Fast Fourier Transform

Overview

- What does it do and why do we use it?
- Implementation in C using MPI
- Performance

Mechanics and Usage

Why do we care?

Fourier Transform

- In essence, it allows us to move from one domain to another
- Creates what is called a “Fourier Series” from an expression
- This series is representative of the original expression and can be reversed
- For our case, we will use it go decompose complex signals into parts for analysis

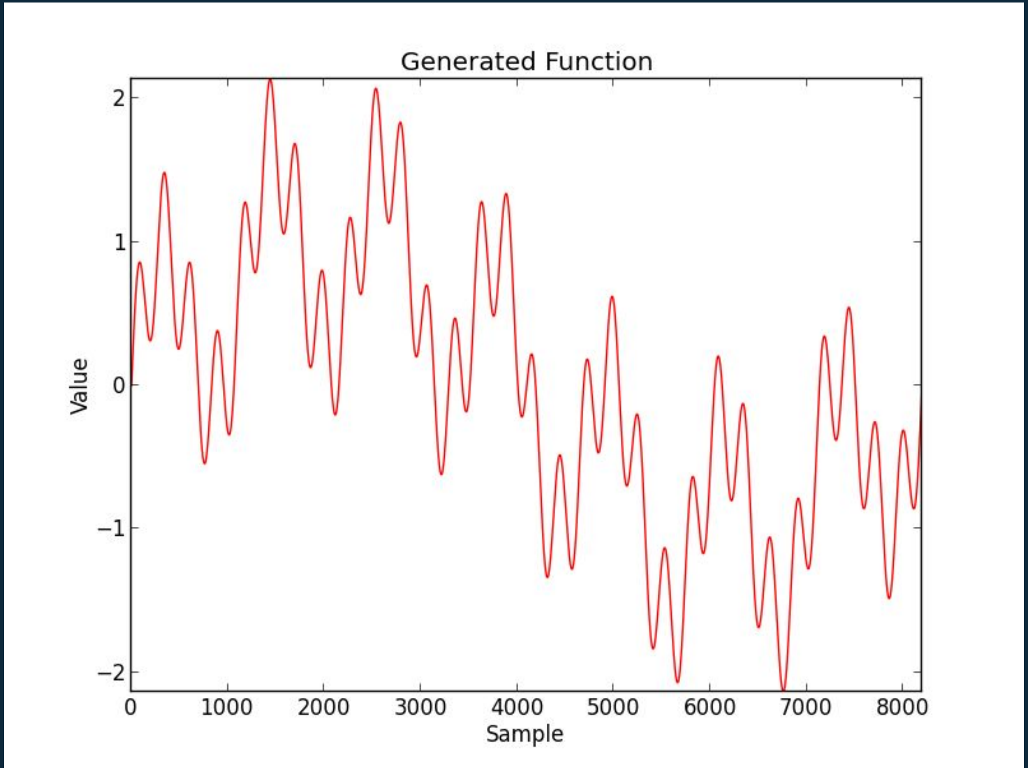
Discretization

- In the real world, we do not have expressions to represent signals
- Given a set of samples, we can run operations on them to achieve the same sort of results.
- This process is of **$O(n^2)$** in a naive DFT

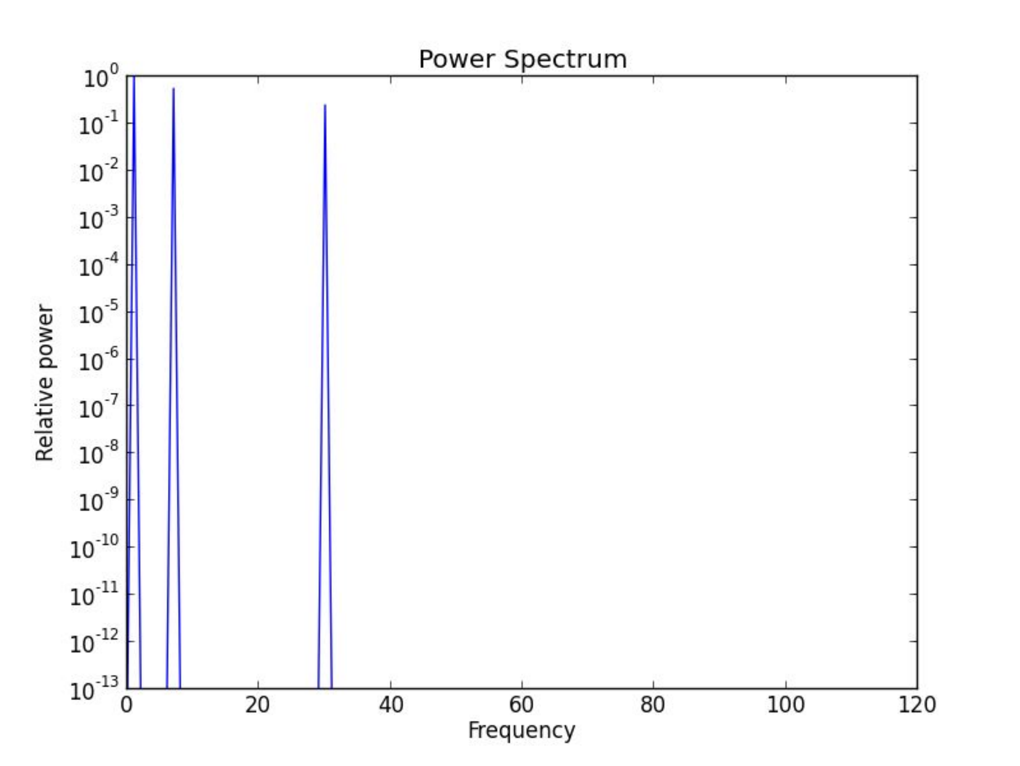
$$X_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{i2\pi k \frac{n}{N}}$$

To find the energy at a particular frequency, spin your signal around a circle at that frequency and average a bunch of points along that path

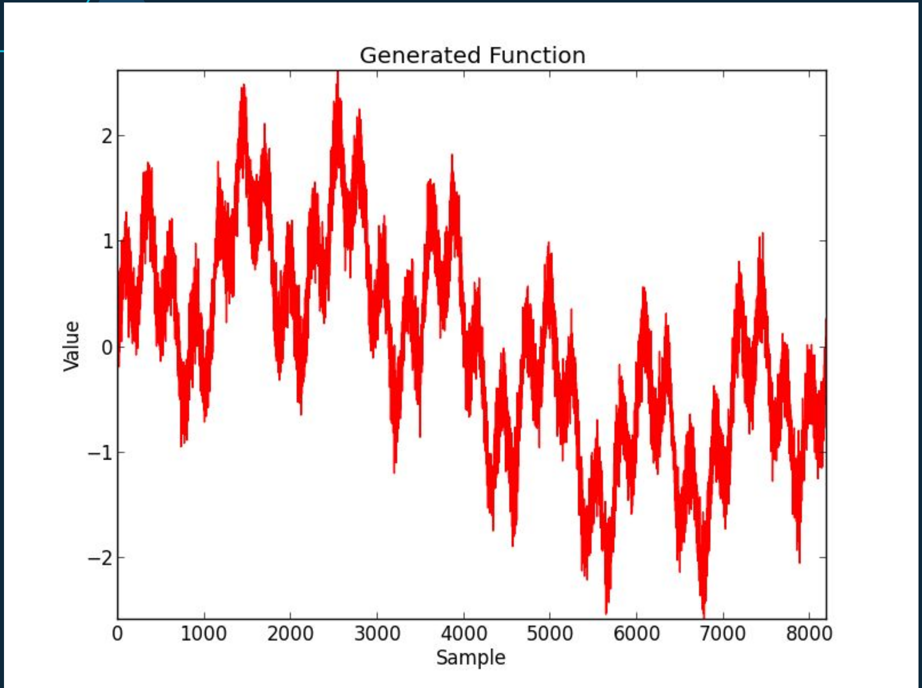
Time Domain Signal

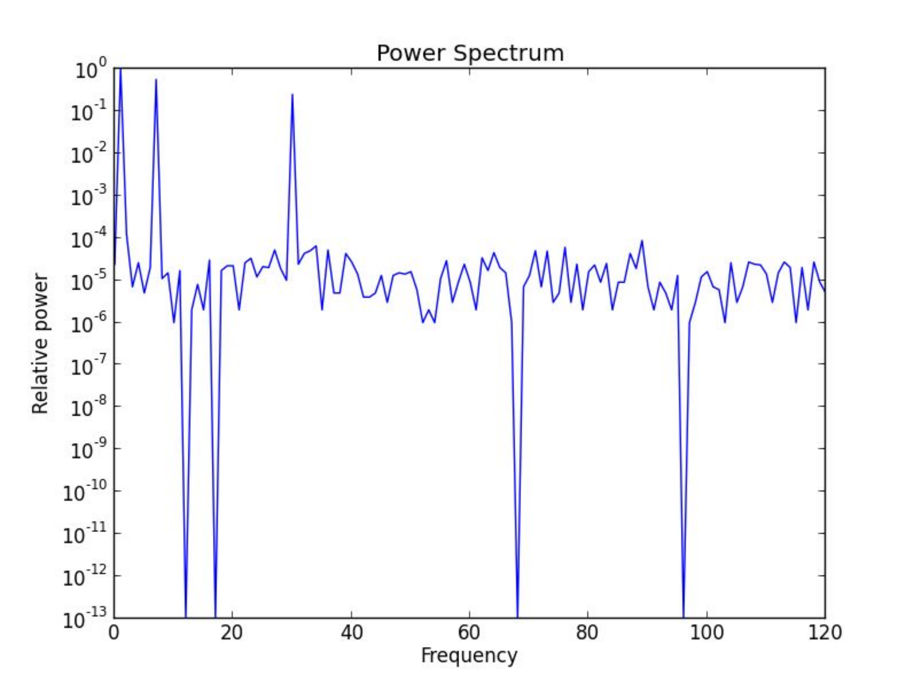


Frequency Domain Signal



Noise Resistance





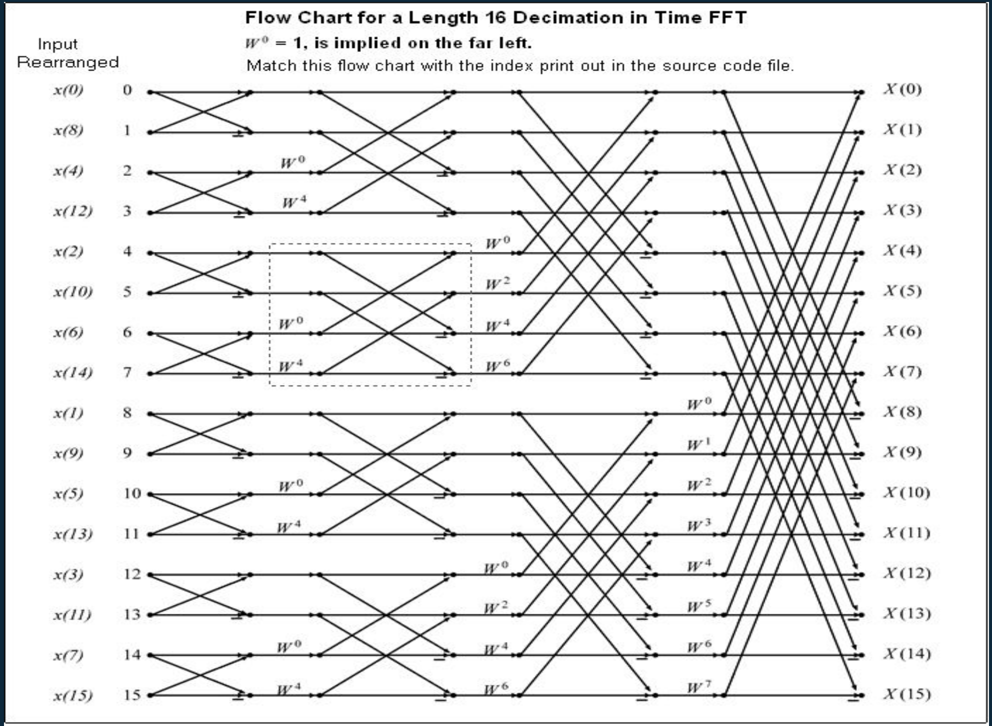
Fast Fourier Transform

- Most commonly used FFT is the Cooley - Tukey Algorithm
- Process is expressed much more succinctly and requires fewer operations
- Is inherently recursive, which can be tricky with large problem sizes
- Reduces work to **O(n * logn)** for 1D FFT
- A 2D FFT can be achieved by applying the operation over all rows, then all columns

Implementation

Serial, OpenMP, and MPI

Butterfly Operations



Burrus, C. Fast Fourier Transforms, Connexions Web site. http://cnx.org/content/col10550/1.22/.

Recursive Algorithm

$X_{0,\ldots,N-1} \leftarrow$ **ditfft2**(x, N, s): *DFT of* $(x_0, x_s, x_{2s}, \ldots, x_{(N-1)s})$:

  if $N = 1$ then

    $X_0 \leftarrow x_0$ *trivial size-1 DFT base case*

  else

    $X_{0,\ldots,N/2-1} \leftarrow$ **ditfft2**$(x, N/2, 2s)$ *DFT of* $(x_0, x_{2s}, x_{4s}, \ldots)$

    $X_{N/2,\ldots,N-1} \leftarrow$ **ditfft2**$(x+\mathrm{s}, N/2, 2s)$ *DFT of* $(x_s, x_{s+2s}, x_{s+4s}, \ldots)$

    for $k = 0$ to $N/2-1$ *combine DFTs of two halves*

      $\mathrm{t} \leftarrow X_k$

      $X_k \leftarrow \mathrm{t} + \exp(-2\pi i\, k/N)\, X_{k+N/2}$

      $X_{k+N/2} \leftarrow \mathrm{t} - \exp(-2\pi i\, k/N)\, X_{k+N/2}$

    endfor

  endif

General Ideas

- We can make our operations easier by sorting in bit reverse order
- This way, we can use an iterative approach, which solves several problems regarding stack and memory
- Requires problem size of 2^K nodes
- Allows for more parallelizable functions later on

OpenMP

- I quickly discovered that this problem benefited very little from parallel optimizations in 1D
- I turned my focus to the 2D problem
- Decomposed problem into “Stripes” of multiple rows of whole
- Ran FFT over each item in chunk, transposed information and repeated
- Still did not see much of a speed up in the OpenMP version

MPI

- Approach was very similar to OpenMP version
- Communication was the most important element of my changes
- Made all operations run based on 1D complex arrays (no complicated structures)
- Special operations were needed to package messages for distribution

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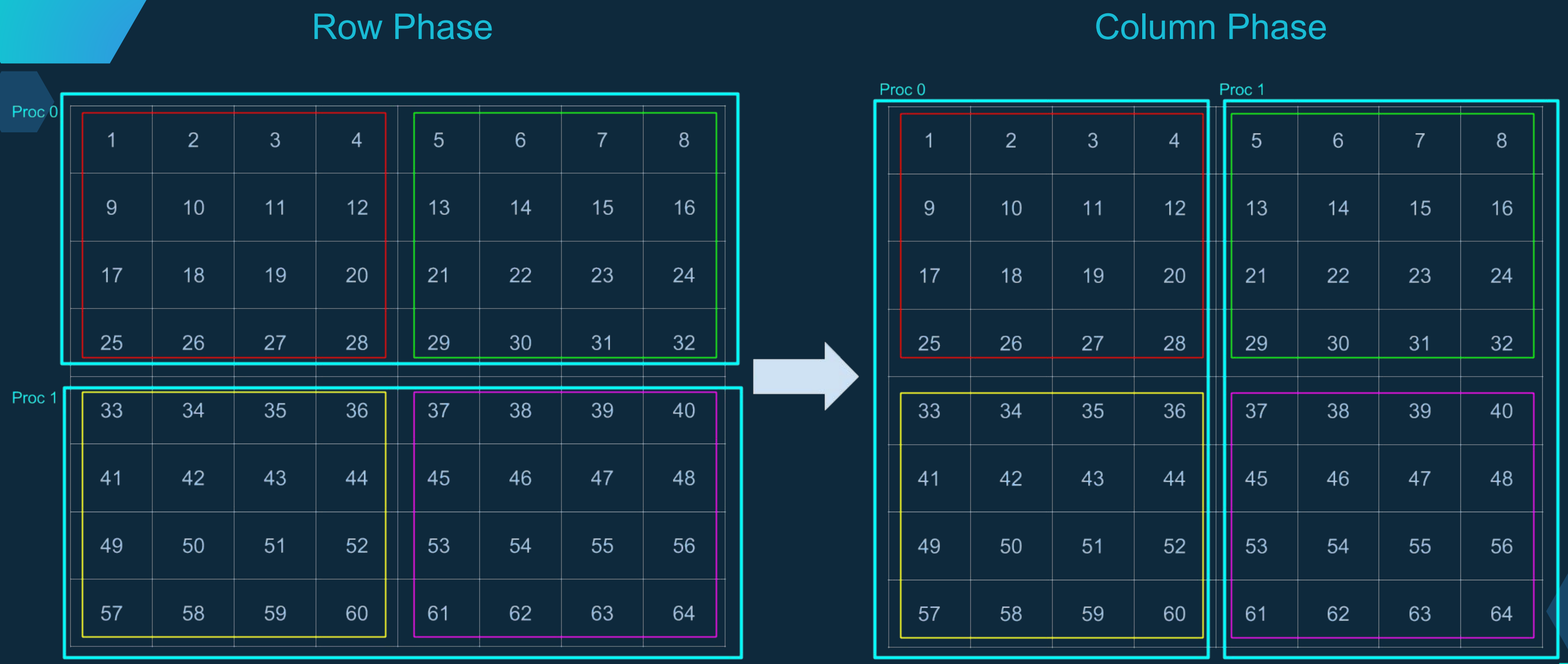
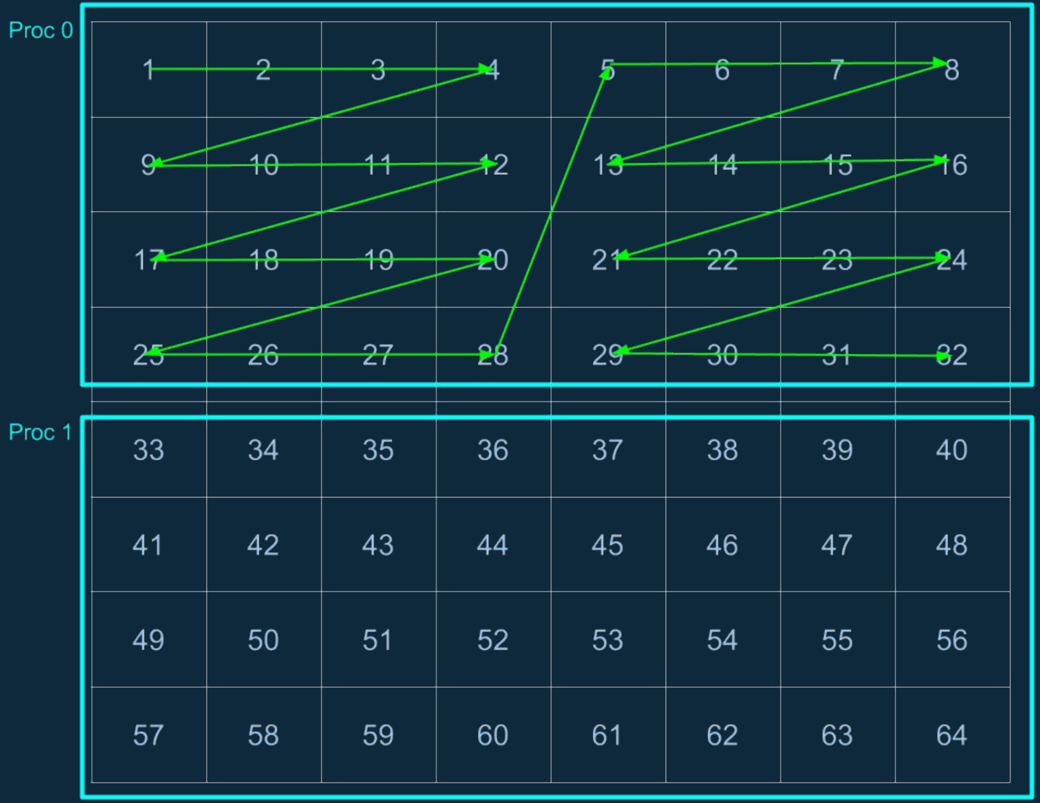

Row Phase
Proc 0
1 2 3 4 5 6 7 8
9 10 11 12 13 14 15 16
17 18 19 20 21 22 23 24
25 26 27 28 29 30 31 32
Proc 1
33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48
49 50 51 52 53 54 55 56
57 58 59 60 61 62 63 64
Column Phase
Proc 0
Proc 1
1 2 3 4 5 6 7 8
9 10 11 12 13 14 15 16
17 18 19 20 21 22 23 24
25 26 27 28 29 30 31 32
33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48
49 50 51 52 53 54 55 56
57 58 59 60 61 62 63 64

Data read pattern for transfer into send buffer

All to all would cut data along jump from 28 to 5 in this case

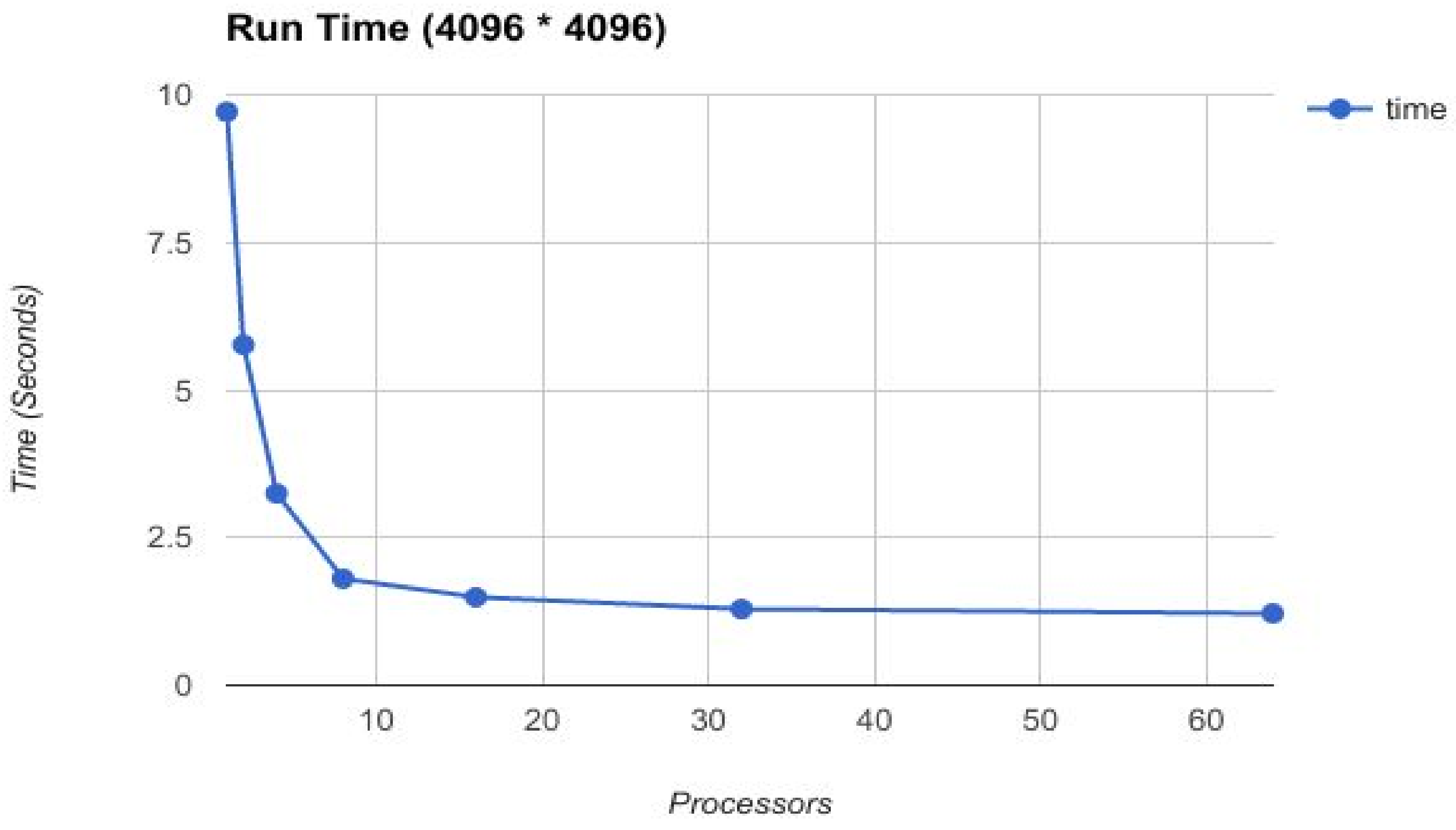


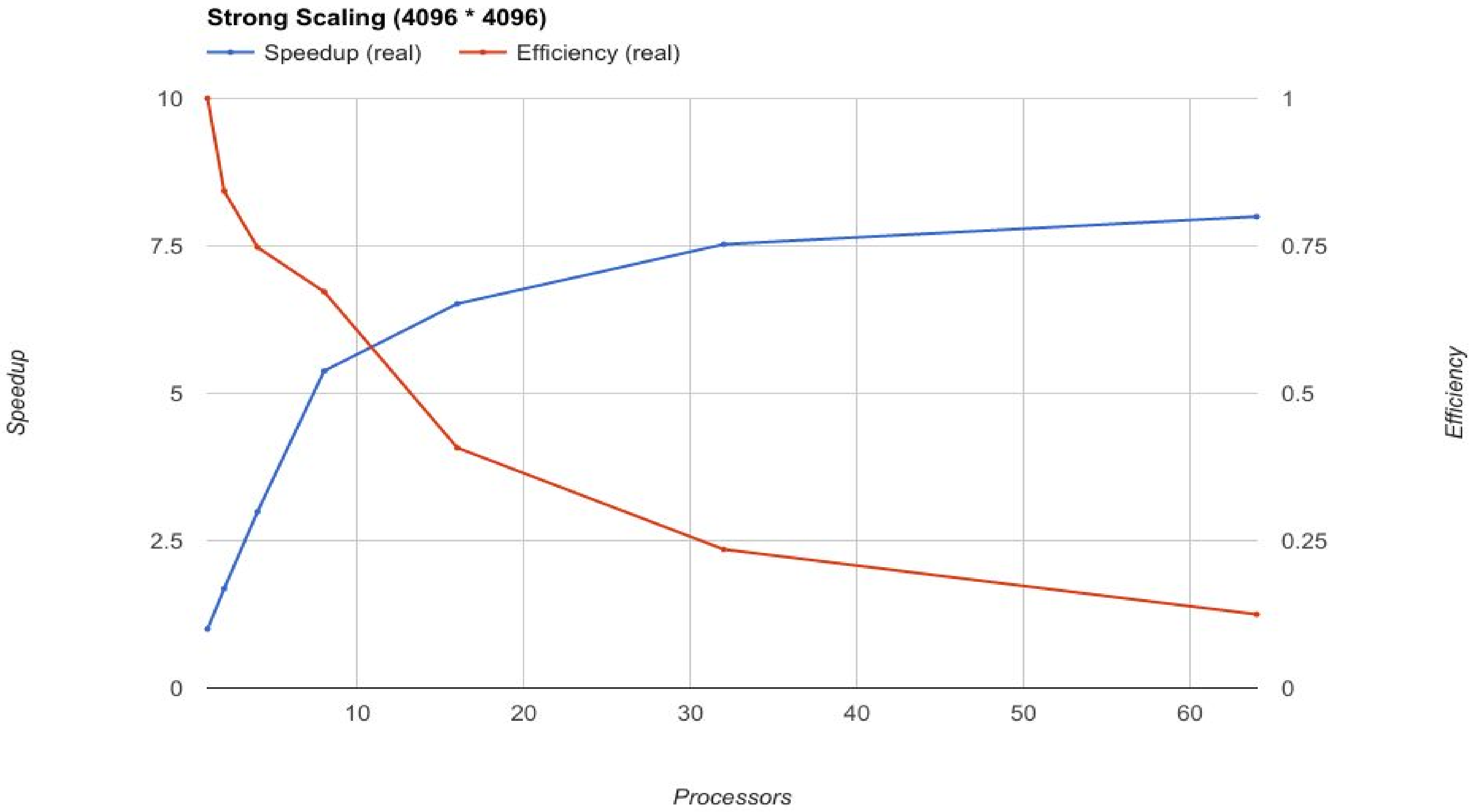
Parallel Performance

Large problems, fast times

Maneframe Strong Scaling

N	N * N	Processors	Time	Speedup (real)	Efficiency (real)
4096	16777216	1	9.712320089	1	1
4096	16777216	2	5.76313591	1.685249184	0.842624592
4096	16777216	4	3.248805046	2.989505357	0.7473763393
4096	16777216	8	1.806185007	5.377256511	0.6721570639
4096	16777216	16	1.490156889	6.517649357	0.4073530848
4096	16777216	32	1.290680885	7.524958493	0.2351549529
4096	16777216	64	1.214707851	7.99560164	0.1249312756


Run Time (4096 * 4096)
10
7.5
5
2.5
0
Time (Seconds)
10
20
30
40
50
60
Processors
time


Strong Scaling (4096 * 4096)
Speedup (real)
Efficiency (real)
Speedup
Efficiency
Processors
10
7.5
5
2.5
0
1
0.75
0.5
0.25
0
10
20
30
40
50
60


Weak Scaling Efficiency (Work per Proc = 65536)
Efficiency
1
0.75
0.5
0.25
0
10
20
30
40
50
60
Processors
Efficiency

Notes on Performance

- Although uncertain, I believe that network topology has a great deal of impact
- Weak scaling was poor, this is due to the fact that communication scales with size
- Due to limitations of size, only powers of two processors can be tested
- Was already very fast in serial, so all speedup is dependent on communications

Final Thoughts

Considerations and Future Work

Moving Forward

- Remove dependance on square matrix inputs
- Add real time system functionality, make callable from other applications
- Interface with image libraries for image signal analysis
- Run tests on higher numbers of procs, larger files
- Look into ways of distributing file I/O for faster load and dump operations

Thank You!

References

[1]"An Interactive Guide To The Fourier Transform – BetterExplained", *Betterexplained.com*, 2017. [Online]. Available: https://betterexplained.com/articles/an-interactive-guide-to-the-fourier-transform/. [Accessed: 08- May- 2017].

[2]"Butterfly Diagram", *Iowahills.com*, 2017. [Online]. Available: http://www.iowahills.com/Example%20Code/FFT%20Butterfly%20Diagram.png. [Accessed: 11- May- 2017].

[3]"Implementing FFTs in Practice", *Cnx.org*, 2017. [Online]. Available: http://cnx.org/contents/ulXtQbN7@15/Implementing-FFTs-in-Practice. [Accessed: 11- May- 2017].