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Applications of Set Covering, Set Packing and Set Partitioning Models: A Survey

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Contents

1 Introduction **575**

2 SP and SPT Models and Their Variants **576**

3 Transformation of the Models **578**
3.1 SPT to SC . . . 579
3.2 SPT to SP . . . 579
3.3 SC to GSP and SP to GSC . . . 580
3.4 Zero-one MDK to GSP . . . 580
3.5 GSP to SP . . . 582
3.6 Zero-one LIP to Zero-one MD Knapsack . . . 585
3.7 Mixed SPT and SP to SP . . . 586

4 Graphs and Networks **586**
4.1 Vertex (Node) Packing Problem . . . 588
4.2 Maximum Matching . . . 589
4.3 Minimum Covering Problem . . . 592
4.4 Chromatic Index and Chromatic Number . . . 592
4.5 Multi-Commodity Disconnecting Set Problem . . . 594
4.6 Steiner Problem in Graphs . . . 596

5 Personnel Scheduling Models **598**
5.1 Days Off or Cyclical Scheduling 599
5.2 Shift Scheduling 601
5.3 Tour Scheduling 602

6 Crew Scheduling **603**

7 Manufacturing **609**
7.1 Assembly Line Balancing Problem 609
7.2 Discrete Lot Sizing and Scheduling Problem 610
7.3 Ingot Size Selection 611
7.4 Spare Parts Allocation 613
7.5 Pattern Sequencing in Cutting Stock Operations 613
7.6 Constrained Network Scheduling Problem 615
7.7 Cellular Manufacturing 617

8 Miscellaneous Operations **618**
8.1 Frequency Planning 618
8.2 Timetable Scheduling 619
8.3 Testing and Diagnosis 620
8.4 Political Districting 622
8.5 Information Retrieval and Editing 623
8.6 Check Clearing 625
8.7 Capital Budgeting Problem 626
8.8 Fixed Charge Problem 627
8.9 Mathematical Problems 628

9 Routing **628**
9.1 Traveling Salesman Problem 629
9.2 Single Depot Vehicle Routing 631
9.3 Multiple Depots and Extensions 633

10 Location **633**
10.1 Plant Location Problem 634
10.2 Lock Box Location Problem 636
10.3 *P*-Center Problem 637
10.4 *P*-Median Problem 638
10.5 Service Facility Location Problem 638

11 Review of Bibliography **640**
11.1 Theory 641
11.2 Transformations 642
11.3 Graphs 642
11.4 Personnel Scheduling 643
11.5 Crew Scheduling 644

11.6 Manufacturing 645
11.7 Miscellaneous Operations 646
11.8 Routing 647
11.9 Location 652

12 Conclusions **658**

References

Abstract

Set covering, set packing and set partitioning models are a special class of linear integer programs. These models and their variants have been used to formulate a variety of practical problems in such areas as capital budgeting, crew scheduling, cutting stock, facilities location, graphs and networks, manufacturing, personnel scheduling, vehicle routing and timetable scheduling among others. Based on the special structure of these models, efficient computational techniques have been developed to solve large size problems making it possible to solve many real world applications. This paper is a survey of the applications of the set covering, set packing, set partitioning models and their variants, including generalizations.

1 Introduction

Set covering (SC), set packing (SP) and set partitioning (SPT) problems are a very useful and important class of linear integer programming models. A variety of practical problems have been formulated as one of these models or their variants. Because of their special structure, it has been possible to develop efficient techniques (see the list of references under the theory) to solve large size problems. Due to the facility to solve large problems and the flexibility to model a variety of systems coupled with the advances in computer technology, these models have been used to solve many real world problems. This paper is a survey of SC, SP and SPT formulations (including their variants) of capital budgeting, crew scheduling, cutting stock, facilities location, graphs and networks, manufacturing, personnel scheduling, vehicle routing and time table scheduling problems among others. In addition, relationships among these models and the conversion of finite linear integer programs (LIP) to one of these models are presented.

Another objective of this paper is to provide an extensive bibliography on both theory and applications of SC, SP and SPT models. For convenience, the reference list is divided into nine groups namely, general, theory, graphs, personnel scheduling, crew scheduling, manufacturing, miscellaneous operations, routing and location. The general list consists of useful references related to general integer programming techniques and concepts including Lagrangian relaxation, surrogate constraints, subgradient methods, tabu search, disjunctive programming, branch and bound and cutting plane methods. These references are selected from the papers listed in the other eight groups which deal with both application and theory of SC, SP and SPT models. The theory list contains articles exclusively devoted to algorithms and mathematical properties of the SC, SP and SPT models. The remaining seven groups deal with papers related to the specific application area the title suggests except the group miscellaneous operations. The miscellaneous operations group contains papers related to a variety of applications including time table scheduling, information retrieval, political redistricting, diagnostic systems, distribution of broadcasting frequencies among others. It should be noted that many papers listed in the application group contain theoretical contributions also. Also some papers in the application group especially in location and routing, may be concerned with nonlinear programming models and general integer programming models. The purpose of including such articles is to provide as comprehensive a bibliography as possible. The list of references included in this paper is by no means complete. The interested reader may review the list of the papers included in the bibliography for additional references. The author apologizes, if some useful and relevant papers are not included.

The next section deals with the basic SC, SP, and SPT models and their variants. The subsequent sections address relationships among these models including LIP and each application group. Several numerical examples are provided throughout the paper to illustrate various applications.

2 SP and SPT Models and Their Variants

Consider a finite set $M = 1, 2, ..., m$ and $M_j, j \in N$, a collection of subsets of the set M where $N = 1, 2, ..., n$. A subset $F \subseteq N$ is called a *cover* of M if $\cup_{j \in F} M_j = M$. The subset $F \subseteq N$ is called a *packing* of M if $M_j \cap M_k = \emptyset$

for all $j, k \in F$ and $j \neq k$. If $F \subseteq N$ is both a cover and packing then it is called a *partitioning*.

Suppose c_j is the cost associated with M_j. Then the set covering problem is to find a minimum cost cover. If c_j is the value or weight of M_j, then the set packing problem is to find a maximum weight or value packing. Similarly the set partitioning problem is to find a partitioning with minimum cost. These problems can be formulated as zero-one linear integer programs as shown below. For all $i \in M$ and $j \in N$ let

$$a_{ij} = \begin{cases} 1 & \text{if } i \in M_j \\ 0 & \text{otherwise} \end{cases}$$

and

$$x_j = \begin{cases} 1 & \text{if } j \in F \\ 0 & \text{otherwise} \end{cases}$$

Then the set covering (1), set packing (2) and set partitioning (3) formulations are given by

$$\begin{array}{lll} \min & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \geq 1 & i = 1, 2, ..., m \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{1}$$

$$\begin{array}{lll} \max & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \leq 1 & i = 1, 2, ..., m \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{2}$$

$$\begin{array}{lll} \max & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j = 1 & i = 1, 2, ..., m \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{3}$$

The following numerical example illustrates the above models.

Example 1 Suppose $M = \{1, 2, 3, 4, 5, 6\}$, $M_1 = \{1, 2\}$, $M_2 = \{1, 3, 4\}$, $M_3 = \{2, 4, 5\}$, $M_4 = \{3, 5, 6\}$, $M_5 = \{4, 5, 6\}$, $c_1 = 5$, $c_2 = 4$, $c_3 = 6$, $c_4 = 2$, and $c_5 = 4$. The formulation of the set covering model is given by

$$\begin{array}{ll} \min & 5x_1 + 4x_2 + 6x_3 + 2x_4 + 4x_5 \\ \text{s.t.} & x_1 + x_2 \geq 1 \end{array}$$

$$
\begin{aligned}
& x_1 + x_3 \geq 1 \\
& x_2 + x_4 \geq 1 \\
& x_2 + x_3 + x_5 \geq 1 \\
& x_3 + x_4 + x_5 \geq 1 \\
& x_4 + x_5 \geq 1 \\
\text{and} \quad & x_1, x_2, x_3, x_4, x_5 \in \{0, 1\}
\end{aligned}
$$

The formulation of the corresponding set packing and set partitioning models is straight forward.

Clearly, the three models are special structured zero-one linear integer programs since all the elements of the constraint coefficient matrix $A = (a_{ij})$ are 0 or 1 and the right hand side (RHS) of the constraints are all unity. For some applications the RHS of the constraints may be not all be unity but positive integers. The corresponding models are called general set covering (GSC), general set packing (GSP) and general set partitioning (GSPT). For these general models, while the variables are required to be non-negative integers, they need not be constrained to zero or one. For some other applications the constraint set may include two types of inequalities or all three types of inequalities. Such models are called mixed models. For example, a model with both less than or equal to and greater than or equal to constraints with all RHS equal to unity is called mixed SC and SP model.

It should be noted that the integer programming formulations of the SC, SP and SPT models including their generalizations are NP-complete, except for a few special cases. The relationships among these models and the transformation of the linear integer programs, including the zero-one multi-dimensional knapsack problem into a SP problem, are presented next.

3 Transformation of the Models

Transformation of the models is useful in comparing various computational techniques to generate the optimal solutions. Several transformations to convert one model to another model including the conversion of a zero-one LIP to a zero-one multi-dimensional knapsack (MDK) problem and the conversion of a MDK problem to a SP problem are explored in this section.

3.1 SPT to SC

Subtracting artificial surplus variables y_i from the ith constraint of the SPT model formulation (3), the constraints of the SPT model can be written as:

$$\sum_{j=1}^{n} a_{ij}x_j - y_i = 1.$$

To insure all artificial surplus variables remain at zero level in any optimal solution (when SPT is feasible), the objective function is changed to

$$\sum_{j=1}^{n} c_j x_j + \theta \sum_{i=1}^{m} y_i$$

where θ is any number greater than $\sum_j c_j$. Substituting for y_i in the objective function and eliminating y_i from the constraints, the following SC formulation yields an optimal solution to the SPT problem.

$$\begin{array}{lll} \min & \sum_{j=1}^{n} c'_j x_j - m\theta & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij}x_j \geq 1 & i = 1, 2, ..., m \\ \text{and} & \quad x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{4}$$

where $c'_j = c_j + \theta \sum_{j=1}^{m} a_{ij}$.

3.2 SPT to SP

Adding artificial slack variables y_i to the ith constraint of the SPT model formulation (3), the constraints of the SPT model can be written as:

$$\sum_{j=1}^{n} a_{ij}x_j + y_i = 1.$$

As noted earlier, the modified objective function

$$\sum_{j=1}^{n} c_j x_j + \theta \sum_{i=1}^{m} y_i$$

guarantees that all artificial slack variables remain at zero level in any optimal solution. Substituting for y_i in the objective function and eliminating

y_i from the constraints, the following formulation yields an optimal solution to the SPT problem.

$$\begin{array}{lll} \min & \sum_{j=1}^{n} c'_j x_j + m\theta & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \leq 1 & i = 1,2,...,m \\ \text{and} & x_j = 0,1 & j = 1,2,...,n \end{array} \tag{5}$$

where $c'_j = c_j - \theta \sum_{i=1}^{m} a_{ij}$. Noting the fact c'_j are negative numbers and changing the objective function to

$$\max \sum_{j=1}^{n} (-c'_j) x_j$$

results in a SP formulation.

3.3 SC to GSP and SP to GSC

Substituting $x_j = 1 - y_j$, the formulation (1) of the SC problem is equi valent to

$$\begin{array}{lll} \max & \sum_{j=1}^{n} c_j y_j - \sum_{j=1}^{n} c_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} y_j \leq b_i & i = 1,2,...,m \\ \text{and} & y_j = 0,1 & j = 1,2,...,n \end{array} \tag{6}$$

where $b_i = \sum_{j=1}^{n} a_{ij}$. Noting the fact that all b_i are nonnegative integers (when SC is feasible), the above is a GSP formulation of the SC problem. A similar transformation can be used to convert a SP problem into a GSC problem.

3.4 Zero-one MDK to GSP

The LIP formulation of a zero-one MDK problem is

$$\begin{array}{lll} \max & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \leq b_i & i = 1,2,...,m \\ \text{and} & x_j = 0,1 & j = 1,2,...,n \end{array} \tag{7}$$

where a_{ij}, c_j, and b_i are all nonnegative numbers. When all numbers involved are rational they can be converted to integers by multiplying them with an appropriate positive integer. In transforming this problem to a GSP problem it is assumed that all a_{ij}, c_j, and b_i are nonnegative integers. Let

$$\begin{aligned} a_j &= \max_{i=1}^{m} a_{ij} \qquad & j &= 1,2,...,n, \text{ and} \\ x_{jk} &= 0,1 & j &= 1,2,...,n \\ & & k &= 1,2,...,a_j \end{aligned}$$

If all x_{jk} are guaranteed to be equal for a given j, clearly (when $a_{ij} > 0$)

$$a_{ij}x_j = \sum_{k=1}^{a_{ij}} x_{jk}.$$

Then an equivalent formulation of the MDK problem is given by

$$\begin{array}{lll} \max & \sum_{j=1}^{n}\sum_{k=1}^{a_j} c_{jk}x_{jk} & \\ \text{s.t.} & \sum_{j=1}^{n}\sum_{k=1}^{a_{ij}} x_{jk} \le b_i & i = 1,2,...,m \\ & x_{jk} + z_j = 1 & j = 1,2,...,n \\ & & k = 1,2,...,a_j \\ & x_{jk} = 0,1 & j = 1,2,...,n \\ & & k = 1,2,...,a_j \\ \text{and} & z_j = 0,1 & j = 1,2,...,n \end{array} \tag{8}$$

where $c_{jk} = c_j/a_j$ for $k = 1,2,...,a_j$. It should be noted that the second set of constraints guarantee that all x_{jk} are equal for a given j. Add artificial slack variables to the equality constraints with a very large negative coefficient in the objective function as in the conversion of the SPT problem to SP problem and eliminate the artificial slack variables from both the objective function and the constraints to obtain the required formulation.

Example 2. Consider the following zero-one two dimensional knapsack problem.

$$\begin{array}{lll} \max & 10x_1 + 4x_2 + 12x_3 + x_4 & \\ \text{s.t.} & 3x_1 + x_2 + 4x_3 + x_4 \le 4 & \\ & x_1 + 2x_2 + 2x_3 + 2x_4 < 3 & \\ \text{and} & x_j = 0,1 & j = 1,2,3,4. \end{array}$$

Since $a_1 = \max(3,1) = 3, a_2 = \max(1,2) = 2, a_3 = \max(4,2) = 4$, and $a_4 = \max(1,2) = 2$, by defining eleven $x_{jk}(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{31}, x_{32}, x_{33}, x_{34},$

x_{41}, and x_{42}) and four $z_j(z_1, z_2, z_3,$ and $z_4)$ zero-one variables, the problem can be reformulated as

$$
\begin{array}{ll}
\max & (10/3)(x_{11}+x_{12}+x_{13})+2(x_{21}+x_{22})+3(x_{31}+x_{32}+x_{33}+x_{34}) \\
 & +(1/2)(x_{41}+x_{42}) \\
\text{s.t.} & x_{11}+x_{12}+x_{13}+x_{21}+x_{31}+x_{32}+x_{33}+x_{34}+x_{41}\le 4 \\
 & x_{11}+x_{21}+x_{22}+x_{31}+x_{32}+x_{41}+x_{42}\le 3 \\
 & x_{1k}+z_1=1 \qquad k=1,2,3 \\
 & x_{2k}+z_2=1 \qquad k=1,2 \\
 & x_{3k}+z_3=1 \qquad k=1,2,3,4 \\
 & x_{4k}+z_4=1 \qquad k=1,2 \\
 & z_j=0,1 \qquad j=1,2,3,4 \\
 & x_{1k}=0,1 \qquad k=1,2,3 \\
 & x_{2k}=0,1 \qquad k=1,2 \\
 & x_{3k}=0,1 \qquad k=1,2,3,4 \\
\text{and} & x_{4k}=0,1 \qquad k=1,2.
\end{array}
$$

The equality constraints can be changed to less than or equal to constraints by subtracting the following expression

$$\theta(\sum_{k=1}^{3} x_{1k}+\sum_{k=1}^{2} x_{2k}+\sum_{k=1}^{4} x_{3k}+\sum_{k=1}^{2} x_{4k}+3z_1+2z_2+4z_3+2z_4)-11\theta$$

from the objective function where $\theta > (10+4+12+1)=27$.

3.5 GSP to SP

The LIP formulation of a GSP problem when the variables are restricted to binary is

$$
\begin{array}{llll}
\max & \sum_{j=1}^{n} c_j x_j & & \\
\text{s.t.} & \sum_{j=1}^{n} a_{ij}x_j \le b_i & i=1,2,...,m & \\
\text{and} & x_j=0,1 & j=1,2,...,n & (9)
\end{array}
$$

where b_i are positive integers. In order to transform this to a SP problem, the ith constraint is replaced by b_i inequality constraints with all right hand sides equal to one. Let

$$\begin{aligned} y_{ijk} &= 0,1 \quad \text{for} \quad j = 1,2,...,n \\ & \qquad\qquad\quad i = 1,2,...,m \\ & \qquad\qquad\quad k = 1,2,...,b_i \\ z_j &= 0,1 \quad \text{for} \quad j = 1,2,...,n \end{aligned}$$

$$\text{and} \quad b = \max_{i=1}^{m} b_i$$

Rearrange the constraints if necessary so that $b_1 = b$ and consider the following mixed SP and SPT formulation.

$$\begin{array}{llll} \max & \sum_{j=1}^{n} c_j \sum_{k=1}^{b_i} y_{1jk} & & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} y_{ijk} \le 1 & i = 1,2,...,m & \\ & & k = 1,2,...,b_i & \\ & \sum_{k=1}^{b_i} y_{ijk} + z_j = 1 & i = 1,2,...,m & \\ & & j = 1,2,...,n & \\ & y_{ijk} = 0,1 & j = 1,2,...,n & \\ & & i = 1,2,...,m & \\ & & k = 1,2,...,b_i & \\ \text{and} & z_j = 0,1 & j = 1,2,...,n & \qquad (10) \end{array}$$

To see the equivalence between these two formulations summing up the inequalities over k for a given i yields

$$\sum_{j=1}^{n} a_{ij} \sum_{k=1}^{b_i} y_{ijk} \le b_i.$$

From the equality constraints it is clear that

$$\sum_{k=1}^{b_i} y_{ijk} = x_j$$

is zero-one for all $i = 1,2,...,m$. It is straight forward to verify that x_j constructed from a feasible solution to the mixed SP and SPT formulation yields a feasible solution to the GSP problem. Now suppose x_j is a feasible solution to the GSP. If $a_{ij} = 0$ and $x_j = 1$ then set any one of the variables

$y_{ijk}, k = 1, 2, ..., b_i$ equal to 1 and the remaining to zero. If $x_j(t)$ and $a_{ij}(t)$ are equal to one for $t = 1, 2, ..., h$ (note that $h \leq b_i$), then set

$$y_{ij(t)t} = 1 \text{ for all } t = 1, 2, ..., h$$

and the rest of the variables to zero. This provides a feasible solution to the mixed SP and SPT formulation with objective values of both formulations the same. Since there is one to one correspondence, the mixed SP and SPT formulation yields an optimal solution to the GSP problem. The equalities can be replaced by inequalities to obtain the SP formulation using the procedure described in coverting SPT to SP.

When x_j are not binary, they can be replaced by binary variables $y_{jk}, k = 1, 2, ..., t_j$ using the transformation

$$x_j = \sum_{k=1}^{t_j} 2^k y_{jk}$$

where t_j is suitably chosen to insure all possible values of x_j are included. Since all x_j are bounded above ($\leq b$) it is possible to select such t_j. Now substituting for x_j, in the GSP formulation yields a zero-one MD knapsack problem which can be converted to a binary GSP problem.

Example 3. Consider the following binary GSP problem.

$$\begin{array}{ll} \max & 4x_1 + 6x_2 + 7x_3 + 8x_4 + 10x_5 \\ \text{s.t.} & x_2 + x_3 + x_4 + x_5 \leq 3 \\ & x_1 + x_2 + x_3 \leq 2 \\ & x_1 + x_3 + x_4 \leq 2 \\ & x_j = 0, 1 \qquad j = 1, 2, 3, 4 \end{array}$$

Using several zero-one variables y_{ijk}, an equivalent mixed SP and SPT formulation is

$$\begin{array}{lll} \max & \sum_{k=1}^{3}(4y_{11k} + 6y_{12k} + 7y_{13k} + 8y_{14k} + 10y_{15k}) & \\ \text{s.t.} & y_{12k} + y_{13k} + y_{14k} + y_{15k} \leq 1 & k = 1, 2, 3 \\ & y_{21k} + y_{22k} + y_{23k} \leq 1 & k = 1, 2 \\ & y_{31k} + y_{33k} + y_{34k} \leq 1 & j = 1, 2 \\ & \sum_{k=1}^{3} y_{1jk} + z_j = 1 & j = 1, 2, ..., 5 \\ & \sum_{k=1}^{2} y_{2jk} + z_j = 1 & j = 1, 2, ..., 5 \end{array}$$

$$
\begin{array}{lll}
 & \sum_{k=1}^{2} y_{3jk} + z_j = 1 & j = 1,2,...,5 \\
 & y_{1jk} = 0,1 & j = 1,3,...,5 \\
 & & k = 1,2,3 \\
 & y_{2jk} = 0,1 & j = 1,2,...,5 \\
 & & k = 1,2 \\
 & y_{3jk} = 0,1 & j = 1,2,...,5 \\
 & & k = 1,2 \\
\text{and} & z_j = 0,1 & j = 1,2,...,5
\end{array}
$$

3.6 Zero-one LIP to Zero-one MD Knapsack

When the variables are bounded above, any LIP can be converted to a Zero-one LIP using the standard binary transformation. Consider the Zero-one LIP

$$
\begin{array}{lll}
\max & \sum_{j=1}^{n} c_j x_j & \\
\text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \leq b_i & i = 1,2,...,m \\
\text{and} & x_j = 0,1 & j = 1,2,...,n
\end{array} \tag{11}
$$

where all $b_i \geq 0$, and integers. When all $a_{ij} > 0$, no changes are necessary to convert the problem to a Zero-one MD Knapsack problem. When some $a_{ij} \leq 0$, let

$$
\begin{array}{ll}
 & a_{ij1} = \max(0, a_{ij}) \geq 0 \\
\text{and} & a_{ij2} = \min(0, a_{ij}) \leq 0
\end{array}
$$

for $i = 1,2,...,m$ and $j = 1,2,...,n$. For each variable x_j (only for j for which some $a_{ij} < 0$)

$$a_{ij} x_j = (a_{ij1} x_{j1} + a_{ij2} x_{j2})$$

when the binary variables x_{j1} and x_{j2} are equal. Replacing x_{j2} with $1 - y_{j2}$, an equivalent formulation of the Zero-one LIP is

$$
\begin{array}{lll}
\max & \sum_{j=1}^{n} c_j x_{j1} & \\
\text{s.t.} & \sum_{j=1}^{n} (a_{ij1} x_{j1} + (-a_{ij2} y_{j2}) \leq b_i + \sum_{j=1}^{n} (-a_{ij2}) & i = 1,2,...,m \\
 & x_{j1} + y_{j2} = 1 & j = 1,2,...,n \\
 & x_{j1} = 0,1 & \\
 & y_{j2} = 0,1 & j = 1,2,...,n
\end{array} \tag{12}
$$

Since all $a_{ij2} \leq 0$, all the coefficients involved are nonnegative including the right hand sides of the inequalities. Using the standard procedure of adding artificial slack variables, the equality constraints can be converted to inequality constraints which yields the equivalent Zero-one Knapsack formulation.

3.7 Mixed SPT and SP to SP

The equality constraints can be converted to less than or equal to constraints, by adding the artificial slack variables and eliminating them from both the constraints and the objective function. Similarly other mixed formula transformations presented in this section. Even though it is possible to convert one model to another it should be noted that some conversions require a large number of additional variables and constraints. In the next section, models related to Graphs and Networks are presented.

4 Graphs and Networks

Let N be a finite set of points and A be a finite set of ordered pair of points, (i, j), from the set N. The pair N and A is called a directed graph and is denoted by $G = (N, A)$. The elements of A are called nodes (or vertices) and the elements of A are called arcs. For any arc $(i, j) \in A$, i is called the beginning node and j is called the ending node. If the elements of A are unordered, they are called edges and the corresponding graph denoted by $G = (N, E)$, is called an undirected graph. Usually no distinction is made between graphs and networks. However, when a subset of nodes are singled out for a specific purpose such as sources and sinks to transport some commodity, the corresponding graph is called a network. A graph is called a bipartite graph if the nodes can be partitioned into two sets such that the beginning node of every arc belongs to one set and the ending node of every arc belongs to the other set. The cardinality of these sets N and A (or E) are denoted by n and m which represent the number of nodes and arcs (edges) respectively.

Two arcs (or two edges) are called adjacent if they have at least one node in common. Similarly two nodes are adjacent to each other if they are connected by an edge or arc. A chain is a sequences of arcs $(A_1, A_2, ...A_r)$ such that each arc has one node in common with its successor and predecessor with the exception of A_1 and A_r which have a common node with the successor and predecessor respectively. If i is the beginning node of A_1 and

j is the ending of A_r, then it is a chain from node i and to node j. If all the nodes encountered are distinct than it is called an elementary chain. If the beginning and end points of an elementary chain are the same then it is called a cycle. A path is a sequence of arcs $(A_1, A_2, ... A_r)$ such that the ending node of every arc in the sequence is the beginning node of the next arc. If i is the beginning node of A_1 and j is the ending node of A_r, then it is a path from node i to j. The path is elementary if all nodes encountered are distinct. If the beginning and ending nodes of a path are same then it is called a circuit. An undirected graph $G = (N, E)$ is called a tree if it has exactly $(n - 1)$ arcs and has no cycles. A directed graph $G = (N, A)$, is called a tree if it contains exactly $m = n - 1$ arcs, has no circuits and every node is the ending node of exactly one arc except one node which is the beginning node of one or many arcs but not the ending node of any arc. The following examples are is used to illustrate the concepts.

Example 4. Suppose $N = \{1, 2, 3, 4, 5\}$ and $A = \{(1, 2),\ (1, 3),\ (4, 1),\ (1, 5),\ (2, 3),\ (4, 2), (3, 4), (3, 5), (4, 5)\}$. In the above graph the arc set $A_1 =$

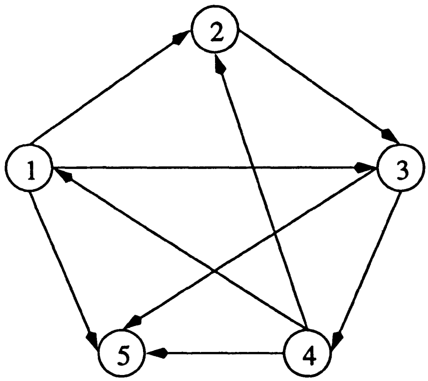

Figure 1: A Directed Graph

$\{(1, 2), (2, 3), (3, 4)\}$ is an elementary path from node 1 to node 5, the arc set $A_2 = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$ is a circuit, the arc set $A_3 = \{(1, 5), (3, 5)\}$ is an elementary chain, and the arc set $A_4 = \{(1, 5), (4, 5), (4, 1)\}$ is a cycle. Clearly in undirected graphs there are only chains and cycles.

Example 5. Suppose $N = \{1, 2, 3, 4, 5, 6\}$ and $A = \{(1, 2),\ (1, 3),\ (3, 4),\ (1, 5),\ (4, 6)\}$. Clearly the above graph is a tree (in fact it is a called rooted tree with root 1).

There are many problems related to graphs and networks such as vertex packing (stability number), maximum matching, minimum covering, chro-

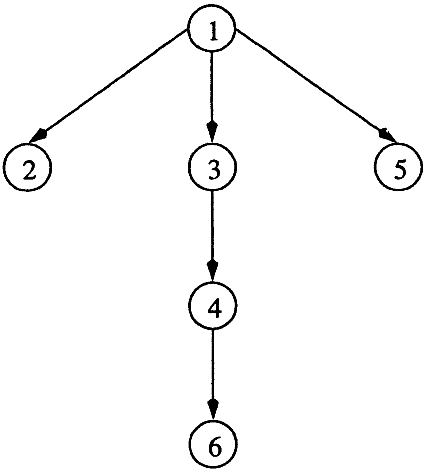

Figure 2: A Directed Tree

matic index, chromatic number, multi- commodity minimum disconnecting set and Steiner problem which can be formulated as one of the SC, SP and SPT models or their variants.

4.1 Vertex (Node) Packing Problem

Consider an undirected graph $G = (N, E)$. A subset of nodes P of N is called a *vertex packing* if no two nodes of the set P are adjacent to each other. The Vertex Packing problem is to find a packing of maximum cardinality. Let $x_i = 1$, if node i is included in a packing and $x_i = 0$ otherwise. Since two nodes connected by an edge cannot be included in a packing a SP formulation of the vertex packing problem is given by

$$\begin{array}{rll} \max & \sum_{i=1}^{n} x_i & \\ \text{s.t.} & x_i + x_j \leq 1 & (i,j) \in E \\ \text{and} & x_j = 0, 1 & i = 1, 2, ..., n \end{array} \tag{13}$$

The maximum value of the objective function is also called the stability number of a graph. To illustrate the usefulness of this model, consider a franchise business whose objective is to maximize the number of profitable franchises in a given area. Certain locations are so close to each other, if franchises are open in both neither will make a profit. Represent each location by a node and connect any two nodes by an edge, if the corresponding locations are unprofitable, when franchises are open in both. To provide another example (even though esoteric) consider placing eight queens on a chessboard so that no queen can capture another queen. In order to determine the feasibility of placing eight queens construct a graph with 64 nodes, each node representing a square on the chessboard and connect two nodes by an edge if the corresponding squares are in the same row or in the same column or in the same diagonal. If the corresponding stability number is eight or more it is possible to place eight queens on a chessboard without one capturing another.

When each node i is assigned a nonnegative weight of c_i and the coefficient of x_i is c_i in the objective function of the formulation (13), then it is called a weighted Vertex Packing Problem. An interesting application of this model is the transformation of a SP problem to a weighted Vertex Packing Problem. To see the connection between these two models, construct an undirected graph with n nodes, each node representing a variable in the formulation (2) and connect two nodes j and k by an edge if there is a constraint i such that $a_{ij} = a_{ik} = 1$. The equivalence between the SP problem and the weighted Vertex Packing Problem generated by the corresponding graph is illustrated below.

Example 6.

$$
\begin{array}{ll}
\max & \sum_{j=1}^{5} c_i x_j \\
\text{s.t.} & x_1 + x_2 + x_3 \leq 1 \\
& x_2 + x_3 + x_4 \leq 1 \\
& x_3 + x_4 + x_5 \leq 1 \\
& x_1 + x_3 + x_5 \leq 1 \\
\text{and} & x_j = 0, 1 \qquad j = 1, 2, ..., 5
\end{array}
$$

The undirected graph with 5 nodes and 8 edges corresponding to the above SP problem is shown below. From the first constraint, it is clear that among the variables x_1, x_2, x_3 no two variables can be found equal to 1 in any feasible solution. This is equivalent to Vertex Packing constraints on nodes (1,2), (2,3) and (1,3).

4.2 Maximum Matching

Consider an undirected graph $G = (N, E)$. Two edges (or arcs) are said to be adjacent to each other if they have a node in common. A subset of

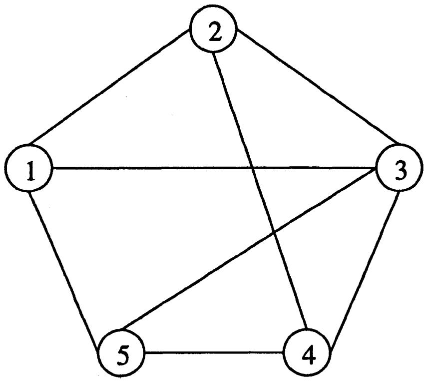

Figure 3: A Set Packing Graph

edges D is called a matching if no two edges in D are adjacent to each other. The maximum matching problem is to determine a matching of maximum cardinality. Let $x_j = 1$, if edge j is included in the matching and $x_j = 0$ otherwise. Also for each edge (i, j), let $a_{ij} = a_{ji} = 1$. Then the SP formulation of the maximum matching problem is given by

$$\begin{array}{lll} \max & \sum_{j=1}^{m} x_j & \\ \text{s.t.} & \sum_{j=1}^{m} a_{ij} x_j \leq 1, & i = 1, 2, ..., n \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., m \end{array} \tag{14}$$

Example 7. To illustrate the above model consider the following graph with numbers on each edge representing the number assigned to each edge. The corresponding maximum matching problem is

$$\begin{array}{lll} \max & \displaystyle\sum_{j=1}^{7} x_j & \\ \text{s.t.} & x_1 + x_2 \leq 1 & \\ & x_1 + x_3 + x_4 \leq 1 & \\ & x_3 + x_5 \leq 1 & \\ & x_4 + x_5 + x_6 \leq 1 & \\ & x_6 + x_7 \leq 1 & \\ & x_2 + x_7 \leq 1 & \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., 7 \end{array}$$

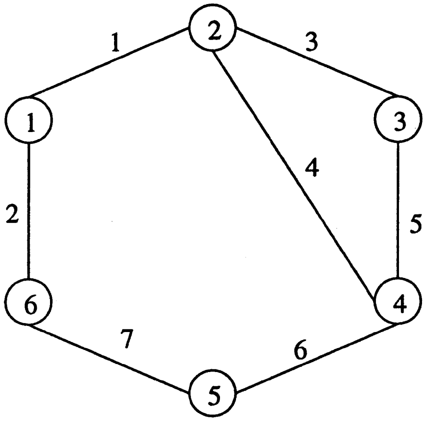

Figure 4: An Undirected Graph

To illustrate the application of this model, suppose there are p workers and q jobs and each worker is trained to perform at least one job. The problem is to determine whether each worker can be assigned to a job for which the individual is qualified. Consider a bipartite graph with p nodes corresponding to the workers in one group and q nodes corresponding to the jobs in the second group. Connect the node in the first group with an edge to a node in the second group if the individual is qualified to perform the corresponding job. If the value of the maximum matching for this graph is p then all workers can be assigned to jobs for which they are qualified.

A closely related problem is the standard assignment problem. Suppose there are n workers and n jobs. It costs c_{ij} if worker i is assigned to job j. The problem is to assign each worker to one job and each job to one worker so that the total cost is a minimum. Let $x_{ij} = 1$, if worker i is assigned to job j and $x_{ij} = 0$ otherwise. The SPT formulation of this problem is given by

$$\begin{array}{lll} \min & \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} & \\ \text{s.t.} & \sum_{j=1}^{n} x_{ij} = 1 & i = 1, 2, ..., n \\ & \sum_{i=1}^{n} x_{ij} = 1 & j = 1, 2, ..., n \\ \text{and} & x_{ij} = 0, 1 & i = 1, 2, ..., n \\ & & j = 1, 2, ..., n \end{array} \tag{15}$$

Efficient techniques have been developed to solve this problem which require polynomial computational time to determine the optimum solution.

4.3 Minimum Covering Problem

Given an undirected graph $G = (N, E)$, an edge covering F is a subset of E such that every node in N is the end point of at least one edge in F. The problem is to determine the minimum number of edges needed to cover all nodes. Let $x_j = 1$, if edge j is in F and $x_j = 0$ otherwise. Also let $a_{ij} = 1$ if node i is an end point of the edge j and $a_{ij} = 0$ otherwise. Then the SC formulation of the minimum covering problem is given by

$$\begin{array}{lll} \min & \sum_{j=1}^{m} x_j & \\ \text{s.t.} & \sum_{j=1}^{m} a_{ij}x_j \leq 1 & i = 1, 2, ..., n \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., m \end{array} \tag{16}$$

For a simple application of this model consider a fort with towers at the endpoints of each wall. A guard stationed at a wall can watch both towers at the end of the wall. The problem is to determine the minimum number of guards needed to watch all towers. Define a node for each tower and connect any two nodes by an edge if they are connected by a wall. Clearly, the minimum number of edges to cover all nodes yields the minimum number of guards needed to watch all the towers. Many more important and useful applications of this model are included in the section related to location problems.

Another related model is to cover all edges by nodes. That is find a subset of nodes P of N such that at least one node of every edge belongs to P. The problem is to determine the minimum number of nodes needed to cover all edges. Let $x_i = 1$ if node i is included in the cover and $x_i = 0$ otherwise. Then the SC formulation of the node covering problem is given by

$$\begin{array}{lll} \min & \sum_{i=1}^{n} x_i & \\ \text{s.t.} & x_i + x_j \geq 1 & (i, j) \in E \\ \text{and} & x_i = 0, 1 & i = 1, 2, ..., n \end{array} \tag{17}$$

It is easy to see the vertex packing problem (13) and the node covering problem (17) are closely related [set $x_i = 1 - y_i$ in formulation (17) to obtain the formulation (13)].

4.4 Chromatic Index and Chromatic Number

The chromatic index of an undirected graph is the minimum number of colors needed to color all edges of the graph so that no two adjacent edges receive the same color. It is clear that no more than m colors are needed since m represents the number of edges. Let $y_k = 1$, if color k is used and $y_k = 0$ otherwise, for $k = 1, 2, ..., m$. Also let $x_{jk} = 1$, if edge j is given color k and $x_{jk} = 0$ otherwise, for $j = 1, 2, ..., m$ and $k = 1, 2, ..., m$. Finally let $a_{ij} = 1$, if node i is an endpoint of edge j and $a_{ij} = 0$ otherwise for $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$. Then an integer programming formulation is given by

$$
\begin{array}{lll}
\min & \sum_{k=1}^{m} y_k & \\
\text{s.t.} & \sum_{k=1}^{m} x_{jk} = 1 & j = 1, 2, ..., m \\
& \sum_{j=1}^{m} a_{ij} x_{jk} \leq y_k & k = 1, 2, ..., m \\
& & i = 1, 2, ..., n \\
& y_k = 0, 1 & k = 1, 2, ..., m \\
\text{and} & x_{jk} = 0, 1 & k = 1, 2, ..., m \\
& & j = 1, 2, ..., m
\end{array}
\qquad (18)
$$

The first set of constraints ensures that every arc is assigned a color and the second set of constraints guarantees that all edges adjacent to a given node receive at most one color if it is used to color some edge. Substituting $z_k = 1 - y_k$, the problem can be transformed to a mixed SP and SPT model.

A related problem is to find the minimum number of colors needed to color all nodes of an undirected graph so that no two adjacent nodes receive the same color. The minimum number of colors needed is called the chromatic number. Let $y_k = 1$, if color k is used and $y_k = 0$ otherwise for $k = 1, 2, ..., n$. Also let $x_{ik} = 1$, if node i given color k and $x_{ik} = 0$ otherwise for $i = 1, 2, ..., n$ and $k = 1, 2, ..., n$. In addition, let $a_{ij} = a_{ji} = 1$, if $(i, j) \in E$ and $a_{ij} = 0$ otherwise and $a_{ii} = 1$ for all $i = 12, ..., n$. Then a linear integer programming formulation of the problem is given by

$$
\begin{array}{lll}
\min & \sum_{k=1}^{n} y_k & \\
\text{s.t.} & \sum_{k=1}^{n} x_{ik} = 1 & k = 1, 2, ..., n \\
& \sum_{j=1}^{n} a_{ij} x_{jk} \leq y_k & i = 1, 2, ..., n \\
& & k = 1, 2, ..., n \\
\text{and} & y_k = 0, 1 & k = 1, 2, ..., n \\
& x_{ik} = 0, 1 & k = 1, 2, ..., n \\
& & i = 1, 2, ..., n.
\end{array}
\qquad (19)
$$

The first set of constraints guarantees that every node receives exactly one color and the second set of constraints ensures that none of the nodes adjacent to a given node receive the same color. Substituting $z_k = 1 - y_k$, this problem also can be transferred to a mixed SP and SPT model.

To illustrate a simple application of this model suppose at the end of an academic year several students must take oral exams from several professors. The problem is to determine the minimum number of periods needed to schedule the oral examinations. During an oral exam only one student can be examined by a professor during any period. To model this problem construct a bipartite graph with N_1 nodes representing the students and N_2 nodes representing the professors. Connect a node $i \in N_1$ and $j \in N_2$ with an edge if student i must be examined by professor j. If the edges are colored so that no two adjacent edges receive the same color, each color can correspond to a period. Clearly, the chromatic index of the graph yields the minimum number of time periods needed to complete all oral examinations. Other models of time table scheduling problems are discussed in the miscellaneous operations section.

4.5 Multi-Commodity Disconnecting Set Problem

Consider a directed network $G = (N, A)$ and let $S = \{s_1, s_2, ... s_k\} \subseteq N$ and $T = \{t_1, t_2, ..., t_k\} \subseteq N$ be the source set and sink set. A set of arcs $D \subseteq A$ is called a disconnecting set which when removed from the network would block all paths from s_i to t_i for $i = 1, 2, ..., k$. To disrupt communications from each s_i to t_i, all arcs in a disconnecting set from the network must be removed. Suppose it costs cj to remove (destroy) the arc c_j, for $j = 1, 2, ..., m$. The problem of interest is to find a disconnecting D which costs the least. Such a disconnecting set is called a multi-commodity minimum disconnecting and is useful in attacking an enemy network to disrupt all communications between the sources and the corresponding sinks. Suppose $P_1, P_2, ..., P_r$ represent all elementary paths from every point in S to the corresponding point in T. Let $x_j = 1$, if the jth edge is selected for removal from the network and $x_j = 0$ otherwise. Also let $a_{ij} = 1$, if path P_i contains the arc j and $a_{ij} = 0$ otherwise for $i = 1, 2, ..., r$ and $j = 1, 2, ..., m$. A SC formulation of the multi-commodity minimum disconnecting problem is given by

$$\begin{array}{lll} \min & \sum_{j=1}^{m} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{m} a_{ij} x_j \geq 1 & i = 1, 2, ..., r \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., m \end{array} \qquad (20)$$

Even for a network of moderate size the number of paths could be prohibitively large. A method called row generation scheme may be used (for $k \geq 3$) to solve this problem which does not require the explicit knowledge of all the constraints. Efficient computational techniques are available when the number of sources k is equal to 1 or 2. The following numerical example is used to illustrate the model.

Example 8. Consider the following network with source set and sink set consisting of three nodes with numbers on each arc representing the arc number assigned to it. There is only one path (elementary) from each source to the corresponding sink and these are listed below.

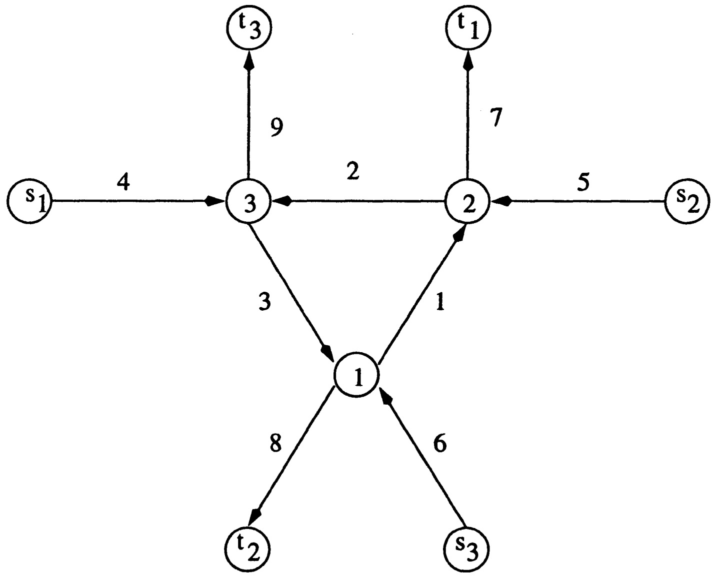

Figure 5: A Multi-Commodity Network

$$
\begin{aligned}
P_1 &= \{(s_1,3),(3,1),(1,2),(2,t_1)\} \\
P_2 &= \{(s_2,2),(2,3),(3,1),(1,t_2)\} \\
P_3 &= \{(s_3,1),(1,2),(2,3),(3,t_3)\}
\end{aligned}
$$

The corresponding SC formulation is given below.

$$
\min \quad \sum_{j=1}^{7} c_j x_j
$$

$$\begin{array}{lll}\text{s.t.} & x_4 + x_3 + x_1 + x_7 \geq 1 & \\ & x_5 + x_2 + x_3 + x_8 \geq 1 & \\ & x_6 + x_1 + x_2 + x_9 \geq 1 & \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., 7\end{array}$$

4.6 Steiner Problem in Graphs

Consider an undirected graph $G = (N, E)$ and cost c_j associated with edge j, $j = 1, 2..., m$. Suppose $N = S \cup P$ and the node set P is designated as the set of Steiner points. The problem is to determine a tree $T = (N_1, E_1)$ such that N_1 and E_1 are subsets of N and E respectively and N_1 contains all nodes in S and the total cost of the edges included in E_1 is a minimum. It should be noted that in a tree every two nodes are connected by a single path. When P is empty, the problem (minimal spanning tree) can be solved very efficiently. Consider a partitioning of the nodes $N = N_1 \cup N_2$ such that both N_1 and N_2 contain some nodes of $S(N_1 \cap N_2 = \emptyset$, $N_1 \cap S \neq \emptyset$ and $N_2 \cap S \neq \emptyset)$. To span all the nodes in S at least one of the edges from the node set N_1 to the node set in N_2 must be included in the tree. Suppose $E_1, E_2, ..., E_r$ represent the edges corresponding to all partitionings (also called cut sets) of the nodes N with the specified property. Let $x_j = 1$, if jth edge is included in E_1 and $x_j = 0$ otherwise for $j = 1, 2, ..., m$. Also let $a_{ij} = 1$ if edge set E_i contains the edge j and $a_{ij} = 0$ otherwise, for all $i = 1, 2, ..., r$ and $j = 1, 2, ..., m$. A SC formulation of the Steiner Problem in graphs is given by

$$\begin{array}{lll}\min & \sum_{j=1}^{m} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{m} a_{ij} x_j \geq 1 & i = 1, 2, ..., r \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., m.\end{array} \tag{21}$$

This problem also can be solved using the row generation scheme which does not require the explicit knowledge of all constraints similar to the multi-disconnecting set problem. Models of this type can be used to determine the minimum cost needed to determine communication links between several locations so that communication is possible between any two pair of locations.

Example 9. Consider the following undirected graph with 5 nodes and 8 edges with numbers on each edge representing the number assigned to it and $S = \{1, 2, 3\}$. The list of all possible partitionings of the nodes and the edges corresponding to each partitioning are given in Table 1. The SC

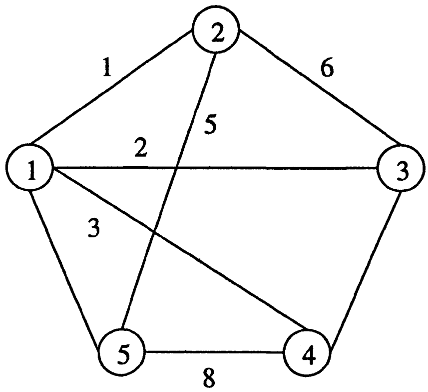

Figure 6: An Undirected Graph for the Steiner Problem

Table 1:

N_1	N_2	*Edges*
(1)	(2, 3, 4, 5)	$E_1 = (1, 2, 3, 4)$
(1, 4)	(2, 3, 5)	$E_2 = (1, 2, 4, 7, 8)$
(1, 5)	(2, 3, 4)	$E_3 = (1, 2, 3, 5, 8)$
(1, 4, 5)	(2, 3)	$E_4 = (1, 2, 5, 7)$
(2)	(1, 3, 4, 5)	$E_5 = (1, 5, 6)$
(2, 4)	(1, 3, 5)	$E_6 = (1, 3, 5, 6, 7, 8)$
(2, 5)	(1, 3, 4)	$E_7 = (1, 4, 6, 8)$
(2, 4, 5)	(1, 3)	$E_8 = (1, 4, 6)$
(3)	(1, 2, 4, 5)	$E_9 = (2, 6, 7)$
(3, 4)	(1, 2, 5)	$E_{10} = (2, 3, 6, 8)$
(3, 5)	(1, 2, 4)	$E_{11} = (2, 4, 6, 7, 8)$
(3, 4, 5)	(1, 2)	$E_{12} = (2, 3, 4, 6)$

formulation consisting of 8 variables, one variable corresponding to each edge and 12 constraints, one constraint corresponding to each partitioning is straight forward.

5 Personnel Scheduling Models

Scheduling personnel is an important activity in many organizations in both manufacturing and service sectors. The scheduling problems arise in a variety of service delivery settings such as scheduling nurses in hospitals, airline and hotel reservations personnel, telephone operators, patrol officers, workers in a postal facility, checkout clerks in supermarkets, ambulance and fire service personnel, toll collectors, check encoders in banks, train and bus crew, personnel in fast food restaurants, and others. The scheduling problems also arise in manufacturing activities especially those requiring continuity in the production process such as chemicals and steel. Operational performance of both service and manufacturing operations such as quality and level of service, labor cost and productivity are effected by employee scheduling decisions. Tardiness, turnover and absenteeism complicates the situation due to unsatisfactory work schedules. Providing satisfactory work schedules to employees, maintaining the required service levels, insuring the human needs such as breaks for lunch and rest, meeting the governmental and union legal requirements, achieving the production goals and controlling the labor costs are some of the major issues in modeling the employee scheduling problem.

The solution to the scheduling problem is relatively simple in organizations which operate five days a week and one standard shift a day, since all employees are required to follow one schedule except possibly lunch breaks. This section deals with scheduling problems which arise in organizations which operate six or more days a week, with one or more shifts per day. There are three basic models called cyclical or days off scheduling, shift scheduling and tour scheduling to structure a variety of personnel scheduling problems. Many criteria such as the total number of employees, total number of hours of labor, total cost of labor, unscheduled labor costs, overstaffing, understaffing, number of schedules with consecutive days off, number of different work schedules utilized may be used in conjunction with the three basic models to suit a particular application.

5.1 Days Off or Cyclical Scheduling

Consider an organization which operates seven days a week with one shift per day. The number of employees required on each day of the week may vary but is stable from week to week. There is only one employee category and the number of employees required on any given day is determined (or estimated) on the basis of the required level of service. Every employee must be given two consecutive days off in a week. The problem of interest is to determine the minimum number of employees required to meet the daily demand for their services. Suppose the number of employees needed on the ith day of the week is $b_i, i = 1, 2, ..., 7$. C learly there are 7 possible schedules which satisfy the two consecutive days off requirement. Let x_i represent the number of employees who start work on ith day and continue work for a total of 5 consecutive days. For example an employee who begins work on the 7th day will also work on the first four days of the week and is off on the 5th and 6th days of the week. This problem can be formulated as a GSC and is given below.

$$\begin{aligned}
\min \quad & \sum_{j=1}^{7} x_j \\
\text{s.t.} \quad & x_1 + x_4 + x_5 + x_6 + x_7 \geq b_1 \\
& x_1 + x_2 + x_5 + x_6 + x_7 \geq b_2 \\
& x_1 + x_2 + x_3 + x_6 + x_7 \geq b_3 \\
& x_1 + x_2 + x_3 + x_4 + x_7 \geq b_4 \\
& x_1 + x_2 + x_3 + x_4 + x_5 \geq b_5 \\
& x_2 + x_3 + x_4 + x_5 + x_6 \geq b_6 \\
& x_3 + x_4 + x_5 + x_6 + x_7 \geq b_7 \\
& \text{and } x_i \text{ is a nonnegative integer for } i = 1, 2, ..., 7
\end{aligned} \tag{22}$$

Since the unused number of man days is given by

$$5\sum_{i=1}^{7} x_i - \sum_{i=1}^{7} b_i$$

minimizing the total employees will also minimize the overstaffing. If the labor costs vary depending upon the days off, it is possible to obtain minimum cost solution by changing the objective function to

$$\sum_{i=1}^{7} c_i x_i$$

where c_i is the cost of the employee who starts work on the ith day of the week. If nonconsecutive days off are permitted, there are a total of 21 possible schedules. The corresponding model can be formulated with 21 variables.

If the optimal solution of the model (22) is implemented, it is possible that some employees may never get a weekend off. Such solutions can be avoided by extending the planning horizon to several weeks and incorporating only those schedules which satisfy the required minimum number of weekends off. For example if the planning horizon consists of four weeks and the schedules are restricted to consecutive days off in each week, there are a total of (7)4 = 2401 possible schedules. Many of these schedules may not include even one weekend off. In addition, some schedules may require a long work stretch. For example, a work schedule with days 1 and 2 off in the first week, days 6 and 7 off in the next three weeks requires an individual to work 10 consecutive days without a break. This schedule also provides 4 consecutive days off if repeated once in four weeks. Such undesirable schedules may be eliminated in formulating the problem. For a given planning horizon consisting of m days, suppose the total number of schedules which meet the requirements is n. Let $a_{ij} = 1$, if the day i in the planning horizon is a work day in the schedule j and $a_{ij} = 0$ otherwise. Also let x_j be the number employees with work schedule j. A GSC formulation of this model is given by

$$\begin{array}{lll} \text{min} & \sum_{j=1}^{n} x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \geq b_i & i = 1, 2, \ldots, m \\ \text{and} & x_j \text{ is a nonnegative integer} & j = 1, 2, \ldots, n \end{array} \tag{23}$$

where b_i is the required number of employees on ith day of the planning horizon. When there are several categories of tasks to be performed and each employee can perform only one task, the scheduling problem can be solved by treating each category of tasks separately. The interesting case is when employees can perform multiple tasks. Models (22) and (23) can easily be extended to incorporate the ability of the employees to perform multiple tasks by defining $a_{ijk} = 1$, if day i of the planning horizon is a work day in schedule j and the employee is required to perform task k. Obviously, the number of schedules, consequently the number of variables in model (23) increase substantially with the length of the planning horizon and the number of tasks.

5.2 Shift Scheduling

Many service facilities and manufacturing companies operate more than one shift a day. For example hospitals operate twenty four hours a day and seven days a week. Shift scheduling deals with problems related to start time, work span, lunch breaks and rest periods in assigning shifts to employees. The work day is divided into several periods of equal duration such as an hour. Based on shift length, constraints on work span (number of periods of continuous work), lunch breaks, rest and start time, several feasible schedules can be generated. For example, if the work day consists of 14 hours from 8 am through 10 pm and is divided into 28 half hour periods, each work schedule can be represented by a sequence of ones and zeroes with one corresponding to work period and zero corresponding to nonwork period. The sequence (0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,1,0,0,1,1,1,1,1,1,0,0, 0,0) corresponds to start work at 12 pm, take one hour break at 4 pm after working for 4 hours, resume work again at 5 pm till 8 pm and quit for the day. Assuming that all employees must start at the beginning of a period and work eight hours at a stretch (including breaks), the number of possible schedules or shifts is 11. However, if there is flexibility in the work stretch many more schedules are possible. If part time employees are permitted, additional schedules can be added to the list of feasible schedules. An important factor in determining the length of the period is the availability of reliable estimates of the personnel requirement during each period of the work day. Suppose b_i represents the number of personnel needed during the ith period and $a_{ij} = 1$, if ith period is a work period in the jth schedule. A GSC formulation of the shift scheduling problem is given by

$$\begin{array}{lll} \text{min} & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \geq b_i & i = 1, 2, ..., m \\ \text{and} & x_j \text{ is a nonnegative integer} & j = 1, 2, ..., n \end{array} \tag{24}$$

where c_j is the cost of schedule j, n is the total number of feasible schedules and m is the number periods in a work day.

While this model provides the number of personnel required to work on each shift during each work day to minimize the total costs, it does not give individual employee schedules for a week including the days off. A model which integrates the shift scheduling and days off scheduling models is more useful.

5.3 Tour Scheduling

The combined model of days-off and shift scheduling is known as tour scheduling. Suppose a planning horizon such as a week or a month is chosen. Each working day of the planning horizon is divided into several periods of equal length. Several feasible tours are selected taking into account the days off, work stretch, starting time of a shift, lunch breaks and rest, legal constraints, part-time employees, and other restrictions. Suppose there are several tasks to be performed during each period of the planning horizon and bik is the number personnel required during period i for task k. Also let $a_{ijk} = 1$, if task k is performed during period i of tour j and $a_{ijk} = 0$ otherwise. Further, suppose m is the total number of periods in the planning horizon, n is the total number of feasible tours and r is the number of tasks. The GSC model of the tour scheduling is problem is given by

$$\begin{array}{lll} \min & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ijk} x_j \geq b_{ik} & i = 1, 2, ..., m \\ & & k = 1, 2, ..., r \\ \text{and} & x_j \text{ is a positive integer} & j = 1, 2, ..., n \end{array} \tag{25}$$

where c_j is the cost of tour j. The number of variables (feasible schedules) increase significantly with the length of the planning horizon.

All the models described so far attempt to determine the number of schedules of each type to minimize some chosen criteria. Individual employees or their preferences are not incorporated. Suppose x_{j*} is the optimal solution to problem (25), w_{ij} is the preference index of the employee i to tour j and the total number of employees is p. Also let $y_{ij} = 1$, if employee is assigned to tour j and $y_{ij} = 0$ otherwise. The GSPT model (also called generalized assignment or transportation problem) to determine the optimal assignment of tours to employees is given by

$$\begin{array}{lll} \max & \sum_{i=1}^{p} \sum_{j=1}^{n} w_{ij} y_{ij} & \\ \text{s.t.} & \sum_{j=1}^{n} y_{ij} = 1 & i = 1, 2, ..., k \\ & \sum_{i=1}^{k} y_{ij} = x_{j*} & j = 1, 2, ..., n \\ \text{and} & y_{ij} = 0, 1 & i = 1, 2, ..., p \\ & & j = 1, 2, ..., n \end{array} \tag{26}$$

It is possible to explicitly incorporate each employee in model (25). Let $x_{tj} = 1$, if employee t is assigned to tour j and $x_{tj} = 0$, otherwise and c_{tj} is the corresponding cost. A mixed GSC and SPT formulation is given by

$$
\begin{array}{llll}
\min & \sum_{t=1}^{p} \sum_{j=1}^{n} c_{tj} x_{tj} & & \\
\text{s.t.} & \sum_{j=1}^{n} x_{tj} = 1 & t = 1, 2, ..., p & \\
& \sum_{t=1}^{p} \sum_{j=1}^{n} a_{ijk} x_{tj} \geq b_{ik} & i = 1, 2, ..., m & \\
& & k = 1, 2, ..., r & \\
\text{and} & x_{tj} = 0, 1 & t = 1, 2, ..., p & \\
& & j = 1, 2, ..., n & (27)
\end{array}
$$

Clearly the size of problem (27) is p times the size of the problem (25) with respect to the number of variables.

Incorporating additional constraints, such as a limit on the ratio of part time employees to the total number of employees, productivity of the employees, requires general integer programming formulation. The following numerical example is used to illustrate tour generation.

Example 10. Consider a facility which operates Monday through Saturday of every week with two shifts per day. Every employee is off on Sunday and must work one shift per day for five days a week. In addition, at least four shifts during a week must be either the first or the second shift. Suppose an employee is off on Saturday. There are two tours which correspond to either all or first shift all are second shift. There are five tours with four first shifts and one second shift. Similarly, there are five tours with four second shifts and one first shift, therefore, the number of possible tours with Saturday off is 12. Since any one of the six days can be selected for day off, the total number of tours is 72. If each shift is treated as a time period, each week consists of twelve time periods. A twelve dimension vector may be used to represent each tour. For example, the vector T = (0,1,0,1,0,1,0,1,1,0,0,0) represents a tour working on the first shift Monday through Thursday, second shift on Friday and is off on Saturday.

6 Crew Scheduling

Crew scheduling is an important problem in many transportation systems such as airlines, cargo and package carriers, mass transit systems, buslines and trains since a significant portion of the cost of operations is due to the payments to the crews which include salaries, benefits and expenses. The primary objective in crew scheduling is to sequence the movements of the crew in time and locations so as to staff the desired vehicle movements at a minimum cost. As in the employee scheduling problem generating a set of feasible schedules is necessary to formulate the corresponding SC and SPT (or their variants) for the crew scheduling problem. However, generating a set of feasible schedules for the crew problem is much more complex, since the crew has to be paired with a specified number of sequence of legs of trips or flights over time and space, in addition to incorporating overnight stay away from home base, return to the home base at least once in a specified number of days, constraints on rest periods, restrictions on work stretch and many other legal and safety rules and regulations. The terminology related to scheduling airline crews is used to describe the crew scheduling problem which can be easily interpreted in the context of train, busline, ship, mass transit and other crew scheduling problems.

A flight leg or flight segment is an airborne trip between an origin and destination city pair. Each flight segment has a specified departure time from the city of origin and a scheduled arrival time at the city of the destination. The duration of the flight segment is the difference between the arrival time and the departure time. Each flight segment must be assigned a crew consisting of a specified number of pilots and flight attendants. Crews reside in various cities called bases. The number and location of the bases depend upon the size of the operation. The union work rules and government regulations for assigning crews vary depending upon the crew type (pilot or flight attendant), crew size, aircraft type and type of operations (international or domestic). A duty period consists of one or several flight segments with a limit on both the duration of the flight and the number of flight segments. A duty period is similar to shift in the employees scheduling problem. During a duty period a crew might be attending to their duties or traveling as passengers to reposition themselves for other assignments which is called deadheading. During deadheading the crew may be assigned flights operated by another carrier. Two duty periods must be separated by rest periods which are called overnights. There are minimum and maximum limits on the duration of the overnights and the minimum limit depends upon the duration of the duty period. Time away from the base which is the elapsed time from departure to the return of the crew base cannot exceed a specified number of days as mandated by the work rules. A pairing is a sequence of flight segments which may be grouped into duty and rest periods, the first flight segment beginning and the las flight segment ending at the crew base.

Calculating the cost of pairing may vary from one organization to another. One may use the salary paid to the crew, plus hotel, per-diem, ground transportation and deadheading fare paid during the rest periods. One may also use the opportunity cost which is obtained by calculating the difference between salary of the crew and the actual salary earned during the flying time plus expenses incurred during rest periods. An adjustment has to be made to account for carry over flights not covered in the current planning horizon. Clearly the planning horizon has a significant impact on the number of pairings, since the number of flight segments included in calculating the pairings increases with the length of the planning horizon.

Determining all pairings is fairly time consuming. All legs are linked together to form resolved legs which must be flown as a unit without changing the crew. Resolved legs are linked together to form trips which can be completed in one duty period. In the third level the trips are linked into pairings. After determining the optimal pairings, they are grouped into bid-line to form monthly schedules for the crew. Suppose the total number of pairings is n and c_j is the cost of jth pairing. Also suppose there are m flight segments during the planning horizon. Let $a_{ij} = 1$, if jth pairing includes the flight segment i and $a_{ij} = 0$ otherwise. Let $x_j = 1$, if jth pairing is elected and $x_j = 0$ otherwise. Then a SC model formulation of the crew scheduling problem is given by

$$\begin{array}{lll} \min & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \geq 1 & i = 1, 2, ..., m \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{28}$$

Inequalities are used in the constraint set to allow deadheading which could be used to move a given crew from one place to another (from one base to another) to reduce the costs. If deadheading is not permitted, an SPT formulation can be obtained by converting the inequalities into equalities. A major problem in solving the crew pairing problem is the enormous number of pairings generated in real life applications.

The crew base concept is not meaningful in a mass-transport system. As in the employee scheduling problem days off is relevant and a crew pairing problem may be viewed as all feasible weekly or monthly schedules depending on the length of the planning horizon.

Example 11. To illustrate the construction of crew pairings consider a small domestic airline operating between three cities with two morning and two afternoon flights between each pair of cities during each day. Suppose the flight time between the pairs 1 and 2 is 5 hours, 1 and 3 is 4 hours and 2 and 3 is 3 hours. The departure and arrival schedules of the daily flights

Table 2:

From City	Departure Time	To City	Arrival Time
1	08	2	13
1	15	2	20
1	10	3	14
1	16	3	20
2	09	1	14
2	14	1	19
2	07	3	10
2	15	3	18
3	08	1	12
3	16	1	20
3	09	2	12
3	18	2	21

are given in Table 2.

The planning horizon is three days which consists of 36 flight legs. Each duty period cannot exceed 12 hours. The minimum duration of the overnight is 8 hours. Time away from the base is limited to three days. Since all flights reach destinations on the same day and considerable time is available for rest (overnight) before returning to duty the following day, duty periods can be constructed treating each day separately. There are eight activities including four departures and four arrivals at each city. The times of these activities during each day for city 1, city 2, and city 3 are (08, 10, 12,14, 15, 16, 19, 20), (07, 09, 12, 13, 14, 15, 20, 21) and (08, 09, 10, 14, 16, 18, 20) respectively. To generate all possible duty periods, first construct a network consisting of 23 nodes representing the departure and arrivals times of each flight and connect the departure node and the corresponding arrival node by an arc. Also, join two consecutive nodes corresponding to each city by an arc as shown in the network below where the first number associated with each node represents the city and the second number the departure or arrival time. Set the length of each arc equal to the difference between the time of the ending node and the beginning node. Clearly the length of each arc represents either the duration of a flight or wait time at an airport to catch another flight. Starting with any departure node enumerate all paths

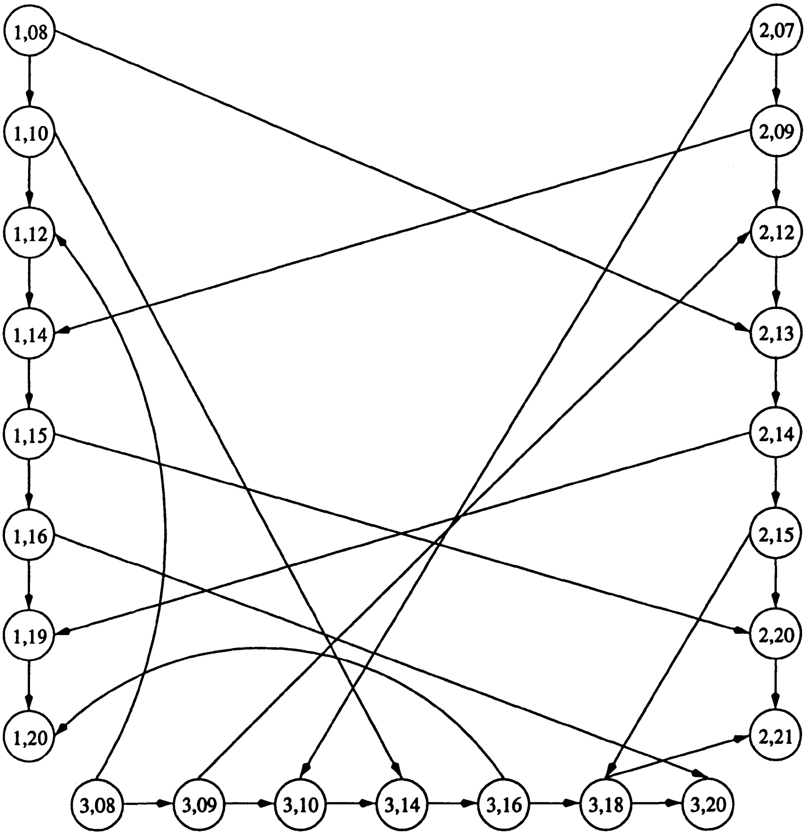

Figure 7: Graph of a Flight Schedule

Table 3:

Duty Period Number	List of Nodes in the Path	Duration of the Path
1	(1,08), (2,13) 5	
2	(1,08), (2,13), (2,14), (1,19)	11
3	(1,10), (3,14)	4
4	(1,10), (3,14), (3,16), (1,20)	10
5	(1,10), (3,14), (3,18), (2,21)	11
6	(1,15), (2,20)	5
7	(1,16), (3,20)	4
8	(2,07), (3,10)	3
9	(2,09), (1,14)	5
10	(2,09), (1,14), (1,15), (2,20)	11
11	(2,09), (1,14), (1,15), (3,20)	11
12	(2,14), (1,19)	5
13	(2,15), (3,18)	3
14	(3,08), (1,12)	4
15	(3,08), (1,12), (1,15), (2,20)	12
16	(3,09), (2,12)	3
17	(3,09), (2,12), (2,14), (1,19)	10
18	(3,09), (2,12), (2,15), (3,18)	9
19	(3,16), (1,20)	4
20	(3,18), (2,21)	3

of length 12 hours or less. Table 3 is a list of 20 duty periods.

Combining duty periods into pairings is a combinatorial problem. To generate all possible pairings, construct a network with 66 nodes representing 20 duty periods for three days, 3 source nodes s_1, s_2, and s_3 and 3 sink nodes t_1, t_2, and t_3. A node on day 1 is connected by an arc to a node on day 2 provided the city of destination on day 1 and the city of origin on day 2 are the same. Similarly the nodes on day 2 and day 3 are connected. Finally connect source node s_i with an arc to all nodes on day 1, with city of origin i and connect all nodes on day 3 with city of destination i to t_i. All paths from (s_i, t_i) for $i = 1, 2, 3$ yield the required pairings. Duty period 5 on day 1, duty period 11 on day 2, and duty period 14 on day 3 is an example of a pairing.

7 Manufacturing

A variety of problems related to manufacturing activity such as assembly line balancing, discrete lot size and scheduling, ingot size selection, spare parts allocation and cutting stock which can be formulated as SC, SP and SPT models or their variants are presented in this section.

7.1 Assembly Line Balancing Problem

In an assembly line balancing problem there are a set of tasks to be performed in a specified order determined by a set of precedence relations. Given the time required for processing each activity, the problem is to determine the minimum number of stations, so that the total time required for processing all activities assigned to a station does not exceed a specified number called cycle time without violating the precedence relations. Suppose the number of tasks is n with processing times $t_i, i = 1, 2, ..., n$. The set of precedence relations P is specified by ordered pairs (i, j) which implies that the task i must be completed before the task j. Also suppose c is the cycle length. Obviously the number of stations required is no more than n. To implement the precedence relations, if $(i, j) \in P$, and tasks i and j are assigned to stations $s(i)$ and $s(j)$, then $s(i) \leq s(j)$.

Let $x_{ik} = 1$, if task i is assigned to station k and $x_{ik} = 0$ otherwise. Also let $y_k = 1$, if station k is open for assigning activities and $y_k = 0$ otherwise. An integer programming formulation of the problem is

$$\begin{array}{lll}
\min & \sum_{k=1}^{n} y_k & \\
\text{s.t.} & \sum_{k=1}^{n} x_{ik} = 1 & i = 1, 2, ..., n \\
& \sum_{t=1}^{n} t_i x_{ik} \leq c y_k & k = 1, 2, ..., n \\
& \sum_{k=1}^{h} x_{ik} \geq x_{jh} & (i, j) \in P \\
& & \text{and } h = 1, 2, ..., n \\
& y_k = 0, 1 & k = 1, 2, ..., n \\
\text{and} & x_{ik} = 0, 1 & i = 1, 2, ..., n \\
& & k = 1, 2, ..., n
\end{array} \tag{29}$$

The first set of constraints insure that every task is assigned exactly to one station. The second set guarantees tasks are assigned only to a station if it is open and the total time of the activities does not exceed the cycle length. The third set maintains the precedence relations. Defining complimentary variables v_k and z_{ik} such that

$$v_k + y_k = 1$$
$$\text{and} \quad x_{ik} + z_{ik} = 1.$$

The second set of constraints can be converted to binary knapsack constraints and the third set of constraints can be transformed to SC constraints. Converting the binary knapsack constraints to SP constraints as noted earlier results in a mixed SP, SC and SPT model. Other formulations of this model are available.

7.2 Discrete Lot Sizing and Scheduling Problem

Consider a production scheduling problem where several items are manufactured on a single machine over a finite planning horizon consisting of several time periods. During each time period either the machine is idle or the entire duration is devoted to the production of a single item. Each item may require a set up time of zero, one or several time periods before production can start if this item is not produced in the previous period. The setup cost and time are item dependent but independent of the item produced in the previous period. Given that an entire duration of a period is devoted to a single item, the demand for each item can be measured in terms of the number of time periods of production needed to satisfy the demand. Inventory cost is incurred when excess production is carried from one time period to the next period. Shortages are not permitted and the setup cost, inventory cost and production cost may vary from period to period. The problem is to determine a production schedule for each item which satisfies the demands at a minimum cost.

Without loss of generality the demand for any item can be assumed to be 0 or 1 during any time period. Clearly, the demand must be an integer since it is measured in terms of the number of time periods of production needed to meet the demand. Suppose the demand is 3 units in time period 6. Since only 1 unit of demand can be met from the production of time period 6, the demand for the remaining two units must be met from the units produced in the first two periods. Make the demand equal to one in period 6 and increase the demands in periods 4 and 5 by 1. Examine the total demand in period 5 and it is more than 1, continue the procedure. Finally in period 1 if the demand is more than 1 and cannot be met by the initial inventory, the problem is infeasible.

Suppose the planning horizon consists of m time periods and the number of items is n. Let k_i be the number of feasible schedules for product i, $x_{ij} = 1$, if jth schedule is selected for product i and $x_{ij} = 0$ otherwise. Also let c_{ij} be the cost of jth schedule of product i, and $a_{ijt} = 1$, if period t is being used either to setup or produce product i in the jth schedule and $a_{ijt} = 0$ otherwise. A mixed SP and SPT formulation of this problem is given by

$$
\begin{array}{llll}
\min & \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} & & \\
\text{s.t.} & \sum_{i=1}^{m} \sum_{j=1}^{k_i} a_{ijt} x_{ij} \leq 1 & t = 1, 2, ..., m & \\
& \sum_{j=1}^{k_i} x_{ij} = 1 & i = 1, 2, ..., n & \\
\text{and} & x_{ij} = 0, 1 & i = 1, 2, ..., n & \\
& & j = 1, 2, ..., k_i & (30)
\end{array}
$$

The first set of constraints insures no more than one product is made during any time period and the second set of constraints guarantees that exactly one schedule is selected for each product i. The number of variables required to formulate this model grows exponentially with the number of time periods.

7.3 Ingot Size Selection

In the steel industry, ingot is an intermediate product which is a mass of metal shaped in a bar or a block. Ingot size or dimensions is an important factor in producing a finished product to customer specifications. In order to make the finished product an ingot must be processed to scale the dimensions resulting in scrap metal. Clearly, producing an optimal ingot size within the technological constraints to manufacture a specific finished product, will reduce the scrap and waste. However, when each finished product requires a different ingot size, producing too many sizes of ingots may result in significant increase in inventory and material handling costs and in logistical problems. One way to deal with the problem is to produce the minimum number of ingots sizes necessary to produce all the finished products. If every finished product can be made from all ingot sizes the problem is trivial; however, this is not usually the case. Suppose the total number of all ingots sizes is n and the number of finished products to be made is m. Let $a_{ij} = 1$, if the finished product i can be produced from ingot of size j and $a_{ij} = 0$ otherwise. Also, let $y_j = 1$, if ingot size j is used to manufacture some finished product and $y_j = 0$ otherwise. Finally let $x_{ij} = 1$ if ingot size j is used to manufacture the finished product i and $x_{ij} = 0$ otherwise. The following formulation provides the minimum number of ingot sizes needed to make all finished products.

$$\begin{array}{lll} \min & \sum_{j=1}^{n} y_i & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_{ij} = 1 & i = 1,2,...,m \\ & x_{ij} leq y_j & i = 1,2,...,m \\ & & j = 1,2,...,n \\ & y_j = 0,1 & j = 1,2,...,n \\ \text{and} & x_{ij} = 0,1 & i = 1,2,...,m \\ & & j = 1,2,...,n \end{array} \tag{31}$$

Substituting $z_j = 1 - y_j$, yields the mixed SP and SPT model. The objective function can be replaced by

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

to minimize the total scrap where c_{ij} is the scrap generated when finished product i is made from an ingot of size j. When $a_{ij} = 0$, the corresponding c_{ij} value can be considered to be infinite (a large number).

A similar formulation may be used to minimize the number of various metallurgical grades of ingots needed to make a variety of steel plates used in production of railroad cars, ships, pipes and boilers. When customer orders are received for steel plates, they are categorized by sales grade based on the required chemical and mechanical properties. The ingots produced in a basic oxygen furnace are also assigned metallurgical grade called met grade based on the chemical or metallurgical composition. Assigning one met grade to each sales grade and producing all customers orders within a sales grade category using only one met grade may require a substantial number of met grades. Producing a variety of met grades can reduce productivity due to change over time required in switching from one met grade to another, in addition to maintaining a large inventory of different metallurgical grades. When more than one met grade can be used to satisfy a customer order, minimizing the number of met grades required to satisfy all customers orders is useful in increasing the productivity and reducing the inventory. If the size is replaced by grade, the formulation of this problem is identical to the above problem.

7.4 Spare Parts Allocation

Consider a repair shop where several types of engines are paired. Each engine may require one or several types of modules to repair. Given the available number of spare modules of each type, the problem is to determine the optimal allocation of modules in order to maximize the number of engines repaired. Suppose m is the number of various types of modules, bi is the number of spares of module type i, n is the number of engines requiring repair and $a_{ij} = 1$, if engine j requires module i and $a_{ij} = 0$ otherwise. Let $x_j = 1$, if engine j is repaired and $x_j = 0$ otherwise. A GSP formulation of this model is

$$\begin{array}{lll} \max & \sum_{j=1}^{n} x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij}x_j \leq b_i & i = 1,2,...,m \\ \text{and} & x_j = 0,1 & j = 1,2,...,m \end{array} \tag{32}$$

When more than one module of the same type is required for repair, the corresponding zero-one MD knapsack formulation can be converted to a GSP problem.

7.5 Pattern Sequencing in Cutting Stock Operations

Certain types of products such as rolls of paper are produced in a variety of large dimensions due to the economies of scale and technological restrictions. These rolls must be cut to specifications using various patterns to meet customer demands. With respect to a given pattern, there is a certain amount of trim loss due to lack of demand for the left over roll because of its small width. Given the demands for various width of rolls, the cutting stock problem is to determine the number of rolls to be cut using a specific pattern to minimize the total trim loss. This problem can be formulated as a linear integer program. Having determined the optimal patterns and the number of rolls to be cut using a specific pattern, determining the sequence in which to cut the different patterns is an important problem.

Suppose there are p optimal patters to meet the demands for m different types of widths. Let d_{ki} equal the number of rolls of width k to be cut in pattern i and $P_i = (d_{1i}, d_{2i}, ..., d_{mi})$. In order to cut a pattern, slitting knives are to be set at appropriate locations. The number of slitter knife settings required to cut pattern i is equal to $b_i = \sum_k d_{ki}$ the number of rolls to be cut. When one pattern is followed by another, the settings of some slitter knives have to be changed to match the required pattern. The objective in sequencing the patterns is to minimize the total number of slitter knife settings required for all patterns combined. The slitter knives can be arranged beginning from the left of the role called single-ended slitting plan where the trim loss occurs at the right end of the roll. One may also use a double-ended plan in which widths may be matched from either or both ends of the roll.

Obviously the maximum number of settings required is $\sum b_i$ which corresponds to slitting each pattern separately. Suppose a single-ended slitting plan is used and consider a subsequence S of patterns $s = (j_1, j_2, ..., j_t)$. Let $h_k(s)$ represent the number of rolls of width k common to all patterns in the subset S which is given by

$$h_k(s) = \min_{j \in s} d_{kj}$$

Also let $c(s)$ represent the total number of widths of all sizes common to all patterns in the subset. Clearly

$$c(s) = \sum_{k=1}^{m} h_k(s)$$

and only subsets for which $c(s) > 0$ contribute to the reduction of slitter knife settings. From these subsets an optimal sequence can be obtained by selecting a combination of subsets in such a way that each pattern appears in exactly one subset. Suppose the number of subsets for which $c(s) > 0$ is n and f_j is the optimal number of slitter knife settings required for the subset j. Also let $a_{ij} = 1$, if the subset j contains the pattern i and $a_{ij} = 0$ otherwise. For feasibility each individual pattern is included in the list of the sets. A SPT formulation of the pattern sequencing problem is

$$\begin{array}{lll} \min & \sum_{j=1}^{n} f_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j = 1 & i = 1, 2, ..., p \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{33}$$

where $x_j = 1$, if the subset j is selected and $x_j = 0$ otherwise.

Example 12. Suppose rolls of 215" width are cut to satisfy the demand for 8 rolls of 64" width, 4 rolls of 60" width, 3 rolls of 48" width, 3 rolls of 45" width, 7 rolls of 33" width, 6 rolls of 32" width and 1 roll of 16" width. Cutting one roll for each one of the 7 patterns listed in Table 4 will satisfy the demand.

Table 4: PATTERNS

Width	No. of Rolls	1 2 3 4 5 6 7
64"	8	2 0 0 1 2 0 3
60"	4	0 3 0 1 0 0 0
48"	3	1 0 1 0 0 1 0
45"	3	0 0 0 2 1 0 0
33"	7	1 0 5 0 1 0 0
32"	6	0 1 0 0 0 5 0
16"	1	0 0 0 0 0 0 1

Consider the subset $s = (1, 3)$ of patterns. The individual number of slitter knives required for patterns 1 and 3 are 4 and 6 respectively. Since the set s has two settings in common the number of settings required for the subset is 8. The subsets, the c orresponding optimal arrangement and the number of setting required are listed in Table 5.

The SPT model corresponding to this example requires 26 variables and 7 constraints.

7.6 Constrained Network Scheduling Problem

Consider a scheduling problem involving m jobs and job i consisting of several tasks. Further suppose that p_{ij}, a nonnegative integer is the time required for processing task j of job i and no preemption of a task is allowed. A set of K resources (or machines) are available to process the tasks and a specified amount of resource of each type is required to process each task of each job. For each job i, there are a set of precedence relations which require that a certain task be completed before processing can begin on another task. The amount of resource k available in period t is R_{kt}, during a planning horizon consisting of T time periods. The cost of completing the job i in f units of time is $g_i(f)$. The problem is to determine the time at which the processing of each task should begin in order to minimize the total cost of completing all jobs in T units of time or less without violating the precedence relations and the resource constraints.

Suppose the number of schedules to complete job i, in T units of time or less without violating the precedence relations is n_i, f_{ih} is the completion time of job i under schedule h and $c_{ih} = g_i(f_{ih})$. Further suppose that a_{ihkt}

Table 5:

Number of Subset	Patterns in the Subset	Optimal Arrangement	Number of Settings (f_j)
1	$(1,3)$	$(1,3)$	8
2	$(1,4)$	$(1,4)$	7
3	$(1,5)$	$(1,5)$	5
4	$(1,6)$	$(1,6)$	9
5	$(1,7)$	$(1,7)$	6
6	$(2,4)$	$(2,4)$	7
7	$(2,6)$	$(2,6)$	9
8	$(3,5)$	$(3,5)$	9
9	$(3,6)$	$(3,6)$	11
10	$(4,5)$	$(4,5)$	6
11	$(4,7)$	$(4,7)$	7
12	$(5,7)$	$(5,7)$	6
13	$(1,3,5)$	$(1,5,3)$	10
14	$(1,3,6)$	$(1,3,6)$	13
15	$(1,4,5)$	$(1,5,4)$	8
16	$(1,4,7)$	$(1,7,4)$	4
17	$(1,5,7)$	$(1,5,7)$	7
18	$(4,5,7)$	$(4,5,7)$	9
19	$(1,4,5,7)$	$(1,5,7,4)$	10
20	(1)	(1)	4
21	(2)	(2)	4
22	(3)	(3)	6
23	(4)	(4)	4
24	(5)	(5)	3
25	(6)	(6)	6
26	(7)	(7)	4

is the amount of resource k required for all tasks of job i in process at time t in schedule h. Let $x_{ih} = 1$, if schedule h is selected for job i and $x_{ih} = 0$ otherwise. An integer programming formulation of the resource constrained network scheduling problem is given by

$$\begin{array}{lcl}
\min & \sum_{i=1}^{m} \sum_{h=1}^{n_i} c_{ih} x_{ij} & \\
\text{s.t.} & \sum_{h=1}^{n_i} x_{ih} = 1 & i = 1, 2, ..., m \\
 & \sum_{i=1}^{m} \sum_{h=1}^{n_i} a_{ihkt} x_{ih} \leq R_{kt} & k = 1, 2, ..., K \\
 & & t = 1, 2, ..., T \\
\text{and} & x_{ih} = 0, 1 & i = 1, 2, ..., m \\
 & & h = 1, 2, ..., n_i
\end{array} \qquad (34)$$

When each task requires the use of a single machine and only one machine of each type is available during each period of the planning horizon, clearly $a_{ihkt} = 0$ or 1 and $R_{kt} = 1$ for all k and t. For this special case (a version of the job-shop problem), the formulation corresponds to a mixed SP and SPT model.

7.7 Cellular Manufacturing

Consider a manufacturing operation where several parts are produced and each part requires processing on a specified set of machines. The machines are grouped into cells and several cells may contain the same machine type. The problem of interest is to determine the minimum number of cells that each part must visit to complete processing on all the required machines. Since there is no interaction between parts, each part can be treated independently. Let $M(i)$, be the set of machines required for processing part i and $a_{jk} = 1$ if cell k contains the machine j and $a_{jk} = 0$ otherwise. Also let $y_{ik} = 1$ if part i visits cell k and $y_{ik} = 0$ otherwise. A SC model to determine the minimum number of cells for part i is

$$\begin{array}{lcl}
\min & \sum_{k=1}^{n} y_{ik} & \\
\text{s.t.} & \sum_{k=1}^{n} a_{jk} y_{ik} \geq 1 & j \in M(i) \\
\text{and} & y_{ik} = 0, 1 & k = 1, 2, ..., n
\end{array} \qquad (35)$$

where n is the total number of cells.

More realistic problems with machine capacity limitations to process the parts can be formulated as linear integer programs including the optimal allocation of machines to cells to minimize the total number of cells visited by all parts combined. The above formulation may be used as a subproblem in developing efficient techniques to solve optimal allocation machines to cells.

8 Miscellaneous Operations

As the title of this section suggests, a variety of unique and unrelated problems are discussed in this section.

8.1 Frequency Planning

Transmit and receive sites, links to connect the sites and frequency bands available for transmission are important components of any satellite communication system. Each ground terminal in a communication system can transmit and receive communications. Because of restrictions on the availability of channels in the region where the transmitter and receiver are located, constraints due to interference and technological limitations of the satellite, the number channels available to a system are limited. A frequency plan is an assignment of a separate frequency interval within the available channels to each link in a communication system.

Suppose r is the number of ground terminals which can be both a transmitter and receiver. Each link j is an ordered pair of stations. Obviously the maximum number of links required is $n = r(r-1)$. If x_j is the frequency assigned to the transmitter of link j then the corresponding frequency of the receiver must be x_j+s where s is a specified number. Due to highly nonlinear form of link inference function, the available range of the frequencies for the entire satellite system is divided into m intervals of equal but small bandwidths. In addition, the link interference constraint requires if a frequency interval i assigned to a link j, then none of the intervals, $i, i+1, ..., i+mj-1$ can be assigned to any other link. This is equivalent to assigning all frequency intervals $(i, i+1, ..., i+mj-1)$ to link j. Suppose p_{ij} is a measure of link interference representing the transmitter and link interference if the interval i is assigned to link j. Let $x_{ij} = 1$, if interval i is assigned to link j and $x_{ij} = 0$ otherwise. A SPT formulation of this model is given by

$$\begin{array}{lll} \min & \sum_{j=1}^{n} \sum_{i=1}^{m-m_j+1} p_{ij}x_{ij} & \\ \text{s.t.} & \sum_{i=1}^{m-m_j+1} x_{ij} = 1 & j = 1, 2, ..., n \\ & \sum_{j=1}^{n} \sum_{i=k}^{m-m_j+1} x_{ij} + s_k = 1 & k = 1, 2, ..., m \end{array}$$

$$
\begin{array}{llll}
& x_{ij} = 0, 1 & j = 1, 2, \ldots, n & \\
\text{and} & s_k = 0, 1 & k = 1, 2, \ldots, m & (36)
\end{array}
$$

where s_k is the slack variable. The first set of constraints insures exactly one bandwidth is assigned to link j and the second set of constraints guarantees that the interference constraints are satisfied. Note that $x_{ij} = 0$, if $j \geq (m - m_j)$ in the above formulation.

8.2 Timetable Scheduling

Scheduling classes and examinations to avoid conflicts, without violating the resources available such as number of class rooms, room capacity and other constraints such as no consecutive examinations are computationally difficult problems. One simple model for each scheduling problem is discussed below.

Suppose there are m classrooms and n classes to be assigned to classrooms each day. Further, suppose each day is divided into t periods and $a_{ik} = 1$ if class i is required to be scheduled during period k and $a_{ik} = 0$ otherwise. Let $x_{ij} = 1$, if class i is scheduled in room j and $x_{ij} = 0$ otherwise. A mixed SP and SPT model of this problem is given by

$$
\begin{array}{llll}
\min & \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} & & \\
\text{s.t.} & \sum_{j=1}^{m} x_{ij} = 1 & i = 1, 2, \ldots, n & \\
& \sum_{i=1}^{n} a_{ik} x_{ij} \leq 1 & j = 1, 2, \ldots, m & \\
& & k = 1, 2, \ldots, t & \\
\text{and} & x_{ij} = 0, 1 & i = 1, 2, \ldots, n & \\
& & j = 1, 2, \ldots, m & (37)
\end{array}
$$

where c_{ij} is the cost of assigning class i to room j. The first set of constraints ensures that every class is assigned a classroom and the second set guarantees that no more than one class is scheduled during any period in any classroom.

Scheduling examinations is a difficult problem in large universities because of the number of students and the number of courses involved. Simple versions of this problem can be typically formulated in one of the two ways.

Given a set of examinations, determine the minimum number of time periods necessary to schedule all examinations with no conflict. A conflict occurs when two examinations are scheduled concurrently and one or more students must take both examinations. A model of this problem is discussed in the section graphs and networks (see Chromatic Number).

A second formulation of the problem is more detailed, having determined the groups of examinations to be scheduled simultaneously called examination block, assign at most one block to a time slot on a given day. Suppose there are m examination blocks and t time periods during a given day. If all examinations are to be completed in D days then $Dt \geq m$. Adding dummy examination blocks if necessary it can be assumed that $Dt = m$. Clearly there are $n = \binom{m}{t}$ possible combinations ($m \geq t$) of examinations schedules on any day. Suppose c_j is the total number of students having two or more examinations in the jth combination. Let $x_j = 1$, if jth combination is selected and $x_j = 0$ otherwise. Also let $a_{ij} = 1$, if examination block i is in jth combination and $a_{ij} = 0$ otherwise. A GSPT formulation of this problem is given by

$$\begin{array}{lll} \min & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j = 1 & i = 1, 2, ..., m \\ & \sum_{j=1}^{n} x_j = D & \\ \text{and} & x_j = 0, 1 & j = 1, 2, .., N \end{array} \tag{38}$$

The first set of constraints insures that every exam block is scheduled and the second constraint guarantees exactly D days are scheduled for all examination blocks.

8.3 Testing and Diagnosis

Testing and diagnosis are very important problems in medicine, repair service, software reliability and others. In this section two models are presented one related blood analysis and the second related to diagnostic inference.

Blood Analysis Several tests have to be performed on a blood specimen. To execute tests several cuvetts are filled with a blood specimen and any necessary testing agents are added. Because of the testing equipment configuration, the tests must be partitioned into r clusters of m tests each with the maximum number of tests $n = rm$. Clearly the total number of possible clusters is $s = \binom{n}{m}$. A priori it is not possible to determine a combination of tests to be performed for a blood specimen. If the clusters are grouped improperly a specimen requiring three tests may require three clusters which is time consuming when compared to all three tests are performed in the same cluster. Given p historical data sets of tests, the problem is to determine the cluster configuration which minimizes the expected number clusters required per specimen.

Suppose the vector $z = (z_1, z_2, ... z_n)$ denotes the test composition of an arbitrary blood specimen where $z_k = 1$, if test k is required and $z_k = 0$ otherwise. Further suppose $z_{ik} = 1$, if test k is performed in sample i and $z_{ik} = 0$ otherwise. Since at least one test is performed for any specimen, the minimum number of clusters required for any specimen is 1. Let $a_{kj} = 1$, if test k is in cluster j and $a_{kj} = 0$ otherwise. The number of tests performed for sample i in cluster j is

$$\sum_{k=1}^{n} z_{ik} a_{kj}$$

and therefore, the fraction of samples using cluster j is equal to

$$c_j = \frac{1}{p} \sum_{k=1}^{p} \min(1, \sum_{k=1}^{n} z_{ik} a_{kj}).$$

Let $x_j = 1$. If cluster j is selected and $x_j = 0$ otherwise. A SPT formulation of the model is given by

$$\begin{array}{lll} \min & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{kj} x_j = 1 & k = 1, 2, ..., n \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., s \end{array} \tag{39}$$

Diagnostic Expert System Diagnostic systems deal with identifying causes or reasons for various symptoms, examination findings on laboratory test results. Such problems of interest in determining the diseases based on various tests or symptoms, repairs needed to correct automobile problems and others.

Suppose a set of disorders or diseases and the manifestations caused by each disorder is specified by experts. Given a set of manifestations the problem is to determine the minimum possible number of disorders causing the manifestations. Let n be number of disorders, m be the number of manifestations and $a_{ij} = 1$, if disorder j causes manifestation i and $a_{ij} = 0$ otherwise. Also let $x_j = 1$, if disorder j is selected and $x_j = 0$, otherwise. An SC formulation of this problem is given by

$$\begin{array}{lll} \min & \sum_{j=1}^{n} x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \geq 1 & i = 1, 2, ..., m \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{40}$$

8.4 Political Districting

Dividing a region such as a state into small areas such as a district to elect political representatives is called political districting. Suppose the region consists of m population units such as counties (or census tracks) and the population units must be grouped together to form r districts. Due to court rulings and regulations, the deviation of the population per district cannot exceed a certain proportion of the average population. In addition, each district must be contiguous and compact. A district is contiguous if it is possible to reach any two places of the district without crossing another district. Compactness essentially means, the district is somewhat circular or a square in shape rather than a long and thin strip. Such shapes reduce the distance of the population units to the center of the district or between two population centers of a district.

Suppose $p_i, i = 1, 2, ..., m$ is the population of the unit i. The mean population is

$$\bar{p} = (\sum_{i=1}^{m} p_i)/m.$$

Then for feasibility every district j must satisfy

$$|p(j) - \bar{p}| \leq a\bar{p}$$

where $p(j)$ is the population of district j and $0 < a < 1$.

Suppose $a_{ij} = 1$, if the unit i is included in district j and $a_{ij} = 0$ otherwise. Clearly $p(j)$ is given by

$$p(j) = \sum_{i=1}^{m} a_{ij}.$$

To test for the contiguity of a district, construct an undirected graph whose nodes a re the units of the district and connect two nodes by an edge if they have a common border. The district is contiguous if there is a path between any two nodes. For compactness, consider any two populations units i and k of a district. If population units i and k are included in a district j, $a_{ij} = a_{kj} = 1$ and

$$d_j = \max_{i,k}(d_{ik} a_{ij} a_{kj})$$

is the distance between two units which are farthest apart where d_{ik} is the distance between units i and k. If A_j is the area of the district j, d_j^2/A_j may be used as a measure of the compactness. Suppose n represents the number of all feasible districts. Suppose c_j a measure of the deviation of population of district j is

$$c_j = |p(j) - \bar{p}|/a\bar{p}.$$

A GSPT formulation of the political districting problem is given by

$$\begin{array}{lll} \min & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j = 1, & i = 1, 2, ..., m \\ & \sum_{j=1}^{n} x_j = r & \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{41}$$

where $x_j = 1$, if the district j is selected and $x_j = 0$ otherwise. If the objective function is changed to

$$\min \max_{j=1}^{n} c_j x_j$$

the problem is called a "bottleneck" problem.

8.5 Information Retrieval and Editing

Consider a multiple file data storage system with a distinct file for each supercategory of information. Each file contains several records with items corresponding to more detailed categories. Multiple files or super categories of files are typically overlapping. A record may contain information relevant to various supercategories. Because of the overlapping nature of information stored, a request for certain specified items of information related to a category can be obtained by interrogating any of the several different files. Given the time required to search a file (depends on the number of records stored) and several requests for information related to various categories the problem is to select the files which provide information in the least amount of time.

Suppose there are n files and f_j is the length of the file $j, j = 1, 2, ..., n$. Also suppose there are m requests and $a_{ij} = 1$, if request i can be met from file j and $a_{ij} = 0$ otherwise. A SC formulation of the model is

$$\begin{array}{lll} \min & \sum_{j=1}^{n} f_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \geq 1 & i = 1, 2, ..., m \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{42}$$

The constraints insure that at least one file is selected for each request.

Table 6:

Edit	Fields					
	1	2	3	4	5	6
1	-	(0,1)	(0)	-	(0,1)	-
2	(1)	-	(1)	(0,1)	-	(2,3)
3	-	(1,2)	-	(1,2,3)	-	-
4	-	(0,2)	-	-	-	(0,1)
5	(1)	-	-	(0)	(1,2)	-

Another application related to data is consider a database generated from surveys or questionnaires. Usually the responses to surveys o r questionnaires contain categorical data meaning that the magnitude of the coded information such as 1 = single, 2 = married, has no intrinsic value. Responses to surveys contain large amounts of incorrect data. To check for the accuracy of data, all data sets are examined through a set of edits or tests. Failed data sets are corrected by making as few changes as possible.

Suppose each questionnaire requires n fields $(y_1, y_2, ..., y_n)$ to represent the data and R_j represents all possible entries in field j. An edit E_i consists of a set of logically unacceptable values. $R_{ij} \subseteq R_j$ for some $j \in F_i \subseteq \{1, 2, ..., n\}$. A data set y fails or inaccurate if $y \in E_i$. Given a set of m edits E_i with corresponding R_{ij} and F_i and $y \in E_i$, the problem is to determine the minimum number of fields to be corrected to obtain a meaningful data.

Let $x_j = 1$, if field j is selected and $x_j = 0$, otherwise. Also let $a_{ij} = 1$, if $j \in F_i$ and $a_{ij} = 0$ otherwise. An SC formulation of the problem is

$$\begin{aligned} \min \quad & \textstyle\sum_{j=1}^{n} x_j \\ \text{s.t.} \quad & a_{ij}x_j \geq 1 \quad i = 1, 2, ..., m \\ & x_j = 0, 1 \quad j = 1, 2, ..., n \end{aligned} \tag{43}$$

Solution to this problem may not generate a feasible record. In generating the constraints in addition to the m edits, all implied edits must be included.

Example 8. Suppose a questionnaire contains 6 fields and the possible values for each field are $R_1 = (0,1)$, $R_2 = (0,1,2)$, $R_3 = (0,1)$, $R_4 = (0,1,2,3)$, $R_5 = (0,1,2,3)$. The five edits selected are listed in Table 6.

Now consider $y = (1,0,0,0,1,0)$. Clearly this data set fails E_1, E_4 and E_5.

Without adding the implied edits the set covering formulation is

$$\begin{array}{ll} \min & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \\ \text{s.t.} & x_2 + x_3 + x_5 \geq 1 \\ & x_2 + x_6 \geq 1 \\ & x_1 + x_4 + x_5 \geq 1 \\ \text{and} & x_j = 0, 1 \qquad j = 1, 2, ..., 6 \end{array}$$

Generating implied edits requires considerable effort due to the combinatorial nature of the problem.

8.6 Check Clearing

Clearing checks is an important activity in commercial banks since cleared checks guarantee the availability of funds to the bank in which the check is deposited. Checks drawn on out-of-town banks (transit) require considerably longer duration for clearance in comparison with checks drawn on local banks or checks drawn on the bank itself. A check not cleared in time costs the bank since funds must be made available to the customer. Various methods are available for check clearance such as clearing the checks through the Federal Reserve System or shipping checks directly to the bank using various transportation modes. Deciding which method to use for clearing is complicated by additional factors such as the time and day of the week the check is deposited and bank availability schedule. Each bank has an availability schedule which outlines the number of days required to clear checks in each region of the country. In addition, checks are grouped based on the drawee check classification and each group or type must be treated separately. The selection of the time period during which a check is sent for clearance is also an important factor.

Suppose the number time periods, the numbers modes for clearance and the number of types of checks are t, m and n respectively. Let $x_{ijk} = 1$, if check type i is sent for clearance by mode j in period k and $x_{ijk} = 0$ otherwise.

Also let $y_{jk} = 1$, if clearing mode j is used in period k and $y_{ik} = 0$ otherwise. Suppose c_{ijk} is the opportunity cost of check type i cleared by mode j in period k, v_j is the variable cost and f_{jk} is the fixed cost for clearing method j in period k and d_{ik} is the number checks of type i available for clearance in period k. Let $a_{ij} = 1$, if mode j can be used to clear check type i and $a_{ij} = 0$ otherwise.

A mixed SP and SPT model of the check clearing problem is given by (substitute $z_{jk} = 1 - y_{jk}$)

$$
\begin{array}{llll}
\min & \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{t} (c_{ijk} + v_{jdjk}) x_{ijk} & + \sum_{j=1}^{m} \sum_{k=1}^{t} f_{jk} y_{jk} & \\
\text{s.t.} & \sum_{j=1}^{m} a_{ij} x_{ijk} = 1 & i = 1,2,...,n & \\
 & x_{ijk} \leq y_{jk} & i = 1,2,...,n & \\
 & x_{ijk} = 0,1 & i = 1,2,...,n & \\
 & & j = 1,2,...,m & \\
 & & k = 1,2,...,t & \\
\text{and} & y_{jk} = 0,1 & j = 1,2,...,m & \\
 & & k = 1,2,...,t & (44)
\end{array}
$$

8.7 Capital Budgeting Problem

Suppose there are n investment projects and c_j is the net - present value of the project j, for $j = 1,2,...,n$. Let a_{ij} be the cash-outlay or capital expenditure required during period i for $i = 1,2,...,m$. Given a budget b_i, for period i, the problem is to determine a subset of projects which maximizes the total net-present value without violating budget restrictions in each period. Let $x_j = 1$, if project j is selected and $x_j = 0$ otherwise. A LIP formulation of the problem is

$$
\begin{array}{llll}
\max & \sum_{j=1}^{n} v_j x_j & & \\
\text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \leq b_i & i = 1,2,...,m & \\
\text{and} & x_j = 0,1 & j = 1,2,...,n & (45)
\end{array}
$$

where v_j is the net-present value of project j.

In this formulation both nonnegative and negative values of a_{ij} are permitted. A positive value implies that the project requires capital expenditure and a negative value corresponds to the situation where the income generated is greated than the capital expenditure. As noted earlier, this problem can be converted to a Zero-one MDK problem which in turn can be transformed to a GSP problem. It is possible to incorporate SP constraints such as

$$x_k + x_j < 1$$

which implies that at most one of the projects k or j may be selected.

8.8 Fixed Charge Problem

A fixed charge bounded linear programming problem may be formulated as

$$\begin{array}{llll} \min & \sum_{j=1}^{n} c_j x_j + \sum_{j=1}^{n} f_j y_j & & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \geq b_i & i = 1, 2, ..., n & \\ & x_j \leq u_j y_j & j = 1, 2, ..., n & \\ & x_j \geq 0 & j = 1, 2, ..., n & \\ \text{and} & y_j = 0, 1 & j = 1, 2, ..., n & (46) \end{array}$$

where u_j is an upper bound on the values of x_j and $f_j > 0$, is the fixed cost which is incurred only when the corresponding $x_j > 0$. In this formulation, it is assumed that all bi are nonnegative. Consider the following related SC problem.

$$\begin{array}{llll} \min & \sum_{j=1}^{n} f_j y_j & & \\ \text{s.t.} & \sum_{j=1}^{n} b_{ij} y_j \geq 1 & i = i_1, i_2, ..., i_t & \\ \text{and} & y_j = 0, 1 & j = 1, 2, ..., n & (47) \end{array}$$

where $b_{ij} = 1$ if $a_{ij} > 0$ and $b_{ij} = 0$ otherwise (corresponding to the constraints $i = (i_1, i_2, ..., i_t)$ for which $b_i > 0$). Clearly any feasible y_j for problem (46) is also a feasible solution to problem (47) and the converse is not true. When an optimal solution from (47) substituted in (46) yields an optimal x, the corresponding value of the objective function yields an upper bound for the fixed charge problem. When the optimal solution to (47) does not lead to a feasible solution to (46), suppose J^* is the set of indices j for which $y_j = 1$ in the current optimal solution and $t_j = 0$ if j in J^* and $t_j = 1$ otherwise. The SC constraint

$$\sum_{j=1}^{n} t_j y_j \geq 1$$

can be added to formulation (47) to eliminate the current optimal solution. The procedure can be continued to generate a good feasible solution. In addition, the formulation (47) can also be embedded in branch and bound algorithms to solve the fixed charge linear programs including the fixed charge transportation problem.

8.9 Mathematical Problems

Suppose $A = (a_{ij})$ is a mxn matrix with all $a_{ij} = 0$ or 1. The a-width of such a matrix is the minimum number of columns of matrix A necessary so that the sum of each row of the resulting submatrix is at least equal to an integer a. Clearly, a cannot exceed

$$a^* = \min_{i=1}^{m} \sum_{j=1}^{n} a_{ij}$$

and GSC formulation of this problem is straight forward.

Suppose S is a (finite) set of integers and $s_i \subseteq s$ for $i = 1, 2, ..., n$ and each s_i is an arithmetic progression. For an arithmetic progression the difference between two consecutive numbers is same. For example (3,5,7,9) is an arithmetic progression which can be expressed as $(2i + 1)$, $i = 1, 2, 3$ and 4. If all elements of S can be covered by the sets $s_i, i = 1, 2, ..., n$ it is called n-cover by arithmetic progressions. Formulation of this problem is also straight forward. Two dimensional version of this problem is useful in production operations of VLSI chips.

9 Routing

In distribution management, strategic decisions regarding the location of plants, warehouses and depots, tactical decisions concerning the fleet size and mix, operational decisions dealing with routing and scheduling of vehicles have a significant impact on the cost of delivery of goods and services to customers and maintaining a satisfactory level of service. Because of considerable capital requirements, it is not possible to relocate facilities and to some extent change the fleet size and mix frequently. Consequently selection of routes and scheduling vehicles is an important problem in adapting to changing market conditions in many operations such as supermarkets, department stores, package delivery, cargo pickup and delivery, newspaper delivery, preventive maintenance tours and others.

Consider a distribution system with one or several depots delivering a product to customers located over a network using several vehicles. Each customer requires a specified amount of the product to be delivered and each vehicle has a capacity which limits the amount of the product that can be delivered in one trip. Usually a vehicle starts from a given depot and must return to the same depot. Given the distances between customers, and the distance between the depots and customers, the routing and scheduling problem is to determine the number of vehicles needed and the assignment of customers to each vehicle without violating the capacity constraints which minimizes the total distance traveled by all vehicles. If the list of customers assigned to each vehicle is known, minimizing the total distance traveled by each vehicle separately and combining the results for all vehicles provides the desired solution. Given a list of customers, each route corresponds to the order in which the customers are visited. Finding the optimal order of visiting customers is the well known Traveling Salesman Problem which is presented next.

9.1 Traveling Salesman Problem

Consider a directed graph $G = (N, A)$. Let c_{ij} be the distance (length or cost) of the arc $(i,j) \in A$. A tour (Hamiltonian cycle) is an elementary circuit which is also equivalent to starting at any given node, visiting every other node exactly once and returning to the starting node. The sum of the distances of the arcs in the circuit is the length of the tour. The objective of the Traveling Salesman Problem (TSP) is to determine a tour of shortest length. Assuming all possible arcs are included in the arc set A, the total number of all possible tours is $n!$ The TSP is a difficult combinatorial problem because of the enormous number of possible tours for a large n. The following mixed SC and SPT formulation of the TSP is useful in developing models for a variety of routing problems. Let $x_{ij} = 1$, if arc (i,j) is in the tour and $x_{ij} = 0$ otherwise.

$$\begin{array}{llll}
\min & \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} & & \\
\text{s.t.} & \sum_{i=1}^{n} x_{ij} = 1 & j = 1, 2, ..., n & \\
 & \sum_{j=1}^{n} x_{ij} = 1 & i = 1, 2, ..., n & \\
 & \sum_{i \in q} \sum_{j \in q} x_{ij} \geq 1 & \text{for all nonempty } q \subseteq N & \\
\text{and} & x_{ij} = 0, 1 & (i,j) \in A & (48)
\end{array}$$

The first two sets of constraints ensure that each node is visited exactly once and the third set of constraints eliminates subtours. When $c_{ij} = c_{ji}$, the problem is called a symmetric TSP. The distance matrix is Euclidean if the distances satisfy the triangle inequality $c_{ij} \leq c_{ik} + c_{kj}$ for all $(i,j), (i,k), (k,j) \in A$. Other formulations of this problem are available. An extension of the problem is called M Traveling Salesman Problem (MTSP) where M salesman are to visit the nodes in such a way so that the total distance traveled by all salesman is a minimum. Each node must be visited by exactly one salesman except the common node. Each salesman must travel along a subtour of the nodes which includes a node common to all salesmen. The MTSP can be formulated as TSP by creating M copies of the common node and connecting each copy of the node with the rest of the nodes as the original node. The M copies of the node are either not connected or connected by an arc with a distance exceeding

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}.$$

The nodes connecting two copies of the node is a subtour which can be assigned to a salesman. A direct integer programming formulation of the MTSP is given by

$$\begin{array}{lll}
\min & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} & \\
\text{s.t.} & \sum_{j=1}^{n} x_{1j} = M & \\
 & \sum_{j=1}^{n} x_{ij} = 1 & i = 2, ..., n \\
 & \sum_{i=1}^{n} x_{i1} = M & \\
 & \sum_{i=1}^{n} x_{ij} = 1 & j = 2, ..., n \\
 & \sum_{i\in q}\sum_{j\in q} x_{ij} \geq 1 & \text{for all nonempty } q \subseteq (N-1) \\
\text{and} & x_{ij} = 0, 1 & (i,j) \in A
\end{array} \tag{49}$$

In this formulation node 1 is assumed to be the common starting node for all salesmen. This formulation is also a mixed GSPT and SC model.

Because of the number of variables and constraints involved the computational effort grows exponentially with the number of nodes to determine the optimal tour using integer programming formulations. Several techniques based on the relaxations of the TSP such as assignment, 2-matching, 1-tree, 1-arborescence and n-path have been successful in generating optimal tours for moderate size problems. Heuristic methods which generate good tours (not necessarily optimal) are useful in practical applications. One approach called the k-opt method seems to work well in generating good tours. Starting with any tour, the k-opt method is a systematic search for better tours by deleting and adding a specified number of arcs. The minimum number of arcs to be replaced to generate a new tour is two. Select any two arcs of the tour and replace them with two new arcs not in the tour if the new tour is better than the current one. After examining all combinations of two arcs, combinations of three arcs can be examined. In a 3-opt method, the search procedure is stopped after examining all combinations of three arcs. Clearly for large values of k, the k-opt method also requires substantial computational effort.

9.2 Single Depot Vehicle Routing

Suppose $G = (N, A)$ is a directed graph and all customers are located at the nodes of the graph. Without loss of generality suppose node 1 is a single depot. If the capacity of a single vehicle is sufficient to satisfy the demand of all customers the problem can be formulated as a TSP. When several vehicles are needed to satisfy the demand and the capacity of the vehicles are different, each individual vehicle must be treated explicitly in developing a model. To get a feel for the size of the problem, a direct integer programming formulation is presented below.

Let $x_{ijk} = 1$, if vehicle k is used to visit node j directly after visiting node i and $x_{ijk} = 0$ otherwise. Let q_i be the demand at node i, Q_k be the capacity of vehicle k and v be the number of vehicles used.

$$
\begin{array}{llll}
\min & \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{v} c_{ijk}x_{ijk} & & \\
\text{s.t.} & \sum_{i=1}^{n}\sum_{k=1}^{v} x_{ijk} = 1 & j = 1,2,...,n & \\
& \sum_{j=1}^{n}\sum_{k=1}^{v} x_{ijk} = 1 & i = 1,2,...,n & \\
& \sum_{j=1}^{n} x_{ijk} \le 1 & k = 1,2,...,v & \\
& \sum_{i=1}^{n} x_{ijk} - \sum_{i=1}^{n} x_{jik} = 0 & k = 1,2,...,v & \\
& & j = 2,...,n & \\
& \sum_{i=1}^{n} q_i \sum_{j=1}^{n} x_{ijk} \le Q_k & k = 1,2,...,v & \\
& \sum_{i\in s}\sum_{j\in s} x_{ijk} \le |s| - 1 & \text{for all nonempty} & \\
& & s \in \{2,...,n\} & \\
& & k = 1,2,...,v & \\
\text{and} & x_{ijk} = 0,1 & i = 1,2,...,n & \\
& & j = 2,2,...,n & \\
& & k = 1,2,...,v & (50)
\end{array}
$$

The first and second set of constraints ensure that a vehicle enters and departs each node. The third set guarantees that each vehicle is used at most once. The fourth set ensures that if a vehicle enters a node it must also depart from that node. The fifth set guarantees that the total demand of the nodes visited by a vehicle is no more than the capacity of the vehicle. The last set corresponds to the usual subtour elimination constraints for each vehicle. Other formulations of this problem including additional constraints on total travel time are available.

It is possible to formulate the above problem as a mixed SP and SPT problem. Suppose r is the total number of feasible tours. For feasibility the combined demand of the nodes in a tour (demand of the tour) must not exceed the maximum capacity of the vehicles. Suppose $a_{ij} = 1$, if node i is included in tour j and $a_{ij} = 0$ otherwise. Let $b_{kj} = 1$, if the vehicle k can carry the demand of tour j and $b_{kj} = 0$ otherwise. Also let c_j be the minimum cost of tour $j, x_j = 1$ if tour is selected and $x_j = 0$ otherwise. Then the problem is

$$\begin{array}{llll} \min & \sum_{j=1}^{r} c_j x_j & & \\ \text{s.t.} & \sum_{j=1}^{r} a_{ij} x_j = 1 & i = 2, 3, ..., n & \\ & \sum_{j=1}^{r} b_{kj} x_j \leq 1 & k = 1, 2, ..., v & \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., r & \qquad (51) \end{array}$$

The first set of constraints ensures that every node is included in exactly one tour and the second set guarantees that each vehicle is assigned no more than one tour.

Generating all feasible tours is a very time consuming process. In addition, one TSP has to be solved for each tour to determine the cost associated with each tour. A variety of heuristics have been developed to generate solutions to the routing problem which can be grouped into four categories namely cluster first-route second, route first-cluster second, savings or insertion and improvement or exchange. In cluster first-route second approach the nodes are first assigned to vehicles and one TSP is solved for each vehicle to determine the optimal tour. In the route first-cluster second approach first a long tour is constructed through all nodes, then the tour is partitioned into pieces or segments which can be assigned to vehicles. In the savings or insertion approach, unassigned nodes are inserted into the existing route or routes based on the least cost or maximum savings, taking into account the vehicle capacity constraints. Having generated a set of feasible routes for each vehicle, improvement or exchange procedures examine exchanging arcs in a given route or exchanging nodes between two routes and leads to an improved feasible solution. Exact procedures to solve the routing problems include specialized branch and bound methods, dynamic programming and cutting plane algorithms.

9.3 Multiple Depots and Extensions

When there are multiple depots a vehicle starting at a specific depot may be required to return to the same depot or permitted to visit another depot. In some applications the delivery of goods at each node must be completed within the time window defined by the earliest and latest time between which the product must be delivered. When the time windows are full days and the delivery must occur on a specified number of days of the planning horizon, the problem is called multi-period vehicle or periodic routing problem. The integer programming formulation (50) can be modified to incorporate multiple depots, and time window constraints. In formulation (51), the additional constraints are taken into account when generating the tours. However, it is no longer feasible to determine the minimum cost of each tour using TSP when time window constraints are imposed. One may have to resort to heuristics to determine the best possible arrangement of each tour. Other extensions include the integrated inventory and routing model and the integrated depot location and routing model which can be formulated as mixed integer and integer programs. The formulation (51) may also be used in scheduling a fleet of ships to pick-up and deliver cargo as well as pick-up passengers and deliver them to their destinations (dial-a-ride problem).

10 Location

The selection of sites for locating facilities has a major impact on both public and private sector operations. Examples of facilities in manufacturing activity include plants, warehouses and retail outlets for producing, distributing and selling products. The location of centers for processing checks (lock box) has significant impact on banking operations. The locations of emergency medical services, fire stations, social service centers, day care centers, post offices, bus stops, shopping centers and hospitals are a major concern of most regional and urban planners. Distance measure, cost structure, time to travel to service centers, supply and demand for services and products are some of the important factors in modeling location problems. When the location of the facilities is unrestricted (can be located on a two dimensional plane) both Euclidean metric and Rectilinear metric can be used to calculate the distance between two points. Given two points $P_1 = (x_1, y_1)$ and

$P_2 = (x_2, y_2)$, the Euclidean distance between the two points is

$$d_{12} = \{(x_1 - x_2)^2 + (y_1 - y_2)^2\}^{1/2}$$

and the rectilinear distance is

$$d_{12} = |x_1 - x_2| + |y_1 - y_2|.$$

If the location of facilities are restricted to the nodes of a graph, the distance between two nodes is the shortest distance between them using the arcs or edges of the graph. When the location of facilities is permitted on the edges (or arcs) of a graph the shortest distance between two points can be calculated using the following approach. Suppose x is a point on the edge (a, b), y is a point on the edge (g, h) and d_{ij} is the shortest distance between nodes i and j. Then the shortest distance between the points x and

$$\begin{aligned} d_{xy} &= \min(d(x,a) + dag + d(g,y), d(x,a) + dah + d(h,y), \\ &\quad d(x,b) + dbg + d(g,y), d(x,b) + dbh + d(h,y)) \end{aligned}$$

where $d(x,a), d(x,b), d(g,y)$ and $d(h,y)$ are the lengths of the edge segments.

Depending on the selection criteria of the locations, distance measure used and the restrictions on the location of the facilities a variety of models have been developed to determine the optimum location of facilities. Models based on the Euclidean distance measure are not included since they require nonlinear programming formulations. In majority of the models, the location of facilities is restricted to the nodes or edges of a graph (which may be used to represent transportation networks) due to the physical travel involved in delivering the goods and services. The location models are divided into five categories, namely plant location problem, lock box location problem, p-center problem, p-median problem and service facilities location problem which are presented next.

10.1 Plant Location Problem

Consider a manufacturing operation with m potential sites for plants to produce a single commodity and ship a specified number of units of the product to each one of the n customers. Suppose fi is the fixed cost of opening a plant at location i, c_{ij} is the unit shipping cost from plant i to customer j, b_j is the demand at customer j and the capacity of the plant is unlimited. The problem is to determine the location of the plants which minimizes the total cost. Since the capacity of each plant is unlimited it is optimal to ship all the quantity to a customer from one location. Let $x_{ij} = 1$, if customer j is supplied from plant i and $x_{ij} = 0$ otherwise. Also let $y_i = 1$, if plant i is open and $y_i = 0$ otherwise. An integer programming formulation of the plant location problem is given by

$$\begin{array}{llll}
\min & \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} b_j x_{ij} + \sum_{i=1}^{m} f_i y_i & & \\
\text{s.t.} & \sum_{i=1}^{m} x_{ij} = 1 & j = 1, 2, ..., n & \\
 & x_{ij} \leq y_i & i = 1, 2, ..., m & \\
 & & j = 1, 2, ..., n & \\
 & x_{ij} = 0, 1 & j = 1, 2, ..., n & \\
 & & i = 1, 2, ..., m & \\
\text{and} & y_i = 0, 1 & i = 1, 2, ..., m & (52)
\end{array}$$

To transform this problem into a SP problem define $z_i = 1 - y_i$ and select a large number M such that

$$M > \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} b_j.$$

Then problem (52) is equivalent to

$$\begin{array}{llll}
\max & \sum_{i=1}^{m} \sum_{j=1}^{n} (M - c_{ij} b_j) x_{ij} & & \\
 & + \sum_{i=1}^{m} f_i z_i - \sum_{i=1}^{m} f_i - mM & & \\
\text{s.t.} & \sum_{i=1}^{m} x_{ij} \leq 1 & j = 1, 2, ..., n & \\
 & x_{ij} + z_i \leq 1 & i = 1, 2, ..., m & \\
 & & j = 1, 2, ..., n & \\
 & x_{ij} = 0, 1 & i = 1, 2, ..., m & \\
 & & j = 1, 2, ..., n & \\
\text{and} & z_i = 0, 1 & i = 1, 2, ..., m & (53)
\end{array}$$

It is also possible to transform problem (53) into a SC problem. Set $c_{ij} b_j = M$, a large number if plant i cannot supply customer j and arrange $c_{ij} b_j$ for each customer in increasing order. Suppose r_{kj} for $k = 1, 2, ..., j*$ are the distinct values of the sequence and $r_{kj} \leq r_{k+1,j}$. Let $z_{kj} = 1$, if customer j is not served by a facility with transportation cost less than or equal to

r_{kj} and $z_{kj} = 0$ otherwise. Also let $a_{ijk} = 1$, if $c_{ij}b_j < r_{kj}$ and $a_{ijk} = 0$ otherwise. A SC formulation is given by

$$
\begin{array}{llll}
\min & \sum_{i=1}^{m} f_i y_i + \sum_{j=1}^{n} \sum_{k=1}^{j^*} (r_{k+1,j} & -r_{kj}) x_{kj} & \\
\text{s.t.} & \sum_{i=1}^{m} a_{ijk} y_i + z_{kj} \geq 1 & k = 1, 2, ..., j^* & \\
 & & j = 1, 2, .., n & \\
 & y_i = 0, 1 & i = 1, 2, ..., m & \\
\text{and} & z_{kj} = 0, 1 & k = 1, 2, ..., j * & \\
 & & j = 1, 2, ..., n & (54)
\end{array}
$$

10.2 Lock Box Location Problem

The number of days required to clear a check drawn on a bank in city i depends upon the city j in which the check is cashed. For a company which pays bills to many clients, it is profitable to maintain accounts at various strategically located banks and pay the clients from checks drawn on one of the banks so that large clearing times can be achieved. It costs the company to maintain an account (lock box) in a bank. Suppose there are n potential lock box locations and m client locations. Suppose c_{ij} is the monetary value per dollar of a check issued in city j and cashed in city i. Suppose b_i is the dollar volume of checks paid in city i. Let $x_{ij} = 1$, if customer in city i is paid from an account in city j. Also let $y_j = 1$, if an account is maintained in city j and $y_j = 0$ otherwise. An integer programming formulation of the lock box location problem is given by

$$
\begin{array}{llll}
\max & \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} b_i x_{ij} - \sum_{j=1}^{n} f_j y_j & & \\
\text{s.t.} & \sum_{j=1}^{n} x_{ij} = 1 & i = 1, 2, ..., m & \\
 & x_{ij} \leq y_j & i = 1, 2, ..., m & \\
 & & j = 1, 2, ..., n & \\
 & x_{ij} = 0, 1 & i = 1, 2, ..., m & \\
 & & j = 1, 2, ..., n & \\
\text{and} & y_j = 0, 1 & j = 1, 2, ..., n & (55)
\end{array}
$$

where f_j is the fixed cost of maintaining an account at location j. This model is similar to the plant location model (47) which can be transformed into a SP model.

10.3 P-Center Problem

Suppose the demand for a service is located at the points $x_i, i = 1, 2, ..., m$ and locations $y = (y_1, y_2, ..., y_p)$ for p service centers are to be selected from a set of points s. For any two points x and y, suppose $d(x, y)$ is the shortest distance between them. For each demand point xi the distance to the closest service center is given by

$$d(x_i, y) = \min_{y_j \in y} d(x_i, y_j).$$

The maximum closest distance between demand points and service centers is

$$d(y) = \max_{x_i} d(x_i, y).$$

The P-Center problem is to determine $y^* \in s$ which minimizes $d(y)$. The problem of locating p emergency service facilities which can be reached from demand points in the shortest possible time is the P-Center problem.

When the number of points in the set S is finite such as the nodes of a graph the optimal locations of the centers can be obtained by solving a series of SC problems. Suppose the number of points in S is $n(n \geq p)$ and h_{ij} is the shortest distance between demand point i and location j. Select any p points from the set s and calculate the corresponding value of the objective function $d_1 = d(y)$. To determine if the objective function can be improved set $a_{ij} = 1$, if $h_{ij} < d_1$ and $a_{ij} = 0$ otherwise and solve the following SC problem

$$\begin{array}{lll} \min & \sum_{j=1}^{n} x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \geq 1 & i = 1, 2, ..., m \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{56}$$

If the value of the objective function of SC is greater than p then the existing solution is optimal. Otherwise calculate d_2 corresponding to the new centers, generate the corresponding SC problem and continue the procedure.

Clearly, when the service centers are restricted to the nodes of a graph, the set of possible locations is finite. Even when the service centers are permitted along the edges, it is possible to reduce the number of possible locations to a finite set. When the locations of service centers are permitted along the edges of a graph the problem is called absolute P-center problem.

10.4 P-Median Problem

Suppose there are m demand points with a user population of a_i at demand point i and there are n possible locations for service centers. Also, suppose d_{ij} is the distance between demand point i and location j. Given a set of permissible locations for each demand point, the problem is to assign one or more service centers to each demand point so that the sum of the population weighted distance for all demands points from the respective service centers is a minimum. To formulate this problem suppose $a_{ij} = 1$ if the demand point i can receive service from location j and $a_{ij} = 0$ otherwise. Also let x_{ij} be the fraction of the population at node i receiving service from location j. When the number of service centers required is p, a mixed integer programming formulation is given by

$$
\begin{array}{lll}
\min & \sum_{i=1}^{m} \sum_{j=1}^{n} a_i d_{ij} x_{ij} & \\
\text{s.t.} & \sum_{j=1}^{n} a_{ij} x_{ij} = 1 & i = 1, 2, ..., m \\
& x_{ij} \leq y_j & i = 1, 2, ..., m \\
& & j = 1, 2, ..., n \\
& \sum_{j=1}^{n} y_j = p & \\
& x_{ij} \geq 0 & i = 1, 2, ..., m \\
& & j = 1, 2, ..., n \\
\text{and} & y_j = 0, 1 & j = 1, 2, ..., n
\end{array} \tag{57}
$$

where $y_j = 1$, if a service center is open in location j and $y_j = 0$ otherwise.

Since service centers have no capacity restrictions, assigning the nearest service center among the selected sites to each demand point is feasible, an optimal solution to problem (57) can always be found with all $x_{ij} = 0, 1$. The model (57) is called generalized p-median problem. When the location of service centers and demand points are restricted to the nodes of a graph, the problem is called a p-median problem. Even when the location of service center is permitted along the edges of the graph, an optimal solution can always be found by restricting the locations to the nodes.

10.5 Service Facility Location Problem

In all the applications discussed in this section the demand points and the service facility locations are restricted to the nodes of a graph. Suppose d_{ij} and t_{ij} represent the shortest distance and time required to travel from node i to node j. A set covering location problem is to find the minimum number facilities required so that every demand point has at least one facility which can be reached with in a specified distance or time or both. Let $a_{ij} = 1$, if the facility located at node j can be reached from the demand point i with in the specified distance or time and $a_{ij} = 0$ otherwise. Also, let $x_j = 1$, if a service facility is located at node j and $x_j = 0$ otherwise. The SC formulation of the this model is straight forward.

A related problem called, maximal covering problem is to maximize the coverage when the number of service centers is restricted to a specified number p. An integer programming formulation of this model is

$$\begin{array}{lll} \max & \sum_{i=1}^{m} y_i & \\ \text{s.t.} & a_{ij}x_j \geq y_i & i = 1, 2, ..., m \\ & \sum_{j=1}^{n} x_j = p & \\ & x_j = 0, 1 & j = 1, 2, ..., n \\ & y_i = 0, 1 & i = 1, 2, ..., m \end{array} \tag{58}$$

Substituting $y_i = 1 - z_i$, the problem can be transformed to a mixed SC and GSPT model.

Another useful model is called hierarchical objective set covering model which has multiple objectives. One is to minimize the number of facility locations to cover all demand points and the second is to maximize excess coverage. A model to maximize excess coverage is given by

$$\begin{array}{lll} \max & \sum_{i=1}^{m} y_i & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij}x_j - y_i \geq 1 & i = 1, 2, ..., m \\ \text{and} & x_j = 0, 1 & j = 1, 2, ..., n \end{array} \tag{59}$$

Both objectives can be incorporated by changing the objective function to

$$\min w \sum_{j=1}^{n} x_j - \sum_{i=1}^{m} y_i$$

where $w > 0$ is some chosen weight. Substituting $y_i = (1 - z_i)$, this model can be transformed to a mixed SPT and GSC model.

Another extension of the set covering location problem is to incorporate backup coverage. Suppose a backup coverage must be located with in a specified distance or time. let $b_{ij} = 1$, if node j is with in the distance or time to provide backup coverage. A SC formulation of this model is

$$\max \quad \sum_{i=1}^{m} y_i$$

$$\begin{array}{llll} \text{s.t.} & \sum_{j=1}^{n} a_{ij}x_j \geq 1 & i = 1,2,...,m & \\ & \sum_{j=1}^{n} b_{ij}x_j - y_i \geq 1 & i = 1,2,...,m & \\ & \sum_{j=1}^{n} x_j = p^* & & \\ \text{and} & x_j = 0,1 & j = 1,2,...,n & \\ & y_j = 0,1 & i = 1,2,...,m & (60) \end{array}$$

where p^* is the optimal number of service centers needed for the primary coverage. The first set of constraints ensures the primary coverage and the second set represents the backup coverage.

The most general extension is the multiple response units model. Suppose multiple response units or several facilities can be located at a given node. Also suppose the demand node i requires r_i response units and the response unit k must be located with in a distance (or time) of h_{ik}. Let x_j be the number response units located at node j and $a_{ijk} = 1$,if location j, can provide response unit k to demand point j and $a_{ijk} = 0$ otherwise. Note that it is reasonable to assume that the maximum distance for the first response unit is less than or equal to the maximum distance for the second response unit, the maximum distance for the second response unit is less than or equal to the maximum distance for the third response unit and so on. This assumption implies that if a location can provide a first response unit, it can also provide second response unit, third response unit and so on. A GSC formulation is given by

$$\begin{array}{llll} \min & \sum_{j=1}^{n} x_j & & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ijk}x_j \geq k & k = 1,2,...,r_i & \\ & & i = 1,2,...,m & \\ \text{and} & x_j \text{ is a nonnegative integer} & j = 1,2,...,n & (61) \end{array}$$

11 Review of Bibliography

The list of references in each category except the general list are divided into subgroups related to the topics discussed in each section. An attempt is made to include articles containing information relevant to more than one subgroup in all appropriate subgroups. A few articles may not have been included in any subgroup due to the inability of the author to secure copies of the articles. Even though the papers have been reviewed carefully, it is very possible that some of them may have been listed in the wrong category. The list of references included in this paper is by no means complete. However, it is the hope of the author that all significant papers have been included. The subgroups and the references related to each subgroup for each category are presented next.

11.1 Theory

Set Covering: Avis (80) Baker (81), Balas and Ng (89), Balas (84), Balas and Ho (80), Balas (80), Balas and Padberg (76, 75b), Beasley (87), Bellmore and Ratliff (71), Benvensite (82) Chang and Nemhauser (85), Chaudary, Moon and McMormick (87), Christofides and Paixo (86), Christofides and Korman (75), Chvatal (79), Comforti, Corneil and Mahjoub (86), Cornuejols and Sassano (89), Crama, Hammer and Ibaraki (90) El-Darzi (88), El-Darzi and Mitra (88a, 88b), Ectheberry (77) Fisher and Kedia (90), Fisher and Wolsey (82), Fowler, Paterson and Tanimoto (81) Garfinkel and Nemhauser (72) Hammer and Simeone (87), Hammer, Johnson and Peled (79), Ho (82), Hochbaum (80) John and Kochenberger (88), Johnson (74) Lawler (66), Leigh, Ali, Ezell and Noemi (88), Lemke, Salkin and Speilberg (71), Lavasz (75) Murty (73) Padberg (79), Peled and Simeone (85), Pierce and Lasky (75), Pierce (68) Roth (69), Roy (72) Salkin (75), Salkin and Koncal (73) Vasko and Wolfe (88), and Vasko and Wilson (86, 84a, 84b)

Set Packing: Balas and Padberg (76, 75b) Chang and Nemhauser (85), Coffman and Leuker (91), Crama, Hammer and Ibaraki (90) Fisher and Wolsey (82), Fowler, Paterson and Tanimoto (81), Fox and Scudder (86) Padberg (79, 73), Pierce (68)

Set Partitioning: Albers (80), Anily and Federgruen (91) Balas and Padberg (76, 75a, 75b) Chan and Yano (92), Coffman and Leuker (91), Crama, Hammer and Ibaraki (90) El-Darzi (88), El-Darzi and Mitra (88a, 88b) Fisher and Kedia (90, 86) Garfinkel and Nemhauser (69) Hammer and Simeone (87), Hwang, Sum and Yao (85) Marsten (74), Michaud (72) Nemhauser, Trotter and Nauss (74) Padberg (79, 73), Pierce and Lasky (75), Pierce (68) Ryan and Falkner (88) Trubin (69)

The constraint co-efficient matrix for all the three models SC, SP and SPT including their generalizations and the mixed models is a matrix of zeroes and ones. Properties of the zero-one matrices in developing solution strategies for linear models with zero-one constraint co-efficient matrix have been explored by Balas and Padberg (76), Balas (72), Berge (72), Fulkerson, Hoffman and Oppenheim (74), Padberg (74a, 74b) and Ryan and Falkner (88).

11.2 Transformations

The transformations and model conversions presented in this section can be found in Balas and Padberg (76), Hammer, Johnson and Peled (79), Lemke, Salkin and Spielberg (71), and Padberg (79) of the list of references on theory and Garfinkel and Nemhauser (75, 73) and Karp (72) of the general list of references.

11.3 Graphs

Vertex Packing: Berge (73) Chang and Nemhauser (85), Chvatal (77) Edmonds (62) Houck and Vemuganti (77) Nemhauser and Trotter (75)

Maximum Matching: Balanski (70), Berge (73) Edmonds (62) Norman and Rabin (59)

Minimum Cover: Balanski (70), Berge (73) Edmonds (62) Norman and Rabin (59) Weinberger (76)

Chromatic Index and Chromatic Number: Berge (73), Brelaz (79), Brown (72) Corneil and Graham (73) Leighton (79) Mehta (81) Salazar and Oakford (74) Wang (74), Wood (69)

Multi-Commodity Disconnecting Set: Aneja and Vemuganti (77) Bellmore and Ratliff (71)

Steiner Problem On Graphs: Aneja (80) Beasley (89, 84) Chopra (92), Cockane and Melzak (69) Dreyfus and Wagner (71) Gilbert and Pollack (68) Hakimi (71), Hanan (66), Hwang and Richards (92) Khoury, Pardalos and Hearn (93), Khoury, Pardalos and Du (93) Maculan (87) Winter (87), Wong (84), Wu, Widmayer and Wong (86)

The formulations and applications of the Vertex Packing, Maximum Matching, Minimum Cover, Chromatic Index and Chromatic Number models can be found in Berge (73), Balanski (70), Edmonds (62), Houck and Vemuganti (77), Nemhauser and Trotter (75) and Weinberger (76). The book by Nemhauser and Wolsey (88) from the general list of references is a good source for additional information on these four models. The Multi-Commodity Disconnecting Set Problem and the Steiner Problem on graphs formulations are due to Aneja and Vemuganti (77) and Aneja (80). Chopra (92) and Khoury, Pardalos and Hearn (93) present many formulations of the Steiner Problem on graphs. Implementation of the Examination Scheduling Problem at Cedar Crest College, Allentown, Pennsylvania is reported in Mehta (81).

11.4 Personnel Scheduling

Days Off (Or Cyclical) Scheduling: Abernathy, Baloff and Hershey (74) Bailey (85), Bailey and Field (85), Baker, Burns and Carter (79), Baker and Magazine (77), Baker (76,74), Baltholdi (81), Bartholdi, Orlin and Ratliff (80), Bartholdi and Ratliff (78), Bechtold (88, 81), Bechtold and Showalter (87, 85), Bennett and Potts (68), Bodin (73), Brown and Tibrewala (75), Brownell and Lowerre (75), Burns and Koop (87), Burns and Carter (85), Burns (78) Emmons and Burns (91), Emmons (85) Howell (66) Koop (86), Krajewski and Ritzman (77) Miller, Pierskalla and Rath (76), Morris and Showalter (83) Rothstein (73, 72) Tibrewala, Phillippe and Browne (72) Vohra (88)

Shift Scheduling: Abernathy, Baloff and Hershey (74), Altman, Beltrami and Rappaport (71) Bailey (85), Bailey and Field (85), Baker (76), Baker, Crabil and Magazine (73), Bartholdi (81), Bechtold and Jacobs (90), Bechtold and Showalter (87, 85), Bodin (73), Browne (79), Byrne and Potts (73). Dantazig (54) Gaballa and Pearce (79) Henderson and Berry (77, 76) Ignall, Kolesar and Walker (72) Keith (79), Koop (88), Krajewski, Ritzman and McKenzie (80), Krajewski and Ritzman (77) Lessard, Rousseu and DuPuis (81), Lowerre (79, 77) Mabert (79), Mabert and Raedels (77), Maier-Rothe and Wolfe (73), Moondra (76), Morris and Showalter (83) Paixo and Pato (89) Segal (74), Shepardson and Marsten (80) Vohra (88)

Tour Scheduling: Abernathy, Baloff and Hershey (74), Abernathy, Baloff, Hershey and Wandell (73) Bailey (85), Bechtold, Brusco and Showalter (91), Bechtold (88), Bechtold and Showalter (87, 85), Bodin (73), Buffa, Cosagrave and Luce (76) Easton and Rossin (91a, 91b) Francis (66) Glover and McMillan (86), Glover, McMillan and Glover (84), Guha and Browne (75) Hagberg (85), Holloran and Byrn (86), Hung and Emmons (90) Krajewski and Ritzman (77) Li, Robinson and Mabert (91) Mabert and Watts (82), Mabert and McKenzie (80), McGinnis, Culver and Deane (78), Megeath (78), Monroe (70), Morris and Showalter (83), Morrish and O'Connor (70) Ozkarahan and Bailey (88), Ozkarahan (87) Papas (67) Ritzman, Krajewski and Showalter (76) Showalter and Mabert (88), Smith (76), Smith and Wiggins (77), Stern and Hersh (80) Taylor and Huxley (89), Tien and Kamiyama (82) Warner (76), Warner and Prawda (72)

Miscellaneous: Abernathy, Baloff and Hershey (71), Ahuja and Sheppard (75) Bechtold (91, 79), Bechtold and Sumners (88), Bechtold, Janaro and Sumners (84) Chelst (81, 78), Chen (78), Church (73) Eilon (64) Gentzler, Khalil and Sivazlian (77), Green and Kolesar (84) Hershey, Abernathy and Baloff (74) Klasskin (73) Linder (69), Loucks and Jacobs (91) McGrath (80) Price (70) Showalter, Krajewski and Ritzman (78) Wolfe and Young (65a, 65b)

The SC formulation of the personnel scheduling problem is due to Dantazig (54). Scheduling models of telephone operators at the General Telephone Company of California and the Illinois Bell Telephone Company are described in Buffa, Cosgrove and Luce (76) and Keith (79). Applications of scheduling models to encode and process checks at the Ohio National Bank, Chemical Bank and the Purdue National Bank are reported in Krajewski, Ritzman and McKenzie (80), Mabert (79), and Mabert and Readels (77). Applications of scheduling models to staffing nursing personnel at the Pediatrics ward of the Colorado General Hospital, Harper Hospital (Detroit) are reported in Megeath (78), and Morrish and O'Connor (70). Models of scheduling patrol officers in San Francisco, aircraft cleaning crews for an international airline, sanitory workers (household refuse collection) in New York and bus drivers in Quebec city are described in Taylor and Huxley (89), Stern and Hersh (80), Altman, Beltrami and Rappaport (71) and Lessard (85). Scheduling of Sales Personnel and Clerical Employees at Qantas Airlines and United Airlines are reported in Gabella and Pearce (79) and Holloran and Byrn (86).

11.5 Crew Scheduling

Airline Crew Scheduling: Anbil, Gelman, Patty and Tanga (91), Arabeyre, Fearnley, Steiger and Teather (69), Arabeyre (66) Baker and Fisher (81), Baker and Frey (80), Baker, Bodin, Finnegan and Ponder (79), Ball and Roberts (85), Barnhart, Johnson, Anbil and Hatay (91), Bronemann (70) Darby-Dowman and Mitra (85) Evers (56) Gerbract (78), Gershkoff (90, 89, 87) Jones (89) Kabbani and Patty (93), Kolner (66) Lavoie, Minoux and Odier (88) Marsten and Shepardson (81), Marsten, Muller and Killion (79), McCloskey and Hansman (57), Minoux (84) Niederer (66) Rannou (86), Rubin (73) Spitzer (87, 61), Steiger (65) Bodin, Golden, Assad and Ball (83) of the Routing references.

Mass Transit Crew Scheduling: Amar (85) Ball, Bodin and Dial (85, 83, 81, 80), Belletti and Davani (85), Bodin, Ball, Duguid and Mitchell (85), Bodin, Rosenfield and Kydes (81), Bodin and Dial (80), Booler (75), Borrett and Roes (81) Carraresi and Gallo (84), Cedar (85) Edwards (80) Falkner and Ryan (87) Hartley (85, 81), Henderson (75), Heurgon and Hervillard (75), Hoffstadt (81), Howard and Moser (85) Keaveny and Burbeck (81), Koutsopoulos (85) Leprince and Mertens (85), Lessard, Rousseau and DuPuis (81), Leudtke (85) Marsten and Shepardson (81), Mitchell (85), Mitra and Darby-Dowman (85), Mitra and Welsh (81) Paixo, Branco, Captivo, Pato, Eusebio and Amado (86), Parker and Smith (81), Piccione, Cherici, Bielli and LaBella (81) Rousseau and Lessard (85), Ryan and Foster (81) Scott (85), Shepardson (85), Stern and Cedar (81), Stern (80) Tykulsker, O'Neil, Cedar and Scheffi (85) Ward, Durant and Hallman (81), Wren, Smith and Miller (85), Wren (81) Bodin, Golden, Assad and Ball (83) of the Routing references.

Application of the Airline Crew Scheduling models at American Airlines, Flying Tiger, Continental Airlines and Swiss Air are reported in Anbil, Gelman, Patty and Tanga (91), Gershkoff (89, 87), Kabbani and Patty (93), Marsten and Shepardson (81), Marsten, Muller and Killion (79), and Steiger (65).

Modelling and implementation of the Mass Transit Crew Scheduling systems in various metropoliton areas (Amsterdam, Christchurch in New Zealand, Hamburg, New York, Helsinki, Los Angeles, Dublin and Rome) are described in Borrett and Roes (81), Falkner and Ryan (87), Hoffstadt (81), Howard and Moser (85), Marsten and Shepardson (81), Mitchell (85), Mitra and Darby-Dowman (85), Piccione, Cherici, Bielli and LaBella (81) and Ryan and Foster (81).

11.6 Manufacturing

Assembly Line Balancing: Baybars (86a, 86b), Bowman (60) Freeman and Jucker (67) Gutjahr and Nemhauser (64) Hackman, Magazine and Wee (89), Hoffman (92) Ignall (65) Johnson (83, 81) Kilridge (62) Patterson and Albracht (75) Salveson (55) Talbot, Patterson and Gehrlein (86), Talbot and Patterson (84) White (61)

Discrete Lot Sizing and Scheduling Problem: Cattrysse, Saloman, Kirk and Van Wassenhove (93), Cattrysse, Maes and Van Wassenhove (90, 88) Dizelenski and Gomory (65) Lasdon and Terjung (71) Manne (58)

Ingot Size Selection: Vasko, Wolfe and Scott (89, 87)

Spare Parts Allocation: Scudder (84)

Pattern Sequencing In Cutting Stock Operations: Pierce (70)

Resource Constrained Network Scheduling Problem: Fisher (73)

Cellular Manufacturing: Stanfel (89)

The Assembly Line Balancing problem formulation (29) is based upon the formulations of Bowman (60) and White (61). Other formulations of this model can be found in the rest of the references listed under this topic. The Discrete Lot Sizing and scheduling model is due to Cattrysse, Saloman, Kirk and Van Wassenhove (93). Generalizations of this model are described in Cattrysse, Maes and Van Wassenhove (90, 88), Dizelenski and Gomory (65), Lasdon and Terjung (71), and Manne (58). Models and formulations of Ingot Size Selection, Spare Parts Allocation, Pattern Sequencing In Cutting Stock Operations, Resource Constrained Network Scheduling Problem and Cellular Manufacturing are described in Vasko, Wolfe and Scott (89, 87), Scudder (84), Pierce (70), Fisher (73), and Stanfell (89). Implementation of the Ingot Size Selection models at the Bethlehem Steel Corporation are reported in Vasko, Wolfe and Scott (89, 87).

11.7 Miscellaneous Operations

Frequency Planning: Thuve (81)

Timetable Scheduling: Almond (69, 66), Arani and Lofti (89), Aubin (89), Aust (76) Barham and Westwood (78), Broder (64) Carter and Tovey (92), Carter (86), Csima and Gotleib (64) Dempster (71), Dewerra (78, 75) Even, Itai and Shamir (76) Ferland and Roy (85) Gans (81), Glassey and Mizrach (86), Gosselin and Trouchon (86), Grimes (70) Hall and Action (67), Hertz (92) Knauer (74) LaPorte and Desroches (84), Lions (67) Mehta (81), Mulvey (82) Tripathy (84, 80) White and Chan (79), Wood (69)

Testing and Diagnosis: Nawijn (88) Reggia, Naw and Wang (83)

Political Districting: Garfinkel and Nemhauser (70)

Information Retrieval and Editing: Day (65) Garfinkel, Kunnathur and Liepins (86)

Check Clearing: Markland and Nauss (83) Nauss and Markland (85)

Capital Budgeting: Valenta (69)

Fixed Charge Problem: Aneja and Vemuganti (74) Frank (72) McKeown (81)

Mathematical Problems: Fulkerson, Nemhauser and Trotter (74) Heath (90)

The Models presented in this section on Frequency Planning, Testing and Diagnosis, Political Districting, Information Retrieval and Editing, Check Clearing, Capital Budgeting, Fixed Charge Problem and Mathematical Problems are based upon Thuve (81), Nawijn (88), Reggia, Naw and Wang (83), Garfinkel and Nemhauser (70), Day (65), Garfinkel, Kunnathur and Liepins (86), Markland and Nauss (83), Valenta (69), McKeown (81) and Fulkerson, Nemhauser and Trotter (74) and Heath (90).

Applications of Timetable Scheduling modles at SUNY, Buffalo, University of Waterloo, Ontario School System and Cedar Crest College are reported in Arani and Lofti (89), Carter (89), Lions (67) and Mehta (81). Implementation of Check Clearing model at the Maryland National Bank is presented in Markland and Nauss (83).

11.8 Routing

Travelling Salesman Problem: Bellmore and Hong (74), Bodin, Golden, Assad and Ball (83) Christofides (85b), Christofides, Mingozzi and Toth (81b), Christofides and Eilon (73) Eilon, Watson-Gandy and Christofides (71) Gavish and Shlifer (78), Golden, Levy and Dahl (81), Golden, Magnanti and Nguyen (77) Held and Karp (70) LaPorte (92b), Lenstra and Rinnooy Kan (75), Lin and Kernighan (73), Lin (65) Magnanti (81), Malandraki and Daskin (89) Russell (77) Solomon and Desrosiers (88)

Single And Multiple Depots: Agarwal, Mathur and Salkin (89), Agin (75), Altinkemer and Gavish (91, 90, 87), Anily and Federgruen (90), Averbakh and Berman (92) Baker (92), Balinski and Quandt (64), Ball, Golden, Assad, Bodin (83), Bartholdi, Platzman, Collins and Warden (83), Beasley (84, 83, 81), Bell, Dalberto, Fisher, Greenfield, Jaikumar, Kedia, Mack and Prutzman (83), Bellman (58), Beltrami and Bodin (74), Bertsimas (92, 88), Bertsimas and Ryzin (91), Bodin, Golden, Assad and Ball (83), Bodin and Golden (81), Bodin and Kursh (79, 78), Bodin (75), Bonder, Cassell and Andros (70), Bramel and Simchi-Levi (93), Bramel, Coffman, Shor and Simchi-Levi (92), Brown and Graves (81), Butt and Cavalier (91) Chard (68), Cheshire, Melleson and Naccache (82), Christofides (85a, 85b), Christofides, Mingozzi and Toth (81a, 81b, 79), Christofides (76, 71), Christofides and Eilon (69), Clark and Wright (64), Crawford and Sinclair (77), Cullen, Jarvis and Ratliff (81), Cunto (78) Daganzo (84), Dantazig and Ramser (59), Doll (80), Dror and Trudeau (90) Eilon, Watson-Gandy and Christofides (71), Eilon and Christofides (69), Etezadi and Beasley (83), Evans and Norbeck (85) Ferebee (74), Ferguson and Dantazig (56), Fisher, Greenfield, Jaikumar and Lester (82), Fisher and Jaikumar (81), Fletcher (63), Fleuren (88), Foster and Ryan (76), Frederickson, Hecht and Kim (78) Garvin, Crandall, John and Spellman (57), Gaskell (67), Gavish and Shlifer (78), Gavish, Schweitzer and Shlifer (78), Gendreau, Hertz and LaPorte (92), Gheysens, Golden and Assad (84), Gillett and Johnson (76), Gillett and Miller (74), Golden and Assad (88, 86a), Golden, Bodin and Goodwin (86), Golden and Baker (85), Golden, Gheysens and Assad (84), Golden and Wong

(81), Golden, Magnanti and Nguyen (77), Golden (77) Haimovich, Rinnooy Kan and Stouge (88), Haimovich and Rinnooy Kan (85), Hauer (71), Holmes and Parker (76), Hyman and Gordon (68) Kirby and McDonald (72), Kirby and Potts (69), Krolak, Felts and Nelson (72) Labbe, LaPorte and Mercure (91), Lam (70), LaPorte (92a), LaPorte, Nobert and Taillefer (88, 87), LaPorte and Nobert (87, 84), LaPorte, Mercure and Nobert (86), LaPorte and Nobert and Derochers (85), LaPorte, Derochers and Nobert (84), Lenstra and Rinnooy Kan (81, 76, 75), Levary (81), Levy, Golden and Assad (80), Li and Simchi-Levi (93, 90), Lecena (86) Magnanti (81), Malandraki and Daskin (89), Male, Liebman and Orloff (77), Marquez Diez- Canedo and Escalante (77), Minas and Mitten (58), Minieka (79), Mole (83, 79), Mole, Johnson and Wells (83), Mole and Jameson (76) Nelson, Nygard, Griffin and Shreve (85), Norbeck and Evans (84) Orloff (76a, 76b, 74a, 74b), Orloff and Caprera (76) Passens (88), Psarafits (89, 88, 83c), Pullen and Webb (67) Robertson (69), Ronen (92) Salvelsbergh (90), Scharge (83), Solomon and Desrosiers (88), Stern and Dror (79), Stewart and Golden (84), Stricker (70), Sumichrast and Markham (93), Sutcliffe and Board (91) Tillman and Cain (72), Tillman and Hering (71), Tillman (69), Tillman and Cochran (68), Turner, Ghare and Foulds (76), Turner and Hougland (75), Tyagi (68) Unwin (68) Van Leeuwen (83) Watson-Gandy and Foulds (72), Webb (72), Williams (82), Wren and Holliday (72) Yellow (70)

Routing With Time Windows: Baker and Schaffer (86), Bodin, Golden, Assad, Ball (83), Bodin and Golden (81) Cassidy and Bennett (72) Desrochers, Desrosiers and Soloman (92), Desrochers, Lenstra, Savelsbergh and Soumis (88), Desrochers, Soumis, Desrosiers and Sauve (85), Desrochers, Soumis, Desrosiers (84), Dumas, Desrosiers and Soumis (91) El-Azm (85) Fleuren (88) Gertsbach and Gurevich (77), Golden and Assad (88, 86a, 86b), Golden and Wasssel (87), Golden and Baker (85), Golden, Magnanti and Nguyen (77) Jaw, Odoni, Psaraftis and Wilson (86) Knight and Hofer (68), Kolen, Rinnooy Kan and Trienekens (87), Koskosidis, Powell and Soloman (92) Malandraki and Daskin (89) Potvin and Rousseau (93), Potvin, Kervahut and Rousseau (92) Salvelsbergh (85), Scharge (83), Sexton and Choi (72), Soloman and Desrosiers (88), Solomon (87, 86)

Periodic Routing Problem: Bodin, Golden, Assad, Ball (83) Cheshire, Melleson and Naccache (82), Christofides (85b, 84) Foster and Ryan (76) Gaudioso and Paletta (92), Golden and Assad (88) Hausman and Gilmour (67) Russel and Igo (79), Raft (82) Sexton and Bodin (85a, 85b), Solomon and Desrosiers (88) Tan and Beasley (84)

Integrated Inventory And Routing: Anily and Federgruen (93, 92, 90a, 90b), Arisawa and Elmaghraby (77a, 77b) Bramel and Simchi-Levi (93) Chien, Balakrishnan and Wong (89), Christofides (85b) Dror and Ball (87), Dror and Levy (86), Dror, Ball and Golden (86) Farvolden, LaPorte and Xu (93), Federgruen and Simchi-Levi (92), Federgruen and Zipkin (84) Golden and Assad (86a), Golden and Baker (85), Golden, Assad and Dahl (84) Hall (91)

Dial-A-Ride Problem: Angel, Caudle, Noonan and Whinston (72) Bennett and Gazis (72), Bodin and Sexton (86), Bodin and Berman (79) Cullin, Jarvis and Ratliff (81) Daganzo (78), Desrosiers, Dumas and Soumis (86), Dulac, Ferland and Forgues (80), Dumas, Desrosiers and Soumis (91) Fleuren (88), Foulds, Read and Robinson (77) Golden and Assad (88) Jaw, Odoni, Psarafitis and Wilson (86) McDonald (72) Newton and Thomas (74, 69) Psaraftis (86, 83a, 83b, 80) Solomon and Desrosiers (88), Stein (78), Stewart and Golden (81, 80)

Location And Routing: Jacobson and Madsen (80) LaPorte, Nobert and Taillefer (88), LaPorte, Nobert and Arpin (86), LaPorte and Nobert (81)

Scheduling A Fleet Of Ships, Aircrafts, Trains and Buses: Applegreen (71, 69), Assad (81, 80) Bartlett (57), Bartlett and Charnes (57), Barton and Gumer (68), Bodin, Golden, Schuster and Romig (80), Brown, Graves and Ronen (87) Ceder and Stern (81), Charnes and Miller (56) Fisher and Rosenwein (89), Florian, Guerin and Bushell (76) Laderman, Gleiberman and Egan (66), Levin (71) Martin-Lof (70), McKay and Hartley (74) Nemhauser (69) Peterson and Fullerton (73), Pierce (69), Pollak (77) Rao and Zionts (68), Richardson (76), Ronen (86, 83) Saha (70), Salzborn (74, 72a, 72b, 70, 69), Simpson (69), Smith and Wren (81), Soumis, Ferland and Rousseau (80), Spaccamela, Rinnooy Kan and Stougie (84), Szpiegel (72) White and Bomberault (69), Wolters (79), Wren (81) Young (70)

The routing problem is introduced by Dantazig and Ramser (59). The SPT formulation of routing problem is due to Balanski and Quandt (64). A variety of routing applications are listed in Table 7.

Table 7: APPLICATIONS

Bartholdi, Platzman, Collins and Warden (3)	Meals-on-Wheels Senior Citizens Inc., Atlanta
Bell, Dalberto, Fisher, Greenfield, Jaikumar, Kedia, Mack and Prutzman (83)	Distribution of Oxygen, Hydrogen etc., at Air Products and Chemicals, Inc.
Bodin and Berman (79)	School Bus Routing at Brentwood School District, Long Island, New York
Bodin and Kursh (78)	Routing and Scheduling of street sweepers in New York City and Washington, D.C.
Brown, Graves and Ronen (87)	Scheduling of Crude Oil Tankers for a major oil company
Brown and Graves (81)	Routing Petroleum tank trucks at Chevron, USA
Cassidy and Bennett (72)	Catering of meals to the schools of the Inner London Education Authority
Ceder and Stern (81)	Scheduling bus trips at Egged, the Israel National Bus Carrier
Crawford and Sinclair (72)	Scheduling beer tankers at WAIKATO Brewers Ltd., Hamilton, Nwe Zealand
Cunto (78)	Routing of boats to sample oil wells at Lake Maracaibo, Venezuela
Evans and Norbeck (85)	Food Distribution at KRAFT
Fisher and Rosenwein (89)	Military Sealift Command of the U.S. Navy
Fisher, Greenfield, Jaikumar and Lester (82)	Distribution of a major product at DUPONT
Golden, Magnanti and Nguyen (77)	Distributing newspaper with large circulation

Golden and Wassil (87)	Distribution of soft drinks at Joyce Beverages, Baltimore Division of Mid-Atlantic Coca-Cola, Pepsi-Cola Bottling Group of Purchase, New York and others
Gavish, Schweitzer and Shlifer (78)	Scheduling buses for large bus company
Jacobsen and Madsen (80)	Designing transfer points and routes for distributing newspaper for a company in Denmark
Jaw, Odoni, Psarafitis and Wilson (86)	Dial-A-Ride Model application at Rufbus GmbhBodenseekreis, Friedrichshafen, Germany
Knight and Hofer (68)	Routing vehicles to collect and deliver small consignments for a contract transport undertaking in London
McDonald (72)	Transporting specimens from a hospital to laboratories
McKay and Hartley (74)	Distribution of bulk petroleum products at the Defence Fuel Supply Center (DFSC) and the Military Sealift Command (MSC)
Salzborn (70)	Scheduling trains at the Adelaide Metropolitan Passenger Service of South Australian Railways
Smith and Wren (81)	Bus scheduling at the West Yorkshire Passenger Transport System
Stern and Dror (79)	Reading Electric Meters in the City of Beersheva, Israel

11.9 Location

Plant (Warehouse) Location and Allocation: Akinc and Khumawala (77), Atkins and Shriver (68) Baker (74), Ballou (68), Barcelo and Casanovas (84), Baumol and Wolfe (58), Bilde and Krarup (77), Brown and Gibson (72), Burstall, Leaver and Sussams (62) Cabot, Francis and Stary (70), Cerveny (80), Cho, Johnson, Padberg and Rao (83), Cho, Padberg and Rao (83), Cohon, ReVelle, Current, Eagles, Eberhart and Church (80), Cooper (64, 63), Cornuejols, Nemhauser and Wolsey (90) Davis and Ray (69), Dearing (85), Drysdale and Sandiford (69), Dutton, Hinman and Millham (74) Efroymson and Ray (66), Ellwein and Gray (71), El-Shaieb (73), Elson (72), Erlenkotter (78, 73), Feldman, Lehrer and Ray (66) Geoffrion and McBride (78), Gelders, Printelon and Van Wassenhove (87), Guignard (80), Guignard and Spielberg (79, 77) Hammer (68), Hoover (67), Hormozi and Khumawala (92) Khumawala, Neebe and Dannenbring (74), Khumawala (73a, 72), Khumawala and Whybark (71), Kolen (83), Koopmans and Beckman (74), Krarup and Pruzan (83), Kuehn and Hamburger (63), Kuhn and Kuenne (62) LaPorte, Nobert and Arpin (86), Levy (67), Louveaux and Peeters (92) Manne (64), Maranzana (64), Marks, ReVelle and Liebman (70) Nambiar, Gelders and Van Wassenhove (89, 81) Perl and Daskin (85, 84), Polopolus (65) ReVelle, Marks and Liebman (70) Sa (69), Saedat (81), Scott (70), Shannon and Ignizio (70), Spielberg (70, 69a, 69b), Smith, Mangtelsdorf, Luna and Reid (89), Swain (74) Tapiero (71) Van Roy and ErlenKotter (82), Vergin and Rogers (67) Wendell and Hurter (73)

Lock Box Location: Cornuejols, Fisher and Nemhauser (77a, 77b) Kramer (66), Kraus, Janssen and McAdams (70) Maier and Vanderwede (76, 74), Malczewski (90), Mavrides (79), McAdams (68) Nauss and Markland (81, 79) Shankar and Zoltners (72), Stancil (68)

***P*-Center Problem:** Aneja, Chandrasekaran and Nair (88) Chandrasekaran and Daugherty (81), Chhajed and Lowe (92), Christofides and Viola (71) Dearing (85), Drenzner (86, 84), Dyer and Frieze (85) Eisemann (62) Garfinkel, Neebe and Rao (77), Goldman (72a, 69) Hakimi, Schmeichel and Pierce (78), Hakimi and Maheshwari (72), Hakimi (64), Halfin (74), Halpern (76), Handler (73), Hansen, Labbe, Peters and Thisse (87), Hooker, Garfinkel and Chen (91) Kariv and Hakimi (79a), Kolen (85) Lin (75) Musuyama, Ibaraki and Hasegawa (81), Minieka (77, 70), Moon and Chaudary (84) Richard, Beguin and Peeters (90) Tansel, Francis and Lowe (83a, 83b) Vijay (85)

***P*-Median Problem:** Chhajed and Lowe (92), Church and Weaver

(86), Church and Meadows (77), Church and ReVelle (76) Dearing (85) Erkut, Francis and Lowe (88) Goldman (72b, 71), Goldman and Witzgall (70) Hakimi (65, 64), Halpern (76), Hansen, Labbe, Peters and Thisse (87), Hooker, Garfinkel and Chen (91) Jarvinen, Rajala and Sinerro (72) Kariv and Hakimi (79a), Khumawala (73b) Mavrides (79), Minieka (77), Mirchandani (79), Moon and Chaudary (84) Narula, Ogbu and Samuelsson (77), Neebe (78) ReVelle and Elzinga (89), ReVelle and Hogan (89b), Richard, Beguin and Peeters (90), Rydell (71, 67) Snyder (71b) Tansel, Francis and Lowe (83a, 83b), Teitz and Bart (68), Toregas, Swain, ReVelle and Bergman (71) Weaver and Church (85) Service Facilities Location Ball and Lin (93), Berlin and Liebman (74) Chrissis, Davis and Miller (82), Church and Meadows (79), Church and ReVelle (76), Current and Storbeck (88) Dee and Liebman (72), Deighton (71), Drezner (86) Erlenkotter (73) Foster and Vohra (92), Francis, Lowe and Ratliff (78) Goodchild and Lee (89), Gunawardane (82) Holmes, Williams and Brown (72) Kolen (85), Kolesar and Walker (74) Marks, ReVelle and Liebman (70), Moon and Chaudary (84), Mukundan and Daskin (91) Neebe (88) Orloff (77) Patel (79) Rao (74), Ratick and White (88), ReVelle (89), ReVelle and Hogan (89a), ReVelle, Toregas and Falkson (76), ReVelle, Marks and Liebman (70), ReVelle and Swain (70), Richard, Beguin and Peeters (90), Rojeski and ReVelle (70) Saatcioglue (82), Saydam and McKnew (85), Schilling, Jayaraman and Barkhi (93), Schilling (82), Schilling, ReVelle, Cohen and Elzinga (80), Schilling (80), Schreuder (81), Slater (81), Storbeck (82) Toregas and ReVelle (73), Toregas, Swain, ReVelle and Bergman (71), Toregas and ReVelle (70) Valinsky (55) Wagner and Falkson (75), Walker (74), White and Case (74), Weaver and Church (83)

Maximal Covering Problem: Balakrishnan and Storbeck (91), Balas (83), Batta, Dolan and Krishnamurty (89), Bennett, Eaton and Church (82) Church and Weaver (86), Church and Roberts (83), Church and Meadows (79), Church and ReVelle (76), Current and O'Kelly (92), Current and Schilling (90), Current and Storbeck (88) Daskin, Haghani, Khanal and Malandraki (89), Daskin (83, 82) Eaton, Daskin, Simmons, Bulloch and Jansma (85) Fuziwara, Makjamroen and Gupta (87) Kalstorin (79) Medgiddo, Zemel and Hakimi (83), Mehrez and Stulman (84, 82), Mehrez (83), Meyer and Brill (88) Pirkul and Schilling (91) ReVelle and Hogan (88) Schilling, Jayaraman and Barkhi (93), Storbeck and Vohra (88)

Hierarchical Objective Set Covering Model: Charnes and Storbeck (80), Church, Current and Storbeck (91), Church and Eaton (87) Daskin, Hogan and ReVelle (88), Daskin and Stern (81) Flynn and Ratick (88) Moore and ReVelle (82), Mukundan and Daskin (91) Plane and Hendrick (77)

Backup Coverage Model: Church and Weaver (86) Daskin, Hogan and ReVelle (88) Hogan and ReVelle (86, 83) Pirkul and Schilling (89) Storbeck and Vohra (88)

Multiple Response Unit Model: Batta and Mannur (80) Marianov and ReVelle (91) Schilling, Elzinga, Cohen, Church and ReVelle (79)

Miscellaneous: Alao (71), Armour and Buffa (63) Beckman (63), Bell and Church (85), Bellman (65), Bertsimas (88), Bindschedler and Moore (61), Bouliane and LaPorte (92) Chan and Francis (76), Chaudary, McCormick and Moon (86), Chaiken (78), Church and Garfinkel (78), Conway and Maxwell (61), Cooper (72, 68, 67), Current and Schilling (89), Current and Storbeck (87), Current, ReVelle and Cohen (85) Dearing and Francis (74a, 74b) Eislet (92), Elzinga, Hearn and Randolph (76), Elzinga and Hearn (73, 72a, 72b), Erkut, Francis, Lowe and Tamir (89), Eyster, White and Wierwille (73) Fitzsimmons and Allen (83), Fitzsimmons (69), Francis and Mirchandani (89), Francis, McGinnis and White (83), Francis and Goldstein (74), Francis and White (74), Francis and Cabot (72), Francis (72, 67a, 67b, 64, 63), Frank (66) Gavett and Plyter (66), Ghosh and Craig (86), Gleason (75), Goldberg and Paz (91), Goldberg, Dietrich, Chen and Mitwasi, Valenzuela and Criss (90) Handler and Mirchandani (79), Hansen, Thisse and Wendell (86), Hitchings (69), Hodgson (90, 81), Hogg (68), Hopmans (86), Hsu and Nemhauser (79), Hurter, Schaeffer and Wendell (75)

Keeny (72), Kimes and Fitzsimmons (90), Kirca and Erkip (88) Larson (75, 72), Lawrence and Pengilly (69), Leamer (68), Love, Morriss and Wesolowsky (88), Love, Wesolowsky and Kraemer (73), Love and Morris (72), Love (72, 69, 67) McKinnon and Barber (72), McHose (61), Mirchandani (80, 79), Mole (73), Mycielski and Trzechiakowske (63) Nair and Chandrasekaran (71) Osleeb, Ratick, Buckley, Lee and Kuby (86) Palermo (61), Picard and Ratliff (78), Price and Turcotte (86), Pritsker (73), Pritsker and Ghare (70) Rand (76), ReVelle and Serra (91), ReVelle (86), Roodman and Schwartz (77, 75), Rosing (92), Ross and Soland (77), Rushton (89) Schaefer and Hurter (74), Schneider (71), Schniederjans, Kwak and Helmer (82), Simmons (71, 69), Snyder (71a), Storbeck (90, 88) Tansel and Yesilkokeen (93), Tansel, Francis and Lowe (80), Taylor (69), Teitz (68), Tewari and Jena (87), Tideman (62) Volz (71) Watson-Gandy (82, 72), Watson-Gandy and Eilon (72), Wesolowsky (73a, 73b, 72), Wesolowsky and Love (72, 71a, 71b), Weston (82), Wirasinghe and Waters (83), Weaver and Church (83) Young (63)

Table 8: APPLICATIONS

Bennett, Eaton and Church (82)	Selecting Sites for Rural Health Workers; Valle del Cauca, Columbai
Cerveny (80)	Location of Bloodmobile Operations
Cohon, ReVelle, Currnt, Eagles, Eberhart and Church	Power plant locations in a six-state region of U.S
Current and O'Kelly (92)	Locating emergency warning sirens in a midwestern city of U.S.
Daskin (82)	Emergency medical vehicles located in Austin, Texas
Daskin and Stern (81)	Emergency medical vehicles location in Austin, Texas
Drysdale and Sandiford (69)	Locating warehouses for R + CA Victor Company Ltd., Canada
Dutton, Hinman and Millham (74)	Locating electrical power generating plants in the Pacific Northwest
Eaton, Daskin, Simmons, Bulloch and Jansma (85)	Emergency medical service vehicle location in Texas
Fitzsimmons and Allen (83)	Selection of out-of-state audit offices
Flynn and Ratic (88)	Air Service locations for small communities in North and South Dakota
Fujiwara, Makjamroen and Gupta (87)	Ambulance deployment - A case study in Bangkok

Goldberg, Dietrich, Chen and Mitwasi, Valenzuela and Criss (90)	Locating emergency medical servicesin Tucson, Arizona
Hogg (68)	Siting of fire stations in Briston County Borough, England
Holmes, Williams and Brown (72)	Location of public day care facilities in Columbus, Ohio
Hopmans (86)	Locating bank branches in Netherlands
Kimes and Fitzsimmons	Selecting profitable sites at La Quinta Inns
Kirca and Erkip (88)	Selecting solid waste transfer points in Turkey
Kolesar and Walker (74)	Dynamic relocation of fire companies in New York City
Nambiar, Gelders and Van Wassenhove (89, 81)	Location of rubber processing factories in Malaysia
Patel (79)	Locating rural social service centers in India
Plane and Hendrick (77)	Location of fire companies for Denver Fire Department
Price and Turcotte (86)	Location of blood bank in Canada
Saedat (81)	Location of grass drying plants in the Netherlands
Schniederjans, Kwak and Helmer (82)	Locating a trucking terminal
Schreuder (81)	Locating fire stations in Rotterdam

Smith, Mangelsdorf, Luna and Reid (89)	Supply centers for Ecuador's health workers just-in-time
Tewari and Jena (87)	Location of high schools in rural India
Volz (71)	Ambulance location in semi-rural areas of Washtenaw County, Michigan
Walker (74)	Location of fire stations in New York City
Weston (82)	Telephone answering sites in a service industry (U.S.)
Wirasinghe and Waters (83)	Location of solid waste transfer points (Canada)

The models and formulations addressed in this section can be found in Dearing (85), Lourveaux and Peeters and Manne (64) (Plant Location), Cornuejols, Fisher and Nemhauser (77a), Kraus, Janseen and McAdams (70), Nauss and Markland (81) (Lock Box), Garfinkel, Neebe and Rao (77) and Minieka (70) (P-Center), Hakimi (64), Narula, Ogbu and Samuelsson (77) and ReVelle and Swain (70) (P-Median), Toregas and ReVelle (70), Toregas, Swain, ReVelle and Bergman (71) (Service Facilities), Church and ReVelle (74) (Maximal Covering), Daskin and Stern (81) (Hierarchical Objective Set Covering), Hogan and ReVelle (86) (Backup Coverage) and Batta and Mannur (90) (Multiple Response Unit).

91

12 Conclusions

In this paper a variety of applications of set covering, set packing, and set partitioning models including their variants and generalizations are presented. In addition, transformations to convert one model to another including the transformation of the MD Knapsack problem and the LIP to one of these models are discussed. It should be noted that some applications such as Travelling Salesman Problem, Multicommodity Disconnecting Problem and Steiner Problem in Graphs require enormous number of constraints where as the formulation of the Personnel Scheduling, Crew Scheduling, and the Routing models require a very large number of variables. The transformations of the MD Knapsack problem and the LIP require both a very large number of variables and constraints.

Moderate size SC, SP and SPT models can be solved efficiently with the existing algorithms and techniques and have been used in many real life situations. Efficient solution techniques to solve very large SC, SP and SPT models will enhance the application of these mpodels to solve many real life lgoistics problems. Clearly, these special structured models are a very useful and important class of linear integer programs and deserve the effort devoted by many researchers.

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