CITY UNIVERSITY LONDON

Agents and Multi-Agents Systems
Coursework
Submission deadline: 6th January 2020, 5:00pm

This written assignment consists of 6 tasks which will be released after the lecture(s) they correspond to, so at the end of Week 1, 3, 5, 8, 9 and 10. To complete this assignment, you will need to develop a good understanding of the concepts introduced during the lectures as well as complete your study with further readings (see references given on Moodle). You can work in teams of two people, except for task 6 - see details at the end of this pdf.

Your report should be clear and precise, and you should provide full justifications and/or computations to explain your answers. **An answer with no explanation will not give you any mark. Presentation of the answers will be taken into account in the marking.**

Only submissions through Moodle will be accepted. Please submit your report **in pdf format**. **No late submissions will be accepted.** Don't leave final submission to the last minute.

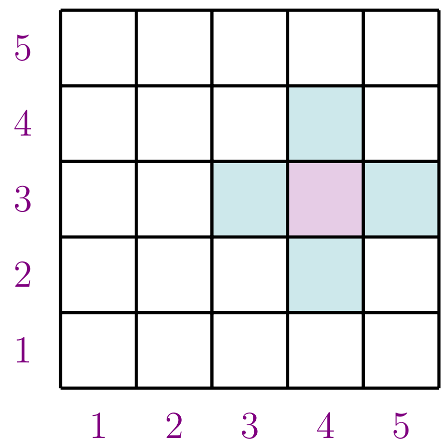
You can get formative feedbacks on preliminary versions of your report - see the Moodle page for info and deadlines.

Tasks

Several companies want to collect underground resources on the same land. Each company sends autonomous robots to explore and collect resources. There are several types of resources, with various monetary values and each company aims at making as much profit as possible.

The land is modelled as a square grid (of dimension $K \times K$). Squares of the grid are called locations and denoted by $(1,1),(1,2),\ldots,(K,K)$, where the first coordinates correspond to the column and the second to the row. We call adjacent locations of a location (ℓ,k) the ones immediately below $(\ell,k-1)$, above $(\ell,k+1)$, on the left $(\ell-1,k)$ and on the right $(\ell+1,k)$ of (ℓ,k). The picture below shows the location

$(4, 3)$ in purple and its four adjacent locations in blue. Note that a location on a corner of the grid has only two adjacent locations and a location on a side only three.



Time is also modelled as a discrete set and durations are expressed as units of time. We start at time 0, the next step is time 1, then time 2... Between time 2 and time 5, 3 units of time have passed.

There are three types of resources r_1, r_2 and r_3. Resources appear randomly in the grid and are destroyed over time (if not collected). Quantities of resources are measured in units.

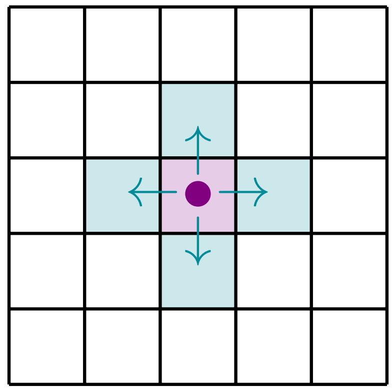
Each location of the land can be as follows:

- be either flat or a hill or a cliff (and only one of them),
- flat locations (and only flat locations) can contain resources: Each location can contain any of the types of resources, each in quantity up to 100 units. For example, location $(4, 3)$, if flat, could contain 93 units of r_1, 42 units of r_2 and no unit of r_3. In each location, one unit of each type of resources is destroyed per unit of time. With the previous example, after one unit of time, location $(4, 3)$ would contain 92 units of r_1, 41 units of r_2 and still no unit of r_3.

Autonomous robots are sent on the land to collect resources. The robots can move to an adjacent location (as in the picture below). This takes one unit of time. The robots only get information when visiting a location but do not have information on locations that have not been visited.

If a robot moves to a location containing a cliff, it will be immediately destroyed. The robots have a special ability which allows them to detect whether a cliff is on an adjacent location, though they do not know which of the adjacent locations contain a cliff. This ability is automatic, they do not have to perform any action to detect whether there are cliffs around.

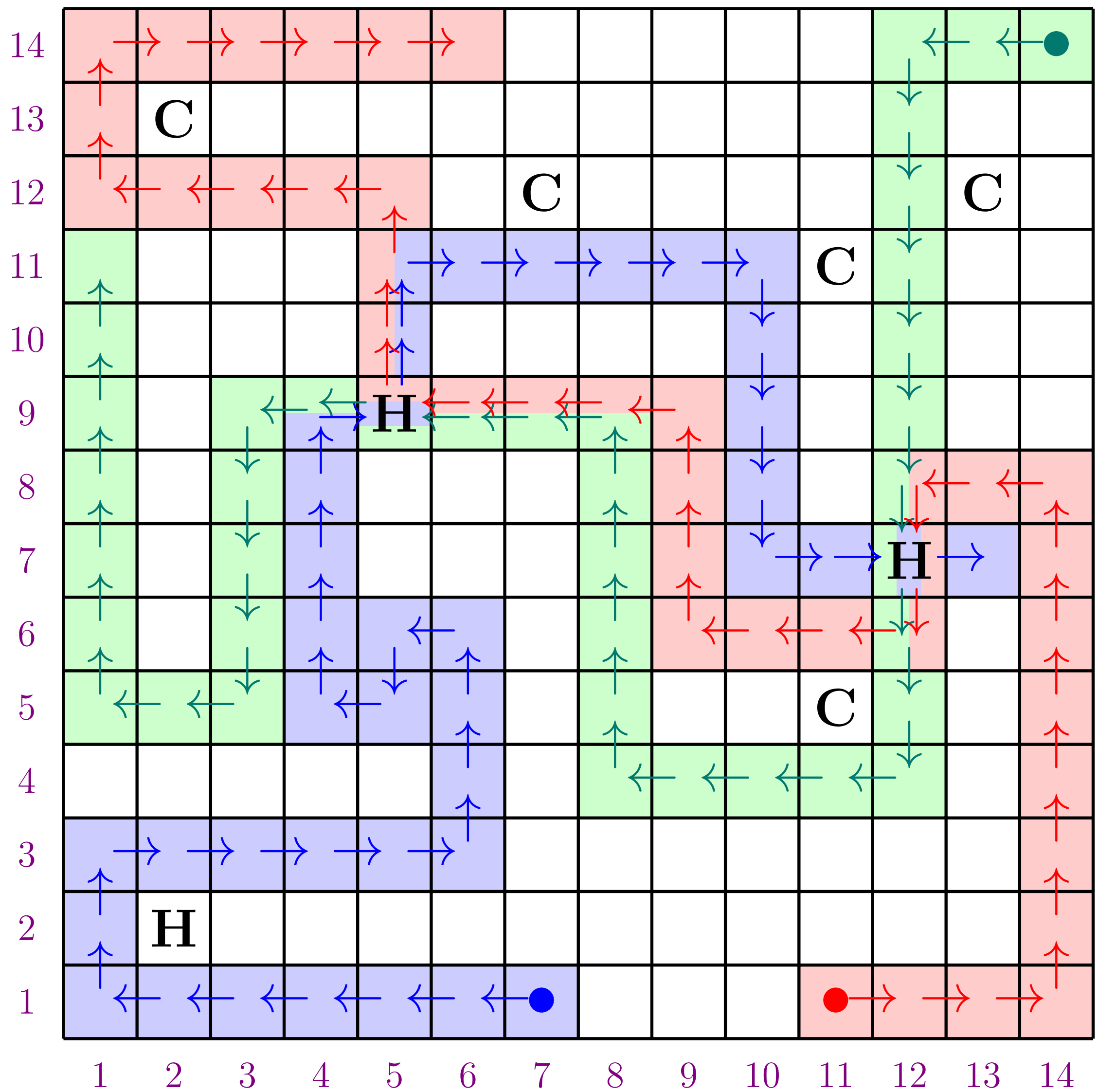
The robots detect the types and quantities of resources contained in the location they are on and can collect them (if on a flat location containing resources).



Task 1

1. What are the states of the environment? How many states are there?
2. Among these characteristics, which ones characterise the environment (and which ones do not)?
   - fully observable
   - partially observable
   - deterministic
   - stochastic
   - static
   - dynamic
   - discrete
   - continuous
3. If you were to design the robots, would you make them reactive, deliberative or hybrid? Why?
4. Consider the following grid, where C represents locations with a cliff, H locations with a hill and that is explored by three robots Red, Blue and Green. We assume that when on the same location at the same time, the robots can detect each other, but cannot communicate with each other.

   The colors represent the paths that robots Red, Blue and Green have followed from time 0 to 38 (at time 0 they were in their departure locations, denoted by a •, at time 1 in the next location,... None of the robots has remained static at any time).



At time 38, among the following facts, which ones are general knowledge, distributed knowledge and/or common knowledge?

F1: There is a cliff on location (2,13).

F2: There is a cliff on location (7,12).

F3: There is a cliff on location (11,11).

F4: There is a cliff on location (11,5).

F5: There is a cliff on location (13,12).

F6: There is a hill on location (2,2).

F7: There is a hill on location (5,9).

F8: There is a hill on location (12,7).

Task 2

For this task, we want to design the robots as logical agents.

1. If you were to use propositional logic to represent the knowledge of the agents, which set of propositional symbols would you choose? Why?

2. What would be the drawbacks and advantages to use propositional logic to model our problem?

3. Using the set of symbols you have chosen in question 1., express the following properties in propositional logic:

   (a) For every location that is a cliff, there is an adjacent location to it that contains some non null quantity of resource r_3.

   (b) For every location that contains some non null quantity of resource r_2, there is exactly one adjacent location that is a hill.

   (c) Resource r_1 can only appear in the corners of the grid (the corners of the grid are the locations $(1,1)$, $(K,1)$, $(1,K)$, (K,K)).

4. Suppose that a robot is on a grid of size 5×5 and knows the following properties:

   (a) for every cliff, at least one of the adjacent locations is flat and contains r_1,

   (b) robot in $(2,2)$ detects a cliff in at least one of the adjacent locations,

   (c) there is no cliff in $(1,2)$ and in $(2,1)$,

   (d) robot in $(4,2)$ does not detect any cliff in any of the adjacent locations,

   (e) there is no resource r_1 in $(1,3)$ and in $(2,2)$,

   (f) either there is no resource r_1 in $(3,3)$ or there is resource r_1 in $(4,3)$.

   Translate these properties into sentences of propositional logic and using the modus ponens, the resolution rule and the and-elimination, show how to deduce from these properties that there is resource r_1 in $(2,4)$ or in $(4,3)$. You can reuse the set of symbols that you chose in question 1. or another one more convenient.

5. Consider the following grid. The symbols C indicates a cliff and r_1 indicates that 100 units of resource r_1 have appeared at time 0. We suppose that no other resource appear later on.

		r_1	C		r_1
					C
	r_1	C			r_1
		r_1	C		
					●

A robot starts in the location marked with • (but does not know where C and r_1 are). Its absolute priority is to not fall from a cliff. Moreover, the robot knows the two following facts:

- For every location that is a cliff, there is an adjacent location that is flat and contains some non null quantity of resource r_1.
- For every location that is flat and contains some non null quantity of resource r_1, exactly one adjacent location is a cliff.

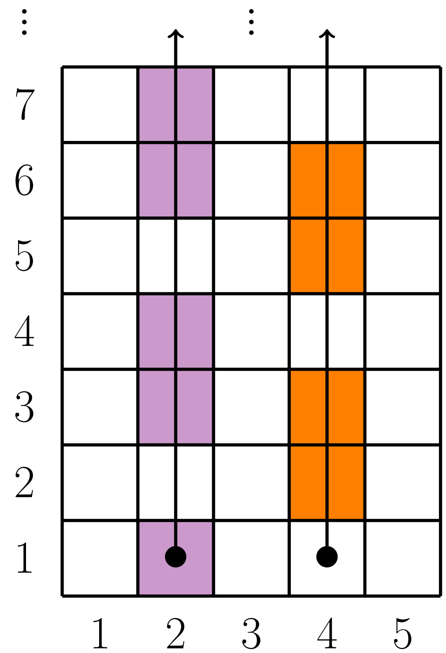
We also remind that the robot can feel if there is a cliff on an adjacent location (but does not know on which location exactly).

What is the maximal quantity of resources that the robot can collect without risking to fall from a cliff?

6. (difficult) Consider the grid below of dimension $5 \times L$. A first robot starts in $(2,1)$ and moves vertically on the grid, while a second robot does similarly starting in $(4,1)$. We assume that there is no hill on the whole grid, and there is neither cliff nor resource r_1 on any of the locations the robots have been through. The locations where robot 1 has detected a cliff around (and only these ones) are pictured in violet: they correspond to the locations of the form $(2, 3i+1)$ and $(2, 3i+3)$ for all $i = 0, 1, 2....$ The locations where robot 2 has detected a cliff around (and only these ones) are pictured in orange: they correspond to the locations of the form $(4, 3i+2)$ and $(4, 3i+3)$ for all $i = 0, 1, 2....$

   Moreover, we have the following properties:

   - For every cliff, there is an adjacent location that contain resource r_1.
   - For every location containing resource r_1, there is at most one adjacent location that is a cliff.



(a) Prove that for all $i = 0, 1, 2...$, there is necessarily a cliff in location $(3, 3i+3)$.

(b) (very difficult) Model the problem in first order logic, and use unifiers and a proof by resolution (using also the modus ponens) to prove it.

Task 3

For this task, we want to use decision theory to design the robots. We assume that the resources can appear in the locations at time $t = 0$ randomly - and if some resource appear in a location, it appears in quantity 100 units - but do not appear later on. For each location (i, j) ($1 \leq i, j \leq K$), and each resource r_k ($k = 1, 2, 3$), we denote the probability that resource r_k appears in location (i, j) at time $t = 0$ in quantity 100 units by $p_{k,i,j}$. We also assume that 1 unit of resource r_1 (resp. r_2, r_3) is worth 20 pounds (resp. 30 pounds, 50 pounds). We remind that the resources disappear over time (if not collected) - see introduction.

1. (a) We first suppose that there is no cliff in the grid. Explain how to design a robot in this setting using decision theory. Be as detailed and precise as you can.

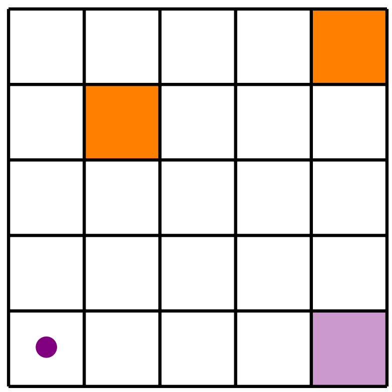
   (b) How would you adapt this design if there were cliffs on the grid (we remind that the robots do not know where the cliffs are but can feel a cliff on an adjacent location), knowing that a robot is worth 30000 pounds.

2. *For the two next questions, take the two last digits of your student's number. Say they are xy. We denote by p the probability $0.xy$.*

Consider the following grid, where a robot is on the location $\bullet$ at time 0 and where:

- on the orange locations, r_1 has probability p to appear, r_2 probability 0.2 and r_3 probability 0.1 at time 0 and in quantity 100 units,
- on the violet location, r_1 has probability 0.5 to appear, r_2 probability p and r_3 probability 0.2 at time 0 and in quantity 100 units.

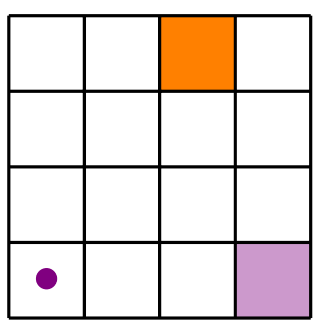
On the other locations, resources have probability 0 to appear.



Which path should the robot follow?

3. Consider the following grid, where a robot is on the location $\bullet$ at time 0 and where:

- on the violet location, r_1 has probability p to appear, r_2 probability 0.2 and r_3 probability x, at time 0 and in quantity 100,
- on the orange location, r_1 has probability 0.7 to appear, r_2 probability x and r_3 probability 0.3 at time 0 and in quantity 100.



For which values of x the robot would start by collecting resources on the orange location?

4. (a bit difficult) For this question, we suppose that the value of one unit of r_1 is 20 pounds, and the value of one unit of r_2 or r_3 is 40 pounds.

   You are going to collect resources on the next location, but you are asked to choose which resource(s) you will collect (before knowing the content of the next location). On the next location, r_1 has probability 0.5 to appear in quantity 100 units when you reach it. Resources r_2 and r_3 have probability 0.75 to appear in total quantity 100 units, but you do not know how many units are of r_2 and how many are of r_3 (but the total is 100 units). You have the two following scenario:

   - Scenario 1: you are asked to choose between collecting resource r_1 or collecting resource r_3.
   - Scenario 2: you are asked to choose between collecting resources r_1 and r_2 or collecting resources r_2 and r_3.

   Explain why, if you prefer to collect resource r_1 in Scenario 1, and prefer to collect r_2 and r_3 in Scenario 2, you would create a paradox.

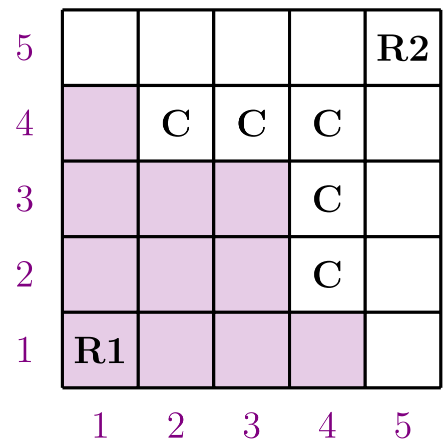
Task 4

1. **Turn-Based Concurrent Games**

   (a) Consider the following grid. Two robots R1 and R2 are on the grid starting respectively in location $(1, 1)$ and $(5, 5)$. The symbol C denotes locations with a cliff and we assume that the robots know where the cliffs are. We also remind that robots cannot move onto a location with a cliff.

   The aim of R1 is to catch R2, i.e. be on the same location as R2 at the same time. The aim of R2 is to reach one of the violet locations (any of them). As soon as one of the robot has reached its aim, the game stops. If both robots reach their aim at the same time, it is a draw. If none of them ever reaches its aim, it is also a draw. The robots take turn: at each turn, they can move to an adjacent location or not move at all. R1 takes the first turn, then R2, then R1...

   Can you prove that R1 has an at least drawing strategy, i.e. has a way to prevent R2 to reach one of the violet locations?



(b) (a bit difficult) We generalise the previous game to any grid (grid as big as we want, with any configurations of cliffs, violet locations and starting points for R1 and R2). The same rules apply: this is a turn-based game and robots are allowed to move to an adjacent location or stay where they are. Robot R1 aims to catch R2 and robot R2 aims to reach a violet locations. Prove that if neither R1 nor R2 has a winning strategy then they both have an at least drawing strategy.

2. **Normal Form Game**

Two robots are trying to decide whether they should exchange some resources. The exchange is modelled as a normal form game: with a single round, robots decide simultaneously which resource they want to give to the other robot. The gain they get from the exchange is given in the picture below: the gains for robot 1 are denoted in blue, the gains for robot 2 are denoted in purple.

Robot 2 gives \ Robot 1 gives	r_1	r_2	r_3
r_1	0 6	3 0	8 1
r_2	4 3	5 1	3 1
r_3	2 7	1 0	3 2

(a) Are there pure Nash equilibria in this game? If so, which ones?

(b) Are there Pareto optimal strategy profiles which are composed with pure strategies? If so, which ones?

(c) (difficult) Is there a Nash equilibrium in this game, which is not a pure Nash equilibrium? If so which ones? *For this question, you might use extra resources. If so, you need to explain fully the computations that you do, and why you do them.*

Task 5

We consider the following grid. Four robots are on the grid, starting in locations marked with $R1$, $R2$, $R3$ and $R4$. There is resource r_1 on the grid in the purple locations, initially in quantity as indicated. We assume that the robots know where r_1 is and in which quantity, and we assume that no other resource appears afterwards. We also remind that one unit of resource in each location is destroyed per unit of time. The aim of each robot is to get as many units of resources as possible. For each location, the robot that is the first to reach it can collect all the resources still in the location. Robots are allowed to form alliances, as long as they agree on how to share the resources. We assume that they can split units of resources to share. For example, if R1 and R2 have 5 resources to share, it would be acceptable that R1 gets 3.5 units and R2 gets 1.5 units.

9									
8		**R1**			11				**R2**
7			7						
6	7								
5			7						
4									
3									
2			7						
1		**R4**			11				**R3**
	1	2	3	4	5	6	7	8	9

1. Model this situation as a coalitional game (i.e. define the players and the surplus). The surplus of each coalition should be the quantity of resource that the coalition can guarantee to get.

2. Which coalition(s) should form? Why?

3. Is the game defined in question 1. balanced?

4. Is the game defined in quesiton 1. convex?

5. Give an example of a coalitional game with five players where exactly two coalitions will form (and such that this is the unique way coalitions will form).

Task 6: This task has to be completed individually.

Five companies are working collaboratively to collect resources and are sharing a robot. They need to decide on a plan of actions. There are five locations that need to be explored and they have to agree on which order the robot should explore them. The locations are denoted by ℓ_1, ℓ_2, ℓ_3, ℓ_4 and ℓ_5.

Each companies have a different plan in mind and they decide to use a voting system to choose an order. The preference orders for each of the companies C_1, C_2, C_3, C_4 and C_5 are given below.

C_1 :	$\ell_3 \succ \ell_5 \succ \ell_2 \succ \ell_1 \succ \ell_4$
C_2 :	$\ell_1 \succ \ell_5 \succ \ell_3 \succ \ell_4 \succ \ell_2$
C_3 :	$\ell_4 \succ \ell_2 \succ \ell_5 \succ \ell_3 \succ \ell_1$
C_4 :	$\ell_4 \succ \ell_1 \succ \ell_2 \succ \ell_5 \succ \ell_3$
C_5 :	$\ell_3 \succ \ell_2 \succ \ell_4 \succ \ell_1 \succ \ell_5$

1. If the companies decide to use a voting system based on the Slater rank, which preference order(s) will be chosen? Why?

2. Explain the advantages and drawbacks of this type of voting system. Justify fully your answer.

3. Give an example with five locations and five companies, all having different preference orders such that a voting system based on the Slater rank would give at least two winners ex-aequo (two preference orders that could both win).

Information

Module mark

For UG students, this written assignment is worth 100% of their final mark. For PG students, this written assignment is worth 70% of their final mark. An oral presentation will be worth the remaining 30%.

Grading

Over 100, the grading for the coursework will be structured as follows:
- Task 1: 12 marks
- Task 2: 22 marks
- Task 3: 16 marks
- Task 4: 22 marks
- Task 5: 16 marks
- Task 6 (individual): 12 marks

Teamwork

You are allowed to work in groups of two on task 1 to 5, or alone. Task 6 has to be completed individually. Working together in groups of two may help you better understand the concepts and learn the skills for each type of task. However, should you prefer to work on your own, this is also acceptable (although this will not give you extra marks). Those working in a team should submit one report each and clearly state at the beginning of their report that they did so, and state the name of their partner. Tasks 1 to 5 should then be identical for the two members of the team, but task 6 should be completed individually. If you decide to work as a team, you should both contribute to all the questions and tasks.

Extenuating circumstances

If you are not able to submit your coursework on time for unforeseen medical reasons or personal reasons beyond your control you should contact the PG/UG Office as soon as possible and fill an Extenuating Circumstances form. Strong evidence in the form of, for instance, medical certificates or legal statements will have to be produced.

Plagiarism

If you copy the work of others (either that of another team or of a third party), with or without their permission, you will score no marks and further disciplinary action will be taken against you. Same applies if you allow others to copy your work.