124
                                                                                            if(!to add[EV[tmp].fr]){
                                                                                                to add[EV[tmp].fr] = true;
                                                                                                                                           inline bool can reach(int fr, int to){
                                                                                                bcc.pb(EV[tmp].fr);
                                                                                                                                               if(dep[to] > dep[fr]) return false;
   6.18 圓方樹 [675aec]
                                                                                            if(!to add[EV[tmp].to]){
                                                                                                                                               int diff = dep[fr] - dep[to];
                                                                                                                                        129
 | #include <bits/stdc++.h>
                                                                                                to add[EV[tmp].to] = true;
                                                                                                                                        130
                                                                                                                                               lp(i,0,mxlg){
                                                                                                bcc.pb(EV[tmp].to);
                                                                                                                                                   if((diff >> j) & 1){
 2 #define lp(i,a,b) for(int i=(a);i<(b);i++)</pre>
                                                                                                                                        131
  #define pii pair<int,int>
                                                                                                                                        132
                                                                                                                                                        fr = jmp[fr][j];
 4 #define pb push back
                                                                                        }while(tmp != ne);
                                                                                                                                        133
                                                                                        for(auto &i:bcc){
 5 #define ins insert
                                                                                                                                        134
                                                                                            to add[i] = false;
 6 #define ff first
                                                                                                                                        135
                                                                                                                                               return fr == to;
                                                                                            F[last special node].pb(j);
  #define ss second
                                                                                                                                        136
 8 #define opa(x) cerr << #x << " = " << x << ", ";</pre>
                                                                                            F[j].pb(last special node);
                                                                                                                                        137
 9 #define op(x) cerr << #x << " = " << x << endl:
                                                                                                                                           int main(){
                                                                                        last special node += 1:
                                                                                                                                               ios::svnc with stdio(false); cin.tie(0);
10 #define ops(x) cerr << x;</pre>
#define etr cerr << endl;</pre>
                                                                                                                                               freopen("test_input.txt", "r", stdin);
```

```
int n, m, q; cin >> n >> m >> q;
142
       lp(i,0,m){}
143
            int u, v; cin >> u >> v;
            E[u].pb(EV.size());
144
145
            EV.pb(edg(u, v));
            E[v].pb(EV.size());
146
147
            EV.pb(edg(v, u));
148
149
       E[0].pb(EV.size());
150
       EV.pb(edg(0, 1));
151
       stack<int> S;
152
       tarjan(0, -1, S);
153
       build_lca();
154
155
       lp(queries,0,q){
            int fr, to, relay; cin >> fr >> to >> relay;
156
157
            if(fr == relay || to == relay){
158
                cout << "NO\n";
159
                continue:
160
            if((can_reach(fr, relay) || can_reach(to, relay)) &&
161
                 dep[relay] >= dep[lca(fr, to)]){
                cout << "NO \ n";
162
163
                continue:
164
165
            cout << "YES\n";</pre>
166
```

# 6.19 最大權閉合圖 [6ca663]

```
1 // 邊 u → v 表示選 u 就要選 v (0-based)
2 // 保證回傳值非負
3 | // 構造:從 S 開始 dfs,不走最小割的邊
4 // 所有經過的點就是要選的那些點。
5 | // 一般圖: O(n²m) / 三分圖: O(m√n)
6 template<typename U>
7 U maximum_closure(vector<U> w, vector<pair<int,int>> EV) {
     int n = w.size(), S = n + 1, T = n + 2;
     Flow G(T + 5); // Graph/Dinic.cpp
     U sum = 0;
     for (int i = 0; i < n; ++i) {</pre>
         if (w[i] > 0) {
            G.add(S, i, w[i]);
            sum += w[i];
         else if (w[i] < 0) {</pre>
            G.add(i, T, abs(w[i]));
     for (auto &[u, v] : EV) { // 請務必確保 INF > Σ/w_i/
         G.add(u, v, INF);
     U cut = G.flow(S, T);
     return sum - cut:
```

#### 6.20 Theorem

- 任意圖
  - 最大匹配 + 最小邊覆蓋 = n (不能有孤點)
  - 點覆蓋的補集是獨立集。最小點覆蓋 + 最大獨立集 = n
  - -w(最小權點覆蓋)+w(最大權獨立集 $)=\sum w_v$

```
- (帶點權的二分圖可以用最小割解,構造請參考 Augment Path.cpp) 40
```

#### 二分圖

- 最小點覆蓋 = 最大匹配 = n 最大獨立集
- 只有邊帶權的二分圖
  - w-vertex-cover (帶權點覆蓋): 每條邊的兩個連接點被選中的次數總和至少要是 $w_e$ 。
  - w-weight matching ( 帶權匹配 )
  - minimum vertex count of w-vertex-cover = maximum weight count of 52 w-weight matching (一個點可以被選很多文·但邊不行) 53
- 點、邊都帶權的二分圖的定理
  - b-matching:假設 v 的點權是  $b_v$  · 那所有 v 的匹配邊 e 的權重都要滿足  $\sum w_e < b_v$  。
  - The maximum w-weight of a b-matching equals the minimum b-weight of vertices in a w-vertex-cover.

#### 7 Math

# 7.1 CRT [682ac6]

```
1 \mid // 求出 d = gcd(a,b) \cdot 並找出 x, y 使 ax + by = d
 tuple<int, int, int> extgcd(int a, int b){
     if (!b) return {a, 1, 0};
      auto [d, x, y] = extgcd(b, a%b);
     return {d, y, x-a/b*y};
 // CRT maybe need use int128
 int CRT m coprime(vector<int> &a, vector<int> &m) {
     int n = a.size(), p = 1, ans = 0;
     vector<int> M(n), invM(n);
      for (int i=0 ; i<n ; i++) p *= m[i];</pre>
     for (int i=0 ; i<n ; i++){</pre>
          M[i] = p/m[i];
          auto [d, x, y] = extgcd(M[i], m[i]);
          invM[i] = x;
          ans += a[i]*invM[i]*M[i];
          ans %= p;
      return (ans%p+p)%p;
 // CRT maybe need use int128
 int CRT m not coprime(vector<int> &a, vector<int> &m) {
     int n = a.size():
      for (int i=1 ; i<n ; i++){</pre>
          int g = __gcd(m[0], m[i]);
          if ((a[i]-a[0])%g!=0) return -1;
          auto [d, x, y] = extgcd(m[0], m[i]);
          x = (a[i]-a[0])*x/g;
          a[0] = x*m[0]+a[0];
          m[0] = m[0]*m[i]/g;
          a[0] = (a[0]\%m[0]+m[0])\%m[0];
```

```
if (a[0]<0) return a[0]+m[0];</pre>
      return a[0];
42 }
43
  // ans = a / b (mod m)
  // ans = ret.F + k * ret.S, k is integer
  pair<int, int> div(int a, int b, int m) {
      int flag = 1:
      if (a < 0) { a = -a; flag *= -1; }</pre>
      if (b < 0) { b = -b; flag *= -1; }
      int t = -1, k = -1:
      int res = extgcd_abc(b, m, a, t, k);
      if (res == INF) return {INF, INF};
      m = abs(m / res);
      t = t * flag;
      t = (t \% m + m) \% m;
      return {t, m};
```

# 7.2 Josephus Problem [e0ed50]

```
1  // 有 n 個人·第偶數個報數的人被刪掉·問第 k 個被踢掉的是誰
int solve(int n, int k){
    if (n=1) return 1;
    if (k<=(n+1)/2){
        if (2*k>n) return 2*k%n;
        else return 2*k;
    }else{
        int res=solve(n/2, k-(n+1)/2);
        if (n&1) return 2*res+1;
        else return 2*res-1;
    }
}
```

#### 7.3 Lagrange any x [1f2c26]

```
1 / /  init: (x1, y1), (x2, y2) in a vector
   struct Lagrange{
       int n;
       vector<pair<int, int>> v;
       Lagrange(vector<pair<int, int>> & v){
           n = _v.size();
           v = v;
       // O(n^2 \log MAX A)
       int solve(int x){
           int ret = 0;
           for (int i=0 ; i<n ; i++){</pre>
                int now = v[i].second;
15
                for (int j=0 ; j<n ; j++){</pre>
                    if (i==j) continue;
17
18
                    now *= ((x-v[j].first)+MOD)%MOD;
19
                    now %= MOD;
20
                    now *= (qp((v[i].first-v[j].first+MOD)%MOD,
                         MOD - 2) + MOD) % MOD;
21
                    now %= MOD;
23
24
                ret = (ret+now)%MOD;
25
26
           return ret:
27
28 };
```

7.4 Lagrange continuous x [57536a]

for (int k=0 ; k<m ; k++){</pre>

```
int up = now * mul[i - 1] % mod;
                                                                                                                                                            ret.arr[i][j] += arr[i][k]*b.arr[k][j]%
1 #include <bits/stdc++.h>
                                                                                 int down = inv fac[m - i] * inv fac[i - 1] % mod;
                                                                                 int tmp = ((long long)v[i - 1] * up % mod) * down 21
                                                                                                                                                            ret.arr[i][j] %= MOD;
  using namespace std;
                                                                                 ret += (neg && tmp) ? (mod - tmp) : (tmp);
  const int MAX N = 5e5 + 10;
                                                                                                                                     23
  const int mod = 1e9 + 7;
                                                                                 ret = (ret >= mod) ? (ret - mod) : ret;
                                                                                                                                     24
                                                                                 now = now * (x - i) % mod:
                                                                                                                                     25
                                                                                                                                                return ret:
  long long inv_fac[MAX_N];
                                                                                 neg ^= 1;
                                                                                                                                     27
  inline int fp(long long x, int y) {
                                                                             return ret;
                                                                                                                                            Matrix pow(int p){
                                                                                                                                                Matrix ret(n, n), mul = *this;
      int ret = 1;
                                                                                                                                                for (int i=0 ; i<n ; i++){</pre>
      for (; y; y >>= 1) {
                                                                     };
                                                                                                                                                    ret.arr[i][i] = 1;
          ret = (y & 1) ? (ret * x % mod) : ret;
          x = x * x % mod;
                                                                     int main() {
                                                                         int n; cin >> n;
                                                                         vector<int> v(n);
      return ret;
                                                                                                                                                for ( ; p ; p>>=1){
                                                                         for (int i = 0; i < n; ++i) {</pre>
                                                                                                                                                    if (p&1) ret = ret*mul;
                                                                             cin >> v[i];
                                                                                                                                                    mul = mul*mul:
  // TO USE THIS TEMPLATE, YOU MUST MAKE SURE THAT THE MOD
                                                                         Lagrange L;
       NUMBER IS A PRIME.
                                                                         L.construct_inv_fac();
19 struct Lagrange {
                                                                                                                                                return ret;
                                                                         L.init(0, v);
      Initialize a polynomial with f(x_0), f(x_0 + 1), ..., f(x_0 + 1)
                                                                         int x; cin >> x;
                                                                         cout << L.sample(x);</pre>
      This determines a polynomial f(x) whose degree is at most 83 }
                                                                                                                                           int det(){
                                                                                                                                                vector<vector<int>> arr = this->arr;
                                                                                                                                               bool flag = false;
      Then you can call sample(x) and you get the value of f(x)
                                                                     7.5 Linear Mod Inverse [ecf71e]
                                                                                                                                                for (int i=0 ; i<n ; i++){</pre>
                                                                                                                                                    int target = -1;
      Complexity of init() and sample() are both O(n).
                                                                     // 線性求 1-based a[i] 對 p 的乘法反元素
                                                                                                                                                    for (int j=i ; j<n ; j++){</pre>
                                                                     vector<int> s(n+1, 1), invS(n+1), invA(n+1);
                                                                                                                                                        if (arr[j][i]){
      int m, shift; // m = n + 1
                                                                     for (int i=1; i<=n; i++) s[i] = s[i-1]*a[i]%p;
      vector<int> v, mul;
                                                                                                                                                            target = j;
                                                                     invS[n] = qp(s[n], p-2, p);
     You can use this function if you don't have inv fac array
                                                                                                                                                            break;
                                                                     for (int i=n; i>=1; i--) invS[i-1] = invS[i]*a[i]%p;
                                                                   6 for (int i=1; i<=n; i++) invA[i] = invS[i]*s[i-1]%p;</pre>
      void construct inv fac() {
          long long fac = 1;
                                                                                                                                                    if (target==-1) return 0;
                                                                                                                                                    if (i!=target){
          for (int i = 2; i < MAX_N; ++i) {</pre>
                                                                     7.6 Lucas's Theorem [b37dcf]
                                                                                                                                     55
                                                                                                                                                        swap(arr[i], arr[target]);
              fac = fac * i % mod;
                                                                                                                                                        flag = !flag;
                                                                   1 \mid // 對於很大的 C^n_{m} 對質數 p 取模,只要 p 不大就可以用。
          inv_fac[MAX_N - 1] = fp(fac, mod - 2);
                                                                   int Lucas(int n, int m, int p){
          for (int i = MAX N - 1; i >= 1; --i) {
                                                                         if (m==0) return 1;
                                                                                                                                                    for (int j=i+1; j<n; j++){</pre>
              inv fac[i - 1] = inv fac[i] * i % mod;
                                                                         return (C(n%p, m%p, p)*Lucas(n/p, m/p, p)%p);
                                                                                                                                                        if (!arr[j][i]) continue;
                                                                                                                                                        int freq = arr[j][i]*qp(arr[i][i], MOD-2)%MOD
  // You call init() many times without having a second
                                                                     7.7 Matrix [8d1a23]
                                                                                                                                                        for (int k=i ; k<n ; k++){</pre>
       instance of this struct.
                                                                                                                                                            arr[i][k] -= freq*arr[i][k];
      void init(int X 0, vector<int> &u) {
                                                                                                                                                            arr[j][k] = (arr[j][k]%MOD+MOD)%MOD;
                                                                   1 | struct Matrix{
          shift = ((1 - X_0) \% mod + mod) \% mod;
                                                                         int n, m;
          if (v.size() == 1) v.push back(v[0]);
                                                                         vector<vector<int>> arr:
          m = v.size();
          mul.resize(m):
                                                                         Matrix(int _n, int _m){
                                                                                                                                               int ret = !flag ? 1 : MOD-1;
                                                                             n = n;
  // You can use sample(x) instead of sample(x \% mod).
                                                                                                                                                for (int i=0 ; i<n ; i++){</pre>
      int sample(int x) {
                                                                             arr.assign(n, vector<int>(m));
                                                                                                                                                    ret *= arr[i][i];
          x = ((long long)x + shift) % mod;
                                                                                                                                                    ret %= MOD;
          x = (x < 0) ? (x + mod) : x;
                                                                         vector<int> & operator [] (int i){
          long long now = 1;
                                                                                                                                     75
                                                                                                                                                return ret;
          for (int i = m; i >= 1; --i) {
                                                                             return arr[i];
                                                                                                                                     76
              mul[i - 1] = now;
              now = now * (x - i) % mod;
                                                                         Matrix operator * (Matrix b){
                                                                                                                                       7.8 Matrix 01 [8d542a]
          int ret = 0:
                                                                             Matrix ret(n, b.m);
          bool neg = (m - 1) & 1;
                                                                             for (int i=0 ; i<n ; i++){</pre>
          now = 1;
                                                                                 for (int j=0 ; j<b.m ; j++){</pre>
                                                                                                                                     1 const int MAX_N = (1LL<<12);</pre>
```

for (int i = 1; i <= m; ++i) {</pre>

x = modmul(x, x, n);

if (res.a[i] >= mod) res.a[i] -= mod;

```
2 | struct Matrix{
                                                                                 if (x==1) return 0;
                                                                                                                                            void resize(int n) {
      int n, m;
                                                                                 if (x==n-1) break;
                                                                                                                                                int len1 = 1;
      vector<bitset<MAX N>> arr:
                                                                                                                                                while (len1 < n) len1 <<= 1;</pre>
                                                                                                                                                int *res = (ll*) calloc(len1, sizeof(ll));
      Matrix(int _n, int _m){
                                                                             if (x==n-1) continue;
                                                                                                                                                for (int i = 0; i < min(len, _n); i++) {</pre>
                                                                             return false;
                                                                                                                                                    res[i] = a[i];
          n = n;
          m = _m;
          arr.resize(n);
                                                                                                                                                len = len1:
                                                                         return true;
                                                                                                                                                deg = n - 1;
                                                                  43 }
                                                                                                                                                free(a);
      Matrix operator * (Matrix b){
                                                                                                                                                a = res:
          Matrix b t(b.m, b.n);
                                                                                                                                     28
                                                                     7.10 Pollard Rho [a5daef]
          for (int i=0 ; i<b.n ; i++){</pre>
                                                                                                                                            Poly& operator=(const Poly rhs) {
              for (int j=0 ; j<b.m ; j++){</pre>
                                                                                                                                                this->len = rhs.len;
                                                                   i | mt19937 seed(chrono::steady clock::now().time since epoch().
                  b_t.arr[j][i] = b.arr[i][j];
                                                                                                                                                this->deg = rhs.deg;
                                                                                                                                                this->a = (11*)realloc(this->a, sizeof(11) * len);
                                                                          count());
                                                                     int rnd(int 1, int r){
                                                                                                                                                copy(rhs.a, rhs.a + len, this->a);
                                                                         return uniform_int_distribution<int>(1, r)(seed);
                                                                                                                                                return *this:
                                                                                                                                     34
          Matrix ret(n, b.m);
                                                                                                                                     35
          for (int i=0 ; i<n ; i++){</pre>
                                                                                                                                            Poly operator*(Poly rhs) {
                                                                                                                                     36
                                                                                                                                                int l1 = this->len, l2 = rhs.len;
              for (int j=0 ; j<b.m ; j++){</pre>
                                                                     // O(n^{1/4}) 回傳 1 或自己的因數、記得先判斷 n 是不是質數
                                                                                                                                     37
                                                                                                                                                int d1 = this->deg, d2 = rhs.deg;
                  ret.arr[i][j] = ((arr[i]&b_t.arr[j]).count()
                                                                          (用 Miller-Rabin)
                                                                                                                                                while (11 > 0 and this->a[11 - 1] == 0) 11--;
                                                                    // c1670c
                                                                                                                                                while (12 > 0 \text{ and } rhs.a[12 - 1] == 0) 12--:
                                                                   8 int Pollard Rho(int n){
                                                                         int s = 0, t = 0;
                                                                                                                                                int 1 = 1;
          return ret;
                                                                                                                                                while (1 < max(11 + 12 - 1, d1 + d2 + 1)) 1 <<= 1;
                                                                         int c = rnd(1, n-1);
                                                                                                                                                int *x, *y, *res;
                                                                                                                                                x = (11*) calloc(1, sizeof(11));
28 };
                                                                         int step = 0, goal = 1;
                                                                                                                                                y = (11*) calloc(1, sizeof(11));
                                                                         int val = 1:
                                                                                                                                                res = (11*) calloc(1, sizeof(11));
  7.9 Miller Rabin [24bd0d]
                                                                                                                                                copy(this->a, this->a + 11, x);
                                                                         for (goal=1;; goal<<=1, s=t, val=1){</pre>
                                                                                                                                                copy(rhs.a, rhs.a + 12, y);
                                                                             for (step=1 ; step<=goal ; step++){</pre>
                                                                                                                                                ntt.tran(1, x); ntt.tran(1, y);
1 // O(k \log^3 n), k = llsprp.size()
2 typedef Uint unsigned long long;
                                                                                 t = ((__int128)t*t+c)%n;
                                                                                                                                                FOR (i, 0, 1 - 1)
                                                                                                                                                    res[i] = x[i] * y[i] % mod;
3 Uint modmul(Uint a, Uint b, Uint m) {
                                                                                 val = ( int128)val*abs(t-s)%n;
                                                                                                                                                ntt.tran(1, res, true);
      int ret = a*b - m*(Uint)((long double)a*b/m);
      return ret + m*(ret < 0) - m*(ret>=(int)m);
                                                                                                                                                free(x); free(y);
                                                                                 if ((step % 127) == 0){
                                                                                                                                                return Poly(1, d1 + d2, res);
                                                                                      int d = __gcd(val, n);
                                                                                     if (d>1) return d;
                                                                                                                                            Poly operator+(Poly rhs) {
 int qp(int b, int p, int m){
                                                                                                                                                int 11 = this->len, 12 = rhs.len;
      int ret = 1;
                                                                                                                                                int 1 = \max(11, 12);
      for ( ; p ; p>>=1){
                                                                                                                                                Poly res;
          if (p&1) ret = modmul(ret, b, m);
                                                                             int d = __gcd(val, n);
                                                                                                                                                res.len = 1:
          b = modmul(b, b, m);
                                                                             if (d>1) return d;
                                                                                                                                                res.deg = max(this->deg, rhs.deg);
                                                                                                                                                res.a = (11*) calloc(1, sizeof(11));
      return ret;
                                                                                                                                                FOR (i, 0, 11 - 1) {
                                                                                                                                                    res.a[i] += this->a[i];
                                                                    7.11 Polynomial [51ca3b]
                                                                                                                                                    if (res.a[i] >= mod) res.a[i] -= mod;
18 vector (int > 11sprp = {2, 325, 9375, 28178, 450775, 9780504,
       1795265022};
                                                                   1 struct Poly {
                                                                                                                                                FOR (i, 0, 12 - 1) {
                                                                         int len, deg;
                                                                                                                                                    res.a[i] += rhs.a[i];
19 bool is prime(int n, vector(int) sprp = llsprp){
      if (n==2) return 1:
                                                                                                                                                    if (res.a[i] >= mod) res.a[i] -= mod;
      if (n<2 || n%2==0) return 0;
                                                                         // len = 2^k >= the original length
                                                                         Poly(): len(0), deg(0), a(nullptr) {}
                                                                                                                                                return res;
                                                                         Poly(int n) {
      int t = 0:
                                                                                                                                            Poly operator - (Poly rhs) {
      int u = n-1;
                                                                             len = 1;
      for ( ; u%2==0 ; t++) u>>=1;
                                                                             deg = _n - 1;
                                                                                                                                                int 11 = this->len, 12 = rhs.len;
                                                                             while (len < n) len <<= 1;</pre>
                                                                                                                                                int 1 = \max(11, 12);
      for (int i=0 ; i<sprp.size() ; i++){</pre>
                                                                             a = (ll*) calloc(len, sizeof(ll));
                                                                                                                                                Polv res:
                                                                                                                                                res.len = 1;
          int a = sprp[i]%n;
                                                                                                                                                res.deg = max(this->deg, rhs.deg);
          if (a==0 || a==1 || a==n-1) continue;
                                                                         Poly(int 1, int d, int *b) {
                                                                                                                                                res.a = (ll*) calloc(l, sizeof(ll));
          int x = qp(a, u, n);
                                                                             len = 1;
                                                                                                                                     79
          if (x==1 || x==n-1) continue;
                                                                             deg = d;
                                                                                                                                                FOR (i, 0, 11 - 1) {
                                                                                                                                     80
                                                                                                                                                    res.a[i] += this->a[i]:
          for (int j=0 ; j<t ; j++){</pre>
                                                                             a = b;
```

```
FOR (i, 0, 12 - 1) {
                res.a[i] -= rhs.a[i];
                if (res.a[i] < 0) res.a[i] += mod;</pre>
            return res:
       Poly operator*(const int rhs) {
            Polv res:
            res = *this:
            FOR (i, 0, res.len - 1) {
                res.a[i] = res.a[i] * rhs % mod;
                if (res.a[i] < 0) res.a[i] += mod;</pre>
           return res;
       Poly(vector<int> f) {
            int _n = f.size();
101
            len = 1:
102
            deg = _n - 1;
            while (len < _n) len <<= 1;</pre>
103
           a = (11*) calloc(len, sizeof(11));
            FOR (i, 0, deg) a[i] = f[i];
105
106
107
       Poly derivative() {
108
            Poly g(this->deg);
109
            FOR (i, 1, this->deg) {
                g.a[i - 1] = this - >a[i] * i % mod;
111
112
            return g;
113
       Poly integral() {
114
            Poly g(this->deg + 2);
115
            FOR (i, 0, this->deg) {
                g.a[i + 1] = this - a[i] * ::inv(i + 1) % mod;
117
118
            return g;
119
120
       Poly inv(int len1 = -1) {
121
            if (len1 == -1) len1 = this->len;
122
            Poly g(1); g.a[0] = ::inv(a[0]);
123
            for (int 1 = 1; 1 < len1; 1 <<= 1) {
124
125
                Poly t; t = *this;
                t.resize(1 << 1);
126
                t = g * g * t;
127
                t.resize(1 << 1);
128
                Poly g1 = g * 2 - t;
129
                swap(g, g1);
130
131
132
            return g;
133
       Poly ln(int len1 = -1) {
134
            if (len1 == -1) len1 = this->len;
135
            auto g = *this;
136
           auto x = g.derivative() * g.inv(len1);
137
138
           x.resize(len1);
139
           x = x.integral();
140
           x.resize(len1);
           return x;
141
142
       Poly exp() {
143
144
            Poly g(1);
            g.a[0] = 1;
145
            for (int 1 = 1; 1 < len; 1 <<= 1) {</pre>
146
                Poly t, g1; t = *this;
147
                t.resize(1 << 1); t.a[0]++;
```

```
g1 = (t - g.ln(1 << 1)) * g;
                g1.resize(1 << 1);
151
                swap(g, g1);
153
           return g;
154
       Poly pow(ll n) {
155
           Poly &a = *this;
156
           int i = 0:
157
158
            while (i <= a.deg and a.a[i] == 0) i++;
           if (i and (n > a.deg or n * i > a.deg)) return Poly(a
                .deg + 1);
            if (i == a.deg + 1) {
160
                Poly res(a.deg + 1);
                res.a[0] = 1;
162
                return res;
           Poly b(a.deg - i + 1);
           int inv1 = ::inv(a.a[i]);
            FOR (j, 0, b.deg)
                b.a[j] = a.a[j + i] * inv1 % mod;
            Poly res1 = (b.ln() * (n % mod)).exp() * (::power(a.a))
            Poly res2(a.deg + 1);
170
           FOR (j, 0, min((ll)(res1.deg), (ll)(a.deg - n * i)))
171
                res2.a[j + n * i] = res1.a[j];
172
173
            return res2;
174
175 };
   7.12 josephus [0be067]
```

#### 7.13 數論分塊 [8ccab5]

# 7.14 最大質因數 [ca5e52]

```
void max_fac(int n, int &ret){
if (n<=ret || n<2) return;</pre>
```

```
if (isprime(n)){
    ret = max(ret, n);
    return;
}
int p = Pollard_Rho(n);
    max_fac(p, ret), max_fac(n/p, ret);
}
```

# 7.15 歐拉公式 [85f3b1]

```
1 | // phi(n) = 小於 n 並與 n 互質的正整數數量。
2 // O(sqrt(n)) · 回傳 phi(n)
  int phi(int n){
      int ret = n;
      for (int i=2 ; i*i<=n ; i++){</pre>
          if (n%i==0){
              while (n%i==0) n /= i;
              ret = ret*(i-1)/i:
      if (n>1) ret = ret*(n-1)/n;
      return ret;
  // O(n Log n) · 回傳 1~n 的 phi 值
  vector<int> phi 1 to n(int n){
      vector<int> phi(n+1);
      phi[0]=0;
      phi[1]=1;
      for (int i=2 ; i<=n ; i++){</pre>
          phi[i]=i-1;
26
      for (int i=2 ; i<=n ; i++){</pre>
          for (int j=2*i ; j<=n ; j+=i){ // 枚舉所有倍數
              phi[j]-=phi[i];
      }
      return phi;
```

#### 7.16 Burnside's Lemma

$$\sum_{k=1}^{n} \frac{c(k)}{n}$$

- n:有多少種置換方式(例如:旋轉方式)
- c(k): 所有可能中,經過 k 次旋轉後,仍不會和別人相同的方式的數量

# 7.17 Catalan Number

任意括號序列:  $C_n = \frac{1}{n+1} \binom{2n}{n}$ 

#### 7.18 Matrix Tree Theorem

目標:給定一張無向圖·問他的生成樹數量。 方法:先把所有自環刪掉·定義 Q 為以下矩陣

$$Q_{i,j} = \begin{cases} \deg(v_i) & \text{if } i = j \\ -(\frac{1}{2}v_iv_j) & \text{otherwise} \end{cases}$$

接著刪掉 Q 的第一個 row 跟 column·它的 determinant 就是答案。 目標:給定一張有向圖·問他的以 r 為根·可以走到所有點生成樹數量。

方法: 先把所有自環刪掉, 定義 Q 為以下矩陣

$$Q_{i,j} = \begin{cases} \deg_{in}(v_i) & \text{if } i = j \\ -(邊v_iv_j \text{ 的數量}) & \text{otherwise} \end{cases}$$

接著刪掉 Q 的第 r 個 row 跟 column · 它的 determinant 就是答案。

# 7.19 Stirling's formula

 $n! \approx \sqrt{2\pi n} (\tfrac{n}{e})^n$ 

#### 7.20 Theorem

- 1.  $1 \sim x$  質數的數量  $\approx \frac{x}{\ln x}$
- 2. x 的因數的數量  $\approx x^{\frac{1}{3}}$
- 3. x 的質因數的數量  $\approx \log \log x$
- 4. p is a prime number  $\Leftrightarrow (p-1)! \equiv -1 \pmod{p}$
- 5. 每個正整數都可以表示成四個整數的平方和
- 6. 任何大於 2 的整數都可以表示成兩個質數的和
- 7.  $n^{k-2} \cdot \prod_{i=1}^k s_i$  n 個點  $\cdot$  k 的連通塊  $\cdot$  加上 k-1 條邊使得變成一個連通圖的方法數  $\cdot$  其中每個連通塊有  $s_i$  個點

## 7.21 二元一次方程式

$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases} = \begin{cases} x = \frac{ed - bf}{ad - bc} \\ y = \frac{af - ec}{ad - bc} \end{cases}$$

若 $x = \frac{0}{0}$ 且 $y = \frac{0}{0}$ .則代表無限多組解。若 $x = \frac{*}{0}$ 且 $y = \frac{*}{0}$ .則代表無解。

#### 7.22 歐拉定理

若a, m互質,則:

$$a^n \equiv a^{n \mod \varphi(m)} \pmod{m}$$

若a, m不互質,則:

$$a^n \equiv a^{\varphi(m) + [n \mod \varphi(m)]} \pmod{m}$$

## 7.23 錯排公式

錯排公式:  $(n \oplus 1)$  個人中,每個人皆不再原來位置的組合數)

$$dp_i = \begin{cases} 1 & i = 0\\ 0 & i = 1\\ (i-1)(dp_{i-1} + dp_{i-2}) & \text{otherwise} \end{cases}$$

# 8 String

#### 8.1 AC automation [018290]

```
struct ACAutomation{
    vector<vector<int>> go:
    vector<int> fail, match, pos;
    int sz = 0; // 有效節點為 [0, sz] · 開陣列的時候要小
    ACAutomation(int n) : go(n, vector<int>(26)), fail(n),
    void add(string s){
        int now = 0:
        for (char c : s){
            if (!go[now][c-'a']) go[now][c-'a'] = ++sz;
            now = go[now][c-'a'];
        pos.push back(now);
    void build(){
        queue<int> que;
        for (int i=0 ; i<26 ; i++){</pre>
            if (go[0][i]) que.push(go[0][i]);
        while (que.size()){
            int u = que.front();
            que.pop();
            for (int i=0 ; i<26 ; i++){</pre>
                if (go[u][i]){
                    fail[go[u][i]] = go[fail[u]][i];
                    que.push(go[u][i]);
                }else go[u][i] = go[fail[u]][i];
   }
    // counting pattern
    void buildMatch(string &s){
        int now = 0;
        for (char c : s){
            now = go[now][c-'a'];
            match[now]++;
        vector<int> in(sz+1), que;
        for (int i=1 ; i<=sz ; i++) in[fail[i]]++;</pre>
        for (int i=1; i<=sz; i++) if (in[i]==0) que.
             push_back(i);
        for (int i=0 ; i<que.size() ; i++){</pre>
            int now = que[i];
            match[fail[now]] += match[now];
            if (--in[fail[now]]==0) que.push back(fail[now]);
8.2 Enumerate Runs [94ca46]
```

2 vector(array(int, 3>> enumerate run(string s){

int n = s.size();

```
SuffixArray sa(s), saBar(string(s.rbegin(), s.rend()));
sa.init lcp(), saBar.init lcp();
set<pair<int, int>> ss;
vector<array<int, 3>> runs;
for (int len=1 ; len<=n ; len++){</pre>
    vector<int> lcp:
    for (int i=0 ; i+len<n ; i+=len){</pre>
        int pos1 = sa.pos[i];
        int pos2 = sa.pos[i+len];
        lcp.push_back(sa.get_lcp(pos1, pos2));
    for (int ll=0, rr=0; ll<lcp.size(); rr++, ll=rr){</pre>
        while (rr<lcp.size() && lcp[rr]>=len) rr++;
        int preLen = 0;
        if (11!=0){
            int p = n-1;
            int pos1 = saBar.pos[p-(ll*len-1)];
            int pos2 = saBar.pos[p-((11+1)*len-1)];
            preLen = saBar.get_lcp(pos1, pos2);
        int sufLen = rr<lcp.size() ? lcp[rr] : 0;</pre>
        int ansL = ll*len-preLen, ansR = (rr+1)*len-1+
             sufLen:
        if (ansL!=ansR && ansR-ansL+1>=2*len && ss.find({
             ansL, ansR+1})==ss.end()){
            ss.insert({ansL, ansR+1});
            runs.push_back({len, ansL, ansR+1});
return runs;
```

#### 8.3 Hash [942f42]

```
i | mt19937 seed(chrono::steady clock::now().time since epoch().
      count());
 int rng(int 1, int r){
     return uniform int distribution<int>(1, r)(seed);
 int A = rng(1e5, 8e8):
 const int B = 1e9+7;
 // 2f6192
 struct RollingHash{
     vector<int> Pow, Pre:
     RollingHash(string s = ""){
         Pow.resize(s.size());
         Pre.resize(s.size());
         for (int i=0 ; i<s.size() ; i++){</pre>
             if (i==0){
                 Pow[i] = 1;
                 Pre[i] = s[i];
             }else{
                 Pow[i] = Pow[i-1]*A%B;
                 Pre[i] = (Pre[i-1]*A+s[i])%B;
```

```
if (x==y) return s.size()-x;
         return:
                                                                      }
                                                                                                                                          if (x>y) swap(x, y);
                                                                                                                                           return st.query(x, y);
                                                                      return a;
     int get(int 1, int r){ // 取得 [l, r] 的數值
         if (l==0) return Pre[r];
                                                                                                                                      // 回傳 [l1, r1] 跟 [l2, r2] 的 lcp·0-based
         int res = (Pre[r]-Pre[1-1]*Pow[r-1+1])%B;
                                                                                                                                      int get lcp(int l1, int r1, int l2, int r2){
                                                                  8.7 Suffix Array [f66629]
                                                                                                                                          int pos 1 = pos[11], len 1 = r1-11+1;
         if (res<0) res += B:
         return res;
                                                                                                                                          int pos_2 = pos[12], len_2 = r2-12+1;
                                                                1 // 注意,當 /s/=1 時, Lcp 不會有值,務必測試 /s/=1 的 case
                                                                                                                                          if (pos 1>pos 2){
                                                                2 struct SuffixArray {
                                                                                                                                               swap(pos_1, pos_2);
                                                                      string s:
                                                                                                                                               swap(len 1, len 2);
                                                                      vector<int> sa, lcp;
 8.4 KMP [7b95d6]
                                                                      // 69ced9
                                                                                                                                          if (11==12){
                                                                      // Lim 要調整成字元集大小· s 不可以有 0
ı|// KMP[i] = s[0...i] 的最長共同前後綴長度·KMP[KMP[i]-1] 可以
                                                                                                                                               return min(len 1, len 2);
                                                                      SuffixArray(string _s, int lim = 256) {
      跳 fail link
                                                                          s = _s;
2 // e5b7ce
                                                                                                                                               return min({st.query(pos 1, pos 2), len 1, len 2
                                                                          int n = s.size()+1, k = 0, a, b;
3 vector<int> KMP(string &s){
                                                                          vector<int> x(s.begin(), s.end()), y(n), ws(max(n,
     vector<int> ret(n);
                                                                               lim)), rank(n);
     for (int i=1; i<s.size(); i++){</pre>
                                                                          x.push back(0);
         int j = ret[i-1];
                                                                                                                                74
                                                                          sa = 1cp = v;
         while (j && s[i]!=s[j]) j = ret[j-1];
                                                                                                                                      // 檢查 [L1, r1] 跟 [L2, r2] 的大小關係 · 0-based
                                                                          iota(sa.begin(), sa.end(), 0);
         ret[i] = j + (s[i] = s[j]);
                                                                                                                                      // 如果前者小於後者,就回傳 <0,相等就回傳 =0,否則回傳
                                                                          for (int j=0, p=0; p<n; j=max(1LL, j*2), lim=p) {</pre>
                                                                              p = i;
     return ret;
                                                                                                                                       // 5h8dh0
                                                                              iota(y.begin(), y.end(), n-j);
                                                                              for (int i=0; i< n; i++) if (sa[i] >= j) v[p++]
                                                                                                                                      int substring cmp(int l1, int r1, int l2, int r2){
                                                                                                                                          int len 1 = r1-l1+1;
                                                                                   = sa[i] - j;
 8.5 Manacher [9a4b4d]
                                                                                                                                          int len 2 = r2-12+1;
                                                                              fill(ws.begin(), ws.end(), 0);
                                                                                                                                          int res = get lcp(l1, r1, l2, r2);
                                                                              for (int i=0 ; i<n ; i++) ws[x[i]]++;</pre>
| string Manacher(string str) {
                                                                              for (int i=1 ; i<lim ; i++) ws[i] += ws[i - 1];</pre>
                                                                                                                                82
     string tmp = "$#";
                                                                                                                                          if (res<len 1 && res<len 2){</pre>
                                                                              for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
     for(char i : str) {
                                                                                                                                               return s[l1+res]-s[l2+res];
                                                                              swap(x, y), p = 1, x[sa[0]] = 0;
         tmp += i;
                                                                              for (int i=1; i<n; i++){
                                                                                                                                           }else if (len 1==res && len 2==res){
         tmp += '#';
                                                                                                                                              return 0:
                                                                                 a = sa[i - 1];
                                                                                                                                           }else{
                                                                                  b = sa[i];
                                                                                                                                               return len_1==res ? -1 : 1;
                                                                                  x[b] = (y[a] == y[b] && y[a + j] == y[b + j])
     vector<int> p(tmp.size(), 0);
                                                                                       ? p - 1 : p++;
     int mx = 0, id = 0, len = 0, center = 0;
     for(int i=1; i<(int)tmp.size(); i++) {</pre>
                                                                          }
         p[i] = mx > i ? min(p[id*2-i], mx-i) : 1;
                                                                                                                                      // 對於位置在 <=p 的後綴·找離他左邊/右邊最接近位置 >p 的
                                                                          for (int i=1; i<n; i++) rank[sa[i]] = i;</pre>
                                                                                                                                           後綴的 Lcp · 0-based
         while(tmp[i+p[i]] == tmp[i-p[i]]) p[i]++;
                                                                          for (int i=0, j ; i<n-1 ; lcp[rank[i++]]=k)</pre>
                                                                                                                                      // pre[i] = s[i] 離他左邊最接近位置 >p 的後綴的 Lcp · 0-
         if(mx<i+p[i]) mx = i+p[i], id = i;</pre>
                                                                              for (k && k--, j=sa[rank[i]-1] ; i+k<s.size() &&</pre>
         if(len<p[i]) len = p[i], center = i;</pre>
                                                                                   j+k<s.size() && s[i+k]==s[j+k]; k++);
                                                                                                                                      // suf[i] = s[i] 離他右邊最接近位置 >p 的後綴的 Lcp · 0-
                                                                          sa.erase(sa.begin());
     return str.substr((center-len)/2, len-1);
                                                                          lcp.erase(lcp.begin(), lcp.begin()+2);
                                                                                                                                      // da12fa
                                                                      }
                                                                                                                                       pair<vector<int>, vector<int>> get left and right lcp(int
                                                                      // f49583
 8.6 Min Rotation [b24786]
                                                                                                                                           vector<int> pre(p+1);
                                                                      vector<int> pos; // pos[i] = i 這個值在 pos 的哪個地方
                                                                                                                                          vector<int> suf(p+1);
                                                                      SparseTable st;
i int minRotation(string s) {
                                                                      void init lcp(){
     int a = 0, n = s.size();
                                                                                                                                          { // build pre
                                                                                                                                100
                                                                          pos.resize(sa.size());
     s += s;
                                                                                                                                101
                                                                                                                                               int now = 0;
                                                                          for (int i=0 ; i<sa.size() ; i++){</pre>
                                                                                                                                               for (int i=0 ; i<s.size() ; i++){</pre>
                                                                                                                                102
                                                                              pos[sa[i]] = i;
     for (int b=0 ; b<n ; b++){</pre>
                                                                                                                                103
                                                                                                                                                  if (sa[i]<=p){</pre>
         for (int k=0 ; k<n ; k++){</pre>
                                                                                                                                104
                                                                                                                                                       pre[sa[i]] = now;
                                                                          if (lcp.size()){
             if (a+k==b || s[a+k]<s[b+k]){
                                                                                                                                                       if (i<lcp.size()) now = min(now, lcp[i]);</pre>
                                                                                                                                105
                                                                              st.build(lcp);
                 b += max(0LL, k-1);
                                                                                                                                106
                 break:
                                                                                                                                107
                                                                                                                                                       if (i<lcp.size()) now = lcp[i];</pre>
                                                                      }
                                                                                                                                108
             if (s[a+k]>s[b+k]){
                                                                                                                                109
                                                                      // 用之前記得 init
                 a = b:
                                                                                                                                110
                                                                      // 查詢「sa 上的位置」的 x 跟 y 的 lcp
                                                                                                                                           { // build suf
                 break;
```

int get\_lcp(int x, int y){

int len = n-sa[i];
int add = len-prePrefix;

```
int now = 0;
112
113
               for (int i=s.size()-1; i>=0; i--){
                                                                   12
                                                                              if (nowRank+add>=k){
114
                   if (sa[i]<=p){</pre>
                                                                   13
                                                                                  return s.substr(sa[i], prePrefix+k-nowRank);
                       suf[sa[i]] = now;
115
                       if (i-1>=0) now = min(now, lcp[i-1]);
116
                                                                              prePrefix = sa.lcp[i];
117
                   }else{
118
                       if (i-1>=0) now = lcp[i-1];
                                                                              nowRank += add;
                   }
119
                                                                   19 }
120
121
122
123
           return {pre, suf};
124
125 };
  8.8 Wildcard Matching [e93074]
  const int B = 27;
  string p;
 4 for (int i=0; i<B; i++) p += ('a'+i);
 5 p += '*';
 6 vector<vector<double>> res(B);
 7 for (int i=0; i<B; i++){</pre>
       vector<double> ss, tt;
       for (auto x : s) ss.push_back(x==p[i] || x=='*');
       for (auto x : t) tt.push_back(x==p[i] || x=='*');
       reverse(tt.begin(), tt.end());
       res[i] = PolyMul(ss, tt);
13 }
14 for (int i=t.size()-1; i+t.size()-1<res[0].size(); i++){</pre>
       int total = 0;
       for (int j=0 ; j<B-1 ; j++) total += (int)abs(round(res[j</pre>
            ][i]));
       total -= 25*(int)abs(round(res[26][i]));
       cout << (total==t.size());</pre>
  8.9 Z Algorithm [9d559a]
 ı|// z[i] 回傳 s[0...] 跟 s[i...] 的 Lcp, z[0] = 0
 vector<int> z_function(string s){
       vector<int> z(s.size());
       int l = -1, r = -1;
       for (int i=1; i<s.size(); i++){</pre>
           z[i] = i >= r ? 0 : min(r-i, z[i-l]);
           while (i+z[i]<s.size() && s[i+z[i]]==s[z[i]]) z[i]++;</pre>
           if (i+z[i]>r) l=i, r=i+z[i];
       return z;
  8.10 k-th Substring1 [61f66b]
 1|// 回傳 s 所有子字串(完全不同)中,第 k 大的
 2 string k_th_substring(string &s, int k){
       int n = s.size();
       SuffixArray sa(s);
       sa.init_lcp();
       int prePrefix = 0, nowRank = 0;
       for (int i=0 ; i<n ; i++){</pre>
```