CMPEN/EE455: Digital Image Processing I
Fall 2018
Project #3

assigned: 3 October 2018
due: Friday 19 October 2018

reading assignment: Same as HW #4

Filtering using Frequency-Domain Analysis

For the procedures below, you can use the following MATLAB functions:

- $F = \text{fft2}(f, M, N)$: Compute the $M \times N$ DFT (FFT) of $M \times N$ image $f(x, y)$ to give $M \times N$ DFT $F(u, v)$.
- $f = \text{ifft2}(F, M, N)$: Compute $M \times N$ DFT^{-1} (IFFT) of $M \times N$ DFT $F(u, v)$ to give $M \times N$ image $f(x, y)$.

  When suitable P and Q are used, both fft2 and ifft2 are useful for zero-padded quantities.
- $H = \text{lpfilter}(TYPE, M, N, sig)$ (from Gonzalez, Woods, and Eddins's MATLAB-based book): defines an $M \times N$ lowpass filter $H(u, v)$ of the given TYPE ('ideal' or 'gaussian') and cutoff frequency sig. The MATLAB .m file for this function is given on CANVAS along with the supplemental .m file for function dftuv(M, N). Finally, file "mainproj3.m" demonstrates many of the ideas used below.

1. *Basic DFT-based Frequency Analysis:* Let f be the $N \times N$ "checker" image.
   (a) Give figures for $|F(u, v)|$ and $\log(1 + |F(u, v)|)$. Next, modulate $f(x, y)$ by $(-1)^{x+y}$ to give an image $f_m(x, y)$, and give figures for $|F_m(u, v)|$ and $\log(1 + |F_m(u, v)|)$. Finally, set $F(0, 0)$ to zero and compute the inverse DFT to give a new image $g(x, y)$. Give observations on all images. In particular:
   i) What do the log and modulation operations do?
   ii) What is the the observable and analytical difference between g and the original f?
   (b) Using only $N \times N$ operations (i.e., you do NOT zero pad anything!), apply a Gaussian lowpass filter $H(u, v)$ with $sig = 15$ to f and denote $g(x, y)$ as the lowpass-filtered output image.
   i) Use the MATLAB function `mesh` in an appropriate way to make a 3-D plot of $|H(u, v)|$.
   ii) Give figures for $|G(u, v)|$ and $g(x, y)$.
   To display the Fourier-transform magnitude figures above, be sure to use appropriate modulation (i.e., multiply $g(x, y)$ by $(-1)^{x+y}$) and scaling (i.e., plot $\log(1 + |G(u, v)|)$ for $|G(u, v)|$).
   (c) Repeat part (b), but now use appropriate zero padding and compute $P \times Q$ quantities.
   (d) For the results of parts (b-c), what impact does a filter have on the output image? Do you observe wraparound error? Discuss the nature of the zero-padded results.

2. *Filtering a Corrupted Image:* Let $f(x, y)$ be the "lenna" image and create the corrupted image $c(x, y)$:

$$c(x, y) = f(x, y) + 32 \cdot \sin\left(\frac{2\pi 32x}{N}\right) + 32 \cdot \cos\left(\frac{2\pi 32y}{N}\right)$$

   Clearly, $c(x, y)$ will have pixels (x, y) having values outside the 8-bit [0,255] range. So, you must perform all processing with sufficient precision!
   (a) Design a suitable notch filter $H(u, v)$ that when applied to $c(x, y)$ gives an image $g(x, y)$ that nearly resembles the original image $f(x, y)$. A notch filter rejects (i.e., sets to 0) a few specific frequencies while passing all others. You must describe how you designed your filter H by giving analysis to back up your design. *Lecture notes L15-15 → L15-18 are very helpful here!*
   (b) Give suitable pictures of:
   i. $f(x, y)$, $|F(u, v)|$, $c(x, y)$, $|C(u, v)|$
   ii. $|H(u, v)|$, $g(x, y)$, $|G(u, v)|$
   iii. The image and Fourier-transform magnitude of the difference image $(f(x, y) - g(x, y))$.
   You, of course, will need to do appropriate modulation and scaling for plotting the DFT magnitudes.
   (c) Is it possible to completely recover f from c? Why or why not?