

Exercises Lectures 2 and 3 with Short Answers

Computer Vision 1, Master AI

1 Exercises (Lectures 2 and 3)

EXERCISE 1:

To calculate the color of light sources, the following intuitive color models are used: intensity I , chromaticity xy , hue H and saturation S . Let's assume, for simplicity reasons, that sunlight S is given by $X = Y = Z = 100$. Further, let $X = 100$, $Y = 100$ en $Z = 150$ be the values for a given artificial lamp A .

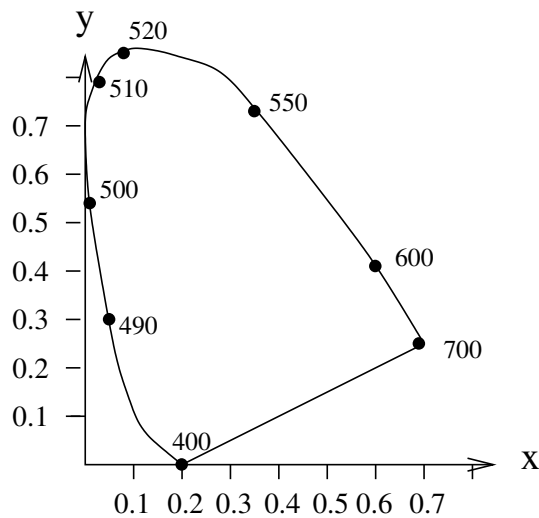


Figure 1: *Chromaticity diagram.*

- (Q.a) Calculate the intensity I of the two light sources S and A .
- (A.a) $I_S = X + Y + Z = 300$, $I_A = 350$
- (Q.b) Calculate the chromaticity values $x = X/(X + Y + Z)$, $y = Y/(X + Y + Z)$ and plot these in the chromaticity diagram given in Figure 1.
- (A.b) $x_S = \frac{1}{3}$, $y_S = \frac{1}{3}$, $x_A = 0.286$, $y_A = 0.286$.
- (Q.c) What is the estimated hue of light source A with S as reference white light.

- (A.c) Around 440 nm
- (Q.d) Rank the light sources with respect to their saturation.
- (A.d) $S < A$
- (Q.e) Plot the colors which are produced through the mixture of S and A .
- (A.e) Line between S and A

EXERCISE 2:

We consider the representation of colors in a color space. In Figure A.1 (see attachment), the color matching functions of the CIE X , Y and Z primary colors are given. Further, in table 1 (see attachment) their spectral values are given with 10 nm interval (e.g. the spectral color of 500 nm has the following tri-stimulus values $\bar{x} = 0.0049$, $\bar{y} = 0.323$ and $\bar{z} = 0.2720$). Given a light source $K(\lambda)$ and an object $\rho(\lambda)$ with certain spectral distributions, then $X = \int_{\lambda} K(\lambda)\rho(\lambda)\bar{x}(\lambda)d\lambda$, $Y = \int_{\lambda} K(\lambda)\rho(\lambda)\bar{y}(\lambda)d\lambda$ and $Z = \int_{\lambda} K(\lambda)\rho(\lambda)\bar{z}(\lambda)d\lambda$. It is assumed that $K(\lambda)$ is a white light source i.e. equal energy distribution over all wavelengths.

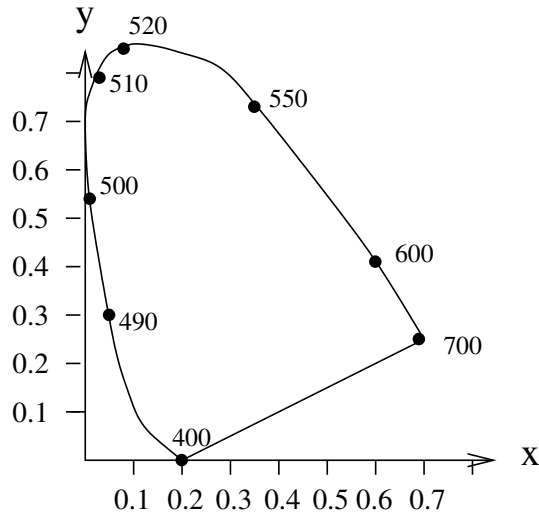


Figure 2: Color scheme.

- (Q.a) Compute X , Y and Z for a given object color A of 500 nm i.e. $\rho(\lambda_{500}) = 1$ and 0 otherwise. Further, calculate the chromaticity coordinates $x = \frac{X}{X+Y+Z}$, $y = \frac{Y}{X+Y+Z}$ and $z = \frac{Z}{X+Y+Z}$ of A .
- (A.a) $X_A = K0.0049, Y_A = K0.323, Z_A = K0.272$ and $x_A = 0.0081, y_A = 0.538, z_A = 0.453$.
- (Q.b) Plot color A as a small circle in the chromaticity diagram given in Figure 2.

- (Q.b) Please plot yourself.
- (Q.c) Given an object color B of 580 nm (i.e. $\rho(\lambda_{580}) = 1$ and 0 otherwise), find X , Y and Z and the chromaticity coordinates x , y en z .
- (A.c) $X_B = K0.9163, Y_B = K0.8700, Z_B = K0.0017$ and $x_B = 0.512, y_B = 0.486, z_B = 0.0$.
- (Q.d) Plot color B as a small cross in the chromaticity diagram.
- (A.d) Please plot yourself.
- (Q.e) Given a color C consisting of the colors A of 500 nm and B of 580 nm. Compute X , Y en Z and the chromaticity coordinates x , y en z .
- (A.e) $x_C = 0.385, y_C = 0.499, z_C = 0.115$.
- (Q.f) Plot the color as a small triangle in the chromaticity diagram.
- (A.f) Please plot yourself.
- (Q.g) If the white light source $K(\lambda)$ varies (only) in intensity what would happen with the values X , Y , Z and x , y and z of the colors A , B and C ? What will be the consequence?
- (A.g) X , Y , Z will change evenly. x , y and z are invariant to intensity changes.
- (Q.h) The tri-stimulus values of a given lamp L are as follows $X = 98.04$, $Y = 100.00$ and $Z = 118.12$. Compute the chromaticity coordinates x , y and z and plot color L with a small rectangle in the chromaticity diagram.
- (A.h) $x_L = 0.31, y_L = 0.316, z_L = 0.373$.
- (Q.i) Indicate, by three different lines, the colors which are generated by the mixture of L with A , B and C respectively.
- (A.i) Please plot yourself.
- (Q.j) What is the hue (dominant wavelength) of C with L as reference white?
- (A.j) Around 540 nm.
- (Q.k) Order the three colors A , B and C with respect to their saturation.
- (A.k) $A = B > C$.
- (Q.l) Given is a color with a spectral power distribution given in Figure A.2 (see attachment). Estimate the hue (dominant wavelength) and describe the amount of the saturation and intensity. What should be the approximated position of this color in the chromaticity diagram?
- (A.l) The hue is around 500 nm.

- (Q.m) Given is a color with spectral power distribution given in Figure A.3. Estimate the hue (dominant wavelength) and describe the amount of the saturation and intensity. What should be the approximated position of this color in the chromaticity diagram?
- (A.m) The hue is around 590 nm.
- (Q.n) Given is a color with spectral power distribution given in Figure A.4. Estimate the hue (dominant wavelength) and describe the amount of the saturation and intensity. What should be the approximated position of this color in the chromaticity diagram?
- (A.n) The hue is around 500 nm. Mixed with white light and hence less peaked/saturated.
- (Q.o) For which of the three spectra a human will perceive the highest intensity? Explain your answer.
- (A.o) Related to Y .

EXERCISE 3:

We consider the color of a matte, dull (not glossy) surface. The color at a specific location on the surface under white light illumination is given by the following simple reflection model $R = Ik_R \cos \theta$, $G = Ik_G \cos \theta$ and $B = Ik_B \cos \theta$, where I is the intensity of the white light source, k_R , k_G and k_B are the amount of red, green and blue reflected by the surface (i.e. color of the surface). Furthermore, $\cos \theta = \vec{n} \cdot \vec{l}$ is the dot product of the two-unit vectors \vec{n} (i.e. surface normal) and \vec{l} (i.e. direction of the light source), see Figure A.5.

- (Q.a) Assume that the surface is flat and homogeneously colored. Explain why the intensity is higher when the surface normal coincides with the direction of the light source than observed under an angle with respect to the direction of the light source.
- (A.a) Dependent on $\cos \theta$.
- (Q.b) Assume that the color of the surface is yellow i.e. $R = 100$, $G = 100$, and $B = 10$. Explain what will happen with the values R , G and B if (only) the intensity of the light source will diminish. Plot the positions of the colors in the RGB -color space.
- (A.b) R , G and B values changes evenly.
- (Q.c) In case of a curved (not flat) surface, indicate where the colors will be positioned in the RGB -color space. Explain your answer.
- (A.c) Same as answer A.b.
- (Q.d) A simple color invariant is given by R/G . Proof that R/G is independent of the (intensity) light source I , object geometry and the direction of the light source.
- (A.d) $\frac{R}{G} = \frac{Ik_R \cos \theta}{Ik_G \cos \theta} = \frac{k_R}{k_G}$

- (Q.e) The values of R , G and B will vary for a curved surface. Give the approximated shapes of the histograms for a homogeneously (curved) surface for R/G .
- (A.e) Only R/G has one value (invariant).
- (Q.f) Consider the same surface. Assume that the surface is glossy (instead of matte). The reflection model is now given by $R = Ik_R \cos \theta + Ik_s \cos^n \alpha$, $G = Ik_G \cos \theta + Ik_s \cos^n \alpha$ and $B = Ik_B \cos \theta + Ik_s \cos^n \alpha$. k_s is the specular reflection coefficient and \cos^n depends on the glossiness and α depends on the viewing condition. Plot the colors of the homogeneously colored (shiny) surface in RGB - and rgb -color space. Proof that $\frac{R-G}{R-B}$ is a color invariant for shiny surfaces.
- (A.f) $\frac{R-G}{R-B} = \frac{k_R - k_G}{k_R - k_B}$

2 Exercises (Lecture 3)

Exercise 1. Image Filtering

Below are four types of filters.

3x3 uniform (box) filter:

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

5x5 uniform (box) filter:

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Another 3x3 filter

$$V = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

The Laplacian using the following matrix values:

$$W = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

- (Q.a) Which of the following would make an image blurrier, a 3x3 or a 5x5 uniform filter? Why?
- (A.a) A 5x5 uniform filter. Average over a larger area.
- (Q.b) What kind of image features are detected by the 3x3 edge filters V and W ?

$$R = \begin{array}{|c|c|c|} \hline 120 & 120 & 120 \\ \hline 120 & 120 & 120 \\ \hline 180 & 180 & 180 \\ \hline \end{array}$$

$$G = \begin{array}{|c|c|c|} \hline 70 & 80 & 70 \\ \hline 70 & 70 & 80 \\ \hline 80 & 230 & 70 \\ \hline \end{array}$$

$$B = \begin{array}{|c|c|c|} \hline 100 & 50 & 30 \\ \hline 90 & 220 & 20 \\ \hline 150 & 120 & 80 \\ \hline \end{array}$$

Table 1: The R , G and B channels of a small image.

- (A.b) V : edges, W : blobs.
- (Q.c) You wish to transform an image by applying a 3x3 uniform filter followed by the 3x3 Laplacian filter W . Show that this can be implemented by a single a 5x5 filter and calculate the elements of this filter.

$$\bullet \text{ (A.c) } W = \begin{array}{|c|c|c|c|c|} \hline 1 & -1 & 0 & -1 & 1 \\ \hline -1 & 1 & 0 & 1 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline -1 & 1 & 0 & 1 & -1 \\ \hline 1 & -1 & 0 & -1 & 1 \\ \hline \end{array}$$

Exercise 2. Color Constancy

Color constancy is an important issue when recognizing an object independent of the color of the light source. Two simple color constancy algorithms are based on the white patch assumption and the grey-world hypothesis. The R , G and B channels of a small image are given in Table 1.

- (Q.a) Give an example of an image for which the white patch method will fail. Please explain.
- (A.a) An image containing a few colorful (object) colors. The assumption is that an achromatic patch is present in the image.
- (Q.b) Calculate the results of both algorithms for the image shown in Table 1.
- (A.b) White patch: $a_1 = \frac{255}{180}$, $a_2 = \frac{255}{230}$, $a_3 = \frac{255}{220}$. Grey-world: $a_1 = \frac{128}{140}$, $a_2 = \frac{128}{91}$, $a_3 = \frac{128}{95.4}$.

Exercise 1. Edges and Corners

Consider the image patches A and B in Table 2.

- (Q.a) Compute the gradient magnitude and the Harris corner response of image patch A (using a simple derivative filter e.g. $[1-1]$).

$A =$	1	1	1	0
	1	1	1	0
	1	1	1	0
	1	1	1	0

$B =$	0	0	0	0
	1	1	1	0
	1	1	1	0
	1	1	1	0

Table 2: Intensity values of image patches A and B .

- (A.a) Gradient: $\nabla f = \sqrt{f_x^2 + f_y^2} =$

0	0	1	0
0	0	1	0
0	0	1	0
0	0	1	0

 Harris corner response $R = \text{Det}M - k(\text{Trace}(M))^2 = -0.64$ where $M = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}$.
- (Q.b) What are the interest points of image patch B ?
- (A.b) Edges and corners.
- (Q.c) Compute the gradient magnitude and the Harris corner response of image patch B (using a simple edge filter e.g. $[1-1]$).
- (A.c) $M = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ and hence $R = 6.56$ which is > 0 and therefore a corner.
- (Q.d) Compute the eigenvalues of M for patch B where M is the 2x2 matrix computed from the image derivatives i.e. second moment matrix (autocorrelation matrix).
- (A.d) Eigenvalues are $\lambda_1 = 4$ and $\lambda_2 = 2$.