

Review of last week

- The Newtonian System
- Newton's Space and Time:
 - Absolute space and absolute time are qualitatively homogeneous.
 - Absolute space (time) is infinite (Eternal)
 - Space and Time are Continuous
 - Absolute space and absolute time are object/mind independent, immutable, and causally inert
 - Absolute space is a void

Introduction

- Newton's approach was largely empirical, focusing on describing and quantifying phenomena through formulas and mathematical laws.
- When it came to gravity, Newton relied on empirical observations and formulated mathematical descriptions to explain its workings.
- Newton refrained from publicly speculating on the nature of gravity, preferring to stick to the formulaic representation.
- In contrast, Newton took a different approach when discussing space and time, asserting the existence of absolute space and time.
- Newton acknowledged that absolute space and time were not directly experienced but believed in their existence as fundamental frameworks for understanding the physical world.

Introduction

- According to the criticism, space and time are not experienced independently but are perceived through our experience of objects and their relations.
- Space is experienced through the presence and movement of objects in relation to each other.
- Time is experienced through the succession of events or processes.
- Critics, such as Leibniz, argued that space and time are not standalone entities but rather relational features that arise from the interactions of physical objects and events.
- This criticism challenges the notion of absolute space and time proposed by Newton and suggests that they are not independent entities but rather depend on the relationships between objects and events.

Gottfried Wilhelm Leibniz



Leibniz — His Life and Time

- Leibniz was born on January 1, 1646, in Leipzig.
- He studied philosophy, mathematics, and law as a student.
- Instead of pursuing an academic career, he chose to become a diplomat and spent much of his adult life in various European Courts.
- Despite his diplomatic duties, Leibniz wrote extensively on philosophy, religion, and mathematics.
- He developed a complex metaphysical view, which he communicated through letters to friends.
- In mathematics, he worked on the theory of infinite series and independently invented the differential calculus around the same time as Newton.

Leibniz – His Life and Time

- Leibniz published his results on the calculus in 1684, three years before Newton's publication in 1687.
- Initially, both Leibniz and Newton were willing to acknowledge each other's contributions to the calculus.
- However, pressure from friends and colleagues led to a bitter priority dispute between the two.
- The dispute extended beyond the calculus and evolved into a philosophical feud, culminating in the Leibniz-Clarke correspondence.
- The correspondence involved a critique by Leibniz and a defense by Clarke, including discussions on Newton's views of space and time.
- The dispute ended abruptly with Leibniz's death in 1716.

- Leibniz's metaphysical system is a complex framework that seeks to unify philosophy, religion, mathematics, and science.
- Unlike Newton's Principia, Leibniz did not present his system in one comprehensive work.
- Instead, his system must be pieced together from his letters and correspondence, which were published posthumously.
- The Monadology is one of the few shorter works where Leibniz provides some insights into his metaphysical ideas.
- The complexity and scattered nature of Leibniz's writings have made it challenging to fully grasp the details of his system.

- Leibniz distinguishes between the world of "phenomena" (physical appearances) and the underlying reality.
- The physical world we experience is a realm of appearances, but it is grounded in a deeper reality.
- According to Leibniz, the fundamental building blocks of reality are called monads or substances.
- Monads are indivisible and have no spatial parts.
- They are self-contained and self-sufficient entities that endure eternally.
- There is an infinite number of monads, each existing independently.

- The monads of Leibniz are not passive and unchanging but are active beings capable of change. One analogy that Leibniz used to describe them is that they are like souls, simple substances which are capable of experiencing or undergoing change.
- Each of these substances has an infinite number of attributes or properties. As they evolve, they are capable of realizing inconsistent or incompatible properties, much as a bar of iron, when heated, is first red, then blue, then white.

- Nicolas Rescher has suggested that one can think of the monads as computer programs with an infinite number of steps which, once started, goes through an infinite number of states in some predetermined order.
- Another way to think of them is in terms of mathematical sequences such as 1, 2, 3,... or 2, 4, 6,... Each monad might be considered as the generator of an infinite series. Each contains within itself the rule which determines what sequence the numbers in the series will appear. Once activated, the monads develop according to some inherent plan just as the numbers in an infinite series are arranged in a certain order according to the rule of the series.

- Each monad, although self-contained, reflects within itself the development of all other monads with which it is compatible.
- Thus, the infinite collection of monads can be partitioned into collections which contain monads which are compossible with one another. These collections of compossible substances or compossible collections of monads constitutes, for Leibniz, different possible worlds.
- A possible world, thus, is a collection of monads and their histories.
 The real world, the world that we live in, is just one among these many possible worlds.

- The monads in each possible world, and hence in the actual world, group together in what Leibniz called **aggregates**. These aggregates are then what is experienced by us human beings in the real world as the objects of everyday experience.
- Thus, the monads are the underlying reality which gives rise to the appearances (aggregates).

Principle of Contradiction

- Each possible world and every monad is subject to a number of logical principles. One, which Leibniz says "is sufficient to demonstrate every part of arithmetic and geometry, that is, all mathematical principles..." Leibniz calls the **Principle of** Contradiction.
- The Principle of Contradiction is the principle that "a proposition cannot be true and false at the same time..." (Alexander, 1956, 15).

Principle of Sufficient Reason

- The second, which in conjunction with the first, Leibniz thought was sufficient to demonstrate all the principles of natural philosophy, i.e., physics, was called the **Principle of Sufficient Reason**.
- This principle is given in various formulations in Leibniz's work, but for our purposes we can treat it as the principle "that nothing happens without a reason why it should be so, rather than otherwise." (Alexander, 1956, 16)

Principle of the Identity of Indiscernibles (PII)

- A third principle, which can be derived from the Principle of Sufficient Reason, is the **Principle of the Identity of Indiscernibles**.
- This principle, like the Principle of Sufficient Reason, comes in a variety of formulations. Read as an epistemological principle, the principle asserts that two states of affairs, E1 and E2, which are indistinguishable, are, in fact, only one. That is, unless there is some mark which distinguishes E1 from E2, then E1 and E2 are really one and the same state of affairs.

Principle of the Identity of Indiscernibles (PII)

- A second version of the principle asserts that there cannot exist indistinguishable things. Thus, at the level of monads, each monad is distinguishable from each of the others in some respect or other. Any two monads not so distinguishable would really in fact be only one.
- Both the **Principle of Sufficient Reason** and the **Principle of the Identity of Indiscernibles** play a role in Leibniz's critique of the Newtonian view of space and time.

- The logical principles enunciated above hold for all possible worlds.
 We have yet to say how one of those possible worlds, namely, the actual real world, is singled out.
- On Leibniz's view, God in His omniscience, is able to see in their totality each and every one of the infinite possible worlds. From this infinite range of possibilities, God creates the real world by actualizing one of the possible worlds.
- Actualizing one of the possible worlds "starts" the programs and what we call the history of the world is the articulation or the development of the inherent principles of development contained within the monads in this world.

The Principle of Perfection

- How does God choose which world to actualize? According to Leibniz, God does not choose a world at random. God chooses which world to actualize by means of, what Leibniz calls, the **Principle of Perfection**.
- God chooses, in effect, the most perfect or best of all possible worlds to actualize. Once God actualizes the most perfect of all possible worlds, God then does not intervene further in the daily day-to-day development of the world. The day-to-day development of world history is governed by the inherent principles of the monads which make up the actualized world.

The Principle of Perfection

- The Principle of Perfection is a maximal principle.
- It says in effect: actualize that world which maximizes goodness, where goodness is understood in terms of *order* (the idea that the laws of nature ought to be simple) and *variety* (the idea that the world ought to be as full of things as possible and ought also to contain the most diverse kinds of things).

The Principle of Perfection

- The idea for such a principle may derive from Leibniz's work on what is today called the calculus of variation, of which Leibniz was one of the originators. That God chose to actualize the real world in accordance with the Principle of Perfection leads to certain consequences.
- Among them is the Principle of Plenitude, from which Leibniz deduces that there can be no vacua. A world in which vacua existed would not be as perfect as one in which they did not because those spaces or times which were empty could have been filled by something or other.

- The real world, so conceived, can be thought of in one of two different ways.
- On the one hand, it is a collection of monads unfolding, as it were, as they develop in accordance with their pre-programmed instructions.
- On the other hand, the real world is the system of phenomenal appearances that constitutes what we would ordinarily take to be the physical world.

- These are not two separate collections of things but rather the same collection but viewed from different perspectives.
- The real world as a set of aggregated monads, i.e., appearances, represents the point of view of human cognizers. What we, from our limited perspective, see as world history is the unfolding of the monads.
- Each monad, recall, is an individual self-contained substance. Yet the fact that the world is a cosmos, a coherent whole, indicates that they all unfold together in an orderly fashion. Since they are self-contained, they cannot interact nor causally influence one another.

- To explain the order and coherence of their development, Leibniz invokes a principle which he calls "pre-established harmony."
- The idea here is that each monad, although completely independent from all the others, is preset in such a way that it unfolds in a coherent way with the unfolding of all the others. That such a harmony should exist is part of the idea that the world is orderly, i.e., subject to laws.

- Thus, the pre-established harmony that is evidenced in the real world is a result of the fact that, of all the possible worlds, God chose to actualize that world which was most perfect.
- The monads themselves are atemporal and do not exist in space.
 Space and time, for Leibniz, are part of the phenomenal reality.

- The connection between Leibniz's metaphysical position and his views on physical theory is **suggestive** rather than deductive.
- The metaphysical principles serve as a constraint on what can count as a legitimate physical theory, but there was no question in Leibniz's mind that those principles were compatible with more than one physical theory.
- In the light of this, one can see that Leibniz's objections to the Newtonian view are based on a number of considerations, some physical, some metaphysical, some epistemological and some theological.

- The basic outline of Leibniz's relational view of space and time can be developed more or less independently of his metaphysical position.
 Certain specific features, such as the continuity of space and time, and the homogeneity of space, do rely for their support on an appeal to the metaphysics.
- Leibniz's relational theory is spelled out in greatest detail in The Leibniz-Clarke Correspondence (1716) and in a short paper "Metaphysical Foundations of Mathematics" (1715).

- The basic idea is that space and time are nothing in themselves, but rather are constituted by relationships between existents.
- Leibniz puts it as follows: "As for my own opinion, I have said more than once, that I hold space to be something merely relative, as time is; that I hold it to be an order of coexistences, as time is an order of successions. For space denotes, in terms of possibility, an order of things which exist at the same time, considered as existing together; without inquiring into their manner of existing. And when many things are seen together, one perceives that order of things among themselves (Alexander, 1951, 25)."

- Space, Leibniz holds, is an "order of coexistences," and time is an "order of successions."
- How are we to understand what this means?
- Suppose we imagine that we have a collection of things that exist. We might, for reasons that will become clear later, call them events or objects. These events or objects can be arranged or related to one another in various ways. What we call "time" and "space" are just two (special) ways in which these existent things relate to one another.

- We can make this concrete with the following illustration.
- Consider some collection of events, e.g., "The birth of you (e1)," "You have entered middle school (e2)," "You have entered high school (e3)
 You have entered CUHK-SZ (e4)." These events can be arranged or ordered in various ways.
- We can arrange them according to which came "earlier" and which "later. " The resulting order is what we call time. Of course, time is not constituted by the ordering in terms of "earlier" and "later" of just these three events, but of all events.

- Leibniz did not stop to consider whether or not this ordering must be unique. He certainly took it to be unique, in the sense that any two observers arranging the events in order of earlier to later would come up with the same sequence, but modern theories of time order suggest this need not be so.
- Have we done away with the notion of time as an independent existent? It might seem that we have cheated. We eliminated time in favor of an ordering of events with respect to "earlier" and "later," but are not these themselves temporal notions?

- Doesn't "a is earlier than b" just mean that "a" occurred at some time before the time at which "b" occurred? If this is correct, that we understand the notions of "earlier" and "later" in terms of their relative positions on a temporal scale, then, of course, we have not gained anything over Newton and the Absolutists, since the concept of time reappears in an essential way in our analysis.
- To succeed, a relational theory of the nature of time must explain time in terms of non-temporal concepts.

- In order for Leibniz's attempt to succeed, he must provide an account of the notion of "earlier" and "later" which does *not* explain these concepts in terms of temporal position.
- Leibniz has such an account, which many modern day commentators think has a great deal of merit and anticipates moves made by 20th century science. It is called the causal theory of time. We will discuss them later.

- First, let us consider what a relational theory of space would look like, on Leibniz's view.
- Consider, again, our small set of events {e1, e2, e3, e4}. Not all of these events can be ordered on a scale of earlier to later. The events e2 and e3 are co-existing or contemporaneous. If we consider all events, there will be a large number of such events, all contemporaneous with e1 and e2.

- Consider, for example, (e5): a flight took off on September 21 in Shenzhen (which we will take to be the day on which e1 and e2 officially occurred), (e6): a traffic accident in Hong Kong, (e7): a murder in Shanghai, etc.
- We could define what we mean by saying that all these events occurred at the same time in the following way: consider any other event e: if e coexists with any one of the events e2, e3, e5, and e7, then it coexists with all of them. (Recall that we are assuming, with Leibniz, that there is a unique ordering of all events, an assumption which Einstein's theory of relativity forces us to abandon.)

- The sequence of events by earlier and later thus turns out to be a sequence of bundles of events. Is there some way in which the events or objects within a given bundle can be ordered? Space, for Leibniz, just is the order of these coexisting objects.
- In the 1715 paper, Leibniz suggests that spatial positions can be relatively determined by considering how many coexisting objects or events can be interposed between two given coexistents.

- Consider the events e5, e6, and e7.
- The event e6 occurs at a place closer to e5 than does e7 in virtue of the fact that every event which occurs at a place between e5 (in Shenzhen) and e6 (in Hong Kong) occurs at a place between e5 (in Shenzhen) and e7 (in Shanghai) but there are other events, occurring in places like Beijing, for example, which occur at places between where e6 occurs and where e7 occurs but *not* between where e5 occurs and where e6 occurs.
- Leibniz gives no details as to exactly how the relative situations of the various coexistents are to be empirically determined and to this extent his account is incomplete.

- Modern analyses suggest that the problem of arranging a spatial "picture" of a collection of contemporaneous existents requires more than can be provided by an ordering relation alone. In addition, some account of measure or magnitude is needed.
- Setting aside the difficulties in providing a complete account of spatial ordering, we have developed a picture of what it means to say that time is the order of successive existents and space is the order of coexistents.
- Space and time are not, however, completely analogous.

- The order which constitutes time, for example, is (or appears to us to be) one-dimentional, whereas the ordering of coexistent objects is three-dimensional.
- A second, more troublesome disanalogy is that once the succession of coexistents is fixed, they remain fixed in their relative order, but the coexistent objects whose relative positions constitutes space are free to move around with respect to one another.
- Does space then change every time the objects constituting it do? If so, then the very idea of motion becomes unintelligible, since something moves if it moves with respect to something else at rest, e.g., if it moves from one *position* to another.

- But, if there is nothing to the idea of spatial position but the relative situation of bodies, then if all bodies are constantly in "motion" the idea of a *fixed* place becomes difficult to define. And without fixed places, it is hard to see how the idea of motion is intelligible.
- Leibniz was aware of this problem and was trying to deal with it in the passage cited above where he says that "space denotes, in terms of possibility, an order of things..." Space, then, is not the actual ordering of bodies at any given instant, since this is subject to constant change, and, space, we want to say, in some sense remains the same through these changes.

- Rather, Leibniz seems to be suggesting that space be identified with a *possible* ordering (his is reinforced by later remarks in his Fifth reply to Clarke, paragraph 47).
- We first come to our idea of space by seeing bodies move with respect to one another. This is basically an *epistemological* point, i.e., a point about how we acquire our *beliefs* about and *knowledge* concerning space.

- Leibniz then goes on to argue that space is to be *identified* with the set of possible places (relative positions) which bodies can occupy with respect to one another. This is a slightly different point, an *ontological* point, about the *nature* of space rather than our knowledge about it.
- The basic point can be illustrated by considering a swarm of bees. They are constantly moving with respect to one another but form a more or less compact, though fluid, unity. Leibniz suggests we understand motion, and, hence, relative position and space in the following way.

- Choose one bee and, at an instant, *imagine* all the other bees to be frozen in position. Now follow the course of this one bee with respect to all the others considered fixed. Then we can say that this one bee is *moving* with respect to the others because his relative position with respect to the (presumed fixed) positions of the others is constantly changing.
- We can now define different positions to be different insofar as their relative position to this (presumed fixed) set of bees is different. Of course, in *actual* fact, none of the bees is at rest with respect to any of the others.

- But, the same argument can be applied to any other bee.
- Follow it and assume the rest (including our original bee) at rest. In this way, one can understand what it means to say that any given bee is moving, even when all the reference bees are also, in fact, moving. That this device works depends on our defining space in terms of *possible* positions rather than *actual* ones, but if one allows this, then one can understand both motion and space from a relationalist point of view.

• It should be pointed out that some commentators are disturbed by this fundamental appeal to "possible positions." They argue, in effect, that understanding possible positions relies implicitly on our prior understanding of *actual* positions, and that Leibniz is, in effect, smuggling in a reference to an absolute spatial framework at this time (see, e.g., Sklar, 1974, p. 171).

- In the 1715 paper, Leibniz gives an account of how a relationalist might handle a metrical concept such as "length." He says: "Quantity or magnitude is that determination of things which can be known in things only through their immediate contemporaneous togetherness (or through their simultaneous observation).
- For example, it is impossible to know what the centimeter and meter are if there is not available an actually given object applied as a standard to compare different objects.

- In effect, the Absolutist contends that objects have *intrinsic* lengths determined by how much Absolute Space they occupy, regardless of the results of any measurement.
- The relationalist, however, wants to argue that lengths, like velocities, are only relative. Just as one cannot speak (so says the relationalist) of the velocity of an object A without specifying with respect to what frame of reference that velocity is being measured, so, one cannot speak of the length of an object A without specifying some usable standard with respect to which the length of A is being measured.

- Because of Leibniz's peculiar metaphysical convictions, he was persuaded, for this reason, to call lengths, and space and time determinations in general, ideal as opposed to real.
- But, we must not be misled here. Leibniz, and the Relationalists, should not be construed as arguing that space, time, lengths, and velocities, etc. are *subjective* as opposed to *objective*.
- Such determinations are as objective as anything in the sense that different observers, using the same standards and in the same (or equivalent) frames of reference, will come up with identical measurements.

- "Time," says Leibniz, "is the order of non-contemporaneous things."
- The basic idea is to define temporal order by ranking events from earlier to later. In order to avoid a circular argument, one must show how the notions of "earlier" and "later" can be understood in nontemporal terms.
- Leibniz's solution, which was the forerunner of many similar attempts, was to try to define the earlier/later relation by means of causal connections. Such a theory is known as a causal theory of time.

- Leibniz's theory is fundamentally a theory of temporal order and must be supplemented by a theory of temporal measure or duration (corresponding to the spatial measure or length).
- Leibniz does say that "Duration is the quantity of time " (Wiener, 1951, 202) and that "Quantity or magnitude is that determination of things which can be known in things only through their immediate contemporaneous togetherness (or through their simultaneous observation)" (Wiener, 1951, 202).

- From these remarks, we may infer that Leibniz would say about duration something similar to what he says about length, namely, that the choice of a fundamental unit of time is a matter of convention and that the duration of some process is not an intrinsic property of the process in question but only a relational property of the process as compared to some standard process, such as the motion of the heavenly sphere or the "regular" swing of a pendulum (or, if he had lived in the 20th century, the "steady" ticking of an atomic clock).
- But, this aspect of Leibniz's theory of time is not developed by him.

- We turn to Leibniz's version of the causal theory of temporal order.
- The key points are made as follows:
- When one of two non-contemporaneous elements contains the ground for the other, the former is regarded as the antecedent, and the latter as the consequent. My earlier state of existence contains the ground for the existence of the later. And since, because of the connection of all things, the earlier state in me contains also the earlier state of the other thing, it also contains the ground of the later state of the other thing, and is therefore prior to it.

- All existing elements may thus be ordered either by the relation of contemporaneity (co-existence) or by that of being before or after in time (succession) (Wiener 1951, 201-2).
- We start with a <u>fundamental conviction that certain events are the</u>
 <u>causes of other</u> events. If A is the cause of B, then, in Leibniz's
 terminology, A is the ground for B. The **concept** of causal connection
 that **is being employed** is taken to be a primitive, unanalyzable
 notion.

- The fundamental intuition is that A causes B when A produces B or when the occurrence of A is a condition which leads to the production of A. Thus, we are inclined to say that striking a match (under normal conditions) causes it to light, heating water causes it to boil, and what we mean is that the one is a productive factor in bringing about the other.
- Leibniz's own example suggests he **was** thinking of a cause as a condition, in the sense that Jones's being 10 years old is a condition or ground for his being 11 years old. These examples are only meant to illustrate the notion of causal connection and not to explain it.

- If A is indeed the cause of B then A must precede B, since B, in some sense, arises out of or because of A. Unstruck matches do not, in general, light by themselves. They must first be struck, then they light. The striking of the match is an *antecedent* physical condition of which the lighting of the match just struck is the *consequence*.
- In general, if A causes B then, not only are A and B noncontemporaneous but A *invariably precedes* B.

- Leibniz himself had no problem with the notion of causality because he thought of it as a 'productive' force ultimately resting on the analogy of a person being able to effect the movement of his body by an effort of will.
- In the 1700's and subsequently, this concept of 'cause' came under increasing attack especially from empiricists such as David Hume and his followers.

- Hume professed not to understand what this sense of causal connection was. He thought that 'A caused B' meant no more (and no less) than that events of type A were constantly associated with events of type B and that A was earlier than B. The notion of a productive connection existing between A and B has disappeared.
- But, now the causal theory of time order is in trouble, for in order to determine that A is the cause of B and not vice versa, we must first say which event came first in time. The temporal order of A and B has been smuggled back in to establish which of the pair is the cause of the other and, hence, the causal order of A and B cannot be used to define their temporal order.

- Most contemporary philosophers are more likely to side with Hume rather than Leibniz. This places a heavy burden on those of them who would like to defend the causal theory of time - Hans Reichenbach, one such 20th century follower of Hume, tried to determine causal order independently of temporal order by a method he called the "mark method."
- Briefly, the idea is to provide a non-temporal criterion of causal precedence by considering that small variations in the cause lead to variations in the effect but not vice versa (you can explore it further and see the problem with such a method).

- Clarke, Newton's spokesman in the debate with Leibniz over Absolute Space and Absolute Time, had three basic criticisms of Leibniz's approach.
- (1) Clarke argued that, on Leibniz's view, if the entire universe were to be moved say, ten feet to the left of its present position then it would be in the same *place* as it is now. But, Clarke argues, this is an "express contradiction" and, hence, the Leibnizian view is absurd (Alexander, 1956, 31).

- Presumably because if the entire universe were first in position A and then moved 10 feet to the left to a new position B, then A and B could not be the same place since they are 10 feet apart. Leibniz had no problem in disposing of this objection (Alexander, 1956, 38).
- Clarke's point is question begging in that it assumes that the notion of Absolute Space makes sense, and, hence, that A and B are different.
- But, this is just what the Relationalist is denying. Clarke's point reduces to arguing that Leibniz is wrong because he is wrong. Hardly telling.

- (2) Clarke's second argument is more telling and Leibniz does not really satisfactorily answer it.
- Clarke asks, in effect, how Leibniz is prepared to deal with the experimental evidence (i.e., the bucket experiment) which "proves" that there is a difference between absolute and merely relative motion and, hence, by implication, between absolute and merely relative space and time.
- A consistent relationalist would have to deny that the experimental result does enable one to distinguish between absolute and relative motion.

- Leibniz does not take this tack. He agrees that there is a distinction between absolute and relative motion but denies that this entails that there is a distinction between absolute and relative space (Alexander, 1956, 74). Clarke professes that surely this is untenable and he presses Leibniz for a fuller explanation (Alexander, 1956, 105).
- Unfortunately, Leibniz died before he could formulate a satisfactory answer and there is no way of knowing if, indeed, he had one.
- Leibniz's position seems to be that absolute motions can be distinguished from purely relative motion by considering the *causes* (forces) which act to set the bodies in motion.

- If A and B are moving in relation to one another, then if an impressed force on A set A in motion, then it is A (rather than B) which is undergoing absolute motion. As far as *the relative motions* of A and B are concerned, there is no distinguishing them.
- When we consider the forces acting on A and B, then, Leibniz suggest, absolute motions can be distinguished from relative motions. In effect, what Leibniz is saying is that kinematically speaking, all motions are purely relative, but dynamically speaking, some motions can be distinguished from others.

- Newton took this difference between kinematic and dynamic effects to be evidence for the existence of Absolute Space.
- Leibniz, rather inconsistently, agreed that the effects were different but arbitrarily denied that they had the implications that Newton tried to draw from them.

- If Leibniz had been a consistent relationalist he could have denied that consideration of forces does make a difference. He would have argued that, *dynamically* speaking, the motions of A and B were indistinguishable as well. He did not, however, make this move.
- It was left to Ernst Mach in the late 1800's to argue for the dynamical equivalence as well as the kinematic equivalence of two observers in relative motion to one another.

- (3) Clarke's third basic criticism of Leibniz was also not adequately answered by Leibniz in the Leibniz-Clarke correspondence.
- Leibniz had argued that space was a system of relations. Clarke urged that space had a quantity as well, but relations could have no quantity and that, therefore, a theory which held space (or time) to be merely relational could not be adequate (Alexander, 1956, 32).

- Leibniz responds by arguing that a relation "also has its quantity".
- Leibniz says: "As for the objection that space and time are quantities, or rather things endowed with quantity; and that situation and order are not so: I answer, that order also has its quantity; there is in it, that which goes before, and that which follows; there is distance or interval. Relative things have their quantity as well as absolute ones. For instance, ratios or proportions in mathematics, have their quantity, and are measured by logarithms; and yet they are relations. And therefore though time and space consist in relations, yet they have their quantity" (Alexander, 1956, 75).

- The basic problem is that specifying a given order of points (objects, events) is not sufficient to determine a unique quantitative measure (length, duration) for them.
- Leibniz tries to give some account of the metrical properties of order in the 1715 paper: "In each of both orders -- in that of time as that of space -- we can speak of a propinquity or remoteness of the elements according to whether fewer or more connecting links are re quired to discern their mutual order...

- ... Two points, then, are nearer to one another when the points between them and the structure arising out of them with the utmost definiteness, present something relatively simpler. Such a structure which unites the points between the two points is the simplest, i.e., the shortest and also the most uniform, path from one to the other; in this case, therefore, the straight line is the shortest one between two neighboring points" (Wiener, 1951, 202).
- The task of defining metrical concepts and characterizing the metrical structure of continuous space and time requires an additional account, not derivable from considerations of order alone.

The Characterization of Space and Time According to Leibniz – Tutorials on Thursday

- In order to facilitate a comparison of Leibniz's view on space and time with that of Newton, it will be useful to construct a table of properties of space and time for the Leibnizian view which parallels that already constructed for the Newtonian view.
- On Thursday, please construct a table of properties of space and time for the Leibnizian view vs. the Newtonian view and discuss them in your group.