

高等数学公式背诵

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1 极限

1.1 两个重要极限

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

1.2 泰勒公式极限应用($x \rightarrow 0$)

$$\sin x = x - \frac{x^3}{6} + o(x^3)$$

$$\arcsin x = x + \frac{x^3}{6} + o(x^3)$$

$$\tan x = x + \frac{x^3}{3} + o(x^3)$$

$$\arctan x = x - \frac{x^3}{3} + o(x^3)$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + o(x^3)$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + o(x^2)$$

2 微分

2.1 反函数导数

$$x'_y = \frac{1}{y'_x}$$

$$x''_{yy} = \frac{-y''_{xx}}{(y'_x)^3}$$

2.2 常用求导公式

$$\begin{aligned}(\sec x)' &= \sec x \tan x & (\arcsin x)' &= \frac{1}{\sqrt{1-x^2}} \\(\csc x)' &= -\csc x \cot x & (\arccos x)' &= -\frac{1}{\sqrt{1-x^2}} \\(\tan x)' &= \sec^2 x & (\arctan x)' &= \frac{1}{1+x^2} \\(\cot x)' &= -\csc^2 x & (\operatorname{arccot} x)' &= -\frac{1}{1+x^2} \\[\ln(x + \sqrt{x^2 + 1})]' &= \frac{1}{\sqrt{x^2 + 1}} & [\ln(x + \sqrt{x^2 - 1})]' &= \frac{1}{\sqrt{x^2 - 1}}\end{aligned}$$

3 常用积分公式

$$\begin{aligned}\int a^x dx &= \frac{a^x}{\ln a} + C & \int \frac{1}{a^2 + x^2} dx &= \frac{1}{a} \arctan \frac{x}{a} + C \quad (a > 0) \\ \int \tan x dx &= -\ln |\cos x| + C & \int \frac{1}{\sqrt{a^2 - x^2}} dx &= \arcsin \frac{x}{a} + C \quad (a > 0) \\ \int \cot x dx &= \ln |\sin x| + C & \int \frac{1}{\sqrt{x^2 + a^2}} dx &= \ln \left(x + \sqrt{x^2 + a^2} \right) + C \\ \int \sec x dx &= \ln |\sec x + \tan x| + C & \int \frac{1}{\sqrt{x^2 - a^2}} dx &= \ln \left| x + \sqrt{x^2 - a^2} \right| + C \quad (|x| > |a|) \\ \int \csc x dx &= \ln |\csc x - \cot x| + C & \int \frac{1}{x^2 - a^2} dx &= \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C \\ \int \tan^2 x dx &= \tan x - x + C & \int \cot^2 x dx &= -\cot x - x + C\end{aligned}$$

4 中值定理

4.1 罗尔定理

$$f'(x) + kf(x) \Rightarrow f(x)e^{kx} \qquad [f^2(x)]' = 2f(x)f'(x) \qquad [f(x)f'(x)]' = [f'(x)]^2 + f(x)f''(x)$$

4.2 泰勒公式

4.2.1 泰勒原式

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)^{n+1}$$

4.2.2 泰勒展开式

$$\begin{aligned}
 e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + o(x^n) \quad (-\infty < x < +\infty) \\
 \sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1}) \quad (-\infty < x < +\infty) \\
 \cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n}) \quad (-\infty < x < +\infty) \\
 \frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n &= 1 + x + x^2 + x^3 + \cdots + x^n + o(x^n) \quad (-1 < x < 1) \\
 \frac{1}{1+x} &= \sum_{n=0}^{\infty} (-1)^n x^n &= 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + o(x^n) \quad (-1 < x < 1) \\
 \ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} &= x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + o(x^n) \quad (-1 < x \leq 1)
 \end{aligned}$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + \cdots + \frac{a(a-1)\cdots(a-n+1)}{n!}x^n + o(x^n) \quad \begin{cases} x \in (-1, 1), & \text{当 } a \leq -1, \\ x \in (-1, 1], & \text{当 } -1 < a < 0, \\ x \in [-1, 1], & \text{当 } a > 0 \text{ 且 } a \notin \mathbb{N}_+, \\ x \in \mathbb{R}, & \text{当 } a \in \mathbb{N}_+. \end{cases}$$

5 微分方程

6 欧拉方程

7 常用极数

$$\text{P极数}(n > 1) \quad \frac{1}{n^p} \begin{cases} p > 1, & \text{收敛,} \\ p \leq 1, & \text{发散.} \end{cases}$$

$$\text{P积分} \quad \int_1^{+\infty} \frac{1}{x^p} dx \begin{cases} p > 1, & \text{收敛,} \\ p \leq 1, & \text{发散.} \end{cases}$$

$$\text{广义P极数} \quad \sum_{n=2}^{\infty} \frac{1}{n \ln^p n} \begin{cases} p > 1, & \text{收敛,} \\ p \leq 1, & \text{发散.} \end{cases}$$

$$\text{广义P积分} \quad \int_2^{+\infty} \frac{1}{x \ln^p x} dx \begin{cases} p > 1, & \text{收敛,} \\ p \leq 1, & \text{发散.} \end{cases}$$

$$\text{等比极数} \quad \sum_{n=1}^{\infty} aq^{n-1} \begin{cases} |q| < 1, & \text{收敛,} \\ |q| \geq 1, & \text{发散.} \end{cases}$$

- 8 曲率半径
- 9 形心公式
- 10 旋转曲面
- 11 空间曲线