Exercises Lectures 2 and 3 with Full Answers
Computer Vision 1, Master AI

# 1 Exercises (Lectures 2 and 3)

**EXERCISE 1:**
To calculate the color of light sources, the following intuitive color models are used: intensity $I$, chromaticity $xy$, hue $H$ and saturation $S$. Let's assume, for simplicity reasons, that sunlight $S$ is given by $X = Y = Z = 100$. Further, let $X = 100$, $Y = 100$ en $Z = 150$ be the values for a given artificial lamp $A$.

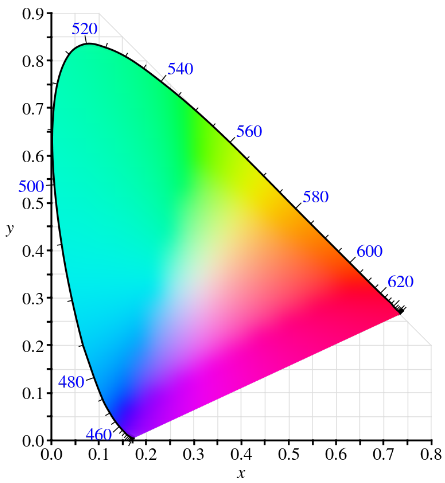


Figure 1: Chromaticity diagram

- (Q.a) Calculate the intensity $I$ of the two light sources $S$ and $A$.

- (A.a) $I = \dfrac{X+Y+Z}{3}$ gives $I_S = \dfrac{300}{3} = 100$, $I_A = \dfrac{350}{3} = 117$
- (Q.b) Calculate the chromaticy values $x = X/(X+Y+Z)$, $y = Y/(X+Y+Z)$ and plot these in the chromaticity diagram given in Figure 1.

- (A.b) $x_S = \dfrac{1}{3}$, $y_S = \dfrac{1}{3}$, and $z_S = 1 - (x_S + y_S) = \dfrac{1}{3}$
$x_A = 0.286, y_A = 0.286$, and $z_A = 1 - (x_A + y_A) = 0.428$

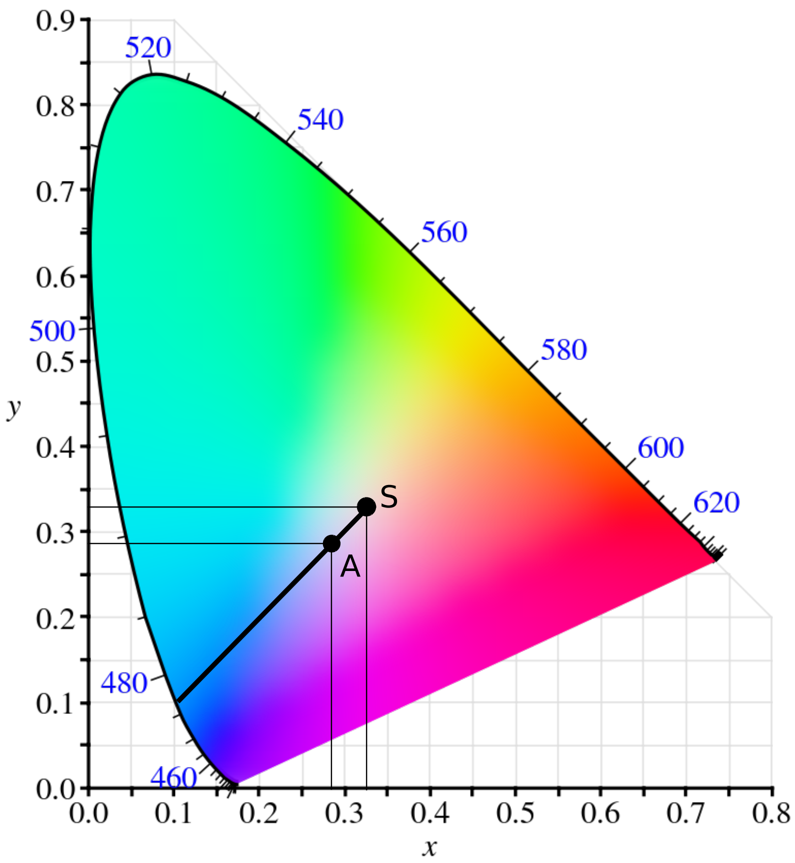


Figure 2: Location of $S$ and $A$ in the chomaticity diagram

See Figure 2

- (Q.c) What is the estimated hue of light source $A$ with $S$ as reference white light.

- (A.c) The line from S through A cuts the boundary at around 476nm[1], which is the hue (dominant wavelength) of A with S as reference white light.
- (Q.d) Rank the light sources with respect to their saturation.
- (A.d) $S < A$
- (Q.e) Plot the colors which are produced through the mixture of $S$ and $A$.
- (A.e) The colors produced by the mixture of $S$ and $A$ lie on the line connecting $S$ and $A$ in the chromaticity diagram.

[1]This was 440nm in the short solution version, due to coarser diagram scale

**EXERCISE 2:**

We consider the representation of colors in a color space. In Figure A.1 (see attachment), the color matching functions of the CIE $X$, $Y$ and $Z$ primary colors are given. Further, in table 1 (see attachment) their spectral values are given with 10 nm interval (e.g. the spectral color of 500 nm has the following tri-stimulus values $\bar{\mathrm{x}} = 0.0049$, $\bar{\mathrm{y}} = 0.323$ and $\bar{\mathrm{y}} = 0.2720$). Given a light source $K(\lambda)$ and an object $\rho(\lambda)$ with certain spectral distributions, then $X = \int_\lambda K(\lambda)\rho(\lambda)\bar{\mathrm{x}}(\lambda)d\lambda$, $Y = \int_\lambda K(\lambda)\rho(\lambda)\bar{\mathrm{y}}(\lambda)d\lambda$ and $Z = \int_\lambda K(\lambda)\rho(\lambda)\bar{\mathrm{z}}(\lambda)d\lambda$. It is assumed that $K(\lambda)$ is a white light source i.e. equal energy distribution over all wavelengths.

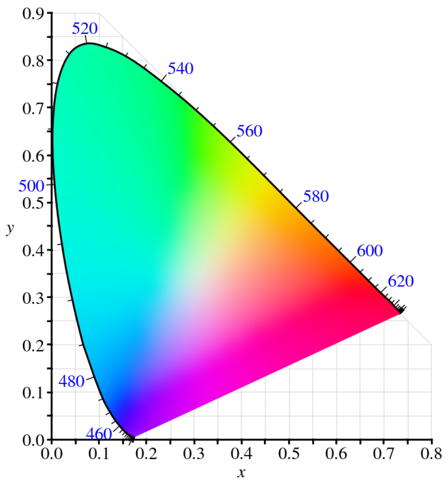


Figure 3: Chromaticity diagram

- (Q.a) Compute $X$, $Y$ and $Z$ for a given object color $A$ of 500 nm i.e. $\rho(\lambda_{500}) = 1$ and 0 otherwise. Further, calculate the chromaticity coordinates $x = \frac{X}{X+Y+Z}$, $y = \frac{Y}{X+Y+Z}$ and $z = \frac{Z}{X+Y+X}$ of $A$.

- (A.a)

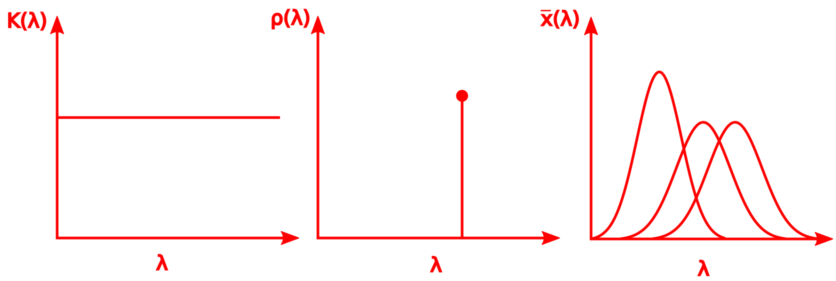


Figure 4: Illustration of $K(\lambda)$, $\rho(\lambda)$ and $\bar{\mathrm{x}}(\lambda)$

$$X_A = \int_\lambda K(\lambda_{500})\rho(\lambda_{500})\bar{\mathrm{x}}(\lambda_{500})d\lambda = K \cdot 1 \cdot \bar{\mathrm{x}}(\lambda_{500}) = K0.0049.$$

Similarly,

$$Y_A = K \cdot \bar{y}(\lambda_{500}) = K0.323, \quad Z_A = K \cdot \bar{z}(\lambda_{500}) = K0.272$$

.

$$x_A = \frac{X_A}{X_A + Y_A + Z_A} = \frac{K0.0049}{K(0.0049 + 0.323 + 0.272)} = 0.0081$$
$$y_A = \frac{Y_A}{X_A + Y_A + Z_A} = \frac{K0.0323}{K(0.0049 + 0.323 + 0.272)} = 0.538$$
$$z_A = \frac{Z_A}{X_A + Y_A + Z_A} = \frac{K0.272}{K(0.0049 + 0.323 + 0.272)} = 0.453$$

- (Q.b) Plot color $A$ as a small circle in the chromaticity diagram given in Figure 3.
- (Q.b) Figure 5

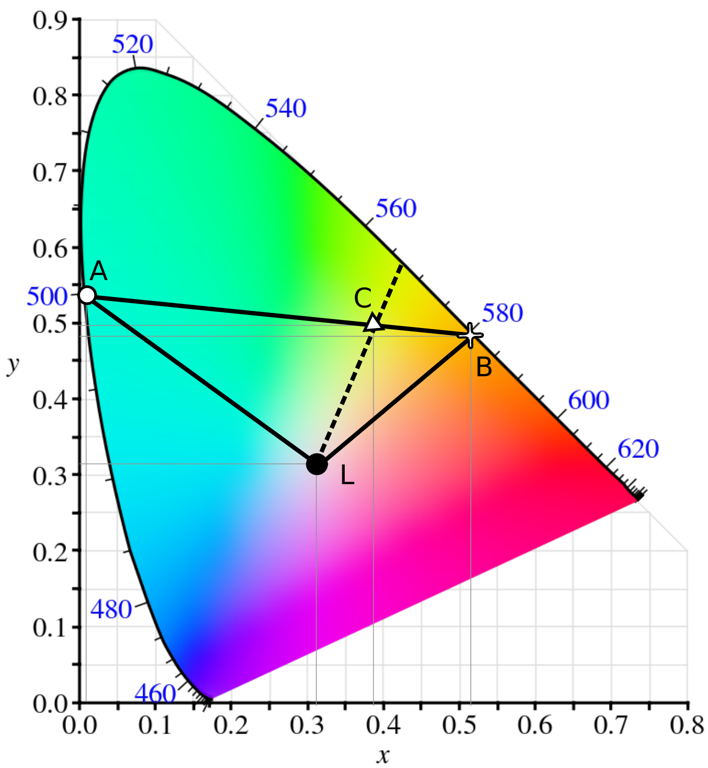


Figure 5: Location of A, C, B, L on the chromaticity diagram.

- (Q.c) Given an object color $B$ of 580 nm (i.e. $\rho(\lambda_{580}) = 1$ and 0 otherwise), find $X$, $Y$ and $Z$ and the chromaticity coordinates $x$, $y$ en $z$.
- (A.c)

$X_B = K \cdot \bar{y}(\lambda_{580}) = K0.9163, \quad Y_B = K \cdot \bar{y}(\lambda_{580}) = K0.8700, \quad Z_B = K \cdot \bar{z}(\lambda_{580}) = K0.0017$

.

$$x_B = \frac{X_B}{X_B + Y_B + Z_B} = \frac{K0.9163}{K(0.9163 + 0.8700 + 0.0017)} = 0.512$$
$$y_B = \frac{Y_B}{X_B + Y_B + Z_B} = \frac{K0.8700}{K(0.9163 + 0.8700 + 0.0017)} = 0.486$$
$$z_B = \frac{Z_B}{X_B + Y_B + Z_B} = \frac{K0.0017}{K(0.9163 + 0.8700 + 0.0017)} = 0.0$$

- (Q.d) Plot color $B$ as a small cross in the chromaticity diagram.
- (A.d) See Figure 5
- (Q.e) Given a color $C$ consisting of the colors $A$ of 500 nm and $B$ of 580 nm. Compute $X$, $Y$ en $Z$ and the chromaticity coordinates $x$, $y$ en $z$.
- (A.e)

$$x_C = \frac{X_A + X_B}{X_A + Y_A + Z_A + X_B + Y_B + Z_B} = \frac{K(0.0049 + 0.9163)}{K(0.0049 + 0.323 + 0.272 + 0.9163 + 0.8700 + 0.0017)} = 0.385$$
$$y_C = \frac{Y_A + Y_B}{X_A + Y_A + Z_A + X_B + Y_B + Z_B} = \frac{K(0.323 + 0.8700)}{K(0.0049 + 0.323 + 0.272 + 0.9163 + 0.8700 + 0.0017)} = 0.499$$
$$z_C = \frac{Y_A + Z_B}{X_A + Y_A + Z_A + X_B + Y_B + Z_B} = \frac{K(0.272 + 0.0017)}{K(0.0049 + 0.323 + 0.272 + 0.9163 + 0.8700 + 0.0017)} = 0.115$$

- (Q.f) Plot the color as a small triangle in the chromaticity diagram.
- (A.f) See Figure 5
- (Q.g) If the white light source $K(\lambda)$ varies (only) in intensity what would happen with the values $X$, $Y$, $Z$ and $x$, $y$ and $z$ of the colors $A$, $B$ and $C$? What will be the consequence?
- (A.g) $X$, $Y$, $Z$ will change evenly. $x$, $y$ and $z$ are invariant to intensity changes.
- (Q.h) The tri-stimulus values of a given lamp $L$ are as follows $X = 98.04$, $Y = 100.00$ and $Z = 118.12$. Compute the chromaticity coordinates $x$, $y$ and $z$ and plot color $L$ with a small rectangle in the chromaticity diagram.
- (A.h)

$$x_L = \frac{X_L}{X_L + Y_L + Z_L} = \frac{98.04}{98.04 + 100.00 + 118.12} = 0.31$$
$$y_L = \frac{Y_L}{X_L + Y_L + Z_L} = \frac{100.00}{98.04 + 100.00 + 118.12} = 0.316$$
$$z_L = \frac{Z_L}{X_L + Y_L + Z_L} = \frac{118.12}{98.04 + 100.00 + 118.12} = 0.373$$

$x_L = 0.31, y_L = 0.316, z_L = 0.373.$

- (Q.i) Indicate, by three different lines, the colors which are generated by the mixture of $L$ with $A$, $B$ and $C$ respectively.
- (A.i) See Figure 5
- (Q.j) What is the hue (dominant wavelength) of $C$ with $L$ as reference white?
- (A.j) The line from L through C cuts the boundary at around 567nm[2], which is the hue of C with L as reference white.
- (Q.k) Order the three colors $A$, $B$ and $C$ with respect to their saturation.
- (A.k) $A = B > C$.
- (Q.l) Given is a color with a spectral power distribution given in Figure A.2 (see attachment). Estimate the hue (dominant wavelength) and describe the amount of the saturation and intensity. What should be the approximated position of this color in the chromaticity diagram?
- (A.l) The hue is around 500 nm.
- (Q.m) Given is a color with spectral power distribution given in Figure A.3. Estimate the hue (dominant wavelength) and describe the amount of the saturation and intensity. What should be the approximated position of this color in the chromaticity diagram?
- (A.m) The hue is around 590 nm.
- (Q.n) Given is a color with spectral power distribution given in Figure A.4. Estimate the hue (dominant wavelength) and describe the amount of the saturation and intensity. What should be the approximated position of this color in the chromaticity diagram?
- (A.n) The hue is around 500 nm. Mixed with white light and hence less peaked/saturated.
- (Q.o) For which of the three spectra a human will perceive the highest intensity? Explain your answer.
- (A.o) Related to $Y$.

**EXERCISE 3:**

We consider the color of a matte, dull (not glossy) surface. The color at a specific location on the surface under white light illumination is given by the following simple reflection model $R = Ik_R \cos\theta$, $G = Ik_G \cos\theta$ and $B = Ik_B \cos\theta$, where $I$ is the intensity of the white light source, $k_R$, $k_G$ and $k_B$ are the amount of red, green and blue reflected by the surface (i.e. color of the surface). Furthermore, $\cos\theta = \vec{n} \cdot \vec{l}$ is the dot product of the two-unit vectors $\vec{n}$ (i.e. surface normal) and $\vec{l}$ (i.e. direction of the light source), see Figure A.5.

[2]This was 540nm in the short solution version, due to coarser diagram scale

- (Q.a) Assume that the surface is flat and homogeneously colored. Explain why the intensity is higher when the surface normal coincides with the direction of the light source than observed under an angle with respect to the direction of the light source.

- (A.a) We have

$$R = Ik_R \cos\theta,$$

$$G = Ik_G \cos\theta,$$

$$B = Ik_B \cos\theta,$$

where $I$ is the light source intensity, $k_R, k_G, k_B$ are constant albedo. Thus, the perceived intensity depends on $\cos\theta$ where $\theta$ is the angle between the surface normal and the light source direction.
Consider 2 cases where (1) the light source direction coincides with the surface normal's and (2) where they are making an angle of 45° (see Figure 6).
The first case gives $\cos\theta = \cos 0° = 1$, which is the maximum, whereas the second case gives $\cos\theta = \cos 45° = \frac{\sqrt{2}}{2}$.

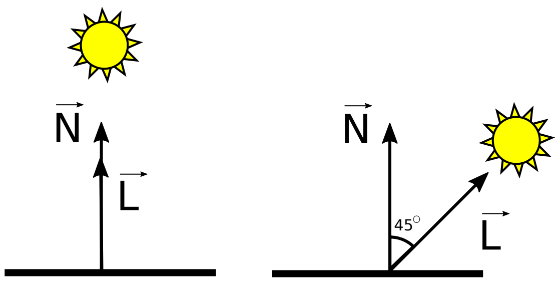


Figure 6: Light source position scenarios: coincidence with surface normal (left), and making an angle of 45° (right)

- (Q.b) Assume that the color of the surface is yellow i.e. $R = 100$, $G = 100$, and $B = 10$. Explain what will happen with the values $R$, $G$ and $B$ if (only) the intensity of the light source will diminish. Plot the positions of the colors in the $RGB$-color space.

- (A.b) We have

$$R = Ik_R \cos\theta,$$

$$G = Ik_G \cos\theta,$$

$$B = Ik_B \cos\theta,$$

where $I$ is the light source intensity. Thus when light source intensity diminish $R$, $G$, and $B$ will proportionally change. The resulted set of colors lies on the line connecting the origin with the point RGB(100, 100, 10) in the RGB-space diagram (Figure 7).

- (Q.c) In case of a curved (not flat) surface, indicate where the colors will be positioned in the $RGB$-color space. Explain your answer.

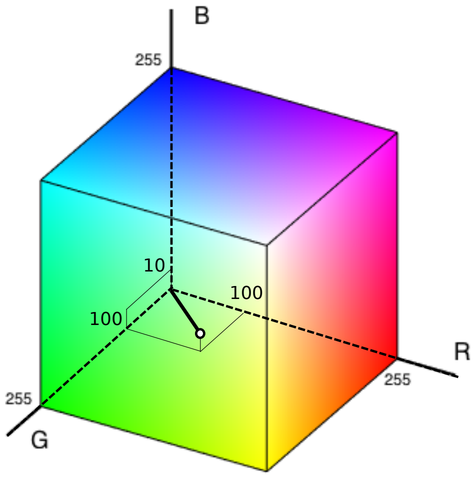


Figure 7: The position of color RGB(100, 100, 10) in the RGB-color space

- (A.c) Same as answer A.b.
- (Q.d) A simple color invariant is given by $R/G$. Proof that $R/G$ is independent of the (intensity) light source $I$, object geometry and the direction of the light source.
- (A.d) $\frac{R}{G} = \frac{Ik_R \cos\theta}{Ik_G \cos\theta} = \frac{k_R}{k_G}$
- (Q.e) The values of $R$, $G$ and $B$ will vary for a curved surface. Give the approximated shapes of the histograms for a homogeneously (curved) surface for $R/G$.
- (A.e) From previous question, $\frac{R}{G} = \frac{k_R}{k_G}$ which stays constant and hence invariant to geometry.
- (Q.f) Consider the same surface. Assume that the surface is glossy (instead of matte). The reflection model is now given by $R = Ik_R cos\theta + Ik_s cos^n\alpha$, $G = Ik_G cos\theta + Ik_s cos^n\alpha$ and $B = Ik_B cos\theta + Ik_s cos^n\alpha$. $k_s$ is the specular reflection coefficient and $cos^n$ depends on the glossiness and $\alpha$ depends on the viewing condition. Plot the colors of the homogeneously colored (shiny) surface in $RGB$- and $rgb$-color space.
  Proof that $\frac{R-G}{R-B}$ is a color invariant for shiny surfaces.
- (A.f)

$$\frac{R-G}{R-B} = \frac{(Ik_R\cos\theta + Ik_s\cos^n\alpha) - (Ik_G\cos\theta + Ik_s\cos^n\alpha)}{(Ik_R\cos\theta + Ik_s\cos^n\alpha) - (Ik_B\cos\theta + Ik_s\cos^n\alpha)} = \frac{Ik_R\cos\theta - Ik_G\cos\theta}{Ik_R\cos\theta - Ik_B\cos\theta} = \frac{k_R - k_G}{k_R - k_B}$$

## 2 Exercises (Lecture 3)

### Exercise 1. Image Filtering

Below are four types of filters.

3x3 uniform (box) filter:

$$T = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

5x5 uniform (box) filter:

$$U = \begin{array}{|c|c|c|c|c|}\hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline \end{array}$$

Another 3x3 filter

$$V = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

The Laplacian using the following matrix values:

$$W = \begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline -2 & 4 & -2 \\ \hline 1 & -2 & 1 \\ \hline \end{array}$$

- (Q.a) Which of the following would make an image blurrier, a 3x3 or a 5x5 uniform filter? Why?
- (A.a) A 5x5 uniform filter. Average over a larger area.
- (Q.b) What kind of image features are detected by the 3x3 edge filters $V$ and $W$?
- (A.b) $V$: edges, $W$: blobs.
- (Q.c) You wish to transform an image by applying a 3x3 uniform filter followed by the 3x3 Laplacian filter $W$. Show that this can be implemented by a single a 5x5 filter and calculate the elements of this filter.
- (A.c) $W = \begin{array}{|c|c|c|c|c|}\hline 1 & -1 & 0 & -1 & 1 \\ \hline -1 & 1 & 0 & 1 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline -1 & 1 & 0 & 1 & -1 \\ \hline 1 & -1 & 0 & -1 & 1 \\ \hline \end{array}$

### Exercise 2. Color Constancy

Color constancy is an important issue when recognizing an object independent of the color of the light source. Two simple color constancy algorithms are based on the white patch assumption and the grey-world hypothesis. The $R$, $G$ and $B$ channels of a small image are given in Table 1.

$R =$

| | | |
|---|---|---|
| 120 | 120 | 120 |
| 120 | 120 | 120 |
| 180 | 180 | 180 |

$G =$

| | | |
|---|---|---|
| 70 | 80 | 70 |
| 70 | 70 | 80 |
| 80 | 230 | 70 |

$B =$

| | | |
|---|---|---|
| 100 | 50 | 30 |
| 90 | 220 | 20 |
| 150 | 120 | 80 |

Table 1: The $R$, $G$ and $B$ channels of a small image.

$A =$

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 |

$B =$

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 |

Table 2: Intensity values of image patches $A$ and $B$.

- (Q.a) Give an example of an image for which the white patch method will fail. Please explain.
- (A.a) An image containing a few colorful (object) colors. The assumption is that an achromatic patch is present in the image.
- (Q.b) Calculate the results of both algorithms for the image shown in Table 1.
- (A.b) White patch: $a_1 = \frac{255}{180}$, $a_2 = \frac{255}{230}$, $a_3 = \frac{255}{220}$. Grey-world: $a_1 = \frac{128}{140}$, $a_2 = \frac{128}{91}$, $a_3 = \frac{128}{95.4}$.

## Exercise 3. Edges and Corners

Consider the image patches $A$ and $B$ in Table 2.

- (Q.a) Compute the gradient magnitude and the Harris corner response of image patch A (using a simple derivative filter e.g. [1-1]).
- (A.a) Assuming that the boundaries are being clamped, Gradient magnitude $\bigtriangledown f = \sqrt{f_x^2 + f_y^2}$, where

$$f_x = f_x^2 = \begin{array}{|c|c|c|c|}\hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline \end{array} \text{ and } f_y = f_y^2 = \begin{array}{|c|c|c|c|}\hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline \end{array}, \text{ hence}$$

$$\bigtriangledown f = \sqrt{f_x^2 + f_y^2} = \begin{array}{|c|c|c|c|}\hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline \end{array}$$

The Harris corner response is computed by $R = \text{Det}M - k(\text{Trace}(M))^2$, where

$$M = \begin{bmatrix} \Sigma f_x^2 & \Sigma f_x f_y \\ \Sigma f_x f_y & \Sigma f_y^2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{and} \quad f_x f_y = \begin{array}{|c|c|c|c|}\hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline \end{array}.$$

Hence $R = \text{Det}M - k(\text{Trace}(M))^2 = -0.64$

- (Q.b) What are the interest points of image patch $B$?
- (A.b) Edges and corners.
- (Q.c) Compute the gradient magnitude and the Harris corner response of image patch $B$ (using a simple edge filter e.g. [1-1]).
- (A.c) Assuming that the boundaries are being clamped, Gradient magnitude $\bigtriangledown f = \sqrt{f_x^2 + f_y^2}$, where

$$f_x = f_x^2 = \begin{array}{|c|c|c|c|}\hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline \end{array} \text{ and } f_y = f_y^2 = \begin{array}{|c|c|c|c|}\hline 0 & 0 & 0 & 0 \\ \hline 1 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline \end{array}, \text{ hence}$$

$$\bigtriangledown f = \sqrt{f_x^2 + f_y^2} = \begin{array}{|c|c|c|c|}\hline 0 & 0 & 0 & 0 \\ \hline 1 & 1 & \sqrt{2} & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline \end{array}$$

The Harris corner response is computed by $R = \text{Det}M - k(\text{Trace}(M))^2$, where

$$M = \begin{bmatrix} \Sigma f_x^2 & \Sigma f_x f_y \\ \Sigma f_x f_y & \Sigma f_y^2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}, \quad \text{and} \quad f_x f_y = \begin{array}{|c|c|c|c|}\hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline \end{array}.$$

Hence $R = \text{Det}M - k(\text{Trace}(M))^2 = 6.56$ $R > 0$ indicating a corner.

- (Q.d) Compute the eigenvalues of $M$ for patch $B$ where $M$ is the 2x2 matrix computed from the image derivatives i.e. second moment matrix (autocorrelation matrix).

- (A.d)

$$
\begin{aligned}
\mathcal{M}x &= \lambda x \\
\mathcal{M}x - \lambda x &= 0 \\
(\mathcal{M} - \lambda I)x &= 0, \text{where I is identity matrix} \\
\text{Det}(\mathcal{M} - \lambda I) &= 0 \\
\text{Det}\left(\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) &= 0 \\
\text{Det}\left(\begin{bmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{bmatrix}\right) &= 0 \\
(3-\lambda)(3-\lambda) - 1 &= 0 \\
\lambda^2 - 6\lambda + 8 &= 0 \\
\lambda = 4 \vee \lambda &= 2
\end{aligned}
$$

Eigenvalues are $\lambda_1 = 4$ and $\lambda_2 = 2$.