COMP/MATH 3531
Final Project

Due: Whenever. Before the end of term.

Choose one of the following:

Monopoly

1. Build a simulation of Monopoly (official rules) using any technique (or combination of techniques) you like.
    a. To simplify the real rules, we will approximate the following:
        i. If someone lands on an unbought space and chooses not to buy it, it is "auctioned off" and bought by a random other player who can afford it.
        ii. Chance and Community Chest don't do anything.
        iii. Properties can't be mortgaged.
        iv. No Houses or Hotels can be built
        v. Free Parking does nothing.
        vi. A player is eliminated from the game when they have a bill they cannot pay.
        vii. The last player with money wins.
    b. Does player order matter? How long is the average game in terms of turns? How many trips around the board, on average, until all properties are bought?
    c. If you include the house rule that:
        i. Free Parking nets you $500
        ii. Properties are not auctioned

        Do the metrics from (b) change? How?

Queues

2. Build a Markov Chain model for two queues with two servers
    a. Customers arrive in queue 1 with probability $\lambda(L_1, L_2) = \lambda \frac{L_1}{L_1+L_2}$ and queue 2 with probability $\lambda_2(L_1, L_2) = \lambda \frac{L_2}{L_1+L_2}$ where L_i is the length of queue i.
    b. Assume both servers work at the same rate μ
    c. Compute the average sojourn time for an individual, and also find the stationary distribution for each line. How long is each server idle for?
    d. Do your results change if $\mu_1 \neq \mu_2$? How?

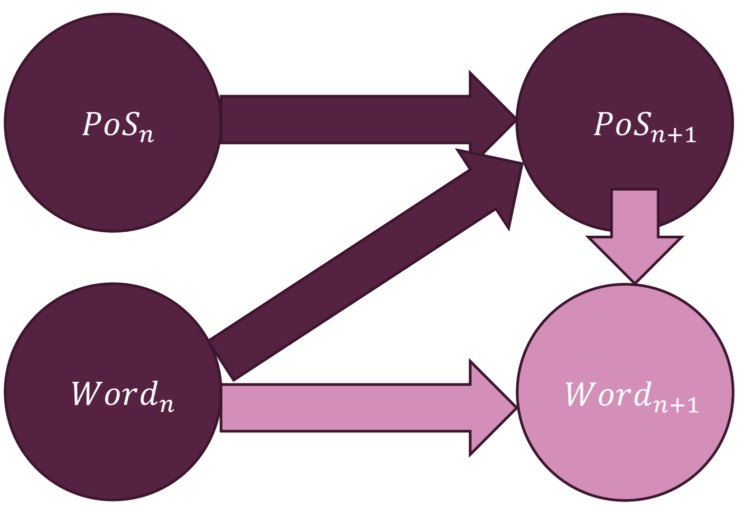
   Extend this to n servers. Compute the average sojourn time and idling time for each server.

   Also mathematically analyze the multi-server Markov chain when you have n servers and individuals all queue together. Is this always better than n independent queues?

   Simulate the both Markov Chains to validate your results.

Words

Build a Hidden Markov Model that uses both Part of Speech and Emitted word to determine the next part of speech and the next emitted word.



You will need to smooth over undefined combinations.

Do this with λ-smoothing or by creating a hierarchy model. Compare the results of both. Is the hierarchichal model better or worse than λ-smoothing?

Mutants

Adaptation happens when the child of an organism has a mutation that allows them to survive better than their parent (the parents are called the wildtype).

Children are better if: They can reproduce more, or they die less.

Starting from the differential equations

$$\frac{dW}{dt} = BW\left(1 - \frac{W + \alpha M}{K}\right) - \delta W$$
$$\frac{dM}{dt} = B(1+s)M\left(1 - \frac{W + \alpha M}{K}\right) - \delta(1-\sigma)M$$

Develop a continuous time Markov Chain model that allows the wildtype to proliferate, but a new progeny has a probability p of being a mutant (p should be small… think 10^{-8} at least. Once a mutant is created, it starts its own Markov process with its own parameters.

Here, K represents the number of individuals that an environment can maintain. The closer the population is to K, the less likely it is to create a new individual.

The question is this: starting from 0.1K wildtype individuals, if $\alpha = 2$, how big does s have to be for the mutant to take hold if $\sigma = 0$. Likewise, how big does σ have to be (acknowledging that it cannot be bigger than 1) in order for the mutant to take hold if s=0?

If we let the model run long enough, does M always dominate W for any values of α, σ and s? What if we start with exactly K wildtype individuals? Is there a difference in time to domination by M? Does M have a bigger chance of failing?