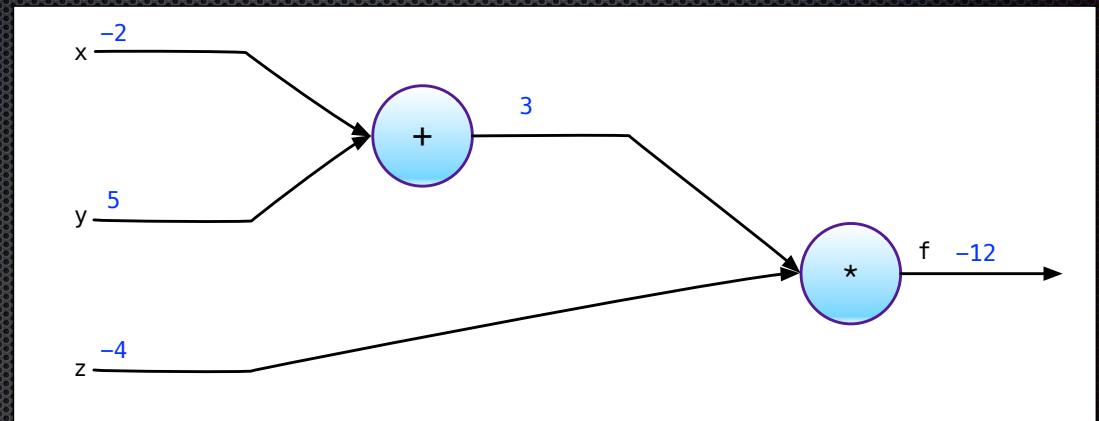


Aprendizaje Automático

Clase 19: Propagación Hacia Atrás
Usando un Grafo Computacional

Propagación Hacia Atrás: Un ejemplo sencillo

$$f(x, y, z) = (x + y)z$$
$$x = -2, \quad y = 5, \quad z = -4$$



Propagación Hacia Atrás: Un ejemplo sencillo

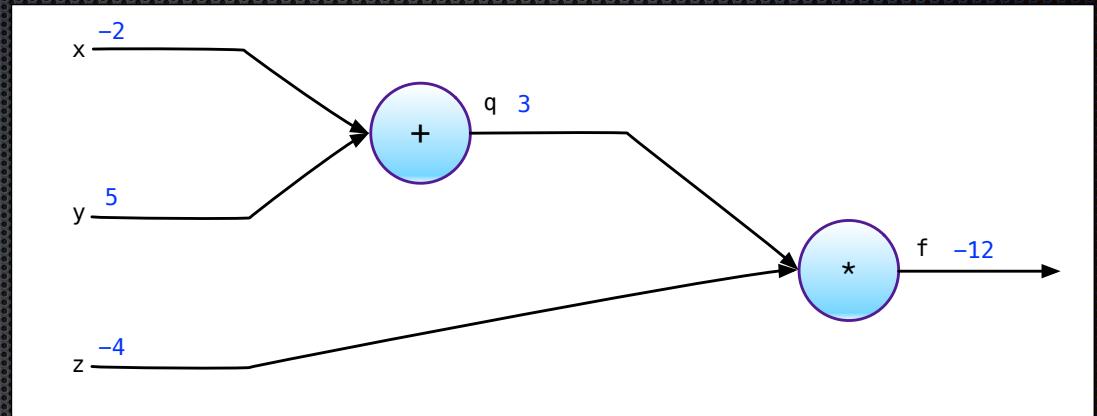
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Encontrar : $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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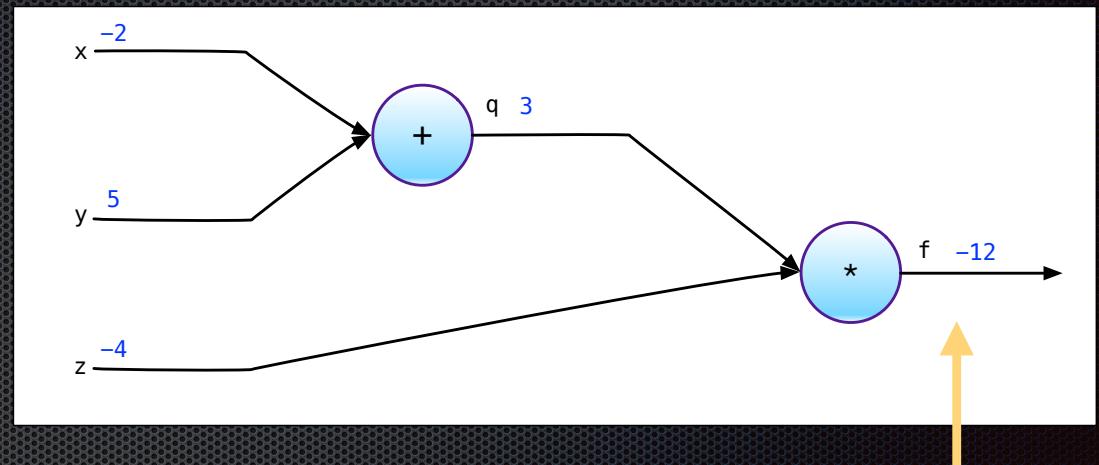
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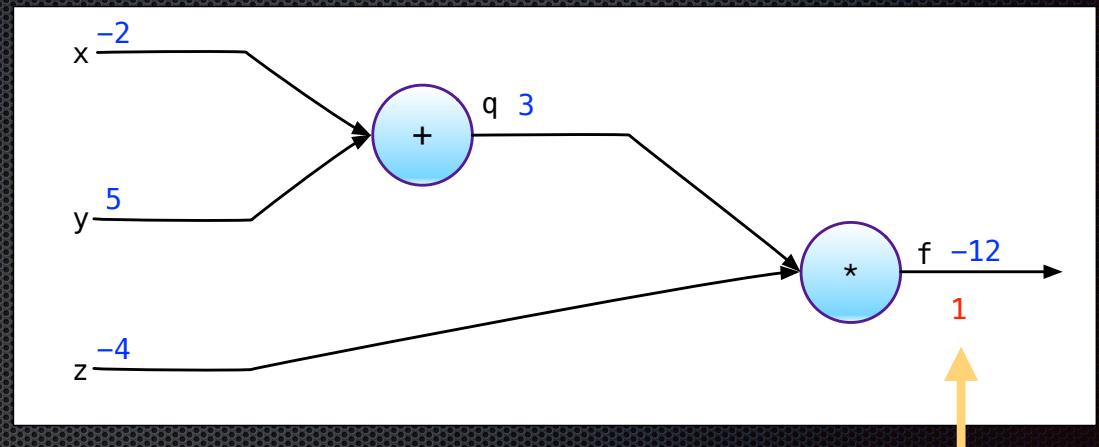
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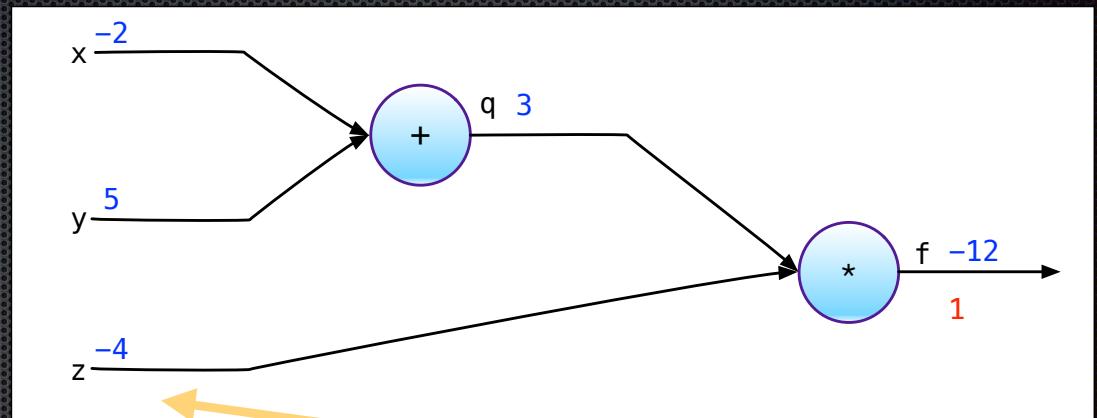
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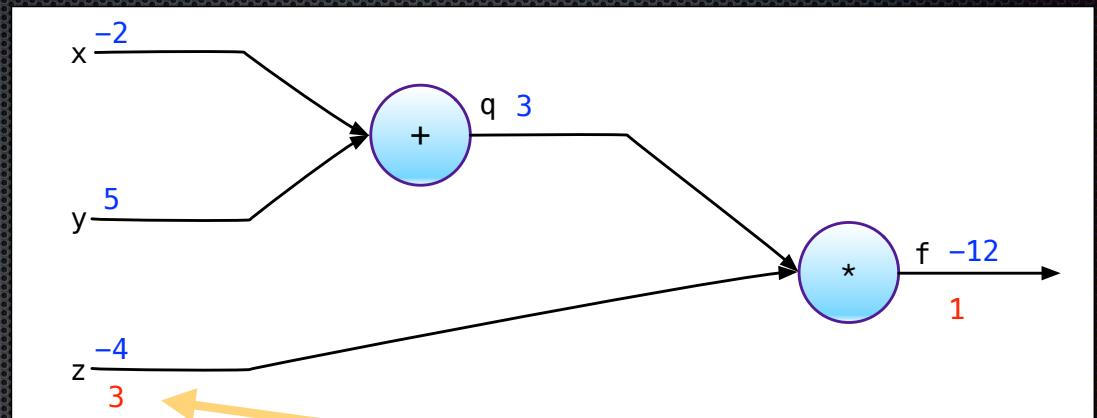
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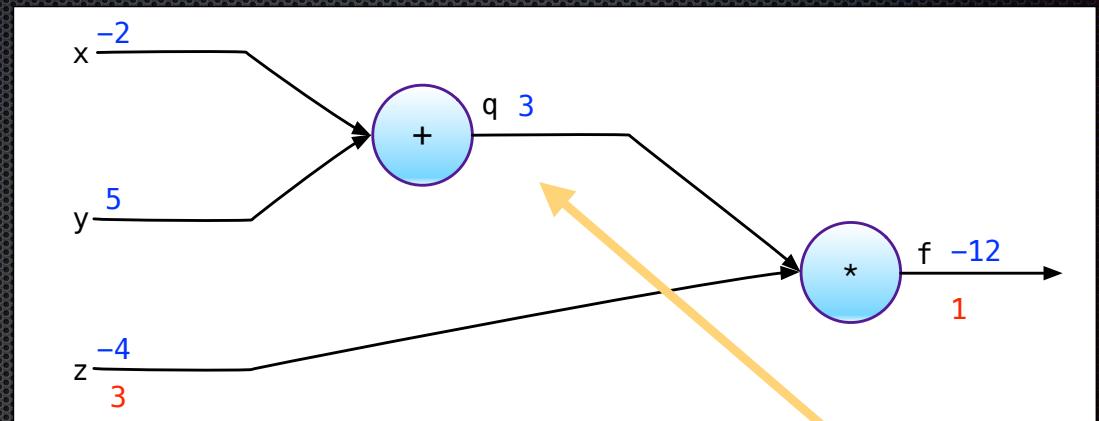
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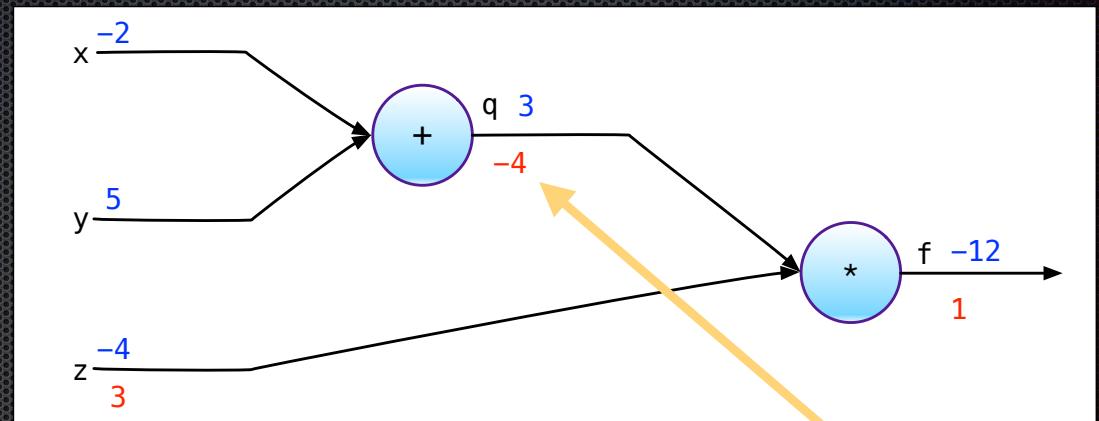
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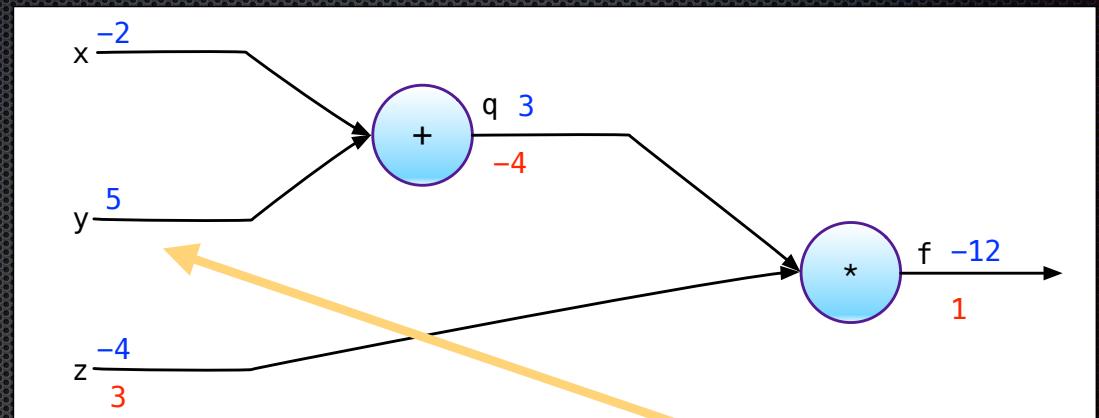
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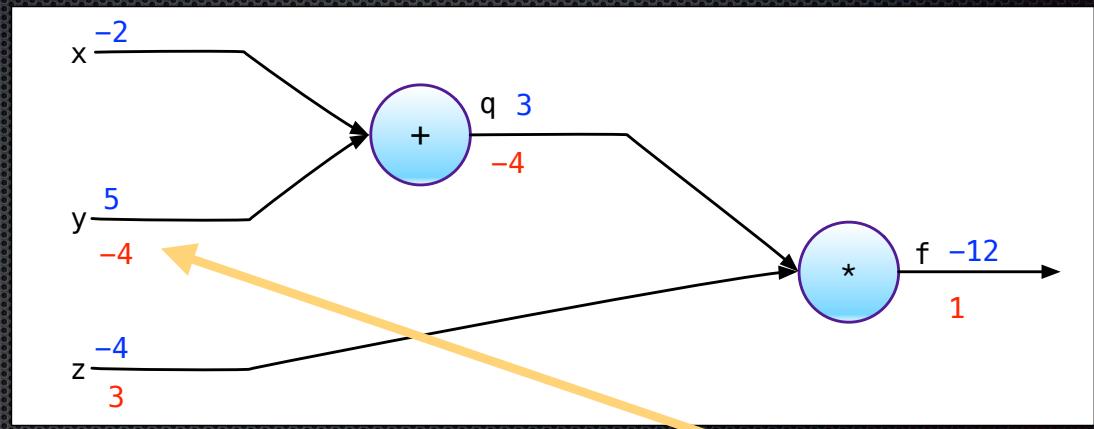
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Regla de la cadena:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

$$\frac{\partial f}{\partial y}$$

Propagación Hacia Atrás: Un ejemplo sencillo

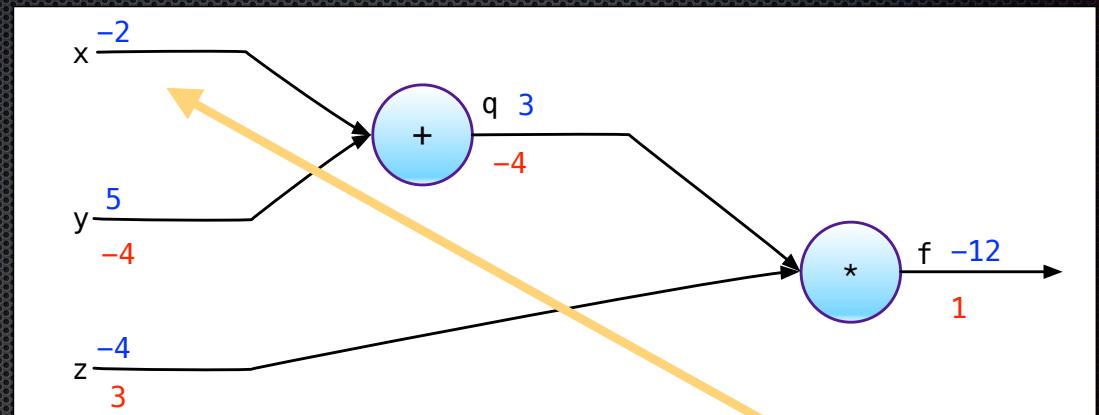
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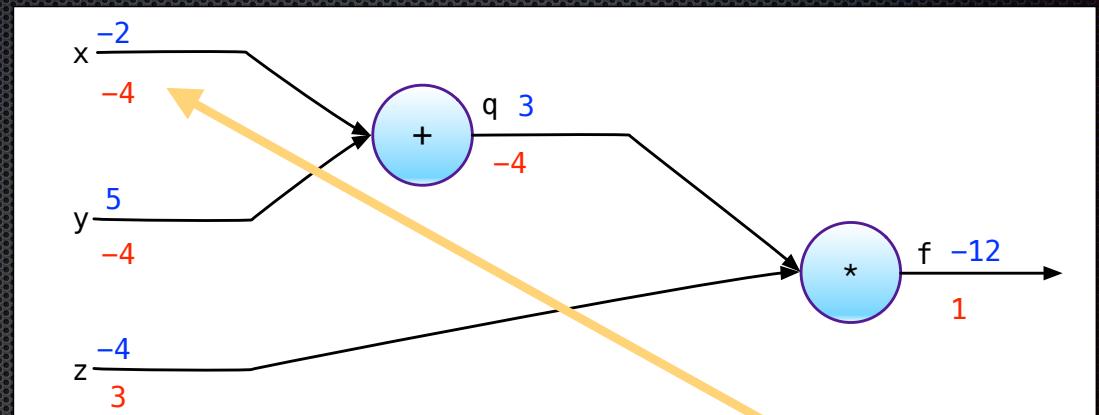
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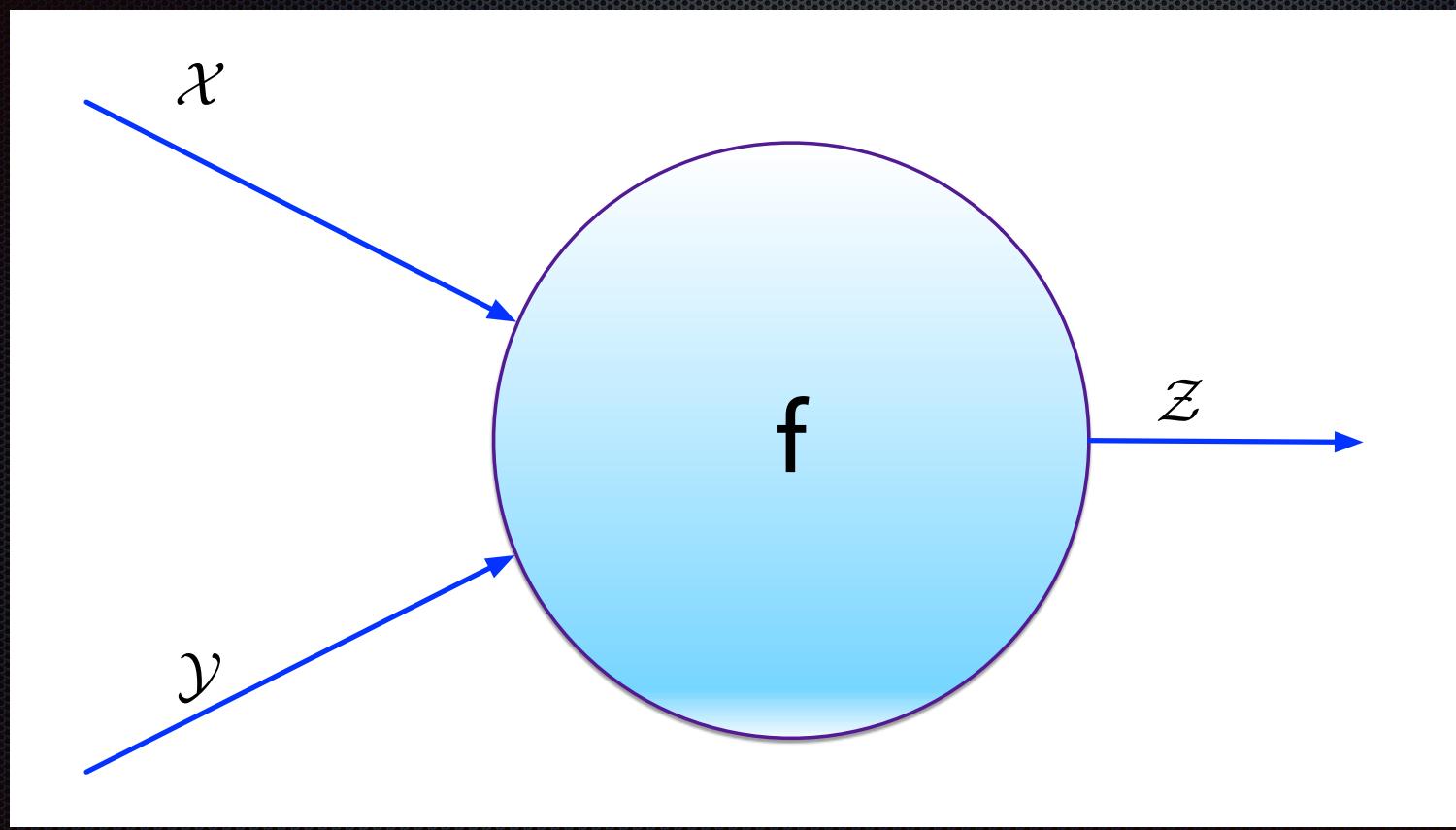


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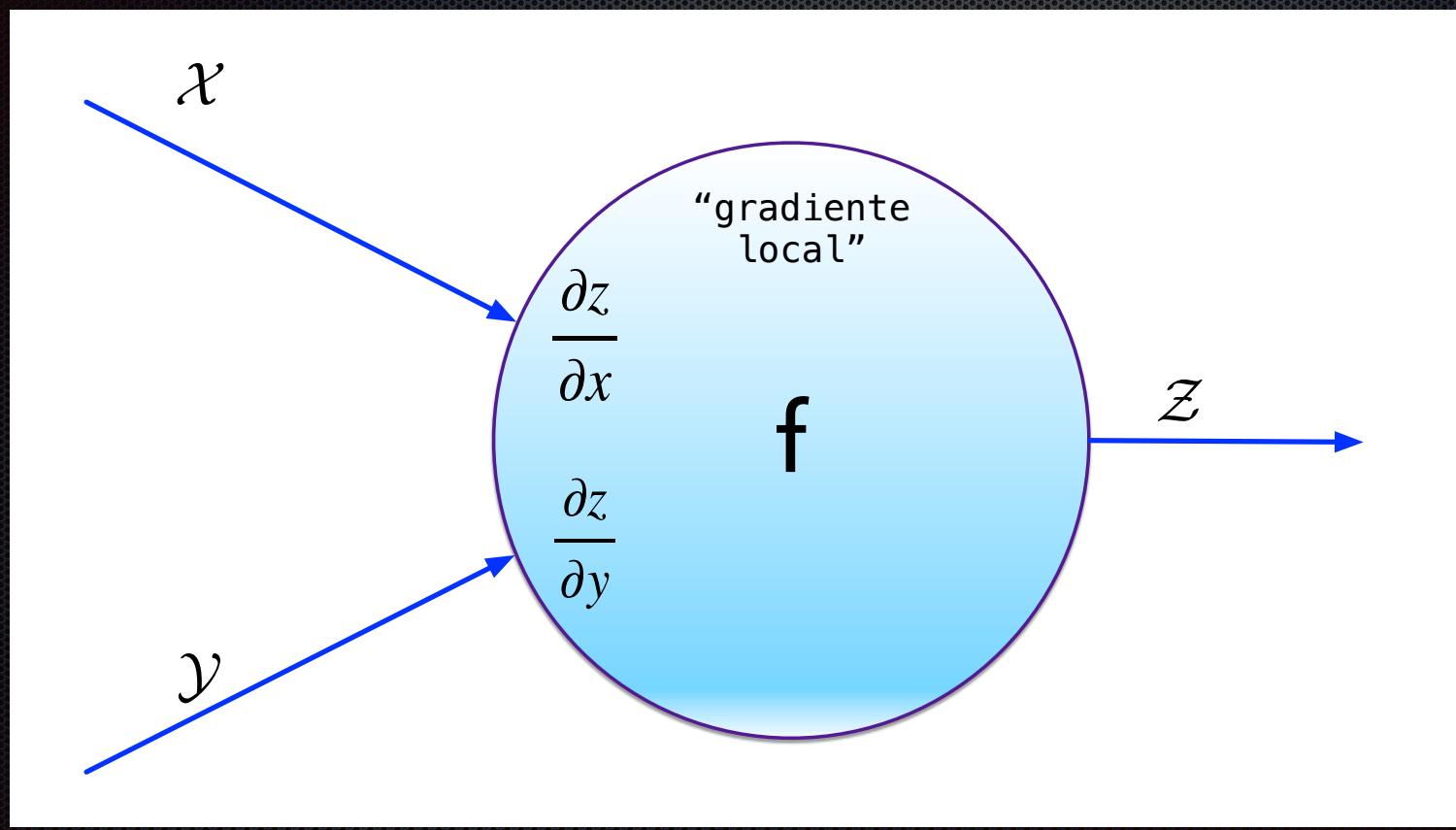
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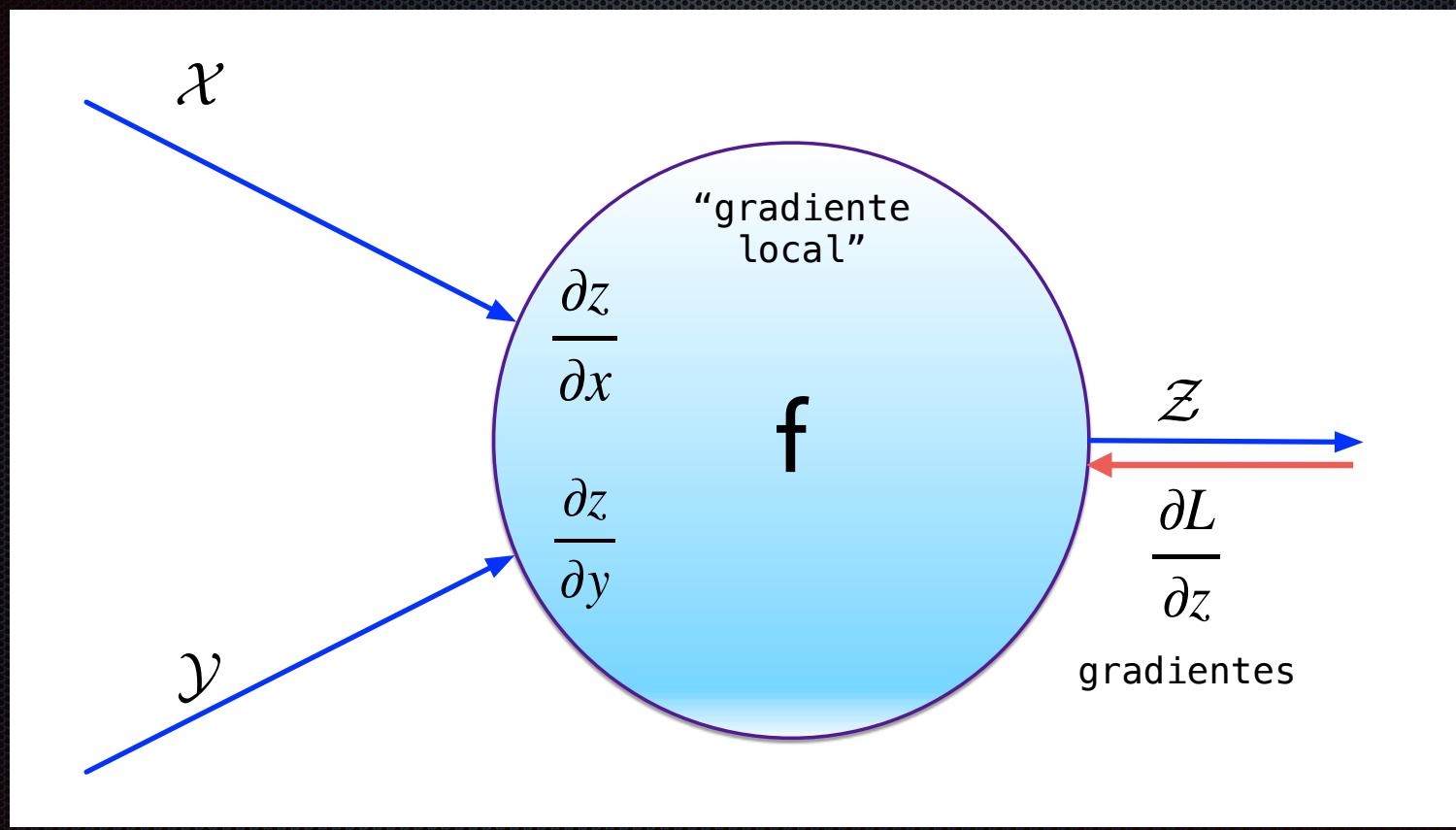
Calculo de los Gradientes en los Nodos



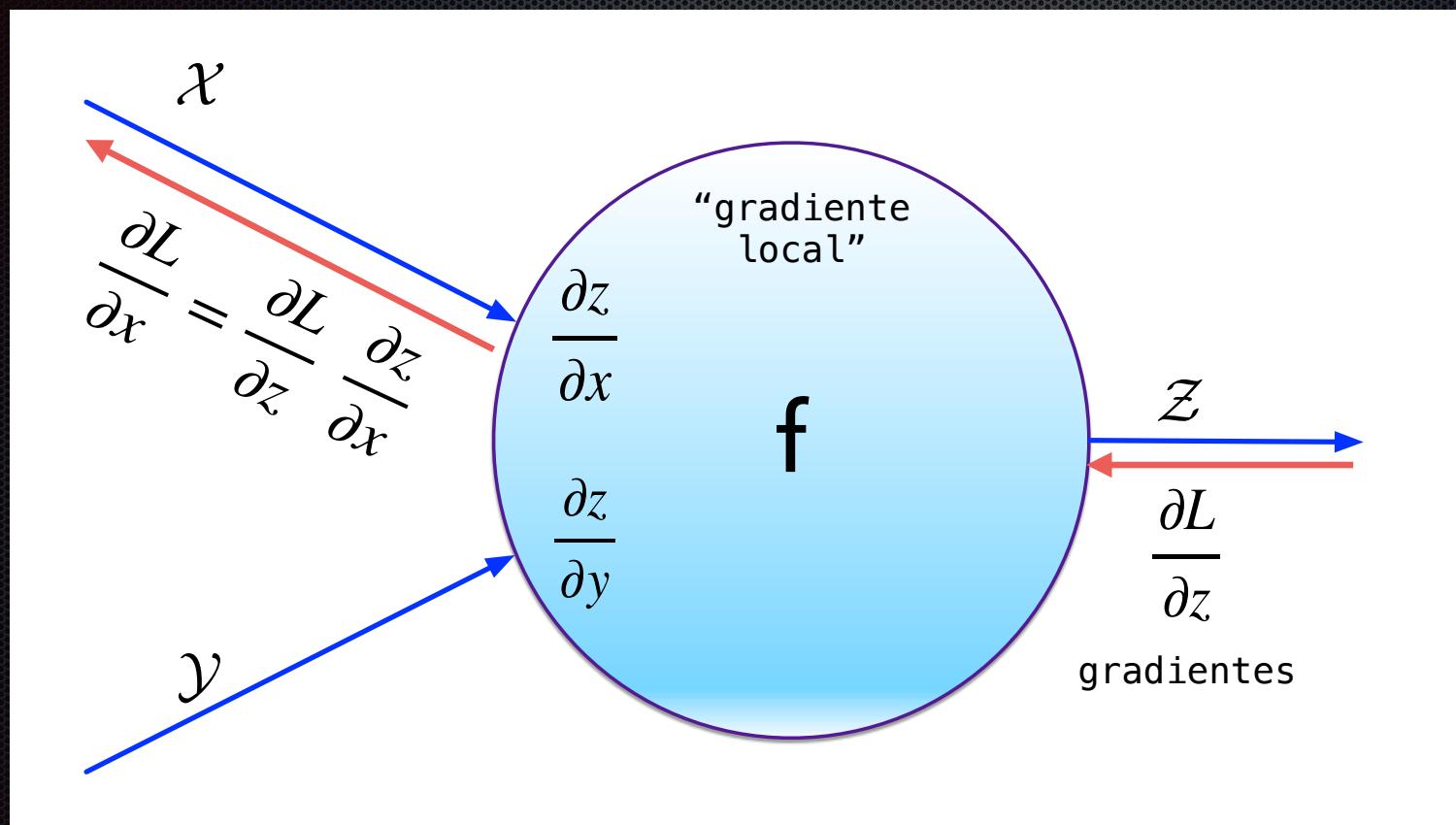
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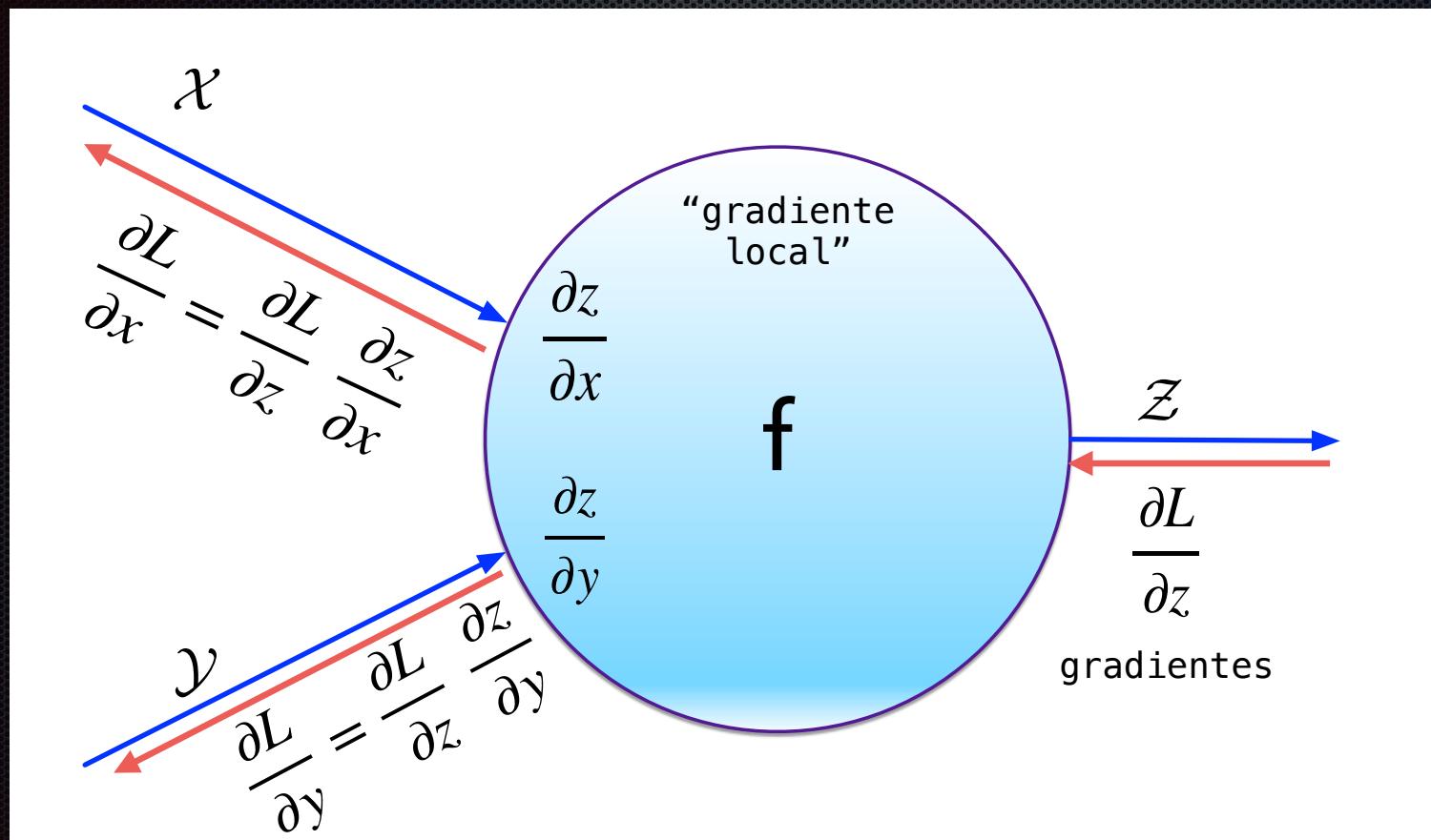
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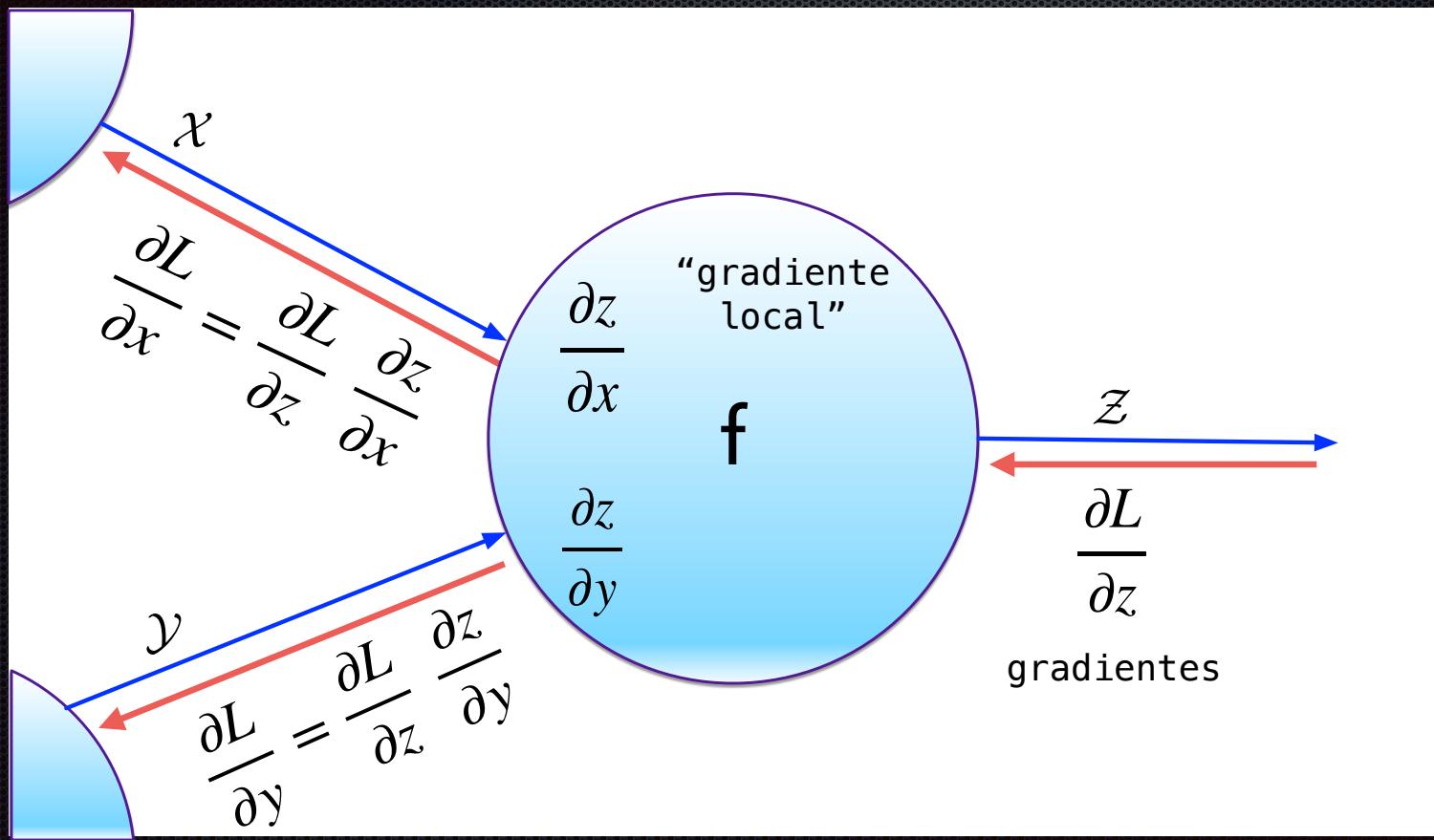
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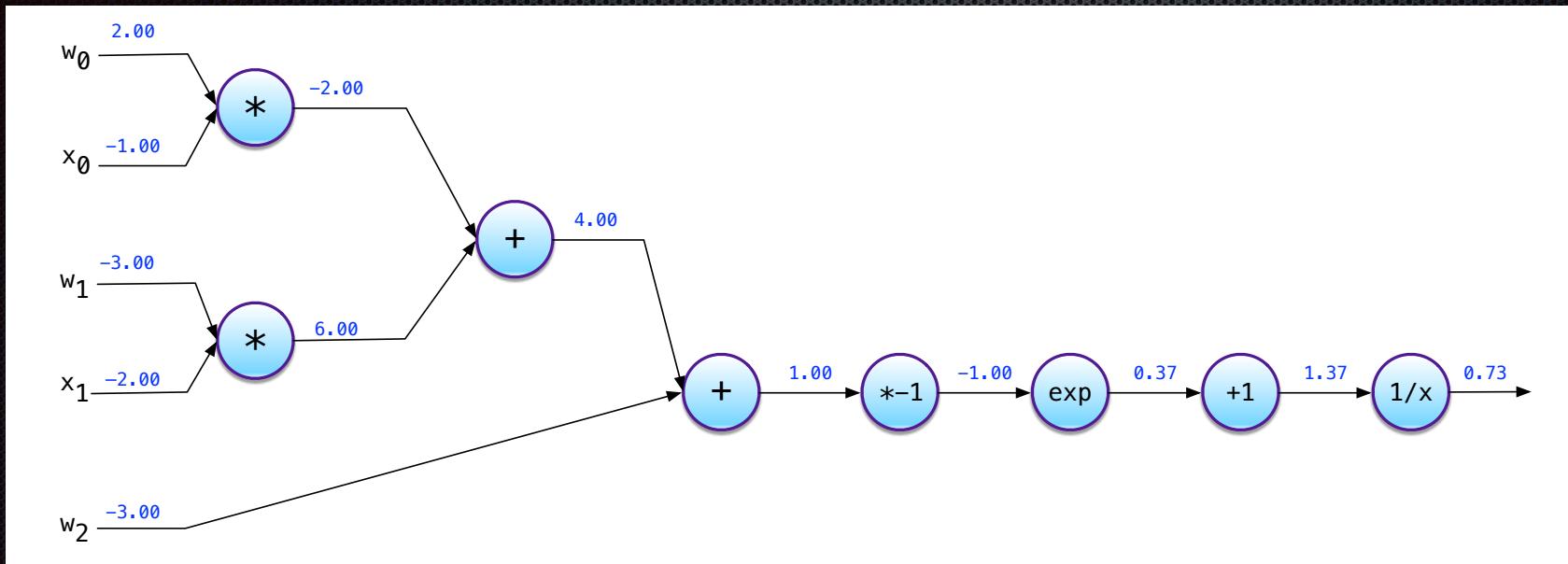


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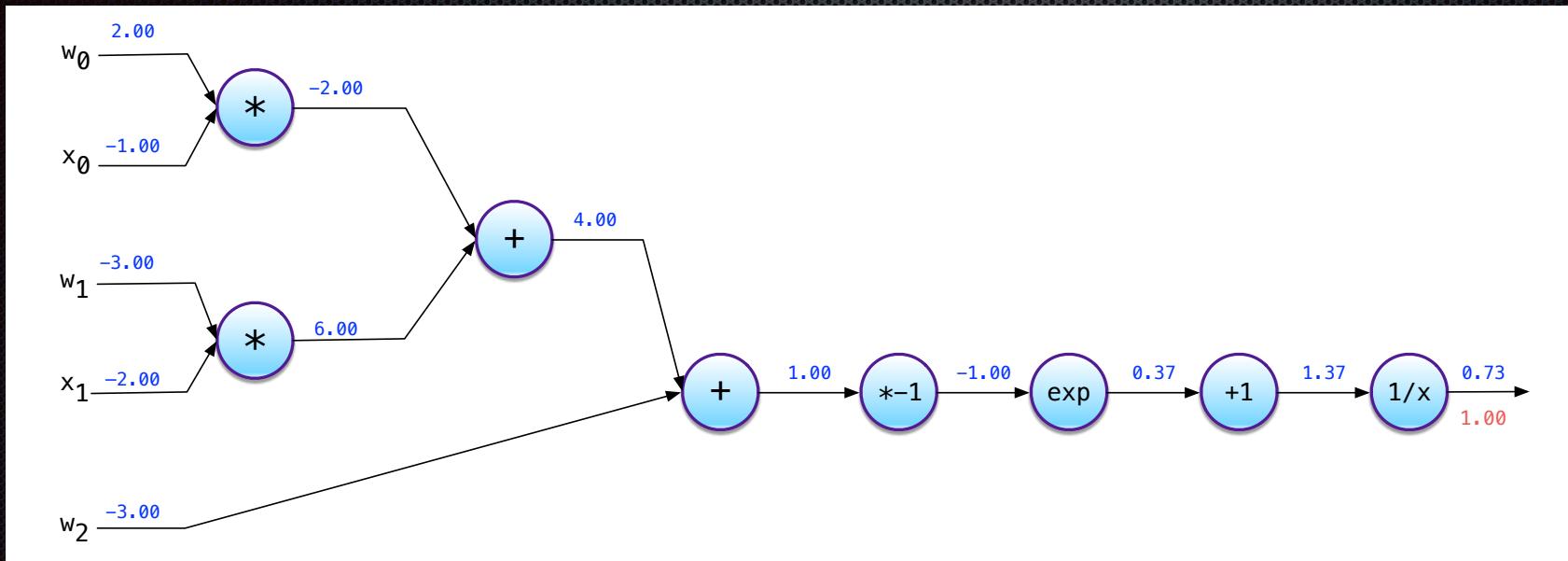
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$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



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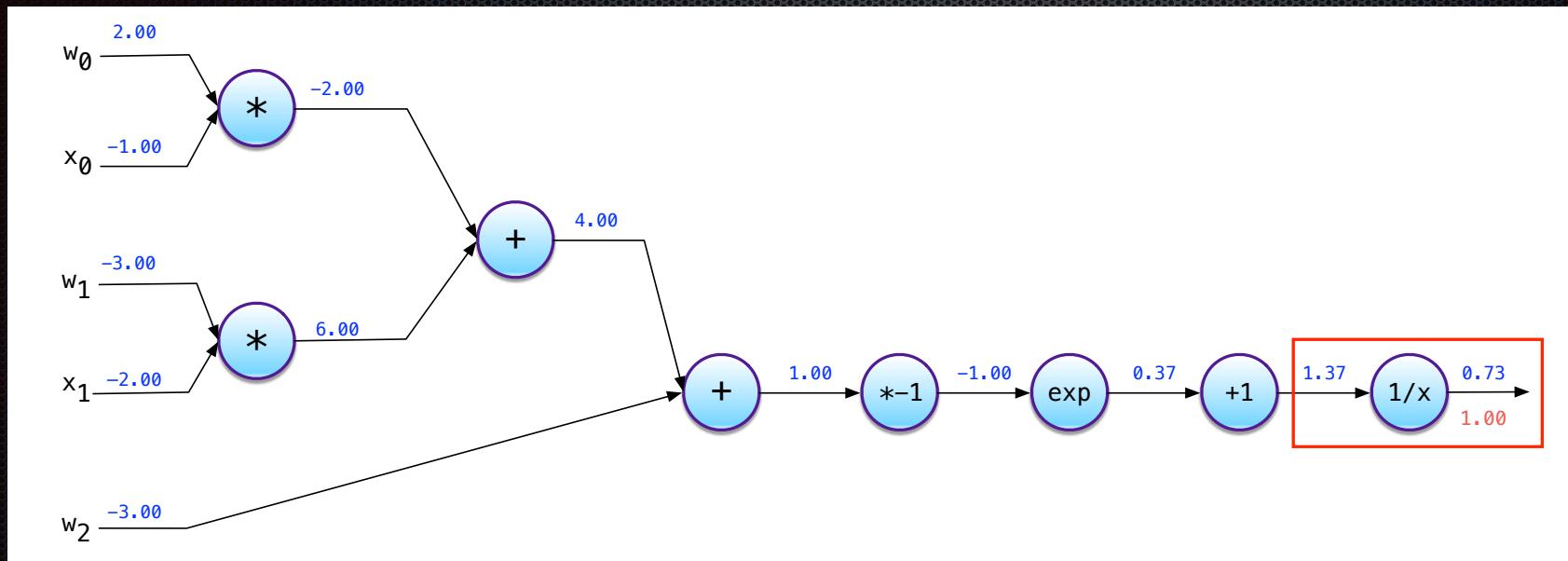
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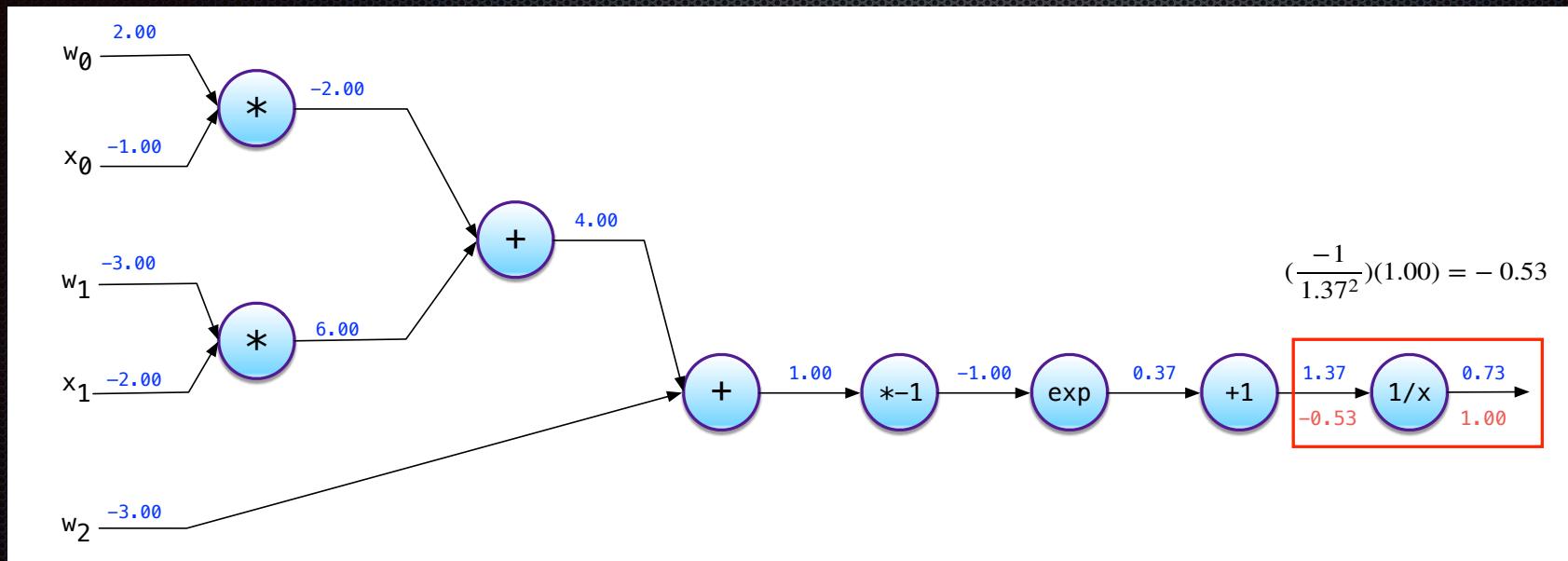
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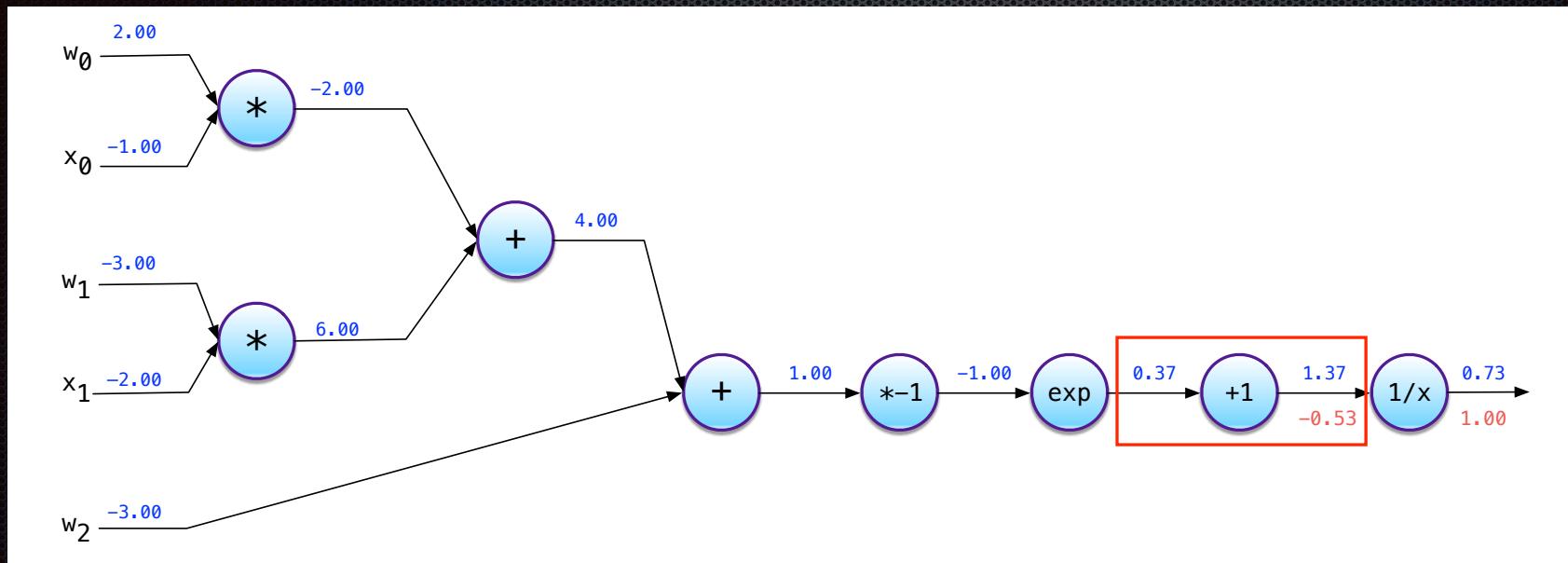
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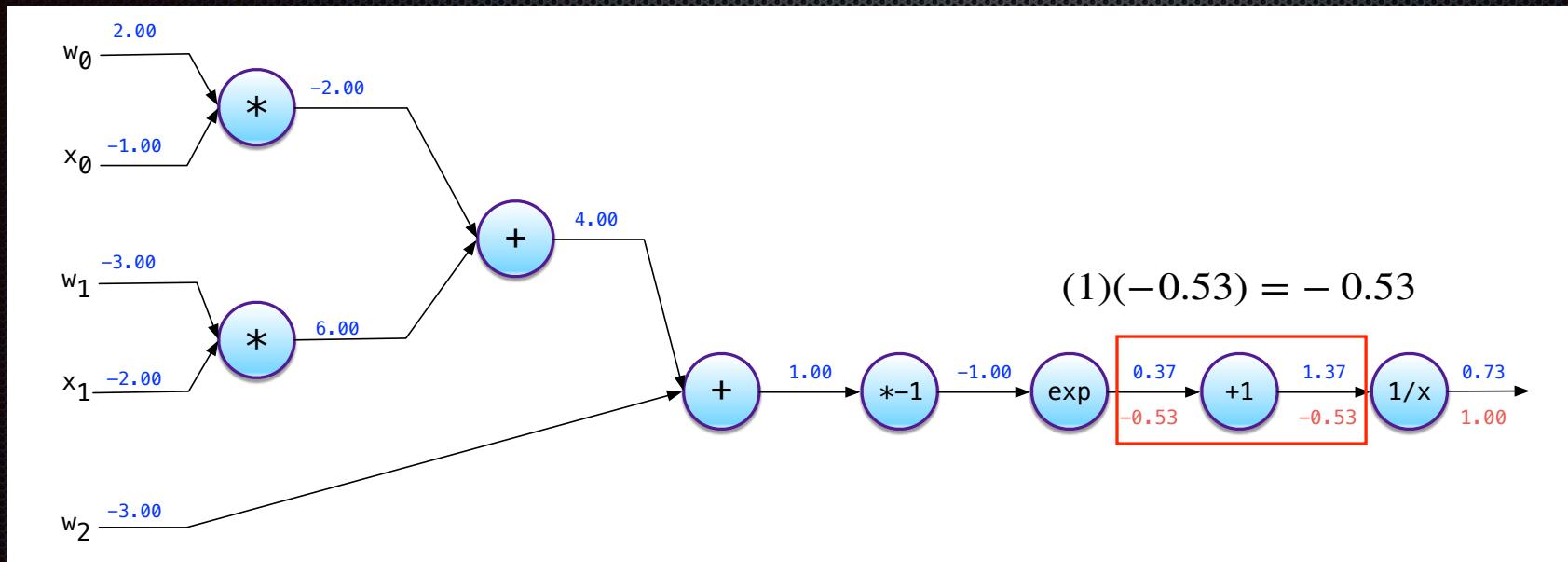
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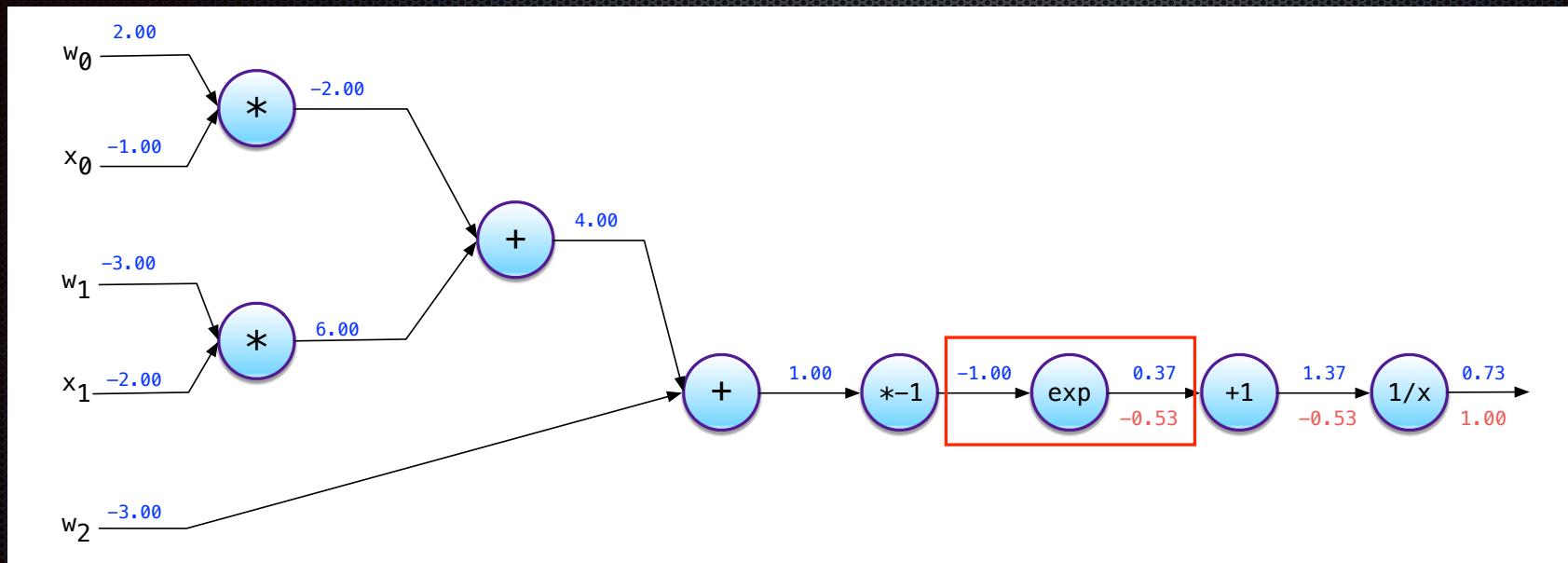
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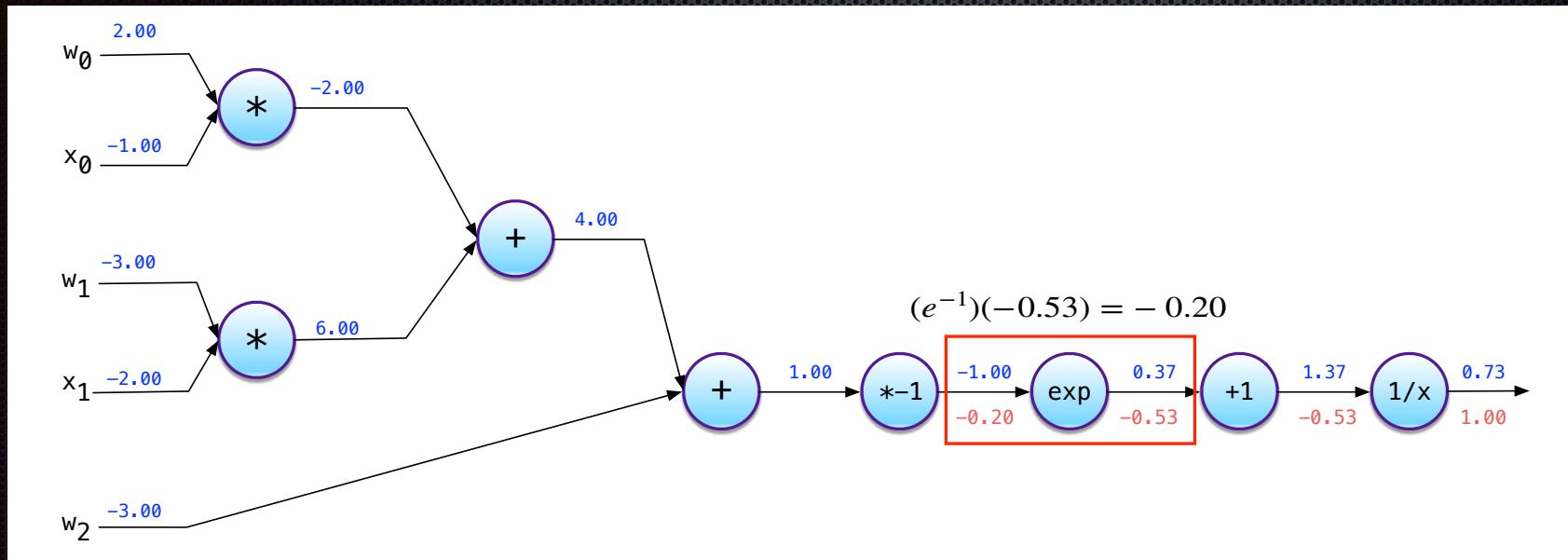
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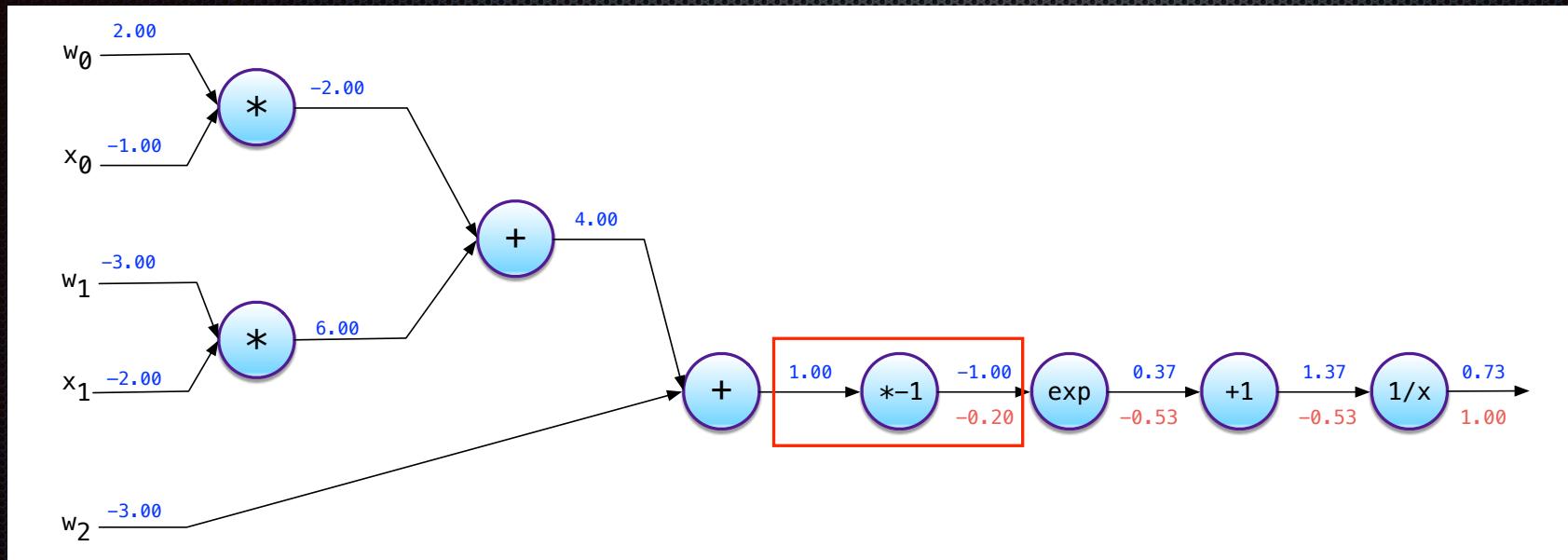
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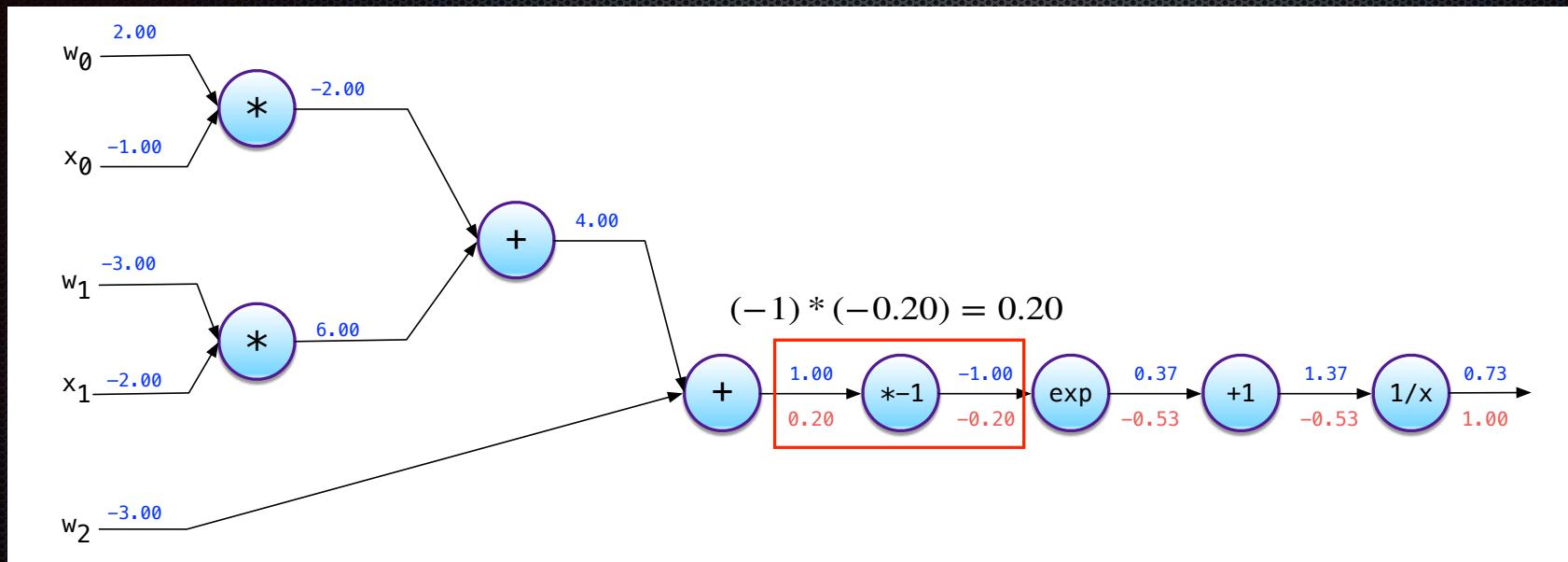
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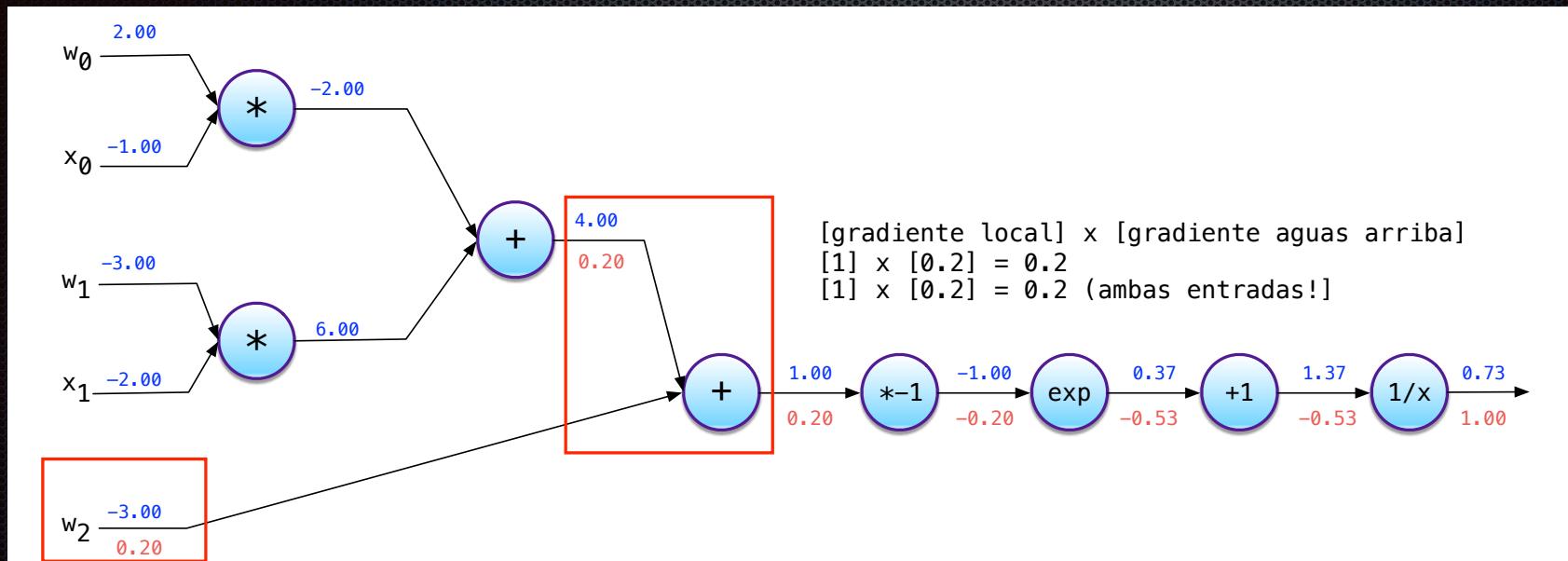
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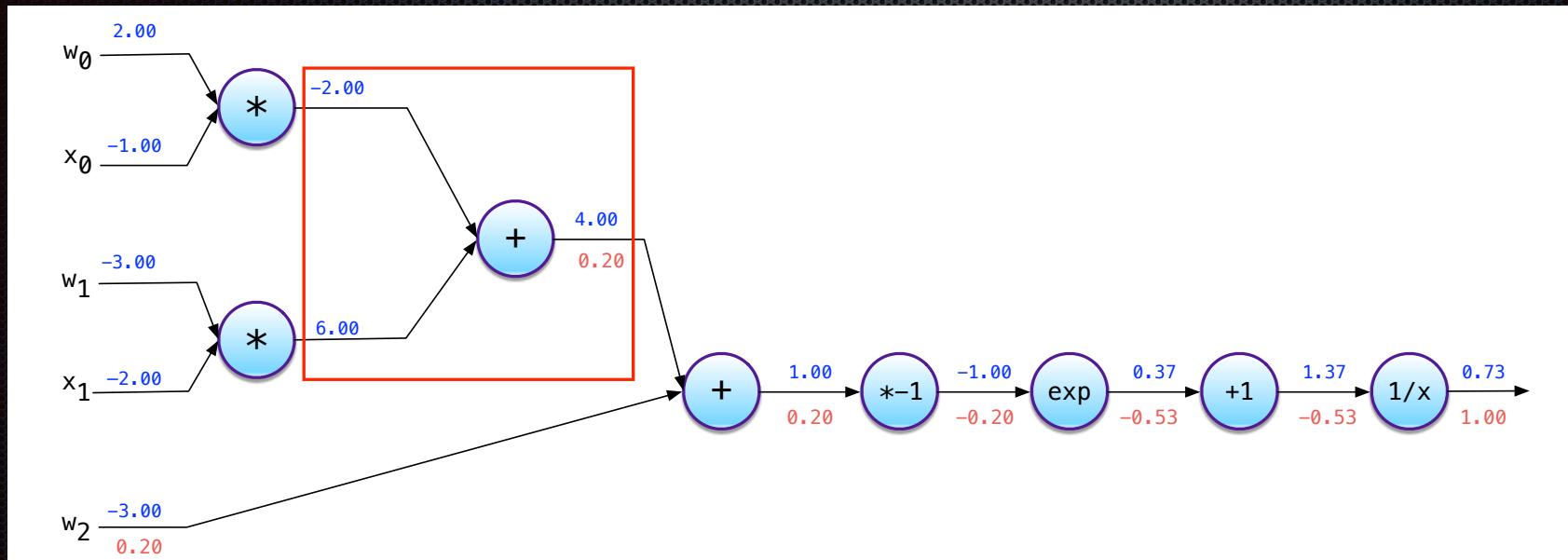
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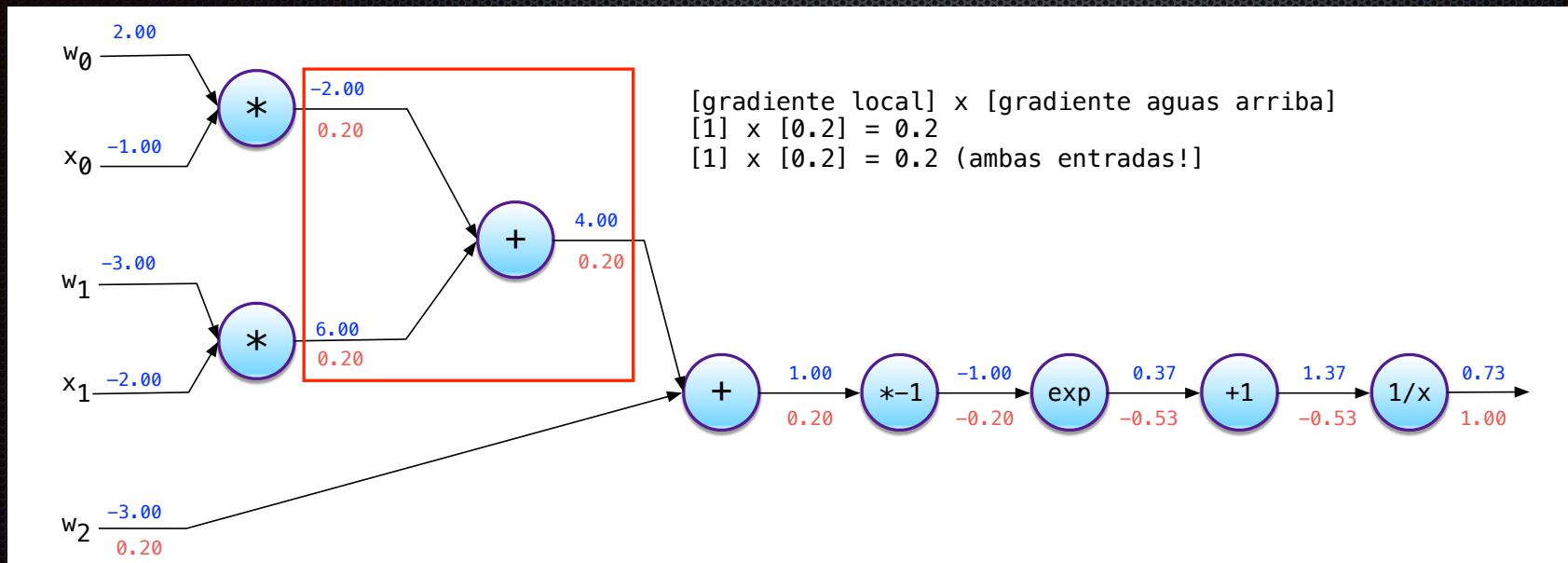
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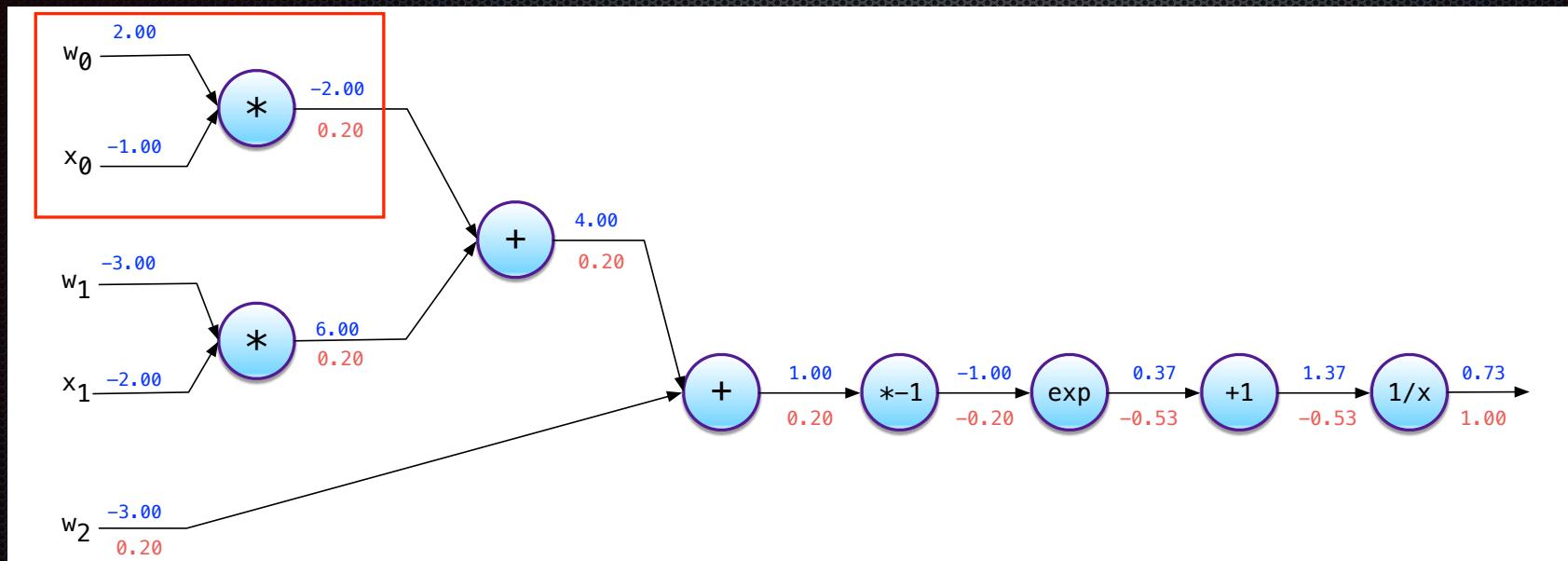
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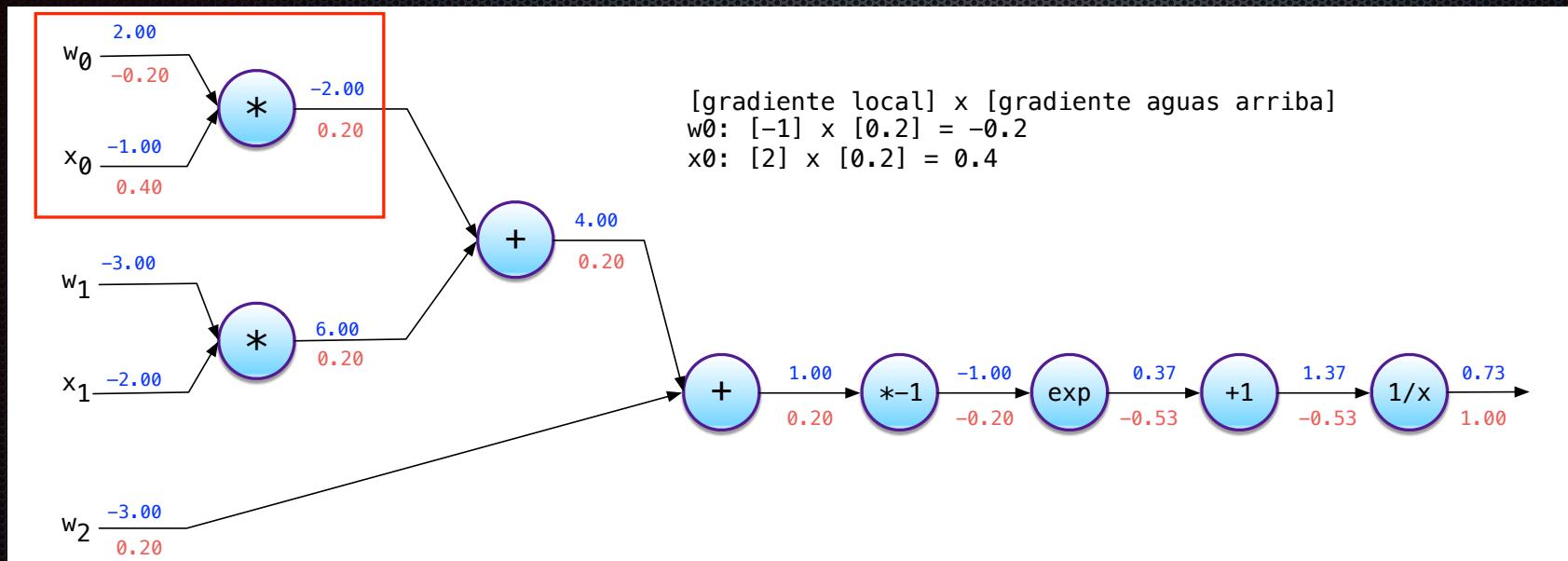
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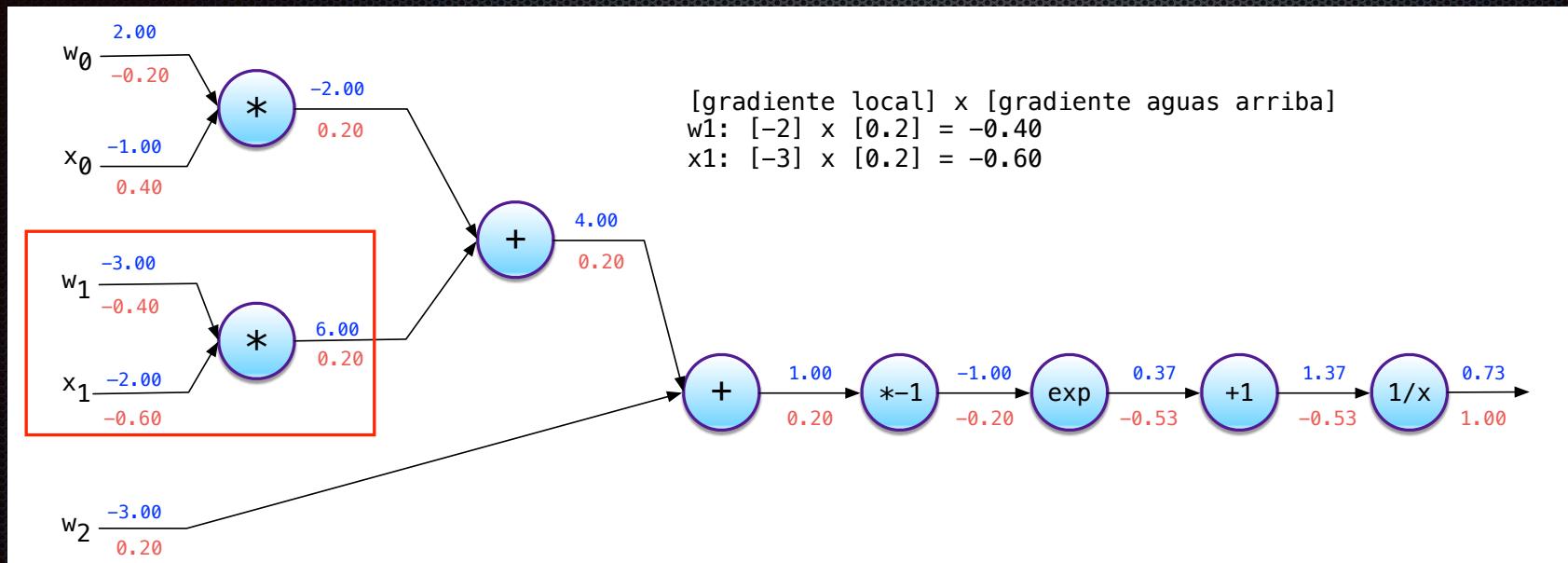
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Patrones del flujo hacia atrás

Compuerta suma: distribuye el gradiente

Compuerta máximo: enruta el gradiente

Compuerta multiplicación: intercambia el gradiente

