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Some Discrete Probability Distribution

5.2 Binomial and Multinomial Distributions

- Binomial Distribution

A Bernoulli trial can result in a success with probability p and a failure with probability $q = 1 - p$. Then, the probability distribution of the binomial random variable X, the number of successes in n independent trials, is

$$b(x; n, p) = \binom{n}{x} p^{x} q^{n-x}, \quad x = 0, 1, 2, \ldots, n.$$

. Theorem 5.1: The mean and variance of the binomial distribution $b(x; n, p)$ are

$$\mu = np \quad \text{and} \quad \sigma^2 = npq.$$

. Multinomial Experiments and Multinomial Distribution

- Multinomial Distribution

If a given trial can result in the k outcomes $E_1, E_2, \ldots, E_k$ with probabilities $p_1, p_2, \ldots, p_k$, then the probability distribution of the random variables $X_1, X_2, \ldots, X_k$, representing the number of occurences for $E_1, E_2, \ldots, E_k$ in n independent trials, is

$$f(x_1, x_2, \ldots, x_k; p_1, p_2, \ldots, p_k, n) = \binom{n}{x_1, x_2, \ldots, x_k} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$$

with

$$\sum_{i=1}^{k} x_i = n \text{ and } \sum_{i=1}^{k} p_i = 1$$

5.3 Hypergeometric Distribution

It following two properties:

1. A random sample of size n is selected without replacement from N items

2. Of the N items, k may be classified as successes and $N-k$ are classified as failures.

- Hypergeometric Distribution in Acceptance Sampling.

- Hypergeometric Distribution

The probability distribution of the hypergeometric random variable X, the number of successes in a random sample of size n selected from N items of which k are labeled success and $N-k$ labeled failure, is

$$h(x; N, n, k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}},$$

with $\max\{0, n-(N-k)\} \le x \le \min\{n, k\}$

5.5 Poisson Distribution and The Poisson Process

Properties of the Poisson Process

1. The number of outcomes occuring in one time interval / specified region of space is independent of the number that occur in any other disjoint time interval or region. In this sense we say that the Poisson process has no memory.

2. The probability that a single outcome will occur during a very short time interval or in a small region is proportional to the length of the time interval or the size of the region and doesn't depend on the number of outcomes occuring outside this time interval or region.

3. The probability that more than one outcome will occur in such a short time interval or fall in such a small region is negligible.

- Poisson Distribution

The probability distribution of the Poisson random variable X, representing the number of outcomes occuring in a given time interval or specified region denoted by t, is

$$p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^{x}}{x!}, \quad x = 0, 1, 2, \ldots$$

where λ is the average number of outcomes per unit time, distance, area, or volume and $e = 2.71828\ldots$

- Theorem 5.4

Both the mean and the variance of the Poisson distribution $p(x;\lambda t)$ are λt

Approximation of Binomial Distribution by a Poisson Distribution.

- Theorem 5.5

Let X be a binomial random variable with probability distribution $b(x;n,p)$. When $n \to \infty$, $p \to 0$, and $np \xrightarrow{n\to\infty} \mu$ remains constant,

$$b(x;n,p) \xrightarrow{n\to\infty} p(x;\mu)$$

- Theorem 5.2 : The mean and variance of the hypergeometric distribution $h(x;N,n,k)$ are

$$\mu = \frac{nk}{N} \text{ and } \sigma^2 = \frac{N-n}{N-1} \cdot n \cdot \frac{k}{N}\left(1-\frac{k}{N}\right)$$

- Relationship to the Binomial Distribution

- Multivariate Hypergeometric Distribution

If N items can be partitioned into the k cells $A_1, A_2, \ldots, A_k$ with $a_1, a_2, \ldots, a_k$ elements, respectively, then the probability distribution of the random variables $X_1, X_2, \ldots, X_k$, representing the number of elements selected from $A_1, A_2, \ldots, A_k$ in a random sample of size n, is

$$f(x_1, x_2, \ldots, x_k; a_1, a_2, \ldots, a_k, N, n) = \frac{\binom{a_1}{x_1}\binom{a_2}{x_2}\cdots\binom{a_k}{x_k}}{\binom{N}{n}}$$

with $\sum_{i=1}^{k} x_i = n$ and $\sum_{i=1}^{k} a_i = N$

5.4 Negative Binomial and Geometric Distributions

→ Negative Binomial Experiments → the probability that the k^{th} success occurs on the x^{th} trial.

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- Negative Binomial Distribution

If repeated independent trials can result in a success with probability p and a failure with probability $q = 1-p$, then the probability distribution of the random variable X, the number of the trial on which the k^{th} success occurs, is

$$b^*(x; k, p) = \binom{x-1}{k-1} p^k q^{x-k}, \quad x = k, k+1, k+2, \ldots$$

- Geometric Distribution

If repeated independent trials can result in a success with probability p and a failure with probability $q = 1-p$, then the probability distribution of the random variable X, the number of the trial on which the first success occurs, is

$$g(x; p) = p q^{x-1}, \quad x = 1, 2, 3, \ldots$$

Theorem 5-3.

The mean and variance of a random variable following the geometric distribution are

$$\mu = \frac{1}{p} \quad \text{and} \quad \sigma^2 = \frac{1-p}{p^2}$$