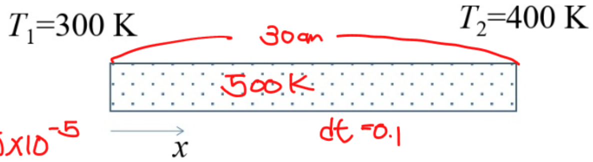
항공우주공학전공 201716418 송재한

MidTerm#2 Unsteady 1-D Heat Conduction

Write a Matlab code to solve the following unsteady, 1-D heat conduction problem.



- **Rectangular bar with length of 30 cm. Initial temperature distribution is T(x)=500K. dt=0.1 sec. for time marching.**
- **Your code must do the following tasks while it runs.**
  1. Plot temperature distribution (T(x)) at time t=3minutes and t=8 minutes.
  2. Plot temperature variation in time (T(t)) at the center (x=15 cm) of the rod.
  3. Plot temperature profile over the rod when the rod reaches a steady-state. Steady-state must be determined when the average relative temperature change over the entire rod between t and t+dt is less than 10^{-6}.
  4. On the terminal screen, print out the real time (physical time) for this rod to reach the steady-state temperature variation.
  5. On the terminal screen, print out the CPU time (computation time).

k		i= 1	2	3	…		…	…	n 번째 → (n-1)등분
		$\frac{30}{n}\times0$: 0cm	$\frac{30}{n}\times1$	$\frac{30}{n}\times2$	…	$\frac{30}{n}\times\frac{n}{2}$: 15cm	…		$\frac{30}{n}\times n$: 30 cm 실제로
1	0초	300K T_1'': $\frac{500-400}{h^2}$ ④	500K T_2'': ⑤	500K T_3'': ⑤	… T_4'': ⑤	500K $T_{\frac{n}{2}}''$: ⑤	…	…	400K T_n'': ⑥
2	0.1초	30	↓	↓	↓	↓			400
3	0.2초	300							400
⋮	⋮	300							400
1801	180초	300							400
	⋮	300							400
4801 번째에	480초	300							400

from Heat diffusion Equation,

$$\rho C_p \frac{\partial T}{\partial t} = k\nabla^2 T + \dot{q}_{gen}$$

we know no heat generation. so $\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\nabla^2 T$,

$\frac{k}{\rho C_p}$ = diffusivity = $\alpha = 8.5\times10^{-5}$

$\therefore \frac{\partial T}{\partial t} = 8.5\times10^{-5}\nabla^2 T$

Because of Assumption about 1D conduction,

$$\frac{\partial T}{\partial t} = 8.5\times10^{-5}\frac{\partial^2 T}{\partial x^2}$$

<pseudo code>

CPU time start

T(x.t) Matrix ⟹ ① T(i.k) ② T''(i,k) ③ T''(i,1) → T(1,2) → T(2.2). └ T''(i,k) → T(i,k+1)

t 이전 값과의 평균 → 10^{-6} 비교

plot T(x, 180), T(x.480)

plot T(15. t)

print (k · dt : real-time)

print (CPU time end)

$$\left.\frac{\partial T}{\partial t}\right|_k = \alpha\nabla^2 T_i$$

$$\frac{T_{k+1} - T_k}{dt} = \alpha\left.\frac{\partial^2 T}{\partial x^2}\right|_i$$

$$T_{k+1} = dt\left(\alpha\left.\frac{\partial^2 T}{\partial x^2}\right|_i\right) + T_k$$

cf. Taylor Series Expansion)

① $\left.\frac{\partial f}{\partial x}\right|_i = \frac{f_{i+1} - f_{i-1}}{2dx} = \frac{f_{i+1} - f_i}{dx} = \frac{f_i - f_{i-1}}{dx} \Rightarrow \left.\frac{\partial T}{\partial t}\right|_k$

(center) (forward) (backward)

② $\left.\frac{\partial^2 f}{\partial x^2}\right|_i = \frac{f_{i+1} - 2f_i + f_{i-1}}{dx^2} = \frac{f_{i+2} - 2f_{i+1} + f_i}{dx^2} = \frac{f_i - 2f_{i-1} + f_{i-2}}{dx^2} \Rightarrow \left.\frac{\partial^2 T}{\partial x^2}\right|_i$

(center) (forward) (backward)