高等数学公式背诵

王泠风

2024 年 10 月 04 日

1 极限

1.1 两个重要极限

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=\mathrm{e}$$

1.2 泰勒公式极限应用(x→0)

$$\sin x = x-\frac{x^3}{6}+o(x^3) \qquad \arcsin x = x+\frac{x^3}{6}+o(x^3)$$

$$\tan x = x+\frac{x^3}{3}+o(x^3) \qquad \arctan x = x-\frac{x^3}{3}+o(x^3)$$

$$\cos x = 1-\frac{x^2}{2}+\frac{x^4}{24}+o(x^4) \qquad \ln(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)$$

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+o(x^3) \qquad (1+x)^a = 1+ax+\frac{a(a-1)}{2!}x^2+o(x^2)$$

2 微分

2.1 反函数导数

$$x'_y=\frac{1}{y'_x} \qquad x''_{yy}=\frac{-y''_{xx}}{(y'_x)^3}$$

2.2 常用求导公式

$$(\sec x)' = \sec x \tan x \qquad (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$
$$(\csc x)' = -\csc x \cot x \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$
$$(\tan x)' = \sec^2 x \qquad (\arctan x)' = \frac{1}{1+x^2}$$
$$(\cot x)' = -\csc^2 x \qquad (\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$
$$[\ln(x+\sqrt{x^2+1})]' = \frac{1}{\sqrt{x^2+1}} \qquad [\ln(x+\sqrt{x^2-1})]' = \frac{1}{\sqrt{x^2-1}}$$

3 常用积分公式

$$\int a^x dx = \frac{a^x}{\ln a} + C \qquad \int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\frac{x}{a} + C \ \ (a>0)$$
$$\int \tan x dx = -\ln|\cos x| + C \qquad \int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{x}{a} + C \ \ (a>0)$$
$$\int \cot x dx = \ln|\sin x| + C \qquad \int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right) + C$$
$$\int \sec x dx = \ln|\sec x + \tan x| + C \qquad \int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left|x+\sqrt{x^2-a^2}\right| + C \ \ (|x|>|a|)$$
$$\int \csc x dx = \ln|\csc x - \cot x| + C \qquad \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$
$$\int \tan^2 x dx = \tan x - x + C \qquad \int \cot^2 x dx = -\cot x - x + C$$

4 中值定理

4.1 罗尔定理

$$f'(x) + kf(x) \Rightarrow f(x)e^{kx} \qquad [f^2(x)]' = 2f(x)f'(x) \qquad [f(x)f'(x)]' = [f'(x)]^2 + f(x)f''(x)$$

4.2 泰勒公式

4.2.1 泰勒原式

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$$

4.2.2 泰勒展开式

$$
\begin{aligned}
e^x &= \sum_{n=0}^{\infty}\frac{x^n}{n!} && = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+o(x^n)\ (-\infty<x<+\infty)\\
\sin x &= \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} && = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+o(x^{2n+1})\ (-\infty<x<+\infty)\\
\cos x &= \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} && = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+o(x^{2n})\ (-\infty<x<+\infty)\\
\frac{1}{1-x} &= \sum_{n=0}^{\infty}x^n && = 1+x+x^2+x^3+\cdots+x^n+o(x^n)\ (-1<x<1)\\
\frac{1}{1+x} &= \sum_{n=0}^{\infty}(-1)^n x^n && = 1-x+x^2-x^3+\cdots+(-1)^n x^n+o(x^n)\ (-1<x<1)\\
\ln(1+x) &= \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} && = x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots+(-1)^{n-1}\frac{x^n}{n}+o(x^n)\ (-1<x\le 1)
\end{aligned}
$$

$$
(1+x)^a = 1+ax+\frac{a(a-1)}{2!}x^2+\cdots+\frac{a(a-1)\cdots(a-n+1)}{n!}x^n+o(x^n)\quad
\begin{cases}
x\in(-1,1), & \text{当}a\le -1,\\
x\in(-1,1], & \text{当}-1<a<0,\\
x\in[-1,1], & \text{当}a>0\text{且}a\notin\mathbb{N}_+,\\
x\in\mathbb{R}, & \text{当}a\in\mathbb{N}_+.
\end{cases}
$$

5 微分方程

6 欧拉方程

7 常用极数

P极数$(n>1)$ $\dfrac{1}{n^p}\begin{cases}p>1, & \text{收敛},\\ p\le 1, & \text{发散}.\end{cases}$

P积分 $\displaystyle\int_1^{+\infty}\frac{1}{x^p}dx\begin{cases}p>1, & \text{收敛},\\ p\le 1, & \text{发散}.\end{cases}$

广义P极数 $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln^p n}\begin{cases}p>1, & \text{收敛},\\ p\le 1, & \text{发散}.\end{cases}$

广义P积分 $\displaystyle\int_2^{+\infty}\frac{1}{x\ln^p x}dx\begin{cases}p>1, & \text{收敛},\\ p\le 1, & \text{发散}.\end{cases}$

等比极数 $\displaystyle\sum_{n=1}^{\infty}aq^{n-1}\begin{cases}|q|<1, & \text{收敛},\\ |q|\ge 1, & \text{发散}.\end{cases}$