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One- and Two-Sample Estimation Problems

9.4 Single Sample: Estimating the Mean.

- Confidence Interval on μ, σ^2 Known.

If $\bar{x}$ is the mean of a random sample of size n from a population with known variance σ^2, a $100(1-\alpha)\%$ confidence interval for μ is given by

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

where $z_{\alpha/2}$ is the z-value leaving an area of $\alpha/2$ to the right

Theorem 9.1 If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error will not exceed $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.

Theorem 9.2 If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error will not exceed a specified amount e when the sample size is

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{e}\right)^2$$

- One-sided Confidence Bounds.
  - One-sided Confidence Bounds on μ, σ^2 Known.

If $\bar{X}$ is the mean of a random variable of size n from a population with variance σ^2, the one-sided $100(1-\alpha)\%$ confidence bounds for μ are given by:

upper one-sided bound: $\bar{x} + z_\alpha \sigma/\sqrt{n}$

lower one-sided bound: $\bar{x} - z_\alpha \sigma/\sqrt{n}$.

- The Case of σ Unknown.
  - Confidence Interval on μ, σ^2 Unknown.

If $\bar{x}$ and s are the mean and standard deviation of a random sample from a normal population with unknown variance σ^2, a $100(1-\alpha)\%$ confidence interval for μ is

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}},$$

where $t_{\alpha/2}$ is the t-value with $v = n-1$ degrees of freedom, leaving an area of $\alpha/2$ to the right.

- Concept of a Large-Sample Confidence Interval
  - Often statisticians recommend that even when normality cannot be assumed, σ is unknown, and $n \geq 30$, s can ~~be~~ replaced σ and the confidence interval

$$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}.$$

may be used. This is often referred to as a large-sample confidence interval.

9.5 Standard Error of a Point Estimate.

- Confidence Limits on μ, σ^2 Unknown.

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = \bar{x} \pm t_{\alpha/2}\,s.e.(\bar{x}).$$

9.6. Prediction Intervals.

- Prediction Interval of a Future Observation, σ^2 Known.

For a normal distribution of measurements with unknown mean μ and known variance σ^2, a $100(1-\alpha)\%$ prediction interval of a future observation x_0 is

$$\bar{x} - z_{\alpha/2}\,\sigma\sqrt{1+\frac{1}{n}} < x_0 < \bar{x} + z_{\alpha/2}\,\sigma\sqrt{1+\frac{1}{n}}$$

where $z_{\alpha/2}$ is the z-value leaving an area of $\alpha/2$ to the right.

- Prediction Interval of a Future Observation, σ^2 Unknown.

  For a normal distribution of measurements with unknown mean μ and unknown variance σ^2, a $100(1-\alpha)\%$ prediction interval of a future observation x_0 is

$$\bar{x} - t_{\alpha/2}\, s\sqrt{1+1/n} < x_0 < \bar{x} + t_{\alpha/2}\, s\sqrt{1+1/n}.$$

where $t_{\alpha/2}$ is the t-value with $v = n-1$ degrees of freedom, leaving an area of $\alpha/2$ to the right

- Tolerance Limit. Unknown μ, Unknown σ

  For a normal distribution of measurements with unknown mean μ and unknown standard deviation σ, tolerance limits are given by $\bar{x} \pm ks$, where k is determined such that one can assert with $100(1-\gamma)\%$ confidence that the given limits contain at least the proportion $1-\alpha$ of the measurements.