Rangkuman Bab 2

Probabilitas.

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2.1 Sample Space

- The set of all possible outcomes of a statistical experiment is called the sample space and is represented by the symbol S.
- Each outcome in a sample space is called an element or a member of the sample space, or simply a sample point.

2.2. Events

- An event is a subset of a sample space
- The intersection of two events A and B, denoted by the symbol $A \cap B$, is the event containing all elements that are common to A and B.
- Two events A and B are mutually exclusive, or disjoint if $A \cap B = \emptyset$, that is, if A and B have no elements in common
- The union of the two events A and B, denoted by the symbol $A \cup B$, is the event containing all the elements that belong to A or B or both.

2.3 Counting Sample Points.

- Rule 1: If an operation can be performed in n_1 ways, and if for each of these ways a second operation can be performed in n_2 ways, then the two operations can be performed together in $n_1 n_2$ ways.
- Rule 2: If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways, and for each of the first two a third operation can be performed in n_3 ways, and so forth, then the sequence of k operations can be performed in $n_1 n_2 \cdots n_k$ ways.
- A <u>permutation</u> is an arrangement of all or part of a set of objects.
- For any non-negative integer n, $n!$ called "n factorial" is defined as

$$n! = (n)(n-1)(n-2)\cdots(2)\cdot(1)$$

with special case $0! = 1$.

- Theorem 2.1: The number of permutations of n objects is $n!$

$$_nP_r = \frac{n!}{(n-r)!} \rightarrow \text{Theorem 2.2}$$

- Theorem 2.3: The number of permutations of n objects arranged in a circle is $(n-1)!$.
- Theorem 2.4: The number of distinct permutations of n things of which n_1 are of one kind, n_2 of a second kind, ..., n_k of a k_{th} kind is

$$\frac{n!}{n_1!\,n_2!\cdots n_k!}$$

- Theorem 2.5: The number of ways partitioning a set of n objects into r cells with n_1 elements in the first cell, n_2 elements in the second, and so forth, is

$$\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1!\,n_2!\,n_3!\cdots n_r!}$$

where $n_1 + n_2 + \cdots + n_r = n$.

- Theorem 2.6. The number of combinations of n distinct objects taken r at a time is

$$_nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

2.4 Probability of An Event.

- The probability of an event A is the sum of the weights of all sample points in A. Therefore,

$$0 \le P(A) \le 1, \quad P(\emptyset) = 0, \text{ and } P(S) = 1$$

Furthermore, if $A_1, A_2, \ldots$ is a sequence of mutually exclusive events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

- Rule 2.3 : If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is

$$P(A) = \frac{n}{N}$$

2.5 Additive Rules.

Theorem 2.7 : If A and B are two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Corollary 2.1 : If A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B)$$

Corollary 2.2 : If $A_1, A_2, \ldots, A_n$ are mutually exclusive, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

Corollary 2.3 : If $A_1, A_2, \ldots, A_n$ is a partition of sample space S, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$
$$= P(S) = 1$$

Theorem 2.8 : For three events A, B, C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Theorem 2.9 : If A and A' are complementary events, then

$$P(A) + P(A') = 1$$

2.6 Conditional Probability, Independence, and The Product Rule

- The conditional probability of B given A, denoted by $P(B|A)$, is defined by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ provided } P(A) > 0.$$

- Two events A and B are independent if and only if ~~iff~~ $P(B|A) = P(B)$ or $P(A|B) = P(A)$, assuming the existence of the conditional probabilities. Otherwise, A and B are dependent

Theorem 2.10: If in an experiment the events A and B can both occur, then

$$P(A \cap B) = P(A)\,P(B|A), \text{ provided } P(A) > 0$$

Theorem 2.11: Two events A and B are independent if and only if

$$P(A \cap B) = P(A)\,P(B).$$

Therefore, to obtain the probability that two independent events will both occur, we simply find the product of their individual probabilities.

Theorem 2.12: If, in an experiment, the events $A_1, A_2, \cdots, A_k$ can occur, then

$$P(A_1 \cap A_2 \cap \dots \cap A_k)$$
$$= P(A_1)\,P(A_2|A_1)\,P(A_3|A_1 \cap A_2) \cdots P(A_k|A_1 \cap \dots \cap A_{k-1})$$

If the events $A_1, A_2, \cdots, A_k$ are independent, then

$$P(A_1 \cap A_2 \cap \dots \cap A_k) = P(A_1)\,P(A_2) \cdots P(A_k)$$

- A collection of events $A = \{A_1, \cdots, A_n\}$ are mutually independent if for any subset of $A_{i_1}, A_{i_2}, \cdots, A_{i_k}$, for $k \le n$, we have

$$P(A_{i_1} \cap \cdots A_{i_k}) = P(A_{i_1}) \cdots P(A_{i_k})$$

2.7 Bayes' Rule

Theorem 2.13: If the events $B_1, B_2, \cdots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \cdots, k$, then for any event A of S,

$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) = \sum_{i=1}^{k} P(B_i)\,P(A|B_i)$$

Theorem 2.14: (Bayes' Rule) If the events $B_1, B_2, \dots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \cdots, k$, then for any event A in S such that $P(A) \neq 0$,

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)\,P(A|B_r)}{\sum_{i=1}^{k} P(B_i)\,P(A|B_i)}$$

for $r = 1, 2, \cdots, k$