

# Final 37810

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*November 1, 2015*

## Question 1

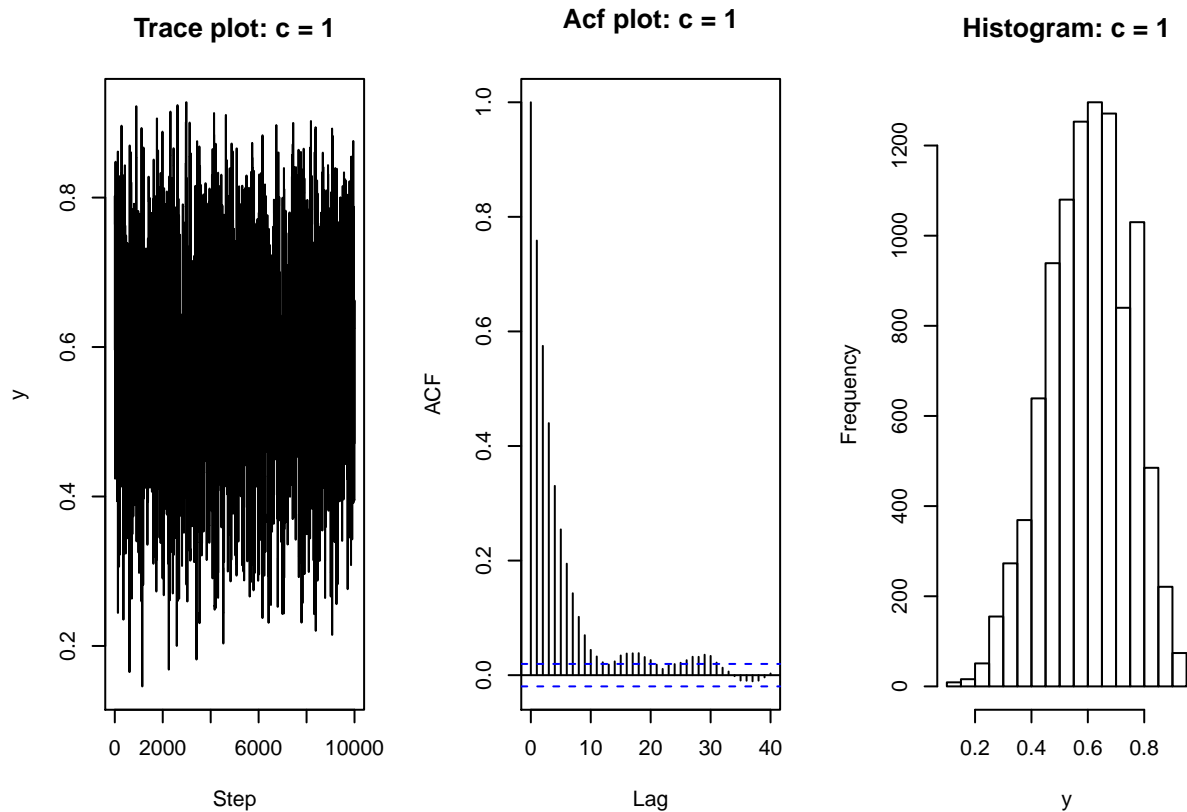
```
library(coda)
Alpha <- 6
Beta <- 4
# phi cannot be 0 or 1
phi <- function(c,oldphi){
  newphi = 0
  while (newphi == 0 || newphi == 1){
    newphi = rbeta(1,c*oldphi,c*(1-oldphi))
  }
  return(newphi)
}

run_metropolis_MCMC <- function(startvalue,c,iterations){
  chain = rep(0, iterations+1)
  chain[1] = startvalue
  for (i in 1:iterations){
    phi = phi(c,chain[i])
    posterior = dbeta(phi,Alpha,Beta)/dbeta(chain[i],Alpha,Beta)
    proposal = dbeta(phi,c*chain[i],c*(1-chain[i]))/dbeta(chain[i],c*phi,c*(1-phi))
    probab = posterior/proposal
    if (runif(1) < probab){
      chain[i+1] = phi
    }else{
      chain[i+1] = chain[i]
    }
  }
  return(chain)
}

startvalue = runif(1)
chain = run_metropolis_MCMC(startvalue,1, 10000)
acceptance = 1-mean(duplicated(chain))

par(mfrow=c(1,3)) #1 row, 3 columns

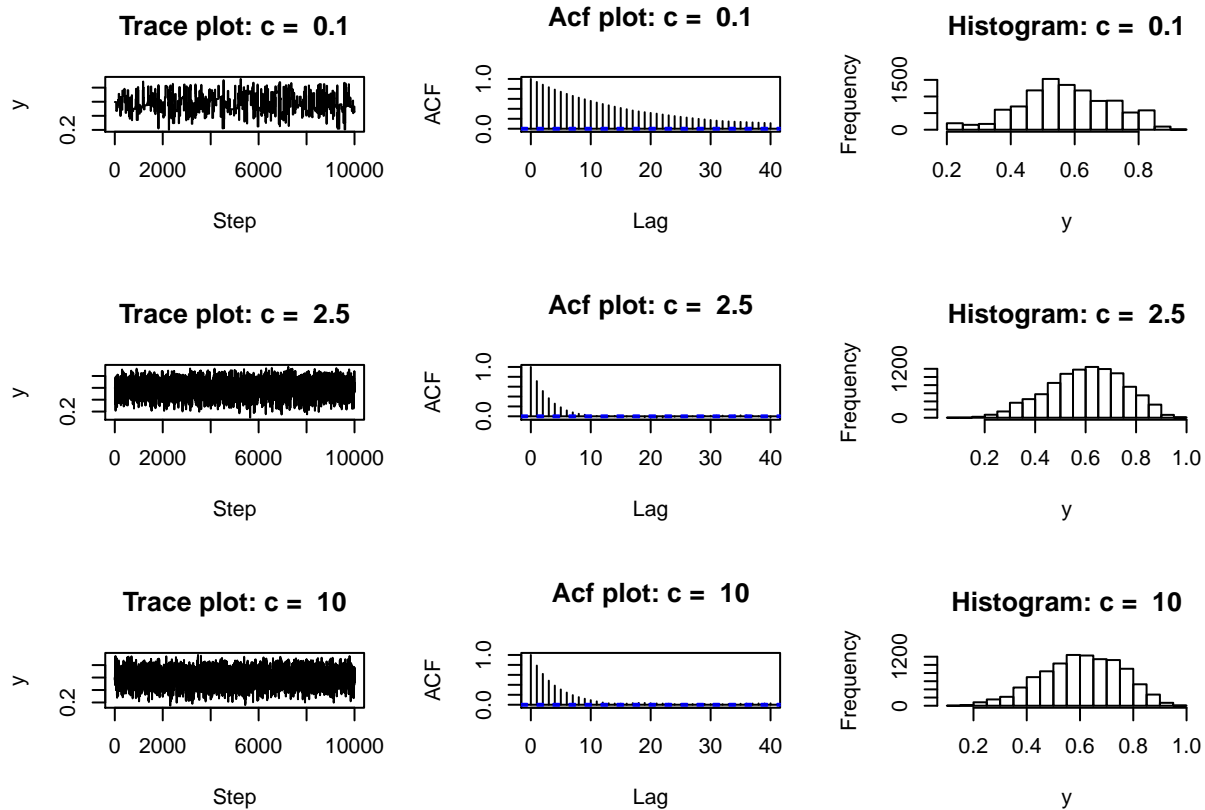
traceplot(as.mcmc(chain), type="l", main = "Trace plot: c = 1", xlab="Step", ylab="y")
acf(chain, main = "Acf plot: c = 1")
hist(chain, main = "Histogram: c = 1", xlab="y")
```



```
Cs <- c(0.1, 2.5, 10)
result <- sapply(Cs, run_metropolis_MCMC, startvalue = startvalue, iterations = 10000)
```

```
graph <- function(Cs){
  n = length(Cs)
  par(mfrow = c(n, 3))
  for( i in 1:n ) {
    traceplot(as.mcmc(result[,i]), type="l", main = paste("Trace plot: c = ", Cs[i]), xlab="Step", ylab="y")
    acf(result[,i], main = paste("Acf plot: c = ", Cs[i]))
    hist(result[,i], main = paste("Histogram: c = ", Cs[i]), xlab="y")
  }
}
```

*# Always got error complaining figure margins too large, but we can get result in R. I'll attach the result*  
`graph(Cs)`



Gibbs Part

## Gibbs Sampling

```
Gibbs_<-function(x0,y0,iterations,B=5,burnIn=0)
## 5 parameters are needed in this function, with B=5 and burnIn=0 by default
{
  if(x0>0&& x0<B&&y0>0&&y0<B)
  ## to check whether the starting values are in the domain.
  {
    x<-c(x0,rep(NA,iterations-1))
    ## Initialize the Markov chain
    y<-c(y0,rep(NA,iterations-1))
    ## Initialize the Markov chain
    for(i in 1:(iterations-1))
    {
      x[i+1]<-(-log(1-runif(1)*(1-exp(-y[i]*B)))/y[i])
      ## use inverse transform sampling to draw sample from conditional distribution
      ## p(x^{i+1}|y^i)
      y[i+1]<-(-log(1-runif(1)*(1-exp(-x[i+1]*B)))/x[i+1])
      ## use inverse transform sampling to draw sample from conditional distribution
      ## p(y^{i+1}|x^{i+1})
    }
    if(burnIn>0)
    {
      x<-x[-(1:burnIn)]
    }
  }
}
```

```

        y<-y[-(1:burnIn)]
        print(length(x))
## discard the first bunch of draws for the burn-in process
    }
    return(data.frame(x,y))
}
else
    stop("Initial values incorrect")
## print the information for incorrect starting values
}

```