Cross-Domain Transfer Theory: Human → Quadruped Biomechanical Mapping

# 1. Problem Statement and Objective

We want a principled mapping f: R^(d\_h) → R^(d\_q) from human locomotor descriptors (biped, two limbs, step cycle) to quadruped locomotor descriptors (four limbs, gait cycles). The mapping must preserve coordination dynamics while operating in IMU feature space.

# 2. Representation

• Task-space features are more transferable than joint-space angles.

• Use phase-space representations (stride phase φ, duty factor, stride frequency).

• Encode symmetry bases: humans have LR anti-phase, quadrupeds have gait-dependent limb couplings.

# 3. Formal Mapping: Constraints and Invariants

Constraints:  
- Temporal invariants: stride frequency, event ordering, phase lags.  
- Spatial/statistical invariants: COM oscillations, duty factors, spectral ratios.  
- Physical feasibility: duty bounds, no-slip during stance, anatomical ranges.

# 4. Practical Retargeting Pipeline

Step 0: IMU frame calibration (gravity and heading alignment).  
Step 1: Event detection & phase estimation.  
Step 2: Feature lift to invariant space (duty, COM spectra, relative phases).  
Step 3: Canonical phase warping (monotone spline).  
Step 4: Invariant-space mapping (linear + constrained).  
Step 5: Optional decoding to limb-level states with HSMM.  
Step 6: Confidence & calibration metrics.

# 5. Human ↔ Quadruped Correspondence

Human L/R legs map to quadruped diagonal or lateral pairs depending on gait:  
- Trot: L ↔ (LF+RH), R ↔ (RF+LH).  
- Pace: L ↔ (LF+LH), R ↔ (RF+RH).  
- Walk: expand half-cycle into 4-beat sequence.

# 6. Mathematical Sketch

We define relative phases θ and coordination manifold M. We seek f mapping human stride invariants to quadruped invariants while projecting into valid gait families and enforcing constraints.

# 7. Learning Strategies

- Two-stage training: synthetic → fine-tuned on real.  
- Cycle consistency mapping.  
- Physics-guided losses.  
- Multi-sensor invariance training.

# 8. Validation Protocol

• Invariant preservation: KL divergence of invariant distributions, phase-lag error, duty RMSE.  
• Contact-sequence validity: ≥95% strides legal.  
• Dynamics & spectrum preservation.  
• Downstream stability in HSMM/FSQ codes.

# 9. Worked Micro-Example: Trot Target

Human step 1.8 Hz → stride 0.9 Hz.  
Map L/R anti-phase to diagonal anti-phase pairs.  
Scale duty factors to dog trot band.  
Warp phase so human heel-strike aligns with diagonal stance.

# 10. Failure Modes and Mitigations

- IMU misalignment → per-stride recalibration.  
- Gait misclassification → gait priors + HSMM family switch.  
- Low SNR collar-only → rely on COM spectral invariants.

# 11. Minimal Implementation Sketch

Inputs: human IMU data → phase estimation → invariant features → constrained mapping → limb decoding.  
Outputs: quadruped-compatible invariants and optional limb states.

# Conclusion

Cross-domain transfer requires invariant-based mapping with biomechanical constraints. This ensures feasible, interpretable, and stable mappings from human to quadruped domains, supporting synchrony analysis in mixed-species datasets.