Rangkuman Bab 6

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Some Continuous Probability Distributions

6.1 Continuous Uniform Distribution

- Uniform Distribution

The density function of the continuous uniform random variable X on the interval $[A,B]$ is

$$f(x; A, B) = \begin{cases} \frac{1}{B-A} & A \le x \le B \\ 0, & \text{elsewhere} \end{cases}$$

- Theorem 6.1 :

The mean and variance of the uniform distribution are

$$\mu = \frac{A+B}{2} \quad \text{and} \quad \sigma^2 = \frac{(B-A)^2}{12}$$

6.2 Normal Distribution

- Normal Distribution

The density of the normal random variable X, with mean μ and variance σ^2 is

$$n(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad -\infty < x < \infty$$

where $\pi = 3,14159\ldots$ and $e = 2.71828$

Properties of the normal curve:

1. The mode, which is the point on the horizontal axis where the curve is a maximum, occurs at $x = \mu$

2. The curve is symmetric about a vertical axis through the mean μ.

3. The curve has its points of inflection at $x = \mu \pm \sigma$: it is concave downward if $\mu - \sigma < X < \mu + \sigma$ and is concave upward otherwise.

• Theorem 6.2

The mean and variance of $n(x; \mu, \sigma)$ are μ and σ^2, respectively. Hence, the standard deviation is σ

6.3 Areas Under the Normal Curve

$Z \rightarrow$ normal random variable

$$Z = \frac{X - \mu}{\sigma}$$

whenever X assumes a value x, the corresponding value of Z is given by $z = (x - \mu)/\sigma$

• The distribution of a normal random variable with mean 0 and variance 1 is called standard normal distribution.

• Using the Normal Curve in Reverse

$$z = \frac{x - \mu}{\sigma} \rightarrow x = \sigma z + \mu$$

6.5 Normal Approximation to the Binomial Distribution

• Theorem 6.3

If X is a binomial random variable with mean $\mu = np$ and variance $\sigma^2 = npq$, then the limiting form of the distribution of

$$Z = \frac{X - np}{\sqrt{npq}},$$

as $n \rightarrow \infty$, is the standard normal distribution $n(z; 0, 1)$

• Normal Approximation to the Binomial Distribution.

Let X be a binomial random variable with parameters n and p. For large n, X has approximately a normal distribution with $\mu = np$ and $\sigma^2 = npq = np(1-p)$ and

$$P(X \leq x) = \sum_{k=0}^{x} b(k; n, p)$$
$$= P\left(Z \leq \frac{x + 0.5 - np}{\sqrt{npq}}\right),$$

and the approximation will be good if np and $n(1-p)$ are greater than or equal to 5.

6.6 Gamma and Exponential Distributions

• The gamma function is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} \, dx \quad \text{for } \alpha > 0$$

Few simple properties of the gamma function

a) $\Gamma(n) = (n-1)(n-2) \cdots (1)\Gamma(1)$, for a positive int n

b) $\Gamma(n) = (n-1)!$, for a positive integer n

c) $\Gamma(1) = 1$.

d) $\Gamma(1/2) = \sqrt{\pi}$

· Gamma Distribution.

The continuous random variable X has a gamma distribution, with parameters α and β, if its density function is given by

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & x>0 \\ 0, & \text{elsewhere} \end{cases}$$

where $\alpha > 0$ and $\beta > 0$

- The special gamma distribution for which $\alpha = 1$ is called exponential distribution.

· Exponential Distributions

The continuous random variable X has an exponential distribution, with parameter β, if its density function is given by

$$f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x>0 \\ 0, & \text{elsewhere} \end{cases}$$

where $\beta > 0$.

· Theorem 6.4: The mean and variance of the gamma distribution are $\mu = \alpha\beta$ and $\sigma^2 = \alpha\beta^2$

· Corollary 6.1: The mean and variance of the exponential distribution are $\mu = \beta$ and $\sigma^2 = \beta^2$