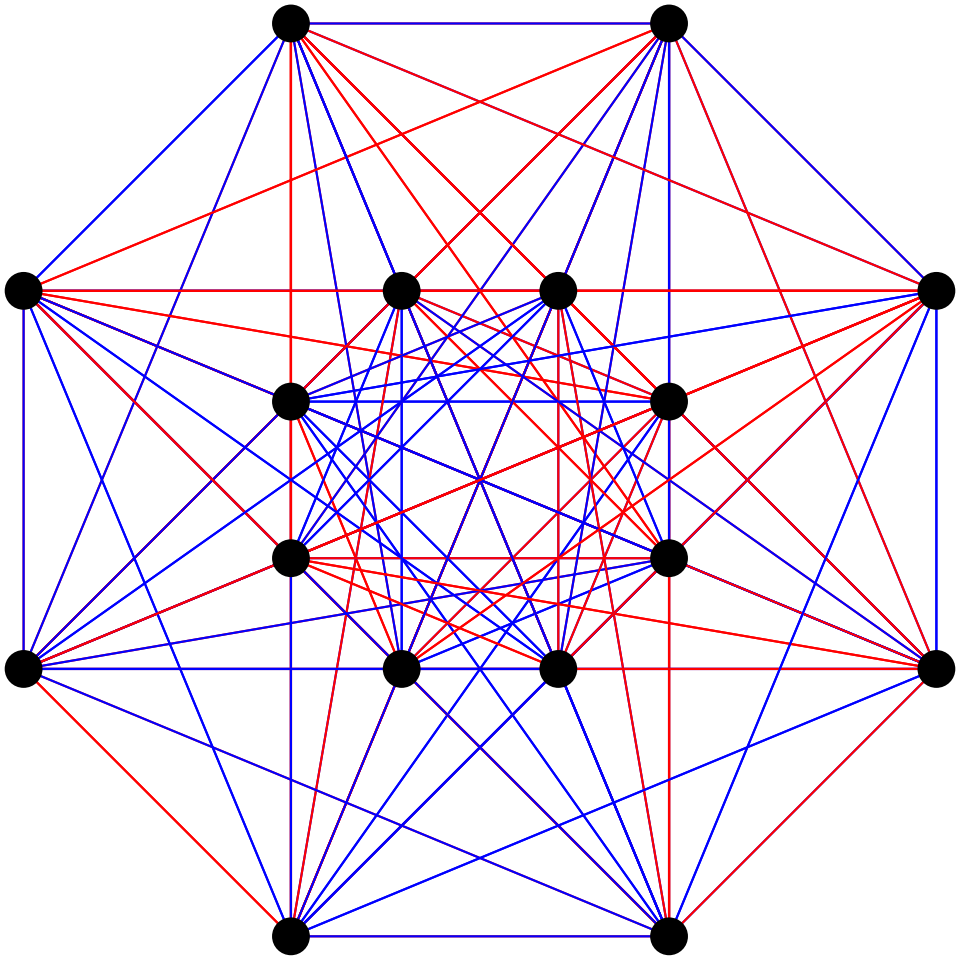

THE LIBRARIAN

Supplement №3, Volume II

Isky Mathews / Adventures in Recreational Mathematics VII / Page 8

On the cover
Benedict Randall Shaw's rendering of a complete graph on the vertices of a hypercube.

Write for *The Librarian.*

Contact the editor for more information.

joshua.loo@westminster.org.uk

Miscellany

On orthographic diversity Despite orthography's limited influence, orthographic diversity has a number of benefits.

Three observations on the Skripal affair The Skripal affair has also revealed a number of pathologies in the present political system, though it is probably not unique in this regard.

Adventures in Recreational Mathematics VII introduces us to large numbers.

On the necessity of trust Beyond the theoretic limits of knowledge, there are practical constraints on the spread of knowledge. A better model to overcome this is needed.

News

Large sections of the library are to be silent, due to public examinations; consult the signs for further information.

A number of old books were sold in the Lecture Room, to clear space. Additionally, many of the art books were sold, due to building work taking place in the Carleton Gallery.

We regret the late publication of this issue; unfortunately public examinations and a number of other factors have caused delays.

The Librarian wishes to congratulate Benedict Randall Shaw, a member of the Library Committee, on his being selected as a member of the British team for the International Mathematics Olympiad.

A number of small typographic changes have occurred; kerning before semi-colons and colons has been increased, footnotes now exclusively reside in the right column, text is now more rigorously and comprehensively kept in a grid, including in mathematic articles where feasible, protrusion has been implemented, and the margins have been slightly altered.

Letters

Policy on letters We urge readers not to write any letters that would make necessary the formulation of a policy more extensive than this.

Submissions Readers may submit letters to the editor, or to Room 5, Lower Corridor, College. Identification is unnecessary; impersonation is prohibited.

Note on letters Letters are reproduced as they were sent. Consequently, *The Librarian* is not responsible for, *inter alia*, any intellectual inadequacies that may arise, or errors of grammar, syntax, or orthography. We are, of course, responsible for any errors of our own creation.

Guidance We suggest that readers in need of advice consult the October 1874 number of *The Elizabethan*.

The Library
librarians@westminster.org.uk
Chair, Library Committee
jonny.heywood@westminster.org.uk
Editor, *The Librarian*
joshua.loo@westminster.org.uk

Sir,

I note that you have ceased to receive letters. This is something of a travesty, compounded by this modern innovation of 'notices'. I therefore write this letter in the hope of inducing other readers to write further letters, preferably differing in content from mine; were this not to occur, that would, at least, be equally humorous, though possibly something of a waste of space.

I beg to subscribe myself,
Lector inscribens

Sir,

I desire to 'employ the medium' of your 'valuable journal' (or whatever it was that your last editorial said) to draw attention to the seeming disappearance of Deus misereatur at Latin prayers. Even those mistaken in preference should be concerned by this dramatic change. If we are to alter our rituals thus, even Ad te levavi's adherents may soon find themselves disappointed.

I am, &c.,
Dies iræ

Sir,

For some reason, I have the impression that the last pages of your article on Fukuyama seem to have disappeared. Will the previous practice of filling pages to reach a multiple of four with sudokus be reinstated? On another note, I was rather distressed to note the inconsistency in styling between the display of metadata in the review of Benedict Randall Shaw's 'algorithmic music' and rest of *The Librarian*.

I remain, &c.,
Senex II

Sir,

By the appearance of your letters, there appear to be more *senes* and Babylonians than there are pupils in your readership.

About *The Librarian*

Note: articles in *The Librarian* do not necessarily reflect the views of any entity, notwithstanding any impression created to the contrary, unless there is some explicit indication to the otherwise.

For the purposes of clarity, 'any entity' refers, *inter alia*, without prejudice as to the to the paragraph above, authors, those connected with them, *The Librarian*, the editors thereof, the Library Committee, the members, the Chair and the Assistant Chair thereof, the library, the librarians, and the school.

The Librarian is the publication of the Library Committee of Westminster School. The existence of a Library Committee dates back to at least December 1879, when the editor of *The Elizabethan* replied to a letter on the 'disgraceful' state of the books in the library, that '[s]ome years ago a regular library committee was in existence'. The present state of the library is far removed from its state in the late 1870s; the employment of four librarians, the Library Committee and the general interest of the rest of the school have all combined to ensure that there is little danger of a lapse into disrepair. The Library Committee broadly exists to support the work of the librarians; some examples of this support include the conveying of pupil views to the librarians, direct support (e.g., in desk duty, and charitable activities), and the publication of *The Librarian*.

Some find that they are unwilling to 'go all the way to the library'. Consequently, *The Librarian* offers a subscription service. Readers may email the editor, with a specified destination, which must either be an email or a physical location. This is, of course, free, as is *The Librarian* in general. Issues are occasionally uploaded to `https://librarian.cf`, which is likely to be increasingly frequently updated. The athletically blessed are encouraged to make the journey to the library. The physical location in most circumstances must be in the school; we do not rule out alternative arrangements, but most would be insufficiently feasible.

The Librarian is typeset in a Bembo-like font using LaTeX 2_ε. Authors retain copyright of their works; rights to everything else remain *The Librarian*'s unless the context makes this repugnant.

We encourage submissions of all kinds. These include, but are not limited to, articles, reviews, letters, puzzles, short stories, poems, compositions, and answers to problems in the Adventures in Recreational Mathematics series. Submissions may be sent to the editor. Readers may also place notices in *The Librarian*, by prior arrangement with the editor.

As *The Librarian* is the publication of the Library Committee, it was initially principally concerned with books, and reviews of works that one might find in the library. However, many other articles have also been found. Thus *The Librarian Supplement* was formed, to house those articles that do not fit in the traditional scope of *The Librarian*. The division is to some extent arbitrary, and so we encourage readers to read both.

A sufficient number of mathematic and scientific articles are published in *The Librarian Supplement* to require the services of Benedict Randall Shaw and Isky Mathews respectively, so correspondence on those subjects should be directed to them.

Recipes sought

for a new culinary column in *The Librarian*.

benedict.randallshaw@westminster.org.uk

On orthographic diversity

Joshua Loo

The Librarian is home to what is certainly unconventional orthography. The use of the diæresis, acute accent, ligatures and '-xion' spellings is often commented upon by readers; many complain that, variously, this is an affectation, it is incorrect, it does not reflect modern usage, and so on.

Any discussion on orthography must recognise that orthography has a relatively limited effect. It does not change the content of what is said; to the extent that one as a reader changes one's mind, this is a choice. There are, however, several reasons why the orthographic unorthodoxy of this publication could potentially be beneficial. They do not seem particularly convincing, because orthography is so unimportant; however, given that most of the criticisms of the use of, for example, the diæresis are that it is 'an affectation', or 'antiquated', they nevertheless are important in relative terms. Five seem particularly pertinent.

The assumption here is that conservative orthography does not inhibit comprehension; were this to occur, that as a harm would outweigh the benefits claimed. It is relatively clear, however, that the diæresis, or acute accent, or ligature, does not impede comprehension. Though it may induce curiosity, there is, at worst, an initial fixed cost. It is relatively trivial for a reader to use the internet to find that 'coördinate' has a diæresis to indicate that 'oo' has two separate syllables, and so on. Where this is too frequent, there is of course a non-negligible cost.

The trade-off might be slightly different in other languages. For example, there is a strong tradition of prescription in French, from the Académie, and there is a tradition of centralisation, if not prescription, in Spanish. This article will not attempt to determine where the balance of advantage lies in languages with strong central authorities; of course, in English, such authority is distributed amongst publications, dictionaries, style guides, &c.

First, the use of older orthography in particular ensures that readers are familiarised with their use, so that when they read older texts, they are not surprised.

Second, some unconventional orthography is helpful. The diæresis, for example, helps those who are not native speakers of English to understand when words are disyllabic. Of course, this demand must be balanced against the need to avoid confusing native speakers too much. Though the placing of a diæresis on, for example, 'react', would be logic, as the 'ea' sound is often monosyllabic, because the spelling 'reäct' is so uncommon as to not appear in dictionaries or corpora, it is not used.

Third, the presence of diverse orthographies in general, that is, local prescription, helps to decrease the prevalence of global prescription. The promotion of local prescription necessarily promotes an understanding and acceptance that, at the very least, there exist multiple acceptable orthographies. By contrast, global prescription either promotes the tyranny of the majority or of a specific privileged minority. Competition between orthographies is far superior to such tyranny.

Indeed, the understanding that such tyranny is often problematic is common even amongst those who most describe themselves as 'descriptivist', for they are the most likely to promote, for example, gender-neutral language. Gender-neutral language is, and certainly was, out of step both with the language of the establishment and the masses. With local prescription comes acceptance of difference, and so the *possibility* of change. Equally, of course, it means that one must accept certain remnants from an age gone by—perhaps, even after gender-neutral language becomes the norm, some will insist on the use of the generic 'he'. Yet this will become fringe usage, and so harm very few; the alternative is a stagnation, where proposed linguistic changes either succeed rapidly or fail. Most of the time, they will fail. Far better is an acceptance of competition between linguistic conventions.

Readers who accept, for example, that some prefer the diæresis, or that some would prefer not to use one on 'daïs', and so on, are far less likely to promote linguistic standards which, for all the improvements thus far, still are problematic, hence the improvement.

Fourth, orthographic diversity acts as a filter. Some potential readers will be too close-minded to continue to read after the first sighting of a spelling or linguistic decision that they dislike. The benefit here is twofold: first, the reader does not waste their time, and second, the publication need not provide or alter itself for them, or publish their letters.

Fifth, choice of orthography influences æsthetic flavour; as æsthetic taste differs, it is only natural that there should be diversity in orthography as well.

Is there a cost? There almost certainly exists one—orthographic diversity may cause confusion or issues when one learns a language. Yet it is hardly the case that the principal cause of Britain's present educational woes with the English language are the product of orthographic diversity, though it may be a minor contributor. The English language has coped without significant ill effect with the gulf between American and Commonwealth orthography; it also manages to proceed despite other differences. Hence, in a comparative between internal and external diversity, and merely external diversity, it makes little difference whether arguments over ligation are between Britons and Americans, or Britons and other Britons; the difference still exists.

More importantly, it is not clear why schools need the rest of the world to use uniform orthography. Perhaps children should initially read books that employ standardised orthography, but, thereafter, it is not clear quite what is inhibited. Indeed, to the extent that there are non-obvious differences in usage, it may be better that people are made aware by visible difference that there is occasionally a difference in usage in other, less visible forms. One particularly pertinent example is the use of 'would' in countries without a long history of English usage. In India, for example, it is common to hear that the government 'would implement a scheme'; it is not that, were something the case, the government would, but rather that the government will.

As contact between speakers of languages from different regions increases, due to the rise of media which are not constrained by geographical distance, such as the internet, understanding of such difference will become increasingly necessary. Hence, even on the metric upon which it seems most likely that orthographic diversity fails, it seems, instead, that such diversity is beneficial.

Three observations seem particularly pertinent in the wake of the of the Skripal affair.

Who?

It is fairly trivial to narrow down possible perpetrators to those who would have a motive to commit such a crime, and those who were capable of developing the sort of chemical weapon in question. The suggestion of the spokeswmoan of the Russian Foreign Ministry, that Sweden, the UK, Czechia and Slovakians were the most likely to have had the capacity to produce such weapons[1] may indeed be true.

What is missing is an explanation as to quite why the Swedes, Czechs, Slovakians, or British should wish to poison a fairly irrelevant spy. In particular, it is fairly trivial to rule out the first three. The Swedes, Czechs, and Slovakians have no interest in the spy; they do not need to demonstrate to anyone that they have chemical weapons capability, and gain far more from the perception that they are responsible international actors than any perception of international power. Equally, the United Kingdom still has one of the strongest armies in Europe[2]; they would not wish to alienate the United Kingdom.

The government equally has little incentive to poison spies under its protection. Most obviously, it is not a very good idea to poison spies whom one has used in the past, as this is likely to discourage future potential informants, who may think that the government is untrustworthy, or even malevolent. Further, its weakness has been shown in its response. Britain has no military leverage over Russia; to the extent that it could punish members of the Russian élite by a reduction of their influence in the United Kingdom, advocacy thereof has been responded to with cries of 'politicisation', 'shame', &c..[3,4] Given the government is so unwilling to remove Russians, even those who have significant connexions with the Putin régime, from London, it is not clear that it had some secret desire to end up in a conflict with the Russian government.

Since Israel is featured in conspiracy theories regularly, it would be surprising for Israel not to have been blamed somewhere; indeed, Craig Murray notes that Israel may have had the capability to carry out the attack. Most obviously, Israel would not desire to cause Russo-American tension, which appear to have been precipitated, because they both need American military aid, and Russian coöperation, in preventing Hezbollah from acquiring particularly threatening Iranian military hardware.

What of the Russians? It is fairly clear that the Russian government might have a motive: Skripal, after all, was described a 'traitor' by Russian state television[5]. There are several clear benefits. First, the attack deters future potential informants. Second, it publicly discredits the British state's intelligence apparatus. Third, it shows, by contrast, their strength.

Equally, however, there are other 'Russian' actors who also would have benefited from the attack. There are many independently operating parts of the Russian state, who could plausibly have wished to demonstrate their loyalty to Putin in the attack. Had it been successful, and killed its target, Putin would have been more likely to reward a particular unit, for example. There are many different law enforcement agencies in Russia,[6] each of which would likely wish to increase their funding, prominence, and power; as the attack can be disowned by the relevant unit, it may have been in the interest of a particular part of the Russian state to attack. Similarly, the Russian state has been associated with criminal organisations, who equally may have wished to please Putin.[7]

It is important to note that these claims are not definitive; there is no evidence to support them. However, establishing that it is *possible* that someone else could simultaneously have had a motive and the ability to attack other than Putin is important. Equally, however, the evidence which suggests that Russia was guilty has not been publicly produced either. Porton Down say that they are unable to verify the location of the novichok.[8] Perhaps other sources have confirmed to the government that the Russian government are to blame. However, given that so much of the argument that the Russian government were to blame has rested upon the specious assertion that no other state or institution could possibly have had any motive other than the Russian government, it is important to rebut it. Of course, in finding the guilty party, one must rely on more than analysis of the motives of different parties.

Another possibility is that a Russian group desired to attack Putin. There was a reasonable risk after the attack of further restrictions on many of Putin's friends and associates in London.

Boris Johnson says that the response of the Russian government does not suggest that it truly believes itself innocent.[9] This is true; most of its statements have been rather combative. However,

---

[1] "Russia Hints UK Lab Was Nerve Agent Source". In: *BBC News. UK* (Mar. 18, 2018). URL: http://www.bbc.co.uk/news/uk-43446312 (visited on 03/18/2018).

[2] Silvija Guzelytė. *National Defence Data 2013-2014 and 2015 (Est.) of the 27 EDA Member States*. European Defence Agency, June 2016. URL: https://eda.europa.eu/docs/default-source/documents/eda-national-defence-data-2013-2014-(2015-est)5397973fa4d264cfa776ff000087ef0f.pdf.

[3] Daniel Capurro. "Front Bench: If Jeremy Corbyn Thought He Could Politicise the Salisbury Poisoning, He Has Miscalculated". In: *The Telegraph* (Mar. 16, 2018). ISSN: 0307-1235. URL: https://www.telegraph.co.uk/news/2018/03/16/front-bench-jeremy-corbyn-thought-could-politicise-salisbury/ (visited on 03/18/2018).

[4] "David Miliband Says Corbyn 'sounded like Donald Trump' over Skripal". In: *The Independent* (Mar. 13, 2018). URL: http://www.independent.co.uk/news/uk/politics/david-miliband-jeremy-corbyn-rusian-spy-poisoning-sergei-skripal-commons-theresa-may-tory-donors-a8252991.html (visited on 03/18/2018).

[5] B. B. C. Monitoring. "Russian State TV Anchor Warns 'Traitors'". In: *BBC News. Europe* (Mar. 8, 2018). URL: http://www.bbc.co.uk/news/world-europe-43330498 (visited on 03/18/2018).

[6] *Russia / Europe / Member Countries / Internet / Home - INTERPOL*. URL: https://www.interpol.int/Member-countries/Europe/Russia (visited on 03/18/2018).

[7] Brian Whitmore. "Putin's Mafia Statecraft". In: *Radio Free Europe/Radio Liberty* (Oct. 27, 2015). URL: https://www.rferl.org/a/putins-mafia-statecraft/27329898.html (visited on 03/18/2018).

[8] Steven Morris and Pippa Crerar. "Porton Down Experts Unable to Verify Precise Source of Novichok". In: *The Guardian* (Apr. 3, 2018). URL: http://www.theguardian.com/uk-news/2018/apr/03/porton-down-experts-unable-to-verify-precise-source-of-novichok (visited on 04/06/2018).

[9] "Russia Spy Response 'Smug Sarcasm'". In: *BBC News* (Mar. 18, 2018). URL: http://www.bbc.co.uk/news/av/uk-politics-43451184/spy-poisoning-boris-johnson-criticises-russia-s-response (visited on 03/18/2018).

even were it not to have ordered the attack, it would still benefit, and indeed is benefiting, from the publicity surrounding the case. It would optically be something of a concession, especially for a state which cares so much about projecting its strength, to grovel before the government, explaining that it had not ordered the attack; such a strategy would require an explanation of how chemical weapons were lost, and international scrutiny of a most unwelcome kind.

Comparisons to Iraq

Comparisons to Iraq are somewhat helpful, but are not perfect. Craig Murray, whose blog is an excellent read, even for those who disagree with its contents, suggests that there is to some extent a rush to blame Russia.[10] It is certainly politically expedient to do so, and so this is a plausible claim. Murray may be lying—it would be unwise to trust him absolutely—but, at the very least, he provides another narrative which is plausible, and so should be considered.

A comparison to Iraq might suggest that there is some sort of conspiracy within the government to start a war with Russia. This is fairly obviously a bad idea. A significant proportion of its instigators would die in a nuclear strike from Russia. There is no effective defence against this. Russia's defence against nuclear attack is a system which uses *nuclear bombs* to cause missiles not to hit their targets. Since the West is less willing to take such risks, we instead have systems which work approximately half the time, when the test projectiles have been fired *in idealised conditions*.[11] Britain is obviously behind the United States in such matters, so there is no reason for it to start a war. Further, as demonstrated above, it was not particularly politically expedient to highlight British weakness at this point in time. Those who could be behind such a conspiracy clearly operate in knowledge of these concerns. There is no motive which could possibly outweigh the risk of nuclear war arising from an attempt to cause war. Even arms manufacturers would prefer to lose business than to initially sell lots of arms before dying in such a war.

Further, if one trusts Murray, one should note that it seems that the Foreign and Commonwealth Office is not being very successful in attempting to prevent leaks. Consequently, were there such a conspiracy, Murray would have noticed this, and published this on his blog. There are a sufficient number of people who read his blog, especially after the revelations, to grant him effective immunity from prosecution; certainly, there would be a great embarrassment for the government were he to be prosecuted for revealing an attempt to deceive parliament in an attempt to start a war.

Discussion

Discussion has been predictably suboptimal. Consider Corbyn's article in *The Guardian*[12]. There are several issues with the view espoused in the article. First, it is not clear how the responses Corbyn outlined—that 'Russian money be excluded from our political system', 'Magnitsky-style sanctions', and 'measures to tackle the oligarchs[sic] and their loot' can be reconciled with a desire to 'reduce tensions and conflict wherever possible.' Clearly, it is possible to do nothing in this situation; Corbyn takes the intellectually lazy option of using the phrase 'wherever possible' to omit from his article that there is necessarily a trade-off between strong action and the avoidance of conflict. The article, rather than providing a mechanism by which the optimal trade-off can be determined, ignores the need for a trade-off, whilst criticising May's.

Second, Corbyn ignores the possibility that we may not find the ultimate perpetrator with a satisfying degree of certainty. Many crimes remain unsolved, leaving a list of suspects; the risk of this always exists, and this attack is no exception. If we do not find the culprit with any certainty, what then? Should we proceed with something less than certainty? Corbyn is right to call for rigour—'the perpetrators . . . must be identified', but equally important is what should happen were one unable to definitively determine the identity of the perpetrator.

Third, Corbyn resorts to rhetoric appealing primarily to his own base *in rebus* Libya and Afghanistan. Certainly, the liberation of Libya was suboptimally executed; it is even possible that Libya is worse off than it would have been had Gadaffi stayed. None of these conclusions are, however, *prima facie* true, at least to a significant number of people. Corbyn's call for calm is an important; the inclusion of not particularly rigorous references to these examples reduces the appeal of this argument.

Criticism of Corbyn has, however, focused on his 'politicisation' of the attack. Quite why a political leader should not 'politicise' an issue, when politics clearly is clearly an important determinant of the outcome of such cases, is unclear. Critics need to explain why the normal democratic process of criticism and replacement is inadequate in this scenario. 'Politicisation', that is, the treating of this affair in a political manner, could expel thousands of Russian oligarchs in London, prevent the use of the London Stock Exchange by large numbers of Russian companies, prevent the use of the financial system to launder and transfer money, prevent the use of financial consultancy services in London, and significantly disrupt the lives of large numbers of Putin's cronies; 'politicisation' could also alter our response, whether by inducing the government to supply samples to the Russians, or the Organisation for the Prohibition of Chemical Weapons, altering the number of diplomats expelled, or a change in policy on chemical weapons. These may be bad ideas; equally, they may be good ideas. Politics, presumably, is about determining whether these are indeed good or bad ideas, and then their implementation, if they are good ideas. It is no use declaring an area affected by politics to be sacrosanct, because there is no particular reason why the *status quo*, almost certainly implemented as a result of some previous political situation, should be superior to any alternative.

As the government has relatively few options, what seems most likely is that we shall soon forget about the affair; occasionally, articles on progress in the investigation will be found, but little further action will be taken. More interesting and difficult to predict will be the effect on Corbyn's reputation, especially in relation to foreign policy; if the affair were to stop his political rise, that would likely be its greatest effect.

---

[10] Craig Murray. *Craig Murray*. URL: https://www.craigmurray.org.uk/ (visited on 03/18/2018).

[11] "Could the U.S. Actually Shoot down a North Korean Missile?" In: *PBS NewsHour* (Nov. 28, 2017). URL: https://www.pbs.org/newshour/world/could-the-u-s-actually-shoot-down-a-north-korean-missile (visited on 03/18/2018).

[12] Jeremy Corbyn. "The Salisbury Attack Was Appalling. But We Must Avoid a Drift to Conflict | Jeremy Corbyn". In: *The Guardian* (Mar. 15, 2018). URL: http://www.theguardian.com/commentisfree/2018/mar/15/salisbury-attack-conflict-britain-cold-war (visited on 03/18/2018).

Adventures in Recreational Mathematics VII: numbers *not* astronomical in size

Isky Mathews

This time we will examine some of mathematics' largest numbers; I feel this is a conceptual area that many find interesting, and even amusing, to think about—some of the numbers I will mention here will be so large that to refer to them as *astronomical* would not just be inaccurate, since there is no object that could found in these quantities within the observable universe, but, frankly, *insulting* to their magnitude.

To demonstrate the previous point, we shall begin by considering the number of baryons in the observable universe. Baryons are particles made up of 3 quarks and interact with the strong nuclear force, e.g. protons or neutrons, and we can calculate how many there are using 4 numbers, 3 of which were obtained using data from the Planck Satellite:

- ρ_{crit}, the critical density of the universe ($= 8.64 \times 10^{-33} kgm^{-3}$)
- Ω_b, the fraction of the universe's energy in baryons ($= 0.0485$)
- L, the radius of the observable universe, which is roughly spherical ($= 4.39 \times 10^{26} cm$)
- m_p, the mass of one proton ($= 1.67 \times 10^{-27} kg$)

Now, since ρ_{crit} is essentially the energy density of the universe, $\rho_{crit} \times \Omega_b$ is the mass stored in baryons per cm^3 of the observable universe on average, making $\rho_{crit} \times \Omega_b \times \frac{4}{3}\pi L^3$ roughly the combined mass of all baryons in the universe. Finally, since a neutron's mass is essentially equivalent to that of a proton, we divide the above expression by m_p to get

$$\frac{\rho_{crit} \times \Omega_b \times \frac{4}{3}\pi L^3}{m_p} = 8.89 \times 10^{79}$$

which is really quite a big number, in comparison to the numbers of things you encounter for everyday life! However, it was small enough to be expressed, to a fair level of precision and concisely, using a notation we are so familiar with that I barely need to name it: that of the *exponential*. For many, if asked to write down quickly the biggest number they could think of at the time, exponentials or stacked exponentials[13] would be their first thought, since it's so simple—for example, just 10^{10^2} is bigger than the number of baryons in the universe. In fact, our first famous number can be expressed as 10^{100}, a *googol*, and the next as $10^{10^{100}}$, a *googolplex*. We shall return to exponentials and the process of stacking them later, for it has great potential to make large numbers.

Primitive Recursive and Non-Primitive Recursive functions

For now, we take ourselves back to near the beginning of the 20th century, where individuals such as **Gödel, Turing** and **Church** were discussing the nature of functions. They realised that the process of calculating the outputs to most functions could be seen as an iterative process that, most importantly, had a predictable number of steps; for example, to calculate $2 + 2$, one could see it as applying $f(n) = n + 1$ to the input 2 twice. Such functions were called *primitive recursive*, because they *could* be written down or represented recursively, i.e. where they were seen as a series of repeated applications of some function, but could also be written down in a single closed form—all polynomials, exponentials and many more that we are familiar with are primitive recursive. The computer scientist Robert Ackermann is most famous for describing an eponymous function, denoted $A(m, n)$, that was still possible to evaluate but was not primitive recursive defined by these conditions:

$$A(m,n) = \begin{cases} n+1 & \text{if } m = 0 \\ A(m-1,1) & \text{if } m > 0 \text{ and } n = 0 \\ A(m-1, A(m, n-1)) & \text{if } m > 0 \text{ and } n > 0 \end{cases}$$

Let us call a *closed-form* representation of a function a form which uses a finite number of operations and without self reference. Then, an amazing fact is that the Ackermann function's above self-referential or *recursive* definition cannot be written out into a closed form, unlike addition or multiplication—this is what it means for it to not be a primitve-recursive function and it grows extremely quickly—try evaluating it for different inputs! Clearly things like $A(0, 3) = 4$ and $A(1, 2) = 4$ are quite small, but then $A(4, 3)$ is an incredible 19729 digit number:

$$A(4,3) = 2^{2^{65536}} - 3$$

In fact, it's often difficult to find examples to demonstrate how large the numbers are that the Ackermann function outputs, because nearly all of them are so big that they either can't be written down in any concise manner or, worse, they couldn't be computed within the lifetime of the universe given all the computing available today. Furthermore, Ackermann and his peers were later able to show that functions of this kind[14] *dominate* all primitive recursive functions, i.e. for any primitive recursive function $f(x)$ and a non-primitive-recursive function $g(x)$, there is some input n so that for all $m > n$, $g(m) > f(m)$.

In order to understand and express just *how* quickly such functions grow, we have to use a lovely typographical system developed some years ago by the famous **Donald Knuth**[15] known as *up-arrow notation*, which is based on the idea of the *hyperoperation hierarchy*. The first operator in the hierarchy is that of the *successor*, an unary operator (meaning that it takes 1 argument) which takes in n and outputs $n + 1$, often written $n++$.[16] Addition can be seen as repeated successorship in that $a + b$ can be seen as denoting $a++$, b times. Multiplication can then be seen as repeated addition in that $a \times b$ represents $a + a$, b times. This continues as we go higher up the hierarchy, with each nth operation $a *_n b$ representing performing the $(n-1)$th operation to a by itself, b times. Knuth created the hyperoperation-notation $a \uparrow b$ which *starts* at exponentiation (as in $a \uparrow b = a^b$) and by writing more arrows, one goes up the hierarchy, so $2 \uparrow\uparrow 4 = 2^{2^{2^2}}$ —the name we give for this operation above exponentiation is "tetration" and $a \uparrow\uparrow\uparrow b$ is

[13] that is to say, those of the form

$$a^{b^{c^{d^{\cdots}}}}$$

[14] As in, those that can be evaluated in a finite amount of time but that are not primitive recursive.

[15] A computer scientist and mathematician, perhaps most famous for his remarkably complicated series of volumes *The Art of Computer Programming* (often referred to as the computer scientist's bible!) but also for the typesetting system TEX, whose offspring, LATEX 2_ε, this very publication uses to format its articles!

[16] This is, interestingly, why C++ is called what it is—it was supposed to be the *successor to C*

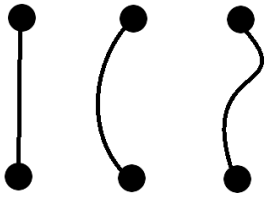

Figure 1: These three 2-point graphs are the same.

called "*a pentated* by *b*" etc. These operations make writing really large numbers simple and if we *index* the arrows, that is say that $\uparrow^n$ denotes n arrows, then we can write down numbers that could never have any practical use—for example, the famous **Graham's number**.

Graham's Number

This number comes out of a question in a somewhat ill-defined area of mathematics known as Ramsey Theory, which purports to comprehend the conditions under which complex structures are forced to appear; **Ronald Graham** and **Bruce Lee Rothschild**, both legends in this field, came up with the question in 1970. The question requires understanding what a *graph* is in pure mathematics; Benedict Randall Shaw has written a helpful article explaining graph theory in a previous issue of *The Librarian*[17], but a summary is that any set of points and lines drawn connecting them is a graph. More formally, a graph is a set of points along with a set of pairings defining connections between those points—thus neither the precise coordinate/relative position of points nor the shape of the lines connecting them matters, only the connections[18].

Given n points, the graph obtained by adding all possible connections between them is called the *complete graph on n vertices*, denoted K_n (e.g. K_3 is like a triangle and K_4 is like a square with its diagonals drawn in). Now, Rothschild and Graham were considering complete graphs of n-dimensional cubes[19], which have 2^n vertices each, and properties of the *colourings* of their edges, i.e. the ways in which you can assign different colours to those edges. In particular, they asked what was the smallest value of n such that every 2-colour colouring, using, for example, red and blue, of the edges of the complete graph on an n-dimensional cube is *forced* to contain a subset S containing exactly 4 of its points such that all the edges between the points in S are the same colour and such that all points in S are *coplanar*[20]. They were able to prove that there is such an n and they knew from checking on paper that $n > 5$ and so they sought to also put an upper-bound on it (Graham's number)[21]. It is constructed as follows:

- Let $G_1 = 3 \uparrow^4 3$ (an amazingly large number, so big that the number of 3s in its power-tower representation couldn't be written in base 10 even if each digit could be assigned to each planck-volume in the observable universe!)
- For each n, let $G_{n+1} = 3 \uparrow^{G_n} 3$
- Then Graham's number is G_{64}.

It is clear from this that uparrow notation becomes inadequate for integers as large as Graham's Number, since there is no way of expressing it concisely if we need to write out all the arrows. Thus, when you have gotten over G_{64}, we must move on to a better framework that will allow us to see just how large it is "in the grand scheme of things".

The Fast-Growing Hierarchy *or* the Grandest of Schemes of Things

The fast-growing hierarchy is a series of functions, built recursively, that grow faster and faster as we go up. We start with the simple function $f_0(x) := x+1$ and we say[22] that $f_1(x) := f_0^x(x)$, or in other words $x + x$. Similarly, $f_2(x) := f_1^x(x) = x \times x$ and in general for any integer $n > 0$, $f_n(x) = f_{n-1}^x(x)$.

So far, there is no difference between this and hyperoperations but now, we can use *ordinals* to give us unbounded growth-rates... There was a previous article[23] introducing readers to the wonderful universe of ordinals but, to simplify their technical definition, they are a clever set-theoretic version of numbers, discovered by **Georg Cantor**, which essentially allows us to have a natural extension of the integers to varying sizes of infinity. The number ω is the ordinal "larger" than all the integers but then we still have a well-defined concept of $\omega + 1$ or $+2$ or $+n$ and much, much more. We call ω the first *limit ordinal*, meaning that it has no specific predecessor, but rather can be reached as a limit of a strictly increasing sequence, and we call $2, 3, 4, n, \ldots$ and $\omega+1, \omega+2, \omega+n$ etc. *successor ordinals* because they *do* have a well-defined predecessor (i.e. they are the successor of some known ordinal). Thus we have the definition that if α is a successor ordinal, then $f_\alpha(x) = f_{\alpha-1}^x(x)$, and if α is a limit ordinal and S_α is a strictly-increasing sequence of ordinals whose limit is α (as in, α is the smallest upper-bound for all the terms in S_α), with $S_\alpha[n]$ denoting the nth term of S_α for some ordinal n, then $f_\alpha(x) = f_{S_\alpha[x]}(x)$.

To give an example[24], $f_\omega(x) = f_x(x)$, since the sequence of integers 1,2,3,...,x,... has the limit ω but since $\omega+1$ is a successor ordinal, $f_{\omega+1} = f_\omega^x(x)$. We can observe from these definitions immediately that $f_\omega(x)$ can't be primitive-recursive, since it grows faster than any f_n for integer n, and thus that it is, in a sense, *beyond uparrows*, since it can't be represented in the form $m \uparrow^k x$, where m, k are fixed integers. In fact, it is possible to show that $f_\omega(x)$ grows at almost exactly the same rate as the Ackermann function that we've seen previously and that $f_{\omega+1}(64) > G_{64}$.[25] Now, you can choose your favourite transfinite ordinal and create a function that grows faster than you can imagine, for example $f_{\omega\times2}$, f_{ω^2}, f_{ω^ω} or, if $\epsilon_0 = \omega^{\omega^{\omega^{\cdots}}}$, then you can even have f_{ϵ_0} and larger!

[17] Benedict Randall Shaw. "An Introduction to Graph Theory". In: *The Librarian Supplement* 1.2 (Nov. 7, 2017). URL: `https://librarian.cf/s2v1/graphtheory.html`.

[18] To be precise, we consider two graphs that have the same number of vertices and the same connections between those vertices but are drawn differently to be distinct graphs or objects but we say they are *isomorphic*, i.e. share all the same graph-theoretic properties.

[19] Benedict Randall Shaw, the mathematic editor, has produced a diagram of such an hypercube in four dimensions, that has been reproduced on the front cover.

[20] i.e. are points on a common plane.

[21] It may be of interest that subsequently we have created a better bound, $2 \uparrow\uparrow\uparrow 6$.

[22] Here, $g^n(x)$, for some integer n and some function $g(x)$, denotes performing g to the input x, n times.

[23] Isky Mathews. "Adventures in Recreational Mathematics V: Cantor's Attic". In: *The Librarian Supplement* 1.1 (Oct. 9, 2017). URL: `https://librarian.cf/`.

[24] Some may notice that this definition only applies for integer x (since there is no 3.2th function in our list, for example)—that's because of the caveat that the fast-growing hiearchy only contains functions defined for ordinal inputs.

[25] They aren't actually comparable in size, since $f_{\omega+1}(64) > f_\omega^{64}(6) > G_{64}$.

A combination of scientific and philosophic observations indicate that there are certain theoretic limitations to one's knowledge. For example, the uncertainty principle shows that there is a trade-off between precision of knowledge of particular momentum and position.[1] Gödel's incompleteness theorems[2] impose certain limits on mathematic knowledge that are beyond the scope of this article.[3]

Some of us will be fortunate enough to reach these limits of knowledge. However, most of us will not; none will reach all these boundaries. Consider, for example, Heisenberg, or Gödel; they were both experts in their respective fields, but in their lives relied upon the expertise of fields outside their considerations. Gödel, at some point in his life, probably consumed some sort of medicine. He was almost certainly intellectually capable of understanding why such medicines worked. Nevertheless, it is possible that he did not. Some questions emerge, separate from the theoretic possibility of knowledge of the workings of these medicines, foremost amongst which is this: should Gödel take the medicine without knowing of its precise workings?

More abstractly, what is one to do with known unknowns? Does one trust those who have studied their field more, even when ignorant of the contents of such study? In most analogous circumstances, the option open to Gödel, that is, understanding how such medicine works, is unavailable, for a variety of reasons: science is often difficult to understand, and not all of us are capable of understanding all necessary fields of science; most of us are insufficiently industrious to learn of the workings of every single apparatus we use; there are limitations on the time we have—should one have to work, one already spends most of one's waking hours doing something other than acquiring such understanding; and so on.

The basis on which societal trust in, for example, medicine, or climate science, or particle physics, emerges, is not scientific; it must explicitly reflect that in most circumstances, for one reason or another, one cannot use the best truth-seeking apparatus available. Instead, it is social. We take medicine not due to trust in science so much as scientists.

Judgement of institutions, though sub-optimal, is often far easier than judgement of the claims that these institutions propagate. It may, for example, be difficult to determine whether Xinhua is always publishing true stories. It is easier, however, to note that Xinhua is funded by the Chinese government, which uses it as a propaganda outlet. These two sorts of judgement are not mutually exclusive. However, the second *per se* is not without its uses.

A similar sort of judgement must necessarily be used in evaluating most reporting, for example. Few of us will have the chance to see many of the events described in newspapers. None of us will see all of them. Coverage of matters that one is unfamiliar with by direct study must be evaluated by other factors. As societies become more complex, epistemic dependence on others increases—that is, dependence on other sources of truth, whose pronouncements' veracity is difficult to directly verify. A similar sort of judgement must necessarily be used in evaluating most reporting, for example. Often, this dependence is not simply optional, but unavoidable. Even a reporter for a newspaper does not know what the other reporters are doing, or whether they too are reporting the truth.

Indeed, such reliance permeates not just the news cycle but many other aspects of life. Academics trust other academics; they certainly trust that data collected by others are accurate, and, because of time constraints, it is inevitable that they will not verify all that their colleagues have found. Many mathematicians will not know of the metamathematic struggles of their colleagues, but simply accept certain axioms; some of their trust is, if not blind, not based on personal verification. One cannot take as axiomatic that academia will produce good results. The replication crisis[4], deliberate fraud[5,6], and *p*-hacking[7], to name a few problems, suggest that, at the very least, academic competence cannot be taken as axiomatic. Systemic concerns are at least as important as internal metrics of academic success.

In schools, and to a lesser extent universities, we trust textbooks, teachers and curricula, without always being able to verify their claims. In history, we can hardly spatially, let alone temporally, be in a position to verify what occurred; even the tools of the modern historian—the analysis of sources and primary research—are absent from the modern textbook-based history course. The same applies to many other subjects—occasionally pupils are encouraged to 'prove' some assertion or other that has been included in the scientific curriculum, but normally one simply gathers a few data, that only with unverified interpretation seem to 'prove' the initial claim.

---

[1] W. Heisenberg. "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik". In: *Zeitschrift für Physik* 43.3-4 (Mar. 1927), pp. 172–198. ISSN: 1434-6001, 1434-601X. DOI: 10.1007/BF01397280. URL: http://link.springer.com/10.1007/BF01397280 (visited on 04/08/2018).

[2] Kurt Gödel. "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I". in: *Monatshefte für Mathematik und Physik* 38-38.1 (Dec. 1931), pp. 173–198. ISSN: 0026-9255, 1436-5081. DOI: 10.1007/BF01700692. URL: http://link.springer.com/10.1007/BF01700692 (visited on 04/08/2018).

[3] Our scientific editor remarks that mathematicians desire to know that their theorems are definitely correct. Logicians define a set of axioms, in a given notation, known as the language of the theory'; they use a series of rules to derive new theorems from previous theorems. These derivation rules are entirely mechanical, such that a computer could check all properly written proofs. A set of axioms that manages to represent numbers and their arithmetic is, by the theorems, unable to prove its own consistency, and will be incomplete. Consistency here means that two contradictory statements are never both proven, and completion means that all statements that are true and can be expressed in the language of the theory must be provable.

[4] Harold Pashler and Eric–Jan Wagenmakers. "Editors' Introduction to the Special Section on Replicability in Psychological Science: A Crisis of Confidence?" In: *Perspectives on Psychological Science* 7.6 (Nov. 2012), pp. 528–530. ISSN: 1745-6916, 1745-6924. DOI: 10.1177/1745691612465253. URL: http://journals.sagepub.com/doi/10.1177/1745691612465253 (visited on 05/06/2018).

[5] John Power. "The Cancer Researcher Catching Scientific Fraud at Rapid Speed". In: *The Atlantic* (Apr. 3, 2018). ISSN: 1072-7825. URL: https://www.theatlantic.com/science/archive/2018/04/jennifer-byrne-science-fraud/557096/ (visited on 05/06/2018).

[6] Daniele Fanelli. "How Many Scientists Fabricate and Falsify Research? A Systematic Review and Meta-Analysis of Survey Data". In: *PLoS ONE* 4.5 (May 29, 2009). Ed. by Tom Tregenza, e5738. ISSN: 1932-6203. DOI: 10.1371/journal.pone.0005738. URL: http://dx.plos.org/10.1371/journal.pone.0005738 (visited on 05/06/2018).

[7] Megan L. Head et al. "The Extent and Consequences of P-Hacking in Science". In: *PLOS Biology* 13.3 (Mar. 13, 2015), e1002106. ISSN: 1545-7885. DOI: 10.1371/journal.pbio.1002106. URL: http://dx.plos.org/10.1371/journal.pbio.1002106 (visited on 05/06/2018).

Quite apart, therefore, from questions as to whether there is an objective reality, are questions as to whether it is possible to reconcile different possible realities from different perspectives. It is at least likely that a significant part of different persons' experiences will overlap, even given something akin to the strong programme: aeroplanes fly, trains move, computers operate, tidal mechanics seem broadly consistent, and so on, suggesting that, at the very least, *modelling* an objective, or reference, reality is useful; this helps in epistemic modelling involving different actors.

These considerations are particularly relevant in light of concerns about false news. Governments worry that some people do not accept the truth, instead consuming false news. This is the implicit epistemic paradigm of at least one body concerned with falsehoods.[8] The sort of model that they espouse involves a dogmatic acceptance of truth from the bodies that have replaced the church as the principal source of information in European countries, and religion in general in others; it must be axiomatic that approved media are correct. Thus the web-page[9] to which the hyper-link at the bottom of BBC News marked '[w]hy you can trust the BBC' says, presumably to confirm that it is trustworthy, that 'the BBC is seen as by far the most trusted and impartial news provider in the UK', that it has 'its own Editorial Guidelines', and so on, ignoring the absurdity of relying upon an organisation to certify itself as trustworthy. Of course, within this model, sometimes they will make mistakes. They are accidental, and do not impugn the bodies that make them except to the extent that procedures must be improved and a few people fired in egregious circumstances. At the same time, it is enough to point to a few falsities from other institutions to justify the other side of the aforesaid axiom: they are, *a priori*, wrong, and these falsities are icing on the cake, yet, at the same time, these falsities are also propagated despite their limitations as evidence, perhaps in a tacit admission that this axiom may be sub-optimal.

What is a better model? Perhaps it is best to first ask why an epistemic model of this sort is needed. Rawls provides a reasonable account of the need for mutually beneficial coöperation. Coöperation necessitates communal decision-making; preferences are created by a combination of value judgements and beliefs about the probability distributions out outcomes. Thus, if there is no agreement on what would be, were a course of action followed, there can be no agreement on what should be, except in the rare case that two differing sets of value judgements and probability distributions were to coïncide in their recommendations. This, of course, is not particularly probable, and so cannot be relied upon. Epistemic bubbles make collective decision-making extremely difficult, since there are no common beliefs as to what is. Hence disagreement spreads not only based on value judgements, but also based on empirical beliefs, that are often moulded to follow value judgements unless they are carefully established otherwise.

Specialisation requires mutual trust: builders must trust engineers, physicists must trust mathematicians, mathematicians logicians, engineers other engineers, and so on. Since each domain is vast, and there are a great number of these domains, it is not possible for an individual to master all of the work that has been done to enable even simple work to occur—a librarian, for example, would struggle to learn both of all the processes that enable the construction of the plastic used in the equipment in the library and of the inner workings of the software used to manage loans. Thus an epistemic model is needed to enable trust, so as to enable coöperation.

How can trust be encouraged? First, social mobility helps to create trust in institutions. One is more likely to trust a relative or friend than a distant bureaucrat who is neither. More importantly, it is far easier to burst or merge epistemic bubbles with greater surfaces of contact. That is to say that if two epistemic bubbles contact each other through large numbers of people, and both have some reasonably good intuitive approximation of the results of the study of probability, the two bubbles are more likely to trust each other. If, for example, one hears one civil servant insisting that the deep state as conceived by some of those who propagate 'conservative news'[10,11] does not exist, it may be that this person is lying, especially on the television, instead of in conversation. If, however, there are many such civil servants, and they all have little reason to lie, as they are acquaintances or friends, both the perceived and true probability of such a conspiracy from the perspective of the acquaintance decrease.

Second, institutions should acknowledge their failings. If common institutions acknowledge that they are fallible, the perceived significance of their mistakes will decrease, thus increasing trust. This is particularly true of intent. The narrative that the media have deliberately attempted to fool the people of the United States is prevalent in part because the lack of acknowledgement by the media seems to suggest that factual mistakes, for example, were deliberate. Thus, as Paxman echoes Heren—'why is this lying bastard lying to me?'—so too Trump echoes Paxman: '[a]nd the FAKE NEWS[sic] winners are …' It is far easier to gain trust when mistakes are seen to be mistakes, not conspiracies.

Third, institutional transparency engenders trust. Legislation such as the Freedom of Information Act enables researchers who might ascribe ulterior motives to harmful government action to discover, for example, that it is not harmful, that it has some other beneficial purpose, that it is the product of accidental maladministration, or that it reflects the capture of government at one level instead of all levels. Sometimes, direct verification is possible. Elementary comprehension of the scientific method, for example, need not be limited to scientists. The risk here is a lapse into dogma; the scientific method works because we are told that it works. The avoidance of such a lapse might even require greater circumspection in mainstream education, and an admission that verification may not always be possible. Yet, though perhaps this might have been a price not worth paying when such dogma was accepted, in the age of an internet that creates equally coherent alternative narratives, it is quite possible that there is no price to pay.

---

[8] *Select Committee on Deliberate Online Falsehoods - Causes, Consequences and Countermeasures | Parliament Of Singapore*. URL: `https://www.parliament.gov.sg/sconlinefalsehoods` (visited on 05/06/2018).

[9] *Learn How the BBC Is Working to Strengthen Trust and Transparency in Online News*. URL: `https://www.bbc.co.uk/news/help-41670342` (visited on 06/02/2018).

[10] *Letter to Social Media Platforms Calling For A Transparency Review*. May 24, 2018, 9:13 a.m. URL: `https://gop.com/letter-to-social-media-platforms-calling-for-a-transparency-review` (visited on 06/02/2018).

[11] Of course, the sort of thing that news outlets report is so far removed from the theoretic limits of knowledge that it does not make sense to distinguish 'conservative' and hypothetical 'liberal' news.

The Library Committee

Dedicated to the betterment of the library.

Thomas Adamo
Morayo Adesina
Anshu Banerjee
James Bithell
Lorna Bo
Robert Doane-Solomon
Ben Goodrick-Green
Jonny Heywood
Shri Lekkala
Joshua Loo
Isky Mathews
Adam Mee
Benedict Mee
Benedict Randall Shaw
Joshua Rosen
Arjun Seth
Brandon Tang
Margaret Teh
Jadd Virji
Jonathan Watts
Ben Weiss
Anna Yang
Michelle Yang
Polina Zakharov