Mikołaj Wielgos

problem 5

Sformułowanie silne

$$\frac{d^2 \varphi}{dx^2} = -\frac{\rho}{\varepsilon_r}$$

$$\varphi'(0) + \varphi(0) = 5$$

$$\varphi(3) = 2$$

$$\rho = 1$$

$$\varepsilon_r = \begin{cases} 10 & dla \ x \in [0,1] \\ 5 & dla \ x \in (1,2] \\ 1 & dla \ x \in (2,3] \end{cases}$$

Przekształcenie do sformułowania słabego (wariacyjnego)

$$\varphi''(x) = -\frac{\rho}{\varepsilon_{r}}$$

$$\varphi''(x)v(x) = -\frac{\rho}{\varepsilon_{r}}v(x)$$

$$\int_{0}^{3} \varphi''(x)v(x) = \int_{0}^{3} -\frac{\rho}{\varepsilon_{r}}v(x)$$

$$[\varphi'(x)v(x)]_{0}^{3} - \int_{0}^{3} \varphi'(x)v'(x) = \int_{0}^{3} -\frac{\rho}{\varepsilon_{r}}v(x)$$

$$\varphi'(3)v(3) - \varphi'(0)v(0) - \int_{0}^{3} \varphi'(x)v'(x) = \int_{0}^{3} -\frac{\rho}{\varepsilon_{r}}v(x)$$

$$-\varphi'(0)v(0) - \int_{0}^{3} \varphi'(x)v'(x) = \int_{0}^{3} -\frac{\rho}{\varepsilon_{r}}v(x)$$

$$-(5 - \varphi(0))v(0) - \int_{0}^{3} \varphi'(x)v'(x) = \int_{0}^{3} -\frac{\rho}{\varepsilon_{r}}v(x)$$

$$-5v(0) + \varphi(0)v(0) - \int_{0}^{3} \varphi'(x)v'(x) = 5v(0) - \int_{0}^{3} \frac{\rho}{\varepsilon_{r}}v(x)$$

$$B(\varphi, v) = L(v)$$

$$\varphi = \widetilde{\varphi} + w$$

$$\widetilde{\varphi} = 2e_n$$

$$B(\widetilde{\varphi} + w, v) = L(v)$$

$$B(w, v) = L(v) - B(\widetilde{\varphi}, v)$$

$$B(w, v) = L(v) - B(2e_n, v)$$

$$B(w, v) = L(v) - 2B(e_n, v)$$

$$B(w, v) = \widetilde{L}(v)$$