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Rangkuman Bab 3

Random Variables &
Probability Distribution.

3.1 Concept of a Random Variables

- A random variable is a function that associates a real number with each element in the sample space.
- If a sample space contains a finite number of possibilities or an unending sequence with as many elements as there are whole numbers, it is called a discrete sample space
- If a sample ~~con~~ space contains an infinite number of possibilities equal to the number of points on a line segment, it is called continuous sample space

→ discrete random variable, continuous random variable.

3.2 Discrete Probability Distributions.

- The set of ordered pairs $(x, f(x))$ is a probability function, probability mass function, or probability distribution of the discrete variable X if, for each possible outcome x,

1. $f(x) \geq 0$

2. $\sum_x f(x) = 1$

3. $P(X=x) = f(x)$.

- The cumulative distribution function $F(x)$ of a discrete random variable X with probability distribution $f(x)$

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t), \text{ for } -\infty < x < \infty$$

3.3 Continuous Probability Distributions

The function $f(x)$ is a probability density function for the continuous random variable X, defined over the set of real numbers, if

1. $f(x) \geq 0$, for all $x \in \mathbb{R}$

2. $\int_{-\infty}^{\infty} f(x)\, dx = 1$

3. $P(a < X < b) = \int_a^b f(x)\, dx$.

- The cumulative distribution function $F(x)$ of a continuous random variable X with density function $f(x)$ is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt, \text{ for } -\infty < x < \infty$$

3.4 Joint Probability Distributions

- The function $f(x,y)$ is a joint probability distribution or probability mass function of the discrete random variables X and Y if

1. $f(x,y) \geq 0$ for all (x,y)

2. $\sum_x \sum_y f(x,y) = 1$.

3. $P(X=x, Y=y) = f(x,y)$

For any region A in the xy plane, $P[(X,Y) \in A]$

$$= \sum\sum_A f(x,y)$$

- The function $f(x,y)$ is a joint density function of the continuous random variables X and Y if

1. $f(x,y) \geq 0$, for all (x,y),

2. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$,

3. $P[(X,Y) \in A] = \iint_A f(x,y)\, dx\, dy$, for any region A in the xy plane

- The marginal distributions of X alone and of Y alone are

$$g(x) = \sum_y f(x,y), \quad h(y) = \sum_x f(x,y)$$

for discrete case, and

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\, dy, \quad h(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$$

for the continuous case.

- Let X and Y be two random variables, discrete or continuous. The conditional distribution of the random variable Y given that $X = x$ is

$$f(y|x) = \frac{f(x,y)}{g(x)}, \text{ provided } g(x) > 0$$

similarly, the conditional distribution of X given that $Y = y$ is

$$f(x|y) = \frac{f(x,y)}{h(y)}, \text{ provided } h(y) > 0.$$

- The random variables X and Y are said to be statistically independent if and only if

$$f(x,y) = g(x)\, h(y)$$

for all (x,y) within their range.

- The random variables $X_1, X_2, \dots, X_n$ are said to be mutually statistically independent if and only if

$$f(x_1, x_2, \dots, x_n) = f_1(x_1)\, f_2(x_2) \cdots f_n(x_n).$$

for all $(x_1, x_2, \dots, x_n)$ within their range.