Forecasting S&P 500 Volatility

An Exploration of Predictive Models

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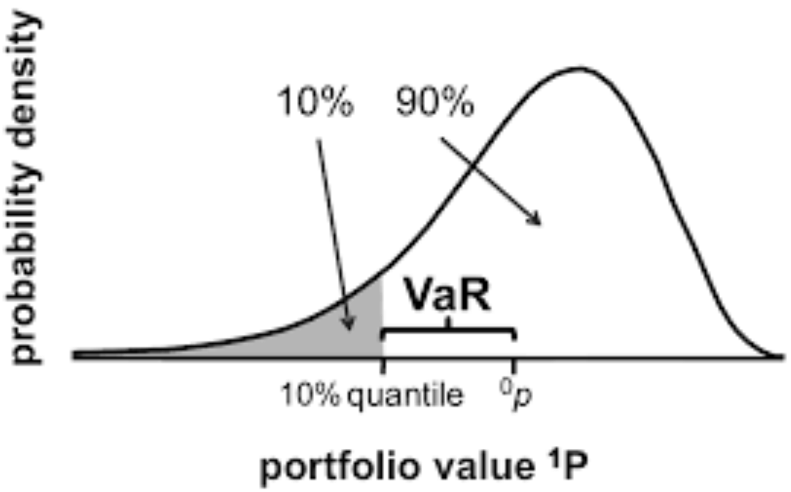
Motivation

* Quantifying volatility is imperative for proper portfolio management
* Forward-looking volatility predictions create better understanding of risk profile
* Useful applications for portfolio managers, volatility traders and retail clients

Volatility Applications

- One application for volatility forecasts is Value-at-Risk (VaR)
- VaR – Risk to portfolio given assumptions about moves in asset prices
- VaR commonly used to gauge maximum losses



Data Overview

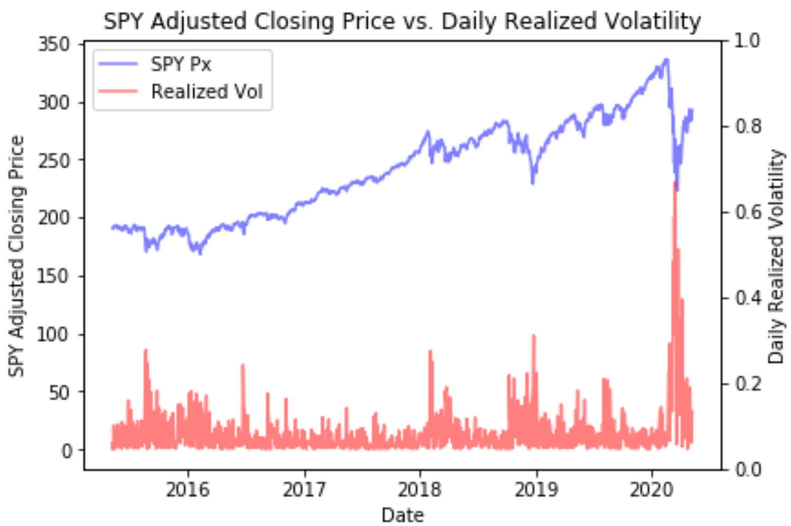
* 5 years S&P 500 ETF (SPY) daily price data
* Date range: May 2015 – May 2020
* Columns: Open, High, Low, Closing, and Adjusted Closing Price as well as Date and Volume Traded
* 1,259 rows and 7 columns
* Source: Yahoo Finance

Data Manipulation

* Calculations required to transform price data into daily changes and then into realized volatility
* Realized volatility – numerical representation for how much a stock moved over a given period of time
* A 1% move in stock corresponds roughly to .1989, or a realized volatility of 19.89

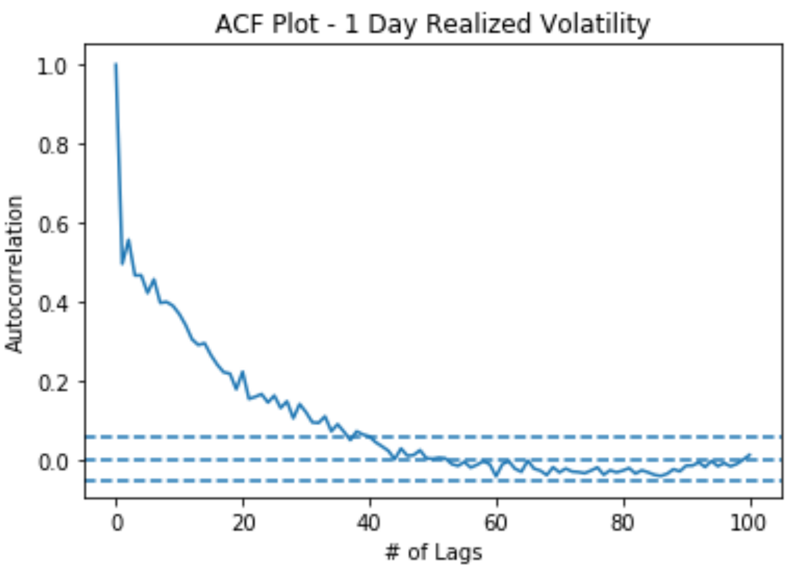
Exploratory Data Analysis (EDA)

* Stocks more volatile going down than up
* Volatility tends to revert to long term average
* Persistence in volatility
* Mean Volatility: .1361
* Std Dev of Volatility: .1905



EDA

* ACF Plot of realized volatility displays significant correlation from day-to-day
* Correlation of a series with itself is called an autoregressive series
* Allows visualization of persistence in volatility
* Past volatility can be predictive of future volatility for over 1 month



EDA Takeaways

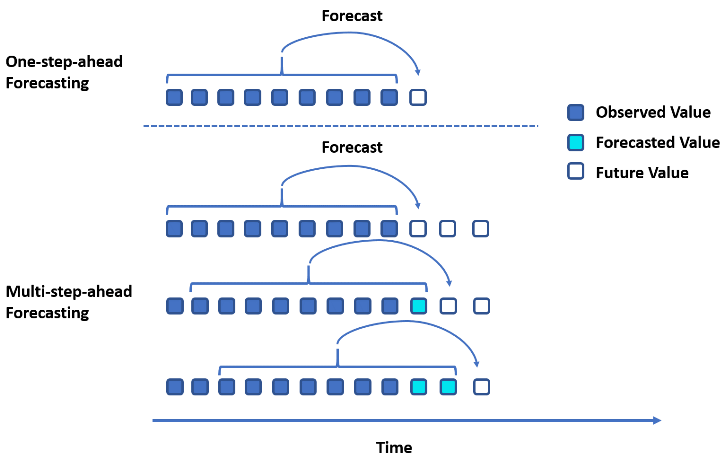
* Since volatility is an autoregressive series, it requires the use of GARCH models, common for forecasting problems in finance
* Several variants of GARCH, a few will be covered in the modeling section
* GARCH models use lagged residuals and lagged variance to make forecasts

Modeling Overview

* Modeling Framework – What is being forecast?
* Data Pre-Processing – What steps are necessary to modify the data before modeling?
* Summary statistics table
* GARCH and related models
* Non-GARCH models – Neural networks, classic machine learning algorithms and symbolic regression
* Modeling conclusions

Modeling Framework

- Models will generate one-step-ahead forecasts
- Forecasts generated from data prior to current forecasting date
- Large forecasting error issues with multi-step-ahead forecasts



Data Pre-Processing: Fixed Rolling Window

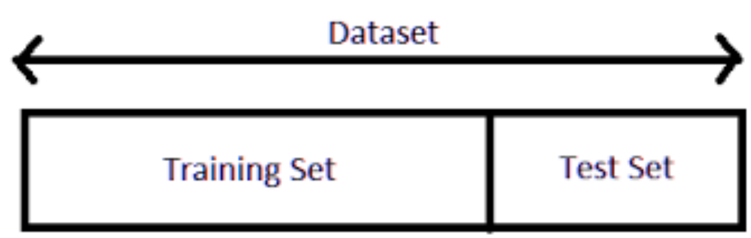
* Fixed Rolling Window – Interval used as input for forecasting is fixed
* Window of fixed length slides forward to the present
* Example: 21-day fixed rolling window uses 21 days of data to predict the current day

Data Pre-Processing: Expanding Window

- Expanding Window – Start date remains the same, new data points are added in
- Easily allows for addition of new data as it becomes available

Data Pre-Processing: Train/Test Split

* Train/Test Split – Part of the data is used for training, the unseen and remaining data used for testing
* All non-GARCH models used this framework
* Realized volatility and high/low price lags as inputs
* 50/50 split used for training and test sets



Modeling Summary Statistics

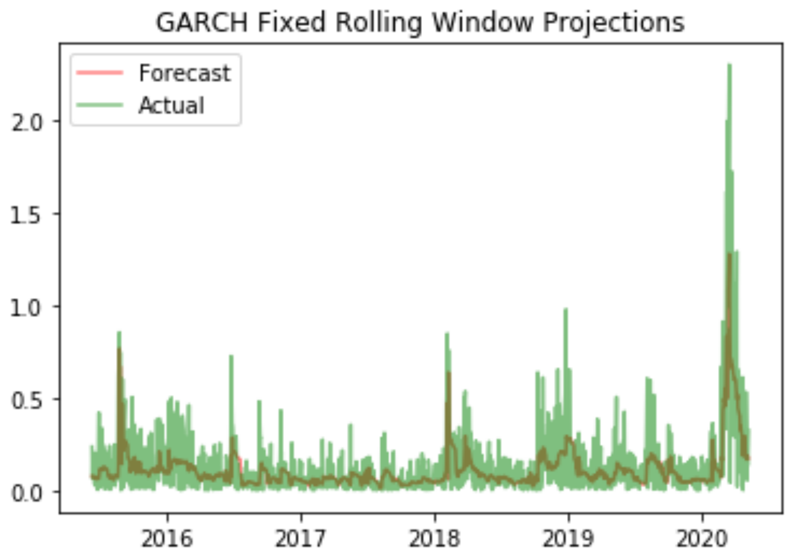
General Model Classification	Model	MAE
Null	Null Model	0.1201
GARCH	GARCH (1,1) Fixed Rolling Window	0.0845
GARCH	GARCH (1,1) Expanding Window	0.0816
GARCH	EGARCH (1,1) Expanding Window	0.0798
GARCH	GJR GARCH (1,1) Expanding Window	0.0802
Neural Network (Recurrent)	LSTM RNN	0.1087
Machine Learning – Random Forest	Random Forest Regressor (5d Lag)	0.1103
Machine Learning – Linear Regression	Linear Regression (5d Lag)	0.1072
Symbolic Regression	Symbolic Regression	0.1095

Null Model

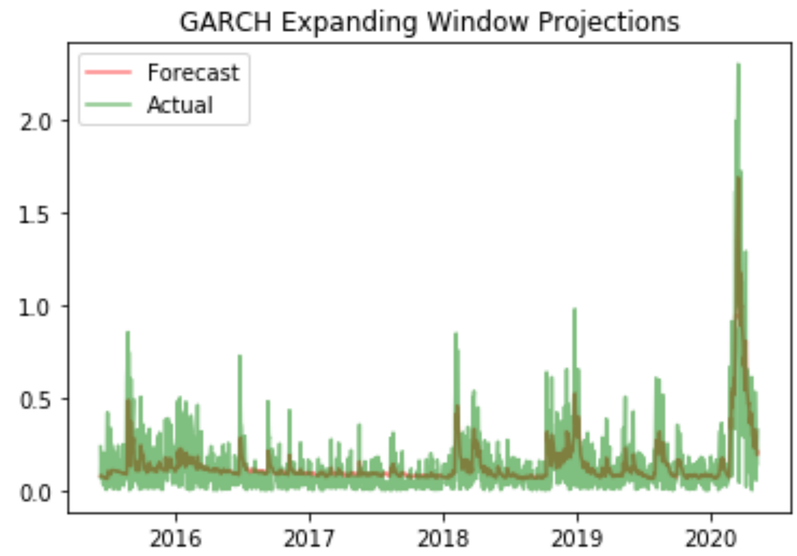
* Null model constructed by using previous day's realized volatility as prediction for next day
* Simple model, but non-trivial
* Volatility is an autoregressive process
* Mean Absolute Error (MAE) – metric for comparing models
    * Easily interpreted: If MAE is .08, then the mean realized volatility prediction is off by .08

GARCH (1,1) Models

Fixed Rolling Window



Expanding Window



GARCH (1,1) Models

Fixed Rolling Window

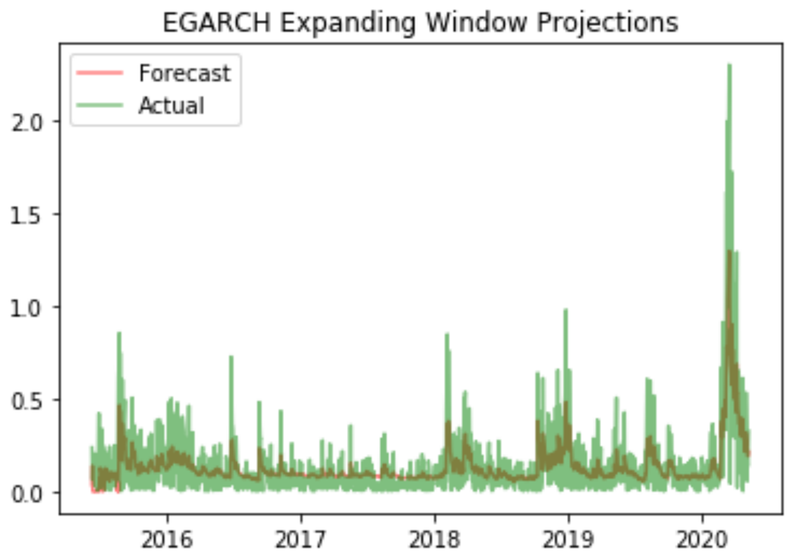
* Slightly higher MAE (0.0845)
* Does not do a great job capturing magnitude of spike in 2020

Expanding Window

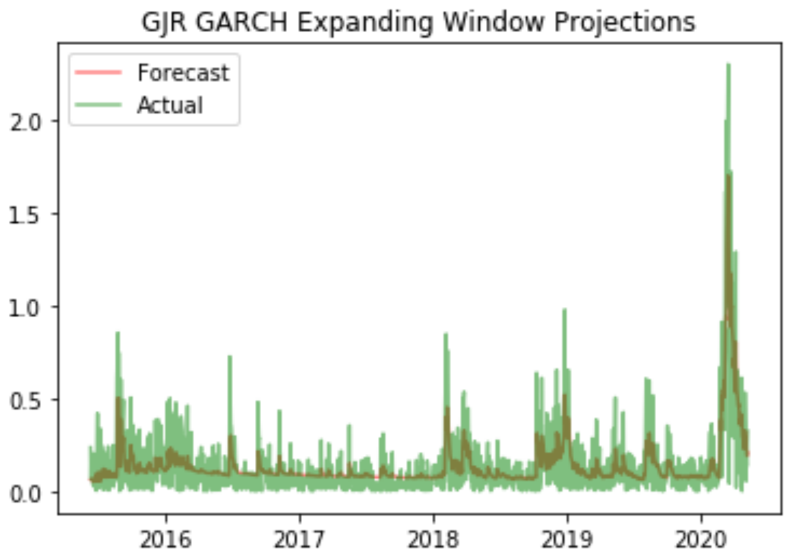
* Slightly lower MAE (0.0816)
* Visually performs better at capturing large volatility spike in 2020

GARCH Variants

EGARCH (1,1)



GJR GARCH (1,1)



GARCH Variants

EGARCH (1,1)

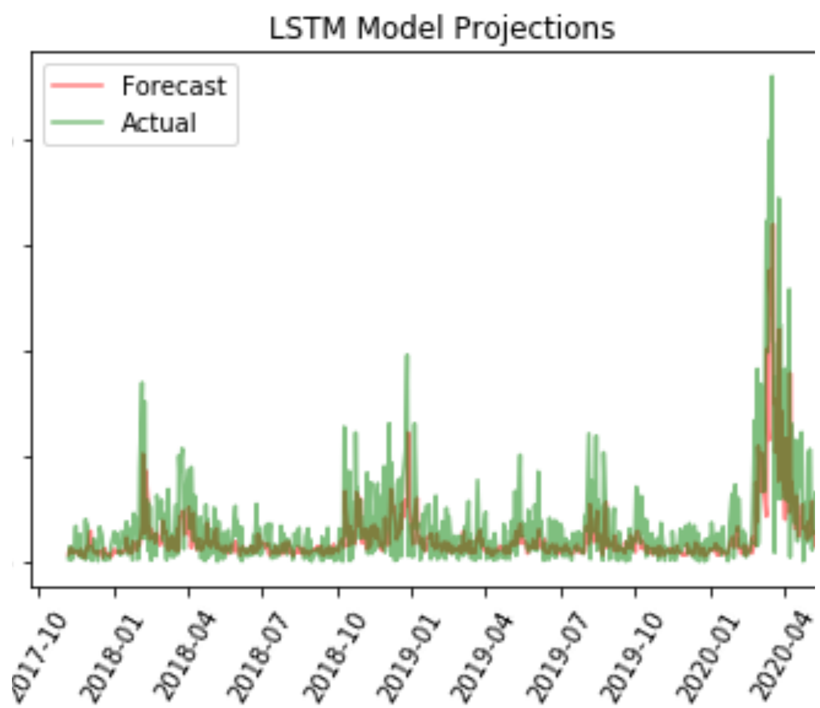
* Best MAE of all GARCH models at 0.0798
* Although best MAE, predictions look more like GARCH (1,1) fixed rolling window
* Does not capture magnitude of 2020 spike

GJR GARCH (1,1)

* Very close to EGARCH MAE of 0.0802
* Predictions similar to the GARCH(1,1) expanding window model
* Reasonably captures 2020 volatility spike and has lower MAE

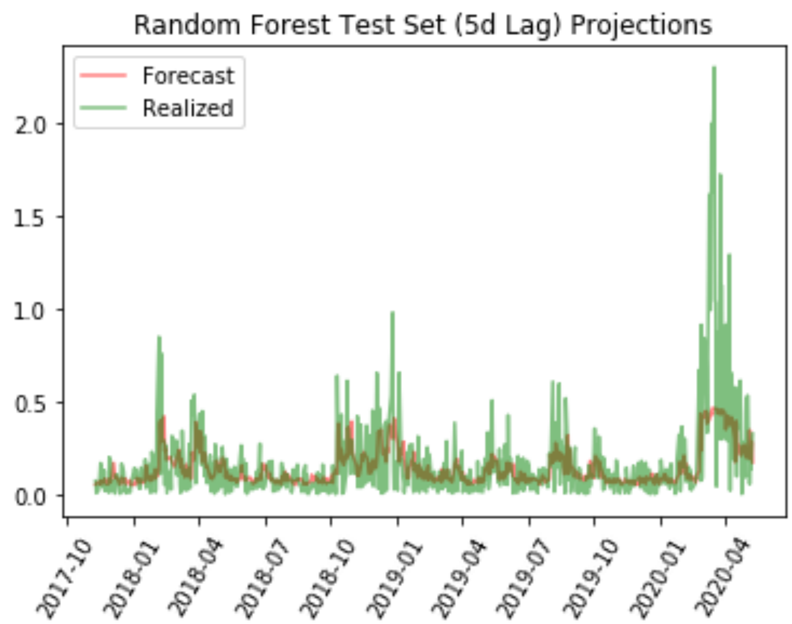
Long Short-Term Memory (LSTM) Recurrent Neural Network (RNN)

- MAE of 0.1087 was higher than any GARCH models
- Still reasonably accurate at predicting magnitude of volatility spikes



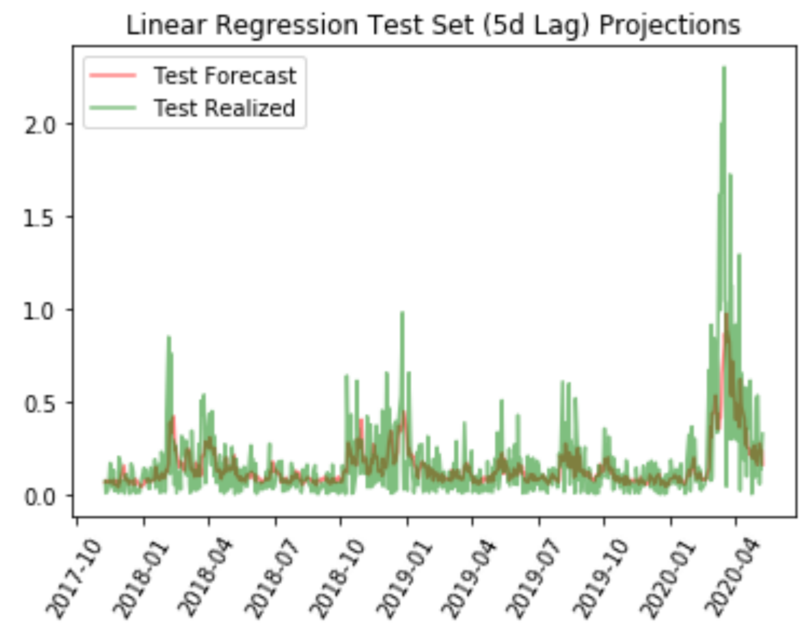
Random Forest Regressor

- All Random Forest models (5, 10, 21d lags) had similar MAE, best was 0.1102
- Visualization of predictions also very similar
- Severe inability to capture magnitude of 2020 spike



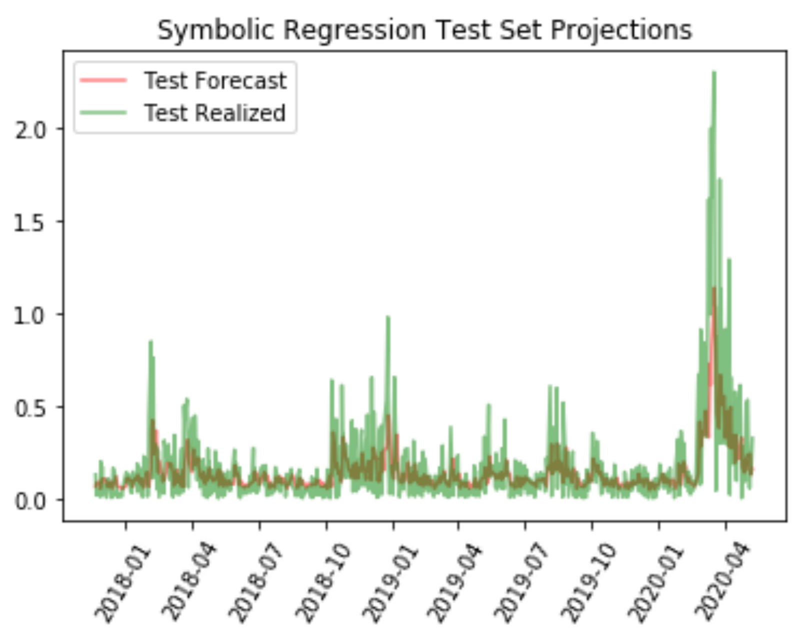
Linear Regression

* All Linear Regression models (5, 10, 21d lags) also had similar MAE, best was 0.1072
* Slightly better than RF models
* Still subpar ability to capture 2020 spike



Symbolic Regression

- MAE of .1095 similar to that of Linear Regression and LSTM models
- Visual predictions are closer to Linear Regression model
- Falls in category of models that does poorly at predicting 2020 spike



Modeling Conclusions

* GARCH and its model variants performed best in predicting realized volatility
* EGARCH and GJR GARCH are two best models in terms of MAE
* GJR GARCH most likely to be used in practice as it more accurately captures spikes in realized volatility
* Visual comparison important! Can have low MAE by getting a lot of the low volatility predictions right, but do poorly on more important high volatility predictions

Practical Uses for Model

* Portfolio managers could adjust portfolio allocations based on volatility predictions
* Volatility traders may choose to use volatility predictions as inputs when evaluating trades
* Retail investors could use model to help make more informed decisions about options strategies
  * Example: Identify optimal times for overwriting equity holdings

Future Work

* More variables as inputs into machine learning and neural network models
* Different underlying assumptions for GARCH models (i.e. use Student’s t-distribution)
* Multi-step forecasting
* Include predictions about volatility distributions
* Extend work to cover more securities, such as single stock equities (i.e. AAPL, MSFT, FB)

Contact Information

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